Negative Natural Properties, Categories, and Mistakes (forthcoming in *Synthese*) *

Thiago Xavier de Melo

Abstract

I develop a theory of naturalness according to which properties are natural relative to categories of objects. This new theory avoids a problem that, I argue, afflicts standard theories: whereas standard theories allow for only one ordering of properties, considerations of the similarity and dissimilarity made by some negative properties—like *having no parts*, *having no members*, *having no determinate mass*, *reflecting no visible light*—require admitting more than one. Additionally, I argue that my view accounts for genuine categories, category mistakes, and negation as privation, and that it accounts for a distinction between characterizing and non-characterizing fundamentality.

Keywords: naturalness, categories, category mistakes, negative properties, relations, properties, relational properties.

1 Introduction

Assume an *abundant* and *intensionalist* conception of properties: for almost any possible n-adic predicate, there is a corresponding n-adic property, and no two of these properties are necessarily co-instantiated.[1] Now, assume *ontological egalitarianism*: all properties exist equally. David Lewis (1983) argued that a lot—including similarity and dissimilarity among objects—could be explained if we assume that properties are ordered by *naturalness*.[2]

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[1]Predicates leading to paradoxes motivate 'almost'. Lewis (1983: 346, 350; 1986: 59-60) identified n-adic properties with standard, paradox-free sets of n-tuples of possibilia. Note that I don't make this identification. (See fn. 28 for more.)

[2]Later, Lewis (2008) preferred 'fundamental' to 'natural'. I use 'natural' while emphasizing that the ordering is nomologically *and* metaphysically rigid. Cf. §2.1 and (Lewis 1986: Ch. 1, n. 44; 2008: n. 2) For a list of

In this paper, I develop and defend a new theory of naturalness. According to this view, *category-relativism*, naturalness is relative to categories of objects. Thus, *being circular* might be natural relative to concrete objects without being natural relative to numbers. I argue that this view is preferable to the standard, category-absolutist (i.e. not category-relativist) view because it offers a better explanation of the *similarity* and *dissimilarity* made by some negative properties; it can generate *multiple orderings* of properties; it can account for *genuine categories*, *category mistakes*, and *negation as privation*; and it can account for a notion of *characterization* and for a distinction between *characterizing* and *non-characterizing fundamentality.*

I start, in §2, by arguing that the standard view doesn't have resources to explain why negative totality properties—properties like *having no parts*, *having no members*, *having no determinate mass*, *reflecting no visible light*—are relevant to the similarity and dissimilarity of some objects but not of others. For example, *reflecting no visible light* is relevant to the similarity and dissimilarity among concrete objects but not among numbers. In §3, I consider four tempting reactions to the problem—postulating a *plurality* of perfect naturalnesses, appealing to a notion of negation as *privation*, appealing to *genuine categories*, and appealing to a notion of *category mistakes*—and I argue that category-relativism is preferable to these reactions. According to this view, it might be that *reflecting no visible light* is natural relative to concrete objects but fails to be natural relative to numbers. I argue that category-relativism solves the problem from negative totality properties, it accounts for notions employed in the other reactions, and it doesn't have the limits that those reactions have. In §4, I develop in more detail a version of category-relativism, a category-relativist account of the problem above, and category-relativist analyses of the notions that figure in the four tempting reactions above.[3]

2 A problem for standard theories

2.1 A Lewisian Standard Theory

Theories of *naturalness* describe an order on properties through three notions. Some properties are *more natural than* others. Some properties are *perfectly natural*—thus, at least as natural as

purported explanatory jobs, see Lewis (1983); for extensions thereof see (Sider 2011; Wildman 2013; Dorr 2019; Bradley 2020), and for an anticipation, (Quinton 1957).

[3]A suggestion by van Roojen (2006: 180-1) and a view by Haug (2011) resemble my category-relativism but differ in crucial ways, such as by making naturalness relative to non-objective structures—namely, scientific disciplines and explanatory goals. McDaniel's (2017) account of category mistakes relies on a notion of 'logical form' which, he suggests (2017: 132), is a generalization of the notion of naturalness. Though McDaniel doesn't develop this further, my notion of naturalness might be what he needs in that context. I defend position-relativism in de Melo (forthcoming) on independent grounds.

any other. And, among the ones that fail to be perfectly natural, many properties are natural to some positive degree, whereas many others don't make the *cutoff*—they are 'not at all natural', they are completely, 'perfectly unnatural'.[4]

According to *standard* theories, all naturalness facts are necessary. For example, if p is perfectly natural, it's necessarily so. And, more generally, naturalness facts aren't relational: if a property is perfectly natural, it's perfectly natural *simpliciter*—not relative to a world, and not relative to a category, object, context, time, etc. Likewise for comparative and cutoff naturalness.[5]

According to *Lewisianism*, perfect naturalness is *primitive*, or at least it is *prior* to comparative and cutoff naturalness.[6] Comparative naturalness is analyzed in terms of 'simpler definition' out of the perfectly natural properties, and the cutoff between some naturalness and complete unnaturalness is analyzed in terms of 'not-too-complicated definition' out of the perfectly natural.[7] We can flesh this out as follows.

Let $\mathscr{L}$ be a *fundamental language*: one with primitive predicates standing for exactly all perfectly natural properties and relations and with only privileged logical connectives. Other properties and relations can be denoted by logically complex predicates formed with a lambda operator. Given a formula ϕ, $\ulcorner \lambda v_1 \dots v_n \phi \urcorner$ is an n-adic, non-primitive predicate in $\mathscr{L}$ that denotes the n-adic property p, where p is such that, necessarily, it is instantiated by objects $o_1 \dots o_n$ in this order iff ϕ is true under an assignment of variables $v_1 \dots v_n$ respectively to $o_1 \dots o_n$. A *fundamental definition* of a property p in $\mathscr{L}$ is any $\mathscr{L}$-predicate standing for p. A property p is *more natural than* q iff some fundamental definition of p in $\mathscr{L}$ is (strictly) c-simpler than any fundamental definition of q in $\mathscr{L}$, for any appropriate measurement c, and p is not completely unnatural. Finally, p is completely unnatural iff it has only excessively complex fundamental definitions (i.e., definitions that are *too* complex). I assume that infinite length is sufficient for being excessively complex.[8]

[4] See (Lewis 1983: 347, 376; 1986: 61; 2008: 204-5; 2020: 525-6, 784); and Nolan (2005: 24).

[5] See Lewis (1986: Ch.1, n.44). Contrast Cameron (2010: 284), Taylor (2016: §IV, spec. n.27), Dunn (1997: 492-3). See Brown and Wildman (2022) for a defense of the necessity of naturalness.

[6] Contrast Hawthorne (2007: 434).

[7] See (Lewis 1983: 347; 1986: 61) This is criticized by Williams (2007) and Hawthorne (2007). See Sider (2011: §7.11) for a defense. Category-relativism promises another defense; see fn. 41.

[8] Cp. Lewis (1986: 61) and Nolan (2005: 24). In my usage, a definition might be more complex than another and yet both be excessively complex. Lewis (1983: 368, 372) suggests that properties lacking *finitely* complex fundamental definitions are completely unnatural. (See also Lewis 1986: 108) See Sider (1995: 364; 2011: 8, 130) for the idea of formulating the definitional conception along these lines. The notion of 'privileged' connectives isn't explicit in Lewis but seems needed to develop the analysis, and is in harmony with inegalitarianism concerning properties. Cf. Williams (2007: n.31), Dorr and Hawthorne (2013: 19).

Which connectives are privileged? Let's start assuming only the ones from classical first-order logic. In particular, classical sentential negation ($\neg$) is the only negation privileged.[9] (I argue against an alternative in §§3.1.) 'Privileged' in which sense? I take it as primitive. But maybe perfect naturalness applies to connectives too.[10]

2.2 Naturalness and Dissimilarity

Let us divide an analysis of dissimilarity in terms of naturalness into two parts. The formal part is the determination of the dissimilarity between two objects in terms of some, call them 'relevant', properties they have—possibly, along with properties of these properties. The material part determines which properties are those, the relevant ones—and possibly properties thereof—in terms of naturalness. My argument against standard theories concerns the second part, but for concreteness, I assume the following as a toy account of the first part.

Jaccard Account Let X be the set of all relevant properties of x; likewise for Y. The dissimilarity between objects x and y is the number of properties in the symmetric difference of X and Y divided by the number of properties in the union of X and Y. The similarity between x and y is one minus their dissimilarity.[11]

This account comes out of the Jaccard (1912) coefficient and the assumption that the dissimilarity between objects is fully determined by the totality of relevant properties they have. Among other advantages, it entails all pseudometric space axioms.[12]

Admittedly, this account has various shortcomings, but as I will indicate throughout the text, these won't be pertinent in our context, and known alternatives add unnecessary complications without being significantly better as well as lead to analogous problems for standard theories anyway.[13]

Given the Jaccard account, we could sometimes restrict or stipulate the domain of properties "relevant" in a given context, thus restricting or stipulating notions of "dissimilarity" and "similarity" accordingly. Call such notions *contextually dependent* and properties relevant in this sense, *contextually relevant*. This is *not* our subject-matter. Naturalness must analyze

[9] Cf. Sider (2011: §10.6).
[10] Cf. Sider's (2011) 'structure'.
[11] That is, $d(x, y) = |(X \setminus Y) \cup (Y \setminus X)| \div |X \cup Y|$. Equivalently, the similarity of x and y is $|X \cap Y| \div |X \cup Y|$.
[12] The axioms are: $d(x, x) = 0$; $d(x, y) = d(y, x)$ (symmetry); $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality). Cf. Gilbert (1972) for the latter.
[13] See subsequent paragraphs, fn. 16, 19, 20, and §2.4.

objective dissimilarity.[14] Thus, hereafter 'relevant' means 'relevant for an account of *objective* dissimilarity'.

Moreover, 'relevance' isn't graded; a property is relevant to an object or not. Alternative formal accounts postulate *weights* for relevant properties, which degrees of naturalness might explain.[15] But, even though attractive, this is an unnecessary complication here, as shown in §2.4.[16]

2.3 The problematic phenomena

Which properties are relevant? Standard theorists cannot deliver the right answers concerning properties such as *having no parts*, *having no members*, *having no determinate mass*, and *reflecting no visible light*. I assume that these properties are, respectively, atomhood, emptiness, zero mass, and (ignoring other chromatic variations) blackness, and more importantly, that they are denoted by predicates of the form $\ulcorner \lambda y \forall x \neg \phi xy \urcorner$, where ϕ is a binary predicate denoting a (maybe less-than-perfectly) natural relation. Call them *negative totality properties.*[17]

What makes these properties problematic for standard theorists is that they are relevant to some but not all of their instances. Emptiness must be relevant to the empty set but not to people, though had by both, because emptiness characterizes the empty set but not people, in the sense that it makes for similarities and dissimilarities among sets but not among people.[18] Likewise, atomhood, zero mass, and blackness are relevant to concreta but not to numbers because they characterize concrete mereological atoms, photons, and black objects, but they are merely instantiated by numbers. Below, I will develop this argument in more detail. In §2.4, I argue that standard theorists can't account for the pertinent phenomena.

In order to better assess whether such properties characterize an object, or are had by but don't characterize it, we can compare situations in which they don't even exist—to be had or not had—with situations where they do. (Below, I explain why the situation where they don't exist is pertinent, even if impossible.) We then ask whether objects become more similar to

[14]Cf. Lewis (1984: 228)

[15]Cf. Dorr (2019: §4.1).

[16]Other shortcomings of the Jaccard account won't be pertinent either. For example, this account doesn't apply to objects with infinitely many relevant properties and it doesn't account for dissimilarity *as* distance in a (pseudo)metric space (cf. Blumson 2018). But the objects in my arguments will not have that many relevant properties and the most a property-based account can hope for is to entail the pseudometric axioms, which the Jaccard account does. See also fn. 20.

[17]Cases from colorlessness, understood as *having no color* (Sommers 1971), and *succeeding no natural number* can be made too.

[18]I further clarify 'characterization' below by contrasting it with 'relevance' and provide an account of it in §4.2.

each other or not from one situation to the other, as follows.

Imagine a situation s_0 where shape and flavor properties exist but not reflection properties. By this, I mean not only that reflection properties aren't instantiated, but also that they aren't there (not even to be referred); objects aren't red or not red, black or not black. We can imagine God has yet to create these properties. (To keep things simple, let's understand 'visible' in absolute terms and ignore other chromatic variations; for example, suppose objects are all opaque.) Now, compare s_0 with a situation s_1, just like s_0, but in which reflection properties are there; God creates reflection, paints objects, and switches lights on. Suppose that in s_0 objects a and b are somewhat dissimilar; they share shape but not flavor. If a and b now reflect red light in s_1, their dissimilarity clearly decreases; analogously, if they now reflect no visible light, their dissimilarity must decrease too. By contrast, from s_0 to s_1, the numbers two and three don't become less dissimilar to each other; yet, they share *reflecting no visible light*.[19]

Let $d^s(x, y)$ denote the dissimilarity between x and y in s. Let c and d denote two and three, and situations s_0 and s_1 be as described above. We can schematically represent the expected result as follows:

Discrepancy $d^{s_0}(a, b)$ is greater than $d^{s_1}(a, b)$, but $d^{s_0}(c, d)$ is the same as $d^{s_1}(c, d)$.

This is the basic phenomenon to be accounted for.

Now, *Discrepancy* does not yet say that *reflecting no visible light*, or blackness, is sometimes relevant, sometimes not. In particular, a property might characterize an object—and thus 'make for similarities', in the weak sense above, among it and other objects—without being relevant to it. For example, *being red and round* characterizes its instances, but it is plausibly irrelevant to them because it is "redundant" on *being red* and *being round*, in that, if, on top of *being red* and *being round*, we also counted *being red and round* as relevant to its instances, we would be double-counting. In §2.4, inter alia, I argue that, although standard theorists can account for a pertinent notion of redundancy here, *reflecting no visible light* is not redundant on further relevant properties of numbers; and likewise for other cases. Meanwhile, given the Jaccard Account, if we admit the relevance of this property to a and b, we explain the decrease of dissimilarity between these objects. And yet, blackness can't be relevant to c and d because they gain it in s_1 without a decrease of dissimilarity between them. Therefore, where p is blackness, consider:

[19]Throughout, I will ignore similarity judgments based on proximities between light spectra, colors, and mass quantities themselves. (Cf. Dorr 2019: §4.1)

Inconstancy p is sometimes relevant, sometimes irrelevant.

Inconstancy—or better, a more specific version of it that specifies when p is relevant—would explain *Discrepancy* transferring the burden to the theory of naturalness to account for how a property can be sometimes relevant, sometimes irrelevant.[20]

Before considering other cases that seem to instantiate *Discrepancy* and *Inconstancy*, I will address six initial objections to the argument above. (In §2.4, I consider further ways in which standard theorists could systematically object to, or accept but analyze, instances of *Discrepancy* or *Inconstancy*.)

First, one might object that, since properties exist necessarily, situation s_0 is metaphysically impossible. However, that this situation is impossible is not important for what the argument aims to establish. In a property-based account, distinct properties make independent contributions to the dissimilarity between objects. Situations s_0 and s_1 represent stages in the calculation of dissimilarity between objects a and b. Imagining these stages as separate situations serves a mere heuristic purpose; it aims to show which results are expected concerning each distinct property. Thus, it's enough that s_0 is theoretically conceivable. Later, in §2.4, we will consider whether the properties added in s_1 are distinct from the ones in s_0 in a pertinent sense, or if there is a relation between new and old properties that could account for these judgments. But the mere impossibility of s_0 isn't important.

Second, one might object to the assumption that *reflecting no visible* light is blackness.[21] But this assumption is not crucial to my argument; indeed, it is most likely false because of other chromatic variations (mentioned above). What is crucial is that at least one negative totality property makes for similarity among concrete objects just as *being red* (or *reflecting red light*) does. Moreover, suppose instead that blackness is a "conjunctive" property like *being concrete and reflecting no visible light* (still ignoring other chromatic variations). Even then, the negative totality property—by hypothesis, *reflecting no visible light*—must make for similarities. For the "conjunctive" property is plausibly a *conjunctive* property, in that its simplest fundamental definition out of the perfectly natural properties and relations is a conjunction of the simplest fundamental definition of *being concrete* and of *reflecting no visible light*. If so, then, this

[20]The Jaccard account explains Discrepancy by considering the objects' *total* number of relevant properties as a divisor. (See fn. 11) Other toy analyses of dissimilarity, like Dorr's (2019: §4.1) 'Division', don't take this into account. But, first, Blumson (2018: p. 32) has convincingly argued against such analyses, and second, the same intuitions that support the argument from the Jaccard account also support analogous arguments from those analyses.

[21]Many thanks to an anonymous referee for helpful discussion on this and the following objections.

conjunctive property must characterize its instances only to the extent that their conjuncts do, and thus, at best, the conjuncts are relevant to their instances, not the conjunctive property itself. Thus, the negative totality property must make for similarities among concrete objects. (See §2.4 for more on this.) Finally, note that the analogue of this objection for the other negative totality properties that I will consider—e.g. the denial of the assumption that emptiness is *having no members*—is highly implausible.

Third, one might observe that 'the number two is black' is deeply infelicitous, a category mistake. On this basis, one could argue, again, against the assumption that blackness is *reflecting no visible light*. But the three points made in the last paragraph will be pertinent here too. Moreover, as I will argue in §4.4, an alternative explanation for this kind of infelicity is available, namely, that the predicate 'is black' denotes a negative totality property—like *reflecting no visible light*—but that saying 'the number two is black' *presupposes* something false—as, for example, that the object is characterizable by the property. More interestingly, based on that linguistic infelicity and on the assumption that blackness is *reflecting no visible light* (or another negative totality property), one might argue that the attribution of this property to the number two is a category mistake and that this *explains* why this property fails to make for similarity among numbers. Indeed, I think that this reaction is onto something. But the phenomenon here cannot be merely linguistic if it is to help in an account of *objective* similarity. This linguistic phenomenon must be tracking an objective, *real* category mistake that makes *reflecting no visible light* irrelevant for the objective similarity among numbers. But, as I argue in §3.1, to develop this explanation we must reach outside the resources of the standard view, and, even then, the resulting account isn't as systematic as the naturalness-based account that I offer in §4.4 in terms of non-standard, category-relative naturalness.

Fourth, one might argue against the intuition that *reflecting no visible light* doesn't make for similarities among numbers. But it would be question-begging to do so on the basis of the claim that, because this property is natural (full stop), it must make for similarities among all its instances, among which are numbers. For such an argument would presuppose the standard view.[22] More promisingly, one might argue that the intuition is in tension with a second

[22]To clarify, what is question-begging is the objection that simply assumes, for example, that *reflecting no visible light* must be natural in the standard way if natural at all, and that rejects the opposing intuition and the force of the argument from negative totality properties on this basis. Contrast this with an argument that gives proper consideration to alternative views but that says that, *despite* the standard theory's counterintuitiveness regarding negative totalities, we have better overall reason to conclude that the standard view is true because it better accommodates other intuitive judgments, and that, based on this, concludes that the contents of the intuitions regarding negative totalities are false. This argument would not be question-begging. For what it

intuition, namely, that the fact that my mug but not the number two reflects visible light contributes to the degree of dissimilarity between my mug and the number two. Indeed, *if* this second intuition could *only* be accounted for by admitting the theoretical claim that *reflecting no visible light* is relevant to the number two (and not had by my mug), then, by analogous reasoning, this property would also be relevant to the number three, and therefore, it would have to make for similarities between the numbers two and three. However, the second intuition can also be accounted for by an alternative theoretical claim, namely, that *reflecting visible light* is relevant to my mug (and not had by the number two), and this claim is not in tension with the intuition that *reflecting no visible light* doesn't make for similarities among numbers. Therefore, by holding this alternative theoretical claim, both intuitions can be admitted and accounted for, and thus, the second intuition doesn't support the denial of the first intuition.

Fifth, one might, perhaps even without argument, simply fail to share the intuition that *reflecting no visible light* doesn't make for similarities among numbers. However, first, I will provide further cases. Defenders of the standard view must deny the pertinent intuitions in *all* those cases, and they must do so without being implicitly driven by the standard view itself. Second, that intuition is particularly strong under multicategorial views, such as that numbers are abstract, Platonic objects whose nature is fundamentally different from the nature of concrete objects (and under analogous views for the other cases we will see). Given such a view, it looks like numbers are just not the sort of thing that is characterized by properties like *reflecting no visible light*.[23] And we cannot exclude from the outset (based just on the theory of naturalness itself) that reality is multicategorial in this way, and thus that some negative totality properties make for similarities among objects of just one sort. But, as I will argue, it is unclear how the standard view itself could accommodate this even as a hypothesis.[24]

Sixth, one might object that the phenomenon is not a metaphysical one, offering instead a debunking, non-metaphysical explanation for why some people, like me, find it unintuitive that *reflecting no visible light* contributes to the similarity between the numbers two and three. My

is worth, I see no pertinent intuition the standard view can accommodate that the category-relativist theory that I will later develop can't accommodate just as well when it is conjoined with the claim that there is only one genuine category. See fn. 24 for more. Moreover, starting in §4, I argue that category-relativism not only accommodates intuitive similarity judgments involving negative totalities but also has other advantages.

[23]Indeed, it should not come as a surprise if Lewis's standard view lacks the resources, as I will argue it does, to account for dissimilarity among objects under multicategorial views of reality given Lewis's commitments to, or hopes of holding, a largely monocategorial view of reality.

[24]By contrast, the category-relativist theory that I will later develop and defend can accommodate all substantive, "first-order" views that the standard one can because, as far as the theory itself goes, it might be that there is only one genuine category of objects, the category of everything.

general response to this sort of objection is that its most cogent versions threaten equally well intuitions that motivate the standard view in the first place. I will support this by considering a specific proposal. The proposed explanation is that, in most contexts, if people understand the words 'two' and 'three' at all, they know that these words refer to things that don't reflect visible light; thus, saying 'two and three reflect no visible light' or 'two and three instantiate *reflecting no visible light*', the explanation goes, is bound to be uninformative, thus violating some pragmatic principle, and this is the reason why it sounds odd to claim that *reflecting no visible light* contributes to the similarity between two and three.

However, this proposal fails to explain why the *last* claim, which involves 'similarity', expresses something unintuitive. That is, what is supposed to express something unintuitive is not '*reflecting no visible light* is instantiated by two and three', but '*reflecting no visible light* contributes to the similarity between two and three', and it is unclear that uttering *this* would violate a pragmatic principle in virtue of the proposition it expresses being known (or presumed to be true) in most contexts. It is not at all clear that we know (or presume) this. Quite the opposite. I would think that we ordinarily presume that this is false; if anything, I would expect its negation to violate pragmatic norms sometimes.

But suppose that the explanation above targeted the right claim, or that another non-metaphysical, debunking explanation is available. The defender of the standard view willing to embrace such an explanation must give a principled reason why the same sort of explanation doesn't overgeneralize—thus debunking intuitions that give traction to the standard view in the first place. For example, in many contexts, given what we know, uttering 'the emeralds are green' violates some pragmatic norms—e.g. in virtue of it being uninformative—and uttering 'the emeralds are grue' violates none—where something is grue iff green and first observed before 2030 or otherwise blue. Still, intuitively, *being green* makes for more similarities than *being grue*—at least under the assumption that 2030 isn't metaphysically special. Standard theorists have been taking this sort of intuition seriously. They take such intuitions to indicate a metaphysical phenomenon that supports a theory of naturalness in virtue of it being explainable by this theory. In general, skeptical, debunking arguments can be deployed against intuitions that the standard view takes seriously. My argument against the standard view continues taking intuitions of that same sort just as seriously. Numbers are just not the sort of thing that *reflecting no visible light* characterizes—at least not under the assumption that the division between abstract and concrete objects is metaphysically special.

Let us now consider other cases that seem to instantiate *Discrepancy* and *Inconstancy*. They involve atomhood, zero mass, and emptiness.

Imagine a situation s_0 where parthood doesn't exist, or where there is no mass-assignment, and where objects a and b are somewhat dissimilar to each other. When God creates parthood, or mass-assignment, and makes a and b atomic, or massless, their dissimilarity decreases, just like the dissimilarity between two concrete objects sharing a part, or having the same positive mass, does. But the dissimilarity between two and three surely stays the same even though two and three have no parts and no determinate masses. Since the only changes involve *having no parts* and *having no determinate mass*, these properties must be relevant to a and b, but not to two and three.

Now, in standard set theory (supplemented with intuitive modal claims[25]), there can't be two empty sets. But note that, as far as set-membership and emptiness themselves go, it is an accident that there necessarily is only one empty set. It is sethood, in particular the criterion of identity associated with it, that is responsible for this limitation. Alternative set theories can allow for many empty sets while preserving everything else of interest. For example, we can imagine sets just like the standard ones except that they have vague borders. In this theory, we could have many empty sets which are nevertheless different because their vague borders are different. Surely, sharing emptiness makes for similarity among these empty sets, but not among Socrates and Plato.[26]

To develop this, let us postulate two primitive, perfectly natural set-membership relations: the standard, determinate one, and a new, indeterminate one. Suppose that in s_0 sets a and b have only vague borders established; God created indeterminate set-membership only. Suppose that a and b are somewhat dissimilar in s_0: one and three are indeterminate-members of a and two and three are indeterminate-members of b. In s_1, God creates standard set-membership and nothing is a standard-member of a or b. Their dissimilarity decreases just like the dissimilarity of sets that share all standard-members decreases. If so, the relevance of emptiness to them explains why. By contrast, the dissimilarity between persons, or numbers, c and d—say, Socrates and Plato, or, aside numbers-as-sets views, two and three—doesn't change when God annihilates or creates emptiness, even though they share emptiness in s_1.[27]

[25] See e.g. Menzel (2018)

[26] We find sets of this sort in Woodruff and Parsons (1999). But the most straightforward way of identifying standard set-membership and emptiness across theories is to start off with two set-memberships in the alternative theory, as I do next.

[27] Some theories other than ZFC won't deliver the same results. Prominently, Quine (1980: 30-2) takes set-

I don't think this is the best theory of vague sets. The point here is just that emptiness would be relevant to such objects. Now, even though the relevance of a property to objects of one sort does *not* entail its relevance to objects of another, the vague sets above are *just like* standard ones except for indeterminate-set-membership. Thus, by analogy, emptiness must be relevant to the standard empty set too.[28]

2.4 Standard theories can't account for Discrepancy

Can standard theorists account for *Discrepancy*? That is, can they explain why, despite appearances, it isn't true; or, can they explain it without *Inconstancy*; or, can they accept *Inconstancy* but then analyze the latter in their own terms?[29]

Generally, standard theories can analyze irrelevance in terms of cutoff naturalness if they embrace this:

No Relevance without Naturalness If a property is relevant to an object, it is natural to some degree, that is, it is not completely unnatural.

And, of course, there is good reason to embrace this. For, besides logical features, the cutoff between some naturalness and complete unnaturalness is the *least* we can hold on to if we don't want to count infinitely many abundant properties as relevant to any objects. Moreover, our very grasp of the notion of naturalness, as in Lewis, ties it to dissimilarity.[30]

However, if naturalness isn't itself relative to objects, or categories thereof, naturalness theorists can't explain, for example, that emptiness is relevant to sets but not to people because it is *natural* relative to sets but not relative to people. Standard theorists must pick a side; each property above is either natural to some degree or completely unnatural. However, whichever side they pick, they can't offer an account of *Discrepancy*—not by denying or analyzing it.[31]

Before presenting this dilemma, note that alternative formal accounts—which would allow us to accept *Discrepancy* without *Inconstancy*—would leave us with comparable options. For

membership to relate individuals to themselves. See fn. 40.

[28]One can't directly translate these arguments into a properties-as-sets, or -classes, view, as Lewis', since we wouldn't be able to conceive of properties without sets, and since set-theoretic properties and relations won't have straightforward sets we can identify them with in ZFC. This is partially the reason why I didn't assume properties-as-sets from the beginning. I insist on the case from emptiness, however, since interesting consequences follow from the study of set-membership within the theory of naturalness. Incidentally, even Lewis (1986: Ch. 1, n. 47) acknowledged that set-membership might be perfectly natural, notwithstanding his view on properties.

[29]I use 'account' in Lewis' (1983: 352) sense, which includes 'denial'. Also, since the project is to *analyze* dissimilarity, standard theorists can't take discrepancies as primitive.

[30]Dorr and Hawthorne (Cp. 2013: 22) on 'joint-carving'.

[31]See fn. 29.

example, given *Discrepancy*, we could try to attribute to emptiness positive but less-than-full weight relative to sets but zero weight relative to people. But then, naturalness theorists don't have to explain *Inconstancy* (of relevance) but must explain inconstancy *of weights.* But if degrees of naturalness determine weights, emptiness will sometimes be natural to some degree, sometimes natural to a lesser degree, contra the standard view. Thus, hereafter, I leave these alternatives aside.

2.4.1 p is unnatural

Suppose p is completely unnatural, where p is atomhood, blackness, or zero mass. If so, given *No Relevance without Naturalness*, p is not relevant; hence, there is no decrease of dissimilarity between two and three from s_0 to s_1. The question is, what about concreta, a and b? How can standard theorists account for the decrease of dissimilarity between them?

Since they, by hypothesis, deny relevance to those properties and since there aren't other shared properties they can blame in the various transitions from s_0 to s_1, they must deny *Discrepancy* by denying that concreta a and b become less dissimilar when they share those properties.

They could insist, for example, that only *reflecting red light*, or *reflecting some visible light*, are relevant. Suppose that, in s_1, a and b are instead *divided* by blackness; a is red, b perfectly black. Their dissimilarity *increases.* The standard theorist could say that only redness is relevant and that this fully explains the total increase. Now, when a and b *share* blackness in s_1, standard theorists could admit that their dissimilarity *seems* to decrease but explain away this appearance, denying a real decrease. What happens, the explanation goes, is that, when God creates reflection properties, everything else which reflects visible light becomes more distant in the space of dissimilarities from the ones which don't. The dissimilarity between a and b decreases *relative to* some other dissimilarities but *not* in absolute terms. In other words, there is an *illusion* of decrease of that distance created by an expansion of the occupied space of dissimilarities.

I claim that when blackness *divides* a and b, the relevance of redness *partially* explains the increase in dissimilarity. But it doesn't *fully* explain it; it doesn't explain its total amount. For, first, the increase must be as great as when a turns red and b blue. Second, and more importantly, blackness must be relevant in the case where it divides a and b because it is relevant when *shared* by them. So, can the latter be explained away as an illusion?

If the explanation for the illusion were complete, then, when God switches lights on, it should at least *seem* like two and three become less dissimilar to each other as well. But it doesn't. Even though every other object that reflects visible light would become farther away in the dissimilarity space from these numbers, *reflecting no visible light* doesn't even seem to make any contribution to the similarity between two and three as it does for concrete objects. Thus, as it is, the explanation why *Discrepancy* merely seems to be the case isn't satisfactory.

An attractive way to complete the explanation is to say that the dissimilarity between concrete objects a and b decreases *relative to* some other *concrete* object's dissimilarities, and that this relation does not hold between dissimilarities among concrete objects and dissimilarities among *numbers*. In other words, the illusion afflicts distances between two objects only if the space occupied by *like* objects expands. But to develop these ideas we need to go beyond the notions of the standard theory, as I consider in §3.1.

Similar remarks can be made concerning the other properties. One could object that we don't need *Inconstancy* because *Discrepancy* is illusory: only properties of the form *having* α *as part*, or *having* α *as member*, or *having* α *as determinate mass*, are relevant. However, we must take the alleged illusion seriously enough to explain it away, which requires giving some metaphysical account of it, even if not in terms of a decrease of dissimilarity. And again, the explanation above can't do this.

Finally, there are independent reasons to deny that emptiness, atomhood, blackness, and zero mass are completely unnatural. First, some objects might have these properties *essentially*—e.g., the empty set, some atoms, black holes, and photons—and all essential properties are natural to some degree.[32] Second, assuming that ϕ has a less-than-excessively complex fundamental definition, standard theorists must either abandon the definitional conception or explain why the quantifier in $\ulcorner \lambda y \forall x \neg \phi xy \urcorner$ makes for excessive complexity.[33]

2.4.2 p is natural

Suppose instead that emptiness, atomhood, blackness, and zero mass, *are* natural to some degree. If so, standard theorists are free to hold that these properties are relevant. But the problem now is to explain why there is *no* decrease in the dissimilarity between two and three

[32] Wildman (2013: 764) argues for this in detail. There is controversy, however. Whereas Wildman's reasoning seems sound, Skiles (2015) offered clear counter-examples. In de Melo (2019) I show that this problem can also be solved by category-relativism.

[33] Cf. Balashov (1999) for further reasons to take zero-value quantities as natural (though without sharing our assumption about their 'negative totality' definition).

when God creates atomhood, blackness, and zero mass, and between Plato and Socrates when God creates emptiness.

Sure, there is naturalness without relevance. Plausibly, red-and-roundness itself detracts nothing from the dissimilarity of objects x and y when we move from a situation where there is no such property to a situation where they share this property. What really detracts, if anything, are the properties of redness and of roundness—that is, provided that these were not in situation zero. If redness and roundness are relevant, red-and-roundness is not relevant, despite its naturalness. Mutatis mutandis for red-or-roundness.

Now, recall that naturalness theorists must analyze relevance. Thus, they must account for the irrelevance of red-and-roundness and the like. However, whereas standard theorists can explain why these are irrelevant, this explanation won't help them concerning emptiness, blackness, atomhood, and zero mass.

An attractive explanation for the case involving red-and-roundness is that this property logically includes, or is logically redundant on, redness and roundness, and thus is innocent, adding nothing to the dissimilarity of objects having it if redness and roundness are already relevant. Naturalness can help flesh this out. We can say that the simplest definition of red-and-roundness shows it to be logically related to redness and roundness in a way that guarantees its irrelevance conditional on the relevance of the latter.

All properties whose simplest definitions are conjunctions, or disjunctions, of relevant ones are irrelevant. And so are conjunctions, or disjunctions, of conjunctions, or of disjunctions, of relevant properties. And so on. Generalizing, given some operation η defined in terms of conjunction and disjunction, and given relevant properties for which $\psi_1 \dots \psi_n$ stand, p is irrelevant to an object x if its simplest definition $\eta(\psi_1 \dots \psi_n)$ is such that there are $\psi_i \dots \psi_j$ among $\psi_1 \dots \psi_n$ and x's satisfaction of $\psi_i \dots \psi_j$ logically entails x's satisfaction of $\eta(\psi_1 \dots \psi_n)$.

Now, if *reflecting no visible light* is irrelevant to numbers in the way that red-and-roundness is irrelevant to red and round objects, its simplest definition must guarantee that, because numbers instantiate other properties, they must instantiate it. Likewise for atomhood and zero mass. But the simplest definitions of these properties don't include predicates for numberhood or for non-concreteness, for example. As far as the simplest definitions of these properties go, it is logically possible for non-concrete numbers to instantiate them or not. Analogously, the simplest definition of emptiness includes no reference to non-sethood, or personhood, for example.

Moreover, whereas my condition for irrelevance above contemplated conjunction and disjunction only, a similar condition that would seem to help us with negative totality properties can't be defended on comparable grounds. For example, since those properties are denoted by predicates of the form $\ulcorner\lambda y\forall x\neg\phi xy\urcorner$, no collection of substitutional instances of such universally quantified predicates—e.g. denoting *not reflecting red*, *not having my desk as part*, or *not having mass one*—would be such that their satisfaction logically entail the simplest definitions of the negative totality properties. Moreover, the properties just listed such as *not reflecting red*, for example, aren't relevant to concreta, and, even if they were, they aren't relevant to numbers. Mutatis mutandis for emptiness and the property of *not having Socrates as a member*.

Finally, a more general lesson can be learned from the way in which the explanation above fails to help us with *Inconstancy*, which is that we should not expect standard theorists to be capable of analyzing *Inconstancy* in *any* way. For the properties in question are relevant or irrelevant depending on the objects, and more specifically, on the *non-logical* features or *categories* of the objects that have them. Clearly, logical properties and relations are of no help to capture this. But neither is standard naturalness given that perfectly natural properties are freely combinable and that simplest definitions of less-than-perfectly natural properties of interest—such as emptiness and personhood—are unlikely to show logical connections between them.

Since the standard theory itself doesn't provide more, we need to appeal to notions that either give up on classical logic or are outside the naturalness theorist's standard tool box.

2.5 Disharmonious Quartets

I articulated the problem above as an explanatory challenge that, alone, the standard theory lacks resources to meet. The problem can be stated in more dramatic terms. Let's suppose that standard theorists attempt to establish sufficient conditions for relevance through a notion of logical inclusion or redundancy, as follows.

No Non-redundant Naturalness without Relevance If a property is natural to some positive degree, then, if it is not redundant on further relevant properties of an object that has it, then it is relevant to that object.

The problem is that, collectively, this claim and *No Relevance without Naturalness* are incompatible, for some p, with:

p's **Relevance** p is relevant to an object x.

p's **Non-redundant Irrelevance** p is irrelevant to an object y but not redundant on other relevant properties of y.

Emptiness, for example, is relevant to the empty set. But it is irrelevant to Socrates and, arguably, is not redundant on other relevant properties of Socrates. Hence, a contradiction: emptiness is natural to some degree and completely unnatural.

The more, and stronger, instances of *Discrepancy* we have that support *Relevance* and *Non-redundant Irrelevance* of some property, the greater the pressure against the two principles connecting naturalness and relevance. I will develop a new theory of naturalness under which the two connecting principles can be revised in a way that avoids the incompatibility above.

3 Toward a Solution

In §3.1, I consider four tempting reactions to the problem from negative totality properties. I find them appropriate, but I argue that they are limited. Thus, I don't take the notions they appeal to as primitives. Instead, I suggest in §3.2 that revising naturalness is the best reaction because it solves the problem, it explains the other tempting reactions without inheriting their limitations, and it promises to do even more. In §§4, I develop this suggestion in detail.

3.1 Tempting Reactions

Many orders: The problem of negative totality properties suggests many orderings. For example, emptiness might be natural in one ordering but completely unnatural in the other. This suggestion faces at least two obstacles. First, it needs to postulate some *connection* between orderings and objects, such that given that the first ordering above is connected to the empty set, but not to Socrates, emptiness characterizes the set but not Socrates, and mutatis mutandis for the second ordering. But which connection is this? Second, what is it that orders properties in each case? Many primitive naturalnesses—emptiness being natural$_1$ but completely unnatural$_2$—isn't attractive because the case for naturalness, as in Lewis' view, is one of unification—*one* primitive, many explanations—but also, because this wouldn't help to explain the connection between naturalness$_1$ and the empty set but not Socrates, for example. (If the connection is taken as primitive too, it should be possible for the ordering that ranks emptiness as natural to be connected to Socrates.)[34]

Privation: Second, we could distinguish ways of *not* having a property. Perhaps, even though Socrates has no members, meaning that it's not the case that he instantiates *being membered*, it's not the case that Socrates is *not*-membered, meaning that it's not the case that he is *privative* of *being membered*. In its turn, the empty set is not-membered; it's *privative* of *being membered*; it properly *fails* to have it, when, in a sense, it 'could have had' members.[35] What's relevant to the empty set is not that it does not have members but that it is not-membered.

However, what makes it so that the empty set is properly privative of *being membered* if not that it does not instantiate it? One could take each privation fact—expressed via a predicate term negation in the fundamental language—as a fundamental fact distinct from a mere denial—perhaps expressed via classical sentential negation. But, first, this requires an expansion of the standard logic at the fundamental level, which must be compared with my proposal of revising naturalness instead. Second, the facts that support *Inconstancy* aren't so inconstant. There is a correlation between mere negations and proper privations that depends on whether the object pertains to a significant class—e.g. of sets for *being membered*, or of concreta for *having no mass*. Thus, a more systematic, unifying explanation that accounts for this pattern seems possible and more attractive.

Alternatively, one could take as a new primitive the modality (expressed with 'could have had' above) in terms of which I elucidated 'privation'. But analogous points apply here too. First, even if one has a standard metaphysical modality (as a primitive or not) in the fundamental language, the intended modality is different since, in the intended sense, the empty set could have had (or better, 'could have been characterized by having') members, even though in all metaphysically possible worlds the empty set doesn't have members. Second, the phenomenon reveals a pattern that invites systematic unification since all, and only, sets 'could have had' members.

Category Mistakes: Third, one can appeal to the notion of category mistake. Emptiness is irrelevant to Socrates, and not to the empty set, because 'Socrates is empty' is a category mistake, and 'the empty set is empty' is not. However, what is a category mistake, and how does it bear on objective similarity and dissimilarity? One could take as brute that a certain object can't correctly combine with some properties. But again, once we realize that this happens in

[34] Schaffer (2004) proposes a pluralism (See also §3.2). Alternatively, one could adopt a notion of contextual naturalness, as Taylor's (2016), which has drawbacks when accounting for *objective* dissimilarity. Cf. §2.2.

[35] Cf. Sommers (1971: 21). Cf. Horn (2001: §1.1.1) on the Aristotelian 'privation'.

a systematic way—only sets can combine with emptiness—a more unifying account, in terms of significant classes of objects, would be more attractive.[36]

Categories: Fourth, appealing to the notion of category would provide the more systematic and unifying explanation that we couldn't find in terms of 'privation' or 'category mistakes'. The 'significant classes' are genuine categories. Emptiness is irrelevant to Socrates because he doesn't belong to the category of sets.[37] However, if 'genuine category' is primitive, why are some properties, like emptiness, *connected to* some categories and not others, e.g., to sets but not concreta, such that it characterizes sets but not concreta? (Again, if the connection is primitive, it should be possible for categories of non-sets to be connected to emptiness.)[38]

3.2 Category-relativism

Instead, I propose a new theory of naturalness. Properties are *natural relative to* categories. For example, emptiness is natural relative to sets but not relative to other categories; hence, emptiness is relevant to the empty set but not to Socrates. Besides solving the problem from negative totality properties, this also vindicates and explains the intuitions of all tempting reactions above. First, we have *many* orderings, but, with Lewis, *one* property of naturalness. It's just that relative to different categories we have different orderings of properties. Second, when a property that is natural relative to a category is (classically, sententially) negated of an object of that category, this negation is a proper *privation*; if the object doesn't pertain to that category, the negation is a *mere* lack. Third, a category is *genuine* when a property or relation is perfectly natural relative to it. Finally, a *category mistake* is an attribution of a property to an object that doesn't belong to a genuine category relative to which that property is natural.

I develop this theory and these accounts in detail for the remainder of this paper. The case for category-relative naturalness is a cumulative one, from unification, too. Solving the problem from negative totality properties gives this theory some advantage over standard theories. Accounting for the notions in the reactions above gives more, especially if those notions are pertinent in yet further theorizing. Here are three potential examples of the latter.

First, as McDaniel (2017: Ch. 4) suggests, one can hope to have a handle on modality via a strong recombination principle given a notion of category mistake. For this principle would

[36] Cf. also Westerhoff (2002) and criticism by McDaniel (2017: §4.3).

[37] Incidentally, as e.g. McDaniel (2017: 120-1) argues, standard theories can't plausibly equate categories with natural properties and can't offer a definition of categories in terms of naturalness.

[38] Appealing to 'modes of being' fails for similar reasons. McDaniel (2017: 132) appeals instead to a notion of logical form to explain the desired connection.

initially say that something is possible if it's a logically possible recombination of perfectly natural properties and relations. However, suppose that set-membership and *being concrete* are perfectly natural. That concreta have members seems logically but not metaphysically possible.[39] Solution: require category correctness too. That concreta have members is a category mistake.[40]

Second, Lewis (1984) has suggested that the more natural a property is, the easier we can refer to it. Thus, where further linguistic facts don't determine the reference of our predicates, naturalness might help doing so. However, as van Roojen (2006) and Schroeter and Schroeter (2013) suggest, under moral realism, one could hope to find references of moral terms given a notion of 'discipline-relative' (van Roojen 2006: 180-1) or 'multi-polar' naturalness (Schroeter and Schroeter 2013: 18). But the first gives up on the idea that naturalness is objective, and the second postulates a plurality of primitive notions of naturalness. As Schroeter and Schroeter (2013: 18-9) argue, both these aspects threaten the idea that naturalness can avoid indeterminacy. Category-relative naturalness provides objective naturalness, multiple orderings, with a unifying notion.[41]

Finally, with Schaffer (2004), perhaps *believing-that-the-snow-is-white* is natural because it makes for great similarity between minds that supervene on different physical constitutions, but unnatural because it is infinitely away from physical properties we know to be very natural. Category-relativism opens the possibility that there are two orderings, say, one relative to minds, the other to physical fundamentalia.

4 Category-relativism: developed

In this section, I will develop a version of category-relativism (§4.1). I will then apply its concept of category-relative complete unnaturalness to solve the problem from negative totality properties and to account for the alternative reactions to this problem (§§4.2–4.4).

[39](See also Wang 2013: 539-41)

[40]As noted (fn. 27), Quine disagrees, but he disagrees by taking individuals as classes (1980: 32). Otherwise, the contrast would be deeper: set-membership would be relevant for non-classes.

[41]Similar problems concern reference to macro-objects that are definitionally too far away from the perfectly natural microphysical properties. Category-relativism offers a way out: multiple pertinent categories. In this way, it also helps defuse objections against definitional conceptions of comparative naturalness. Cf. Williams (2007) and Hawthorne (2007) For another problem to which category-relativism might offer similar solutions, see Sider (2011: 48) on Hawthorne's Ural mountains case.

4.1 Category-relativism in Many Languages

The standard view takes perfect naturalness to be a property of properties and relations. Category-relativism can take perfect naturalness to be, instead, a non-symmetric relation that properties and relations can bear to categories of objects. Here, I use 'category' in an abundant sense; indeed, a category is just a property. A *genuine category* is a category relative to which a property or relation is perfectly natural. Like the standard view, category-relativism takes category-relative perfect naturalness facts to be metaphysically necessary, contextually independent, unrelated to time, and fundamental. Likewise, it takes category-relative perfect naturalness to be primitive.[42]

In the standard view, a property or relation is natural, as opposed to completely unnatural, iff there is a less-than-excessively complex analysis of it in terms of perfectly natural properties and relations. In §2.1, we made sense of this in terms of a fundamental language in which complex predicates can be formed out of primitive predicates that stand for perfectly natural properties and relations. According to category-relativism, a property or relation is natural, as opposed to completely unnatural, *relative to a category* k iff there is a less-than-excessively complex analysis of it in terms of perfectly natural properties and relations that are natural relative to k. As a first pass, we can imagine *multiple* languages just like that of §2.1, but one for each genuine category.

Let $\mathscr{L}_k$ be a *k-fundamental language*, one with primitive predicates for exactly all properties and relations that are perfectly natural relative to a category k, and with the same logical resources as the standard fundamental language $\mathscr{L}$ of §2.1. Concepts of *fundamental definition*, *comparative naturalness*, and *complete unnaturalness* can be defined just like before, except that they are now relative to k-fundamental languages. In particular, we can define complete unnaturalness as follows.

k-Relative Complete Unnaturalness A property or relation is *completely unnatural* relative to k iff it has only excessively complex $\mathscr{L}_k$-predicates denoting it.

But some fine-tuning is needed. Suppose that the category of sets is genuine. Given that we can't define a term for *Socrates* out of predicates that stand for properties and relations that are perfectly natural relative to sets, the relational property of *having Socrates as a member*

[42] Another version of category-relativism takes perfect naturalness to be a non-distributive plural property of properties and relations in order to make better sense of the independence between perfect naturalness facts. But since the latter has no direct consequences for our current concerns, I will keep working with this simpler, relational version of category-relativism.

will be completely unnatural relative to sets. And yet, *having Socrates as a member* is surely relevant to some sets, and thus, it must be natural relative to sets.

Here I will adopt a simple solution to this problem, which is to add names denoting non-sets, like Socrates, to the *sets*-fundamental language. In general, k-fundamental languages must include names denoting objects of any categories. Likewise, note that the logical resources of k-fundamental languages must include—this time not unlike the standard fundamental language—a quantifier that ranges over things of any categories. For if the *sets*-language did not include a quantifier that ranges over Socrates, for example, emptiness would lack a *sets*-fundamental definition, and thus, it would be completely unnatural relative to sets.[43]

Below, I will show how the account of complete unnaturalness above helps to solve the problem from negative totality properties and to account for the other tempting reactions to this problem.

4.2 Characterizability, Characterization, and Privation

First, we can define the notion of characterization in terms of which I first introduced the problem from negative totality properties back in §2.3 as follows.

Characterization A property p characterizes an object o iff p is had by o and o belongs to a genuine category relative to which p isn't completely unnatural.

That is, for a property to make for similarities among some objects, it must be natural relative to a genuine category to which those objects belong. Likewise, we can account for a notion of privation in terms of which, I agree, it is appropriate to explain, for example, the relevance (for an account of *objective* similarity) of emptiness to the empty set but not to Socrates.

Privation An object o is privative of a property p iff p is not had by o and o belongs to a genuine category relative to which p isn't completely unnatural.

[43] Adding names to the fundamental language may be problematic, inter alia, if perfect naturalness applies to objects (relative to categories); for in this case, we may want the addition of names to represent these facts instead. (Dorr and Hawthorne 2013: n. 30) But that some objects belong to genuine categories may play some of the theoretical role intended by the postulation of perfectly natural objects. Moreover, alternative approaches are possible, such as one whose only fundamental language assigns "sorts" to different predicates (or argument-places of predicates) to represent different genuine categories. Indeed, I think that this alternative is preferable in some contexts, such as when category-relativism is combined with position-relativism—the view that naturalness is attributable to position of relations (on which see de Melo (forthcoming))—and I believe that a fully general account of privation and category mistakes supports this view. (See fn. 49) But since my space here is scarce, and since the 'many languages' approach is simpler to present than others and sufficient for our purposes of developing and defending category-relativism over the standard view, I will keep working with the many languages approach here.

But it will also be useful to introduce the following notion of characterizability.

Characterizability A property p *can characterize* an object o iff o belongs to a genuine category relative to which p isn't completely unnatural.

This accounts for the modality in terms of which I elucidated the intended notion of privation back in §3.1. Indeed, we can account for privation and characterization in terms of characterizability: an object is privative of a property iff it lacks that property when that property could have characterized it; likewise, a property characterizes an object iff it is had by, and it can characterize, that object. As I will show, this modality will also help in accounting for real category mistakes.[44]

Thus, since *having members* is definable out of properties and relations that are perfectly natural relative to sets but not relative to, say, people, *having members* can characterize sets but it cannot characterize beings that are not sets, like people. Thus, some sets, like the singleton {Socrates}, are characterized by *having members* and the empty set is privative of *having members*. Even though the latter property *could* also characterize the empty set, it does not, because, as it happens, the empty set doesn't have this property. Likewise, emptiness can characterize sets, and it does characterize the empty set since this set has it. Meanwhile, since *having members*, and likewise emptiness, cannot characterize non-sets, Socrates isn't privative of *having members* and isn't characterized by emptiness even though he lacks the first and has the second. Likewise, the light-reflection relation, if perfectly natural, must be such relative to, say, concreta, but not relative to non-concrete numbers; hence, *reflecting no visible light* isn't completely unnatural relative to concreta as it is relative to (categories that include) numbers, and thus, even though it can characterize concreta, it can't characterize numbers.

Recall that negation in k-fundamental languages was assumed to be the standard classic sentential operator. A negation of a sentence *contradicts* that sentence—that sentence and its negation can't both be true and can't both be false. But, as intended, 'privation' generates only *contraries*, for sometimes an object doesn't have a monadic property but isn't privative of that property. For example, Socrates neither has members nor is he privative of them.

[44] A notion of characterizability is definable in the standard view too. A completely unnatural property p that I lack would have failed to characterize me even if I had it. I am not privative of it because it is not the sort of property that characterizes objects like me, because (given the standard view) it is not the sort of property that characterizes *any* of its instances.

4.3 Harmonious Relevances for Dissimilarity

The principles connecting relevance for dissimilarity to naturalness can now be stated adequately.

No Relevance without Characterization If a property is relevant to an object, it characterizes that object.

No Non-redundant Characterization without Relevance If a property characterizes an object and is not redundant on further relevant properties of that object, then it is relevant to that object.[45]

Emptiness may be relevant to the empty set and not to Socrates, even if it is not redundant on further relevant properties of Socrates, because, even though emptiness can, and does, characterize the empty set, it doesn't (because it cannot) characterize Socrates. Likewise, *reflecting no visible light* may be relevant to black holes and not to numbers because it isn't completely unnatural relative to concrete objects as it is relative to numbers.

In other words, what the problem of negative totality properties shows, and category-relativism meets, is the need to accommodate *fundamental yet non-characterizing* facts. These are instantiations of a property by an object such that, first, the property can't properly characterize (and thus, isn't relevant to) the object; and yet, the property is *not redundant* on further relevant (thus characterizing) properties of the object, and the property deserves to be denoted by a finite predicate in the fundamental language, because of its relevance to some other object. (Recall *p's Non-redundant Irrelevance* and *Relevance.*)

4.4 Real Category Mistakes

In many contexts, uttering 'the number two is an empty atom with zero mass' would sound infelicitous—even if granted that it is true by definition. Likewise for 'Socrates is empty', but not for 'the empty set is empty'. I think that the linguistic phenomenon in this case—i.e. that feeling that the utterance is odd—tracks important metaphysical facts. Category-relativism helps to account for these facts.

Call the broad linguistic phenomena 'category mistakes'. We can distinguish a context-dependent sense of it and a narrower, objective sense—just like we did concerning dissimilarity in §2.2. In certain contexts, we make salient a pertinent set of categories—regardless of their true

[45] 'Characterization' may be replaced with 'Category-relative Naturalness'.

metaphysical status—relative to which mistakes are mistaken. But, if the notion of category mistake has any bearing on metaphysics, and especially on objective similarity and dissimilarity, it is in the objective sense. That is, sure, one could change the conversation in such a way that saying 'Socrates is empty' doesn't feel odd. But the tempting reaction gestured at in §3.1 must be that, whereas saying 'The empty set is empty' is metaphysically ok, 'Socrates is empty' is a *real* category mistake. My purpose here is to answer what is a real category mistake and what's mistaken about it.

My answer follows Ofra Magidor's (2013) view concerning the broader linguistic phenomenon in natural languages.[46] In her view, category mistakes aren't meaningless, truth-valueless, or always false. The sentence 'The number two isn't green' is meaningful and true. When an utterance of this sentence is a category mistake, it is odd because it triggers a presupposition that is assumed in the context to be false. In particular, Magidor claims that the predicates in the following subject-predicate sentences trigger the following presuppositions about the respective objects:

1. 'Two is green' that the referred object is *colored*;
2. 'My toothbrush is pregnant' that the object is *a female*;
3. 'The chair is dreaming' that the object has *mental states*, or that it is *able to dream*; and
4. 'The theory of relativity is prime' that the object is *either prime or composite.* (See Magidor 2013: 132, 142-3)

The same presuppositions are triggered by negations of these sentences.

However, Magidor's view alone doesn't systematically determine which presuppositions are triggered in each case. Moreover, the view doesn't offer a characterization of category mistakes as opposed to presupposition failures in general. Her view is only that the oddness of those utterances are better explained in terms of presupposition failures.[47]

But both greater systematicity and a characterization of category mistakes can be achieved about *real* category mistakes if we supplement Magidor's view with the category-relativist notion of characterizability, as follows.

Real Category Mistake Let S be a sentence that attributes a property p to an object o or that denies p of that object. Saying S in a context C is a *real category mistake* iff i) saying

[46] See Magidor (2013: Ch. 5, spec. §§3.2 and 4).

[47] Magidor makes both points clear e.g. in (2013: 141, 146)

S in C triggers the presupposition that p can characterize o but ii) p cannot characterize o.[48]

Since the number two cannot be characterized by *being green*, saying (1) is a real category mistake in any contexts where the presupposition of characterizability is triggered. Likewise for a toothbrush and *being pregnant*, a chair and *dreaming*, and a theory and *being prime*. The same can be said of negations of those sentences.[49]

If we assume that utterances of 'Socrates is empty' and 'the empty set is empty' trigger the presupposition that emptiness, the property denoted by 'is empty', can characterize the referred object, then, since emptiness isn't natural at all relative to categories to which Socrates belongs, saying 'Socrates is empty' is a real category mistake, but saying 'the empty set is empty' isn't a real category mistake. Now, it is plausible that this presupposition is triggered in theoretical contexts where the objective similarity and dissimilarity among objects is in question. For this reason, the explanation goes, the reaction that emptiness is irrelevant to Socrates because saying 'Socrates is empty' is a real category mistake, is appropriate. A similar explanation is available with regard to the oddness of saying 'the number two is black' in contexts where this presupposes that the number two can be characterized by having the property denoted by 'is black' even if we assume, as I did in §2.3, that 'is black' denotes a negative totality property had by the number two.

Conclusion

I have argued that the standard theory of naturalness cannot account for the fact that some negative totality properties are relevant to the objective similarity and dissimilarity of some objects but not of others. Category-relativism can. Moreover, category-relativism can account for concepts involved in intuitive formulations of the problem from negative totality properties and

[48]Note that real category mistakes are partially, but only partially, context-independent. An account of a *broader*, fully context-sensitive phenomenon suggests itself, namely, one that makes the second clause context-sensitive by adopting an appropriate, context-sensitive notion of characterizability. (This, in turn, could be defined by a generalization of category-relative naturalness, one that is context-sensitive (Cp. Taylor 2016) but also category-relative.) Meanwhile, a *stricter* notion of 'category mistake' could take (ii) as necessary and sufficient.

[49]Magidor briefly suggests (2013: 142, 146) that greater systematicity and a general characterization of category mistakes could be achieved through some modality for 'can' such that all presuppositions are of the form 'can be p'. My proposal follows roughly this route. But, first, it does so concerning *real* category mistakes only. Second, I suspect that even the account of the broader phenomenon suggested in fn. 48 would not comprehend all cases that Magidor is interested in. Finally, I believe that a proper generalization of the accounts of characterization, privation, and real category mistakes, to the relational case requires position-relativism. (See fn. 43 and cp. e.g. Magidor (2013: 1-2, 144-5, 27 n. 5) respectively, on the phenomenology of category mistakes, relational predicates, and argument-place-relativity of selectional features.)

in tempting reactions to the problem: multi-order naturalness, negation as privation, genuine categories, characterization, non-characterizing fundamentality, and category mistakes.

Thus, category-relativism not only provides a better account of something the standard view is supposed to account for, but it also expands the list of phenomena a theory of naturalness can explain.

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