Unit 3: Coordinate geometry

Subunit 3.2: Circles

12 A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]

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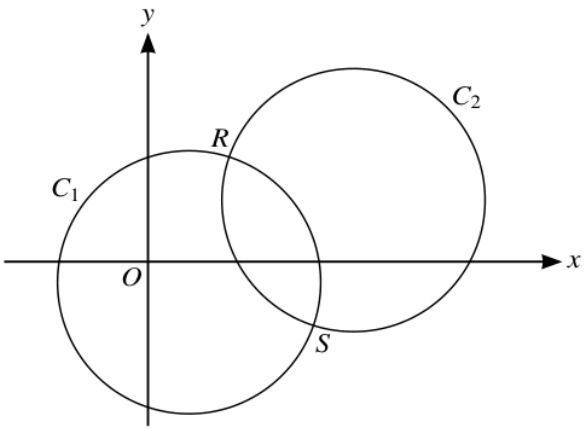

The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

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The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

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8 The points $A\,(7,\,1)$, $B\,(7,\,9)$ and $C\,(1,\,9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

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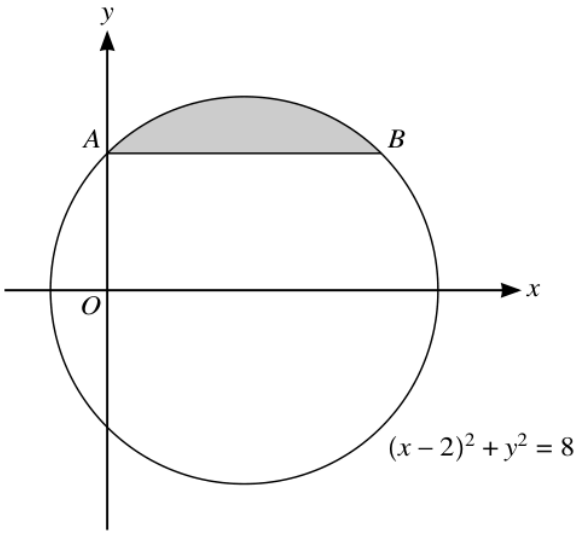

The diagram shows the circle with equation $(x-2)^2+y^2=8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

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(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

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5 Points A (7, 12) and B lie on a circle with centre (−2, 5). The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

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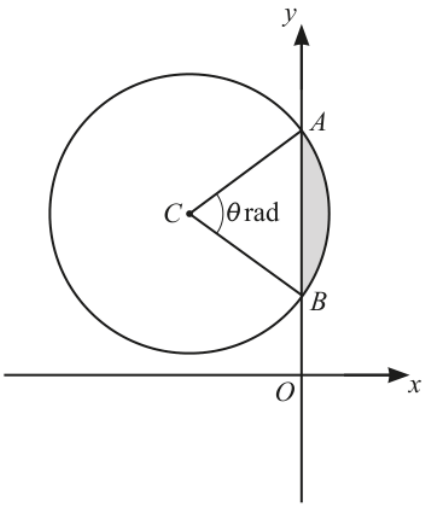

The diagram shows the circle with centre $C(-4, 5)$ and radius $\sqrt{20}$ units. The circle intersects the y-axis at the points A and B. The size of angle ACB is θ radians.

(a) Find the equation of the tangent to the circle at the point $(-6, 9)$. [3]

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(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [2]

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(c) Find the value of θ correct to 4 significant figures. [3]

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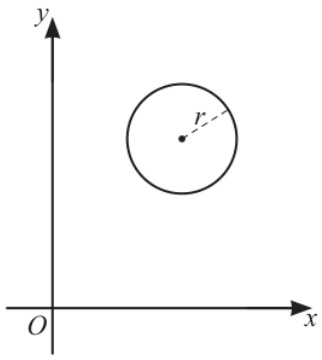

The diagram shows a circle C of radius r, where $x > 0$ and $y > 0$ for all points on C. The least distance between any point on C and the x-axis is 8 units, and the least distance between any point on C and the y-axis is 5 units.

(a) State the coordinates of the centre of the circle in terms of r. [1]

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(b) Given that the distance between the origin and the centre of the circle is 15 units, find the value of r. [3]

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(c) The point on the circle furthest from the origin is denoted by P.

Find the gradient of the tangent to the circle at P. [2]

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$8\sin^2\theta - 5\sin^4\theta$

10 The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

11 The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

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The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

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The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

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10 **(a)** The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

10 The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

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(b) Find the point of intersection of the tangents to the circle at A and B. [6]

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7 The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

10 Points $A\,(-2,\ 3)$, $B\,(3,\ 0)$ and $C\,(6,\ 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

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(b) Hence state the coordinates of D. [1]

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(c) Find an equation of the circle. [2]

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The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

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9 The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

8 The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A\,(1,\ 1)$ and $B\,(2,\ -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

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(b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p, q and k are integers. [4]

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Topical Question No: 15

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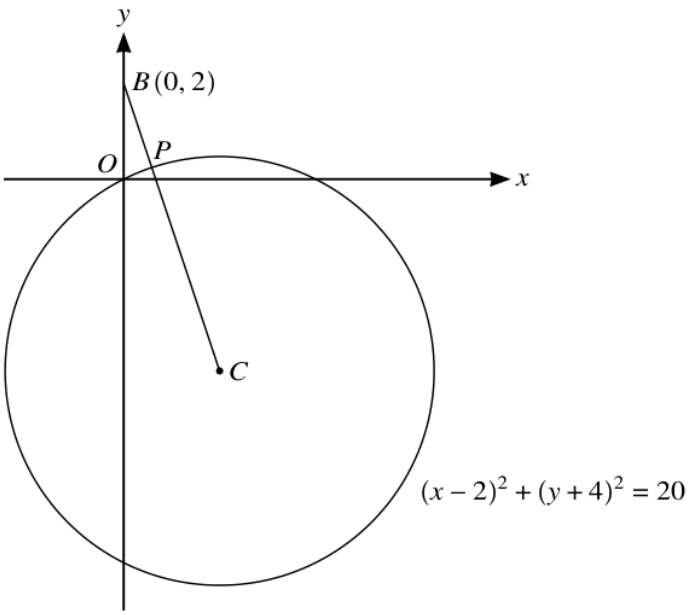

The diagram shows the circle with equation $(x-2)^2+(y+4)^2=20$ and with centre C. The point B has coordinates $(0, 2)$ and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

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(b) Hence find the coordinates of P, giving your answer in exact form. [5]

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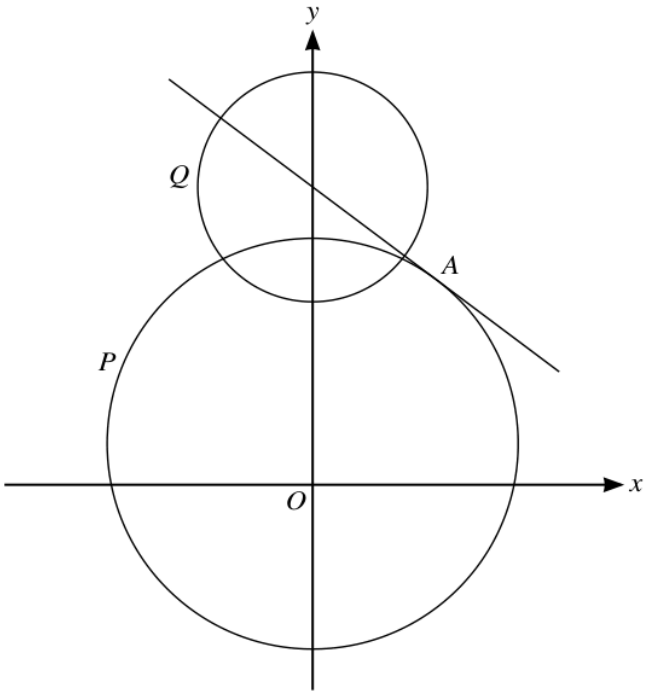

The diagram shows a circle P with centre (0, 2) and radius 10 and the tangent to the circle at the point A with coordinates (6, 10). It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

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(b) Find the equation of the tangent to the circle P at A. [2]

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(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

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10 The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

10 The equation of a circle is $(x-3)^2+y^2=18$. The line with equation $y=mx+c$ passes through the point $(0,-9)$ and is a tangent to the circle.

Find the two possible values of m and, for each value of m, find the coordinates of the point at which the tangent touches the circle. [8]

7 The equation of a circle is $(x-6)^2+(y+a)^2=18$. The line with equation $y=2a-x$ is a tangent to the circle.

(a) Find the two possible values of the constant a. [5]

(b) For the greater value of a, find the equation of the diameter which is perpendicular to the given tangent. [3]

8 A circle with equation $x^2 + y^2 - 6x + 2y - 15 = 0$ meets the y-axis at the points A and B. The tangents to the circle at A and B meet at the point P.

Find the coordinates of P. [8]

8 The circle with equation $x^2+y^2-6x+10y-27=0$ intersects the line $x=-2$ at the points P and Q.

Find the area of the triangle formed by the tangents to the circle at P and Q, and the line $x=-2$. [8]