A novel approach to the priority vector deriving from intuitionistic fuzzy preference relations ☆

Fang Liu*, Xu Tan

School of Mathematics and Information Science, Guangxi University, Nanning, Guangxi 530004, China

Abstract

In the typical analytic hierarchy process (AHP), consistency analysis of matrix is very important, so it is very important to analyze the consistency of intuitionistic fuzzy matrix, when decision makers construct matrix by intuitionistic fuzzy number. In this paper, a new approximation-consistency of intuitionistic fuzzy preference relation is proposed. Then several properties related to the consistency of the new definition are given. In considering the randomness of pairwise comparison of alternatives, a new method for calculating intuitionistic fuzzy matrix weight vector is proposed. A new algorithm of solving the decision making problem with intuitionistic fuzzy matrix is proposed. Finally, two numerical examples are calculated by the proposed new definition, and the calculated results are compared with the known ones.

Keywords: Intuitionistic fuzzy matrix; Interval weight; Approximate consistency; Permutation of alternatives

1. Introduction

In 1965, Zadeh[1] proposed fuzzy set theory. The fuzzy set is an extension of the classical Cantor set. At present, the research on fuzzy set theory has been

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*Corresponding author

Email addresses: fang272@126.com (Fang Liu), 895657316@qq.com (Xu Tan)

very mature. Scholars at home and abroad consider how to expand the theory of fuzzy set, and various expansion forms emerge, such as interval fuzzy number, intuitionistic fuzzy number, trapezoidal fuzzy number, etc. Among them, intuitionistic fuzzy set theory is the most active. Intuitionistic fuzzy sets were proposed by Atanassov [2]. A series of definitions and algorithms proposed by Atanassov [2]laid the foundation of intuitionistic fuzzy set theory. The disadvantage of fuzzy set theory is that it has only one membership function. When the decision is disapproved for scheme, it cannot describe. For example, a voting model with yes, no, and no votes. The introduction of intuitionistic fuzzy set solves this problem well. The membership function, non-membership function and the third parameter of intuitionistic fuzzy set can express the three attitudes of support, opposition and neutrality. Since intuitionistic fuzzy set theory was proposed, some scholars have combined it with Saaty's[3] analytic hierarchy process. A large number of scholars at home and abroad have studied the intuitionistic fuzzy AHP theory. In 2007, Xu [4] proposed the intuitionistic fuzzy matrix and solved the decision-making problem of intuitionistic fuzzy. In analytic hierarchy process, it is very important to obtain the weight of matrix. Xu[5] constructed a linear programming model to obtain an accurate weight. Liao[6] obtained the precise weights by introducing relaxation variables. Considering that intuitionistic fuzzy set are mathematically equivalent to interval fuzzy set, Gong[7] obtained the priority of interval values by constructing the minimum variance model, and Xu[8] obtained the priority weight of interval value of intuitionistic fuzzy preference relation by error analysis. From the characteristics of intuitionistic fuzzy set itself, Xu[9] obtained the intuitionistic fuzzy priority weight through normalized sequential summation. By constructing linear programming model, Wang[10] obtained intuitionistic fuzzy weights. Liao[11] obtained the intuitionistic fuzzy priority weights by constructing a fractional programming model. In short,there are three main forms to obtain the weights of intuitionistic fuzzy preference relation. The first is the exact weight, which is the traditional real weight. The second is the interval weights, which indicates the membership range of the importance of the scheme. The third is intuitionistic fuzzy weights, which represent the membership and non-membership of the importance of the scheme by intuitionistic fuzzy number. In analytic hierarchy process (AHP), the consistency analysis of matrix is also important. The consistency of matrix determines whether the final weight is credible or not. Therefore, it is very important to study the consistency of intuitionistic fuzzy matrix. At home and abroad, the additive consistency, multiplicative consistency, satisfied consistency and weak transfer of intuitionistic fuzzy matrix are mainly studied. The research on additive consistency and multiplicative consistency of intuitionistic fuzzy matrix is the most active. Xu[5] transforms the intuitionistic fuzzy matrix into the interval additive reciprocal matrix by using the transformation formula between the intuitionistic fuzzy preference relation and the interval fuzzy preference relation. Xu[5] think the original intuitionistic fuzzy matrix with additive consistency or multiplicative consistency,if the interval additive reciprocal matrix with additive consistency or multiplicative consistency. Gomg [7] proposed the additive consistency of intuitionistic fuzzy matrix by using interval weight vector. According to the definition of additive consistency of additive reciprocal matrix, Wang [10] proposed additive consistency only considering membership degree. At the same time, Wang[10] introduced the concept of standardized intuitionistic fuzzy weight vector and defined additive consistency. According to the transformation relation between intuitionistic fuzzy preference relation and interval fuzzy preference relation, Gong [12] proposed the multiplicative consistency. Xu [13] gives a piecewise multiplicative consistency. Liao[11]defined multiplicative consistency by introducing the concept of standardized intuitionistic fuzzy weight vector. There are few studies on the satisfied consistency and weak consistency of intuitionistic fuzzy matrix. Xu[14] constructed the completely multiplicative consistent matrix, and then constructed the consistency index by using the distance between the general intuitionistic fuzzy matrix and the completely multiplicative consistent matrix. When the consistency index reached the requirements of the decision maker, Xu[14] believed that the intuitionistic fuzzy matrix was satisfied consistency. Xu[15]defined the weak transitivity of intuitionistic fuzzy matrix according to the weak transitivity of additive reciprocal matrix. In general,there are three kinds of research on consistency of intuitionistic fuzzy matrix. Some people[10] [13] [11] turn intuitionistic fuzzy matrix into interval additive reciprocal matrix, according to the consistency of interval additive reciprocal matrix, consistency of intuitionistic fuzzy matrix is given. Some[7][12] directly generalize the consistency of additive reciprocal matrix to the intuitionistic fuzzy matrix . Some[4]construct the consistency matrix based on the relationship between the intuitionistic fuzzy matrix and its weight vector.

According to the search of literatures at home and abroad, we find that the consistency definition of intuitionistic fuzzy matrix is mostly given from the traditional additive reciprocal matrix or interval additive reciprocal matrix, but rarely from the characteristics of intuitionistic fuzzy matrix itself. Scholars have given various definitions of consistency without considering the randomness of scheme arrangement. For example, the consistency index of the intuitionistic fuzzy matrix proposed by Xu[14]changes with the change of the order of alternative schemes. When we construct a consistency indicator, we should require that it should not change under any arrangement. In this article, through the transformation formula of between intuitionistic fuzzy preference relation and interval additive preference relation,we transform the intuitionistic fuzzy matrix into the interval additive reciprocal matrice, and put forward the approximate consistency of intuitionistic fuzzy matrix considering the arrangement of schemes, and give a new method to obtain the interval weight vector of intuitionistic fuzzy matrix. A new algorithm for solving decision-making problems with intuitionistic fuzzy matrix has been given. Two examples have been carried out to illustrate the proposed definition, algorithm and some comparisons have been made.

The structure of this paper is shown as follows. In Section 2, we analyze the definition of consistent of intuitionistic fuzzy matrix. In Section 3, a new concept of approximation-consistency of intuitionistic fuzzy matrix is proposed. In Section 4, the method of obtaining the interval weight vector of intuitionistic fuzzy matrix is given. In Section 5,we offer a new algorithm to the decision-making problem with intuitionistic fuzzy matrix. Two examples are carried out to illustrate the new definition and algorithm. The main conclusions are shown in Section 6.

2. Preliminaries

In 1965, Zadeh[1] proposed the fuzzy set theory, and in 1986, Atanassov[2] extended the fuzzy set and proposed the intuitionistic fuzzy set. Firstly, we give the concept of fuzzy set and intuitionistic fuzzy set:

Definition 1. [1] Let X is a non-empty set, F is constituted by μ_F whose value is [0,1],where $\mu_F : X \rightarrow [0,1]$, we call F as fuzzy set on X.

Definition 2. [2] Let X is a non-empty set, we call
$A = \{< x, \mu_A(x), v_A(x) > | x \in X\}$ is the intuitionistic fuzzy set,where $0 \leq \mu_A(x) \leq 1, \quad 0 \leq v_A(x) \leq 1, \quad 0 \leq \mu_A(x) + v_A(x) \leq 1.$

among them, $\mu_A(x)$ and $v_A(x)$ are the membership and non-membership of the element x, and $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ represents the hesitation x belonging to A.

Definition 3. [15] Let $\alpha = (\mu_\alpha \nu_\alpha)$ an intuitionistic fuzzy number, where $0 \leq \mu_\alpha \leq 1$ $0 \leq v_\alpha \leq 1, 0 \leq \mu_\alpha + v_\alpha \leq 1$. The score function and accurate function are defined as

$$S(\alpha) = \mu_\alpha - v_\alpha, H(\alpha) = \mu_a + v_a. \tag{1}$$

Based on the above functions, Xu[15] proposed the following way to compare the sizes of intuitionistic fuzzy number $a_i = (\mu_i, \nu_i)$ and $a_j = (\mu_j, \nu_j)$:
Step 1. Calculate the S values of the IFVs $a_i = (\mu_i, \nu_i)$ and $a_j = (\mu_j, \nu_j)$ using 1.
Step 2. If $S(\alpha_i) < S(\alpha_j)$, then $\alpha_i < \alpha_j$, then go to Step 3.
Step 3. Calculate the accuracy degrees of these IFVs using the accuracy function $H(\alpha) = \mu_a + v_a$, and then rank the IFVs according to the following principles:
(1)If $H(\alpha_i) > H(\alpha_j)$,then $\alpha_i > \alpha_j$;

(2)If $H\left(\alpha_i\right)=H\left(\alpha_j\right)$,then $\alpha_i=\alpha_j$.

Beliakov[?] pointed out that the above ordering scheme is not closed to the scalar product of intuitionistic fuzzy numbers.In other words, $\alpha_i < \alpha_j$ does not necessarily mean $\lambda\alpha_i < \lambda\alpha_j$, where λ is a scalar. Based on the similarity function and the exact function, Zhang[?] proposed a full order scheme to compare any two intuitionistic fuzzy numbers.

Definition 4. [?] Let $\alpha = (\mu_\alpha \nu_\alpha)$ an intuitionistic fuzzy number, where $0 \leq \mu_\alpha \leq 1$ $0 \leq v_\alpha \leq 1, 0 \leq \mu_\alpha + v_\alpha \leq 1$. The similarity function and accurate function are defined as

$$L(\alpha) = \frac{1-v_\alpha}{1+\pi_\alpha}, H(\alpha) = \mu_a + v_a. \tag{2}$$

Based on the above functions, Zhang[?] proposed the following way to compare the sizes of intuitionistic fuzzy number $a_i = (\mu_i, \nu_i)$ and $a_j = (\mu_j, \nu_j)$:

Step 1. Calculate the L values of the IFVs $a_i = (\mu_i, \nu_i)$ and $a_j = (\mu_j, \nu_j)$ using 2.

Step 2. If $L\left(\alpha_i\right) < L\left(\alpha_j\right)$, then $\alpha_i < \alpha_j$, then go to Step 3.

Step 3. Calculate the accuracy degrees of these IFVs using the accuracy function $H(\alpha) = \mu_a + v_a$, and then rank the IFVs according to the following principles:

(1)If $H\left(\alpha_i\right) > H\left(\alpha_j\right)$, then $\alpha_i > \alpha_j$;

(2)If $H\left(\alpha_i\right) = H\left(\alpha_j\right)$, then $\alpha_i = \alpha_j$.

The algorithm for intuitionistic fuzzy number is given below:

Definition 5. [15] If $a_{ij} = (\mu_{ij}, \nu_{ij})$ and $a_{kl} = (\mu_{kl}, \nu_{kl})$ are two intuitionistic fuzzy number, then,we have the follow algorithm :

$$\bar{a}_{ij} = \left(v_{ij}, \mu_{ij}\right). \tag{3}$$

$$a_{ij} + a_{kl} = \left(\mu_{ij} + \mu_{kl} - \mu_{ij} \cdot \mu_{kl}, v_{ij} \cdot v_{kl}\right). \tag{4}$$

$$a_{ij} \cdot a_{kl} = \left(\mu_{ij} \cdot \mu_{kl}, v_{ij} + v_{kl} - v_{ij} \cdot v_{kl}\right). \tag{5}$$

$$\lambda a_{ij} = \left(1 - \left(1 - \mu_{ij}\right)^\lambda, v_{ij}^\lambda\right), \lambda > 0. \tag{6}$$

$$a_{ij}^{\lambda} = \left(\mu_{ij}^{\lambda}, 1 - (1 - v_{ij})^{\lambda}\right), \lambda > 0. \tag{7}$$

In what follow, for the set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, the definition of the additive reciprocal matrix $B = (b_{ij})_{n\times n}$ and its consistency are given:

Definition 6. [16] $B = (b_{ij})_{n\times n}$ is called as an additive reciprocal preference relation, if $b_{ij} + b_{ji} = 1$, $0 \leq b_{ij} \leq 1$, and $b_{ii} = 0.5$ for all $i, j = 1, 2, \ldots\ n$.

Then by considering the additive transitivity, we has the following definition of additive consistency:

Definition 7. [16]Additive reciprocal preference relation $B = (b_{ij})_{n\times n}$ is additively consistent, if $b_{ij} = b_{ik} - b_{jk} + 0.5$, for all $i, j, k \in \{1, 2, \ldots, n\}$.

In addition, the multiplicative consistency of additive reciprocal matrix $B = (b_{ij})_{n\times n}$ is defined as follows:

Definition 8. [16] Additive reciprocal matrix $B = (b_{ij})_{n\times n}$ is multiplicative consistency, if

$$b_{ij} \cdot b_{jk} \cdot b_{ki} = b_{ik} \cdot b_{kj} \cdot b_{ji}, \tag{8}$$

for all $i, j, k \in \{1, 2, \ldots, n\}$.

The interval additive reciprocal matrix is a generalization of the additive reciprocal matrix. We know that an interval additive reciprocal matrix can be used to express the opinions of decision makers, the corresponding definition is given as follows:

Definition 9. [17] An interval additive reciprocal matrix $R = (r_{ij})_{n\times n}$ is defined as

$$R = (r_{ij})_{n\times n} = \begin{bmatrix} [0.5, 0.5] & [r_{12}^-, r_{12}^+] & \cdots & [r_{1n}^-, r_{1n}^+] \\ [r_{21}^-, r_{21}^+] & [0.5, 0.5] & \cdots & [r_{2n}^-, r_{2n}^+] \\ \vdots & \vdots & \ddots & \vdots \\ [r_{n1}^-, r_{n1}^+] & [r_{n2}^-, r_{n2}^+] & \cdots & [0.5, 0.5] \end{bmatrix}, \tag{9}$$

where $r_{ij}^- \leq r_{ij}^+$, $r_{ii}^- = r_{ii}^+ = 0.5$, $r_{ij}^- + r_{ji}^+ = 1$ and $r_{ij}^+ + r_{ji}^- = 1$ for $i, j = 1, 2, \ldots\ n$.

In what follow, we introduce the intuitionistic fuzzy matrix.A finite set of objects $X = \{x_1, x_2, \cdots, x_n\}$, $a_{ij} = (\mu_{ij}, \nu_{ij})$ is an intuitionistic fuzzy number,where μ_{ij} is the preference intensity of the alternatives x_i to x_j, and ν_{ij} is the preference intensity of the alternatives x_j to x_i, where

$$\mu_{ij}, v_{ij} \in [0,1], \mu_{ij} + v_{ij} \leq 1. \tag{10}$$

In addition, $\pi_{ij} = 1 - \mu_{ij} - v_{ij}$ is a hesitancy degree for decision maker,all preference values a_{ij} constitute decision matrix $A = (a_{ij})_{n\times n}$.

We use intuitionistic fuzzy matrix $A = (a_{ij})_{n\times n}$ to express the opinions of decision makers,the corresponding definition is given as follows:

Definition 10. [4] An intuitionistic fuzzy matrix $A = (a_{ij})_{n\times n}$is defined as

$$A = (a_{ij})_{n\times n} = \begin{pmatrix} (0.5,0.5) & (\mu_{12}, v_{12}) & \cdots & (\mu_{1n}, v_{1n}) \\ (\mu_{21}, v_{21}) & (0.5, 0.5) & \cdots & (\mu_{2n}, v_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{n1}, v_{n1}) & (\mu_{n2}, v_{n2}) & \cdots & (0.5, 0.5) \end{pmatrix}, \tag{11}$$

where

$$\mu_{ij}, v_{ij} \in [0,1], \mu_{ij} + v_{ij} \leq 1, \mu_{ij} = v_{ji}, \mu_{ji} = v_{ij}, \mu_{ii} = v_{ii} = 0.5. \tag{12}$$

We know that $\mu_{ik} \leq 1 - v_{ik} (i, k = 1, 2, \cdots, n)$ is always true for any intuitionistic fuzzy number,so we can transform the intuitionistic fuzzy matrix

$$A = (a_{ik})_{n\times n} = \begin{pmatrix} (0.5,0.5) & (\mu_{12}, v_{12}) & \cdots & (\mu_{1n}, v_{1n}) \\ (\mu_{21}, v_{21}) & (0.5, 0.5) & \cdots & (\mu_{2n}, v_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{n1}, v_{n1}) & (\mu_{n2}, v_{n2}) & \cdots & (0.5, 0.5) \end{pmatrix}, \tag{13}$$

into the interval additive reciprocal matrix

$$R = (r_{ik})_{n\times n} = \begin{bmatrix} [0.5, 0.5] & [r_{12}^-, r_{12}^+] & \cdots & [r_{1n}^-, r_{1n}^+] \\ [r_{21}^-, r_{21}^+] & [0.5, 0.5] & \cdots & [r_{2n}^-, r_{2n}^+] \\ \vdots & \vdots & \ddots & \vdots \\ [r_{n1}^-, r_{n1}^+] & [r_{n2}^-, r_{n2}^+] & \cdots & [0.5, 0.5] \end{bmatrix}, \tag{14}$$

where

$$r_{ik}^{-} = \mu_{ik}, r_{ik}^{+} = 1 - v_{ik}, \tag{15}$$

Then we arrive at the following theorem:

Theorem 1. *Let $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i, k = 1, 2, \cdots, n)$ the intuitionistic fuzzy preference relation,for interval additive reciprocal matrix $R = (r_{ik})_{n\times n}$ by (15). If and only if $\mu_{ik}+\mu_{kt}+\mu_{ti} = \mu_{tk}+\mu_{ki}+\mu_{it}, \nu_{ik}+\nu_{kt}+\nu_{ti} = \nu_{tk} + \nu_{ki} + \nu_{it}$,we have $r_{ik}^{-} + r_{kt}^{-} + r_{ti}^{-} = r_{tk}^{-} + r_{ki}^{-} + r_{it}^{-}, r_{ik}^{+} + r_{kt}^{+} + r_{ti}^{+} = r_{tk}^{+} + r_{ki}^{+} + r_{it}^{+}, i, t, k = 1, 2, \cdots, n$.*

Proof. When $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i, k = 1, 2, \cdots, n)$ is the intuitionistic fuzzy preference relation, we can obtain that interval additive reciprocal matrix $R = (r_{ik})_{n\times n}$ by (15). It is noted from Definition (10) that

$$r_{ik}^{-} = \mu_{ik}, r_{ik}^{+} = 1 - v_{ik}.$$

If

$$r_{ik}^{-} + r_{kt}^{-} + r_{ti}^{-} = r_{tk}^{-} + r_{ki}^{-} + r_{it}^{-}, r_{ik}^{+} + r_{kt}^{+} + r_{ti}^{+} = r_{tk}^{+} + r_{ki}^{+} + r_{it}^{+},$$

We have

$$\mu_{ik} + \mu_{kt} + \mu_{ti} = r_{ik}^{-} + r_{kt}^{-} + r_{ti}^{-} = r_{tk}^{-} + r_{ki}^{-} + r_{it}^{-} = \mu_{tk} + \mu_{ki} + \mu_{it},$$

$$\nu_{ik}+\nu_{kt}+\nu_{ti} = 1-r_{ik}^{+}+1-r_{kt}^{+}+1-r_{ti}^{+} = 1-r_{tk}^{+}+1-r_{ki}^{+}+1-r_{it}^{+} = \nu_{tk}+\nu_{ki}+\nu_{it}.$$

On the contrary,if

$$\mu_{ik} + \mu_{kt} + \mu_{ti} = \mu_{tk} + \mu_{ki} + \mu_{it}, \nu_{ik} + \nu_{kt} + \nu_{ti} = \nu_{tk} + \nu_{ki} + \nu_{it}.$$

We have

$$r_{ik}^{-} + r_{kt}^{-} + r_{ti}^{-} = \mu_{ik} + \mu_{kt} + \mu_{ti} = \mu_{tk} + \mu_{ki} + \mu_{it} = r_{tk}^{-} + r_{ki}^{-} + r_{it}^{-},$$

$$r_{ik}^{+}+r_{kt}^{+}+r_{ti}^{+} = 1-\nu_{ik}+1-\nu_{kt}+1-\nu_{ti} = 1-\nu_{tk}+1-\nu_{ki}+1-\nu_{it} = r_{tk}^{+}+r_{ki}^{+}+r_{it}^{+}.$$

In the process of transforming interval additive reciprocal judgment matrix into intuitionistic fuzzy judgment matrix, the formula is very effective if some good properties can be preserved.For example, The advantages and disadvantages of the scheme and the weak transitivity are preserved. Probability is a way of comparing the sizes of interval numbers.So let's give you a definition of what is possible:

Definition 11. [17] Let $\widetilde{w}_i = \left[w_i^-, w_i^+\right]$ and $\widetilde{w}_j = \left[w_j^-, w_j^+\right]$ are the interval number.

$$p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) = \max\left\{1 - \max\left\{\frac{w_j^+ - w_i^-}{w_i^+ - w_i^- + w_j^+ - w_j^-}, 0\right\}, 0\right\} \tag{16}$$

is the probability of $\widetilde{w}_i \geq \widetilde{w}_j$

Possible degrees $p\left(\widetilde{w}_i \geq \widetilde{}w_j\right)$ satisfy the following properties:

$$0 \leq p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) \leq 1. \tag{17}$$

$$p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) + p\left(\widetilde{w}_j \geq \widetilde{w}_i\right) = 1. \tag{18}$$

$$p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) = 1 \Leftrightarrow w_j^+ \leq w_i^-. \tag{19}$$

$$p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) = 0 \Leftrightarrow w_i^+ \leq w_j^-. \tag{20}$$

$$p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) \geq 0.5, p\left(w_j \geq w_k\right) \geq 0.5 \Rightarrow p\left(w_i \geq w_k\right) \geq 0.5 \tag{21}$$

$$p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) > 0.5 \Leftrightarrow w_i^+ + w_i^- > w_j^+ + w_{j*}^-. \tag{22}$$

$$p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) = 0.5 \Leftrightarrow w_i^+ + w_i^- = w_j^+ + w_{j*}^-. \tag{23}$$

$P\left(\widetilde{w}_i \geq \widetilde{w}_j\right) > 0.5$ means that w_i is better than w_j, $p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) < 0.5$ means that w_i is worse than w_j, $p\left(\widetilde{w}_i \geq \widetilde{w}_j\right) = 0.5$ means that there is no difference between w_i and w_j.

By means of probability, we can describe the advantages and disadvantages of pairwise comparison of schemes and have the following theorems:

Theorem 2. *Let interval additive reciprocal matrix $R = (r_{ik})_{n\times n}$, for the intuitionistic fuzzy preference relation $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i, k = 1, 2, \cdots, n)$ is constructed by (15).*

(1) If $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) > 0.5$, then $S_{ij} - S_{ji} > 0$.

(2) If $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) = 0.5$, then $S_{ij} - S_{ji} = 0$.

(3) If $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) < 0.5$, then $S_{ij} - S_{ji} < 0$.

Proof. Under the condition that $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) > 0.5$, we obtain $u_{ij}^- + u_{ij}^+ > u_{ji}^- + u_{ji}^+$. Since $r_{ij}^- = \mu_{ij}, r_{ij}^+ = 1 - v_{ij}$, $u_{ij}^- + u_{ij}^+ > u_{ji}^- + u_{ji}^+$ is equal to $\mu_{ij} - v_{ij} > \mu_{ji} - v_{ji}$. It means that $S_{ij} - S_{ji} > 0$. In the same way, under the condition that $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) = 0.5$ and $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) < 0.5$, we have $S_{ij} - S_{ji} = 0$ and $S_{ij} - S_{ji} < 0$

Theorem 3. *Let interval additive reciprocal matrix $R = (r_{ik})_{n\times n}$,for the intuitionistic fuzzy preference relation $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i, k = 1, 2, \cdots, n)$ is constructed by (15).*

(1) If $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) > 0.5$, then $L_{ij} - L_{ji} > 0$.

(2) If $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) = 0.5$, then $L_{ij} - L_{ji} = 0$.

(3) If $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) < 0.5$, then $L_{ij} - L_{ji} < 0$.

Proof. Since $L_{ij} > L_{ji}$ is equal to

$$\frac{1 - v_{ij}}{2 - \mu_{ij} - v_{ij}} > \frac{1 - v_{ij}}{2 - \mu_{ji} - v_{ji}},$$

$$\Downarrow$$

$$(1 - v_{ij})(2 - \mu_{ji} - v_{ji}) > (1 - v_{ji})(2 - \mu_{ij} - v_{ij}),$$

$$\Downarrow$$

$$\mu_{ij} - v_{ij} > \mu_{ji} - v_{ji}.$$

Under the condition that $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) > 0.5$, we obtain $u_{ij}^- + u_{ij}^+ > u_{ji}^- + u_{ji}^+$. It means that $L_{ij} - L_{ji} > 0$. In the same way, under the condition that $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) = 0.5$ and $p\left(\left[u_{ij}^-, u_{ij}^+\right] \geq \left[u_{jt}^-, u_{ji}^+\right]\right) < 0.5$, we have $L_{ij} - L_{ji} = 0$ and $L_{ij} - L_{ji} < 0$

Theorem 4. *Let interval additive reciprocal matrix $R=(r_{ik})_{n\times n}$,for the intuitionistic fuzzy preference relation $A=(a_{ik})_{n\times n}=(\mu_{ik},v_{ik})_{n\times n}\,(i,k=1,2,\cdots,n)$ is constructed by (15).When the interval additive reciprocal matrix is converted into fuzzy complementary judgment matrix, the weak transitivity remains by (1).*

Proof. Since $R=(r_{ik})_{n\times n}$ satisfies weak transfer, under the condition

$$p\left(\left[u_{ij}^{-},u_{ij}^{+}\right]\geq\left[u_{ji}^{-},u_{ji}^{+}\right]\right)\geq 0.5,p\left(\left[u_{jk}^{-},u_{jk}^{+}\right]\geq\left[u_{kj}^{-},u_{kj}^{+}\right]\right)\geq 0.5,$$

we obtain

$$p\left(\left[u_{ij}^{-},u_{ij}^{+}\right]\geq\left[u_{ji}^{-},u_{ji}^{+}\right]\right)\geq 0.5.$$

According to the theorem 2, we have $S_{ik}\geq 0.5$, $S_{kj}\geq 0.5$, $S_{ik}\geq 0.5$. So the weak transitivity remains (1).

Theorem 5. *Let interval additive reciprocal matrix $R=(r_{ik})_{n\times n}$,for the intuitionistic fuzzy preference relation $A=(a_{ik})_{n\times n}=(\mu_{ik},v_{ik})_{n\times n}\,(i,k=1,2,\cdots,n)$ is constructed by (15).When the interval additive reciprocal matrix is converted into fuzzy complementary judgment matrix, the weak transitivity remains by (2).*

Proof. Since $R=(r_{ik})_{n\times n}$ satisfies weak transfer,under the condition

$$p\left(\left[u_{ij}^{-},u_{ij}^{+}\right]\geq\left[u_{ji}^{-},u_{ji}^{+}\right]\right)\geq 0.5,p\left(\left[u_{jk}^{-},u_{jk}^{+}\right]\geq\left[u_{kj}^{-},u_{kj}^{+}\right]\right)\geq 0.5,$$

we obtain

$$p\left(\left[u_{ij}^{-},u_{ij}^{+}\right]\geq\left[u_{ji}^{-},u_{ji}^{+}\right]\right)\geq 0.5.$$

According to the theorem 3, we have $L_{ik}\geq 0.5$, $L_{kj}\geq 0.5$, $L_{ik}\geq 0.5$. So the weak transitivity remains (2).

The study of consistency is very important in the decision-making problem,according to the consistency of additive reciprocal matrix or interval additive reciprocal matrix, scholars put forward the consistency of intuitionistic fuzzy matrix, the relevant definitions are as follows:

Definition 12. [5]Let $A=(a_{ik})_{n\times n}=(\mu_{ik},v_{ik})_{n\times n}\,(i,k=1,2,\cdots,n)$ the intuitionistic fuzzy preference relation,we call $A=(a_{ik})_{n\times n}$ additive consistency, if $\mu_{ik} \leq 0.5\left(\omega_i - \omega_k + 1\right) \leq 1 - v_{ik}$, $i = 1, 2, \cdots, n-1, k = i+1, \cdots, n$, where $\omega = \left(\omega_1, \omega_2, \cdots, \omega_n\right)^{\mathrm{T}}$ is an interval vector.

In addition, Gong[7] believed that the condition for the additive consistency of the interval additive reciprocal matrix $R = (r_{ik})_{n\times n}$ is the existence of the interval weight vector

$$\widehat{\omega} = \left(\widehat{\omega}_1, \widehat{\omega}_2, \cdots, \widehat{\omega}_n\right)^{\mathrm{T}} = \left(\left[\omega_1^l, \omega_1^u\right], \left[\omega_2^l, \omega_2^u\right], \cdots, \left[\omega_n^l, \omega_n^u\right]\right)^{\mathrm{T}},$$

where

$$r_{ik} = \left(0.5 + 0.2\log_3 \omega_i^l/\omega_k^u, 0.5 + 0.2\log_3 \omega_i^u/\omega_k^l, i, k = 1, 2, \ldots n\right).$$

Based on between the intuitionistic fuzzy preference relation $A = \left(a_{ik}\right)_{n\times n} = \left(\mu_{ik}, v_{ik}\right)$ and interval fuzzy preference relation $R = \left(r_{ik}\right)_{n\times n}$ conversion formula. According to the constraint relationship between the interval weight vector and the additive consistency of $A = \left(a_{ik}\right)_{n\times n}$,Gong [7] proposed the following additive consistency:

Definition 13. [7]Let $A = (a_{ik})_{n\times n} = \left(\mu_{ik}, v_{ik}\right)_{n\times n}(i, k = 1, 2, \cdots, n)$ the intuitionistic fuzzy preference relation, we call $A = \left(a_{ik}\right)_{n\times n}$ additive consistency, if $\mu_{ik} = 0.5 + 0.2\log_3 \omega_i^l/\omega_k^u, \quad v_{ik} = 0.5 + 0.2\log_3 \omega_k^l/\omega_i^u, \quad i, k = 1, 2, \cdots, n,$ where

$$\tilde{\omega} = \left(\tilde{\omega}_1, \tilde{\omega}_2, \cdots, \tilde{\omega}_n\right)^{\mathrm{T}} = \left(\left(\omega_1^\mu, \omega_1^v\right), \left(\omega_2^\mu, \omega_2^v\right), \cdots, \left(\omega_n^\mu, \omega_n^v\right)\right)^{\mathrm{T}},$$

is an interval weight vector.

Both definition 12 and 13 are obtained by converting intuitionistic fuzzy preference relation to interval additive reciprocal preference relation. In addition, some scholars give the definition of consistency from the characteristics of intuitionistic fuzzy number. Wang[10] by considering the membership and non-membership of intuitionistic fuzzy number to define the additive consistency of intuitionistic fuzzy preference relation:

Definition 14. [10] Let $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i,k = 1,2,\cdots,n)$ the intuitionistic fuzzy preference relation,we call $A = (a_{ik})_{n\times n}$ additive consistency. If

$$\mu_{ik} + \mu_{kt} + \mu_{ti} = \mu_{tk} + \mu_{ki} + \mu_{it}, \quad i,t,k = 1,2,\cdots,n.$$

In addition to the additive consistency, the additive reciprocal matrix also has the multiplicative consistency. By extending the multiplicative consistency of additive reciprocal matrix,Xu[5] has proposed the multiplicative consistency of the intuitionistic fuzzy matrix:

Definition 15. [5] Let $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i,k = 1,2,\cdots,n)$ the intuitionistic fuzzy preference relation,we call $A = (a_{ik})_{n\times n}$ multiplicative consistency. If

$$\mu_{ik} \le \frac{\omega_i}{\omega_i + \omega_k} \le 1 - v_{ik}, \quad i = 1,2,\cdots,m-1, \quad k = i+1,\cdots,n,$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ is a set of interval weight vector.

According to the transformation rule between intuitionistic fuzzy preference relation $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}$and interval fuzzy preference relation $R = (r_{ik})_{n\times n}$, Gong[12] put forward multiplicative consistency of intuitionistic fuzzy preference relation:

Definition 16. [12] Let $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i,k = 1,2,\cdots,n)$

the intuitionistic fuzzy preference relation,we call $A = (a_{ik})_{n\times n}$ multiplicative consistency, if

$$\mu_{ik} = \frac{\omega_i^l}{\omega_i^l + \omega_k^u}, \quad v_{ik} = \frac{\omega_k^l}{\omega_k^l + \omega_i^u}, \quad i,k = 1,2,\cdots,n,$$

where

$$\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \cdots, \tilde{\omega}_n)^{\mathrm{T}} = \left(\left(\omega_1^\mu, \omega_1^v\right), \left(\omega_2^\mu, \omega_2^v\right), \cdots, \left(\omega_n^\mu, \omega_n^v\right)\right)^{\mathrm{T}},$$

is a set of interval weight vector.

In addition, Liao[11] directly defined the multiplicative consistency of intuitionistic fuzzy preference relation by membership degree and non-membership degree of intuitionistic fuzzy number:

Definition 17. [11] Let $A = (a_{ik})_{n\times n} = (\mu_{ik}, v_{ik})_{n\times n}\,(i, k = 1, 2, \cdots, n)$ the intuitionistic fuzzy preference relation, we call $A = (a_{ik})_{n\times n}$ multiplicative consistency, if $\mu_{ik}\cdot\mu_{kt}\cdot\mu_{ti} = v_{ik}\cdot v_{kt}\cdot v_{ti}, \quad i, t, k = 1, 2, \cdots, n$

Consistency is a very ideal case. In practice, we consider more of acceptable consistency. In the literature[14], the construction of completely multiplicative consistency matrix and acceptable multiplicative consistency are proposed, the definition of completely multiplicative consistency matrix is given as follows:

Definition 18. [14] In a fixed set of $X = \{x_1, x_2, \cdots, x_n\}$, intuitionistic fuzzy matrix $A = (a_{ij})_{n\times n} = (\mu_{ij}, v_{ij})\,, i, j \in I_n$. We construct completely multiplicative consistency intuitionistic fuzzy matrix $\overline{A} = (\overline{a}_{ij})_{n\times n}$, where

$$\overline{\mu}_{ij} = \begin{cases} \dfrac{\sqrt[j-i-1]{\prod_{t=i+1}^{j-1}\mu_{it}\mu_{tj}}}{\sqrt[j-i-1]{\prod_{i=i+1}^{j-1}\mu_{it}\mu_{tj}} + \sqrt[j-i-1]{\prod_{t=t+1}^{j-1}(1-\mu_{it})(1-\mu_{tj})}}, j > i+1 \\ \mu_{ij}, j = i+1 \\ 0.5, j = i \\ \overline{\nu}_{ji}, j < i \end{cases},$$

$$\overline{\nu}_{ij} = \begin{cases} \dfrac{\sqrt[j-i-1]{\prod_{t=i+1}^{j-1}\nu_{it}\nu_{tj}}}{\sqrt[j-i-1]{\prod_{i=i+1}^{j-1}\nu_{it}\nu_{tj}} + \sqrt[j-i-1]{\prod_{t=t+1}^{j-1}(1-\nu_{it})(1-\nu_{tj})}}, j > i+1 \\ \nu_{ij}, j = i+1 \\ 0.5, j = i \\ \overline{\mu}_{ji}, j < i \end{cases}.$$

By analyzing definition 18, we can find that different permutation σ will construct different completely multiplicative consistency intuitionistic fuzzy matrice, which is illustrated by the following example:

Example 1.[14] Consider the intuitionistic fuzzy matrice $R_{\sigma(123)}$ as follow:

$$R_{\sigma(123)} = \begin{pmatrix} & A_1 & A_2 & A_3 \\ A_1 & (0.5, 0.5) & (0.7, 0.2) & (0.65, 0.2) \\ A_2 & (0.2, 0.7) & (0.5, 0.5) & (0.55, 0.25) \\ A_3 & (0.2, 0.65) & (0.25, 0.55) & (0.5, 0.5) \end{pmatrix}, \tag{24}$$

we have $\overline{R}_{\sigma(123)}$ by definition 18

$$\overline{R}_{\sigma(123)} = \begin{array}{cc} & \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{pmatrix} (0.5, 0.5) & (0.7, 0.2) & (0.7404, 0.0769) \\ (0.2, 0.7) & (0.5, 0.5) & (0.55, 0.25) \\ (0.0769, 0.7404) & (0.25, 0.55) & (0.5, 0.5) \end{pmatrix} \end{array}, \quad (25)$$

Switching the positions of x_2 and x_3, we have

$$R_{\sigma(132)} = \begin{array}{cc} & \begin{array}{ccc} A_1 & A_3 & A_2 \end{array} \\ \begin{array}{c} A_1 \\ A_3 \\ A_2 \end{array} & \begin{pmatrix} (0.5, 0.5) & (0.65, 0.2) & (0.7, 0.2) \\ (0.2, 0.65) & (0.5, 0.5) & (0.25, 0.55) \\ (0.2, 0.7) & (0.55, 0.25) & (0.5, 0.5) \end{pmatrix} \end{array}, \quad (26)$$

You can also get $\overline{R}_{\sigma(132)}$

$$\overline{R}_{\sigma(132)} = \begin{array}{cc} & \begin{array}{ccc} A_1 & A_3 & A_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{pmatrix} (0.5, 0.5) & (0.65, 0.2) & (0.3824, 0.2340) \\ (0.2, 0.65) & (0.5, 0.5) & (0.25, 0.55) \\ (0.2340, 0.3824) & (0.55, 0.25) & (0.5, 0.5) \end{pmatrix} \end{array}, \quad (27)$$

Through observation, we can find that different permutations σ can construct different completely multiplicative consistency intuitionistic fuzzy matrices.In what follow, approximate consistency of intuitionistic fuzzy matrix is put forward, and present a new method to obtain the interval weight vector of intuitionistic fuzzy matrix.

3. approximation-consistency of intuitionistic fuzzy matrixs

It is assumed that $X = \{x_1, x_2, \ldots, x_n\}$ is a set of alternatives. The interval preference degree r_{ij} is produced by comparing the alternative x_i to the alternative x_j. τ is a one by one mapping from $I = \{1, 2, \cdots, n\}$ to $I = \{1, 2, \cdots, n\}$ defined as

$$\tau : k \to i_k, \forall k, i_k \in \{1, 2, \ldots, n\}.$$

The interval number $r^\sigma_{ij} = [r^-_{\sigma(i)\sigma(j)}, r^+_{\sigma(i)\sigma(j)}]$ $(\forall i, j \in I)$ is the preference intensity of the alternatives $x_{\sigma(i)}$ to $x_{\sigma(j)}$.Therefore, a permutation σ corresponds to an interval additive reciprocal matrix R^σ. The definition of the interval additive reciprocal matrix with a permutation σ can be given as follows:

Definition 19. [18] Let σ is a permutation of $\{1, 2, \ldots, n\}$. R^σ is an interval additive reciprocal matrix with a permutation σ:

$$R^\sigma = (r^\sigma_{ij})_{n\times n} =$$

$$\begin{array}{cc}
 & \begin{array}{cccc} x_{\sigma(1)} & x_{\sigma(2)} & \cdots & x_{\sigma(n)} \end{array} \\
\begin{array}{c} x_{\sigma(1)} \\ x_{\sigma(2)} \\ \vdots \\ x_{\sigma(n)} \end{array} &
\begin{bmatrix}
[0.5, 0.5] & [r^-_{\sigma(1)\sigma(2)}, r^+_{\sigma(1)\sigma(2)}] & \cdots & [r^-_{\sigma(1)\sigma(n)}, r^+_{\sigma(1)\sigma(n)}] \\
\left[r^-_{\sigma(2)\sigma(1)}, r^+_{\sigma(2)\sigma(1)}\right] & [0.5, 0.5] & \cdots & [r^-_{\sigma(2)\sigma(n)}, r^+_{\sigma(2)\sigma(n)}] \\
\vdots & \vdots & \ddots & \vdots \\
\left[r^-_{\sigma(n)\sigma(1)}, r^+_{\sigma(n)\sigma(1)}\right] & [r^-_{\sigma(n)\sigma(2)}, r^+_{\sigma(n)\sigma(2)}] & \cdots & [0.5, 0.5]
\end{bmatrix}
\end{array}.$$

Liu[18] defined the additive approximation-consistency of interval additive reciprocal matrix $R^\sigma = (r^\sigma_{ij})_{n\times n}$ .First of all, Liu[18] defined the additive reciprocal matrix $P^\sigma = (p^\sigma_{ij})_{n\times n}$ and $Q^\sigma = (q^\sigma_{ij})_{n\times n}$, where

$$p^\sigma_{ij} = \begin{cases} r^-_{\sigma(i)\sigma(j)}, & i < j, \\ 0.5, & i = j, \\ r^+_{\sigma(i)\sigma(j)}, & i > j, \end{cases} \quad , \qquad q^\sigma_{ij} = \begin{cases} r^+_{\sigma(i)\sigma(j)}, & i < j, \\ 0.5, & i = j, \\ r^-_{\sigma(i)\sigma(j)}, & i > j. \end{cases} \quad . \tag{28}$$

It is noted that the definition of approximation-consistency of interval additive reciprocal matrices have been proposed in [18]. That is:

Definition 20. [18] It is supposed that R is an interval additive preference relation. R^σ is a matrix derived from R by applying a permutation σ.We call R^σ is additive approximation-consistency, if there is a permutation σ such that P^σ and Q^σ determined by (28) are all additively consistent,

Definition 21. [18] It is supposed that R is an interval additive preference relation. R^σ is a matrix derived from R by applying a permutation σ.We call R^σ is multiplicative approximation-consistency, if there is a permutation σ such that P^σ and Q^σ determined by (28) are all multiplicative consistency.

In what follows, through the transformation formula of between intuitionistic fuzzy preference relation and interval additive preference relation,we propose the concept of approximation-consistency of intuitionistic fuzzy matrix with permutations σ:

Definition 22.

$A = (\ a_{ij})_{n\times n}$ is an intuitionistic fuzzy matrix, interval additive reciprocal matrix $R = (\ r_{ij})_{n\times n}$ is constructed by (15). Assume that σ is a permutation of $\{1, 2, \ldots, n\}$. R^{σ} is an interval additive reciprocal matrix with a permutation σ as follow:

$$R^{\sigma} = (r^{\sigma}_{ij})_{n\times n} =$$

$$\begin{array}{c} \\ x_{\sigma(1)} \\ x_{\sigma(2)} \\ \vdots \\ x_{\sigma(n)} \end{array}
\begin{array}{c}
\begin{array}{cccc} x_{\sigma(1)} & x_{\sigma(2)} & \cdots & x_{\sigma(n)} \end{array} \\
\begin{bmatrix}
[0.5, 0.5] & [r^{-}_{\sigma(1)\sigma(2)}, r^{+}_{\sigma(1)\sigma(2)}] & \cdots & [r^{-}_{\sigma(1)\sigma(n)}, r^{+}_{\sigma(1)\sigma(n)}] \\
\left[r^{-}_{\sigma(2)\sigma(1)}, r^{+}_{\sigma(2)\sigma(1)}\right] & [0.5, 0.5] & \cdots & [r^{-}_{\sigma(2)\sigma(n)}, r^{+}_{\sigma(2)\sigma(n)}] \\
\vdots & \vdots & \ddots & \vdots \\
\left[r^{-}_{\sigma(n)\sigma(1)}, r^{+}_{\sigma(n)\sigma(1)}\right] & [r^{-}_{\sigma(n)\sigma(2)}, r^{+}_{\sigma(n)\sigma(2)}] & \cdots & [0.5, 0.5]
\end{bmatrix}
\end{array}.$$

where

$$r^{-}_{\sigma(ij)} = \mu_{\sigma(ij)}, \quad r^{+}_{\sigma(ij)} = 1 - v_{\sigma(ij)}, r^{-}_{\sigma(ij)} + r^{+}_{\sigma(ji)} = r^{+}_{\sigma(ij)} + r^{-}_{\sigma(ji)} = 1. \tag{29}$$

Letting $C_{\sigma} = (c^{\sigma}_{ij})_{n\times n}$ and $D_{\sigma} = (d^{\sigma}_{ij})_{n\times n}$, where

$$c^{\sigma}_{ij} = \begin{cases} r^{+}_{\sigma(i)\sigma(j)}, & i < j \\ 0.5, & i = j \\ r^{-}_{\sigma(i)\sigma(j)}, & i > j \end{cases}, \quad d^{\sigma}_{ij} = \begin{cases} r^{-}_{\sigma(i)\sigma(j)}, & i < j \\ 0.5, & i = j \\ r^{+}_{\sigma(i)\sigma(j)}, & i > j \end{cases}. \tag{30}$$

Definition 23. $A = (\ a_{ij})_{n\times n}$ is an intuitionistic fuzzy matrix, $R = (\ r_{ij})_{n\times n}$ is constructed by (15). Assume that σ is a permutation of $\{1, 2, \ldots, n\}$. R^{σ} is an interval additive reciprocal matrix with a permutation σ, it is said that $A = (\ a_{ij})_{n\times n}$ is multiplicative approximation-consistency, if $C_{\sigma} = (c^{\sigma}_{ij})_{n\times n}$ and $D_{\sigma} = (d^{\sigma}_{ij})_{n\times n}$ are all multiplicative consistency through (19).

Definition 24.

$A = (\ a_{ij})_{n\times n}$ is an intuitionistic fuzzy matrix, $R = (\ r_{ij})_{n\times n}$ is constructed by (15). Assume that σ is a permutation of $\{1, 2, \ldots, n\}$. R^{σ} is an interval additive reciprocal matrix with a permutation σ, it is said that $A = (\ a_{ij})_{n\times n}$ is additive approximation-consistency, if $C_{\sigma} = (c^{\sigma}_{ij})_{n\times n}$ and $D_{\sigma} = (d^{\sigma}_{ij})_{n\times n}$ are all additive consistent through (19).

In what follow, we propose acceptable approximate consistency of intuitionistic fuzzy matrix:

Definition 25. We call intuitionistic fuzzy matrice $A = (a_{ij})_{n\times n}$ is a acceptable approximate multiplicative consistency. If there is a permutation σ of $(1, 2, ..., n)$, the distance between the intuitionistic fuzzy matrice $A = (a_{ij})_{n\times n}$ and the completely multiplicative consistency intuitionistic fuzzy matrice $\overline{A} = (\overline{a}_{ij})_{n\times n}$ satisfy that

$$d(A, \overline{A}) < \tau, \tag{31}$$

the τ is consistency check index.

According to the distance formula of Hamming[19], $d(A, \overline{A})$ can be expressed by the Hamming distance formula as follows:

$$d_h(A, \overline{A}) = \frac{1}{2(n-1)(n-2)} \sum_{i=1}^{n}\sum_{j=1}^{n} \left(\left|\mu_{ij} - \overline{\mu}_{ij}\right| + \left|v_{ij} - \overline{v}_{ij}\right| + \left|\pi_{ij} - \overline{\pi}_{ij}\right|\right).$$

As shown in the above analysis, we have defined the approximation- consistency of intuitionistic fuzzy matrix. Next, for the sake of applications in decision making problems, we introduce the weight vector of intuitionistic fuzzy number.

4. Methods of obtaining interval weights

There have been a lot of researches on the weight of intuitionistic fuzzy number, mainly including precise weight, interval value weight and intuitionistic fuzzy weight. In what follow, we define the interval weight vector of alternatives by considering the permutations of alternatives:

It is supposed that σ is any permutation of $\{1, 2, \ldots, n\}$ with $\sigma(i) = k$ $(i, k \in \{1, 2, \ldots, n\})$. where $\omega_k(C^\sigma)$ and $\omega_k(D^\sigma)$ stand for the weights of the alternative k derived from C^σ and D^σ, respectively. We define the interval weight vector of the alternative k as

$$\omega_k^\sigma = [\min\{\omega_k(C^\sigma), \omega_k(D^\sigma)\}, \max\{\omega_k(C^\sigma), \omega_k(D^\sigma)\}]. \tag{32}$$

Moreover, by considering the randomness of the permutation σ, the expected value of ω_k^σ is calculated as the final weight of the alternative k. That is, one has

$$\omega_k = \frac{1}{n!} \sum_\sigma \omega_k^\sigma.$$

Then the final wight vector is given as

$$\omega = (\omega_1, \omega_2, \ldots, \omega_n). \tag{33}$$

In the next section, we illustrate the new method by carrying out numerical examples and offer some comparisons.

5. Algorithm and illustrative examples

Based on the proposed definition and methods, it is requisite to consider their applications in decision making problems. Now we offer the algorithm to solve the decision making problem with intuitionistic fuzzy matrix:

Step 1:Consider a decision-making problem with a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$. The decision maker provides an intuitionistic fuzzy matrix A , $R_\sigma = (\ r_{\sigma(i)\sigma(j)})_{n\times n}$ with a permutation σ of $\{1, 2, \ldots, n\}$ is constructed by (29).

Step 2: A series of R^{σ_i} are obtained with $i = 1, 2, \cdots, n!/2$. Then we achieve additive reciprocal matrices C^{σ_i} and D^{σ_i} by using (30).

Step 3: Additive approximation-consistency of R^{σ_i} is checked by testing the additive consistency of C^{σ_i} and D^{σ_i}. If there is a permutation σ such that C^σ and D^σ determined by using (30) are all additive additive consistency, one can go directly to the next step. Otherwise, any one of them can be adjusted to that with additive consistency by the consistency improving methods [20]

Step 4: Based on (33), the interval weights of alternatives are given.

Step 5: By applying the possibility degree formula in [19], the possibility degree matrix is given.

Step 6: According to the possibility degree matrix, a method such as that in [21] is used to generate the ranking of alternatives.

Step 7: End.

Example 1: [8][11][12][22] Suppose that a decision maker provides his preference information over a collection of alternatives x_1, x_2, x_3 with the following IFPR:

$$A = \begin{pmatrix} (0.5, 0.5) & (0.2, 0.6) & (0.6, 0.4) \\ (0.6, 0.2) & (0.5, 0.5) & (0.7, 0.1) \\ (0.4, 0.6) & (0.1, 0.7) & (0.5, 0.5) \end{pmatrix}.$$

We have the following result from (1):

$$R = \begin{bmatrix} [0.5, 0.5] & [0.2, 0.4] & [0.6, 0.6] \\ [0.6, 0.8] & [0.5, 0.5] & [0.7, 0.9] \\ [0.4, 0.4] & [0.1, 0.3] & [0.5, 0.5] \end{bmatrix}.$$

First, we check the additive approximation-consistency of the interval additive reciprocal matrix. Applying any permutation σ of alternatives to R, it is found that C^σ andD^σ are not additively consistent, meaning that R has not additive approximation-consistency.

$$C^\sigma = \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.6 & 0.5 & 0.9 \\ 0.4 & 0.1 & 0.5 \end{bmatrix},$$

$$D^\sigma = \begin{bmatrix} 0.5 & 0.2 & 0.6 \\ 0.8 & 0.5 & 0.7 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}.$$

Second, we choose randomly a permutation of alternatives such as $\sigma = (1,2,3)$to adjust R to that with additively consistent. Here the improving method in [19] is applied to adjust C^{σ} andD^{σ} to C'_{σ} andD'_{σ}

$$C'_{\sigma} = \begin{bmatrix} 0.5 & 0.3333 & 0.6667 \\ 0.6667 & 0.5 & 0.8333 \\ 0.333 & 0.1667 & 0.5 \end{bmatrix},$$

$$D'_{\sigma} = \begin{bmatrix} 0.5 & 0.2667 & 0.5333 \\ 0.7333 & 0.5 & 0.7667 \\ 0.4667 & 0.2333 & 0.5 \end{bmatrix}.$$

Third, according to (33), the interval weight vector can be computed as $w1 = [0.4889, 0.5722], w2 = [0.4332, 0.5056], w3 = [0.4555, 0.5444]$.

Finally, the matrix of possibility degrees is determined by using the method in [21] as

$$P = \begin{bmatrix} 0.5 & 0.9769 & 0.7920 \\ 0.0231 & 0.5 & 0.195 \\ 0.2080 & 0.8050 & 0.5 \end{bmatrix},$$

the ranking of the alternative is given as $x_1 > x_2 > x_3$. The obtained result is in agreement with that in [11]. As compared to those in [11], here the permutations of alternatives are considered and the weights are given by using the expected value.

Example 2: [6]Suppose that a decision maker provides his preference information over a collection of alternatives x_1, x_2, x_3 with the following IFPR:

$$A = \begin{pmatrix} (0.5, 0.5) & (0.8, 0.1) & (0.6, 0.4) \\ (0.1, 0.8) & (0.5, 0.5) & (0.7, 0.1) \\ (0.4, 0.6) & (0.1, 0.7) & (0.5, 0.5) \end{pmatrix}.$$

We have the following result from (1):

$$R = \begin{bmatrix} [0.5, 0.5] & [0.8, 0.9] & [0.6, 0.6] \\ [0.1, 0.2] & [0.5, 0.5] & [0.7, 0.9] \\ [0.4, 0.4] & [0.1, 0.3] & [0.5, 0.5] \end{bmatrix}.$$

First, we check the additive approximation-consistency of the interval additive reciprocal matrix. Applying any permutation σ of alternatives to R, it is found that C^{σ} andD^{σ} are not additively consistent, meaning that R has not additive approximation-consistency.

$$C^{\sigma} = \begin{bmatrix} 0.5 & 0.9 & 0.6 \\ 0.1 & 0.5 & 0.9 \\ 0.4 & 0.1 & 0.5 \end{bmatrix},$$

$$D^{\sigma} = \begin{bmatrix} 0.5 & 0.8 & 0.6 \\ 0.2 & 0.5 & 0.7 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}.$$

Second, we choose randomly a permutation of alternatives such as $\sigma = (1, 2, 3)$to adjust R to that with additively consistent. Here the improving method in [19] is applied to adjust C^{σ} andD^{σ} to C'_{σ} andD'_{σ}

$$C'_{\sigma} = \begin{bmatrix} 0.5 & 0.6667 & 0.8333 \\ 0.3333 & 0.5 & 0.6667 \\ 0.1667 & 0.3333 & 0.5 \end{bmatrix},$$

$$D'_{\sigma} = \begin{bmatrix} 0.5 & 0.6667 & 0.7333 \\ 0.3333 & 0.5 & 0.5667 \\ 0.2667 & 0.4333 & 0.5 \end{bmatrix}.$$

Third, according to (33), the interval weight vector can be computed as $w1 = [0.4721, 0.5333], w2 = [0.4721, 0.5278], w3 = [0.4666, 0.5333]$.

Finally, the matrix of possibility degrees is determined by using the method in [21] as

$$P = \begin{bmatrix} 0.5 & 0.5449 & 0.5412 \\ 0.4506 & 0.5 & 0.4959 \\ 0.4588 & 0.5041 & 0.5 \end{bmatrix},$$

the ranking of the alternative is given as $x_1 > x_3 > x_2$. The best plan is the same with that in [6].

6. Conclusions

In real decision-making problems, in order to solve the complexity and uncertainty of problems, scholars put forward intuitionistic fuzzy number, which solved the practical problems well.In this paper, we review the consistency definition of intuitionistic fuzzy matrix and point out the shortcomings of the definition. In order to solve the shortage of traditional definition, we put forward the concept of approximate consistency of intuitionistic fuzzy matrix.Through the transformation formula of intuitionistic fuzzy preference relation and interval additive preference relation,we transform the intuitionistic fuzzy matrix into the interval additive reciprocal matrice, and put forward the approximate consistency of intuitionistic fuzzy matrix considering the arrangement of schemes, and give a new method to obtain the interval weight vector of intuitionistic fuzzy matrix.A new algorithm for solving decision-making problems with intuitionistic fuzzy matrix has been given. Two examples have been carried out to illustrate the proposed definition and algorithm and some comparisons have been made.

In the future, the developed definition and methods will be extended to address group decision making problems with intuitionistic fuzzy matrix.

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