Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

[Turn over

1 The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

2 The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

3 Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

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(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

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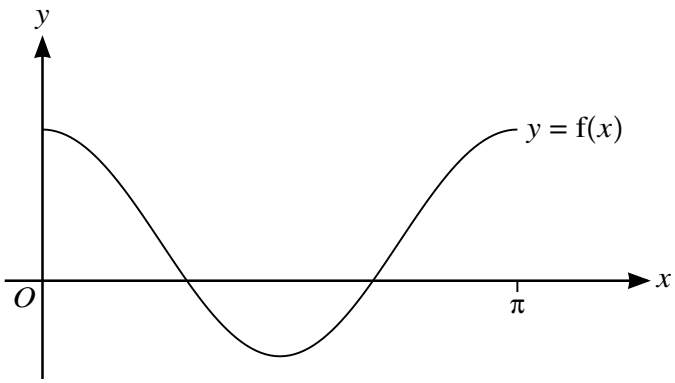

The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

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A function g is such that $\mathrm{g}(x) = \mathrm{f}(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = \mathrm{g}(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

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(c) State the equation of the curve which is the reflection of $y = \mathrm{f}(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

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5 The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

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(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

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6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \tfrac{1}{2}x - a,$$
$$\mathrm{g} : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $\mathrm{gg}(2) = 10$ and $\mathrm{f}^{-1}(2) = 14$, find the values of a and b. [4]

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(b) Using these values of a and b, find an expression for $\mathrm{gf}(x)$ in the form $cx + d$, where c and d are constants. [2]

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7 **(a)** Prove the identity $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}\equiv\frac{2}{\cos\theta}$. [3]

(b) Hence solve the equation $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=\frac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

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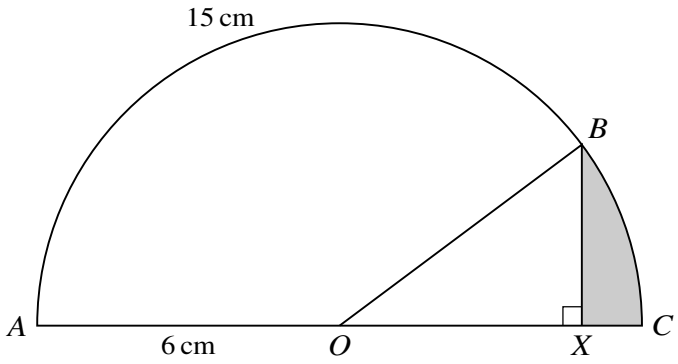

In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

9 The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each stationary point. [2]

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10 The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

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(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

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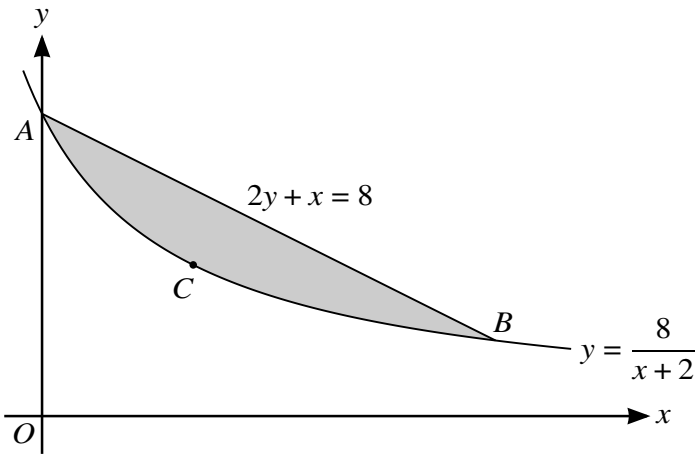

The diagram shows part of the curve $y = \frac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

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(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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