

## **Unit 2: Functions**

### **Subunit 2.3: Inverse functions**

Topical Question No: 1

9 (a) Express  $2x^2 + 12x + 11$  in the form  $2(x + a)^2 + b$ , where  $a$  and  $b$  are constants. [2]

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The function  $f$  is defined by  $f(x) = 2x^2 + 12x + 11$  for  $x \leq -4$ .

(b) Find an expression for  $f^{-1}(x)$  and state the domain of  $f^{-1}$ . [3]

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The function  $g$  is defined by  $g(x) = 2x - 3$  for  $x \leq k$ .

(c) For the case where  $k = -1$ , solve the equation  $fg(x) = 193$ . [2]

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Topical Question No: 2

7 Functions  $f$  and  $g$  are defined as follows:

$$f : x \mapsto x^2 + 2x + 3 \text{ for } x \leq -1,$$

$$g : x \mapsto 2x + 1 \text{ for } x \geq -1.$$

(a) Express  $f(x)$  in the form  $(x + a)^2 + b$  and state the range of  $f$ . [3]

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(b) Find an expression for  $f^{-1}(x)$ . [2]

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*Topical Question No: 3*

- 9** The function  $f$  is defined by  $f(x) = -3x^2 + 2$  for  $x \leq -1$ .

(a) State the range of  $f$ .

[1]

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(b) Find an expression for  $f^{-1}(x)$ .

[3]

[illegible]

The function  $g$  is defined by  $g(x) = -x^2 - 1$  for  $x \leq -1$ .

(c) Solve the equation  $fg(x) - gf(x) + 8 = 0$ .

[5]

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*Topical Question No: 4*

- 5** The function  $f$  is defined for  $x \in \mathbb{R}$  by

$$f : x \mapsto a - 2x,$$

where  $a$  is a constant.

- (a) Express  $ff(x)$  and  $f^{-1}(x)$  in terms of  $a$  and  $x$ . [4]

[illegible]

- (b)** Given that  $\text{ff}(x) = f^{-1}(x)$ , find  $x$  in terms of  $a$ . [2]

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Topical Question No: 5

9 The functions  $f$  and  $g$  are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$
$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express  $f(x)$  in the form  $(x - a)^2 + b$ . [2]

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It is given that  $f$  is a one-one function.

(b) State the smallest possible value of  $c$ . [1]

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It is now given that  $c = 5$ .

(c) Find an expression for  $f^{-1}(x)$  and state the domain of  $f^{-1}$ . [3]

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Topical Question No: 6

9 Functions  $f$  and  $g$  are defined as follows:

$$f(x) = (x - 2)^2 - 4 \text{ for } x \geq 2,$$

$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where  $a$  is a constant.

(a) State the range of  $f$ . [1]

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(b) Find  $f^{-1}(x)$ . [2]

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(c) Given that  $a = -\frac{5}{3}$ , solve the equation  $f(x) = g(x)$ . [3]

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(d) Given instead that  $ggf^{-1}(12) = 62$ , find the possible values of  $a$ . [5]

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*Topical Question No: 7*

**6** The function  $f$  is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for  $f^{-1}(x)$ . [3]

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(b) Show that  $1 - \frac{8}{x^2 + 4}$  can be expressed as  $\frac{x^2 - 4}{x^2 + 4}$  and hence state the range of  $f$ . [4]

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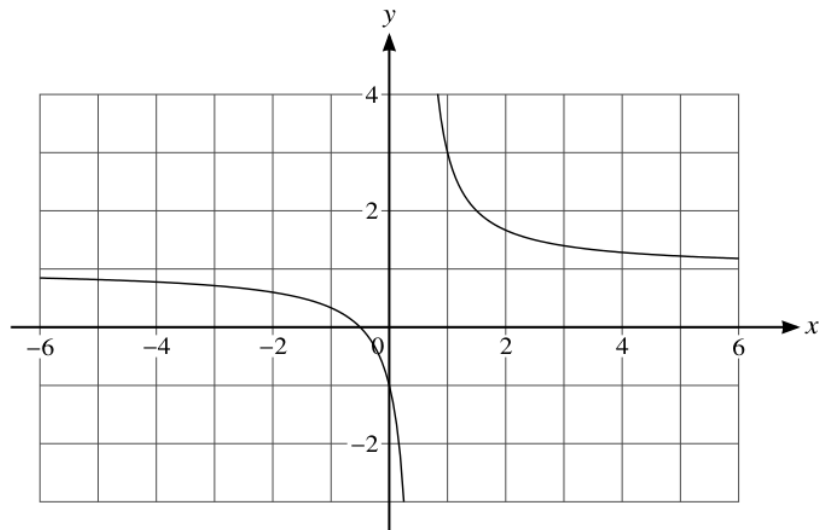
Topical Question No: 8

10 Functions  $f$  and  $g$  are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \frac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of  $y = f(x)$ .

State the domain of  $f^{-1}$ . [1]

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(b) Find an expression for  $f^{-1}(x)$ . [3]

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(c) Find  $gf^{-1}(3)$ . [2]

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(d) Explain why  $g^{-1}(x)$  cannot be found. [1]

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(e) Show that  $1 + \frac{2}{2x-1}$  can be expressed as  $\frac{2x+1}{2x-1}$ . Hence find the area of the triangle enclosed by the tangent to the curve  $y = f(x)$  at the point where  $x = 1$  and the  $x$ - and  $y$ -axes. [6]

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Topical Question No: 9

8 The functions  $f$  and  $g$  are defined as follows, where  $a$  and  $b$  are constants.

$$f(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$
$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that  $f(7) = \frac{5}{2}$  and  $gf(5) = 4$ , find the values of  $a$  and  $b$ . [4]

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For the rest of this question, you should use the value of  $a$  which you found in (a).

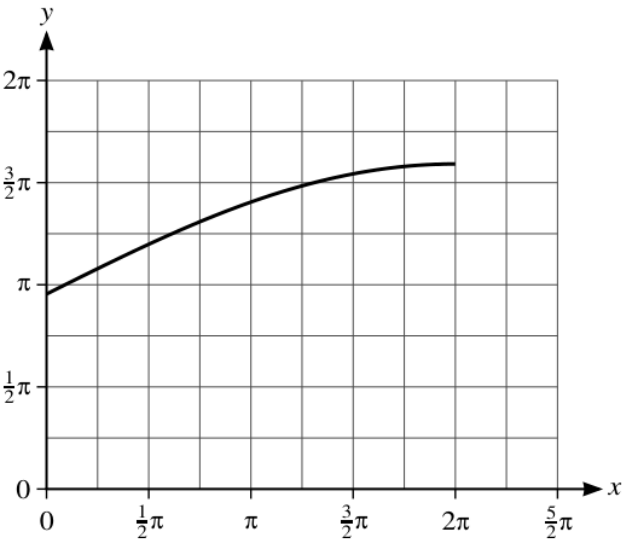
(b) Find the domain of  $f^{-1}$ . [1]

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The diagram shows the graph of  $y = f(x)$  where the function  $f$  is defined by

$$f(x) = 3 + 2 \sin \frac{1}{4}x \text{ for } 0 \leq x \leq 2\pi.$$

(a) On the diagram above, sketch the graph of  $y = f^{-1}(x)$ . [2]

(b) Find an expression for  $f^{-1}(x)$ . [2]

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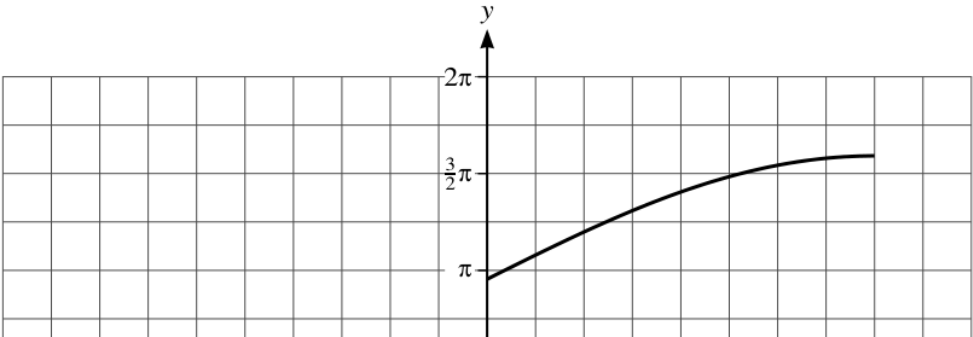
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(c)



Topical Question No: 11

7 The function  $f$  is defined by  $f(x) = 2 - \frac{5}{x+2}$  for  $x > -2$ .

(a) State the range of  $f$ . [1]

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(b) Obtain an expression for  $f^{-1}(x)$  and state the domain of  $f^{-1}$ . [4]

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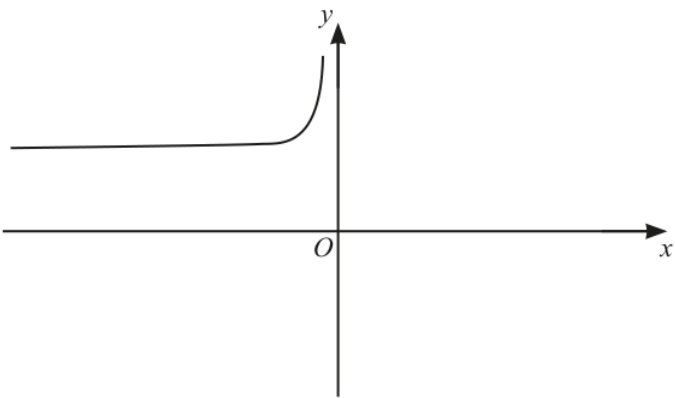
The function  $g$  is defined by  $g(x) = x + 3$  for  $x > 0$ .

(c) Obtain an expression for  $fg(x)$  giving your answer in the form  $\frac{ax+b}{cx+d}$ , where  $a, b, c$  and  $d$  are integers. [3]

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The function  $f$  is defined by  $f(x) = \frac{2}{x^2} + 4$  for  $x < 0$ . The diagram shows the graph of  $y = f(x)$ .

(a) On this diagram, sketch the graph of  $y = f^{-1}(x)$ . Show any relevant mirror line. [2]

(b) Find an expression for  $f^{-1}(x)$ . [3]

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(c) Solve the equation  $f(x) = 4.5$  . [1]

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(d) Explain why the equation  $f^{-1}(x) = f(x)$  has no solution. [1]

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Topical Question No: 13

4 The function  $f$  is defined as follows:

$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for  $f^{-1}(x)$ . [1]

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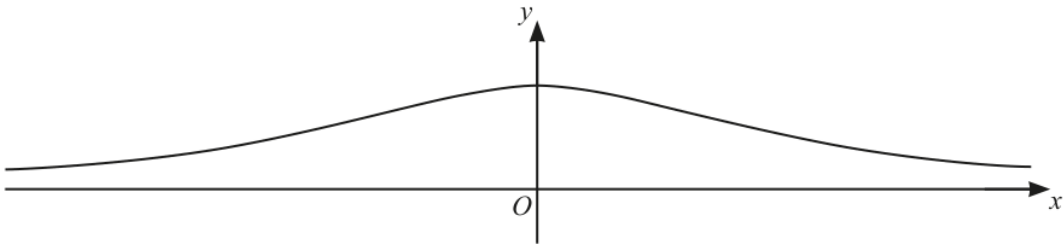
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The diagram shows the graph of  $y = g(x)$  where  $g(x) = \frac{1}{x^2 + 2}$  for  $x \in \mathbb{R}$ .

(b) State the range of  $g$  and explain whether  $g^{-1}$  exists. [2]

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The function  $h$  is defined by  $h(x) = \frac{1}{x^2 + 2}$  for  $x \geq 0$ .

(c) Solve the equation  $hf(x) = f\left(\frac{25}{16}\right)$ . Give your answer in the form  $a + b\sqrt{c}$ , where  $a$ ,  $b$  and  $c$  are integers. [4]

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Topical Question No: 15

10 The functions  $f$  and  $g$  are defined by

$$f(x) = \sqrt{x} \qquad \text{for } x \geq 0,$$

$$g(x) = 3\sqrt{x+2} - 5 \qquad \text{for } x \geq -2.$$

- (a) Describe fully a sequence of transformations which transforms the graph of  $y = f(x)$  to the graph of  $y = g(x)$ . You should make clear the order in which the transformations are applied. [5]

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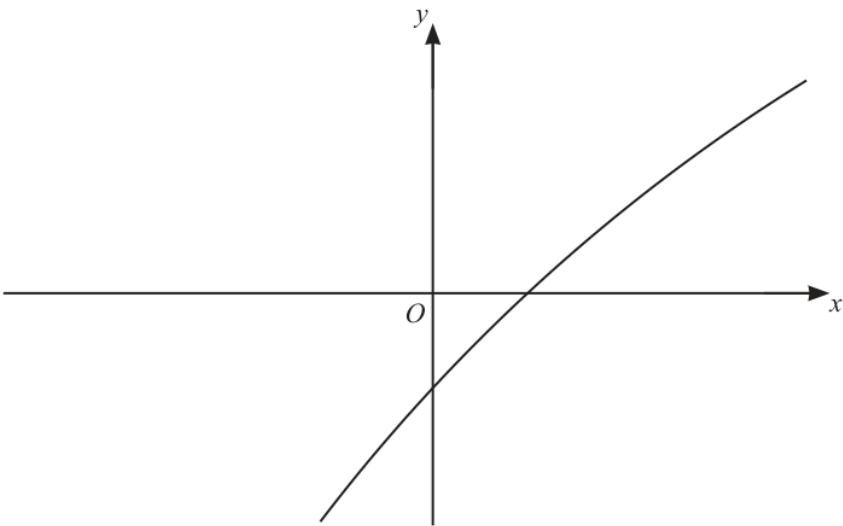
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The diagram shows the graph of  $y = g(x)$ .

- (b) On the diagram sketch the graph of  $y = g^{-1}(x)$  together with any relevant mirror line. [2]

- (c) Find an expression for  $g^{-1}(x)$ . [2]

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