

Unit 6: Series

Subunit 6.2: Arithmetic and geometric progressions

Topical Question No: 1

9 The first term of a progression is $\cos \theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\frac{1}{\cos \theta}$.

(i) Show that the second term is $\cos \theta \sin^2 \theta$. [3]

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(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

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(b) For the case where the progression is arithmetic, the first two terms are again $\cos \theta$ and $\cos \theta \sin^2 \theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

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Topical Question No: 2

- 4 The first term of a geometric progression and the first term of an arithmetic progression are both equal to a .

The third term of the geometric progression is equal to the second term of the arithmetic progression.

The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a . [6]

This image shows a full page of a worksheet designed for handwriting practice. It consists of multiple rows of horizontal dotted lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

Topical Question No: 3

- 4 The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.
- (a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

- (b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

[illegible]

Topical Question No: 4

- 8 (a)** An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression.

[4]

This image shows a full page of white paper with ten evenly spaced horizontal dashed lines, typical of primary school handwriting practice paper. The lines extend across the entire width of the page, leaving margins at the top and bottom. There are no other markings or text on the page.

- (b) A geometric progression a_1, a_2, a_3, \dots is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by S . The sum to infinity of the even-numbered terms (i.e. a_2, a_4, a_6, \dots) is denoted by S_E .

Find the values of S and S_E .

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical Question No: 5

- 5** An arithmetic progression has first term 5 and common difference 6.

For this progression, find the sum of all the terms that lie between 150 and 400.

[6]

[illegible]

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Topical Question No: 6

- 1** The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression.

[4]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

Topical Question No: 7

- 3 Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.
- (a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- (b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

[illegible]

Topical Question No: 8

- 4** The n th term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical Question No: 9

8 The first term of a progression is $\sin^2 \theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2 \theta \cos^2 \theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

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It is now given instead that the progression is arithmetic.

(b) (i) Find the common difference of the progression in terms of $\sin \theta$. [3]

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Topical Question No: 10

- 2 The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dashed lines, creating a series of uniform gaps for letter height. The lines are evenly spaced across the entire page, providing a guide for consistent letter formation. There is no text or other markings on the page.

Topical Question No: 11

- 8 The first, second and third terms of an arithmetic progression are a , $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a , 18 and $b + 3$ respectively.

(a) Find the values of a and b . [5]

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(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

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Topical Question No: 12

- 9 (a)** A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio.

[3]

[illegible]

- (b) An arithmetic progression P has first term a and common difference d . An arithmetic progression Q has first term $2(a + 1)$ and common difference $(d + 1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } O} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } O} = \frac{2}{3}.$$

Find the value of a and the value of d .

[6]

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Topical Question No: 13

- 2** The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15 .

Find the sum of the first 50 terms of the progression.

[5]

[illegible]

$$(1 - k^2)^5$$

Topical Question No: 14

- 4** The first, second and third terms of an arithmetic progression are k , $6k$ and $k + 6$ respectively.

(a) Find the value of the constant k .

[2]

[illegible]

(b) Find the sum of the first 30 terms of the progression.

[3]

[illegible]

Topical Question No: 15

6 The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p .

(a) Given that the common difference of the progression is not zero, find the value of p . [3]

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(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

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Topical Question No: 16

- 9 The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

(b) Show that the n th term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the n th term of the other geometric progression. [4]

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Topical Question No: 17

- 8 A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.

- (a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480 . [5]

Topical Question No: 18

- 8 (a) The first three terms of an arithmetic progression are 25 , $4p - 1$ and $13 - p$, where p is a constant.

Find the value of the tenth term of the progression.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) The first three terms of a geometric progression are 25 , $4q - 1$ and $13 - q$, where q is a positive constant.

Find the sum to infinity of the progression.

[4]

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Topical Question No: 19

- 5 The first and second terms of an arithmetic progression are $\tan \theta$ and $\sin \theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

- (a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

[illegible]

The first and second terms of a geometric progression are $\tan \theta$ and $\sin \theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

- (b) (i)** Find the sum to infinity of the progression in terms of θ . [2]

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Topical Question No: 20



- 10 The geometric progression a_1, a_2, a_3, \dots has first term 2 and common ratio r where $r > 0$.
It is given that $\frac{9}{2}a_5 + 7a_3 = 8$.

(a) Find the value of r . [3]

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(b) Find the sum of the first 20 terms of the geometric progression. Give your answer correct to 4 significant figures. [2]

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(c) Find the sum to infinity of the progression a_2, a_5, a_8, \dots . [3]

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(a) Find the common difference.

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(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100.

[illegible]

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- (a) Find the tenth term of the progression. Give your answer correct to 3 significant figures. [5]

[illegible]

- (b) Find the exact value of the sum to infinity of the progression. [2]

[illegible]