Unit 4: Mechanics (for Paper 4)

Subunit 4.2: Kinematics of motion in a straight line:

Topical Question No: 1

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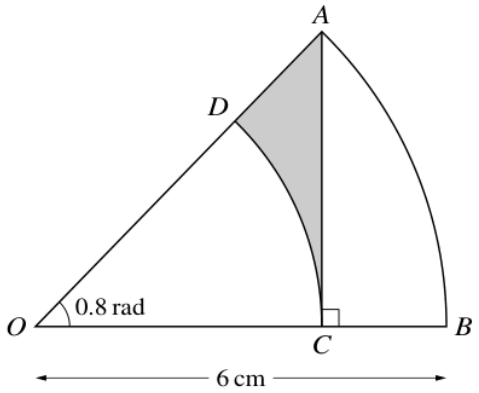

The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

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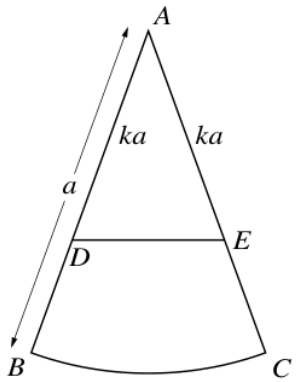

The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

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(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\dfrac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

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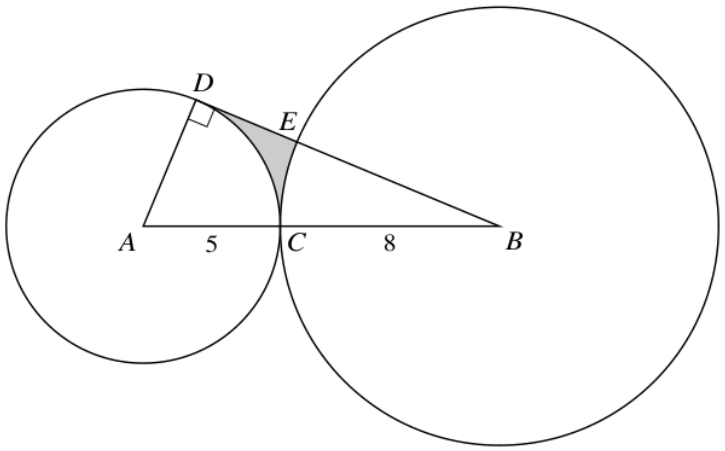

The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B.

(a) Find the perimeter of the shaded region. [5]

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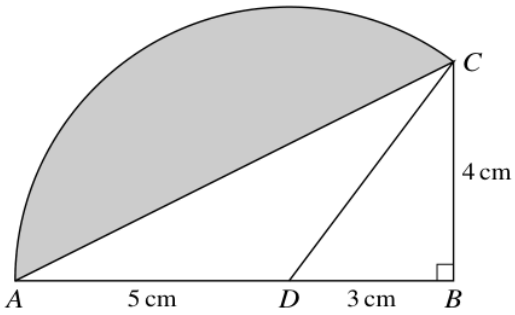

The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D.

(a) Find the perimeter of the shaded region. [5]

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(b) Find the area of the shaded region. [3]

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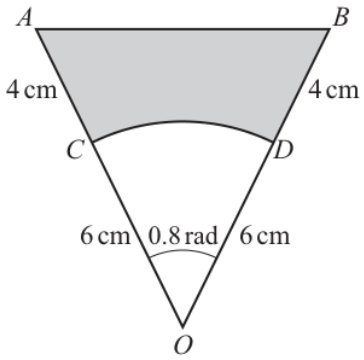

The diagram shows a triangle OAB where $OA = OB = 10$ cm and angle $AOB = 0.8$ radians.
Points C and D on OA and OB respectively are such that the arc CD is part of a circle with centre O and radius 6 cm. The shaded region is bounded by the arc CD and the line segments CA, AB and BD.

(a) Find the perimeter of the shaded region. [3]

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(b) Find the area of the shaded region. [3]

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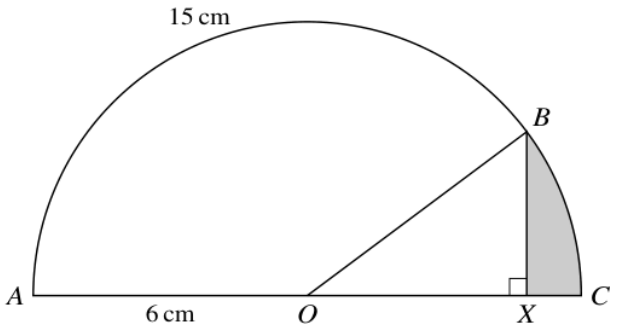

In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

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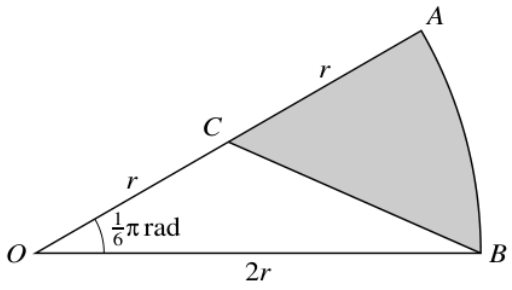

In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

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(b) Find the exact perimeter of the shaded region. [2]

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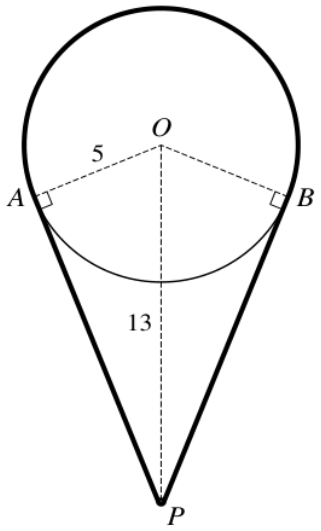

The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

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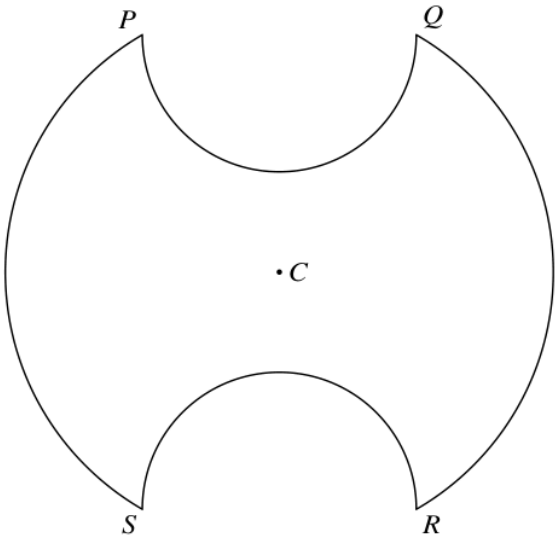

The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

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(b) Find the exact perimeter of the plate. [3]

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(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)$ cm^2. [5]

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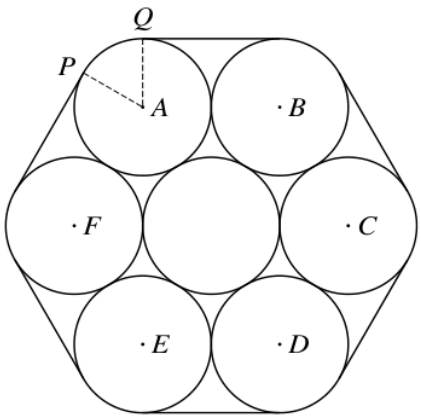

The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

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(b) Find the length of the rope. [4]

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(c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

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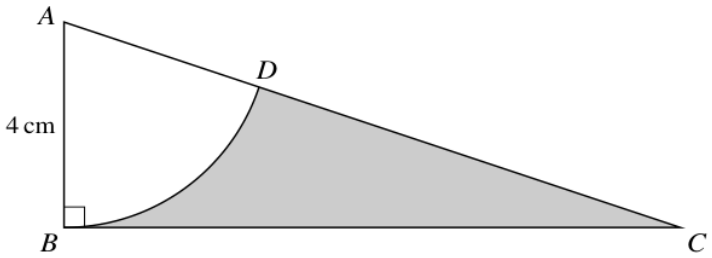

The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

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(b) Find the perimeter of the shaded region. [4]

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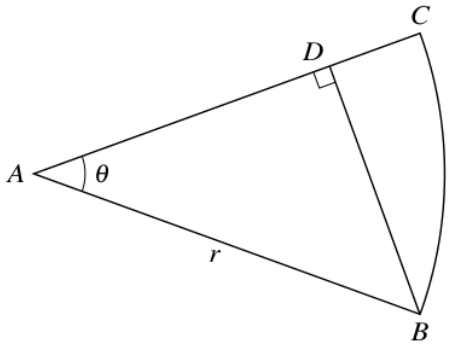

The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r. [3]

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(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r. [4]

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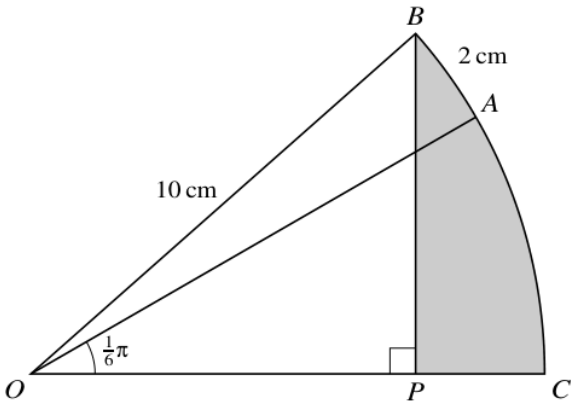

The diagram shows a sector $OBAC$ of a circle with centre O and radius 10 cm. The point P lies on OC and BP is perpendicular to OC. Angle $AOC = \frac{1}{6}\pi$ and the length of the arc AB is 2 cm.

(a) Find the angle BOC. [2]

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(b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

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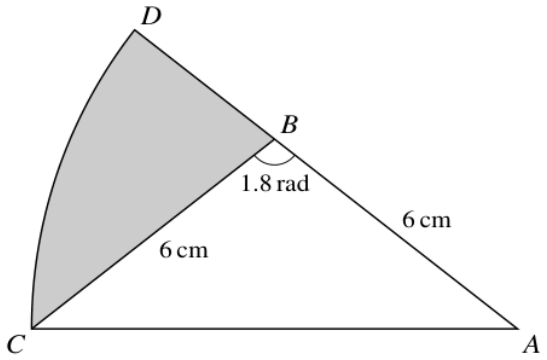

The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. [5]

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(b) Find the area of the shaded region. [3]

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Topical Question No: 15

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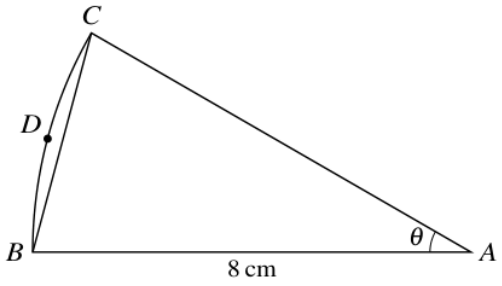

The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi$ cm^2. The point D lies on the arc BC.

Find the perimeter of the segment BCD. [4]

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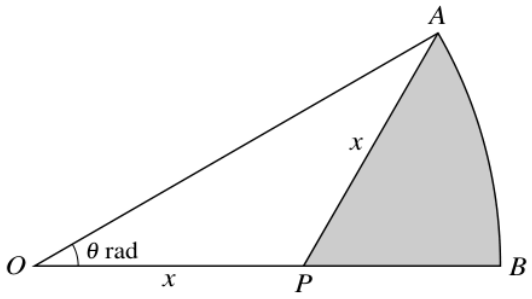

The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$. [2]

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(b) Find the area of the shaded region APB in terms of x and θ. [4]

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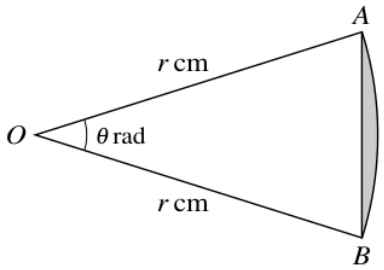

The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2.

(a) Find the area of the shaded region. [5]

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(b) Find the perimeter of the shaded region. [2]

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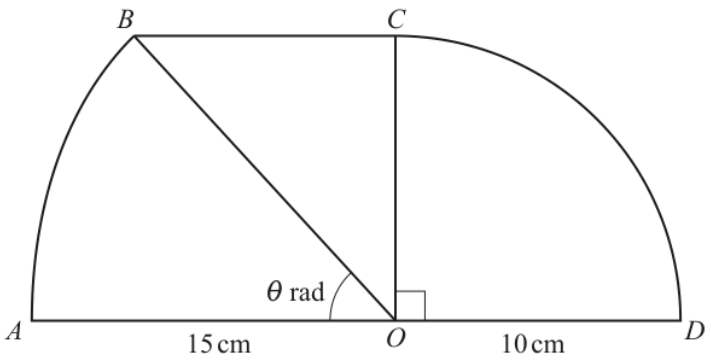

In the diagram, AOD and BC are two parallel straight lines. Arc AB is part of a circle with centre O and radius 15 cm. Angle $BOA = \theta$ radians. Arc CD is part of a circle with centre O and radius 10 cm. Angle $COD = \frac{1}{2}\pi$ radians.

(a) Show that $\theta = 0.7297$, correct to 4 decimal places. [1]

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(b) Find the perimeter and the area of the shape $ABCD$. Give your answers correct to 3 significant figures. [7]

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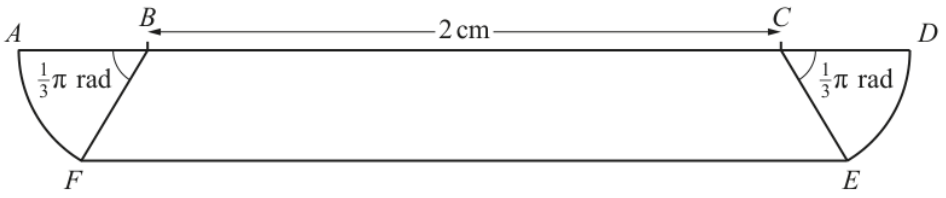

The diagram shows a symmetrical plate *ABCDEF*. The line *ABCD* is straight and the length of *BC* is 2 cm. Each of the two sectors *ABF* and *DCE* is of radius *r* cm and each of the angles *ABF* and *DCE* is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that $r = 0.4$ cm.

(i) Show that the length $EF = 2.4$ cm. [2]

(ii) Find the area of the plate. Give your answer correct to 3 significant figures. [4]

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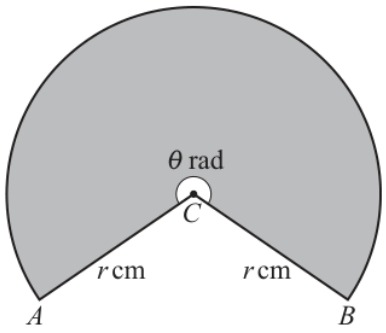

The diagram shows a sector of a circle with centre C. The radii CA and CB each have length r cm and the size of the reflex angle ACB is θ radians. The sector, shaded in the diagram, has a perimeter of 65 cm and an area of 225 cm^2.

(a) Find the values of r and θ. [4]

(b) Find the area of triangle ACB. [2]

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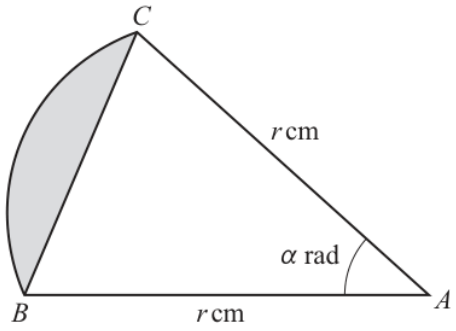

The diagram shows a sector ABC of a circle with centre A and radius r cm. The angle BAC is α radians, where $0 < \alpha < \frac{1}{2}\pi$.

(a) It is given that the area of the triangle ABC is $4\,\text{cm}^2$ and the area of the sector ABC is $8\alpha\,\text{cm}^2$.

Find the exact area of the shaded segment. [4]

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(b) It is given instead that the length of the chord BC is $\frac{1}{\sqrt{2}}r$ cm but the area of the triangle ABC is still $4\,\text{cm}^2$.

Find the area of the shaded segment. Give your answer correct to 3 significant figures. [4]

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