Fund. Fis. Tema 1. Problemas. Soluciones

1.

$$|\vec F_e| = k_e \cdot \frac{Q_1 \cdot Q_2}{r^2}$$

$$|\vec F_g| = G \cdot \frac{m_1 \cdot m_2}{r^2}$$

$$\boxed{\frac{|\vec F_e|}{|\vec F_g|} = \frac{k_e}{G} \cdot \frac{Q_e^2}{m_e \cdot m_{p^+}} \approx 10^{39}}$$

$|\vec F_e| = -3{,}7 \cdot 10^{-8}$ N

$|\vec F_g| = -3{,}6 \cdot 10^{-47}$ N

$G = 6{,}67 \cdot 10^{-11}\ \frac{N \cdot m^2}{kg^2}$

2.

$Q_1 + Q_2 = 5 \cdot 10^{-5}$ C

$|\vec F_{12}| = 1{,}0$ N

$d = 2{,}0$ m

$Q_1 = ?$, $Q_2 = ?$

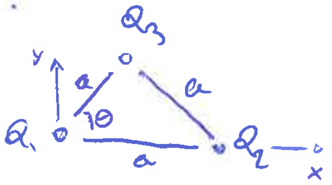
$$|\vec F_e| = k_e \cdot \frac{Q_1 Q_2}{r^2} \Rightarrow Q_1 Q_2 = \frac{|\vec F_e| r^2}{k_e} \Rightarrow Q_1 = \frac{1}{Q_2} \cdot \frac{|\vec F_e| r^2}{k_e}$$

$$Q_1 + Q_2 = \frac{1}{Q_2} \cdot \frac{|\vec F_e| \cdot r^2}{k_e} + Q_2 = Q_T \Rightarrow Q_2^2 - Q_T Q_2 + \frac{|\vec F_e| \cdot r^2}{k_e} = 0$$

$\Rightarrow Q_2 = \begin{cases} + \ 3{,}84 \cdot 10^{-5}\,C \\ - \ 1{,}16 \cdot 10^{-5}\,C \end{cases} \Rightarrow Q_1 = \begin{cases} 1{,}16 \cdot 10^{-5}\,C \\ 3{,}84 \cdot 10^{-5}\,C \end{cases} \Rightarrow$

$$\boxed{\begin{aligned} Q_2 &= 1{,}16 \cdot 10^{-5}\,C \\ Q_2' &= 3{,}84 \cdot 10^{-5}\,C \end{aligned}}$$

3.



$Q_1 = 1\mu C$

$Q_2 = +2\mu C$

$Q_3 = +3\mu C$

$\vec r(Q_1) = (0,0)$

$\theta = 60°$

$a = 1mm$

a) $\vec F_{T,3} = ?$

b) $\vec r_3(\vec F = 0) = ?$

a)

$$\vec F_{T,3} = \vec F_1 + \vec F_2$$

$$\vec F_1 = k_e \frac{Q_1 Q_3}{a^2} \hat u_{r_1}, \quad \hat u_{r,1} = \cos\theta\, \hat u_x + \sin\theta\, \hat u_y$$

$$\vec F_2 = k_e \frac{Q_2 Q_3}{a^2} \hat u_{r_2}, \quad \hat u_{r,2} = -\cos\theta\, \hat u_x + \sin\theta\, \hat u_y$$

$$\Rightarrow \vec F_T = k_e \frac{Q_3}{a^2} \left[(Q_1 - Q_2) \cos\theta\, \hat u_x + (Q_1 + Q_2) \sin\theta\, \hat u_y \right]$$

$$\Rightarrow \boxed{\vec F_T} = 2{,}7 \cdot 10^{10} \left[(-1\mu) \cos 60°\, \hat u_x + (3\mu) \sin 60°\, \hat u_y \right] N =$$

$$\boxed{= -1{,}35 \cdot 10^{4}\, \hat u_x + 7{,}01 \cdot 10^{4}\, \hat u_y \ N}$$

b)

$$\vec F_{T,3} = 0 \Rightarrow \vec F_1 = -\vec F_2 \Rightarrow F_{1,x} = F_{2,x}, \quad F_{1,y} = F_{2,y}.$$

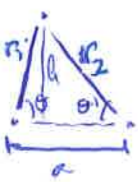
$$\Rightarrow \left. \begin{aligned} k_e \frac{Q_1 Q_3}{r_1^2} \cos\theta &= k_e \frac{Q_2 Q_3}{r_2^2} \cos\theta' \\ k_e \frac{Q_1 Q_3}{r_1^2} \sin\theta &= k_e \frac{Q_2 Q_3}{r_2^2} \sin\theta' \end{aligned} \right\} \Rightarrow \left. \begin{aligned} \frac{Q_1}{r_1^2} \cos\theta &= \frac{Q_2}{r_2^2} \cos\theta' \\ \frac{Q_1}{r_1^2} \sin\theta &= \frac{Q_2}{r_2^2} \sin\theta' \end{aligned} \right\} \Rightarrow \operatorname{tg}\theta' = \operatorname{tg}\theta \Rightarrow$$

$$\Rightarrow \theta' = n\pi + \theta$$

$$\Rightarrow \boxed{\theta' = \theta}$$

$$\Rightarrow k_e \frac{Q_1 Q_3}{r_1^2} \cos\theta = k_e \frac{Q_2 Q_3}{r_2^2} \cos\theta \Rightarrow \frac{Q_1}{r_1^2} = \frac{Q_2}{r_2^2} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{Q_1}{Q_2} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{Q_1}{Q_2}} \Rightarrow r_1 = r_2 \sqrt{\frac{Q_1}{Q_2}}$$

$\downarrow r_1 + r_2 = a$

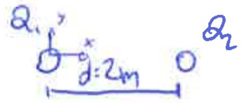


$$\cos\theta + \cos\theta' = \frac{x_1}{r_1} + \frac{x_2}{r_2} = a \Rightarrow \frac{x_1}{r_2} \sqrt{\frac{Q_2}{Q_1}} + \frac{x_2}{r_2} = \frac{1}{r_2}\left(x_1 \sqrt{\frac{Q_1}{Q_2}} + x_2 \right) = a$$

$$(r_1^2 - h^2)^{\frac{1}{2}} + (r_2^2 - h^2)^{\frac{1}{2}} = a$$

$$\boxed{\begin{aligned} r_2 &= a \cdot \frac{1}{1 + \sqrt{\frac{Q_1}{Q_2}}} = 0{,}59\,mm \\ r_1 &= 0{,}41\,mm \end{aligned}}$$

4.



$Q_1 = +9\mu C$
$Q_2 = -4\mu C$
$Q_3 = +1\mu C$
$\vec{r}(Q_3) = ?$

a) $\vec{F}_1 = -\vec{F}_2$, $\vec{F}_1 = k_e \frac{Q_1 Q_3}{x^2}\hat{u}_x$, $\vec{F}_2 = k_e \frac{Q_2 Q_3}{x_2^2}\hat{u}_x$

$|\vec{F}_1| = |\vec{F}_2| : \begin{cases} x>d: \frac{|Q_1|}{x^2} = \frac{|Q_2|}{(x-d)^2} \Rightarrow \boxed{x} = \begin{cases} 1{,}2\,m : x<d. \text{ No vale} \\ \boxed{6{,}0\,m} \end{cases} \\ x<0: \frac{|Q_1|}{x^2} = \frac{|Q_2|}{(x+d)^2} \Rightarrow x = \begin{cases} -6{,}0\,m \\ -1{,}2\,m \end{cases} \end{cases}$ Sin solución factible porque usamos los módulos al igual $\vec{F}$'s

$x = +6{,}0\,m$

b) Ahora:
$Q_1 = 6{,}0\mu C$
$Q_2 = 15{,}0\mu C$
$Q_3 = -10\mu C$

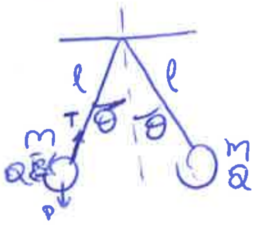
(Si $\vec{r}_{e3} = (1,0)$: $\vec{F}_T = k_e Q_3\left(\frac{Q_1}{r_{13}^2} - \frac{Q_2}{r_{23}^2}\right)\hat{u}_x = +8{,}1\cdot 10^{-19}\hat{u}_x N$)

$\vec{F}_{13} = -\vec{F}_{23} \Rightarrow \left|\frac{Q_1}{r_{13}^2}\right| = \left|\frac{Q_2}{r_{23}^2}\right| \Rightarrow r_{13} = r_{23}\sqrt{\frac{Q_1}{Q_2}}$

$r_{13} + r_{23} = d \Rightarrow d = r_{23}\left(1+\sqrt{\frac{Q_1}{Q_2}}\right) \Rightarrow \boxed{r_{23} = 1{,}23\,m,\ r_{13} = 0{,}77\,m}$

Para a): $r_{13} = d + r_{23}$ $x>0 \Rightarrow r_{13} = +6\,m$
$r_{23} = d + r_{13}$ $x<0 \Rightarrow r_{13} = -6\,m \rightarrow$ sin sentido, $r \geq 0$.

5.



$m = 3{,}0\cdot 10^{-2} kg$
$l = 0{,}15\,m$
$\theta = 5{,}0°$
$Q = ?$

a) En eq. $\sum_i \vec{F}_i = 0$:

eje y: $P = T\cos\theta \Rightarrow T = \frac{P}{\cos\theta} = \frac{mg}{\cos\theta} = \frac{3\cdot 10^{-2}\cdot 9{,}81}{\cos(5°)}$.

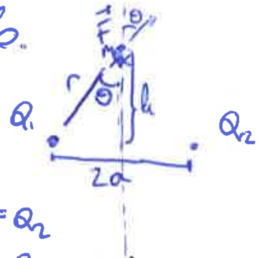
eje x: $F_e = T\,\text{sen}\,\theta \Rightarrow F_e = \frac{P}{\cos\theta}\text{sen}\,\theta = P\,\text{tg}\,\theta = k_e\frac{Q^2}{4l^2\text{sen}^2\theta} \Rightarrow$

$\Rightarrow \boxed{Q = 2l\,\text{sen}\,\theta\sqrt{\frac{P\,\text{tg}\,\theta}{k_e}} \simeq 2{,}21\cdot 10^{-8}C = 44{,}2\,nC}$

b) $\boxed{\theta = \theta'}$, ~~[illegible]~~

ya que: $P_1 = P_2 \Rightarrow T_{1,y} = T_{2,y} \Rightarrow \theta_1 = \theta_2$.

6.



$Q_1 = Q_2$
$h = ?$
$F(h) = max.$

$\vec{E}_T = \vec{E}_1 + \vec{E}_2$, $\vec{E}_1 = k_e\frac{Q}{r_1^2}\hat{u}_r = k_e\frac{Q}{r_1^2}(\cos\theta\,\hat{u}_y + \text{sen}\,\theta\,\hat{u}_x)$

$\Rightarrow \vec{E}_{T_{max}} = (E_{1,y} + E_{2,y})_{max} = 2k_e\frac{Q}{r^2}\cos\theta\,\hat{u}_y$.

$r^2 = a^2 + h^2$, $\cos\theta = \frac{h}{r} = \frac{h}{(a^2+h^2)^{1/2}}$

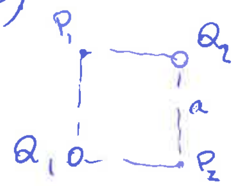
$\Rightarrow \vec{E}_{max} = 2k_e\frac{Q}{(a^2+h^2)}\cdot\frac{h}{(a^2+h^2)^{1/2}} = 2k_e Q\frac{h}{(a^2+h^2)^{3/2}}\hat{u}_y$

$\Rightarrow \frac{\partial \vec{E}_T}{\partial h} = 2k_e Q\left[\frac{1\cdot(a^2+h^2)^{3/2} - h\cdot\frac{3}{2}(a^2+h^2)^{1/2}\,2h}{(a^2+h^2)^3}\right] \underset{h_{max}}{=} 0 \Rightarrow$

$\Rightarrow (a^2+h^2)^{3/2} - 2\cdot\frac{3}{2}h^2(a^2+h^2)^{1/2} = (a^2+h^2)^{1/2}\left[(a^2+h^2)^1 - 3h^2\right] = 0 \Rightarrow \begin{cases} a^2+h^2 = 0 \Rightarrow a^2 = -h^2 \Rightarrow h = ai \text{ irreal} \\ a^2 - 2h^2 = 0 \Rightarrow \boxed{h = a/\sqrt{2}} \end{cases}$

7) e^- en $\vec{E} = 4\cdot10^4\,N/C\ \hat{u}_x$ $\Rightarrow$ $\boxed{\vec{F} = \vec{E}\,Q = 4\cdot10^4\cdot(-1{,}6\cdot10^{-19})\,N\,\hat{u}_x = -6{,}4\cdot10^{-15}\,\hat{u}_x\ N}$

8)



$Q_1 = 1{,}6\,nC$

$Q_2 = -2{,}4\,nC$

$a = \frac{1}{2}\,m$

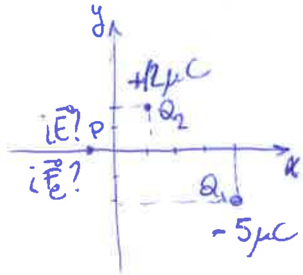
$\vec{E} = \vec{E}_1 + \vec{E}_2$

en P_1: $\vec{E}_1 = k_e \frac{Q_1}{a^2}\hat{u}$, $\vec{E}_2 = +k_e \frac{Q_2}{a^2}\hat{u}_x$

$\boxed{\vec{E}_{T,P_1} = k_e\cdot\frac{1}{a^2}\left(|Q_1|\,\hat{u}_y + |Q_2|\,\hat{u}_x\right) = (57{,}6\,\hat{u}_x + 86{,}4\,\hat{u}_y)\ N/C}$

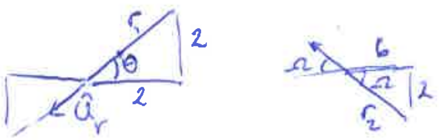
en P_2: $\vec{E}_{T,P_2} = -\vec{E}_{T,P_1}\,(Q_1 \to Q_2) \Rightarrow$ $\boxed{\vec{E}_{T,P_2} = k_e\,\frac{1}{a^2}\left(+|Q_1|\,\hat{u}_x + |Q_2|\,\hat{u}_y\right) = (86{,}4\,\hat{u}_x + 57{,}6\,\hat{u}_y)\ N/C}$

9)



$r_{Q_2P} = \sqrt{(2)^2+(2)^2} = \sqrt{8}$ $\qquad \hat{u}_{r_2} = -\cos\theta\,\hat{u}_x - \operatorname{sen}\theta\,\hat{u}_y$

$r_{Q_1P} = \sqrt{(2)^2+(5)^2} = \sqrt{29}$ $\qquad \hat{u}_{r_1} = -\cos\alpha\,\hat{u}_x + \operatorname{sen}\alpha\,\hat{u}_y$



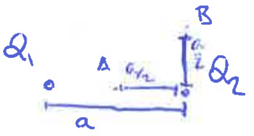
A) $\boxed{\vec{E}_p} = K_e\left[-\left(\frac{Q_2}{r_2^2}\cos\theta + \frac{Q_1}{r_1^2}\cos\alpha\right)\hat{u}_x + \left(\frac{Q_1}{r_1^2}\operatorname{sen}\alpha - \frac{Q_2}{r_2^2}\operatorname{sen}\theta\right)\hat{u}_y\right]$

$= k_e\left[-\left(\frac{+12\mu}{8}\cdot\frac{2}{\sqrt{8}} + \frac{-5\mu}{29}\cdot\frac{5}{\sqrt{29}}\right)\hat{u}_x + \left(\frac{-5\mu}{29}\cdot\frac{2}{\sqrt{29}} - \frac{+12\mu}{8}\cdot\frac{2}{\sqrt{8}}\right)\hat{u}_y\right]$

$= (-8{,}11\cdot10^3\,\hat{u}_x - 10{,}12\cdot10^3\,\hat{u}_y)\ N/C$

B) $\boxed{\vec{F}_e = \vec{E}\cdot q_e = (-8{,}11\cdot10^3\,\hat{u}_x - 10{,}12\cdot10^3\,\hat{u}_y)(-1{,}6\cdot10^{-19}\,C) = (1{,}3\cdot10^{-15}\,\hat{u}_x + 1{,}6\cdot10^{-15}\,\hat{u}_y)\ N}$

10)



$Q_1 = 25\,nC$

$Q_2 = -10\,nC$

$a = 10\,cm$

A) $\vec{E}_A$?, $\vec{E}_B$?

B) $W(a'=2a)$?

A) en A: $\vec{E}_T = \vec{E}_1 + \vec{E}_2$; $\vec{E}_1 = +k_e\frac{|Q_1|}{(\frac{a}{2})^2}\hat{u}_x$, $\vec{E}_2 = +k_e\frac{|Q_2|}{(\frac{a}{2})^2}\hat{u}_x$

$\Rightarrow$ $\boxed{\vec{E}_{T,A} = \frac{k_e}{(\frac{a}{2})^2}\left(|Q_1| + |Q_2|\right)\hat{u}_x = 1{,}26\cdot10^5\ V/m}$

en B: $\vec{E}_T = \vec{E}_1 + \vec{E}_2$; $\vec{E}_2 = -k_e\frac{|Q_2|}{(\frac{a}{2})^2}\hat{u}_y$, $\vec{E}_1 = k_e\frac{|Q_1|}{r^2}(\cos\theta\,\hat{u}_x + \operatorname{sen}\theta\,\hat{u}_y)$

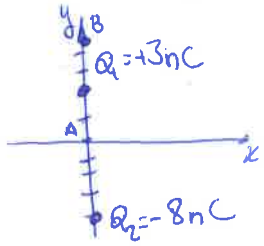
$\vec{E}_1 = k_e\frac{|Q_1|}{a^2+(\frac{a}{2})^2}\left[\frac{a}{[a^2+(\frac{a}{2})^2]^{1/2}}\hat{u}_x + \frac{a/2}{[a^2+\frac{a^2}{4}]^{1/2}}\hat{u}_y\right] \Rightarrow$ $\boxed{\vec{E}_{T,B} = (1{,}61\,\hat{u}_x + 4{,}41\,\hat{u}_y)\cdot10^4\ V/m}$

B) $\Delta E_p = E_{p,f} - E_{p_0} = k_e\frac{Q_1Q_2}{a'} - k_e\frac{Q_1Q_2}{a} = k_e Q_1Q_2\left(\frac{1}{2a} - \frac{1}{a}\right) = k_e QQ_2\cdot\frac{-1}{2a} = +1{,}125\cdot10^{-5}\,J$

$\Rightarrow$ $W = -1{,}125\cdot10^{-5}\,J$: realizado por el sistema.

$W = +1{,}125\cdot10^{-5}\,J$: realizado por un agente externo

11.



A) $\vec{E}_T=\vec{E}_1+\vec{E}_2$; $\vec{E}_1=-k_e\frac{|Q_1|}{r_1^2}\hat{u}_y$, $\vec{E}_2=-k_e\frac{|Q_2|}{r_2^2}\hat{u}_y$

$\Rightarrow \vec{E}_T=-k_e\left(\frac{|Q_1|}{r_1^2}+\frac{|Q_2|}{r_2^2}\right)\hat{u}_y$.

$\boxed{\vec{E}_T=-11,25\ V/m\ \hat{u}_y}$

B) $\vec{E}_T=\vec{E}_1+\vec{E}_2$, $\vec{E}_1=+k_e\frac{|Q_1|}{r_1^2}\hat{u}_y$, $\vec{E}_2=-k_e\frac{|Q_2|}{r_2^2}\hat{u}_y$

$\Rightarrow \boxed{\vec{E}_T=+k_e\left(\frac{|Q_1|}{r_1^2}-\frac{|Q_2|}{r_2^2}\right)\hat{u}_y=+2,1\ \hat{u}_y\ V/m}$

C) $\boxed{V_A=V_{1A}+V_{2A}=k_e\frac{Q_1}{r_1}+k_e\frac{Q_2}{r_2}=k_e\left(\frac{|Q_1|}{r_1}-\frac{|Q_2|}{r_2}\right)=-4,5\ V}$

$\boxed{V_B=V_{1B}+V_{2B}=k_e\frac{Q_1}{r_1}+k_e\frac{Q_2}{r_2}=k_e\left(\frac{|Q_1|}{r_1}-\frac{|Q_2|}{r_2}\right)=+1,0\ V}$

12.

$\vec{E}=10\ \hat{u}_x\ N/C$.

$V=?$

$\boxed{\Delta V=\int\vec{E}\cdot d\vec{l}=-\int 10\ \hat{u}_x\cdot d\vec{l}=-10x\ V}$

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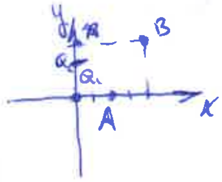
$V=100-25\cdot x\ V/m$

$\vec{E}=?$

$\boxed{\vec{E}=-\nabla V=-\left(\frac{\partial V}{\partial x},\frac{\partial V}{\partial y},\frac{\partial V}{\partial z}\right)=+25\ \hat{u}_x\ V/m}$

14.



$Q_2(0,2)$

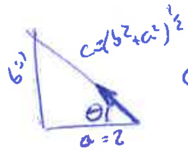
$r_{Q_1}(0,0)$

$Q_1=+1nC$

$Q_2=-20nC$

$A(2,0)$

$B(4,2)$



A) $|\vec{E}_T|=?$ $\boxed{\begin{array}{l}|\vec{E}_1|=k_e\frac{|Q_1|}{r_1^2}=2,25\ V/m\\ |\vec{E}_2|=k_e\frac{|Q_2|}{r_2^2}=36\ V/m\end{array}}$

B) $\vec{E}_T=?$ $\vec{E}_1=E_1\cdot\hat{u}_x$

$\vec{E}_2=E_2\cdot(\cos\theta\, u_x+\sin\theta\, u_y)$

$\Rightarrow \boxed{\vec{E}_T=-29,95\ \hat{u}_x+16,1\ \hat{u}_y\ N/C}$

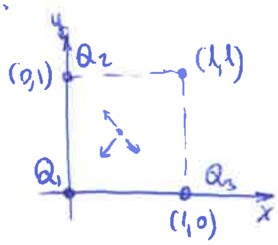
C) $V_A=V_1+V_2=k_e\frac{Q_1}{r_1}+k_e\frac{Q_2}{r_2}=k_e\cdot\left(\frac{1\cdot10^{-9}}{2}-\frac{20\cdot10^{-9}}{(4+4)^{1/2}}\right)=-76\ V$

$V_B=V_1+V_2=k_e\left(\frac{1\cdot10^{-9}}{(16+4)^{1/2}}-\frac{20\cdot10^{-9}}{(16+1)^{1/2}}\right)=-41,6\ V$

$\Rightarrow \Delta V=V_B-V_A=34,36\ V \Rightarrow \Delta E_p=3\cdot C\cdot 34,36V=103,1\ J\Rightarrow \boxed{W=-103,1J}$

$\Rightarrow$ $\boxed{\text{Un agente externo debe realizar } +103,1\ J}$

15.



$Q_1 = -2\mu C$

$Q_2 = Q_3 = +1\mu C$

A) $|\vec{E}(1,1)| = ?$ $\quad \vec{E}_T = \vec{E}_1 + \vec{E}_2 + \vec{E}_3.$

$$E_1 = k_e \frac{Q_1}{r_1^2} = k_e \frac{Q}{2} \ ; \quad E_2 = k_e \frac{Q_2}{r_2^2}, \quad E_3 = k_e \frac{Q_3}{r_3^2}$$

$$\Rightarrow \vec{E}_1 = -k_e \frac{|Q_1|}{2}\left(\frac{\sqrt{2}}{2}\hat{u}_x + \frac{\sqrt{2}}{2}\hat{u}_y\right); \quad \vec{E}_2 = +k_e \frac{|Q_2|}{1}\hat{u}_x ; \quad \vec{E}_3 = +k_e \frac{|Q_3|}{1}\hat{u}_y$$

$$\Rightarrow \vec{E}_T = k_e\left(\left[-\frac{|Q|\cdot\sqrt{2}}{4} + \frac{|Q_2|}{1}\right]\hat{u}_x + \left[-\frac{|Q|\sqrt{2}}{4} + \frac{|Q_3|}{1}\right]\hat{u}_y\right) N/C \Rightarrow$$

$$\Rightarrow \boxed{\vec{E}_T = (+2{,}636\,\hat{u}_x + 2{,}636\,\hat{u}_y)\ kN/C}$$

$|\vec{E}(\tfrac{1}{2},\tfrac{1}{2})| = ?$ $\quad \vec{E}_T = \vec{E}_1 + \vec{E}_2 + \vec{E}_3, \quad \vec{E}_2 = -\vec{E}_3 \Rightarrow \vec{E}_T = \vec{E}_1.$

$$\boxed{E_1 = k_e \frac{Q_1}{r^2} = k_e \frac{Q_1}{[2(\frac{1}{2})^2]^{\frac{1}{2}}} = 25.455{,}8\ N/C}$$

B)

$$V_{(1,1)} = V_1 + V_2 + V_3 = k_e\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right) = k_e\left(\frac{Q_1}{r_1} + 2\frac{Q_2}{r_2}\right) = k_e\left(\frac{-2\mu}{\sqrt{2}} + 2\cdot\frac{1\mu}{1}\right) = 5{,}27\,kV$$

$$V_{(\frac{1}{2},\frac{1}{2})} = V_1 + V_2 + V_3 = k_e\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right) = k_e\left(\frac{Q_1}{r_1} + 2\frac{Q_2}{r_2}\right) = k_e\left(\frac{-2\mu}{\sqrt{2(\frac{1}{2})^2}} + 2\cdot\frac{1\mu}{\sqrt{2\cdot(\frac{1}{2})^2}}\right) = 0\ V$$

$$\boxed{\Delta V = V_{\frac{1}{2}\frac{1}{2}} - V_{1,1} = -5{,}27\,kV}$$

16.

$Q_1 = +15{,}0\mu C$

$Q_2 = +6{,}0\mu C$

$a = 2{,}0\,m$

$Q_3 = -10\mu C$

$x_3 = \frac{a}{2}$

A)
$$\boxed{\vec{F}_T} = \vec{F}_1 + \vec{F}_2 = \left(-k_e \frac{Q_1 Q_3}{(\frac{a}{2})^2} + k_e \frac{Q_2 Q_3}{(\frac{a}{2})^2}\right)\hat{u}_x\ N = k_e \frac{Q_3}{(\frac{a}{2})^2}(|Q_1| + |Q_2|)\hat{u}_x\ N =$$

$$\boxed{= +8{,}1\cdot 10^{-19}\ \hat{u}_x\ N}$$

B)
$$\vec{F}_T = 0 \Rightarrow \vec{F}_1 = -\vec{F}_2 \Rightarrow k_e \frac{Q_1 Q_3}{x^2} = k_e \frac{Q_3 Q_2}{(a-x)^2} \Rightarrow \frac{Q_1}{Q_2} = \frac{x^2}{(a-x)^2} \Rightarrow$$

$$\Rightarrow \frac{x}{a-x} = \left(\frac{Q_1}{Q_2}\right)^{\frac{1}{2}} \Rightarrow x = \left(\frac{Q_1}{Q_2}\right)^{\frac{1}{2}}(a-x) \Rightarrow x\left(1 + \left(\frac{Q_1}{Q_2}\right)^2\right) = a\cdot\left(\frac{Q_1}{Q_2}\right)^2 \Rightarrow$$

$$\Rightarrow \boxed{x = \frac{a\left(Q_1/Q_2\right)^2}{1 + \left(Q_1/Q_2\right)^2} = 1{,}72\,m}$$

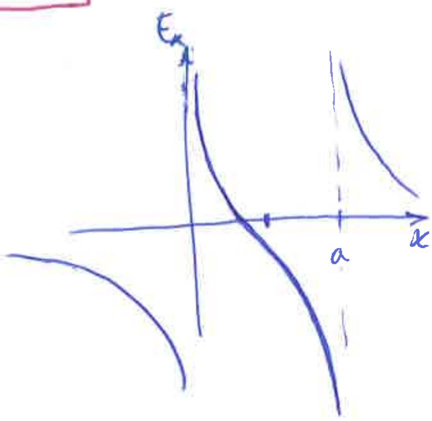
C) $\vec{E}_T = \vec{E}_1 + \vec{E}_2$

E_x:

$x > a$: $\quad E_{Tx} = E_{1x} + E_{2x} = k_e\left(\frac{Q_1}{x^2} + \frac{Q_2}{(x-a)^2}\right) N/C$

$x < 0$: $\quad E_{Tx} = -(E_{1x} + E_{2x}) = -k_e\left(\frac{Q_1}{x^2} + \frac{Q_2}{(x+a)^2}\right) N/C$

$a < x < 0$: $\quad E_{Tx} = +E_{1x} - E_{2x} = k_e\left(\frac{Q_1}{x^2} - \frac{Q_2}{(a-x)^2}\right) N/C$



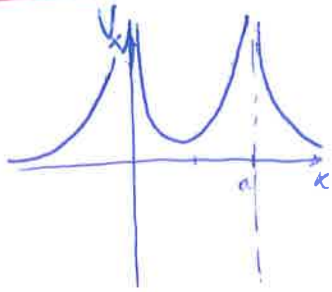
16.

D) $V_T = V_1 + V_2$.

$x > a$: $\boxed{V_T = k_e \frac{Q_1}{x} + k_e \frac{Q_2}{x-a} = k_e\left(\frac{Q_1}{x} + \frac{Q_2}{x-a}\right) V}$

$x < 0$: $\boxed{V_T = k_e\left(\frac{Q_1}{x} + \frac{Q_2}{x+a}\right) V}$

$0 < x < a$: $\boxed{V_T = k_e\left(\frac{Q_1}{x} + \frac{Q_2}{a-x}\right) V}$



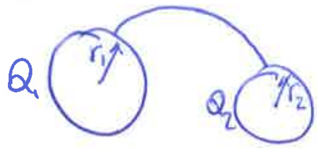
También integrando: $\Delta V = -\int \vec{E}\cdot d\vec{x}$

$x>a$: $\Delta V = -\int k_e\left(\frac{Q_1}{x^2} + \frac{Q_2}{(x-a)^2}\right)dx = -k_e\int \frac{Q_1}{x^2}dx - k_e\int \frac{Q_2}{(x-a)^{-2}} = -k_eQ_1(-x^{-1}) - k_eQ_2(-(x-a)) =$
$= k_e\left(\frac{Q_1}{x} + \frac{Q_2}{x-a}\right)$

$x<0$: $\Delta V = -\int k_e\left(\frac{Q_1}{x^2} + \frac{Q_2}{(x+a)^2}\right)dx = k_e\left(\frac{Q_1}{x} + \frac{Q_2}{x+a}\right)$

$0<x<a$: $\Delta V = -\int k_e\left(\frac{Q_1}{x^2} - \frac{Q_2}{(a-x)^2}\right)dx = k_e\left(\frac{Q_1}{x} + \frac{Q_2}{a-x}\right)$

17.



$r_1 = 6cm$, $Q_1 = 1\mu C$
$r_2 = 9cm$, $Q_2 = 1\mu C$

A) $V_1(r_1) = k_e \frac{Q_1}{r_1} V$, $V_2(r_2) = k_e \frac{Q_2}{r_2} V$

$\boxed{V_1(r_1) = 150 \text{ kV}}$ $\boxed{V_2 = 100 \text{ kV}}$

B) $V_1 = V_2$, $k_e \frac{Q_1}{r_1} = k_e \frac{Q_2}{r_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{r_2}{r_1}$. $Q_1 + Q_2 = 2\mu C$.

$\Rightarrow Q_1 + \frac{r_2}{r_1}Q_1 = Q_1\left(1 + \frac{r_2}{r_1}\right) = Q_T \Rightarrow Q_1 = \frac{Q_T}{1 + \frac{r_2}{r_1}} \Rightarrow Q_2 = Q_T - Q_1$

$\Rightarrow \boxed{V_1 = k_e \frac{Q_1}{r_1} = 120 \text{ kV}}$ $\boxed{Q_1 = 0{,}8\mu C \Rightarrow Q_2 = 1{,}2\mu C}$

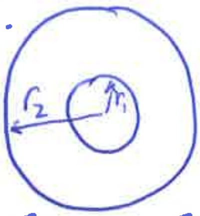
18.

$r_1 = 10cm$.
$V(r_1) = 1kV$
$Q = ?$
$V(r=90cm) = ?$

A) $V = k_e \frac{Q_1}{r} \Rightarrow \boxed{Q_1 = V \cdot \frac{r_1}{k_e} = 11{,}1 nC}$

B) $\boxed{V(r=90cm) = k_e \cdot \frac{Q_1}{r_{90}} = k_e \cdot \frac{Q_1}{(90+10)cm} = 100 V}$

19.



$r_1 = 20cm$, $Q_1 = +2\mu C$
$r_2 = 50cm$, $Q_2 = -4\mu C$

A) $\underset{r=0}{V_T} = V_1 + V_2$. $V_1 = k_e \frac{Q_1}{r_1}$, $V_2 = k_e \frac{Q_2}{r_2} \Rightarrow V_T = k_e\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right) \Rightarrow$

$\Rightarrow \boxed{V_T = k_e \cdot \left(\frac{2\mu}{0{,}2} - \frac{4\mu}{0{,}5}\right) = 18 \text{ kV}}$

B) $\underset{r=35cm}{V_T} = V_1 + V_2$. $\boxed{V_T = k_e \cdot \frac{Q_1}{r} + k_e \frac{Q_2}{r_2} = k_e\left(\frac{2\mu}{0{,}35} - \frac{4\mu}{0{,}5}\right) = -20{,}57 \text{ kV}}$

C) $\underset{r=0{,}6m}{V_T} = V_1 + V_2$ $\boxed{V_T = k_e\left(\frac{2\mu}{0{,}6} - \frac{4\mu}{0{,}6}\right) = -k_e \frac{2\mu}{0{,}6} = -30 \text{ kV}}$

20.

$m = 6,64\cdot 10^{-27}\,kg$

$Q = +3,204\cdot 10^{-19}\,C$

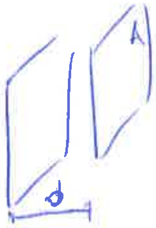
$\Delta V = 100\,kV$

$E_{c,0} = 0\ ;\ \Delta E_p = \Delta V\cdot Q = 100.000\cdot 3,204\cdot 10^{-19}\ J$

$E = cte \Rightarrow \cancel{E_{c,0}} + E_{p,0} = E_{c,f} + \cancel{E_{p,f}} \Rightarrow \Delta E_c = \Delta E_p = \frac{1}{2} m v^2 \Rightarrow$

$$\Rightarrow \boxed{v = \left(\frac{2\Delta E_c}{m}\right)^{1/2} = 3,11\cdot 10^{6}\ m/s}$$

21.



$A = 150\ cm^2$

$d = 2\ mm$

$\Delta V = 1\ kV$

$Q = ?\ ,\ E_p ?$

$$C = \frac{Q}{\Delta V} = \frac{Q}{E\cdot d} = \frac{\varepsilon_0\cdot A}{d} \Rightarrow \boxed{Q = \Delta V\cdot\frac{\varepsilon_0 A}{d} = 66,4\,nC}$$

$$\boxed{E_p = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{Q}{\Delta V}(\Delta V)^2 = \frac{1}{2}Q(\Delta V) = 33,2\ \mu J}$$

22.

$Q_b = -Q_a$

$Q_a = Q$

$$C = \frac{Q}{\Delta V}\ .\ \Delta V = V_b - V_a\ .\ V_a = k_e\frac{Q}{r_a} - k_e\frac{Q}{r_b}\ ,\ V_b = k_e\frac{Q}{r_b} - k_e\frac{Q}{r_b} = 0$$

$$\Rightarrow \Delta V = k_e Q\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = k_e Q\left(\frac{r_a - r_b}{r_b r_a}\right)$$

$$\Rightarrow \boxed{C = \frac{Q}{\Delta V} = \frac{1}{k_e}\cdot\frac{r_b r_a}{r_a - r_b}}$$

otra forma: $\Delta V = -\int \vec{E}\,d\vec{r} = -\int_{r_a}^{r_b} k_e\frac{Q}{r}\,dr = +k_e Q\cdot\frac{1}{r^2}\Big]_{r_a}^{r_b} = +k_e Q\cdot\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = k_e Q\cdot\left(-\frac{1}{r_a} + \frac{1}{r_b}\right) = k_e Q\,\frac{r_a - r_b}{r_b r_a}$

23.

$n_e = ?$

Cu

$I = 6A$

$$I = \frac{dQ}{dt} = \frac{|e^-|\,dn}{dt} \Rightarrow dn = \frac{I}{|e|}dt \Rightarrow n_e = \int\frac{I}{|e|}dt = \frac{I}{|e|}\Delta t \Rightarrow$$

$$\Rightarrow \boxed{n_e(t=3s) = \frac{6A}{1,6\cdot 10^{-19}C}\cdot 3s = 1,12\cdot 10^{20}}$$

24.

$\Delta V = 3V$

$I = 6mA$

$R = ?$

$$I = \frac{\Delta V}{R} \Rightarrow \boxed{R = \frac{\Delta V}{I} = \frac{3V}{6\cdot 10^{-3}A} = \frac{1}{2}\,k\Omega = 500\,\Omega}$$

25.

$\rho_{Cu} = 1,72\cdot 10^{-8}\ \Omega\cdot m$

$A = 1\ mm^2$

$R = 4\ m\Omega$

$$\boxed{R = \rho\cdot\frac{l}{A} \Rightarrow l = R\cdot\frac{A}{\rho} = 0,233\,m}$$

26.

A) $R = 50\,\Omega$, $\Delta V = 120V$ $\Big\}$ $\boxed{I = \frac{\Delta V}{R} = \frac{120}{50}\frac{V}{\Omega} = 2,4A}$

B) $R = ?$, $\Delta V = 8V$, $I = 4mA$ $\Big\}$ $\boxed{R = \frac{\Delta V}{I} = \frac{8V}{4mA} = 2k\Omega}$

27.

$\rho_{Al.} = 2{,}8 \cdot 10^{-8}\ \Omega \cdot m$

$I = ?$ $\quad l = 2km$

$A = 1mm^2$, $\Delta V = 50V$.

$I = \frac{\Delta V}{R}$; $R = \rho \frac{l}{A} \Rightarrow I = \frac{\Delta V}{\rho \frac{l}{A}} = \frac{\Delta V \cdot A}{\rho \cdot l} \Rightarrow$

$\Rightarrow \boxed{I = \frac{50 \cdot (1 \cdot 10^{-3})^2}{2{,}8 \cdot 10^{-8} \cdot 2 \cdot 10^3} = 0{,}89 A}$

28.

$r = \frac{1}{4} mm$

$l = 1{,}1 m$

$\Delta V = 12V$

$I = 3{,}75 A$

$\rho = ?$

$I = \frac{\Delta V}{R} = \frac{\Delta V}{\rho \frac{l}{A}} \Rightarrow \rho = \frac{\Delta V \cdot A}{l \cdot I}$

$\Rightarrow \boxed{\rho = \frac{12 \cdot \pi \cdot (\frac{1}{4} \cdot 10^{-3})^2}{1{,}1 \cdot 3{,}75} = 5{,}71 \cdot 10^{-7}\ \Omega \cdot m}$

29.

$\Delta V = 120V$

$I = 9A$

$\boxed{P = I \cdot \Delta V = 9 \cdot 120 = 1080 W}$

30.

$P = 500W$

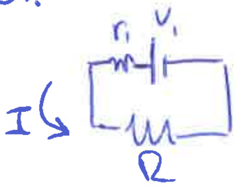
$I = 10 A$

$R = ?$

$P = I \cdot \Delta V = I^2 \cdot R \Rightarrow$

$\Rightarrow \boxed{R = \frac{P}{I^2} = \frac{500}{100} = 5 \Omega}$

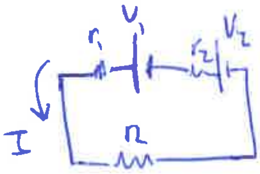
31.



$P = I^2 R$. $\quad I(r_1 + R) = \Delta V \Rightarrow I = \frac{\Delta V}{R + r_1} \Rightarrow P = \frac{(\Delta V)^2}{(R+r_1)^2} \cdot R$

$\frac{\partial P}{\partial R} = \frac{\Delta V^2}{(R+r_1)^2} + R \cdot \frac{(-2) \cdot (\Delta V)^2}{(R+r_1)^3} = 0 \Rightarrow (\Delta V)^2 + R \cdot \frac{(-2)(\Delta V)^2}{(R+r_1)} = 0 \Rightarrow$

$\Rightarrow (\Delta V)^2 \left[1 - \frac{2R}{R+r_1} \right] = 0 \Rightarrow \frac{2R}{R+r_1} = 1 \Rightarrow 2R = R + r_1 \Rightarrow \boxed{R = r_1}$



$P = I^2 \cdot R$

$I(R + r_1 + r_2) = V_1 + V_2 \Rightarrow I = \frac{V_1 + V_2}{r_1 + r_2 + R} \Rightarrow P = \left(\frac{V_1 + V_2}{r_1 + r_2 + R} \right)^2 R$

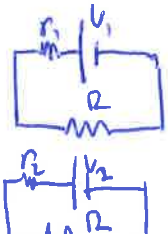
$\frac{\partial P}{\partial R} = \frac{(V_1+V_2)^2 \cdot (r_1 + r_2 + R)^2 - (V_1 + V_2)^2 R \cdot 2(r_1 + r_2 + R)}{(r_1 + r_2 + R)^4} = 0 \Rightarrow$

$\Rightarrow (r_1 + r_2 + R)^2 - 2R(r_1 + r_2 + R) = (r_1 + r_2 + R) \cdot [(r_1 + r_2 + R) - 2R] = 0$

$\Rightarrow R = -(r_1 + r_2)$: sin sentido

$r_1 + r_2 + R - 2R = r_1 + r_2 - R = 0 \Rightarrow \boxed{R = r_1 + r_2}$

32.



$V_1 = 2V$, $r_1 = 1\Omega$

$V_2 = 2{,}5V$, $r_2 = 2\Omega$

$\left. \begin{array}{l} I(r_1 + R) = \Delta V_1 \\ I(r_2 + R) = \Delta V_2 \end{array} \right\} \Rightarrow \frac{r_1 + R}{r_2 + R} = \frac{\Delta V_1}{\Delta V_2} \Rightarrow (r_1 + R)\Delta V_2 = (r_2 + R)\Delta V_1 \Rightarrow$

$\Rightarrow \boxed{R = \frac{\Delta V_1 r_2 - r_1 \Delta V_2}{\Delta V_2 - \Delta V_1} = \frac{2 \cdot 2 - 1 \cdot 2{,}5}{2{,}5 - 2} = \frac{3/2}{1/2} = 3 \Omega}$

$\Rightarrow \boxed{I = \frac{\Delta V_1}{r_1 + R} = \frac{2}{1+3} = \frac{2}{4} = 0{,}5 A}$