TRAINMULTINOMIALNB($\mathbb{C}, \mathbb{D}$)
1 $V \leftarrow$ EXTRACTVOCABULARY($\mathbb{D}$)
2 $N \leftarrow$ COUNTDOCS($\mathbb{D}$)
3 **for each** $c \in \mathbb{C}$
4 **do** $N_c \leftarrow$ COUNTDOCSINCLASS($\mathbb{D}, c$)
5 $prior[c] \leftarrow N_c / N$
6 $text_c \leftarrow$ CONCATENATETEXTOFALLDOCSINCLASS($\mathbb{D}, c$)
7 **for each** $t \in V$
8 **do** $T_{ct} \leftarrow$ COUNTTOKENSOFTERM($text_c, t$)
9 **for each** $t \in V$
10 **do** $condprob[t][c] \leftarrow \frac{T_{ct}+1}{\sum_{t'}(T_{ct'}+1)}$
11 **return** $V, prior, condprob$

APPLYMULTINOMIALNB($\mathbb{C}, V, prior, condprob, d$)
1 $W \leftarrow$ EXTRACTTOKENSFROMDOC(V, d)
2 **for each** $c \in \mathbb{C}$
3 **do** $score[c] \leftarrow \log prior[c]$
4 **for each** $t \in W$
5 **do** $score[c] \mathrel{+}= \log condprob[t][c]$
6 **return** $\arg\max_{c \in \mathbb{C}} score[c]$

▶ **Figure 13.2** Naive Bayes algorithm (multinomial model): Training and testing.

assign a high probability to the *UK* class because the term Britain occurs. The problem is that the zero probability for WTO cannot be "conditioned away," no matter how strong the evidence for the class *UK* from other features. The estimate is 0 because of *sparseness*: The training data are never large enough to represent the frequency of rare events adequately, for example, the frequency of WTO occurring in *UK* documents.

SPARSENESS

ADD-ONE SMOOTHING

To eliminate zeros, we use *add-one* or *Laplace smoothing*, which simply adds one to each count (cf. Section 11.3.2):

$$\hat{P}(t|c) = \frac{T_{ct}+1}{\sum_{t' \in V}(T_{ct'}+1)} = \frac{T_{ct}+1}{(\sum_{t' \in V} T_{ct'}) + B}, \tag{13.7}$$

where $B = |V|$ is the number of terms in the vocabulary. Add-one smoothing can be interpreted as a uniform prior (each term occurs once for each class) that is then updated as evidence from the training data comes in. Note that this is a prior probability for the occurrence of a *term* as opposed to the prior probability of a *class* which we estimate in Equation (13.5) on the document level.