CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$a+3d=15$ and $a+7d=25$	M1	Or forming any valid equations which can be used to find *d* or *a*.
	Finding *a* and *d* $\left[d=\frac{5}{2},\ a=\frac{15}{2}\right]$	DM1	Or any valid method to find a using *their* d or d using *their* a or finding u_{30} directly from either u_4 or u_8 and *d*.
	$u_{30}=\frac{15}{2}+29\times\frac{5}{2}=80$	A1	
		3	

Question	Answer	Marks	Guidance
2	$\cos\left(\frac{\pi}{6}\right)+\tan 2x+\frac{\sqrt{3}}{2}=0 \Rightarrow \tan 2x=-\sqrt{3}$	M1	Making $\tan 2x$ the subject. $\tan 2x=0$ is M0. Accept decimals and one sign error.
	$\Rightarrow 2x=-\frac{\pi}{3} \Rightarrow x=-\frac{\pi}{6}$	A1	May come from non-exact working. Ignore answers outside the given range.
		2	

Question	Answer	Marks	Guidance
3(a)	$x^3:\ \binom{5}{3}3^2(-ax)^3\left[-10\times3^2\times a^3\right]$ or $x^4:\ \binom{5}{4}3(-ax)^4\left[5\times3\times a^4\right]$	M1	Allow for either term, allow sign error and combination notation.
	$x^3:\ -90a^3$	A1	Allow in the full expansion.
	$x^4: 15a^4$	A1	Allow in the full expansion.
		3	

Question	Answer	Marks	Guidance
3(b)	Coefficient of x^4 is $a \times their -90a^3 + 7 \times their\ 15a^4 \quad \left[=15a^4\right]$	**M1**	Must select two appropriate terms only.
	$15a^4 = 240$	**DM1**	Reducing to a simple quartic equation in a.
	$\Rightarrow a^4 = 16 \quad \Rightarrow a = 2$	**A1**	A0 if $a = -2$ is given as a solution.
		3	

Question	Answer	Marks	Guidance
4	Let $x = \sin^2\theta$ $(2x+7)(2x-1) = 0$ or $\left(2\sin^2\theta + 7\right)\left(2\sin^2\theta - 1\right)$	**M1**	Or equivalent method.
	$\Rightarrow \sin^2\theta = \frac{1}{2} \Rightarrow \sin\theta = [\pm]\frac{1}{\sqrt{2}}$	**M1**	Finding $\sin^2\theta$ and then $\sin\theta$ (may be implied).
	$\theta = 45°,\ 135°,\ 225°,\ 315°$	**A1 A1**	A1 for any two correct values. A1 for all correct and no others within the range. For answers in radians, A1 only for all 4 angles. If no (correct) working, then **SC B1** for all 4 solutions.
		4	

Question	Answer	Marks	Guidance
5(a)	Reflection [in] *y*-axis	**B1 B1**	B1 for reflection B1 mention of *y*-axis, OE. **SC B2** for stretch, SF –1, parallel to *x*-axis.
	Translation or shift $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$	**B1***	B1 for 'translation' and a correct vector/description. Do not accept 'left'/'right'. If two translations then B0 and B0 for the order.
	Stretch, factor 2, parallel to *y*-axis	**B2,1,0**	B2 all correct OE. B1 any 2 parts correct. This can be at any point in the sequence.
	Correct order and three correctly named transformations only	**DB1**	If a fourth transformation is given this mark is not awarded and no marks are given for the two transformations of the same type, except where the reflection is described as a stretch. If any transformation is incorrectly named this cannot be given. If translation is not $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ then DB0 is given.
	Alternative Solution for first 3 marks		
	Translation or shift $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	**B1***	B1 for 'translation' and correct vector/description.
	Reflection [in] *y*-axis	**B1 B1**	B1 for 'reflection', B1 for 'in *y*-axis'.
	Alternative solutions		
	There are alternative solutions which can be marked in the same way e.g. the given stretch, translation $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$, reflect in $x = -2.5$		
		6	

Question	Answer	Marks	Guidance
5(b)	$g(x) = 2f(-x-1)$ or $a = 2,\ b = -1,\ c = -1$	B1	First B1 for $a = 2$ and no additional terms added to the function. $a = -2$ is B0.
		B1	Second B1 for $b = -1$ and $c = -1$.
		2	

Question	Answer	Marks	Guidance
6	$\dfrac{\frac{10(1-r^8)}{1-r}}{\frac{10(1-r^4)}{1-r}} = \dfrac{17}{16}$ $\left[a\dfrac{(1-r^8)}{(1-r)} = \dfrac{17}{16} \times a\dfrac{(1-r^4)}{(1-r)}\right]$	M1*	OE, i.e. substituting p and q expressions into ratio $\dfrac{17}{16}$. $16 = a\dfrac{(1-r^4)}{(1-r)},\ 17 = a\dfrac{(1-r^8)}{(1-r)}$ gets M0 unless recovered later.
	Simplifying to $16r^8 - 17r^4 + 1\ [=0]$ (or equivalent form)	DM1	Or $\dfrac{(1-r^8)}{(1-r^4)} = (1+r^4) = \dfrac{17}{16}$.
	$\left[(16r^4-1)(r^4-1)=0\right] \Rightarrow r = \pm\dfrac{1}{2}$	A1	Or $r^4 = \dfrac{1}{16} \Rightarrow r = \pm\dfrac{1}{2}$ (condone extra $r = \pm 1$ solution).
	$S_\infty = \dfrac{10}{1-\left([\pm]\frac{1}{2}\right)}$	DM1	Use of correct sum to infinity formula with either of *their* r values providing $\lvert r \rvert < 1$.
	$S_\infty = 20$ and $\dfrac{20}{3}$	A1	Allow 6.67 or better. A0 if there is only one or more than two S_∞ values.
		5	

Question	Answer	Marks	Guidance
7(a)	Area of sector *BOF* = $\frac{1}{2}\times 20^2\times(2\pi-2.4)$ $[=776.63\ldots]$	**M1**	Or combination of large semi-circle and small sector: $\frac{1}{2}\times 20^2\times\pi+\frac{1}{2}\times 20^2\times(\pi-2.4)$.
	Length *BD* = *DF* = $2\times 20\sin 0.6$ or $\sqrt{20^2+20^2-2\times 20\times 20\cos 1.2}$ $[=22.58\ldots]$	**M1***	Length of radius of small circles is acceptable for M1.
	Area of two semicircles = $\pi\times(20\sin 0.6)^2$ $[=400.64\ldots]$	**DM1**	
	Area of triangles = $2\times\frac{1}{2}\times 20\times 20\sin 1.2$ $[=372.81\ldots]$	**M1**	
	Total area = $1550\,[\text{cm}^2]$	**A1**	Expect 1550.09 but accept AWRT to 3sf.
		5	
7(b)	$\frac{1}{2}\pi r^2=50\pi\Rightarrow r=10$	**B1**	May be seen as $20\sin\frac{\theta}{2}$, where $\theta=\frac{\pi}{3}$.
	$\Rightarrow\ \theta=\frac{\pi}{3}$	**M1***	OE Finding θ using *their* r. Allow working in degrees.
	Arc length of sector *BOF* = $20\times\left(2\pi-\textit{their}\,\frac{2\pi}{3}\right)$	**DM1**	
	Total perimeter = $20\times\left(2\pi-\textit{their}\,\frac{2\pi}{3}\right)+2\pi\times\textit{their}\,10$	**DM1**	Dependent on the first dM1.
	$\frac{140\pi}{3}$ or $46\frac{2}{3}\pi$	**A1**	Must be a single exact term.
		5	

Question	Answer	Marks	Guidance
8(a)	$3(x-2)^2+2$ or $a=-2,\ b=2$	**B1 B1**	
		2	
8(b)	2 or $k=2$ or $k\geq 2$	**B1FT**	FT on *their a*. Do not accept $x = 2$ or $x \geqslant 2$.
		1	
8(c)	$3(x-2)^2+14-12=y \Rightarrow (x-2)^2=\dfrac{y-2}{3}$	**M1**	Using *their* completed square form.
	$x=[\pm]\sqrt{\dfrac{y-2}{3}}+2$	**DM1**	
	$\mathrm{f}^{-1}(x)=\sqrt{\dfrac{x-2}{3}}+2$	**A1**	OE, e.g. $y=\dfrac{\sqrt{3x-6}}{3}+2$.
		3	

Question	Answer	Marks	Guidance
8(d)	Finding $f^{-1}(29)$ [= 5]	**M1**	Or solving f(x) = 29 [using *their* completed square form, OE].
	Finding $f^{-1}($ *their* $5)$	**M1**	Or solving f(x) = *their* 5.
	$x=3$	**A1**	If using f(x) method, x = 1 must be discarded.
	Alternative solution for Question 8(d)		
	$3\left(3(x-2)^2+2\right)-2)^2+2=29$ using *their* completed square form	**M1**	Or $3\left(3x^2-12x+14\right)^2-12\left(3x^2-12x+14\right)+14=29$. Allow if the $'=29'$ appears later in the working.
	Solving as far as $9(x-2)^4=9$ or $x^2-4x+3=0$	**DM1**	OE Or $[27]\left(x^4-8x^3+24x^2-32x+15\right)=0$.
	$x=3$ only	**A1**	WWW Only dependent on the first M1.
		3	

Question	Answer	Marks	Guidance
9(a)	$y=x^3-3x+3$ and $y=2x^3-4x^2+3 \Rightarrow x^3-4x^2+3x[=0]$	**M1**	Reducing to 3-term cubic or quadratic if x cancelled.
	$[x](x-1)(x-3)[=0]$	**DM1**	Factorising the cubic or quadratic.
	$x=0$, 1 and 3 {x = 0 may be seen in the working}	**A1**	**SC B1** for $x=1,\ 3$ only, with no M marks awarded.
		3	

Question	Answer	Marks	Guidance
9(b)	Attempt at integration of both functions. Can be before or after subtraction of the functions or integrals	**M1**	Expect integration of $\int\left(\left(x^3-3x+3\right)-\left(2x^3-4x^2+3\right)\right)\mathrm{d}x$ or $\int\left(-x^3+4x^2-3x\right)\mathrm{d}x$. At this stage, subtraction can be done either way.
	$=\pm\left(-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\right)$ or $\pm\left\{\left(\frac{x^4}{4}-\frac{3}{2}x^2+3x\right)-\left(\frac{2}{4}x^4-\frac{4}{3}x^3+3x\right)\right\}$	**A1**	OE ± covers A1 being awarded to those who subtract the 'other' way.
	$=\left[\left(-\frac{81}{4}+\frac{108}{3}-\frac{27}{2}\right)-\left(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2}\right)\right]$, or $\left(\frac{81}{4}-\frac{27}{2}+9\right)-\left(\frac{1}{4}-\frac{3}{2}+3\right)-\left\{\left(\frac{81}{2}-\frac{108}{3}+9\right)-\left(\frac{1}{2}-\frac{4}{3}+3\right)\right\}$	**DM1**	OE Minimum required is $\left(\frac{63}{4}-\frac{7}{4}\right)-\left(\frac{27}{2}-\frac{13}{6}\right)$, i.e. four fractions. Correctly apply limits *their* 1 and 3. Do not allow if $x=0$ used. Need at least one correct substitution in every bracket. If two integrals, need to see substitution into both. Allow one sign error only in each expression, if brackets are not shown.
	$=\frac{8}{3}$	**A1**	Accept if this comes from use of limits $f(1)-f(3)$ or $\int\left(x^3-4x^2+3x\right)\mathrm{d}x$, if $\left\lvert\frac{-8}{3}\right\rvert$ used. Only dependent on the first method mark. Accept AWRT 2.67.
		4	

Question	Answer	Marks	Guidance
10(a)	Gradient of $AB = \frac{-5-3}{8-4}$ $[=-2]$	**M1***	
	Midpoint $AB = \left(\frac{8+4}{2}, \frac{-5+3}{2}\right)$ $[(6,-1)]$	**M1**	
	Gradient of normal $= -\frac{1}{-2}$ $\left[=\frac{1}{2}\right]$ and an attempt to find the required equation	**DM1**	Must be used to find equation of perpendicular through *their* $(6,-1)$.
	Equation of perpendicular bisector is $y+1=\frac{1}{2}(x-6)$, so $y=\frac{1}{2}x-4$	**A1**	WWW AG – working involving the perpendicular bisector must be seen.
	Alternative Method for Question 10(a)		
	$AC^2=(a-4)^2+(b-3)^2$, $BC^2=(a-8)^2+(b+5)^2$ both expanded	**M1***	
	Solving $AC = BC$ [= 10]	**DM1**	Only allow a single sign error.
	Eliminating a^2 and b^2	**DM1**	May be awarded before the previous DM1.
	$a=2b+8$, concluding $y=\frac{x}{2}-4$	**A1**	WWW
		4	

Question	Answer	Marks	Guidance
10(b)	Using the centre as $\left(a,\ \frac{1}{2}a-4\right)$	**M1**	May see centre as $(2y+8,\ y)$ OE. May be seen in an incorrect equation.
	$(4-a)^2+(3-0.5a+4)^2=100$	**M1**	Sub in $(4, 3)$ or $(8, -5)$. Could use circle with $(6,-1)$ and $r=\sqrt{80}$.
	$1.25a^2-15a-35\ [=0] \Rightarrow a^2-12a-28\ [=0]\ \left(\text{or } b^2+2b-15\ [=0]\right)$	**DM1**	Obtain a 3-term quadratic in *their* x or y.
	$\left[(a-14)(a+2)=0\right] \Rightarrow a=14,\ a=-2$	**A1**	Or $\left[(b-3)(b+5)=[0]\right] \Rightarrow b=3,\ b=-5$.
	$\Rightarrow (x-14)^2+(y-3)^2=100$ and $(x+2)^2+(y+5)^2=100$	**A1**	
	Alternative Method 1 for the first 3 marks:		
	Make a or b the subject from a circle centre (a,b) using A or B	**M1**	E.g. $b=\sqrt{100-(y-3)^2}+4$ from circle through A. These equations may have been found in part **(a)**.
	Form an equation in a or b only	**M1**	Substitute *their* a or b into their second circle equation.
	Simplify to a quadratic in a or b	**DM1**	Expect $a^2-12a-28=0$ or $b^2+2b-15=0$, OE.
	Alternative Method 2 for the first 3 marks:		
	Obtaining CM (C, centre; M, mid-point of AB)	**M1**	Expect $\sqrt{80}$. Must be clear this is CM, not AB.
	Using the triangle CMT, where CT is parallel to the x-axis, to find the vertical distance of C from M, MT	**DM1**	Expect $MT=4$.
	Using the triangle CMT, where MT is parallel to the y-axis, to find the horizontal distance of C from M, CT	**DM1**	Expect $CT=8$.
		5	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}} - 8x$	**B1**	
	$\frac{d^2y}{dx^2} = -\frac{1}{4}kx^{-\frac{3}{2}} - 8$	**B1**	
		2	
11(b)	$x^{-\frac{1}{2}} - 8x = 0 \Rightarrow 1 - 8x^{\frac{3}{2}} = 0$ or $x^{-1} = 64x^2$ $\left[\Rightarrow x^3 = \frac{1}{64} \text{ or } 8x^{\frac{3}{2}} = 1\right]$ Setting their $\frac{dy}{dx}$ to zero and solving, providing their only error(s) are incorrect coefficients	**M1**	OE Award if working leads to $x = \frac{1}{4}$ WWW. Squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ to $x^{-1} - 64x^2 = 0$ gets M0.
	$x = \frac{1}{4}$ only	**A1**	If $x = 0$ included, A0 and max of 3/4. **SC B1** only for $x = \frac{1}{4}$ only from squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ directly to $x^{-1} - 64x^2 = 0$ (**SC B1** replacing the M1A1).
	$y = \frac{11}{4}$	**A1**	**SC B1** for $y = \frac{11}{4}$ from squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ to $x^{-1} - 64x^2 = 0$.
	$\frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}} - 8$ which is negative, so maximum	**B1 FT**	WWW FT *their* x-value and *their* $\frac{d^2y}{dx^2}$. No FT if $x = 0$ is the only solution.
		4	

Question	Answer	Marks	Guidance
11(c)	When $x=1$, attempting to find $y=k-2$ and gradient = $\frac{1}{2}k-8$	**M1***	OE **SC B1** if both correct gradients only, or both correct y-coordinates only.
	Equation of tangent is $y-k+2=\left(\frac{1}{2}k-8\right)(x-1)$	**A1**	OE, e.g. $y=\left(\frac{k}{2}-8\right)x+\frac{k}{2}+6$ or $y=\frac{k}{2}x-8x+\frac{k}{2}+6$.
	When $x=\frac{1}{4}$,attempting to find $y=\frac{1}{2}k+1.75$ and gradient = $k-2$	**M1***	OE
	Equation of tangent is $y-\frac{1}{2}k-1.75=(k-2)(x-0.25)$	**A1**	OE, e.g. $y=(k-2)x+\frac{k}{4}+\frac{9}{4}$ or $y=kx-2x+\frac{k}{4}+\frac{9}{4}$.
	Meet at $\left(\frac{1}{2}k-8\right)(0.6-1)+k-2=(k-2)(0.6-0.25)+\frac{1}{2}k+1.75$ Equate two tangent equations and substitute $x=0.6$	**DM1**	OE, e.g. $\left(\frac{k}{2}-8\right)0.6+\frac{k}{2}+6=(k-2)0.6+\frac{k}{4}+\frac{9}{4}$. M0 if constants in both equations are the same.
	$\Rightarrow\ [-0.2k+k+3.2-2=0.35k-0.7+0.5k+1.75]$ $\Rightarrow\ 0.05k=0.15$ $k=3$	**A1**	
		6	