

Q 19 : $g: A \rightarrow M$

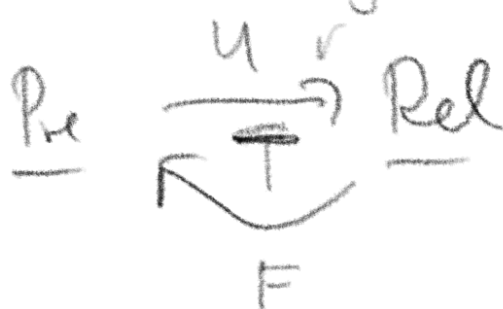
$\varphi_g: LA \rightarrow M$

$\varphi_g [a_0, \dots, a_n] = g(a_0) + \dots + g(a_n)$

$\varphi_g \{a_0, \dots, a_n\} = ?$

$\varphi_g \{a_0, a_n\} = g(a_0) + g(a_n)$

Ex 21



$FR = R^* = \text{refl, trans closure}$

$Pre(R^*, S) \cong Rel(R, S)$

S preorder

$\underbrace{x R^* y \rightarrow \exists x S fy}_{\uparrow \text{use}}$

$$\hookrightarrow x R y \rightarrow f x S f y \quad \text{that } S \text{ is provable}$$

Ex 22

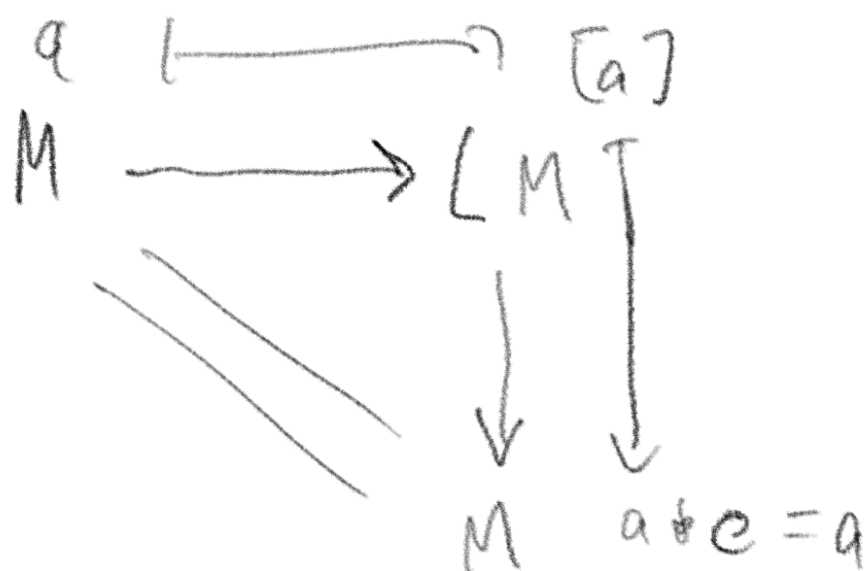
$$\eta : A \rightarrow \underbrace{U(FA)}_{\text{List } A}$$

$$\eta a = [a]$$

$$\varepsilon : \underbrace{F(U M)}_{\text{List } M} \rightarrow M$$

$$\varepsilon [a_0, a_1, \dots, a_n] = a_0 + a_1 + \dots + a_n$$

$$\begin{array}{ccc}
 [a_0, a_1, \dots, a_n] & \xrightarrow{\quad} & [[a_0], [a_1], \dots, [a_n]] \\
 \downarrow L\eta & & \downarrow \varepsilon \\
 LA & \xrightarrow{\quad} & L^2 A \\
 & \searrow & \downarrow \varepsilon \\
 & & LA \\
 & & \downarrow \\
 & & [a_0, a_1, \dots, a_n]
 \end{array}$$



Ex 23 $\underline{C}(LA, B) \xrightarrow{\varphi} \underline{D}(A, RB)$

$\xleftarrow{\psi}$

$$\eta = \varphi \text{ id}$$

$$\varepsilon = \psi \text{ id}$$

$$\varepsilon \circ L \eta = (\psi \text{ id}) \circ L(\varphi \text{ id})$$

$$\begin{array}{ccc}
 C(LA, B) & \xleftarrow{\psi} & D(A, RB) \\
 \downarrow h \mapsto h \circ Lf & & \downarrow k \mapsto h \circ f
 \end{array}$$

$$\begin{array}{c}
 \downarrow f \\
 A
 \end{array}
 \quad
 \begin{array}{c}
 \downarrow \\
 C(LC, B)
 \end{array}
 \xleftarrow[\psi]{}
 \begin{array}{c}
 \downarrow \\
 D(C, RB)
 \end{array}$$

$k: D(A, RB)$
 $l: D(C, A)$

$$\psi \circ k \circ l \overset{id}{=} \psi \circ \overset{\varphi}{=} \overset{id}{=} \psi(k \circ l)$$

$$\begin{aligned}
 &= \psi(id \circ \varphi \circ id) \\
 &= \psi(\varphi \circ id) = (\psi \circ \varphi) \circ id \\
 &= id
 \end{aligned}$$

other case is symmetric

Given M, ε

define

$$\begin{array}{ccc}
 & D(LA, LR) & \\
 \swarrow \varepsilon \circ \eta & \nwarrow L & \\
 C(LA, B) & \xleftarrow[\varphi]{} & D(A, RB) \\
 \searrow R & & \nearrow \varepsilon \circ \eta
 \end{array}$$

$C(LA, B) \xrightarrow{\quad} D(A, RB)$

$R \xrightarrow{\quad} C(LA, B)$

...

Ex 24

$$\gamma_A : \underline{C}^{\text{op}} \rightarrow \underline{S}t$$

$$(\gamma_A) B = C(B, A)$$

$$\begin{array}{l} f : \\ \underline{C}(B, C) \\ \quad \cup \\ \underline{C}^{\text{op}}(C, B) \end{array} \quad (\gamma_A) f : (\gamma_A) C \rightarrow (\gamma_A) B$$

$$\underline{C}(C, A) \rightarrow \underline{C}(B, A)$$

$$g \mapsto g \circ f$$

$$(\gamma_A) \text{id } g = g \circ \text{id} = g$$

$$(\gamma_A) (f \circ h) g = g \circ (f \circ h)$$

$$((\gamma_A) h \circ (\gamma_A) f) g$$

$\underbrace{\hspace{10em}}_{g \circ f}$

$$(g \circ f) \circ h$$

Q85

Ex 25

$$f: \underline{C}(A, B)$$

$$\forall f: \text{PSH}(\underline{C}) (\forall A, \forall B)$$

$$\int \underbrace{\forall A X}_{\underline{C}(X, A)} \longrightarrow \forall B \underbrace{X}_{\underline{C}(X, B)}$$

$$\forall f g = f \circ g$$

...

Ex 27

$$F: \underline{C} \longrightarrow \underline{D}_F$$

$$F: \underline{C}(A, B) \xrightarrow{\sim} \underline{D}(FA, FB)$$

$$F^{-1} \circ id \hookleftarrow$$

$$F^{-1}$$

$$\varphi$$

$$\text{id} \circ A \xrightarrow{\psi} B \quad FA \simeq FB$$

\swarrow $F^{-1}\psi$
 \searrow γ

$$F^{-1} \text{id} = F^{-1}(F \text{id})$$

$$= \text{id}$$

$$F^{-1}(f \circ g) = F^{-1}(F(F^{-1}f \circ F^{-1}g))$$

$$\underbrace{F(F^{-1}f) \circ F(F^{-1}g)}_{F^{-1}(f \circ F^{-1}g)} \quad \parallel$$