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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6441974659

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

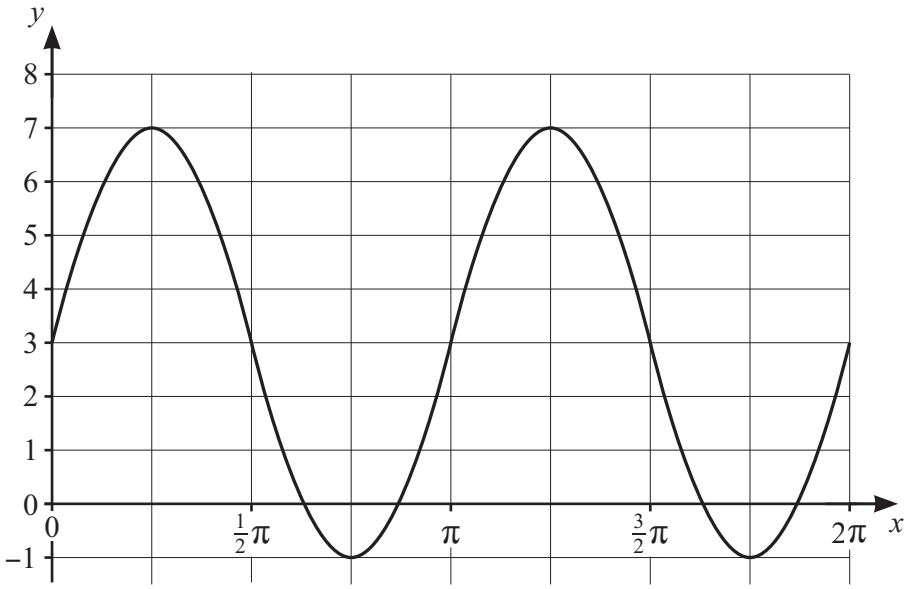
INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

[Turn over

1



The diagram shows the curve with equation $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$, where a, b and c are positive constants.

(a) State the values of a, b and c. [3]

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(b) For these values of a, b and c, determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations:

(i) $a\sin(bx) + c = 7 - x$ [1]

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(ii) $a\sin(bx) + c = 2\pi(x - 1)$. [1]

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2 The first term of an arithmetic progression is -20 and the common difference is 5.

(a) Find the sum of the first 20 terms of the progression. [2]

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It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

(b) Find the value of k. [3]

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3 The equation of a curve is $y = 2x^2 - 3$. Two points A and B with x-coordinates 2 and $(2 + h)$ respectively lie on the curve.

(a) Find and simplify an expression for the gradient of the chord AB in terms of h. [3]

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(b) Explain how the gradient of the curve at the point A can be deduced from the answer to part **(a)**, and state the value of this gradient. [2]

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4 Find the term independent of x in the expansion of each of the following:

(a) $\left(x+\frac{3}{x^2}\right)^6$ [2]

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(b) $(4x^3-5)\left(x+\frac{3}{x^2}\right)^6$. [4]

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5 The function f is defined by $\mathrm{f}(x) = \frac{2x+1}{2x-1}$ for $x < \frac{1}{2}$.

(a) (i) State the value of $\mathrm{f}(-1)$. [1]

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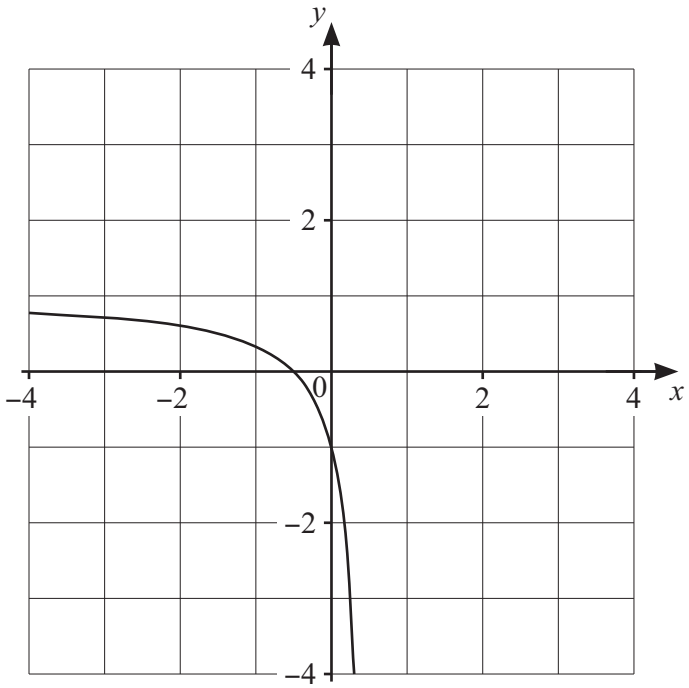
(ii)



The diagram shows the graph of $y = \mathrm{f}(x)$. Sketch the graph of $y = \mathrm{f}^{-1}(x)$ on this diagram. Show any relevant mirror line. [2]

(iii) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of the function f^{-1}. [4]

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The function g is defined by $g(x) = 3x + 2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

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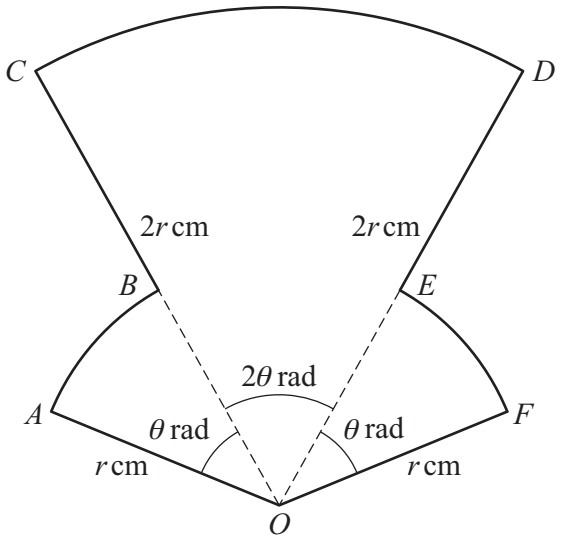

The diagram shows a metal plate $OABCDEF$ consisting of sectors of two circles, each with centre O. The radii of sectors AOB and EOF are r cm and the radius of sector COD is $2r$ cm.
Angle AOB = angle $EOF = \theta$ radians and angle $COD = 2\theta$ radians.

It is given that the perimeter of the plate is 14 cm and the area of the plate is $10\,\text{cm}^2$.

Given that $r > \frac{3}{2}$ and $\theta < \frac{3}{4}$, find the values of r and θ. [6]

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7 **(a)** By expressing $-2x^2+8x+11$ in the form $-a(x-b)^2+c$, where a, b and c are positive integers, find the coordinates of the vertex of the graph with equation $y=-2x^2+8x+11$. [3]

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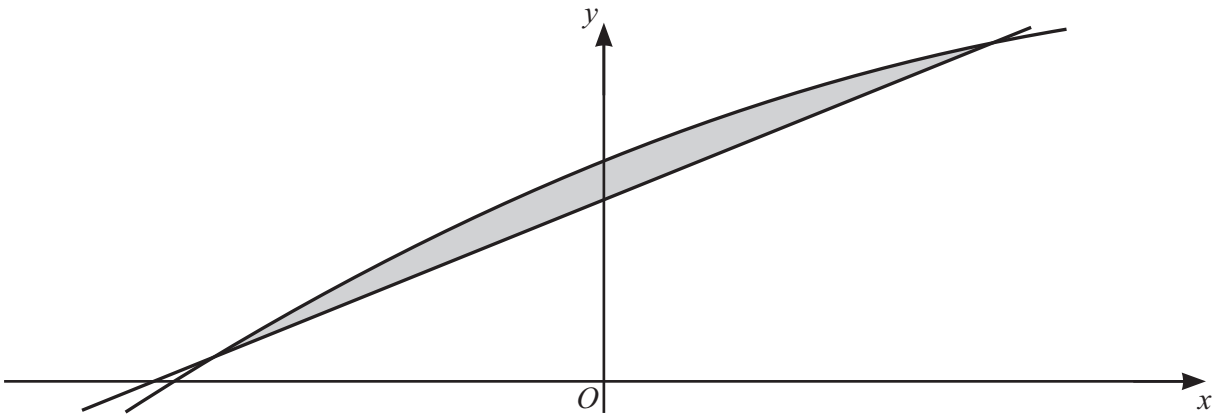
(b)



The diagram shows part of the curve with equation $y=-2x^2+8x+11$ and the line with equation $y=8x+9$.

Find the area of the shaded region. [5]

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8 The equation of a circle is $x^2 + y^2 + px + 2y + q = 0$, where p and q are constants.

(a) Express the equation in the form $(x-a)^2 + (y-b)^2 = r^2$, where a is to be given in terms of p and r^2 is to be given in terms of p and q. [2]

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The line with equation $x + 2y = 10$ is the tangent to the circle at the point $A(4, 3)$.

(b) (i) Find the equation of the normal to the circle at the point A. [3]

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(ii) Find the values of p and q. [5]

9 The equation of a curve is $y = \frac{1}{2}k^2x^2 - 2kx + 2$ and the equation of a line is $y = kx + p$, where k and p are constants with $0 < k < 1$.

(a) It is given that one of the points of intersection of the curve and the line has coordinates $\left(\frac{5}{2}, \frac{1}{2}\right)$.

Find the values of k and p, and find the coordinates of the other point of intersection. [7]

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(b) It is given instead that the line and the curve do **not** intersect.

Find the set of possible values of p. [3]

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10 A function f with domain $x > 0$ is such that $f'(x) = 8(2x-3)^{\frac{1}{3}} - 10x^{\frac{2}{3}}$. It is given that the curve with equation $y = f(x)$ passes through the point (1, 0).

(a) Find the equation of the normal to the curve at the point (1, 0). [3]

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(b) Find $f(x)$. [4]

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It is given that the equation $f'(x) = 0$ can be expressed in the form

$$125x^2 - 128x + 192 = 0.$$

(c) Determine, making your reasoning clear, whether f is an increasing function, a decreasing function or neither. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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