Unit 5: Trigonometry

Subunit 5.5: Solving trigonometric equations

Topical Question No: 1

11 **(a)** Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

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(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

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Topical Question No: 2

5 Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

Topical Question No: 3

3 Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

7 **(a)** Show that $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} \equiv \dfrac{4}{5\cos^2\theta - 4}$. [4]

(b) Hence solve the equation $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} = 5$ for $0° < \theta < 180°$. [3]

Topical Question No: 5

7 **(a)** By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta\sin\theta = 1$$

for $0° < \theta < 360°$. [5]

(b) Show that $\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

7 (a) Show that $3\tan^2\theta + 5\sin^2\theta \equiv \dfrac{8\sin^2\theta - 5\sin^4\theta}{1-\sin^2\theta}$. [3]

(b) Hence solve the equation $3\tan^2\theta + 5\sin^2\theta = 9$ for $0° < \theta < 270°$. [4]

Topical Question No: 7

2 **(a)** Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

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(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

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7 **(a)** Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

4 **(a)** Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$ [2]

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(b) Hence express $\cos x$ in terms of k. [2]

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(c) Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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8 **(a)** The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30°\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

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(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

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Topical Question No: 11

11 The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leqslant x \leqslant 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$. [5]

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(b) Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

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Topical Question No: 12

1 Solve the equation $4 \sin \theta + \tan \theta = 0$ for $0° < \theta < 180°$. [3]

7 **(a)** **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

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(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

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(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta + \sin\theta} + \dfrac{1 - \cos\theta}{\cos\theta - \sin\theta} \equiv \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$. [3]

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4 **(a)** Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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3 **(a)** Show that the equation $\frac{7\tan\theta}{\cos\theta}+12=0$ can be expressed as

$$12\sin^2\theta-7\sin\theta-12=0.$$

[3]

(b) Hence solve the equation $\frac{7\tan\theta}{\cos\theta}+12=0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

4 **(a)** Show that the equation $\cos\theta(7\tan\theta - 5\cos\theta) = 1$ can be written in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are integers to be found. [3]

(b) Hence solve the equation $\cos 2x(7\tan 2x - 5\cos 2x) = 1$ for $0° < x < 180°$. [3]

1 Solve the equation $6\sin\theta = 1 + \dfrac{2}{\sin\theta}$ for $-180° < \theta < 180°$. [4]