CÁLCULO II.
1^o DE GRADO EN MATEMÁTICAS Y DOBLE GRADO INFORMÁTICA-MATEMÁTICAS. Curso 2022/23
DEPARTAMENTO DE MATEMÁTICAS

Hoja 4

1.- Sea $F(x,y) = f(u(x,y), v(x,y))$ con $u = \dfrac{x-y}{2}, v = \dfrac{x+y}{2}$. Aplicar la regla de la cadena para calcular $\nabla F(x,y)$ en función de las derivadas parciales de f, $\dfrac{\partial f}{\partial u}$ y $\dfrac{\partial f}{\partial v}$.

SOL:
$\nabla F(x,y) = \left(\dfrac{\partial F}{\partial x}, \dfrac{\partial F}{\partial y}\right) = \left(\dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial x} + \dfrac{\partial f}{\partial v}\dfrac{\partial v}{\partial x}, \dfrac{\partial f}{\partial u}\dfrac{\partial u}{\partial y} + \dfrac{\partial f}{\partial v}\dfrac{\partial v}{\partial y}\right) = \left(\dfrac{1}{2}\dfrac{\partial f}{\partial u} + \dfrac{1}{2}\dfrac{\partial f}{\partial v}, -\dfrac{1}{2}\dfrac{\partial f}{\partial u} + \dfrac{1}{2}\dfrac{\partial f}{\partial v}\right)$

2.- Sean $f(x,y) = x^2 + y$, $g(u) = (\operatorname{sen} 3u, \cos 8u)$ y $h(u) = f(g(u))$. Calcular $\dfrac{dh}{du}$ en $u = 0$ tanto de forma directa como usando la regla de la cadena.

SOL:
- De forma directa:

$$h(u) = f(g(u)) = f(\operatorname{sen} 3u, \cos 8u) = (\operatorname{sen} 3u)^2 + \cos 8u \Rightarrow \frac{dh}{du} = 6 \operatorname{sen} 3u \cos 3u + -8 \operatorname{sen} 8u \Rightarrow \frac{dh}{du}(0) = 0$$

- Usando la regla de la cadena:

$$\frac{dh}{du}(0) = \frac{\partial f}{\partial x}(g(0))\frac{dg_1}{du}(0) + \frac{\partial f}{\partial y}(g(0))\frac{dg_2}{du}(0) = \frac{\partial f}{\partial x}(0,1)\frac{dg_1}{du}(0) + \frac{\partial f}{\partial y}(0,1)\frac{dg_2}{du}(0) = 0$$

3.- Las relaciones $u = f(x,y)$, $x = x(t)$ e $y = y(t)$ definen u como función escalar de t, digamos $u = u(t)$. Aplicar la regla de la cadena para la derivada de u respecto de t cuando

$$f(x,y) = e^{xy} \cos xy^2, \quad x(t) = \cos t, \quad y(t) = \operatorname{sen} t.$$

SOL:

$$\frac{\partial u}{\partial t} = \frac{\partial f}{\partial x}(x(t),y(t))\frac{dx}{dt}(t) + \frac{\partial f}{\partial y}(x(t),y(t))\frac{dy}{dt}(t) =$$

$$= \left(e^{x(t)y(t)} y(t) \cos(x(t)(y(t))^2) - e^{x(t)y(t)} \operatorname{sen}(x(t)(y(t))^2)(y(t))^2\right)(-\operatorname{sen} t) +$$

$$+ \left(e^{x(t)y(t)} y(t) \cos(x(t)(y(t))^2) - e^{x(t)y(t)} \operatorname{sen}(x(t)(y(t))^2) 2x(t)y(t)\right)(\cos t) =$$

$$= e^{\cos t \operatorname{sen} t}\left(\cos(\cos t \operatorname{sen}^2 t)(\cos^2 t - \operatorname{sen}^2 t) + \operatorname{sen}(\cos t \operatorname{sen}^2 t)\operatorname{sen}^3 t - 2\cos^2 t \operatorname{sen} t\right)$$

4.- La sustitución $t = g(x,y)$ convierte $F(t)$ en $f(x,y) = F(g(x,y))$. Calcular la matriz $Df(x,y)$ en el caso particular en que $F(t) = e^{\operatorname{sen} t}$ y $g(x,y) = \cos(x^2+y^2)$.

SOL:
Por una parte, $DF(t) = F'(t)$, de forma que $DF(g(x,y))) = e^{\operatorname{sen}(\cos(x^2+y^2))}\cos(\cos(x^2+y^2)$. Por otra, $Dg(x,y) = \left(\dfrac{\partial g}{\partial x}(x,y), \dfrac{\partial f}{\partial y}\right) = (2x \operatorname{sen}(x^2+y^2), 2y \operatorname{sen}(x^2+y^2)$. Luego

$$Df(x,y) = DF(g(x,y))Dg(x,y) = e^{\operatorname{sen}(\cos(x^2+y^2))}\cos(\cos(x^2+y^2)(2x \operatorname{sen}(x^2+y^2), 2y \operatorname{sen}(x^2+y^2) =$$

$$= \left(2xe^{\operatorname{sen}(\cos(x^2+y^2))}\cos(\cos(x^2+y^2)\operatorname{sen}(x^2+y^2), 2ye^{\operatorname{sen}(\cos(x^2+y^2))}\cos(\cos(x^2+y^2)\operatorname{sen}(x^2+y^2)\right)$$