Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

2108969358

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A curve is such that its gradient at a point (x, y) is given by $\frac{dy}{dx} = x - 3x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, 1)$.

Find the equation of the curve. [4]

2 The circle with equation $(x-3)^2 + (y-5)^2 = 40$ intersects the y-axis at points A and B.

(a) Find the y-coordinates of A and B, expressing your answers in terms of surds. [2]

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(b) Find the equation of the circle which has AB as its diameter. [2]

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3 **(a)** Show that the equation

$$5\cos\theta - \sin\theta\tan\theta + 1 = 0$$

may be expressed in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $5\cos\theta - \sin\theta\tan\theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

4 **(a)** Expand the following in ascending powers of x up to and including the term in x^2.

(i) $(1+2x)^5$. [1]

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(ii) $(1-ax)^6$, where a is a constant. [2]

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In the expansion of $(1+2x)^5(1-ax)^6$, the coefficient of x^2 is -5.

(b) Find the possible values of a. [4]

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5 The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p. [3]

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(b) One of the values of p found in **(a)** is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

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6 A line has equation $y = 6x - c$ and a curve has equation $y = cx^2 + 2x - 3$, where c is a constant. The line is a tangent to the curve at point P.

Find the possible values of c and the corresponding coordinates of P. [7]

7 The function f is defined by $\mathrm{f}(x) = 1 + \dfrac{3}{x-2}$ for $x > 2$.

(a) State the range of f. [1]

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(b) Obtain an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $\mathrm{g}(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $\mathrm{gf}(x)$. [2]

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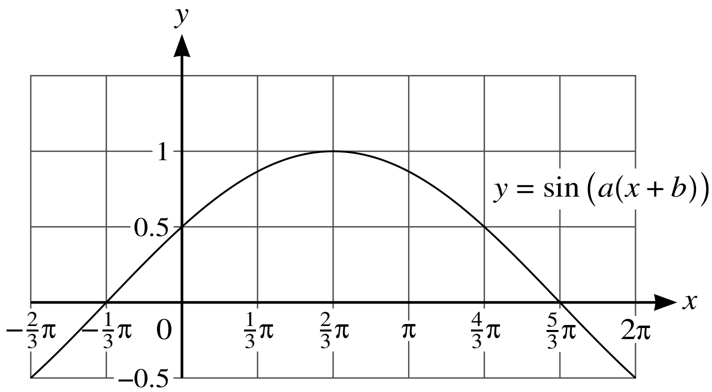

The diagram shows part of the graph of $y = \sin\big(a(x + b)\big)$, where a and b are positive constants.

(a) State the value of a and one possible value of b. [2]

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Another curve, with equation $y = \mathrm{f}(x)$, has a single stationary point at the point (p, q), where p and q are constants. This curve is transformed to a curve with equation

$$y = -3\mathrm{f}\left(\tfrac{1}{4}(x + 8)\right).$$

(b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q. [3]

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9 A curve has equation $y = 2x^{\frac{1}{2}} - 1$.

(a) Find the equation of the normal to the curve at the point $A\,(4,\ 3)$, giving your answer in the form $y = mx + c$. [3]

A point is moving along the curve $y = 2x^{\frac{1}{2}} - 1$ in such a way that at A the rate of increase of the x-coordinate is $3\,\text{cm}\,\text{s}^{-1}$.

(b) Find the rate of increase of the y-coordinate at A. [2]

At A the moving point suddenly changes direction and speed, and moves down the normal in such a way that the rate of decrease of the y-coordinate is constant at $5\,\text{cm}\,\text{s}^{-1}$.

(c) As the point moves down the normal, find the rate of change of its x-coordinate. [3]

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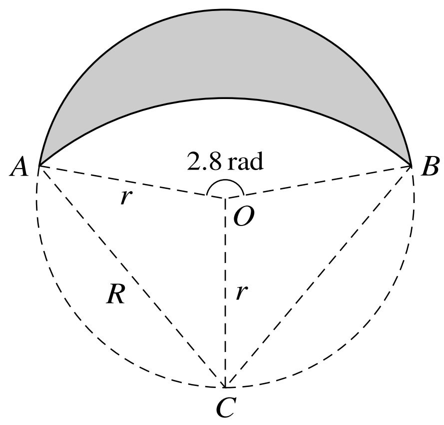

The diagram shows points A, B and C lying on a circle with centre O and radius r. Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r. The lower arc is part of a circle with centre C and radius R.

(a) State the size of angle ACO in radians. [1]

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(b) Find R in terms of r. [1]

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(c) Find the area of the shaded region in terms of r. [7]

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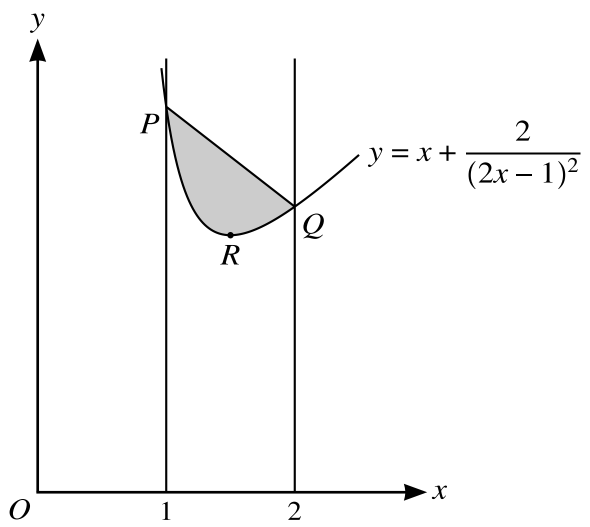

The diagram shows part of the curve with equation $y = x + \dfrac{2}{(2x-1)^2}$. The lines $x = 1$ and $x = 2$ intersect the curve at P and Q respectively and R is the stationary point on the curve.

(a) Verify that the x-coordinate of R is $\frac{3}{2}$ and find the y-coordinate of R. [4]

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(b) Find the exact value of the area of the shaded region. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.