Unit 3: Coordinate geometry

Subunit 3.3: Intersection of lines and curves

Topical Question No: 1

4 A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

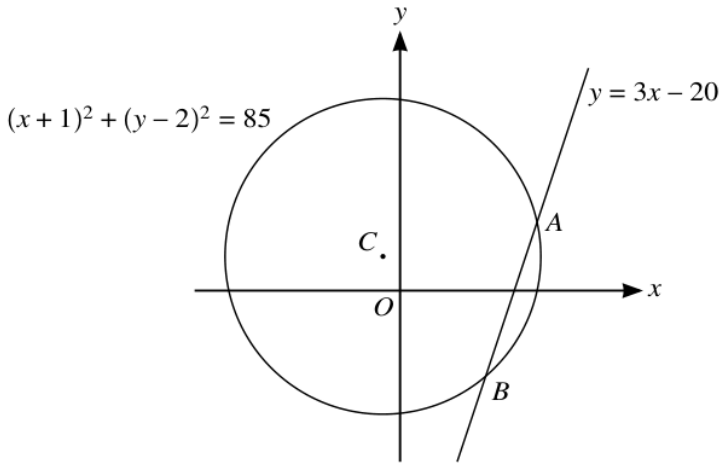
Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

Topical Question No: 2

2 A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]

6



The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

(a) Find, by calculation, the coordinates of A and B. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

.....

.....

.....

.....

Topical Question No: 4

1 A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k. [4]

Topical Question No: 5

7 The straight line $y = x + 5$ meets the curve $2x^2 + 3y^2 = k$ at a single point P.

(a) Find the value of the constant k. [4]

(b) Find the coordinates of P. [2]

Topical Question No: 6

1 A curve has equation $y = 5 + 3x - 2x^2$ and a straight line has equation $y = kx + 13$, where k is a constant.

Find the set of values of k for which the curve and the line do **not** meet. [4]

5 The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Topical Question No: 8

1 Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

6 Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

3 A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

5 The equation of a curve is $y = 4x^2 - kx + \frac{1}{2}k^2$ and the equation of a line is $y = x - a$, where k and a are constants.

(a) Given that the curve and the line intersect at the points with x-coordinates 0 and $\frac{3}{4}$, find the values of k and a. [4]

(b) Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]

5 The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

5 A circle has equation $(x - 1)^2 + (y + 4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find an equation of the circle with diameter AB. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 The equation of a curve is $2x^2 - kxy + 2 = 0$ and the equation of a line is $y = px + 3$, where k and p are constants.

(a) Given that $k = 2$ and $p = 11$, find the coordinates of the points of intersection of the curve and the line. [4]

(b) Given instead that $p = 4$, find the set of values of k for which the curve and the line do not intersect. [5]