Unit 2: Functions

Subunit 2.2: Composition of functions and range

9 Functions f, g and h are defined as follows:

$$\text{f} : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$
$$\text{g} : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$
$$\text{h} : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $\text{f}(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

(b) Given that $\text{f}(x) \equiv \text{gh}(x)$, find the values of m and n. [4]

9 The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = (3x-2)^2 + k \quad \text{and} \quad \mathrm{g}(x) = 5x - 1,$$

where k is a constant.

(a) Given that the range of the function gf is $\mathrm{gf}(x) \geqslant 39$, find the value of k. [4]

(b) For this value of k, determine the range of the function fg. [2]

6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \tfrac{1}{2}x - a,$$
$$\mathrm{g} : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $\mathrm{gg}(2) = 10$ and $\mathrm{f}^{-1}(2) = 14$, find the values of a and b. [4]

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(b) Using these values of a and b, find an expression for $\mathrm{gf}(x)$ in the form $cx + d$, where c and d are constants. [2]

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5 The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

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(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

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8 Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 - 1 \text{ for } x < 0,$$
$$\mathrm{g} : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $\mathrm{fg}(x) = 3$. [4]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]