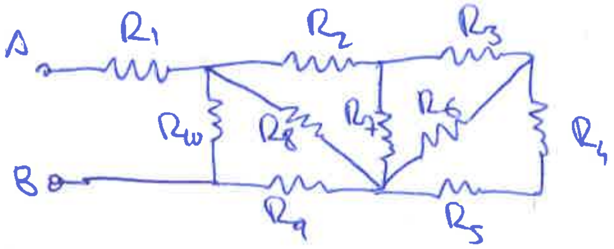
Fund. Fis. Tema 2. Problemas. Soluciones.

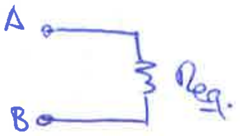
1.



$\rightarrow R_{(4,5)} = R_4 + R_5 = 3\,\Omega$

$\rightarrow R_{6,(4,5)} = \left(\frac{1}{R_6} + \frac{1}{R_{4,5}}\right)^{-1} = 2\,\Omega$

$\rightarrow R_{3,(6,(4,5))} = R_3 + R_{6,(4,5)} = 12\,\Omega$

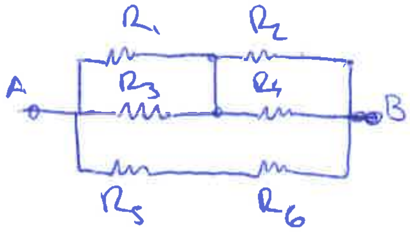


$\rightarrow R_{7,(3...} = \left(\frac{1}{R_7} + \frac{1}{R_{3,(6...}}\right)^{-1} = 4\,\Omega$

$\rightarrow R_{2,(7...} = R_2 + R_{7,(3...} = 6\,\Omega \rightarrow R_{8,(2...} = \left(\frac{1}{R_8} + \frac{1}{R_{2,(7...}}\right)^{-1} = 3\,\Omega$

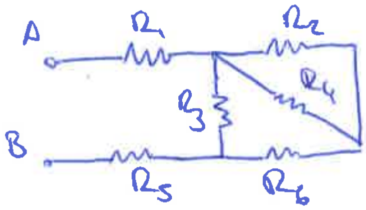
$\rightarrow R_{9,(8...} = R_9 + R_{8,(2...} = 12\,\Omega \rightarrow R_{10,(9...} = \left(\frac{1}{R_{10}} + \frac{1}{R_{9,(8...}}\right)^{-1} = 3\,\Omega$

$\rightarrow \boxed{R_{eq.} = R_1 + R_{10,(9...} = 5\,\Omega}$



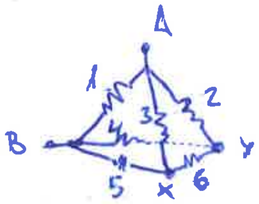
$\rightarrow R_{5,6} = R_5 + R_6 \;;\; R_{1,3} = \left(\frac{1}{R_1} + \frac{1}{R_3}\right)^{-1} \;;\; R_{2,4} = \left(\frac{1}{R_2} + \frac{1}{R_4}\right)^{-1};$

$\rightarrow R_{(1,3),(2,4)} = R_{1,3} + R_{2,4} \rightarrow \boxed{R_{eq.} = \left(\frac{1}{R_{5,6}} + \frac{1}{R_{(1,3)(2,4)}}\right)^{-1}}$

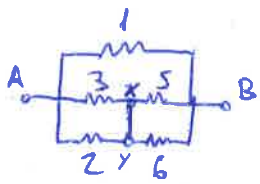


$R_{2,4} = \left(\frac{1}{R_2} + \frac{1}{R_4}\right)^{-1} \rightarrow R_{6,(2,4)} = R_6 + R_{2,4} \rightarrow$

$\rightarrow R_{3,6,(2,4)} = \left(\frac{1}{R_3} + \frac{1}{R_{6,(2,4)}}\right)^{-1} \rightarrow \boxed{R_{eq.} = R_1 + R_{3,6,(2,4)} + R_5}$



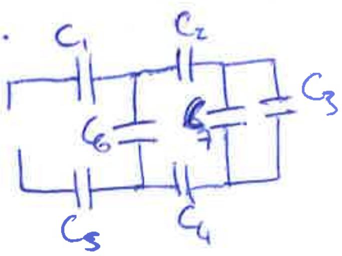
$V_x = V_y \Rightarrow$



$\Rightarrow$ Igual que el segundo caso.

$R_1 = R_2 = R_3 = R_4 = R_5 = R_6$

2.

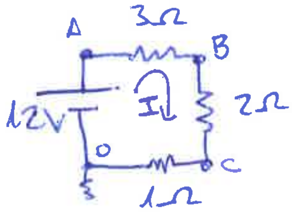


$C_{3,7} = C_3 + C_7 = 2\,\mu F \rightarrow C_{2,(7,3),4)} = \left(\frac{1}{C_2} + \frac{1}{C_{3,7}} + \frac{1}{C_4}\right)^{-1} = \frac{2}{3}\,\mu F$

$\rightarrow C_{6,(2...} = C_6 + C_{2,...} = 1\,\mu F \rightarrow C_{eq.} = \left(\frac{1}{C_1} + \frac{1}{C_{6,(2...}} + \frac{1}{C_5}\right)^{-1}$

$\rightarrow \boxed{C_{eq.} = \frac{1}{3}\,\mu F}$

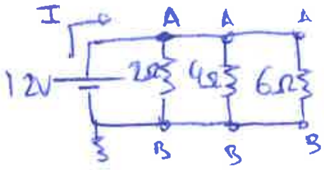
3.



$\boxed{R_{eq} = 3+2+1\,\Omega = 6\,\Omega}$

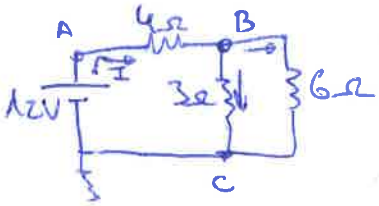
$\boxed{I = I_{3\Omega} = I_{2\Omega} = I_{1\Omega} = \frac{12}{6} = 2\text{ A}}$

$\boxed{V_{AB} = I\cdot 3 = 6\text{ V},\quad V_{BC} = I\cdot 2 = 4\text{ V},\quad V_{CD} = I\cdot 1 = 2\text{ V}.}$



$\boxed{R_{eq} = \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)^{-1} = \frac{12}{11}\,\Omega}$, $\boxed{V_{AB} = 12\text{ V}}$.

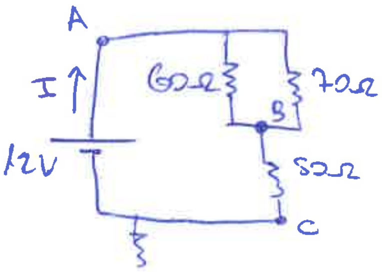
$\boxed{I = \frac{12}{\frac{12}{11}} = 11\text{ A}}$, $\boxed{I_{2\Omega} = \frac{12}{2} = 6\text{ A}}$, $\boxed{I_{4\Omega} = \frac{12}{4} = 3\text{ A}}$, $\boxed{I_{6\Omega} = \frac{12}{6} = 2\text{ A}}$



$\boxed{R_{eq} = \left(\frac{1}{3}+\frac{1}{6}\right)^{-1} + 4 = 6\,\Omega} \Rightarrow \boxed{I = \frac{12}{6} = 2\text{A} = I_{4\Omega}}$

$I = \frac{V_A - V_B}{4\Omega} = \frac{12 - V_B}{4\Omega} \Rightarrow \boxed{V_B = 4\text{V}} \Rightarrow \boxed{V_{AB} = 8\text{V}}\ \boxed{V_{BC} = 4\text{ V}}$,

$\boxed{I_{3\Omega} = \frac{V_B}{3\Omega} = \frac{4}{3}\text{ A}}$, $\boxed{I_{6\Omega} = \frac{V_B}{6} = \frac{2}{3}\text{ A}}$.

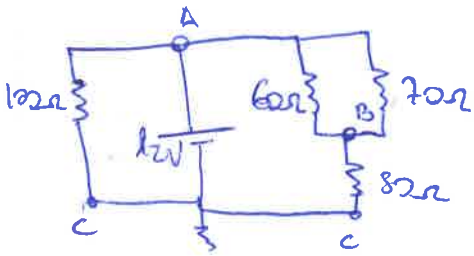


$\boxed{R_{eq} = \left(\frac{1}{60}+\frac{1}{70}\right)^{-1} + 50 = \frac{1070}{30} \sim 82{,}3\,\Omega}$

$\boxed{I = \frac{V}{R_{eq}} \simeq \frac{12}{82{,}3} \simeq 145{,}8\text{ mA} = I_{50\Omega}}$

$I = \frac{V_B - V_C}{50\Omega} = \frac{V_B}{50\Omega} \Rightarrow V_B \simeq 7{,}29\text{ V} \Rightarrow \boxed{V_{BC} = 7{,}29\text{V},\ V_{AB} = 4{,}71\text{V}}$.

$\Rightarrow \boxed{I_{60\Omega} = \frac{V_{AB}}{60\Omega} \sim 78{,}5\text{ mA}}$, $\boxed{I_{70\Omega} = \frac{V_{AB}}{70} \sim 67{,}3\text{ mA}}$



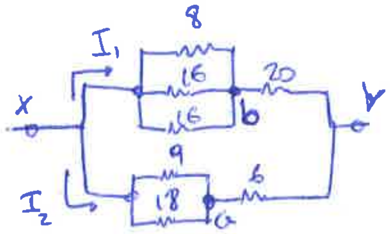
$R_{eq}^{-1} = \left[\left(\frac{1}{60}+\frac{1}{70}\right)^{-1} + 50\right]^{-1} + \frac{1}{100} \Rightarrow \boxed{R_{eq} \simeq 45{,}1\,\Omega}$

$\boxed{I = \frac{V}{R_{eq}} = \frac{12}{45{,}1} \sim 266\text{ mA}}$, $\boxed{I_{100\Omega} = \frac{V_A - V_C}{100} = \frac{12}{100} = 0{,}12\text{A}}$

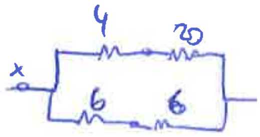
$\Rightarrow \boxed{I_{AC} = 146\text{ mA} = I_{50\Omega}} \Rightarrow \boxed{V_{BC} = 7{,}3\text{V} \Rightarrow V_{AB} = 4{,}7\text{ V}}$

$\Rightarrow \boxed{I_{60\Omega} = \frac{V_{AB}}{60} = 78{,}\overline{3}\text{mA}}$, $\boxed{I_{70\Omega} = \frac{V_{AB}}{70} \simeq 67{,}1\text{ mA}}$

4.



R_{eq}:



$\Rightarrow \boxed{R_{eq} \simeq 8\ \Omega}$.

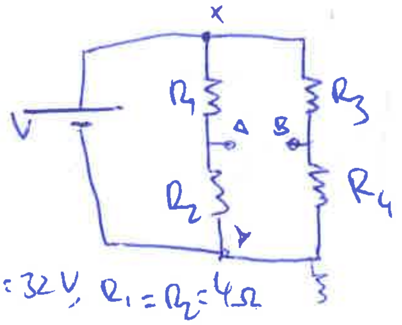
$$I_8 = \frac{V_{xb}}{8} \Rightarrow V_{xb} = \frac{1}{2}\cdot 8 = 4V \Rightarrow I_{16} = \frac{V_{xb}}{16} = \frac{1}{4}A$$

$$\Rightarrow I_1 = I_8 + I_{16} + I_{16} = \frac{1}{2} + 2\cdot\frac{1}{4} = 1A \Rightarrow I_{20} = \frac{V_{by}}{20} \Rightarrow V_{by} = 20V$$

$$\Rightarrow V_{xy} = 24V \Rightarrow I_2 = \frac{V_{xy}}{R_{eq_{1,2}}} = \frac{V_{xy}}{6+6} = \frac{24}{12} = 2A \Rightarrow I_2 = \frac{V_{ay}}{6} \Rightarrow V_{ay} = 12V$$

$$\Rightarrow V_{xa} + V_{ay} = V_{xy} \Rightarrow \boxed{V_{xa} = V_{xy} - V_{ay} = 24-12 = 12V}$$

5.



$V = 32V$, $R_1 = R_2 = 4\Omega$

$R_4 = 8V$.

a) $V_{ab} = 0 \Rightarrow R_3 = ? \Rightarrow V_A = V\cdot\frac{R_2}{R_1+R_2} = V_B = V\cdot\frac{R_4}{R_3+R_4} \Rightarrow \boxed{R_3 = R_4\cdot\frac{R_1}{R_2} = 8\Omega}$

b) $R_3 = 2\Omega$, $V_{ab} = ?$

$$I_{1,2} = \frac{V_x - V_a}{R_1} = \frac{V_a - \cancel{V_y}^{0}}{R_2} = \frac{V_x - \cancel{V_y}^{0}}{R_1+R_2} \Rightarrow -V_a = \frac{R_1}{R_1+R_2}\cdot V - V \Rightarrow$$

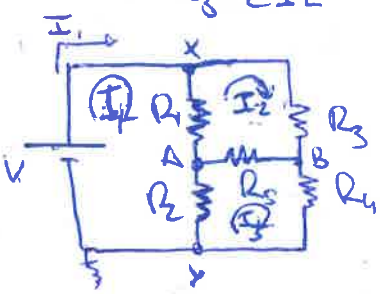
$$\Rightarrow V_a = V\left(1 - \frac{R_1}{R_1+R_2}\right) = 16V$$

$$\Rightarrow I_{3,4} = \frac{V_x - V_b}{R_3} = \frac{V_b - V_y}{R_4} = \frac{V_x - V_y}{R_3+R_4} = \frac{12}{2+8} = 1,2A$$

$$\Rightarrow V_b = I_{3,4}\cdot R_4 = \frac{12}{10}\cdot 8 = 9,6V \Rightarrow \boxed{V_{ab} = V_a - V_b = 6,4V}$$

c) $R_5 = 4\Omega$ 1) $R_3 = 4\Omega \Rightarrow \boxed{I_{R_5} = \frac{V_a - V_b}{R_5} = \frac{0}{R_5} = 0A}$. $\boxed{I_{1,2} = 1,5A,\ I_{3,4} = 1A}$

2) $R_3 = 2\Omega$



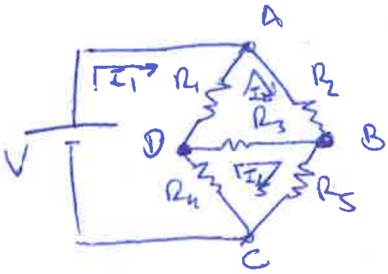
Por el método de las corrientes de malla:

$$\left.\begin{array}{ll} I_1: & +V - I_1R_1 - I_1R_2 + I_2R_1 + I_3R_2 = 0 \\ I_2: & -I_2R_1 - I_2R_3 - I_2R_5 + I_1R_1 + I_3R_5 = 0 \\ I_3: & -I_3R_2 - I_3R_5 - I_3R_4 + I_2R_5 + I_1R_2 = 0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -4I_1 + I_2 + 2I_3 = -16 \\ 4I_1 - 10I_2 + I_3 = 0 \\ 4I_1 + 4I_2 - 16I_3 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} I_1 = \frac{156}{31}A \quad I_2 = \frac{20}{31}A \\ I_3 = \frac{44}{31}A \end{array}$$

$$\boxed{\begin{array}{c} I_{R_1} = I_1 - I_2 \simeq 4,39A,\ I_{R_3} = I_2 = 0,65A,\ I_{R_4} = I_3 = 1,42A,\ I_{R_2} = I_1 - I_3 = 3,61A \\ I_{R_5} = I_3 - I_2 = 0,77A \end{array}}$$

6.



Por el método de las corriente de malla:

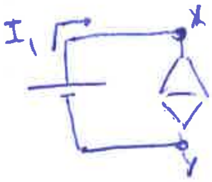
$$\left.\begin{array}{l} +V - I_1R_1 - I_1R_4 + I_2R_1 + I_3R_4 = 0 \\ -I_2R_1 - I_2R_2 - I_2R_3 + I_1R_1 + I_3R_3 = 0 \\ -I_3R_4 - I_3R_3 - I_3R_5 + I_1R_4 + I_2R_5 = 0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{pmatrix} -7 & 3 & 4 \\ 3 & -9 & 5 \\ 4 & 5 & -11 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 0 \end{pmatrix} \Rightarrow I_1 = \frac{148}{31}A, \quad I_2 = \frac{106}{31}A$$

$$I_3 = \frac{102}{31}A$$

$$\Rightarrow \boxed{\begin{array}{l} I_{R_1} = 1{,}35A, \; I_{R_2} = 3{,}42A, \; I_{R_3} = 0{,}129A = I_2 - I_3 \Rightarrow V_B > V_D \\ I_{R_4} = 1{,}48A, \; I_{R_5} = 3{,}29A \end{array}}$$

$$I_{R_3} = \frac{V_{BD}}{R_3} \Rightarrow V_{BD} = I_{R_3} \cdot R_3 \Rightarrow \boxed{V_{DB} = -V_{BD} = -0{,}645\,V}$$



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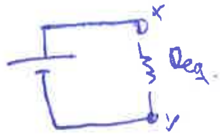
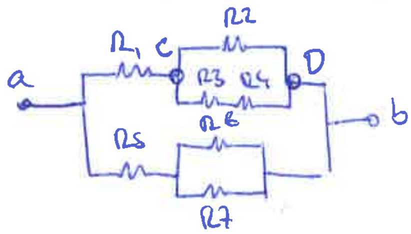


$$\Rightarrow V_{xy} = I_{Req.} \cdot Req. \Rightarrow \boxed{Req. = \frac{V_{xy}}{I_{Req.}} = \frac{V}{I_1} = \frac{12}{\frac{148}{31}} = 2{,}51\,\Omega}$$

7.



R3, R4 en serie. R2, $R_{(3,4)}$ en paralelo.

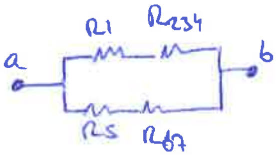
R_1, $R_{2(3,4)}$ en serie.

R6 y R7 en paralelo. R5, $(R_{6,7})$ en serie.

$R_{1,2,3,4}$ y $R_{5,6,7}$ en paralelo.

$$\Rightarrow \boxed{Req. = 4\,\Omega}.$$

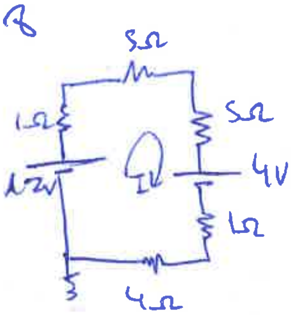
$$V_{ab} = I \cdot Req \Rightarrow \boxed{I = \frac{V_{ab}}{Req.} = \frac{20}{4} = 5A}.$$



$$\begin{array}{l} R_1 + R_{234} = 8 \\ R_5 + R_{67} = 8 \end{array} \Rightarrow I_{R_1} = I_{R_5} \Rightarrow I = 2I_{R_1} \Rightarrow I_{R_1} = 2{,}5A \Rightarrow \boxed{P_{R_1} = I^2R_1 = 37{,}5\,W}$$

$$V_{ac} = I_{R_1} \cdot R_1 \Rightarrow V_{ab} = V_{ac} + V_{cD} \Rightarrow \boxed{V_{cD} = V_{ab} - V_{ac} = 20 - 2{,}5 \cdot 6 = 5\,V}$$

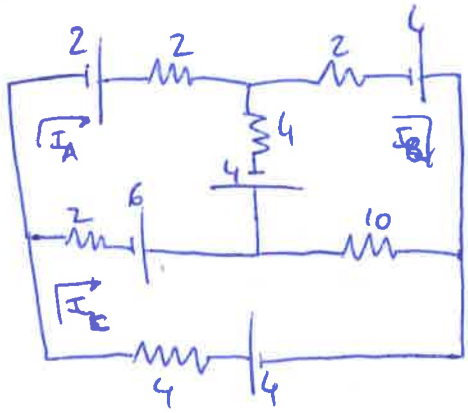
8



$$+12 - I - 5I - 5I - 4 - I - 4I = 0 \Rightarrow I = \frac{1}{2}A$$

$$\boxed{V_{1\Omega} = I \cdot R_{1\Omega} = \frac{1}{2}V, \quad V_{5\Omega} = I \cdot R_{5\Omega} = \frac{5}{2}V, \quad V_{4\Omega} = I \cdot R_{4\Omega} = \frac{4}{2}V}$$

9.



Por el método de las corrientes de malla:

$\boxed{I_A}$

$$+2-2I_A-4I_A+4-6-2I_A+4I_B+2I_C=0$$
$$-4I_A+2I_B+I_C=0$$

$\boxed{I_B}$

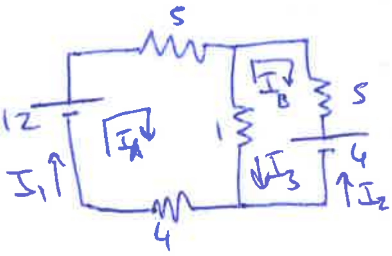
$$-2I_B+4-10I_B-4-4I_B+4I_A+10I_C=0$$
$$4I_A-16I_B+10I_C=0$$

$\boxed{I_C}$

$$-2I_C+6-10I_C+4-4I_C+2I_A+10I_B=0$$
$$1I_A-8I_B+5I_C+5=0$$

$$\Rightarrow \begin{pmatrix}-4 & 2 & 1\\ 2 & -8 & 5\\ 1 & 5 & -8\end{pmatrix}\begin{pmatrix}I_A\\ I_B\\ I_C\end{pmatrix}=\begin{pmatrix}0\\ 0\\ -5\end{pmatrix} \Rightarrow I_A=\frac{15}{16}\,A,\quad I_B=\frac{55}{48}\,A,\quad I_C=\frac{280}{192}\,A$$

$$\boxed{\begin{array}{lll} I_1=I_A=0{,}938\,A & I_3=I_C-I_A=0{,}521\,A & I_5=I_C-I_B=0{,}313\,A\\ I_2=I_A-I_B=-0{,}208\,A & I_4=I_B=1{,}146\,A & I_6=I_C=1{,}458\,A\end{array}}$$



$\boxed{I_A}$ $+12-5I_A-I_A-4I_A+I_B=0$

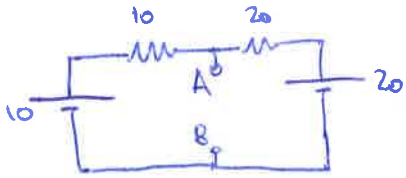
$\boxed{I_B}$ $-I_B-5I_B-4+I_A=0$

$$\Rightarrow \begin{pmatrix}-10 & 1\\ 1 & -6\end{pmatrix}\begin{pmatrix}I_A\\ I_B\end{pmatrix}=\begin{pmatrix}-12\\ +4\end{pmatrix}$$

$$\Rightarrow I_B=-\frac{28}{59}\,A,\quad I_A=+\frac{68}{59}\,A$$

$$\boxed{I_1=I_A=1{,}15\,A,\quad I_3=I_A-I_B=1{,}63\,A,\quad I_2=-I_B=0{,}47\,A}$$

10.



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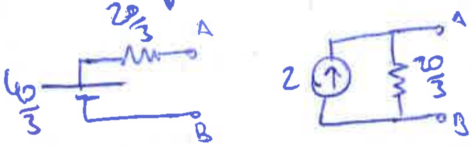
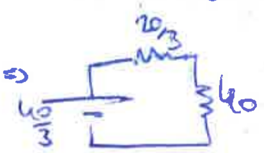
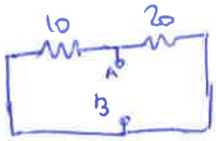


Si entre A y B: R=40Ω



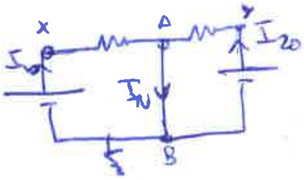
$$\Rightarrow \boxed{I=0{,}286\,A}$$

$\underline{R_{eq}}$:



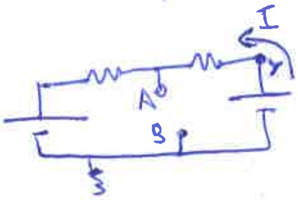
$$\Rightarrow \boxed{R_{eq}=\left(\frac{1}{10}+\frac{1}{20}\right)^{-1}=\frac{20}{3}\,\Omega}$$

$\underline{I_N}$:



$$I_{10}=\frac{V_x-V_A}{10}=\frac{10-V_A}{10}=\frac{10}{10}=1\,A$$
$$I_{20}=\frac{V_y-V_A}{20}=\frac{20-V_A}{20}=\frac{20}{20}=1\,A$$
$$\Rightarrow \boxed{I_N=I_{10}+I_{20}=2A}$$

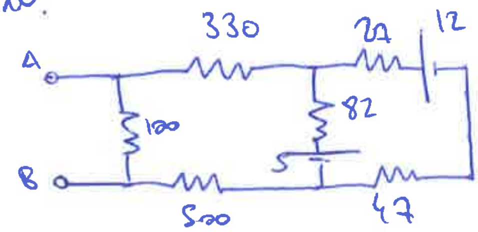
$\underline{V_{Th}}$:



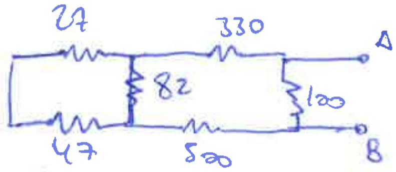
$$+20-20I-10I-10=0 \Rightarrow I=\frac{1}{3}\,A$$
$$I=\frac{V_y-V_A}{R_{20}}=\frac{20-V_A}{20}=\frac{1}{3}\Rightarrow V_A=\frac{40}{3}\,V$$

$$\Rightarrow \boxed{V_{Th}=\frac{40}{3}\,V}$$

10.

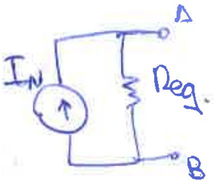


$R_{eq.}$:



$$\boxed{R_{eq.} = 89{,}68\ \Omega}$$

idéntico

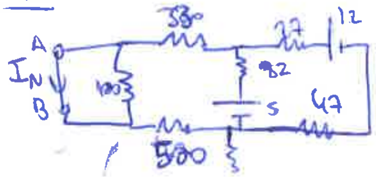


Si entre A y B $R = 40\ \Omega$

$$\Rightarrow I = \frac{V_{Th}}{R_{eq.}+40}$$

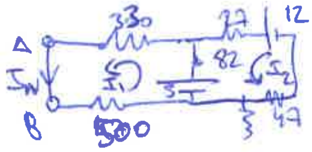
$$\Rightarrow \boxed{I = 8{,}999\ mA}$$

I_N:



Por $R = 100\ \Omega$ no circula corriente.

$\Rightarrow$



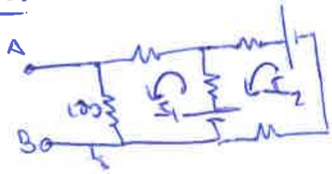
I_1: $+5 - 82I_1 - 330I_1 - 500I_1 + 82I_2 = 0$

I_2: $+12 - 27I_2 - 82I_2 - 5 + 82I_1 - 47I_2 = 0$

$$\begin{pmatrix} -912 & 82 \\ 82 & -156 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -5 \\ -12 \end{pmatrix} \Rightarrow \begin{matrix} I_1 = 13{,}01\ mA \\ I_2 = 83{,}76\ mA \end{matrix}$$

$$\boxed{I_N = I_1 = 13{,}01\ mA}$$

V_{Th}.



I_1: $+5 - 82I_1 - 330I_1 - 100I_1 - 500I_1 + 82I_2 = 0$

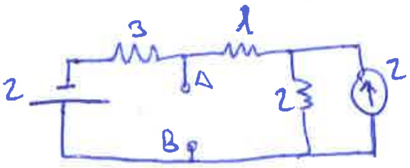
I_2: $+12 - 27I_2 - 82I_2 - 47I_2 + 82I_1 = 0$ $\Rightarrow$

$$\Rightarrow \begin{pmatrix} -1012 & 82 \\ 82 & -156 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -5 \\ -12 \end{pmatrix} \Rightarrow \begin{matrix} I_1 = 11{,}67\ mA \\ I_2 = 83{,}05\ mA \end{matrix}$$

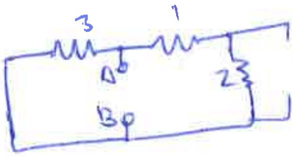
$$\Rightarrow V_{AB} = I_1 \cdot R_{100} = 11{,}67 \cdot 10^{-3} \cdot 10^2 = 1{,}167\ V \Rightarrow V_{Th} = 1{,}167\ V$$

$$\Rightarrow \boxed{V_{Th.} = I_N \cdot R_{eq} = 13{,}01 \cdot 10^{-3} \cdot 89{,}68 = 1{,}167\ V}$$

11.

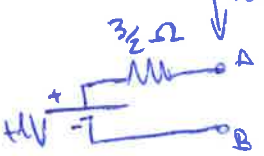


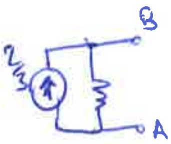
$R_{eq.}$:



$$\Rightarrow R_{eq} = \frac{3}{2}\ \Omega$$

idéntico



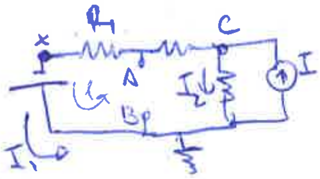


Si entre A y B conecto $R = 1\ \Omega$:

$$\Rightarrow I = \frac{V}{R_T} \Rightarrow$$

$$\Rightarrow \boxed{I = \frac{1}{\frac{3}{2}+1} = 0{,}4\ A}$$

V_{Th}:

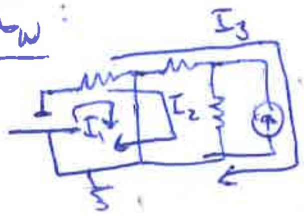


$$\left.\begin{matrix} (c) & I = I_1 + I_2 \\ (1) & +2 - 2I_2 - 4I_1 = 0 \end{matrix}\right\} \Rightarrow \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\Rightarrow I_1 = -1\ A$. Si $V_B = 0 \Rightarrow V_{Th} \equiv V_A$.

$$\Rightarrow I_1 = \frac{V_x - V_A}{R_1} = \frac{-2 - V_A}{3} \Rightarrow V_A = +1\ V$$

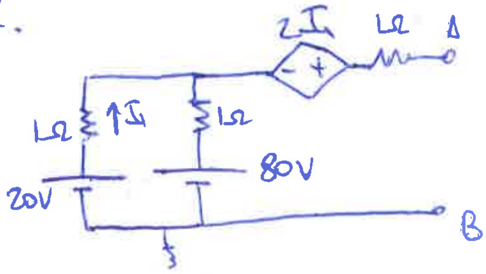
I_N:



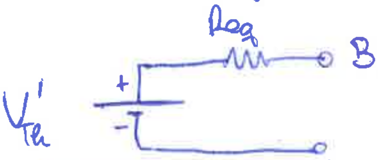
$$\left.\begin{matrix} -V - R_1(I_1+I_2+I_3) = 0 \\ -V - R_1(I_1+I_2+I_3) - R_2(I_2+I_3) = 0 \\ -V - R_1(I_1+I_2+I_3) - R_2(I_2+I_3) - R_3 I_2 = 0 \end{matrix}\right\}$$

$$\Rightarrow I_1 = -\frac{2}{3}\ A \equiv I_N.$$

12.



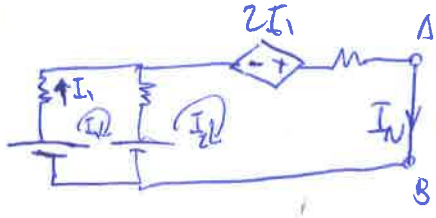
idéntico



$$\boxed{\begin{array}{l} V'_{Th} = +10\,V \\ R_{eq} = \dfrac{V_{Th}}{I_N} = \dfrac{10}{20} = \dfrac{1}{2}\,\Omega \end{array}}$$

a) Al tener la fuente de tensión dependiente, no puedo calcular Req. directamente. Debo calcular I_N y V_{Th}.

I_N:



$\Rightarrow I_N = I_2$

I_1: $+20 - I_1 - I_1 - 80 + I_2 = 0$

I_2: $+80 - I_2 + 2I_1 - I_2 + I_1 = 0$

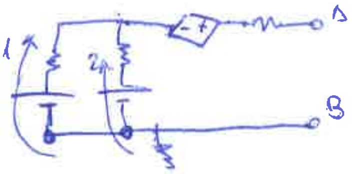
$\Rightarrow$

$-60 - 2I_1 + I_2 = 0$

$80 + 3I_1 - 2I_2 = 0$

$\Rightarrow I_1 = -40\,A$, $I_2 = -20\,A$

V_{Th}:

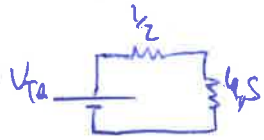


① $V_B + 20 - I + 2I = V_A$

② $V_B + 80 - I + 2I = V_A$

$\Rightarrow I = -30\,A \Rightarrow V_A = -10\,V \Rightarrow V_{Th} = -10\,V$

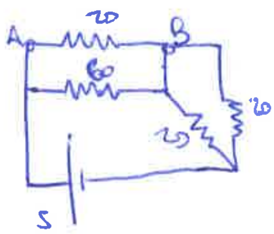
b) $P = I^2 \cdot R$



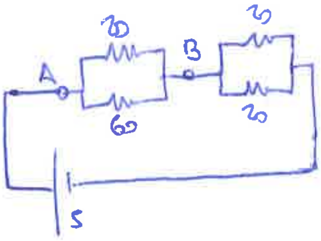
$\Rightarrow I = \dfrac{V_{Th}}{5} = 2A$

$\Rightarrow \boxed{P = 2^2 \cdot 4{,}5 = 18\,W}$

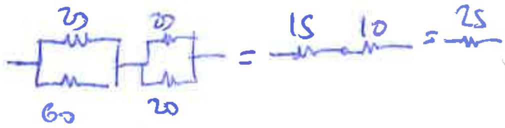
13.



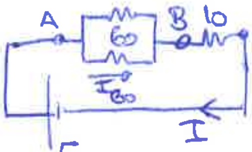
$=$



$\Rightarrow$ a) $P = I \cdot V \Rightarrow I = ?$



b) $I_{60} = ?$



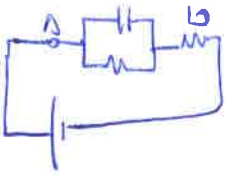
$I = \frac{1}{5}A \Rightarrow V_B = \frac{1}{5} \cdot 10 = 2V$

$\Rightarrow \boxed{I_{60} = \dfrac{V_A - V_B}{R_{60}} = \dfrac{5-2}{60} = 0{,}05A}$

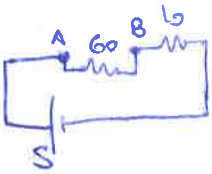
$I = \frac{5}{25} = \frac{1}{5}A$

$\Rightarrow \boxed{P = I \cdot V = \frac{1}{5} \cdot 5 = 1W}$

c) $E_p = \frac{1}{2} V^2 C = ?$



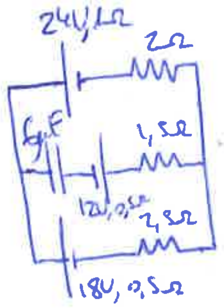
t muy grande



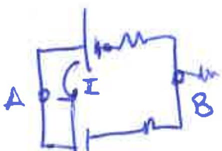
$V_A - V_B = I \cdot 60 = \dfrac{V_A}{60+10} \cdot 60 = \dfrac{5 \cdot 60}{70} = 4{,}28\,V$

$\Rightarrow \boxed{E_p = \frac{1}{2} V^2 \cdot C = \frac{1}{2} \cdot 4{,}28^2 \cdot 20 \cdot 10^{-12} = 0{,}18\,nJ}$

14.



a) En el estado estacionario



$-18 - \frac{I}{2} - \frac{5}{2}I - 2I - I + 24 = 0$

$\Rightarrow I = 1A$

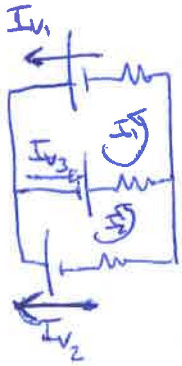
$\Delta V_C = V_A - V_B$;

$V_B + I \cdot \frac{5}{2} + \frac{I}{2} + 18 = V_A$

$\Rightarrow V_A = 21\,V \Rightarrow \boxed{\Delta V_C = 21V}$

b) $\boxed{E_p = \frac{1}{2} C V^2 = \frac{1}{2} \cdot 6 \cdot 10^{-6} \cdot 21^2 = 1{,}323\,mJ}$

14 c)



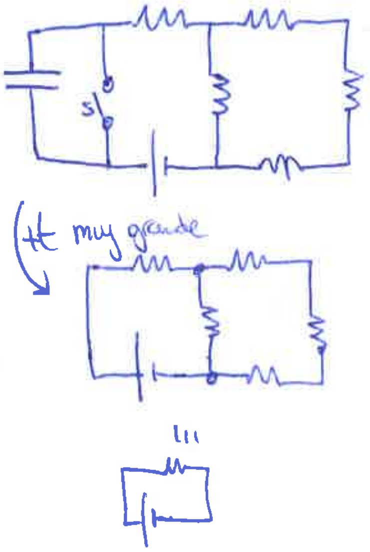
$\circlearrowleft I_1$: $-0{,}5I_1 + 12 - 1{,}5I_1 - 2I_1 - I_1 + 2I_2 + 0{,}5I_2 + 1{,}5I_2 = 0$

$\circlearrowleft I_2$: $-18 - 0{,}5I_2 - 2{,}5I_2 - 1{,}5I_2 - 12 - 0{,}5I_2 + 0{,}5I_1 + 1{,}5I_1 = 0$

$$\Rightarrow \left.\begin{array}{l} -30 - 5I_2 + 2I_1 = 0 \\ 36 + 2I_2 - 5I_1 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} I_1 = +\frac{40}{7}\,A \\ I_2 = -\frac{26}{7}\,A \end{array}$$

$$\boxed{I_{V_1} = I_1 = \frac{40}{7}A,\quad I_{V_2} = -I_2 = +\frac{26}{7}A,\quad I_{V_3} = I_{V_1} + I_{V_2} = I_1 - I_2 = \frac{66}{7}A}$$

15.



a) Con S cerrado, $Q_c = 0$ porque $\Delta V_c = 0 \Rightarrow Q_c = C \cdot \Delta V_c$.

b)

$$R_{eq} = 8k + \left[\frac{1}{30k} + \frac{1}{5k + 10k + 5k}\right]^{-1} = 20\,k\Omega$$

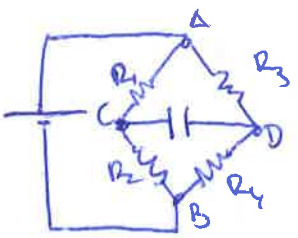
$$\Rightarrow \boxed{I = \frac{V}{R_{eq}} = \frac{20}{20k}\,A = 1\,mA} \Rightarrow \boxed{P = I \cdot V = 20\,mW}$$

c) Para el proceso de carga de un condensador:

$$Q(t) = V \cdot C \cdot \left(1 - e^{-\frac{t}{RC}}\right) = 20 \cdot 4 \cdot 10^{-6} \cdot \left(1 - \exp\left(-\frac{0{,}05}{20 \cdot 10^3 \cdot 6 \cdot 10^{-6}}\right)\right)$$

$$\Rightarrow \boxed{Q(t = 0{,}05\,s) = 3{,}72 \cdot 10^{-5}\,C}$$

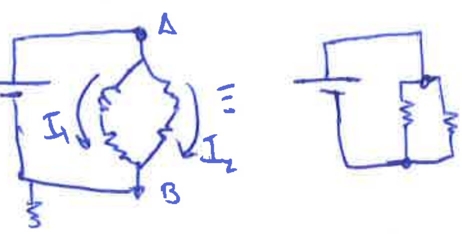
16.



a)

$$\boxed{\begin{array}{l} I_1 = \dfrac{V_A}{R_1 + R_2} = \dfrac{36}{90} = 0{,}4\,A \\ I_2 = \dfrac{V_A}{R_3 + R_4} = \dfrac{36}{60} = 0{,}6\,A \end{array}}$$

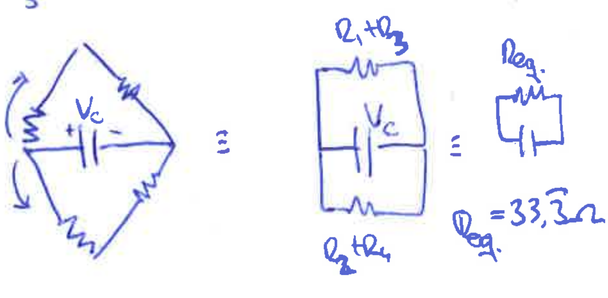
estado estacionario



b) $\Delta V_c = V_c - V_D$; $I_1 = \frac{V_c}{R_2} \Rightarrow V_c = 0{,}4 \cdot 80 = 32\,V$

$I_2 = \frac{V_D}{R_4} \Rightarrow V_D = 0{,}6 \cdot 20 = 12\,V$

$$\Rightarrow \boxed{\Delta V_c = 20\,V}$$



En la descarga de un condensador:

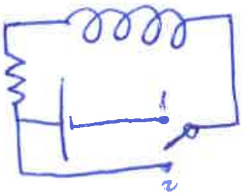
$$I(t) = \frac{Q_0}{RC} e^{-\frac{t}{RC}} \rightarrow V(t) = \frac{Q_0}{C} e^{-\frac{t}{RC}} \Rightarrow$$

$$\Rightarrow V(t = ?) = 1\,V \Rightarrow \frac{C}{Q_0} = e^{-\frac{t}{R_{eq}C}} \Rightarrow \ln\frac{C}{Q_0} = -\frac{t}{R_{eq}C} \Rightarrow$$

$$\Rightarrow \boxed{t = -R_{eq} \cdot C \cdot \ln\frac{C}{Q_0} = 0{,}998\,ms}$$

$Q_0 = C \cdot \Delta V_c$

17.



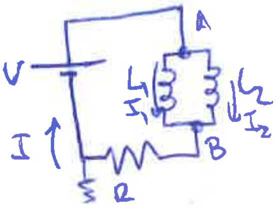
estado estacionario conectado por 1 $\longrightarrow$

$$I = \frac{V}{R} = \frac{80}{20} = 4A.$$

Al desconectar la batería: $I(t) = I_0 e^{-\frac{R}{L}t}$

$$\Rightarrow I(t) = \frac{I_0}{e} = I_0 e^{-\frac{R}{L}t} \Rightarrow 1 = e^{1-\frac{R}{L}t} \Rightarrow \frac{R}{L}t = 1 \Rightarrow \boxed{t = \frac{L}{R} = 2{,}5\,ms}$$

18



$$\left.\begin{aligned} +V - L_1\frac{dI_1}{dt} - IR &= 0 \\ +V - L_2\frac{dI_2}{dt} - IR &= 0 \\ I_1 + I_2 &= I \end{aligned}\right\} \Rightarrow \left.\begin{aligned} V - L_1\frac{dI_1}{dt} - (I_1+I_2)R &= 0 \\ V - L_2\frac{dI_2}{dt} - (I_1+I_2)R &= 0 \end{aligned}\right\}$$

$$\longrightarrow V_{AB} = -L_1\frac{dI_1}{dt} = -L_2\frac{dI_2}{dt} \Rightarrow dI_2 = \frac{L_1}{L_2}dI_1 \Rightarrow I_2 = \frac{L_1}{L_2}I_1.$$

$$\Rightarrow \left.\begin{aligned} V - L_1\frac{dI_1}{dt} - \left(1+\frac{L_1}{L_2}\right)I_1R &= 0 \\ V - L_2\frac{dI_2}{dt} - \left(1+\frac{L_2}{L_1}\right)I_2R &= 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} V - L_1\frac{dI_1}{dt} - \left(\frac{L_1+L_2}{L_2}\right)I_1R &= 0 \\ V - L_2\frac{dI_2}{dt} - \left(\frac{L_1+L_2}{L_1}\right)I_2R &= 0 \end{aligned}\right\}$$

$\longrightarrow$ Resolvemos la homogénea:

$$-L_{1,2}\frac{dI_{1,2}}{dt} - \left(\frac{L_2+L_1}{L_{2,1}}\right)I_{1,2}R = 0 \Rightarrow \int\frac{dI_{1,2}}{I_{1,2}} = -\int\frac{L_2+L_1}{L_1L_2}R\,dt \Rightarrow$$

$$\Rightarrow \ln I_{1,2} = -\frac{L_2+L_1}{L_1L_2}Rt + cte \Rightarrow I_{1,2} = Ae^{-\frac{L_2+L_1}{L_1L_2}Rt}$$

$\longrightarrow$ Resolvemos la inhomogénea:

$$A = A(t) \Rightarrow \dot{I}_{1,2} = \dot{A}e^{-\frac{L_2+L_1}{L_1L_2}Rt} - A\cdot\frac{L_2+L_1}{L_1L_2}Re^{-\frac{L_2+L_1}{L_1L_2}Rt} \Rightarrow$$

$$\Rightarrow V - L_{1,2}\cdot\dot{A}e^{-\frac{L_2+L_1}{L_1L_2}Rt} + AL_{1,2}\frac{L_2+L_1}{L_1L_2}Re^{-\frac{L_2+L_1}{L_1L_2}Rt} - \frac{L_1+L_2}{L_{2,1}}\cdot Ae^{-\frac{L_1+L_2}{L_1L_2}Rt}\cdot R = 0 \longrightarrow$$

$$\longrightarrow V - L_{1,2}\cdot\dot{A}e^{-\frac{L_2+L_1}{L_1L_2}Rt} + A\frac{L_2+L_1}{L_{2,1}}Re^{-\frac{L_1+L_2}{L_1L_2}Rt} - \frac{L_1+L_2}{L_{2,1}}RAe^{-\frac{L_1+L_2}{L_1L_2}Rt} = 0 \Rightarrow$$

$$\Rightarrow V - L_{1,2}\dot{A}e^{-\frac{L_2+L_1}{L_1L_2}Rt} = 0 \Rightarrow dA = \frac{V}{L_{1,2}}e^{\frac{L_2+L_1}{L_1L_2}Rt}dt \Rightarrow A = \frac{V}{R}\cdot\frac{L_{2,1}}{L_2+L_1}e^{\frac{L_2+L_1}{L_2L_1}Rt} + cte.$$

$$\Rightarrow I_{1,2}(t) = \frac{V}{R}\cdot\frac{L_{2,1}}{L_1+L_2}e^{\frac{L_2+L_1}{L_2L_1}Rt}\cdot e^{-\frac{L_1+L_2}{L_1L_2}Rt} + cte\cdot e^{-\frac{L_2+L_1}{L_1L_2}Rt}.$$ Con condición de cont. $I(0) = 0 \Rightarrow$

$$I_{1,2}(t=0) = \frac{V}{R}\cdot\frac{L_{2,1}}{L_1+L_2} + cte = 0 \Rightarrow cte = -\frac{V}{R}\cdot\frac{L_{2,1}}{L_1+L_2} \Rightarrow \boxed{I_{1,2}(t) = \frac{V}{R}\cdot\frac{L_{2,1}}{L_1+L_2}\left(1 - e^{-\frac{L_1+L_2}{L_1L_2}Rt}\right)}$$

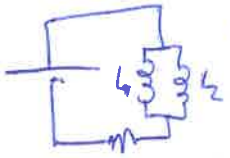
18.

$$I_{L_1}(t) = \frac{V}{R}\cdot\frac{L_2}{L_1+L_2}\left(1-e^{-\frac{L_1+L_2}{L_1L_2}Rt}\right), \quad I_{L_2}(t) = \frac{V}{R}\cdot\frac{L_1}{L_1+L_2}\left(1-e^{-\frac{L_1+L_2}{L_1L_2}Rt}\right)$$

$$I_R(t) = I_{L_1}(t) + I_{L_2}(t) = \frac{V}{R}\cdot\frac{1}{L_1+L_2}\left[L_2+L_1\right]\cdot\left(1-e^{-\frac{L_1+L_2}{L_1L_2}Rt}\right)$$

$$\Rightarrow \boxed{I_R(t) = \frac{V}{R}\left(1-e^{-\frac{L_1+L_2}{L_1L_2}Rt}\right)}$$

→ Comprobaciones:



equivalente

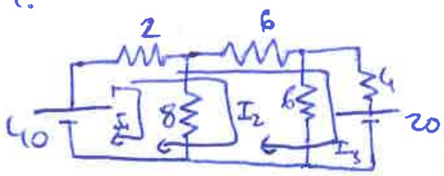
$$\frac{1}{L_{eq}} = \frac{1}{L_1}+\frac{1}{L_2} = \frac{L_1+L_2}{L_1L_2} \Rightarrow L_{eq} = \frac{L_1L_2}{L_1+L_2}$$

$$I(t) = \frac{V}{R}\left(1-e^{-\frac{R}{L_{eq}}t}\right) = \frac{V}{R}\left(1-e^{-\frac{L_1+L_2}{L_1L_2}Rt}\right) = I_R \quad ✓$$

En $t\to\infty$ $\boxed{I = \frac{V}{R} = \frac{24}{15} = 1{,}6\,A}$.

→ Ojo: $I_{L_1}(t) \neq I_{L_2}(t) \Rightarrow I_{L_1}(t) \neq \frac{I_R}{2}$

19.



$$\left.\begin{array}{ll} I_1: & +40-2(I_1+I_2+I_3)-8I_1=0 \\ I_2: & +40-2(I_1+I_2+I_3)-6(I_3+I_2)-6I_2=0 \\ I_3: & +40-2(I_1+I_2+I_3)-6(I_3+I_2)-4I_3-20=0 \end{array}\right\}$$

$\Rightarrow 6I_2 = 4I_3+20 \Rightarrow I_2 = \frac{1}{3}(2I_3+10)$.

$8I_1 = 6(I_3+I_2)+6I_2 = 6I_3+12I_2 = 6I_3+4(2I_3+10) = 14I_3+40 \Rightarrow I_1 = \frac{7}{4}I_3+5$.

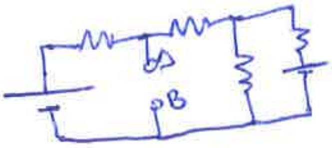
$40-2I_1-2I_2-2I_3-8I_1 = 40-\frac{7}{2}I_3-10-\frac{4}{3}I_3-\frac{20}{3}-2I_3-14I_3-40=0$

$\Rightarrow \frac{125}{6}I_3 = -\frac{50}{3} \Rightarrow \boxed{I_3 = -\frac{4}{5}\,A,\ I_2 = \frac{14}{5}\,A,\ I_1 = \frac{18}{5}\,A}$

$\Rightarrow P_{20V} = |I_{20}\cdot V_{20}| = |I_3\cdot V_{20}| = \frac{4}{5}\cdot 20 = 16\,W$. $P_{40} = |I_{40}\cdot V_{40}| = |(I_1+I_2+I_3)\cdot V_{40}| = \frac{28}{5}\cdot 40 = 224\,W$

→ $\boxed{P_{20} = 16\,W,\quad P_{40} = 224\,W}$

19.

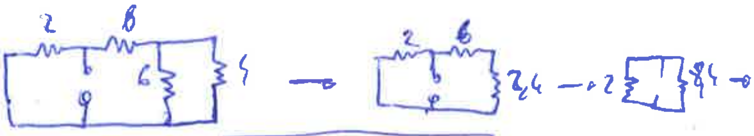


idéntico

Si entre A y B pongo 8Ω:

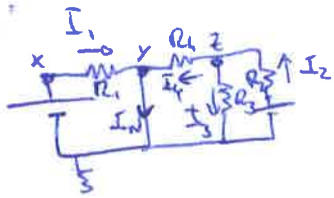
$$I_8 = \frac{V_{Te}}{R_{eq}+8} = 3,6\text{ A}.$$

Req.



$$\Rightarrow \boxed{R_{eq} = 1,615\,\Omega}$$

I_N:



$$I_1 = \frac{V_x - V_y}{R_1} = \frac{40-0}{2} = 20\text{ A}.$$

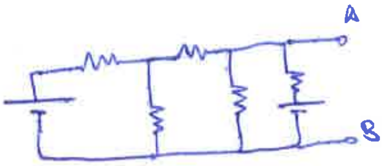
$$I_2 = \frac{20-V_z}{R_2}, \quad I_3 = \frac{V_z}{R_3}, \quad I_4 = \frac{V_z}{R_4}$$

$$I_2 = I_3 + I_4 \Rightarrow \frac{20-V_z}{4} = \frac{V_z}{6} + \frac{V_z}{6} \Rightarrow V_z \simeq 8,57\text{ V}.$$

$$\Rightarrow I_N = I_1 + I_4 = 20 + \frac{8,57}{6} = 21,43\text{ A}$$

V_{Te}:

$$\boxed{V_{Te} = I_N \cdot R_{eq} = 34,60\text{ V}}$$



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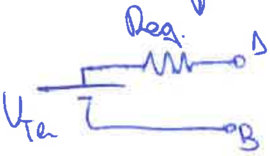
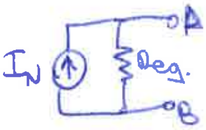
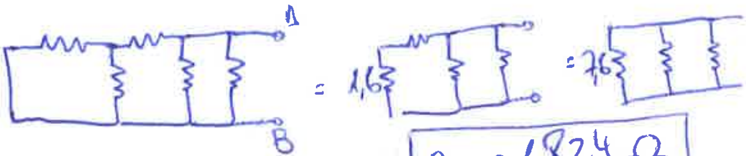


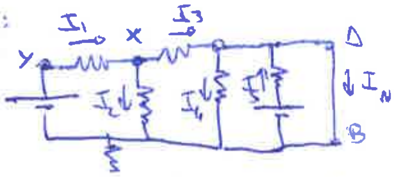


Req.



$$\boxed{R_{eq} = 1,824\,\Omega}$$

I_N:



$$I_1 = \frac{40-V_x}{2}, \quad I_2 = \frac{V_x}{8}$$

$$I_3 = \frac{V_x}{6}, \quad I_4 = \frac{V_A}{6}, \quad I_5 = \frac{20}{4} = 5\text{ A}$$

en (x) $I_1 = I_2 + I_3$

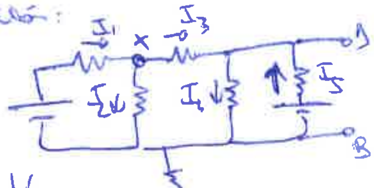
en (A) $I_3 + I_5 = \cancel{I_4} + I_N$ (0)

$$\frac{40-V_x}{2} = \frac{V_x}{8} + \frac{V_x}{6} \Rightarrow V_x = 25,26\text{ V}$$

$$\Rightarrow I_N = \frac{V_x}{6} + 5 = 9,21\text{ A}$$

$$\Rightarrow \boxed{V_{Te.} = I_N \cdot R_{eq} = 9,21 \cdot 1,824 = 16,8\text{ V}}$$

→ Comprobación:



$V_A = V_{Te}.$

$$I_1 = \frac{40-V_x}{2} \qquad I_4 = \frac{V_A}{6}$$

$$I_2 = \frac{V_x}{8} \qquad I_5 = \frac{20-V_A}{4}$$

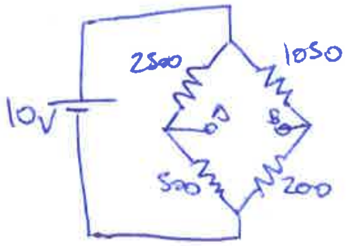
$$I_3 = \frac{V_x - V_A}{6}$$

en (x) $I_1 = I_2 + I_3 \longrightarrow \frac{40-V_x}{2} = \frac{V_x}{8} + \frac{V_x - V_A}{6} \Rightarrow V_A = \frac{19}{4}V_x - 120$

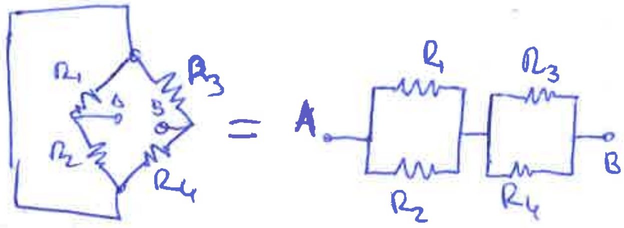
en (A) $I_4 = I_3 + I_5 \Rightarrow \frac{V_A}{6} = \frac{V_x - V_A}{6} + \frac{20-V_A}{4} \Rightarrow 75 = \frac{125}{48}V_x \Rightarrow$

$$\Rightarrow V_x = +28,8\text{ V} \Rightarrow V_A = \frac{19}{4} \cdot 28,8 - 120 = 16,8\text{ V}.$$

20.

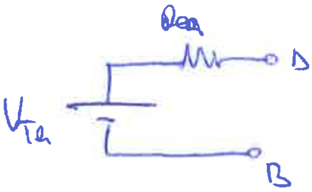


Req:

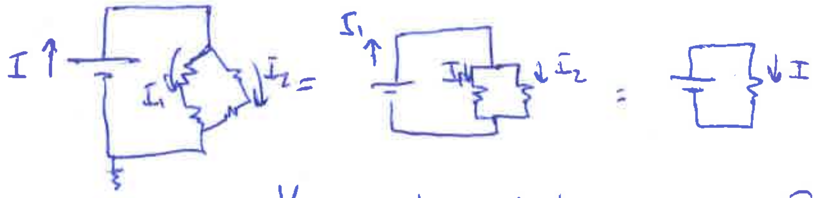


$$\Rightarrow R_{eq} = 584,\hat{6}\ \Omega$$

idéntico



V_{Th}:



$$I = \frac{V}{R'_{eq}} = V\cdot\frac{1}{R'_{eq}} = V\cdot\left(\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}\right) = 11,\hat{3}\ mA$$

$$I_1 = \frac{V}{R_1+R_2} = \frac{V-V_A}{R_1} = \frac{V_A}{R_2} \Rightarrow V_A = \frac{R_2}{R_1+R_2}\cdot V$$

$$I_2 = \frac{V}{R_3+R_4} = \frac{V-V_B}{R_3} = \frac{V_B}{R_4} \Rightarrow V_B = \frac{R_4}{R_3+R_4}V$$

$$\Rightarrow V_{Th} = V_A - V_B = \left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right)V = 66,\hat{6}\ mV$$

Si entre A y B coloco $R = 100\Omega$

$$\Rightarrow I_{100\Omega} = \frac{V_{Th}}{R_{eq}+100} = 9,737\cdot 10^{-5} A$$

La sensibilidad del galvanómetro es:

$$S = 0,5\ \frac{\mu A}{mm} \Rightarrow S^{-1} = 2\ \frac{mm}{\mu A} \Rightarrow$$

$$\Rightarrow \boxed{\Delta L = S^{-1}\cdot I = 2\cdot 97,37 = 19,5\ cm}$$