Unit 6: Series

Subunit 6.2: Arithmetic and geometric progressions

9 The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\dfrac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

4 The first term of a geometric progression and the first term of an arithmetic progression are both equal to a.

The third term of the geometric progression is equal to the second term of the arithmetic progression.

The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a. [6]

4 The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

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(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

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8 **(a)** An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression. [4]

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(b) A geometric progression $a_1, a_2, a_3, ...$ is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by S. The sum to infinity of the even-numbered terms (i.e. $a_2, a_4, a_6, ...$) is denoted by S_E.

Find the values of S and S_E. [4]

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Topical Question No: 5

5 An arithmetic progression has first term 5 and common difference 6.

For this progression, find the sum of all the terms that lie between 150 and 400. [6]

Topical Question No: 6

1 The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

3 Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

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(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

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4 The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

8 The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

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It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

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2 The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

8 The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

9 **(a)** A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a + 1)$ and common difference $(d + 1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

Topical Question No: 13

2 The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is −15.

Find the sum of the first 50 terms of the progression. [5]

4 The first, second and third terms of an arithmetic progression are k, $6k$ and $k + 6$ respectively.

(a) Find the value of the constant k. [2]

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(b) Find the sum of the first 30 terms of the progression. [3]

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6 The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p.

(a) Given that the common difference of the progression is not zero, find the value of p. [3]

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(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

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9 The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

8 A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

8 **(a)** The first three terms of an arithmetic progression are $25, 4p-1$ and $13-p$, where p is a constant.

Find the value of the tenth term of the progression. [4]

(b) The first three terms of a geometric progression are $25, 4q-1$ and $13-q$, where q is a positive constant.

Find the sum to infinity of the progression. [4]

5 The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(b) (i) Find the sum to infinity of the progression in terms of θ. [2]

10 The geometric progression $a_1, a_2, a_3, \ldots$ has first term 2 and common ratio r where $r > 0$.
It is given that $\frac{9}{2}a_5 + 7a_3 = 8$.

(a) Find the value of r. [3]

(b) Find the sum of the first 20 terms of the geometric progression. Give your answer correct to 4 significant figures. [2]

(c) Find the sum to infinity of the progression $a_2, a_5, a_8, \ldots$. [3]

7 The first term of an arithmetic progression is 1.5 and the sum of the first ten terms is 127.5 .

(a) Find the common difference. [2]

(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100. [5]

3 The third term of a geometric progression is 18 and the sum of the first three terms is 26. It is given that the common ratio is negative.

(a) Find the tenth term of the progression. Give your answer correct to 3 significant figures. [5]

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(b) Find the exact value of the sum to infinity of the progression. [2]

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