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October 6, 2025

Dear committee members,

I am writing this letter in strong support of the proposal “Foundations of directed type theory” under consideration by the EPSRC. I would be eager to collaborate with Altenkirch, Buchholtz, and Kraus on the work described here.

Recent discoveries over the past several decades have led to a new proposed foundation for mathematics commonly referred to as *homotopy type theory* or *univalent foundations*. The three co-PIs are all experts in this area. This new formal system is bi-interpretable with traditional foundations but has several advantages, in that it makes “higher structures” much easier to access. In traditional foundations, it is easy to make definitions and constructions involving sets or propositions. In homotopy type theory, the sets and propositions are viewed as just two layers of an infinite hierarchy, which includes additional types at a higher homotopy level.

Somewhat surprisingly, from the point of view of this summary, higher *categories* are not so easy to describe within these new foundations, which is the motivation for the current proposal. There is growing enthusiasm for a redevelopment of these ideas in a natively *directed* setting, in which the primitive “paths” might be non-reversible, mimicking the behavior of arrows in a category. The proposal gives a comprehensive overview of the state of the art (that among other things, highlights the PIs’ central role in some of these developments).

I would be eager to collaborate on WP2 on the syntax and semantics of what the proposal calls “ $\infty$ -DTT.” I was involved in early work on this variant of directed type theory, proposing a formal system that goes by the name *simplicial type theory*, and the semantics of that core system are fairly well understood. But as the proposal describes, the current vision of  $\infty$ -DTT is more expansive, including for instance various modalities proposed by Buchholtz and others. There are further desiderata such as better computational behavior, such as in the higher observational type theory of Altenkirch et al, which would require rebasing some foundational developments of  $(\infty, 1)$ -topos theory on *cubical* rather than *simplicial sets*.

These are difficult and important questions that are attracting a lot of interest, as indicated by the large number of enthusiastic collaborators this proposal has attracted. The PIs have prepared a compelling and ambitious proposal that befits their collective experience. I support it strongly.

I propose to spend five days per year over the first two years of the grant period and ten days per year over the final two years of this grant period contributing to this project. This averages to an in-kind contribution of about £6300 per year.

Please don’t hesitate to let me know if there is any other information that I might provide.

Sincerely

Emily Riehl  
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Department of Mathematics