Michael Shulman
Professor of Mathematics
University of San Diego
San Diego, California, 92117
United States of America
shulman@sandiego.edu

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Subject: Letter of support for the project proposal "Foundations of Directed Type Theory"

To Whom It May Concern,

I am writing to express my support for the above research proposal submitted by Thorsten Altenkirch, Ulrik Buchholtz, and Nicolai Kraus.

The focus of my own research is on the development of improved type theories for homotopy theory and higher category theory, including their syntax, semantics, metatheory, and implementation. With Riehl, I developed the simplicial type theory that is the current state of the art in synthetic type theories for higher category theory and has since been implemented in the proof assistant Rzk. With Kolomatskaia, I developed displayed type theory, which allows an internal definition of higher categories in homotopy type theory. With Altenkirch, Kaposi, and others I have developed higher observational type theory, which provides more direct access to the higher identity structure of types through parametricity. I have also written the Narya proof assistant, which implements both displayed type theory and higher observational type theory.

Developing a directed type theory, which can synthetically represent categories and higher categories rather than just groupoids, has been a difficult problem for many years, due partly to the apparent need for variance and modalities to control varieties of type dependency. I am excited about the ideas in the proposed project, specifically the use of parametricity-like structures as in higher observational type theory, to develop new and more usable directed type theories. A good directed type theory can be expected to have many applications both in mathematics and computer science, as detailed in the project proposal.

My collaboration would naturally focus on WP2, WP5, and WP6, based on my previous work on semantics, parametricity, and higher observational type theory. Altenkirch and I have previously worked together on the latter theory, and I expect no difficulties in extending that collaboration to the directed case. In our successful collaborations we have used e-mail and videoconferences, and we may plan one or more visits in person for this project as well.

Overall, I think this is an excellent proposal, and I will be very happy to collaborate with the research team. Together with my postdoc in San Diego, I am proposing to spend about 10 days a year on this project, which corresponds to an in-kind contribution of around GBP 9,000.

Michael Shulman