Cambridge Assessment International Education

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2923681341

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

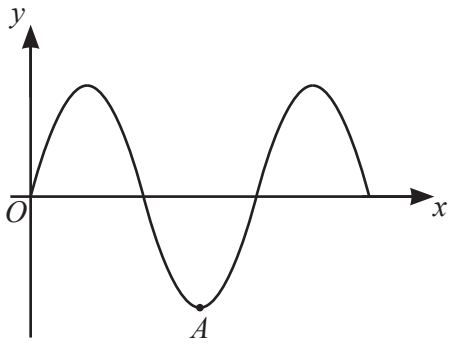
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

[Turn over

1 Find the exact value of $\int_3^\infty \frac{2}{x^2}\,\mathrm{d}x$. [3]

2



The diagram shows part of the curve with equation $y = k \sin \frac{1}{2}x$, where k is a positive constant and x is measured in radians. The curve has a minimum point A.

(a) State the coordinates of A. [1]

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(b) A sequence of transformations is applied to the curve in the following order.

Translation of 2 units in the negative y-direction

Reflection in the x-axis

Find the equation of the new curve and determine the coordinates of the point on the new curve corresponding to A. [3]

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3 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3(4x+5)^{\frac{1}{2}}$. It is given that the points $(1, 9)$ and $(5, a)$ lie on the curve.

Find the value of a. [5]

4 **(a)** Prove that $\dfrac{(\sin\theta+\cos\theta)^2-1}{\cos^2\theta} \equiv 2\tan\theta$. [3]

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(b) Hence solve the equation $\dfrac{(\sin\theta+\cos\theta)^2-1}{\cos^2\theta} = 5\tan^3\theta$ for $-90° < \theta < 90°$. [3]

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5 A curve has the equation $y = \dfrac{3}{2x^2 - 5}$.

Find the equation of the normal to the curve at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

6 It is given that the coefficient of x^3 in the expansion of

$$(2+ax)^4(5-ax)$$

is 432.

Find the value of the constant a. [5]

7 The straight line $y = x + 5$ meets the curve $2x^2 + 3y^2 = k$ at a single point P.

(a) Find the value of the constant k. [4]

(b) Find the coordinates of P. [2]

8 **(a)** An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression. [4]

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(b) A geometric progression $a_1, a_2, a_3, ...$ is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by S. The sum to infinity of the even-numbered terms (i.e. $a_2, a_4, a_6, ...$) is denoted by S_E.

Find the values of S and S_E. [4]

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9 The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = (3x-2)^2 + k \quad \text{and} \quad \mathrm{g}(x) = 5x - 1,$$

where k is a constant.

(a) Given that the range of the function gf is $\mathrm{gf}(x) \geqslant 39$, find the value of k. [4]

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(b) For this value of k, determine the range of the function fg. [2]

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(c) The function h is defined for all real values of x and is such that $\mathrm{gh}(x) = 35x + 19$.

Find an expression for $\mathrm{g}^{-1}(x)$ and hence, or otherwise, find an expression for $\mathrm{h}(x)$. [3]

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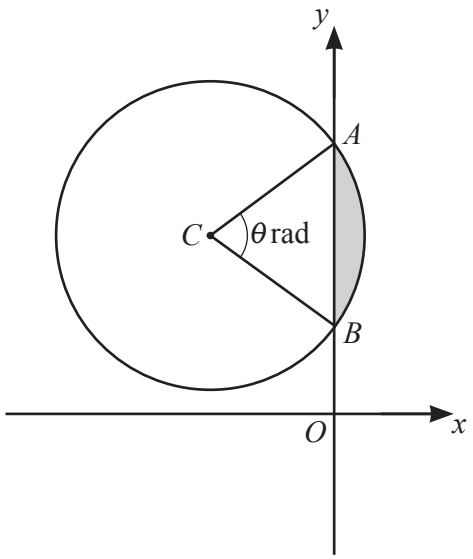

The diagram shows the circle with centre $C(-4, 5)$ and radius $\sqrt{20}$ units. The circle intersects the y-axis at the points A and B. The size of angle ACB is θ radians.

(a) Find the equation of the tangent to the circle at the point $(-6, 9)$. [3]

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(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [2]

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(c) Find the value of θ correct to 4 significant figures. [3]

(d) Find the perimeter and area of the segment shaded in the diagram. [4]

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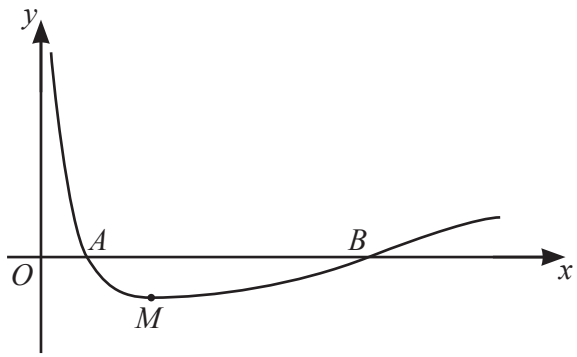

The diagram shows the curve with equation $y = 2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1$ for $x > 0$. The curve crosses the x-axis at points A and B and has a minimum point M.

(a) Find the exact coordinates of M. [4]

(b) Find the area of the region bounded by the curve and the line segment AB. [7]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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