Q 19 :

$$\frac{g : A \to M}{\varphi_g : LA \to M}$$

$$\varphi_g\, [a_0, \dots, a_n] = g(a_0) + \dots + g(a_n)$$

$$\varphi_g\, \{a_0, \dots, a_n\} = \ ?$$

$$\varphi_g\, \{a_0, a_1\} \quad g(a_0) + g(a_1) \ ? \quad g(a_1) + g(a_0)$$

Ex 21

$$\begin{tikzcd} \underline{\mathrm{Pre}} \arrow[r, "U"] \arrow[r, phantom, shift right=3, "\top"] & \underline{\mathrm{Rel}} \arrow[l, bend left, "F"] \end{tikzcd}$$

$$FR = R^* = \text{refl. trans closure}$$

$$\mathrm{Pre}(R^*, S) \cong \mathrm{Rel}(R, S)$$

S preorder

$$x R^* y \to fx\ S\ fy$$

use

$xRy \to fx \, S \, fy$ that S is preorder

Ex 22

$$\eta : A \to \underbrace{U(FA)}_{\text{List } A}$$

$$\eta\, a = [a]$$

$$\varepsilon : \underbrace{F(U M)}_{\text{List } M} \to M$$

$$\varepsilon\, [a_0, a_1, \dots, a_n] = a_0 + a_1 \dots + a_n$$

$$[a_0, a_1 .. a_n] \longmapsto [[a_0], [a_1], \dots, [a_n]]$$

$$\begin{tikzcd} LA \arrow[r, "L\eta"] \arrow[dr, equal] & L^2A \arrow[d, "\varepsilon"] \\ & LA \end{tikzcd}$$

$$[a_0, a_1 .. a_n]$$

$$a \longmapsto [a]$$

$$\begin{tikzcd}
M \arrow[r] \arrow[dr, equal] & {[M]} \arrow[d] \\
 & M
\end{tikzcd}
\qquad [a] \longmapsto a * e = a$$

<u>Ex 23</u> $\underline{C}(LA, B) \underset{\psi}{\overset{\varphi}{\cong}} \underline{D}(A, RB)$

$\eta = \varphi\, id$

$\varepsilon = \psi\, id$

$\varepsilon \circ L\eta = (\psi\, id) \circ L(\varphi\, id)$

$$\begin{tikzcd}
C(LA, B) \arrow[d, "h \mapsto h \circ Lf"] & D(A, RB) \arrow[l, "\psi"'] \arrow[d, "k \mapsto h \circ f"] \\
{} & {}
\end{tikzcd}$$

$$\begin{tikzcd} \cdot \arrow[d, "f"] & \cdot \arrow[d] & & \cdot \arrow[d] \\ A & C(LC, B) & & D(C, RB) \arrow[ll, "\psi"'] \end{tikzcd}$$

$k : D(A, RB)$

$f : D(C, A)$

$$\psi \, \overset{\mathrm{id}}{k} \circ L\overset{\varphi\,\mathrm{id}}{f} = \psi(k \circ f)$$

$$= \psi(\mathrm{id} \circ \varphi\, \mathrm{id})$$

$$= \psi(\varphi\, \mathrm{id}) = (\psi \circ \varphi)\, \mathrm{id}$$

$$= \mathrm{id}$$

other case is symmetric

Given η, ε

define

$$\begin{tikzcd} & D(LA, LR) \arrow[dl, "- \circ \eta"'] & \\ C(LA, B) \arrow[rr, bend right, "\varphi"'] \arrow[dr, "R"'] & & D(A, RB) \arrow[ll, bend right, "\psi"'] \arrow[ul, "L"'] \\ & C(RLA, RB) \arrow[ur, "\varepsilon \circ -"'] & \end{tikzcd}$$

$C(RCA, R \ldots$

...

Ex 24

$$YA : \underline{C}^{op} \to \underline{Set}$$

$$(YA)\,B = C(B,A)$$

$f:$

$\underline{C}(B,C)$

$\shortparallel$

$\underline{C}^{op}(C,B)$

$$(YA)\,f : (YA)C \to (YA)B$$

$$\underline{C}(C,A) \to \underline{C}(B,A)$$

$$g \mapsto g \circ f$$

$$(YA)\,\mathrm{id}\,g = g \circ \mathrm{id} = g$$

$$(YA)(f \circ h)\,g = g \circ (f \circ h)$$

$$((YA)h \circ \underbrace{(YA)f)\,g}_{g \circ f}$$

$(g\circ f)\circ h$ ass

Ex 25

$$\frac{f : \underline{C}(A,B)}{Yf : PSh(\underline{C})(YA, YB)}$$

$$\int_X \underbrace{YAX}_{\underline{C}(X,A)} \longrightarrow \underbrace{YBX}_{\underline{C}(X,B)}$$

$$Yf\,g = f\circ g$$

...

Ex 27

$$F : \underline{C} \longrightarrow \underline{D}$$

$$F : \underline{C}(A,B) \underset{F^{-1}}{\overset{F}{\xrightarrow{\cong}}} \underline{D}(FA,FB)$$

$F^{-1}\varphi$ id φ

$$\begin{tikzcd} A \arrow[loop left, "\mathrm{id}"] & B \arrow[l, bend left, "F^{-1}\psi"] \end{tikzcd} \qquad \begin{tikzcd} FA \arrow[r, phantom, "\cong"] & FB \arrow[l, bend left, "\psi"] \end{tikzcd}$$

$$F^{-1}\,\mathrm{id} = F^{-1}(F\,\mathrm{id})$$
$$= \mathrm{id}$$

$$F^{-1}(f \circ g) = F^{-1}(F(F^{-1}f \circ F^{-1}g))$$

$$f \circ g = F(F^{-1}f) \circ F(F^{-1}g)$$

$$F^{-1}(F(F^{-1}f \circ F^{-1}g)) = F^{-1}f \circ F^{-1}g$$