Department of Computer Science
Aarhus University

September 12, 2025

Letter of Support for "Foundations of Directed Type Theory"

To whom it may concern,

I am writing to express my support for the EPSRC proposal "Foundations of Directed Type Theory" put forward by Thorsten Altenkirch and Ulrik Buchholtz from Nottingham University. During my PhD I studied the syntactic and semantic aspects of modal dependent type theory. In particular, together with collaborators I introduced *multimodal dependent type theory*: the first framework for dependent type theories which integrated multiple interacting modalities while also enjoying all the desirable properties of ordinary type theory. Since 2023, have been working with Ulrik Buchholtz and Jonathan Weinberger to apply these results to simplicial type theory—a variant of directed type theory. We have used this fusion of simplicial and modal type theory to successfully define and study the ∞-categories of spaces and categories.

The goal of simplicial type theory is to apply type-theoretic reasoning to ∞-category theory, but it is imperfect in this regard. A major advance would be a fully directed type theory where all types faithfully modeled ∞-categories. This has proven to be a substantial challenge, with both the syntax and the semantics of such type theories introducing many novel complications. However, the potential utility of such a type theory is extremely compelling and the results of the last decade make this proposal to develop its foundations very timely.

I think I would be most closely integrated with WGs 2 and 3. My research primarily focuses on the syntax and semantics of both modal and simplicial type theory which are likely to play a role in directed type theory. I have also cultivated experience proving metatheorems with multimodal type theory and hope to apply this to directed type theory. In addition, I have enjoyed my collaboration with Ulrik Buchholtz (together with Jonathan Weinberger) and believe we can continue to successfully collaborate on these topics.

In conclusion, I think this is an outstanding proposal and would be delighted to collaborate with these researchers on directed type theory. Together with my doctoral student here in Aarhus, I am happy to spend 10 days per year working on this project. I would value this as an in-kind contribution of roughly 8.000GBP.

Yours sincerely

Daniel Gratzer PhD