Unit 6: Series

Subunit 6.1: Binomial expansion

Topical Question No: 1

6 The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

Topical Question No: 2

1 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(1 + x)^5$. [1]

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(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^6$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(1 + x)^5(1 - 2x)^6$. [2]

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3 Find the term independent of x in each of the following expansions.

(a) $\left(3x + \frac{2}{x^2}\right)^6$ [3]

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(b) $\left(3x + \frac{2}{x^2}\right)^6 (1 - x^3)$ [3]

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6 In the expansion of $\left(\frac{x}{a}+\frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a. [6]

6 It is given that the coefficient of x^3 in the expansion of

$$(2+ax)^4(5-ax)$$

is 432.

Find the value of the constant a. [5]

3 **(a)** Find the complete expansion of $\left(2x-\frac{3}{x}\right)^4$. [4]

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(b) Hence determine the coefficient of x^2 in the expansion of $\left(x^2+5\right)\left(2x-\frac{3}{x}\right)^4$. [2]

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2 The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

Topical Question No: 8

1 **(a)** Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

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(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

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4 **(a)** Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

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(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

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3 **(a)** Find the first three terms in the expansion of $(3 - 2x)^5$ in ascending powers of x. [3]

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(b) Hence find the coefficient of x^2 in the expansion of $(4 + x)^2(3 - 2x)^5$. [3]

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4 The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

7 **(a)** Write down the first four terms of the expansion, in ascending powers of x, of $(a - x)^6$. [2]

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(b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20, find in exact form the possible values of the constant a. [5]

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3 The coefficient of x^4 in the expansion of $\left(2x^2 + \frac{k^2}{x}\right)^5$ is a. The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b.

(a) Find a and b in terms of the constant k. [3]

(b) Given that $a + b = 216$, find the possible values of k. [3]

Topical Question No: 14

1 The coefficient of x^4 in the expansion of $(3 + x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x + \frac{a}{x}\right)^6$.

Find the value of the positive constant a. [4]

2 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(2 + 3x)^4$. [2]

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(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

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2 The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

3 **(a)** Give the complete expansion of $\left(x+\frac{2}{x}\right)^5$. [2]

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(b) In the expansion of $(a+bx^2)\left(x+\frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b. [4]

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Topical Question No: 18

3 The coefficient of x^3 in the expansion of $(3+ax)^6$ is 160.

(a) Find the value of the constant a. [2]

(b) Hence find the coefficient of x^3 in the expansion of $(3+ax)^6(1-2x)$. [3]

Topical Question No: 19

1 The coefficient of x^2 in the expansion of $(1-4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2+ax)^5$.

Find the value of the positive constant a. [3]

Topical Question No: 20

1 Find the coefficient of x^2 in the expansion of

$$(2-5x)(1+3x)^{10}.$$ [4]

5 **(a)** Find the first three terms, in ascending powers of x, in the expansion of each of the following expressions.

(i) $(2-px)^5$ [2]

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(ii) $\left(1-\frac{1}{2}x\right)^4$ [2]

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(b) Given that the coefficient of x^2 in the expansion of $(2-px)^5\left(1-\frac{1}{2}x\right)^4$ is 93, find the possible values of the constant p. [3]

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