Unit 8: Integration

Subunit 8.4: Applications of integration (Area and Volume)

Topical Question No: 1

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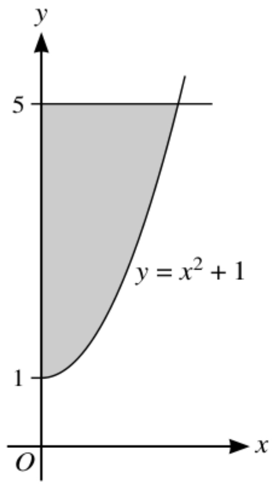

The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the ***y*-axis**.

Find the volume obtained. [4]

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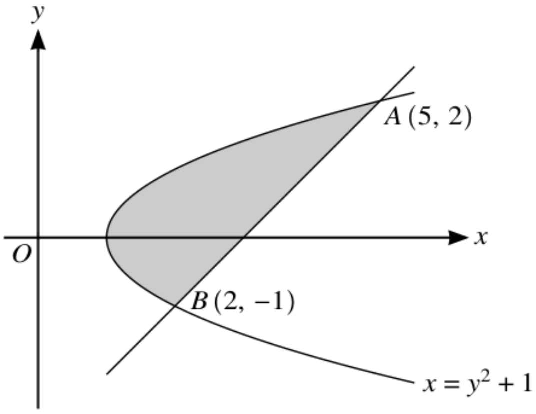

The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5,\ 2)$ and $B\,(2,\ -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

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(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the ***y*-axis**. [9]

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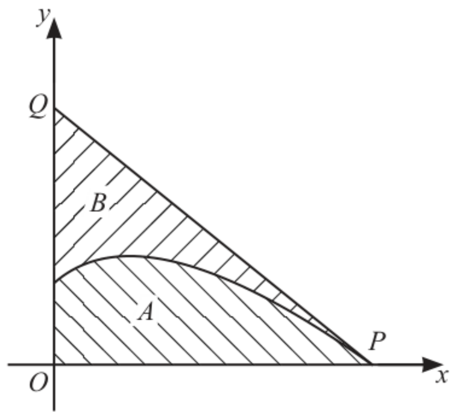

The diagram shows the curve with equation

$$y = 4(3x + 4)^{\frac{1}{2}} - 2x - 6$$

for values of x such that $0 \leqslant x \leqslant 7$. The tangent to the curve at the point $P\ (7, 0)$ meets the y-axis at the point Q. Region A is bounded by the curve and the two axes. Region B is bounded by the curve, the line segment PQ and the y-axis.

(a) Find the area of region A. [4]

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(b) Find the area of region B. [5]

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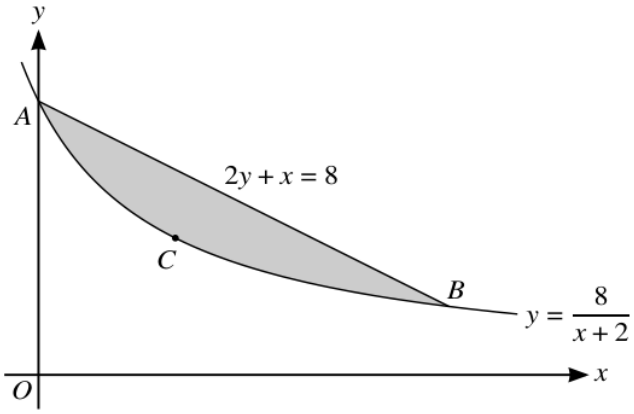

The diagram shows part of the curve $y = \dfrac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

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(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

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3 A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of $600\,\text{cm}^3$ per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

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(b) Find the rate of increase of the radius after 30 seconds. [3]

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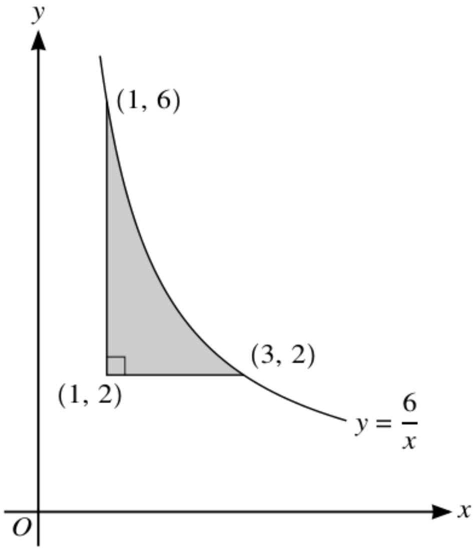

The diagram shows part of the curve $y = \frac{6}{x}$. The points $(1, 6)$ and $(3, 2)$ lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the ***y*-axis**. [5]

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(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

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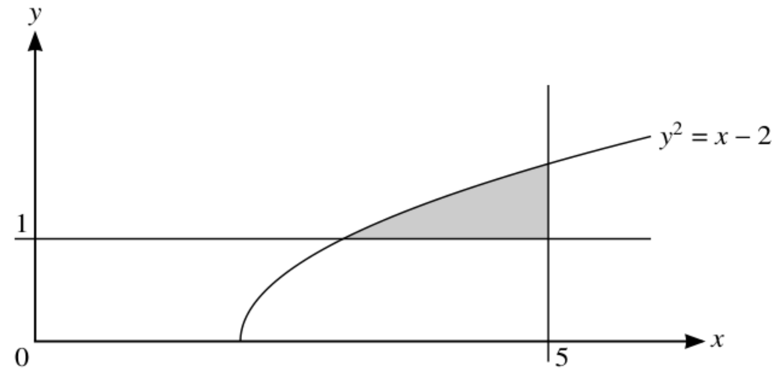

The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

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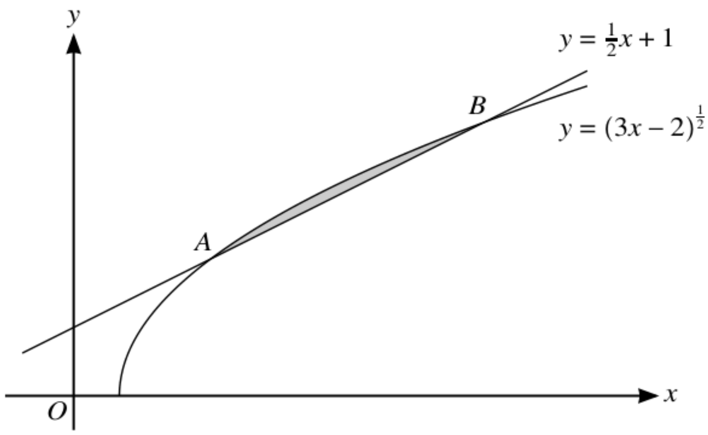

The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

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(b) Hence find the area of the region enclosed between the curve and the line. [5]

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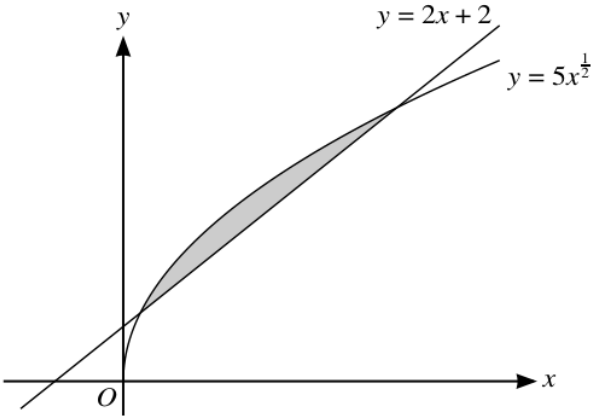

The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

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Topical Question No: 10

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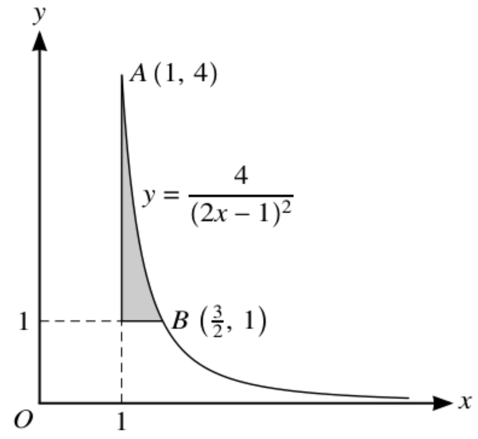

The diagram shows part of the curve with equation $y = \dfrac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\,(1, 4)$ and $B, \left(\frac{3}{2}, 1\right)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

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(b) A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

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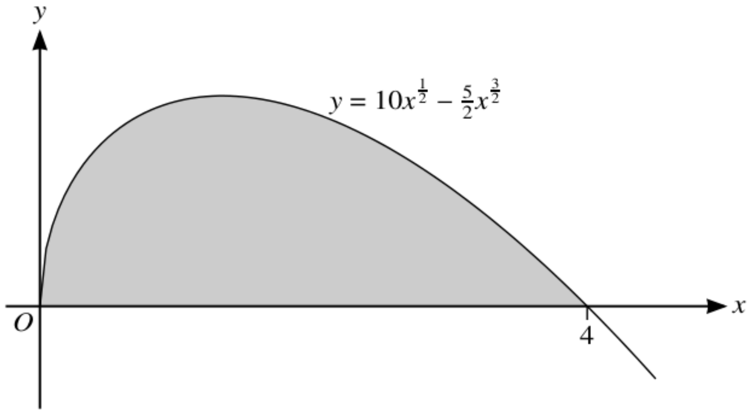

The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points $(0, 0)$ and $(4, 0)$.

Find the area of the shaded region. [4]

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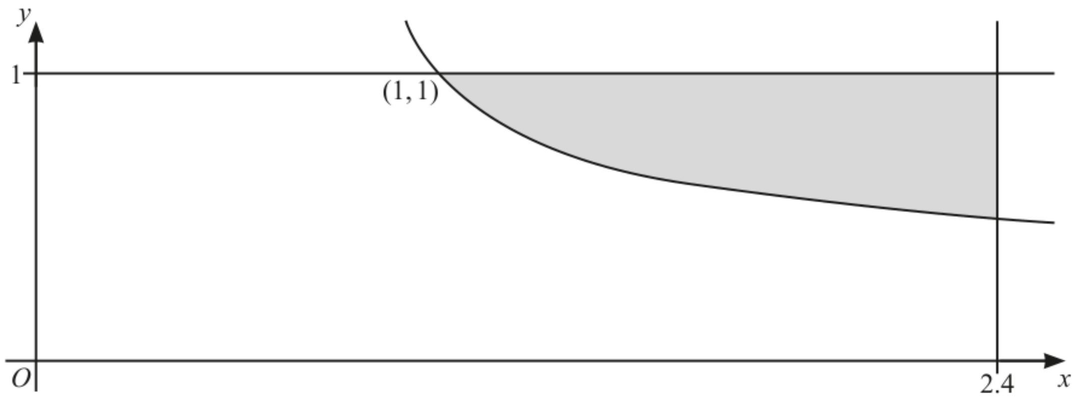

The diagram shows part of the curve with equation $y = \dfrac{1}{(5x-4)^{\frac{1}{3}}}$ and the lines $x = 2.4$ and $y = 1$. The curve intersects the line $y = 1$ at the point $(1, 1)$.

Find the exact volume of the solid generated when the shaded region is rotated through $360°$ about the x-axis. [6]

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6 The curve with equation $y = 2x - 8x^{\frac{1}{2}}$ has a minimum point at A and intersects the positive x-axis at B.

(a) Find the coordinates of A and B. [4]

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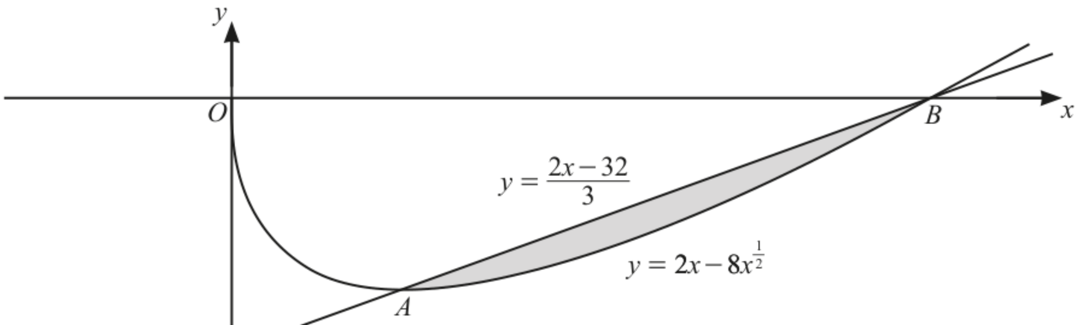

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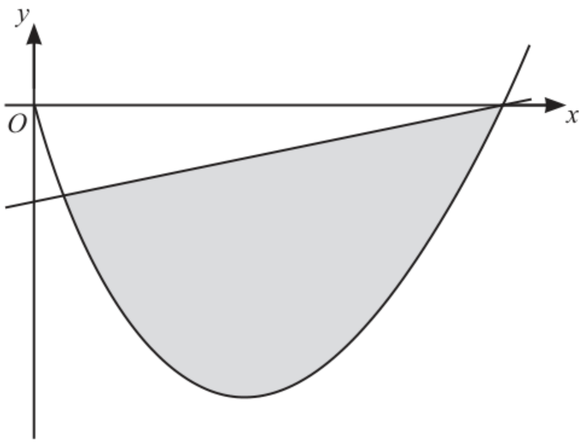

The diagram shows the curve with equation $y = 5x^{\frac{3}{2}} - 20x$ and the line with equation $y = x - 16$. The x-coordinates of the points of intersection of the curve and line are 1 and 16.

Find the area of the shaded region between the curve and the line. [5]