The Logarithmic Filter

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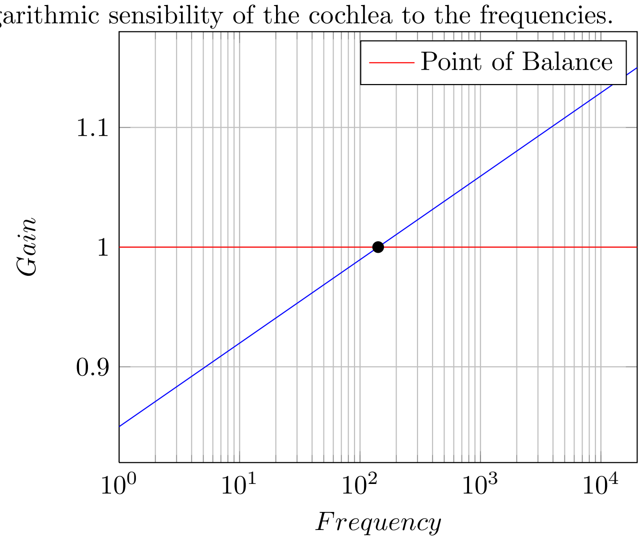
November 4, 2013

Abstract

This document gathers technical information for building a logarithmic filter.

1 Goal

In the aim of making a linear filter for the ear we have to take into account the logarithmic sensibility of the cochlea to the frequencies.



$Gain = 1$ Point of Balance

2 Equations

$$G(f) = m \log_s f + G_0 - m \log_s F_{MIN} \tag{1}$$

where

G	:	Gain
G_0	:	Initial Gain or $G(F_{MIN}) = 1 - \delta$
s	:	Scale
f	:	Frequency
F_{MIN}	:	Lower frequency
F_{MAX}	:	Upper frequency
B	:	Point of Balance
m	:	slope

Or taking C as $C = G_0 - m \log_s F_{MIN}$

$$G(f) = m \log_s f + C$$

or even expressed as the natural logarithm,

$$G = m \frac{\ln f}{\ln s} + C$$

If G is defined as $1 \pm \delta$ we get, some useful equations.

$$2\delta = m \log_s \frac{F_{MAX}}{F_{MIN}} \tag{2}$$

$$m = \frac{2\delta}{\log_s \frac{F_{MAX}}{F_{MIN}}} \tag{3}$$

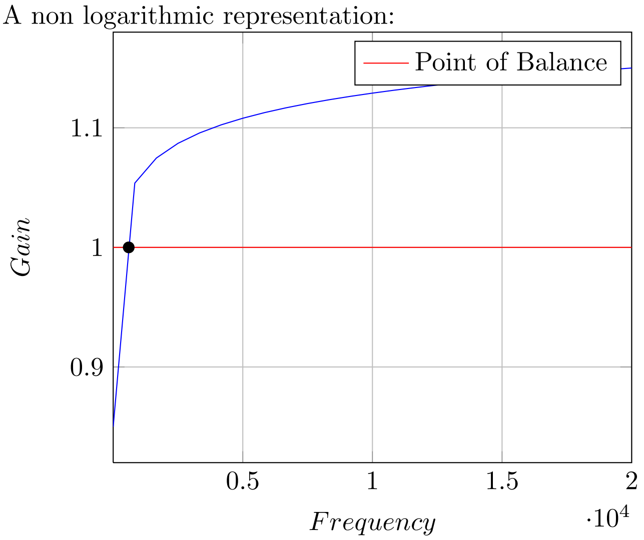
$$\ln B = \frac{1}{2} \log_s F_{MAX} \cdot F_{MIN} \tag{4}$$

$$B = \sqrt{F_{MAX} \cdot F_{MIN}} \tag{5}$$

m and s work in pair, $s = s(m)$.

$$\ln s = \frac{2\delta}{m \ln \frac{F_{MAX}}{F_{MIN}}}$$

A non logarithmic representation:



3 Transfer Function

Let's suppose,

$$H(j\omega) = m \ln j\omega + 1 - \delta - m \ln F_{MIN} \tag{6}$$

or just

$$H(j\omega) = m \ln j\omega + C \tag{7}$$

and let's us assume

$$s = j\omega$$

$$H(s) = m \ln s + C$$

The Inverse Laplace transformation $h(t) = \mathcal{L}^{-1}\{F(s)\}$ can be obtained by knowing:

$$\begin{aligned}
\mathcal{L}\{\alpha f(t) + \beta g(t)\} &= \alpha F(s) + \beta G(s) && (8)\\
\mathcal{L}\{-t \cdot f(t)\} &= \frac{d}{ds} F(s) && (9)\\
\mathcal{L}\{\delta(t)\} &= 1 && (10)\\
\mathcal{L}\{u(t)\} &= \frac{1}{s} && (11)
\end{aligned}$$

So,

$$\mathcal{L}^{-1}\{m \ln s + C\} = \mathcal{L}^{-1}\{m \ln s\} + C \cdot \delta(t)$$

and

$$\begin{aligned}
t \cdot h(t) &= \mathcal{L}^{-1}\{\frac{d}{ds}(m \ln s)\} && (12)\\
&= m \cdot \mathcal{L}^{-1}\{\frac{1}{s})\} && (13)\\
&= m \cdot u(t) && (14)
\end{aligned}$$

all together,

$$\boxed{h(t) = \frac{m \cdot u(t)}{t} + C \cdot \delta(t)} \tag{15}$$

4 Approach

So we suggest to try the convolution

$$y(t) = h(t) * x(t)$$

in a discretized way.