

## **Unit 5: Trigonometry**

### **Subunit 5.5: Solving trigonometric equations**

Topical Question No: 1

11 (a) Solve the equation  $3 \tan^2 x - 5 \tan x - 2 = 0$  for  $0^\circ \leq x \leq 180^\circ$ . [4]

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(b) Find the set of values of  $k$  for which the equation  $3 \tan^2 x - 5 \tan x + k = 0$  has no solutions. [2]

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(c) For the equation  $3 \tan^2 x - 5 \tan x + k = 0$ , state the value of  $k$  for which there are three solutions in the interval  $0^\circ \leq x \leq 180^\circ$ , and find these solutions. [3]

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*Topical Question No: 2*

**5** Solve the equation

$$\frac{\tan \theta + 3 \sin \theta + 2}{\tan \theta - 3 \sin \theta + 1} = 2$$

for  $0^\circ \leq \theta \leq 90^\circ$ .

[5]

[illegible]

*Topical Question No: 3*

- 3** Solve the equation  $\frac{\tan \theta + 2 \sin \theta}{\tan \theta - 2 \sin \theta} = 3$  for  $0^\circ < \theta < 180^\circ$ .

[4]

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### Topical Question No: 4

7 (a) Show that  $\frac{\sin \theta + 2 \cos \theta}{\cos \theta - 2 \sin \theta} - \frac{\sin \theta - 2 \cos \theta}{\cos \theta + 2 \sin \theta} \equiv \frac{4}{5 \cos^2 \theta - 4}$ . [4]

[illegible]

(b) Hence solve the equation  $\frac{\sin \theta + 2 \cos \theta}{\cos \theta - 2 \sin \theta} - \frac{\sin \theta - 2 \cos \theta}{\cos \theta + 2 \sin \theta} = 5$  for  $0^\circ < \theta < 180^\circ$ . [3]

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*Topical Question No: 5*

- 7 (a) By first obtaining a quadratic equation in  $\cos \theta$ , solve the equation

$$\tan \theta \sin \theta = 1$$

for  $0^\circ < \theta < 360^\circ$ .

[5]

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- (b) Show that  $\frac{\tan \theta}{\sin \theta} - \frac{\sin \theta}{\tan \theta} \equiv \tan \theta \sin \theta$ .

[3]

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[illegible]

### Topical Question No: 7

- 2 (a)** Express the equation  $3 \cos \theta = 8 \tan \theta$  as a quadratic equation in  $\sin \theta$ . [3]

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- (b) Hence find the acute angle, in degrees, for which  $3 \cos \theta = 8 \tan \theta$ . [2]

[illegible]



### Topical Question No: 8

7 (a) Show that  $\frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} \equiv \frac{2}{\sin \theta \cos \theta}$ . [4]

[illegible]

(b) Hence solve the equation  $\frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} = \frac{6}{\tan \theta}$  for  $0^\circ < \theta < 180^\circ$ . [4]

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Topical Question No: 9

4 (a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where  $k$  is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k. \tag{2}$$

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(b) Hence express  $\cos x$  in terms of  $k$ . [2]

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(c) Hence solve the equation  $\frac{\tan x + \sin x}{\tan x - \sin x} = 4$  for  $-\pi < x < \pi$ . [2]

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*Topical Question No: 10*

- 8 (a)** The curve  $y = \sin x$  is transformed to the curve  $y = 4 \sin(\frac{1}{5}x - 30^\circ)$ .

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

[illegible]

- (b)** Find the exact solutions of the equation  $4 \sin(\frac{1}{5}x - 30^\circ) = 2\sqrt{2}$  for  $0^\circ \leq x \leq 360^\circ$ . [3]

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*Topical Question No: 11*

**11** The function  $f$  is given by  $f(x) = 4\cos^4 x + \cos^2 x - k$  for  $0 \leq x \leq 2\pi$ , where  $k$  is a constant.

(a) Given that  $k = 3$ , find the exact solutions of the equation  $f(x) = 0$ . [5]

[illegible]

(b) Use the quadratic formula to show that, when  $k > 5$ , the equation  $f(x) = 0$  has no solutions. [5]

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*Topical Question No: 12*

**1** Solve the equation  $4 \sin \theta + \tan \theta = 0$  for  $0^\circ < \theta < 180^\circ$ .

[3]

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Topical Question No: 13

7 (a) (i) By first expanding  $(\cos \theta + \sin \theta)^2$ , find the three solutions of the equation

$$(\cos \theta + \sin \theta)^2 = 1$$

for  $0 \leq \theta \leq \pi$ . [3]

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(ii) Hence verify that the only solutions of the equation  $\cos \theta + \sin \theta = 1$  for  $0 \leq \theta \leq \pi$  are 0 and  $\frac{1}{2}\pi$ . [2]

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(b) Prove the identity  $\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} \equiv \frac{\cos \theta + \sin \theta - 1}{1 - 2 \sin^2 \theta}$ . [3]

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*Topical Question No: 14*

- 4 (a)** Show that the equation

$$3 \tan^2 x - 3 \sin^2 x - 4 = 0$$

may be expressed in the form  $a \cos^4 x + b \cos^2 x + c = 0$ , where  $a$ ,  $b$  and  $c$  are constants to be found. [3]

[illegible]

- (b)** Hence solve the equation  $3 \tan^2 x - 3 \sin^2 x - 4 = 0$  for  $0^\circ \leq x \leq 180^\circ$ . [4]

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*Topical Question No: 15*

- 3 (a)** Show that the equation  $\frac{7 \tan \theta}{\cos \theta} + 12 = 0$  can be expressed as

$$12 \sin^2 \theta - 7 \sin \theta - 12 = 0. \quad [3]$$

[illegible]

- (b) Hence solve the equation  $\frac{7 \tan \theta}{\cos \theta} + 12 = 0$  for  $0^\circ \leq \theta \leq 360^\circ$ . [3]

[illegible]



Topical Question No: 16



- 4 (a) Show that the equation  $\cos \theta (7 \tan \theta - 5 \cos \theta) = 1$  can be written in the form  $a \sin^2 \theta + b \sin \theta + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers to be found. [3]

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- (b) Hence solve the equation  $\cos 2x (7 \tan 2x - 5 \cos 2x) = 1$  for  $0^\circ < x < 180^\circ$ . [3]

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