

Position-relative Naturalness

(forthcoming in *The Journal of Philosophy*)^{*}

Thiago Xavier de Melo

Abstract

I develop and defend a new theory of naturalness according to which relations can be natural to different degrees relative to their different positions. Set-membership, for example, is more natural relative to its set-position than to its member-position. I call this view position-relativism. The alternative view, position-absolutism, implies that existential derivatives of the same non-symmetric relation—such as *being a member of something* and *having something as a member*—must always have the same degree of naturalness. But this is false. Position-relativism avoids this problem and promises to do more.

Keywords: naturalness; relations; properties; positions; slots.

1 Introduction

Let us assume that properties exist in abundance but differ in naturalness. Green things, for example, are objectively more similar to each other than grue things because *being green* is more natural than *being grue*. Theories of naturalness typically consider the polyadic case as a mere extension of the monadic one: relations are more or less natural just like properties are. But I will argue that this is false. I will defend *position-relativism*, the view that relations can be natural relative to positions; some n -adic relations are natural to different degrees relative to their different positions. The loving relation, for example, might be more natural relative to its lover-position than to its beloved-position. Properties constitute a limit case where naturalness

*This paper is forthcoming in The Journal of Philosophy. Please cite their version. (I thank Scott Dixon, Jan Dowell, Mark Heller, Arturo Javier-Castellanos, Michaela McSweeney, Isaiah Lin, Michael Rieppel, Byron Simmons, J. Robert G. Williams, two anonymous referees for The Journal of Philosophy, and two referees of another journal, for helpful comments and discussion on previous versions of this paper. Very special thanks to Kris McDaniel for his continuous encouragement and valuable feedback on various versions of this paper.)

is relativized to one sole position, and this is the reason why they can be natural to a degree *simpliciter*.¹

David Lewis suggested that *perfect* naturalness might be primitive, and he endorsed a *definitional conception* of comparative naturalness, according to which the more complex it is to define (a predicate that expresses) a property or relation out of the (predicates that express) perfectly natural ones, the less natural that property or relation is.² The position-relativism I will defend is Lewisian in this sense, but it adopts position-relativized notions of perfect naturalness and complexity.

Position-relativism is preferable to position-absolutism because position-absolutism implies a false—and thus far unnoticed—consequence, namely, that genuine existential derivatives of non-symmetric relations—such as *loving something* and *being loved by something*—are *always* equally natural. I will show that this consequence is false by appealing to connections between naturalness and similarity. Roughly, some such pairs of derivatives—like *being a member* and *having a member*—must differ in naturalness because they differ concerning their relevance to the similarity among their instances or concerning the degree of similarity among their instances. Moreover, I will argue, connections between naturalness and other phenomena don’t make things better for absolutism, and adopting position-relativism is preferable to abandoning the definitional conception.

In §2, I explain the position-absolutist (standard) Lewisian view on naturalness and show how to apply it to existential derivatives of relations. In §3, I raise two new puzzles against this view. In §4, I develop and defend position-relativism as a solution to those puzzles. Specifically, in §4.1, I argue that one of the puzzles is solved by a position-relativized conception of complexity of definitions. In §4.2, I argue that the other puzzle is solved by a position-relativized perfect naturalness. In §5, I defend the Lewisian, definitional conception of naturalness, and in §6, I indicate further potential work for position-relativized naturalness.

¹I assume that relations have ‘positions’ in *some* sense. When John loves Mary the loving relation relates John to Mary in such a way that John occupies the lover-position while Mary occupies the beloved-position. But nothing here hinges on the nature of positions or on whether they are *sui generis*.

²(See Lewis 1983, 346-7; 1984, 228; 1986, 60-4) I follow Lewis and others in talking about definitions of properties and relations as short for definitions of predicates that express these properties and relations in a fundamental language. See §2.1.

2 Standard Lewisianism on Derivatives

2.1 Position-absolutism and Lewisianism

Let's assume that for almost any possible predicate there is a property or relation that that predicate expresses, and that no two of those properties or relations are necessarily co-instantiated.³ According to theories of *naturalness*, some properties and relations are *more natural than* others; some are maximally, *perfectly* natural; and, while many are natural to some degree, most are ‘not at all natural’; they are completely, ‘perfectly unnatural’.⁴ In *standard* theories, such facts aren't relative to anything—not to worlds, times, objects, or laws, for example.⁵ In particular, standard theories are *position-absolutists*: relations are perfectly natural *simpliciter* as opposed to relative to positions. Likewise for comparative naturalness and complete unnaturalness.

Lewisian theories take perfect naturalness as primitive and analyze the other notions in terms of complexity of definitions and perfect naturalness.⁶ The perfectly natural properties and relations make for similarities, carve at the joints, and are just enough to characterize things completely and without redundancy.⁷ Once we have perfectly natural properties and relations, others are natural in a derivative way, to some degree, iff related to the perfectly natural ones by ‘not-too-complicated’ definitions;⁸ moreover, we can compare their naturalnesses in terms of the complexity of their definitions out of the perfectly natural properties and relations.⁹

Since naturalness is a worldly matter, definability and complicatedness can't be merely linguistic. These notions must somehow track reality. Lewis' analysis seems to require a combination of a privileged language, an appropriate method for generating definitions, and an appropriate conception of complexity. Let's say that a *fundamental language*, \mathcal{L} , has primitive predicates expressing each perfectly natural property or relation and that it contains the logical resources of standard first-order languages and a lambda operator, with which predicates standing for less-than-perfectly natural properties and relations can be formed. Given an interpreted formula ϕ , $\Gamma(\lambda v_1 \dots v_n. \phi)^\sqcap$ is an n -adic predicate in \mathcal{L} that expresses the n -adic property P

³Lewis' view of properties (and relations) as classes or sets of (tuples of) possibilia is one such theory. (Lewis 1983, 344–6) But I won't assume it here.

⁴See (Lewis 1983, 347, 376; 2008, 204–5). See also fn. 8 and 12

⁵See (Lewis 1986, Ch. 1, fn. 44; 2008, fn. 2).

⁶Lewis himself was sometimes open to perfect naturalness being defined (1986, 64; 1983, 347–8, spec. fn. 9), but not in terms of comparative naturalness or complete unnaturalness. Contrast (Hawthorne 2007, 434); more in §5.

⁷(Lewis 1986, 60), Cf. also (Schaffer 2004, 93–4)

⁸(See Lewis 1986, 61). As (Nolan 2005, 24) notices, ‘not-too-complicated’ suggests a ‘cut-off’, which I take to analyze the cut-off between the ones that are natural to some degree, because appropriately connected to the perfectly natural, and the ones that aren't. See also (Lewis 1984, 228), and fn. 4 and 12.

⁹(Lewis 1983, 376; 1984, 228; 1986, 61)

that is such that, necessarily, it is instantiated by objects $o_1 \dots o_n$, in this order, iff ϕ is true (in that interpretation) under an assignment of variables $v_1 \dots v_n$, respectively, to $o_1 \dots o_n$.¹⁰

Given such a fundamental language and a conception of complexity to be specified, a *fundamental definition* of P is any \mathcal{L} -predicate that expresses P ; and a *most* fundamental (or ‘simplest’) definition of P is a least complex fundamental definition. Primitive predicates are definitions with zero complexity.¹¹ Finally:

Definitional Conception A property or relation P is *completely unnatural* iff all its fundamental definitions are too complex. A property or relation P is *more natural than* a property or relation Q iff P is not completely unnatural and a most fundamental definition of P is less complex than any fundamental definition of Q .¹²

Not any combination of language and complexity will do. Compare *being red and round* and *being red or round* under the assumption that *being red* and *being round* are perfectly natural. The first property is more natural than the second, and yet, their most fundamental definitions have the same length and the same number (of occurrences) of predicates and connectives if conjunction and disjunction are in the fundamental language.¹³ In this case, our conception of complexity must be egalitarian about logical connectives. Other things equal, disjunctions contribute with more complexity than conjunctions. And, since length still matters, a full theory of comparative naturalness must specify how length and disjunctiveness weigh against each other.¹⁴

Thus, different versions of the definitional conception may adopt different combinations of language and concepts of complexity. We’ll assume the version above, which includes a first-order language with disjunction and egalitarianism.¹⁵ The arguments below apply to any

¹⁰Cf. Sider (1995, 364; 2011, 8, 130) for the definitional conception in terms of ‘fundamental language’. Hawthorne (2007; 2006, 236) and Williams (2007, 374–8) raised skepticism about this conception. I defend it in §5. See also Sider (2011, 130).

¹¹Otherwise, the account wouldn’t entail intended claims involving perfectly natural properties.

¹²Nothing will hinge on precise conditions for being too complex. Without complete unnaturalness, the standard view would have fewer resources. (See the principles in §3.1.1). Moreover, to underpin complete unnaturalness, it’s enough that some properties have no fundamental definition. Consider, for example, the property had by one of two indiscernible objects. (See Lewis (1986, 63).) Finally, a sufficient condition for being too complex is also enough to underpin complete unnaturalness. See (Lewis 1983, 372; 1986, 108) on infinite unpatterned length, and (2020, 525–6, 784) on the objectivity of complete unnaturalness.

¹³See *Similarity to Naturalness*, in §3.1. See Lewis (1986, 61; 1983, 376) on disjunctiveness and loss of naturalness. See also (Sider 1995, 364).

¹⁴Cp. (Sider 1995, 364)

¹⁵Alternatively, the language could include conjunctions and negations, from which to define disjunctions, which will be more complex in terms of length and number of connectives. This indicates a way to specify how length and disjunctiveness might weigh against each other in the egalitarian proposal: the complexity of a disjunction is its length if it were defined. Alternatively, Sider (1995, 363–4) suggests adopting a notion of ‘logical distance’.

version except position-relativized ones.

2.2 Existential Derivatives

Some properties, like *loving something*, are existential derivatives of relations, like *loving*.¹⁶

Plausibly, the degree of naturalness of an existential derivative depends on the degree of naturalness of the relation it derives from. (More in §5)

The definitional conception has the potential to explain this dependence between relations and their derivatives. A *predicate* is an *existential derivative* of a n -adic predicate $\Gamma(\lambda v_1 \dots v_n. \phi)^\sqcap$ if it is formed by prefixing ϕ with one or more existential quantifiers for variables $v_1 \dots v_n$ free in ϕ and, then, by prefixing the resulting formula with lambda-operators for the remaining free variable(s). For example, one existential derivative of ' $(\lambda xy.(Fx \ \& \ Gy))$ ' is ' $(\lambda x.\exists y(Fx \ \& \ Gy))$ '. The definitional complexity of such a derivative is a function of the complexity of the predicate from which it's derived plus the complexity added by the quantifier. Since 'existential derivative' here applies to predicates, let's say that a *property* P is a *genuine existential derivative* of a relation R iff a most fundamental definition of P is an existential derivative of a most fundamental definition of R . (Intuitively, a simplest path from the perfectly natural to P passes through R .) Given the definitional conception, the degree of naturalness of genuine existential derivatives of a relation R is determined by the degree of naturalness of R minus whatever complexity is added by the existential quantifiers.

Given this analysis, the definitional conception would explain the dependence between the naturalnesses of genuine existential derivatives and of the relations from which they derive. However, I will show that this analysis has undesirable consequences.

3 Two Puzzles about Non-symmetric Relations

Position-absolutism generates two independent puzzles, which I develop in the next subsections. But both puzzles follow the same schema. I start by supposing that a fundamental language contains a binary predicate $\Gamma(\lambda xy. \phi)^\sqcap$ denoting a non-symmetric relation, and existential derivative predicates $\Gamma(\lambda x.\exists y\phi)^\sqcap$ and $\Gamma(\lambda y.\exists x\phi)^\sqcap$. On the one hand, I argue that, whatever ϕ is, position-absolutism implies that the derivatives are equally complex. On the other hand, I argue that in many cases the derivatives express properties that aren't equally natural.¹⁷ The puzzle schema

¹⁶(Hawthorne 2001, 399).

¹⁷I focus on binary relations, but the problem can be generalized.

is this: given that $\Gamma(\lambda x.\exists y\phi)^\sqsupset$ and $\Gamma(\lambda y.\exists x\phi)^\sqsupset$ are equally complex, and that they are most fundamental definitions of the properties they express, by Definitional Conception, these properties must have the same degree of naturalness. However, this contradicts the claim that the properties $(\lambda x.\exists y\phi)$ and $(\lambda y.\exists x\phi)$ have non-equivalent naturalnesses.

This schema can be instantiated in two independent ways: one where $\Gamma(\lambda xy.\phi)^\sqsupset$ expresses a perfectly natural relation, and one where it expresses a less-than-perfectly natural one. Solutions to each of the puzzles will give us different but complementary conceptions of position-relativized naturalness and independent arguments for position-relativism.

3.1 A puzzle from perfectly natural relations

This instance of the puzzle arises provided that, among the perfectly natural relations, at least one has a pair of genuine derivatives that differ in naturalness. I assume that standard set-membership is one of the perfectly natural relations.¹⁸ I argue, in §3.1.1, that the existential derivatives of set-membership have non-equivalent naturalness: *having members* (i.e. *having something as a member*) is more natural than *being a member (of something)*. In §3.1.2, I argue that, where ‘ \in ’ is a primitive denoting set-membership, ‘ $(\lambda y.\exists x x \in y)$ ’ and ‘ $(\lambda x.\exists y x \in y)$ ’ are most fundamental definitions of the properties they express and there is no non-arbitrary conception of complexity that can differentiate them. In §3.1.3, I argue that the mere conceivability of set-membership being ‘fundamental’ is already problematic for position-absolutists.

3.1.1 The puzzle’s first part: non-equivalent naturalnesses

I offer two arguments from connections to similarity to the claim that *having members* is more natural than *being a member*. In stating these connections, my purpose is not to offer a full, specific account of similarity. Quite the opposite. The connections are meant to be weak enough to move naturalness theorists whatever their specific accounts might be.¹⁹

Roughly, the first argument rests on the claims that natural properties make for similarities and that *being a member* doesn’t make for similarities *at all*.²⁰

¹⁸Here, we ignore the view that properties are sets, but note that even Lewis admitted that set-membership could be perfectly natural. (1986, Cf. 1, fn. 47). See also Sider (2011, Ch. 13). Later, Lewis (1991) developed a theory where set-membership isn’t primitive. But there would still be non-symmetric relations, like parthood and maybe member-singleton. Since my argument isn’t particularly directed against Lewis, I keep the standard hypothesis that set-membership is primitive—hence, a good candidate for perfect naturalness. Importantly, theories of naturalness shouldn’t be contingent on which set-theory is best. See §3.1.3.

¹⁹I also couldn’t offer a full account of similarity here because I don’t think that position-relative naturalness alone is sufficient. I develop other essential components elsewhere.

²⁰Some theories, as NBG, characterize sets as classes that instantiate *being a member*. (Cf. Holmes 2017) (See

Let's adopt a provisional notion of a property being *relevant to* an object, and assume that, if objects x and y share some but not all of their relevant properties, subtracting one from the number of relevant properties they share (while maintaining the others equal) decreases their degree of similarity. For concreteness, we can assume the Jaccard coefficient as part of a toy account that incorporates this assumption. Let X be the set of all relevant properties of x ; likewise, for Y . The Jaccard similarity between objects x and y is the number of properties in the intersection of X and Y divided by the number of properties in the union of X and Y . The dissimilarity between them is one minus their similarity—or equivalently, the number of properties in the symmetric difference of X and Y divided by the number of properties in the union of X and Y .²¹

Can a theory of naturalness determine which properties are relevant for similarity? Given standard theories, the following are plausible.

No Relevance without Naturalness If a property is relevant to an object, then it is not completely unnatural.

No Non-redundant Naturalness without Relevance If a property had by an object is not completely unnatural, then, if it is not (in some pertinent sense) *redundant* relative to further relevant properties of that object, then it must be relevant to that object.

The intuitive idea behind the ‘non-redundancy’ condition is this. Since *being green and cold* is, let's assume, redundant relative to *being green* and *being cold*, if the latter are also relevant to an object, *being green and cold*, even though not completely unnatural, doesn't add to the number of relevant properties of that object.²²

Let's assume for now that *being a member* is a genuine derivative of set-membership. (I support this in §3.1.2.) If so, this property must be completely irrelevant to Socrates and to

also Lewis 1991, 18) Given position-relativism, these theories will plausibly disagree about the naturalness of set-membership. I assume standard set theory instead.

²¹See (Jaccard 1912), and see (Blumson 2018, 32) for arguments to prefer this to accounts of dissimilarity that rely only on the symmetric difference between X and Y —and thus not on their union. The Jaccard coefficient doesn't contemplate weights—whereby a property may contribute more similarity than another. It is, however, good enough for our purposes since, in this first argument, the pertinent intuitions are meant to support the claim that *being a member* doesn't make for similarities *at all*. Jaccard similarity also doesn't apply to objects with infinitely many relevant properties, but presumably at least some objects don't have infinitely many relevant properties—like Socrates and Plato in the argument below. Note that this account distinguishes two versions of the assumption above: the first concerns cases where the relevant property is also removed from the union of X and Y , and the second concerns cases where this is not the case. The first is the most pertinent for the argument below.

²²This informal understanding of ‘redundancy’ is sufficient for our purposes, but I assume that Lewisians can offer some analysis here too. One is analogous to Lewis' (2001) analysis of disjunctive properties, which appeals to comparative naturalness. Or, naturalness theorists could appeal to ‘simplest definitions’ more directly. Roughly, P is *redundant relative to* $Q \dots R$ of x iff, by simplest definition, x has P because it has $Q \dots R$.

Plato. For intuitively, the similarity between Socrates and Plato does not depend on their sharing a set-theoretic relation to sets. Consider some thought experiments: ignore the view of properties as sets and imagine that God could annihilate all sets, thereby preserving all properties of Socrates and Plato except for set-theoretic ones; or imagine that we find set-theory to be utter nonsense, there being no truth to the claim that Socrates is a member or to the claim that he fails to be a member. Intuitively, the similarity between Socrates and Plato wouldn't decrease; or it wouldn't be judged to decrease in light of that finding. Now, given the Jaccard account, there are three hypotheses that would explain this intuition. The first hypothesis is that *being a member* is relevant to Socrates and Plato but that they are perfectly similar to each other, which is obviously false. The second hypothesis is that *being a member* is relevant to Socrates and Plato but so is its negation (*failing to be a member*) which is gained by them when they lose the first property. The third hypothesis is that *being a member* isn't relevant to Socrates and to Plato. But the second hypothesis is less plausible than the third. For, if the second hypothesis were true, there should be a difference in the degree of similarity between Socrates and Plato in two situations: the situation where they instantiate *failing to be a member* (perhaps because God annihilated all sets but not the set-membership relation itself), and the situation where there is no set-membership relation to be borne or not (perhaps because God annihilated set-membership too or because set-theory is utter nonsense). But there seems to be no such difference between these situations. Just as, intuitively, bearing the relationship to sets doesn't seem to contribute to how similar Socrates and Plato are, failing to bear that relation to sets would also fail to contribute to the similarity between them. Moreover, note that in a final, weighted account of similarity, the analogue of the second hypothesis is that both the property and its negation are relevant and have the *same* weight, which is again implausible (especially given that this weight is supposed to be neither zero nor maximal). Thus, I conclude that *being a member* must be irrelevant to Socrates and Plato.²³

Moreover, *being a member* isn't redundant relative to further relevant properties of Socrates. For even if it was redundant relative to, say, *being a member of {Socrates, Aristotle}* relative to Socrates, the latter isn't relevant to Socrates, for consider God's annihilating act again. More

²³Elsewhere, I develop a theory that implies that the sort of thing Socrates is makes set-membership and all its derivatives irrelevant to him. But besides non-standard, this theory doesn't solve the current problem since *being a member* isn't relevant for the similarity among sets. Incidentally, note that *being a member* would be relevant to Socrates if necessarily co-instantiated with, say, *being an entity* and the latter was perfectly natural and non-redundant. But it's controversial that the latter is perfectly natural, and this would amount to rejecting that *being a member* is a genuine existential derivative of set-membership, which we are assuming for now. (See §3.1.2)

generally, candidate collections of properties of Socrates relative to which *being a member* could be thought to be redundant seem to be doomed to contain at least one irrelevant property. For example, *being a member* could be thought to be redundant relative to the pair of properties expressed by fundamental language equivalents of ‘is a person’ and ‘is such that every person is a member of something’; but the second property seems irrelevant to Socrates. Every such collection must include a property that, roughly speaking, “connects” the instantiation of a property that is relevant to Socrates (e.g., *being a person*) to the instantiation of *being a member* and such connective properties, were God to annihilate them, would, just like *being a member*, seem to decrease nothing in the similarity between Socrates and other instances of it, such as, Plato.²⁴

But then, given that *being a member* is not redundant relative to further relevant properties of Socrates and given that it is not relevant to Socrates, we can infer, via *No Non-Redundant Naturalness without Relevance*, that *being a member* must be completely unnatural.²⁵

Now, a similar argument can’t be made for *having members*, for this property is relevant for similarities and dissimilarities among sets. That is, this property is relevant to, say, the singleton {Socrates} because the fact that the singleton, but not the empty set, instantiates *having members* must contribute to their dissimilarity and because the fact that two singletons share *having members* must contribute to their similarity. Moreover, *having members* cannot be completely unnatural precisely because it is relevant to some objects. Therefore, *having members* is more natural than *being a member*.²⁶

The second argument to this conclusion doesn’t depend on one existential derivative being completely irrelevant for similarity (as *being a member* is). It merely compares the derivatives concerning similarity as follows. Since the overall similarity among possible instances of *having members* is greater than the overall similarity among possible instances of *being a member*, *having members* is more natural than *being a member*. (By ‘possible instances of *P*’ I don’t mean ‘all things that could have been *P*’, but ‘all *P* things that could have been’.²⁷) I develop

²⁴The talk of ‘connection’ here can be made precise, again, in terms of simplest definition in analyses of ‘redundancy’ such as the one suggested in fn. 22.

²⁵Object derivatives—like *being a member of {Socrates, Aristotle}*—generate puzzles analogous to ours. Moreover, such derivatives require an explanation of how objects, like sets themselves, can enter the definitional picture. (See §6)

²⁶Note that, since these are genuine derivatives, they aren’t redundant relative to further properties of Socrates either. For the existentially quantified sentence with which *being a member* is defined isn’t merely a “big” disjunction of its instances. (Cf. fn. 22) Hence, this property isn’t redundant relative to *being a member of {Socrates, Aristotle}*. Analogously for *having members*.

²⁷Or, ‘all *P* things across possible worlds’, though see fn. 3.

this below without overgeneralizing the connection between similarity and naturalness.

The overall similarity among possible instances of *having members* is greater than the one among possible instances of *being a member*. For the instances of the first property are all instances of the latter but not vice-versa: whereas all sorts of things—including you, my desk, volcanoes, and sets—are members of something, only some such things have members.²⁸ We can further support this, for example, if we assume that the degree to which things in a class are similar to each other is determined by the minimum degree of similarity among a pair of its objects.²⁹ For the similarity between any two possible membered sets is surely greater than the similarity between, say, the singleton of the empty set and the lava of Mount Etna.

We don't have to assume a specific or direct connection between the naturalness of properties and the overall similarity among their possible instances. For most theorists are committed to *some* version of the following:

Similarity to Naturalness If the overall similarity among possible instances of *P* is greater than the overall similarity among possible instances of *Q*, then, other factors being equal or absent, *P* is more natural than *Q*.

Here, the ‘other factors’ condition is schematic—that is, I am initially open to there being different plausible specifications of it in different theories of naturalness. But, if one *explains*, for example, that green things are more similar to each other than grue things by saying that *being green* is more natural than *being grue*, then one is committed to some version of this principle. Of course, such explanation doesn't commit one to the unqualified principle that, for any properties *P* and *Q* whatsoever, if the overall similarity among possible instances of *P* is greater than the overall similarity among possible instances of *Q*, then *P* is more natural than *Q*. But, if two properties don't differ in naturalness but satisfy the antecedent of this conditional—just like *being green* and *being grue* do—there must be some further condition that distinguishes them from *being green* and *being grue*. Otherwise, the explanation for the case of *being green* and *being grue* would be ad hoc. Whatever one thinks this further condition is, they can read it into the ‘other factors’ clause—thus giving rise to a specific version of *Similarity to Naturalness*.³⁰

²⁸Some theories count proper classes as ‘things’ that aren't members of anything. (Cf. Maddy 1983, Holmes 2017). Others, like (Quine 1980, 30-2), take individuals to be members of themselves. But both claims are controversial. See fn. 20. Moreover, I assume that a theory of naturalness must be compatible at least with the conceivability of standard set theories. See §3.1.3.

²⁹For reasons given in (Dorr and Hawthorne 2013, 22–23) and discussed below, one cannot take these measurements, directly, as measurements of *naturalness*, but this is not our thesis.

³⁰Lewis (2001, §VI) endorses a version of this principle. Given a conception of properties as regions in a

Could one's specific theory of naturalness be committed to a (non-arbitrary) specification of the 'other factors' condition that blocks the inference that *having members* is more natural than *being a member*?³¹

One important motivation for the 'other factors' condition is that we need something to the effect that when P is, intuitively speaking, a conjunction of (... a conjunction of) properties among which is Q , P might still fail to be more natural than Q . For example, a complete profile like *being a green-cold-solid-...-cube* and *being green* satisfy the antecedent of *Similarity to Naturalness*—the first “makes” for greater similarity than the second—and yet the complete profile isn't more natural than *being green*.³²

Now, first, the best explanation naturalness theorists can offer for this arguably takes the talk about conjunctions literally, via 'simplest definitions'. Let's say that P is a conjunction of Q and R iff a most fundamental definition of P is of the form $\Gamma(\lambda x.(\phi \ \& \ \psi))^\neg$ where $\Gamma(\lambda x.\phi)^\neg$ expresses Q and $\Gamma(\lambda x.\psi)^\neg$ expresses R . *Similarity to Naturalness* could then be further specified as follows:

Similarity to Naturalness* If the overall similarity among possible instances of P is greater than the overall similarity among possible instances of Q , then, if P isn't a conjunction of (... conjunctions of) properties among which is Q , other factors being equal or absent, P is more natural than Q .

Plausibly, whereas *being a green-cold-solid-...-cube* and *being green*, respectively, satisfy the first condition of *Similarity to Naturalness**, they don't satisfy the second. By contrast, *being green* and *being grue* satisfy both conditions. Likewise, *having members* and *being a member* satisfy both conditions. Hence, like *being green* and *being grue*, *having members* is, other factors equal or absent, more natural than *being a member*. Second, even assuming that the explanation above isn't the best, and thus that *Similarity to Naturalness** isn't the best version of the schematic principle to explain the case of conjunctive properties, since the phenomena involving conjunctive properties and the phenomena involving existential derivatives are different, there is no reason to think that one's explanation of the case of conjunctive properties

similarity space, he assumed that other things equal the more spread or scattered a region is, the less natural is the property. Based on this, he argued for various comparative claims. (2001, 391-4) Likewise, we can argue that since *being a member* is more spread (but not less scattered) than *having members*, the first is less natural than the second; that is, as far as similarity goes. (Cf. 2001, 395)

³¹By considering further factors, we put away, to the extent needed here, the worry that *ceteris paribus* clauses render this sort of principle too vague. (Dorr and Hawthorne 2013, 23)

³²(Dorr and Hawthorne 2013, fn. 34)

non-arbitrarily explains why, despite differing in similarity, genuine derivatives of relations don't differ in naturalness.

*Similarity to Naturalness** still has an unspecified ‘other factors’ qualification. Consider *being a member of {Socrates}*. Based on unqualified versions of the two principles above, one could wrongly infer that this property is very natural from the truth that the overall similarity among possible instances of it is just the similarity between Socrates and himself, hence maximal. In comparisons with other properties, the ‘conjunction’ clause of *Similarity to Naturalness** doesn’t block the inference since *being a member of {Socrates}* is plausibly a genuine “object derivative” of set-membership instead of a conjunction of any properties.³³ What explains the mistake here is that other factors are not equal. First, this property doesn’t ‘make for dissimilarities’ at all. For someone distinct from Socrates doesn’t instantiate *being a member of {Socrates}* but may be maximally similar to Socrates. Second, and in particular, this property divides Socrates and his duplicates.

But when it comes to *having members* and *being a member*, further possible factors one could appeal to—stemming from connections between naturalness and *dissimilarity*, *duplication*, or *completeness*—either don’t defeat our non-equivalence claim or support it. First, *being a member* doesn’t make for more *dissimilarity* or ‘carve more at the joints’ than *having members*. The opposite is true. For nothing fails to be a member of something, and *having members* divides non-empty sets from everything else. Second, sharing of *being a member* doesn’t have a better claim for accounting for *duplicates* than *having members*. Finally, *being a member* isn’t needed for a complete description of reality once we have set-membership, which we already assumed to be perfectly natural.³⁴ Thus, it’s unclear how specific commitments of a position-absolutist could defeat our non-equivalent naturalness claim.³⁵

I take the connections of naturalness to the notions above to be of foremost importance. A

³³See fn. 25.

³⁴I’m relying on intuitive understandings of the formulations found in Lewis (1983) and Dorr and Hawthorne (2013). Of course, there is much to say about all these criteria. But very little would be uncontroversial, and, again, a face-value reading favors our non-equivalence claim.

³⁵To further clarify the challenge that the standard view faces, consider the following suggestion: the overall similarity among things that have members is greater than the overall similarity among things that are members not because *having members* is more natural than *being a member* but because the fact that things have members implies that they have fairly natural properties whereas the fact that things are members doesn’t imply that they have as many (equally) natural properties. However, notice that an analogous suggestion can be made with regard to *being green* and *being grue*, but, as argued, in this case we take the difference in overall similarity among their instances to support a difference in naturalness between the properties. This second argument challenges the standard view to find and motivate a difference between the pair of existential derivatives and paradigmatic pairs of properties such as of *being green* and *being grue*. (I thank an anonymous referee for helpful discussion here.)

theory of naturalness that isn't appropriately connected to them doesn't deserve its name.³⁶ But one might want to reject our non-equivalence claim by considering further explanatory jobs that a theory of naturalness is supposed to do, some of which are very Lewisian (accounting for laws, causation, and mental content) and others less (e.g., essence). I won't go over these, but in §6 I consider further potential uses of the notion of position-relativized naturalness that aren't available for position-absolutists. For now, we have enough to admit that *having members* must be more natural than *being a member*.

3.1.2 The puzzle's second part: equivalent complexities

Now for the second part of the puzzle. No conception of complexity can distinguish $\Gamma(\lambda x.\exists y\phi xy)\vdash$ and $\Gamma(\lambda y.\exists x\phi xy)\vdash$ if ϕ is primitive, which is the case if ϕ expresses a perfectly natural relation. (For the case where ϕ isn't primitive, see §3.2.) In particular, where ' \in ' expresses set-membership, ' $(\lambda x.\exists y x \in y)$ ' and ' $(\lambda y.\exists x x \in y)$ ' must be equally complex. One could try, for example, to stipulate that binding a quantifier to the first argument-place of a predicate is always less complicated than binding it to the second. But this attempt should be accompanied by an arbitrary stipulation about an appropriate representation of perfectly natural relations by primitive n -ary predicates: the order of the argument-places of each predicate should match the intended results concerning the first, the second, ..., and the n -th existential derivative of the relation it expresses. Moreover, this excludes the possibility of non-symmetric relations whose existential derivatives are equally natural. Our conception of naturalness shouldn't commit us to such a universal substantive thesis about the naturalness of all possible relations and all its existential derivatives.

Now, assume that ' $(\lambda x.\exists y x \in y)$ ' and ' $(\lambda y.\exists x x \in y)$ ' are most fundamental definitions of the existential derivatives of set-membership. Since they are equally complex, by Definitional Conception, *having members* and *being a member* will have the same degrees of naturalness. This result, however, contradicts the previous claim that they have non-equivalent naturalnesses.

The puzzle assumes that those properties are *genuine* existential derivatives of set-membership. But one can imagine that *having members* is more natural than *being a member* in virtue of a simpler definition. For example, if a) *having members* is itself perfectly natural, then it's not a genuine existential derivative of set-membership; or, if b) *being a set* is a perfectly natural

³⁶Cf. also Schaffer's (2004, 93) distinction between 'qualifications for' and 'responsibilities of' naturalness. (Cf. also Bennett 2017, 126).

property expressed by ‘ S ’, and if ‘ \emptyset ’ is a name for the empty set, then ‘ $(\lambda y.(Sy \& y \neq \emptyset))$ ’ expresses *having members*; or, if c) the language has second-order quantifiers and an operator ‘ ϵ ’ that, concatenated to a predicate, forms a term for the predicate’s extension, then ‘ $(\lambda y.\exists x\exists Z(y = \epsilon Z \& Zx))$ ’ expresses *having members*.³⁷

However, alternative (a) makes the collection of perfectly natural properties dependent on one another since, by hypothesis, set-membership is perfectly natural. Moreover, if both the relation and its derivative are perfectly natural, then their degrees of naturalness aren’t dependent on one another, which is implausible. (See §5 for more.) Alternative (b) depends on the fundamental language having a primitive for (non-)identity and a primitive name for the empty set, which are controversial theses. Lewis (1983, 345), for example, listed identity among the less-than-perfectly natural. And, if the language contains proper names for sets, then, as Dorr and Hawthorne (2013, fn. 30) observe, all less-than-perfectly natural properties will be very, and equally, natural. For let ‘ I ’ be a predicate for *instantiating* (or *set-membership* itself), and ‘ f ’ be the name for a property (set), the predicate ‘ $(\lambda x.Ixf)$ ’ will express that property. (Position-relativism solves this; cf. §6) Alternative (c), besides containing identity, is plausibly more complex—hence not most fundamental—since it includes two quantifiers. These, of course, don’t exhaust the conceivable alternatives. But similar considerations will be relevant elsewhere.

On the other hand, one may question whether *being a member* is a genuine derivative of set-membership. This property is necessarily co-instantiated with *being self-identical*,³⁸ and therefore, given intensionalism—the view that no two properties or relations are necessarily co-instantiated—it is the same property as *being self-identical*. If a') *being self-identical*, that is, *being a member*, is itself perfectly natural, then it is not a genuine derivative of set-membership; or, if b') the relation of identity, expressed by ‘ $=$ ’, is perfectly natural, then ‘ $(\lambda x.x = x)$ ’ expresses *being a member* and, if this predicate is less complex than ‘ $(\lambda x.\exists y x \in y)$ ’, the property fails to be a genuine derivative of set-membership. But, analogously to (a), (a') makes the collection of perfectly natural properties and relations dependent on one another since, by hypothesis, set-membership is perfectly natural. Alternative (b') depends on the fundamental language having a primitive for identity, which, as I indicated above, is controversial too. Moreover, as we will see in §4.1, reflexivizations of polyadic predicates must be more complex than

³⁷ See (Burgess 2005, 130)

³⁸ See fn. 20 and fn. 28.

those predicates, and it is unclear how complex reflexivizations are. Finally, different from the hypotheses about *having a member*, note that if *being a member* turns out to be more natural than *having a member*, this may recommend a revision of our intuitions about similarity with regard to *being a member* (conditional on a final account of similarity). In principle, this is not problematic. As I will argue in §4.3, the theory of naturalness itself should make room for this hypothesis. But the fact that it is implausible puts pressure against the claim that *being a member*, that is, *being self-identical*, is very natural.³⁹

My overall argument above depends on the plausibility of a specific claim about the derivatives of set-membership, namely, that these are genuine derivatives and differ in naturalness. But note that the standard view has a great burden, which is that whenever two existential derivatives of *any* perfectly natural relation differ in naturalness, the most natural derivative cannot be a genuine derivative of the relation—that is, it must *always* be the case that further suitable properties and relations are perfectly natural out of which *the most natural* derivative must be definable in a simpler way. Thus, the standard view depends on the plausibility of a much stronger claim.

Based on these arguments from plausibility, I conclude that a theory of naturalness must allow for the possibility of perfectly natural non-symmetric relations whose genuine existential derivatives differ in naturalness and that the definitional conception conflicts with this under the standard view.

3.1.3 Conceivability and Further Cases

Given a relatively pre-theoretical understanding of essence, Fine (1994) argued that, plausibly, *being a member of {Socrates}* is necessary but not essential to Socrates even though the relationship to Socrates is essential to the singleton. From this, he argued against some modal analyses of essence. But as he notices, besides plausibility, the mere conceivability of such claims is already problematic for those modal analyses. For an account of essence “should not settle, as a matter of definition, any issue which we are inclined to regard as a matter of substance.” (1994, 5)

I assumed that set-membership is perfectly natural. The expression ‘perfect naturalness’

³⁹Note that position-relativism doesn’t require intensionalism, and that, if intensionalism had not been assumed, the objections I have just addressed would not come up, and thus, the argument for position-relativism would be in this respect easier to make. I assumed intensionalism to show that this argument doesn’t depend on rejecting the criterion of identity assumed by Lewis.

is a theoretical term. But the notion that some properties and relations are, what we may call, ‘fundamental’, that is, that they are part of the ultimate fabric of reality, that they are needed for a complete and perspicuous characterization of reality, is relatively pre-theoretical. This notion must be *accounted for* in a theory of naturalness. Position-absolutism says that the fundamental properties and relations are the perfectly natural ones.

Call ‘position-biased’ relations whose genuine existential derivatives are such that only one of them seems relevant (in the sense of §3.1.1) to its instances. The mere conceivability of relations being fundamental and position-biased is already problematic for position-absolutists. A theory of fundamentality of relations should not exclude from the outset views whose fundamental relations are position-biased if they are conceivable. Given this, one is invited to consider views whose relations may be fundamental and position-biased. For example, one may consider parthood, some relations between numbers and concreta (like mass-assignment), and some intentional relations (like believing).⁴⁰

3.2 A puzzle from less-than-perfectly natural relations

Let’s turn now to less-than-perfectly natural relations. *Loving* might be one of them. Intuitively, lovers are more similar to each other than the objects of their love are. Likewise, the minimum possible degree of similarity among lovers is greater than the minimum possible degree of similarity among beloved ones, for the class of beloveds across possible worlds includes, for example, inanimate objects—like philosophy, art, and watches—which are incapable of loving even though objects of love, whereas the class of lovers includes only subjects capable of loving. Thus, *loving something* seems more natural than *being loved by something*. Suppose these are both genuine existential derivatives of *loving*. By Definitional Conception, they must be equally natural. For, take whatever is the most fundamental definition, $\Gamma(\lambda xy.\phi xy)\vdash$, of *loving*, we can define both existential derivatives almost in the same way— $\Gamma(\lambda y.\exists x\phi xy)\vdash$ and $\Gamma(\lambda x.\exists y\phi xy)\vdash$.

Thus, the puzzle schema seems to have instances involving less-than-perfectly natural relations. But let’s *not* rest our case on this. For there could be a simpler predicate in the fundamental language that expresses *loving something*. Moreover, maybe the puzzle here is due to fundamental non-symmetric relations with which *loving* is defined. If so, a solution to the first puzzle would solve this one, too. I will argue, however, that even if there are no perfectly

⁴⁰Indeed, I believe that these relations may fit the description but I won’t argue for this here due to space limitations.

natural non-symmetric relations, there are pairs of genuine existential derivatives that differ in naturalness. To be able to query for fundamental definitions of the derivatives involved, I'll hereafter build non-symmetric relations and their derivatives out of (place-holders for) perfectly natural properties and symmetric relations.

Suppose that *being circular*, *being triangular*, and *being square*, are perfectly natural properties expressed, respectively, by ‘*C*’, ‘*T*’, and ‘*S*’. Consider *being an x and a y such that x is circular and y is triangular or square* and its two derivatives, respectively expressed by R_1 , E_{11} , and E_{12} :

$$R_1) (\lambda xy.(Cx \ \& \ (Ty \vee Sy)))$$

$$E_{11}) (\lambda x.\exists y(Cx \ \& \ (Ty \vee Sy)))$$

$$E_{12}) (\lambda y.\exists x(Cx \ \& \ (Ty \vee Sy)))$$

On the one hand, the property expressed by E_{11} seems more natural than the one expressed by E_{12} . For the most dissimilar things that share the first at least share their shape, circular, whereas the most dissimilar things that share the second fail to share even their shape: one is triangular, and the other is a square. Given that the first derivative isn't a conjunction of the second, *Similarity to Naturalness** suggests that the first is more natural than the second. But, given position-absolutism, there is no principled way of classifying the fundamental definition of the first derivative as less complicated than the second. That is, since E_{11} and E_{12} have the same number of occurrences of predicates, quantifiers, conjunctions, and disjunctions, position-absolutism implies that E_{11} and E_{12} are equally complex.⁴¹

Here is another case. Let ‘*A*’ and ‘*B*’ be primitive predicates denoting the dyadic and symmetric relations of, respectively, *intersecting* and *touching*. (Assume that an object may intersect another without touching it, and vice-versa.) Now, consider *being an x and a y such that x intersects and touches something that touches y* and its two derivatives, expressed by R_2 , E_{21} , and E_{22} :

$$R_2) (\lambda xy.\exists z((Axz \ \& \ Bxz) \ \& \ Bzy))$$

$$E_{21}) (\lambda x.\exists y\exists z((Axz \ \& \ Bxz) \ \& \ Bzy))$$

$$E_{22}) (\lambda y.\exists x\exists z((Axz \ \& \ Bxz) \ \& \ Bzy))$$

⁴¹The quantifiers are world-bound. And I ignore similarities between shapes since these are just place-holders for perfectly natural properties.

E_{21} and E_{22} have equally complex definitions since they have the same number of primitive predicates, conjunctions, and quantifiers. But the first derivative, *intersecting and touching something that touches something* (expressed by E_{21}) seems more natural than *touching something that intersects and touches something* (expressed by E_{22}). For things that share the first property, in addition to touching something, also intersect something—namely, that very thing which it touches—whereas things that share the second may or may not intersect something. Given *Similarity to Naturalness* (and its starred version), it's plausible that those properties have non-equivalent naturalnesses.⁴²

4 Position-relativism

At this point, one might be tempted to blame the definitional conception. In §5, I defend this conception and argue that, even if we abandoned it, an unexplained phenomenon would still remain.

Here, I present a solution to the puzzles by developing a conception of naturalness according to which relations can be natural to different degrees relative to their different positions. *Loving*, for example, might be more natural relative to its lover-position than relative to its beloved-position, which would explain why *loving something* is more natural than *being loved by something* (assuming these to be genuine derivatives of *loving*). The rough idea is that when we “plug” the existential quantifier into the beloved-position to define *loving something*, we leave the more natural position (the lover-position) free, which explains why the resulting property is more natural than *being loved by something*—which, by contrast, is defined by “plugging” the quantifier into the lover-position and leaving the less natural position (the beloved-position) free.

4.1 Variable-relative Complexity

In this section, I take steps, first, towards making some sense of variable-relative complexity—i.e. complexity of an n -adic predicate $\ulcorner(\lambda v_1 \dots v_n.\phi)\urcorner$ relative to a variable v_i —with which we

⁴²One might think that the differences in similarity-making that I have been claiming between pairs of existential derivatives all concern similarity in ‘intrinsic’ respects, and in such respects only, and that an account of this concept could perhaps explain the pertinent similarity claims while maintaining that derivatives in those pairs are equally natural. The main difficulties for this suggestion stem from the fact that the pertinent relations can hold between non-overlapping objects, such that intuitively, it is not the case that objects instantiate those derivatives just in virtue of how their parts are as opposed to how they are related to other objects. (I thank an anonymous referee for helpful discussion here.)

will understand position-relative comparative naturalness, and, second, towards showing how this can help explain the differences in naturalness between existential derivatives of the less-than-perfectly natural relations of §3.2. Such steps fall short of a precise definition of variable-relative complexity, and thus, of comparative naturalness. Instead, I explain the differences in naturalness in terms of a couple of heuristic principles. In doing so, my ultimate aim is to argue that position-relativism (with variable-relative complexity) has better resources to explain those differences than position-absolutism has.

The role of the heuristic principles can be clarified with an example that, incidentally, is also dialectically pertinent when comparing position-relativism and -absolutism. To introduce the position-absolutist conception of complexity in §2, I mentioned two principles: that the lengthier a definition is, the more complex it is, and that disjunctions make for more complexity than conjunctions. Based on these principles, we can explain why a) ‘ $(\lambda x.(Fx \ \& \ Gx))$ ’ is less complex than b) ‘ $(\lambda x.((Fx \ \& \ Gx) \ \& \ Hx))$ ’ and than c) ‘ $(\lambda x.(Fx \ \vee \ Gx))$ ’—since (a) has fewer occurrences of connectives than (b) and contains a conjunction where (c) contains a disjunction. In turn, based on these comparisons, we can understand the corresponding comparative claims of naturalness involving properties expressed by these predicates—the property expressed by (a) is more natural than the ones expressed by (b) and (c).

Clearly, these principles fall short of a precise and successful analysis of complexity. The analysis they immediately suggest is that the complexity of a predicate is the weighted number of occurrences of connectives in that predicate. But, as it is, this analysis leaves open how exactly the weights of conjunctions and disjunctions compare to each other—e.g., to determine which one is more complex, (b) or (c). More importantly, even assuming that a promising weight function is specified, and even assuming that I am wrong about the existence of pairs of derivatives that differ in naturalness, the analysis fails. For consider binary predicates ‘ $(\lambda xy.Rxy)$ ’ and ‘ $(\lambda xy.(Fx \ \& \ Gy))$ ’, and their respective reflexivizations ‘ $(\lambda x.Rxx)$ ’ and ‘ $(\lambda x.(Fx \ \& \ Gx))$ ’. The original, dyadic predicates have the same number of connectives as their reflexivizations, and yet they must be less complex than their reflexivizations. For otherwise, the account can’t deliver expected results, such as that the loving relation is more natural than self-love.⁴³ On the other hand, whenever a relation expressed by a binary predicate (e.g., ‘ $(\lambda xy(Px \ \& \ Py))$ ’) is less natural than the property expressed by its reflexivization (‘ $(\lambda x(Px \ \& \ Px))$ ’), there is

⁴³The relation is more natural than its reflexivization since the latter supervenes on the former, but not vice-versa, and since a predicate for the property is definable from a predicate for the relation, but not vice-versa.

an alternative predicate that expresses the latter which is not the reflexivization predicate, and the expected result—that the property expressed by the reflexivization is more natural—can be accounted for in terms of that alternative predicate (e.g., ‘ (λxPx) ’). Thus, a conception of complexity that renders reflexivizations as more complex than corresponding binary predicates seems preferable. And yet, the analysis of complexity as the weighted number of occurrences of connectives doesn’t support this.

I think that there are deeper lessons to be drawn from the case of reflexivizations. But what I want to observe here is just that, despite the failure above, those principles still help us make some sense of the notion of complexity of definitions on which the definitional conception and position-absolutism rest. Even if the principles don’t provide the final explanations, they provide *some* explanation for why the properties expressed by (a)-(c) compare as they (plausibly) do when it comes to naturalness. Even though position-absolutists have never developed a more promising definition of complexity, one hopes that such a definition would ultimately vindicate the above explanations. I intend the principles below to have a similar status.

Consider the following predicates again:

$$E_{11} (\lambda x. \exists y (Cx \ \& \ (Ty \vee Sy)))$$

$$E_{12} (\lambda y. \exists x (Cx \ \& \ (Ty \vee Sy)))$$

First, in E_{11} the lambda-variable (‘ x ’) occurs only in the scope of a conjunction. In E_{12} , the lambda-variable (‘ y ’) occurs in the scope of a disjunction, which in turn occurs in the scope of that conjunction. The first principle says that the complexity of a monadic predicate increases when the lambda-variable occurs in the scope of disjunctions.

This inegalitarianism about variables within disjunctions shouldn’t be surprising. As we noticed, disjunctive properties tend to be less natural, and Lewis’ definitional conception can accommodate this via an inegalitarianism about conjunctions and disjunctions. Lewis himself often correlated disjunctiveness to unnaturalness.⁴⁴ What’s different here is that now we know that it’s not only the mere occurrence of disjunctions in a fundamental definition of a property that matters. It also matters whether the given lambda-variable appears in the scope of that disjunction.

Second, consider the definitions of E_{21} and E_{22} :

$$E_{21} (\lambda x. \exists y \exists z ((Axz \ \& \ Bxz) \ \& \ Bzy))$$

⁴⁴See (Lewis 1986, 61; 1986, 376).

$$E_{22}) (\lambda y. \exists x \exists z ((Axz \ \& \ Bxz) \ \& \ Bzy))$$

Here, the lambda-variable of E_{21} (' x ') occurs in the scope of more logical notions than the variable of E_{22} (' y '), and yet, we want to say that the property expressed by E_{21} is more natural than the one expressed by E_{22} . The other difference between these predicates is that their variables occur concatenated to different occurrences of predicates. In E_{21} , ' x ' occurs in argument-places of occurrences of both ' A ' and ' B '. In E_{22} , ' y ' occurs only in one argument-place, of the second occurrence of ' B '. This is a difference in the number of occurrences of primitive predicates in whose argument-places the respective variable occurs versus ones where it doesn't.

This case suggests as a principle that the complexity of a non-primitive monadic predicate increases with the number of atomic formulas in which the lambda-variable does *not* occur. Again, this isn't surprising since the following intuitive reasoning supports it. The more properties and relations we *need* to define a new property that, nevertheless, do *not* directly characterize objects that have that new property, the less natural the property is, and thus the more complex that definition must be. Thus, *intersecting* is needed to define *touching something that intersects and touches something* (E_{22}), but it doesn't directly characterize things that instantiate the newly defined property since things that instantiate the latter may or may not bear *intersecting* to something. By contrast, *intersecting* directly characterizes things that instantiate *intersecting and touching something that touches something* (E_{21}).

Now, if we have a position-relativized conception of complexity to deal with monadic existential derivatives of relational predicates, we can understand position-relativized complexity of relational predicates themselves. A predicate like R_1 , ' $(\lambda xy. (Cx \ \& \ (Ty \vee Sy)))$ ', has a complexity relative to ' x ', and a complexity relative to ' y '. Whatever principles govern the appropriate notion of complexity so that the pair of monadic existential derivatives are dealt with, they also apply to the definitions of relations relative to specific variables.

In a complete theory of comparative naturalness as definitional complexity, the principles mentioned above must be weighed in against each other. For whereas the lambda-variable of E_{11} is not connected to two primitive predicates, the lambda-variable of E_{12} isn't connected to just one primitive predicate. If we considered only the last principle—according to which complexity increases with the number of predicates in which the lambda-variable does not occur— E_{12} would seem to be less complex than E_{11} . But this is not the intended result since E_{12} is not more natural than E_{11} .

Again, I don't offer a full theory of comparative complexity here. My ultimate purpose is to argue that position-relativism is preferable to position-absolutism. For this aim, it's enough to observe, first, that no matter how exactly those principles are weighed against each other, a definitional conception of comparative naturalness must adopt a version of position-relativism to avoid our puzzle from less-than-perfectly natural relations. Secondly, as seen in §2, any conception of complexity—including position-absolutist conceptions—must provide a way of comparing different principles against each other. Finally, even putting aside the arguments of §3, the case of reflexive predicates and properties suggest that even a position-absolutist conception of complexity may make useful reference to occurrences of lambda-variables within argument-places. For maybe the greater complexity of reflexive predicates rests on facts about, say, the number of occurrences of lambda-variables.

4.2 Position-relativism about Perfect Naturalness

No notion of complexity can *per se* avoid the first puzzle—the one raised from perfectly natural non-symmetric relations. To solve that puzzle, I'll develop a position-relativized conception of perfect naturalness.

Lewis developed his conception of perfect naturalness by introducing perfect naturalness as a property of properties, and then, by extending this conception to relations.⁴⁵ Now, the extension of such conception to relations is straightforward given Lewis' conception of properties as classes of individuals and his conception of relations as classes of *n*-tuples. As he says, 'any relation is a [monadic] property of the pairs (or triples, or whatnot) that instantiate it.' (Lewis 1986, Ch. 1, fn. 40) Perfectly natural *n*-adic relations turn out to be mere perfectly natural properties of *n*-tuples. Since they are both species of classes, once perfect naturalness is intelligible concerning one, it must be intelligible concerning the other.

But now, since the perfect naturalness of properties isn't relative to positions, the perfect naturalness of relations didn't turn out to be relative to positions either. Moreover, since the perfect naturalness of properties is understood as making for similarities among the members of the class, the perfect naturalness of relations must now be understood as making for similarities *among the n-tuples*. Finally, since perfectly natural properties are needed to characterize their instances in a minimal but complete characterization of reality, the perfectly natural relations would seem to be needed to characterize *n*-tuples in such a complete characterization of reality.

⁴⁵(Cp. Lewis 1986, 60–1)

However, roughly speaking, the puzzle from non-symmetric relations suggests that relations also make for similarities *among occupants* of one or another position, and that the positions of a relation may sometimes make for similarities independently of one another. For example, set-membership makes for similarities among sets, not members. Moreover, a relation may be needed to characterize things that occupy one of its positions without being needed to characterize things that occupy the other position. Set-membership is certainly not needed to characterize members of sets, but it's needed to characterize sets.

To solve the puzzle, I reverse Lewis' order of development. I start with the naturalness of relations: position-relativized naturalness. The monadic case is just a limit case where we have only one position. But the position-relativized conception of perfect naturalness is elucidated in a way that is similar to Lewis' own conception of perfectly natural properties, except that now it is relative to positions.

The conception is the following. An n -adic relation R with positions $p_1 \dots p_n$ is *perfectly natural relative to its position p_i* when it makes for similarities among occupants of p_i and it's needed to characterize them in a complete characterization of reality.⁴⁶ Given this, set-membership might be perfectly natural relative to its set-position, but not relative to its member-position, for, intuitively, it might make for similarities among occupants of its set-position, and it might be needed to characterize everything completely and without redundancy if it's needed to characterize things that occupy the set-position.

Now, our first puzzle showed that existential derivatives of a non-symmetric relation can differ in naturalness even when the relation is among the fundamental ones. And since, according to position-absolutism, such relations must be perfectly natural (*simpliciter*), position-absolutism incorrectly implies that its existential derivatives must be equally natural.

Position-relativism allows for fundamental non-symmetric relations whose existential derivatives aren't equally natural. Let's say that a relation is *fundamental* iff it's perfectly natural relative to at least one of its positions; call these positions *perfectly natural*. Fundamental relations are needed for a complete characterization of reality, but their positions needn't all be perfectly natural. Thus, set-membership may be a fundamental relation in virtue of it being perfectly natural relative to only one of its positions, the set-position.

⁴⁶When I say that a relation makes for similarities 'among occupants' of a position p_i , I mean to state something about R and p_i such that, as a consequence, the monadic existential derivative of R that leaves only p_i "free" makes for similarity among its instances. However, neither this nor the explanation in the main text is an analysis of position-relativist perfect naturalness. This notion is primitive just like Lewis suggested his notion could be.

4.3 Fundamental Definitions

Standardly, a property or relation is less-than-perfectly natural iff it is related to the perfectly natural ones by not-too-complex definitions; moreover, the less complex its most fundamental definition is, the more natural it is.⁴⁷ Likewise, given position-relativism about perfect naturalness, a position is less-than-perfectly natural iff it is related to the perfectly natural positions by not-too-complex definitions; moreover, the less complex its most fundamental definition is, the more natural it is. I develop this below.

A *fundamental language* has primitive predicates for exactly all fundamental properties and relations. But some argument-places of such predicates are syntactically different, say, underlined, not by an arbitrary stipulation, but because they stand for perfectly natural positions. For example, the primitive ‘ $\in \underline{_}$ ’ can express set-membership while its underlined argument-place stands for the set-position. Indeed, each primitive predicate of such a language is such that all and only argument-places that stand for perfectly natural positions are underlined. A definition of a relation R that is fundamental relative to its position p , or more simply, a *fundamental definition* of p of R , is any predicate ϕ of the fundamental language that expresses R and whose lambda-variable that stands for p occurs in underlined argument-places and only in such argument-places. Now, let the complexity of a definition of R relative to a *position* p of R be the complexity of that definition relative to the variable standing for p . Among the fundamental definitions of p of R , a *most* fundamental one is one that is least complex relative to p .

Position-relative Definitional Conception A relation R is *completely unnatural* relative to its position p iff every fundamental definition of p is too complex relative to p . A relation R is *more natural* relative to its position p than a relation S is natural relative to its position q iff R isn’t completely unnatural relative to p and a most fundamental definition of p of R is simpler relative to p than any fundamental definition of q of S is to q .

Thus, consider predicates P_1 and P_2 below.

$$P_1) (\lambda y. \exists x x \in \underline{y})$$

$$P_2) (\lambda x. \exists y x \in \underline{y})$$

⁴⁷See §2.1, fn. 8 and 9

Whereas P_1 is a fundamental and not-too-complex definition of (the only position of) *having members*, P_2 is not a fundamental definition of (the only position of) *being a member*. Thus, as far as P_2 goes, *being a member* would be completely unnatural (to its only position).⁴⁸ And this property will, in fact, be completely unnatural unless there is some not-too-complex fundamental definition of it. Likewise, as far as ' $(\lambda xy.x \in \underline{y})$ ' goes, set-membership would be completely unnatural relative to its member-position, for ' $(\lambda xy.x \in \underline{y})$ ' is not a fundamental definition relative to the member-position. And set-membership will, in fact, be completely unnatural relative to its member-position unless there is some definition of this relation that is fundamental and not too complex relative to the member-position.⁴⁹

Note that the absence of such a definition of the member-position concurs with the complete unnaturalness of *being a member*, which explains the latter's irrelevance for similarity. (See the first argument of §3.1.1) Conversely, if we found such a definition, it should convince us that *being a member* is, despite appearances, relevant. Even then, however, note that the complexity of ' $(\lambda xy.x \in \underline{y})$ ' relative to ' \underline{y} ' is zero,⁵⁰ and there is no zero-complexity fundamental definition of the member-position. Thus, set-membership is more natural to its set-position than to its member-position, which explains the greater naturalness of *having a member*. (See the second argument of §3.1.1) Therefore, position-relativism can deliver the desired results concerning existential derivatives, and to this extent, it is preferable to position-absolutism.⁵¹

5 The definitional conception

One may think that to escape the puzzles, we should reject the definitional conception rather than reject position-absolutism.⁵² This can be supported by noting a tension between the definitional conception, Lewis' minimalism, and the application of naturalness to a theory of

⁴⁸The notion of ‘too complex’ here is the Lewisian one. (See fn. 12) Alternatively, position-relativists could count any definition of a relation as fundamental to any position but attribute this complete unnaturalness to a special notion of ‘too complex’.

⁴⁹Alternative views are conceivable. For example, every relation could have one ‘canonical’ definition—maybe simplest in some position-absolutist sense—by reference to which all positions of a relation must be compared.

⁵⁰See fn. 11.

⁵¹If relations have directions, set-membership is distinct from member-sethood, which relates the same things but always in the opposite direction. One could then i) attribute perfect naturalness to set-membership but not to member-sethood, ii) let *having members* be definable from set-membership, and iii) deny that *being a member* is definable from set-membership. The resulting view is close to position-relativism, but it is arbitrary because of (i). See Fine (2000, 2-7) and Dorr (2004, §4) for analogous arbitrariness claims.

⁵²I won’t address arguments against fundamental non-symmetric relations such as Dorr’s (2004). Even if successful, they would not avoid position-relativism concerning less-than-perfectly natural relations. See also Whittle (2010) and MacBride (2015) on Dorr’s argument.

meaning.⁵³ According to Lewis' minimalism, the only actual perfectly natural properties are the ones identified by fundamental physics. Moreover, naturalness can provide an eligibility constraint to a theory of meaning: *ceteris paribus*, the more natural a property is, the better is the interpretation that assigns it to a predicate. But consider two interpretations of our utterances that assign radically different references to our predicates but that are equally good concerning everything except eligibility (e.g., the number of utterances that are true is nearly the same in both). Suppose that among other differences, one interpretation assigns *being a speaker* to 'is a speaker', whereas the second assigns a crazy, intuitively unintended, property *C* to this predicate.⁵⁴ The eligibility constraint should settle for the intended interpretation based on, among others, the claim that *being a speaker* is more natural than *C*. However, since speakers could have had many microphysical structures different from ours, the definition of *being a speaker* might be infinitely complex. The worry is that it won't be significantly simpler than the definition of *C*.

Given the tension above, Hawthorne (2007, 434) rejects the definitional conception and suggests that comparative naturalness might be primitive. I reject Lewis' minimalism.

There are compelling arguments for rejecting minimalism. Schaffer (2004), for example, showed that Lewis' minimalism is in tension with the claim that perfectly natural properties make for similarities (and ground causal powers). He offered a scientific conception according to which, even if microphysics provides a supervenience base for every other truth, properties of higher levels of science might be perfectly natural if they make for similarities (and ground causal powers) to high degrees. *Being a speaker* might be one of the perfectly natural properties in this conception, or at least it must be closer to one of them than *C* is. Notice, however, that a scientific conception can still endorse a significant non-redundancy criterion so long as it's relative to a level of nature. According to this understanding, redundancy would be allowed between, but not within, levels of nature.⁵⁵

Thus, I don't take the first tension as a good reason to reject the definitional conception. But the puzzles from non-symmetric relations depend on the much less controversial claim that naturalness must account for similarity—as opposed to meaning. Moreover, they don't depend on Lewis' minimalism. For we can raise our first and second puzzles from properties already in

⁵³See (Hawthorne 2007, 433–4), (Williams 2007, 374–8), and (Sider 1995, 363–4; 2011, §3.2).

⁵⁴(Hawthorne 2007, 433–4).

⁵⁵Schaffer (2004) is silent on such versions of non-redundancy. But nothing he says goes against them. This is important because I appealed to a non-redundancy requirement before, in §3.1. I defend elsewhere that naturalness, and non-redundancy, must be relativized to *sorts* of things, which avoids the tension above.

the second and third levels of our chain of definitions—rather than in allegedly infinitely high levels. Therefore, if one was after an objection to the definitional conception, a stronger one is in §3 of this paper. But I don't think the right lesson is to reject the definitional conception here either.

There are two options: either stick with the definitional conception and adopt position-relativism, or adopt, as Hawthorne suggested, a primitivism about comparative naturalness, saving position-absolutism. The costs of adopting position-relativism aren't higher. For position-relativism about perfect naturalness is as intelligible as position-absolutism; likewise, the notion of position-relativized complexity doesn't require more resources than a position-absolutist one. Thus, we must compare the definitional conception to primitivism about comparative naturalness. I favor the definitional conception for the following reasons.

First, there is a relation of dependence between the naturalness of properties such as *being green or blue*, and *being green* and *being blue*. The definitional conception has the virtue of explaining this dependence. Taking comparative naturalness as primitive leaves it unexplained.

Second, the same point applies to relations and their derivatives. In §2, I argued that the definitional conception has the potential to explain why and how the degrees of naturalness of genuine existential derivatives depend on the degrees of naturalness of the relations they derive from. I take this explanandum as an independently plausible phenomenon. Hawthorne, who is not fond of the definitional conception, assumes this dependence when he infers that ‘If [a relation] R is highly natural, [its] existential derivative may not be quite as natural. But it will be pretty natural.’ (Hawthorne 2001, 399) Of course, a result of this paper is that Hawthorne's claim must be qualified; it must be relativized to positions. The idea remains, however, that their naturalnesses are somehow dependent on one another. Taking comparative naturalness as primitive doesn't explain this fact. The definitional conception—supplemented with position-relativism—does.

Finally, suppose we reject the definitional conception. Our puzzles would be avoided. But something puzzling would remain. We would avoid the unintended result concerning the degree of similarity among the instances of existential derivatives but would be left with something that asks for an explanation anyway. For if the dependence between the naturalness of derivatives and their original relations is independently plausible, then it's independently puzzling that, whereas set-membership is *perfectly* natural and one of its existential derivatives is *very* natural, the other existential derivative is highly *unnatural*. The definitional conception—with position-

relativism—explains this. Primitivism about comparative naturalness doesn't.

6 Further Work

My argument thus far has been that position-relativism is preferable to its alternative because it is better connected to similarity than its alternative. Here, I would like to briefly indicate a few further contexts where position-relativism seems preferable to position-absolutism.

Consider *intrinsicality*. Langton and Lewis (1998) suggested that the basic intrinsic properties are independent of accompaniment, not disjunctive, and not negations of disjunctive properties. Something is accompanied (by x) iff it coexists but doesn't overlap with a contingent object (x). A property P is disjunctive iff it can be expressed by disjunctions of (conjunctions of) properties that are more natural than P . Marshall and Parsons (2001) raised this counterexample: *being such that there are cubes*. This isn't intrinsic but it would seem to fit Langton and Lewis' criteria since, Marshall and Parsons argue, even though it can be expressed by a disjunctive predicate, namely, ' $(\lambda x.(Cx \vee \exists y(Axy \& Cy)))$ ', where ' C ' expresses *being a cube* and ' A ' *being accompanied by*, it is not a disjunctive property, for the property corresponding to the second disjunct, *being accompanied by a cube* (expressed by ' $(\lambda x.\exists y(Axy \& Cy))$ '), isn't more natural than *being such that there are cubes*.

But how can the last comparison be substantiated under the definitional conception? Of course, Marshall and Parsons cannot substantiate their claim on a comparison between the predicates ' $(\lambda x.\exists y(Axy \& Cy))$ ' and ' $(\lambda x.(Cx \vee \exists y(Axy \& Cy)))$ ', for the second is more complex ('by a disjunction'⁵⁶) than the first. But the predicate ' $(\lambda x.\exists yCy)$ '⁵⁷ can easily substantiate Marshall and Parsons' comparison, for this predicate is (under position-absolutism) a fundamental definition of *being such that there is a cube* and it is less complex than ' $(\lambda x.\exists y(Axy \& Cy))$ '. Given position-relativism, however, ' $(\lambda x.\exists yCy)$ ' isn't a fundamental definition of *being such that there are cubes* since its only lambda-variable (' x ') doesn't occur in underlined argument-places. Likewise, predicates of the form $\ulcorner(\lambda x.(\phi x \& \exists yCy))\urcorner$ plausibly fail to be fundamental—e.g., if ϕ is a simple, primitive predicate but true of everything like ' $x = x$ '⁵⁸—or fail to be simpler than ' $(\lambda x.\exists y(Axy \& Cy))$ '.⁵⁹

⁵⁶Langton and Lewis argue: "it seems to us (1) that *being accompanied by a cube* is less natural than *being a cube*, and (2) that *being either a cube or accompanied by a cube* is less natural still by a disjunction." (2001, 354)

⁵⁷Cp. Marshall (2012, 538).

⁵⁸See §3.1.2 on identity.

⁵⁹Relatedly, consider Geach's (1969, 66, 71–2) distinction between Cambridge and real change. Imagine that God annihilates the singleton {Socrates}: a mere Cambridge change for Socrates, but a real one for sets containing

Second, consider *haecceistic properties*. As Dorr and Hawthorne (2013, fn. 30) observed, if the fundamental language doesn't have names at all, *living in Oxford* is unlikely to be ranked by naturalness. But, if it has names for everything, it has names for abundant properties themselves, and thus, all properties that aren't perfectly natural may turn out to be very, and equally, natural. For let ' I ' be a predicate for *instantiating*, whatever property the name ' f ' denotes, the (very simple) predicate ' $(\lambda x.Ix f)$ ' expresses that property. Either way, there is a problem.

However, assume position-relativism. Given that the second horn above assumes an abundant conception of properties, the referred-to *instantiation* relation is analogous, or identical, to set-membership. Thus, there is pressure to take it as natural relative to its property-position but not to its instance-position. Accordingly, let ' $(\lambda xy.Ixy)$ ' express this relation. The predicate ' $(\lambda x.Ix f)$ ' fails to be a fundamental definition of the property named by ' f ' since the lambda-variable (' x ') occurs in non-underlined argument-places. Thus, the second horn doesn't look as problematic here as it is under position-absolutism.

Finally, consider *essence*. According to Wildman's (2013) sparse modalism, x is essentially related to y iff x is necessarily related to y and the relation is perfectly natural. Fine (1994) argued that, whereas the singleton $\{\text{Socrates}\}$ necessarily and essentially has Socrates as a member, Socrates is necessarily but not essentially a member of the singleton. According to position-absolutists, relations are perfectly natural or not *simpliciter*. Thus, position-absolutist naturalness alone can't help the project of analyzing essence in terms of necessity when it comes to Fine's counter-example. But sparse modalism with position-relativist naturalness can do a better job. For it may contain as a necessary condition not that the relation is perfectly natural *simpliciter* but that the relation must be perfectly natural relative to the position occupied by x . Therefore, someone who is already attracted to a theory of naturalness and wants to account for a Finean notion of essence in terms of it has some reason to prefer position-relativism to its alternative.

Of course, the mere adoption of position-relativism isn't enough to support the abovementioned accounts. More must be said, and rival accounts and further cases must be considered.⁶⁰ What I wanted to show was just this: the potential of position-relativism to solve problems that

the singleton. Position-relativists can explain this asymmetry. A necessary condition for genuine change for x could be that x loses (or gains) a property or relation that is natural to a non-zero degree relative to the position previously (or now) occupied by x .

⁶⁰Even among naturalness theories there are other possibilities; see Lewis (2001) and Weatherson (2001) for the problem involving intrinsicality. See also Skiles (2015) for another objection to Wildman (2013).

under position-absolutism seem difficult to solve.

7 Conclusion

The puzzles from genuine existential derivatives constitute a thus far overlooked problem for the standard Lewisian view on naturalness. The view entails that, given any binary relation, its two genuine existential derivatives are equally natural. However, this is false, as shown by facts about similarity among objects that instantiate derivatives of some non-symmetric relations. I have argued that an obvious reaction to the problem—namely, that of abandoning the Lewisian definitional analysis of comparative naturalness in favor of a primitivism about it—would leave too much unexplained. Instead, I developed position-relativism—the view that relations can be natural to different degrees relative to different positions. I developed the view in two complementary ways, concerning perfect naturalness and concerning complexity, and I showed how this view can explain the relevant phenomena in various instances of the problem from genuine existential derivatives. To further support taking position-relativism seriously, I argued that it is preferable to position-absolutism in a few further contexts—namely, contexts involving some accounts of intrinsicality, haecceitic properties, and essence.

This leaves us with two general questions. First, whether the position-relativized notion can still do the other explanatory jobs that the position-absolutist one was supposed to do. Second, whether it can do even more. Like position-absolutism, however, position-relativism still owes a precise analysis of comparative naturalness.

References

- Bennett, K. (2017). *Making Things Up*. Oxford University Press, Oxford.
- Blumson, B. (2018). Two Conceptions of Similarity. *The Philosophical Quarterly*, 68(270):21–37.
- Burgess, J. P. (2005). *Fixing Frege*. Princeton Monographs in Philosophy. Princeton University Press, Princeton, NJ.
- Dorr, C. (2004). Non-symmetric Relations. In Zimmerman, D., editor, *Oxford Studies in Metaphysics*, pages 155–192.
- Dorr, C. and Hawthorne, J. (2013). Naturalness. In Bennett, K. and Zimmerman, D., editors, *Oxford Studies in Metaphysics, Vol. 8*, pages 1–77. Oxford University Press.

- Fine, K. (1994). Essence and Modality. *Philosophical Perspectives*, 8:1–16.
- Fine, K. (2000). Neutral Relations. *The Philosophical Review*, 109(1):1–33.
- Geach, P. T. (1969). *God and the Soul*. St. Augustine's Press.
- Hawthorne, J. (2001). Intrinsic Properties and Natural Relations. *Philosophy and Phenomenological Research*, 63(2):399–403.
- Hawthorne, J. (2006). Quantity in Lewisian Metaphysics. In *Metaphysical Essays*. Clarendon, Oxford.
- Hawthorne, J. (2007). Craziness and Metasemantics. *The Philosophical Review*, 116(3):427–440.
- Holmes, M. R. (2017). Alternative Axiomatic Set Theories. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Winter 2017 edition.
- Jaccard, P. (1912). The Distribution of the Flora in the Alpine Zone.1. *New Phytologist*, 11(2):37–50.
- Langton, R. and Lewis, D. (1998). Defining 'Intrinsic'. *Philosophy and Phenomenological Research*, 58(2):333–345.
- Langton, R. and Lewis, D. (2001). Marshall and Parsons on 'Intrinsic'. *Philosophy and Phenomenological Research*, 63(2):353–355.
- Lewis, D. (1983). New work for a theory of universals. *Australasian Journal of Philosophy*, 61(4):343–377.
- Lewis, D. (1984). Putnam's paradox. *Australasian Journal of Philosophy*, 62(3):221–236.
- Lewis, D. (1986). *On the Plurality of Worlds*. Blackwell Publishers, Malden Mass.
- Lewis, D. (1991). *Parts of Classes*. Blackwell.
- Lewis, D. (2001). Redefining 'Intrinsic'. *Philosophy and Phenomenological Research*, 63(2):381–398.
- Lewis, D. (2008). Ramseyan Humility. In Braddon-Mitchell, D. and Nola, R., editors, *Conceptual Analysis and Philosophical Naturalism*, pages 203–218. The MIT Press.
- Lewis, D. K. (2020). *Philosophical Letters of David K. Lewis*, volume 1. Oxford University Press, Oxford, UK.
- MacBride, F. (2015). On The Origins of Order: Non-Symmetric or Only Symmetric Relations? In Loux, M. J. and Galuzzo, G., editors, *The Problem of Universals in Contemporary Philosophy*, pages 173–94. Cambridge University Press.
- Maddy, P. (1983). Proper Classes. *The Journal of Symbolic Logic*, 48(1):113–139.

- Marshall, D. (2012). Analyses of Intrinsicality in Terms of Naturalness. *Philosophy Compass*, 7(8):531–542.
- Marshall, D. and Parsons, J. (2001). Langton and Lewis on “Intrinsic”. *Philosophy and Phenomenological Research*, 63(2):347–351.
- Nolan, D. (2005). *David Lewis*. Acumen, Chesham.
- Quine, W. V. O. (1980). *Set Theory and Its Logic*. Belknap Pr, Cambridge, Mass, rev. ed. edition.
- Schaffer, J. (2004). Two conceptions of sparse properties. *Pacific Philosophical Quarterly*, 85(1):92–102.
- Sider, T. (1995). Sparseness, Immanence, and Naturalness. *Noûs*, 29(3):360–377.
- Sider, T. (2011). *Writing the Book of the World*. Oxford University Press, Oxford:.
- Skiles, A. (2015). Essence in Abundance. *Canadian Journal of Philosophy*, 45(1):100–112.
- Weatherson, B. (2001). Intrinsic Properties and Combinatorial Principles. *Philosophy and Phenomenological Research*, 63(2):365–380.
- Whittle, B. (2010). There are Brute Necessities. *The Philosophical Quarterly*, 60(238):149–159.
- Wildman, N. (2013). Modality, Sparsity, and Essence. *The Philosophical Quarterly*, 63(253):760–782.
- Williams, J. R. G. (2007). Eligibility and Inscrutability. *Philosophical Review*, 116(3):361–399.