Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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2894789041

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 The coefficient of x^2 in the expansion of $(1-4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2+ax)^5$.

Find the value of the positive constant a. [3]

2 The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

3 **(a)** Show that the equation $\frac{7\tan\theta}{\cos\theta} + 12 = 0$ can be expressed as

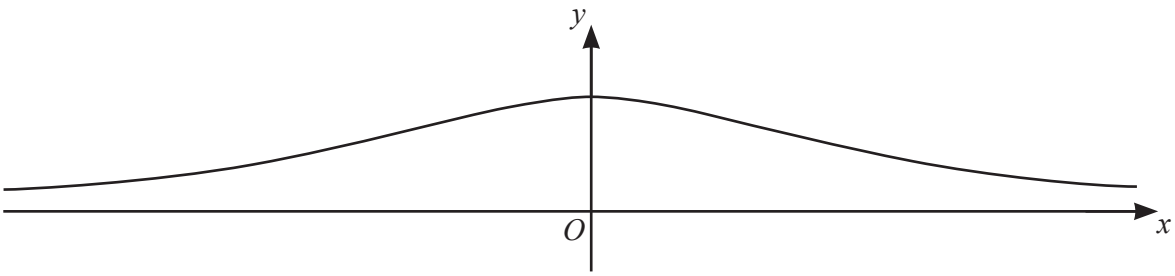
$$12\sin^2\theta - 7\sin\theta - 12 = 0.$$ [3]

(b) Hence solve the equation $\frac{7\tan\theta}{\cos\theta} + 12 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

4 The function f is defined as follows:

$$\mathrm{f}(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [1]



The diagram shows the graph of $y = \mathrm{g}(x)$ where $\mathrm{g}(x) = \dfrac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists. [2]

The function h is defined by $\mathrm{h}(x) = \dfrac{1}{x^2+2}$ for $x \geqslant 0$.

(c) Solve the equation $\mathrm{hf}(x) = \mathrm{f}\left(\dfrac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where a, b and c are integers. [4]

5 The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(b) (i) Find the sum to infinity of the progression in terms of θ. [2]

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(ii) Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures. [3]

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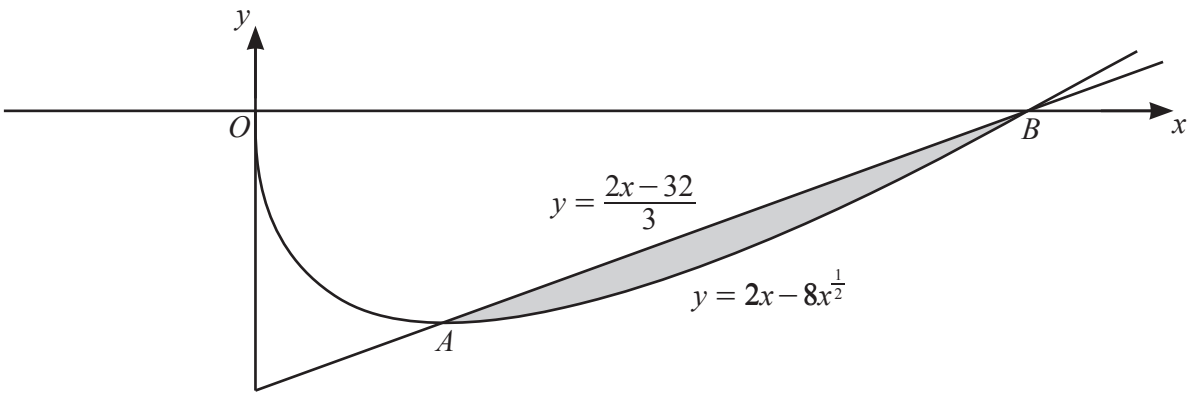
6 The curve with equation $y = 2x - 8x^{\frac{1}{2}}$ has a minimum point at A and intersects the positive x-axis at B.

(a) Find the coordinates of A and B. [4]

(b)



The diagram shows the curve with equation $y = 2x - 8x^{\frac{1}{2}}$ and the line AB. It is given that the equation of AB is $y = \frac{2x-32}{3}$.

Find the area of the shaded region between the curve and the line. [5]

7 The equation of a circle is $(x-6)^2+(y+a)^2=18$. The line with equation $y=2a-x$ is a tangent to the circle.

(a) Find the two possible values of the constant a. [5]

(b) For the greater value of a, find the equation of the diameter which is perpendicular to the given tangent. [3]

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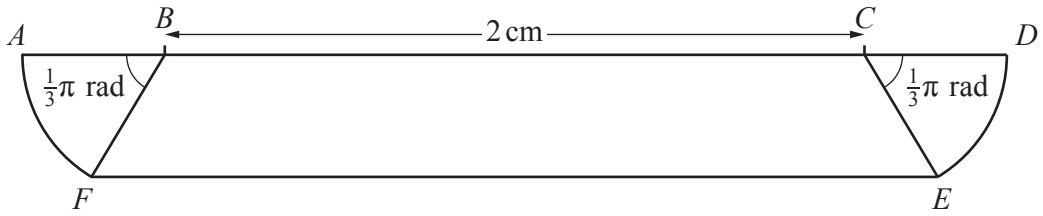

The diagram shows a symmetrical plate $ABCDEF$. The line $ABCD$ is straight and the length of BC is 2 cm. Each of the two sectors ABF and DCE is of radius r cm and each of the angles ABF and DCE is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that $r = 0.4$ cm.

(i) Show that the length $EF = 2.4$ cm. [2]

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(ii) Find the area of the plate. Give your answer correct to 3 significant figures. [4]

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(b) It is given instead that the perimeter of the plate is 6 cm.

Find the value of r. Give your answer correct to 3 significant figures. [4]

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9 A function f is such that $f'(x) = 6(2x-3)^2 - 6x$ for $x \in \mathbb{R}$.

(a) Determine the set of values of x for which $f(x)$ is decreasing. [4]

(b) Given that $f(1) = -1$, find $f(x)$. [4]

10 The equation of a curve is $y = (5-2x)^{\frac{3}{2}} + 5$ for $x < \frac{5}{2}$.

(a) A point P is moving along the curve in such a way that the y-coordinate of point P is decreasing at 5 units per second.

Find the rate at which the x-coordinate of point P is increasing when $y = 32$. [4]

(b) Point A on the curve has y-coordinate 32. Point B on the curve is such that the gradient of the curve at B is -3.

Find the equation of the perpendicular bisector of AB. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

Additional page

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