Unit 7: Differentiation

Subunit 7.4: Stationary points

Topical Question No: 1

10 The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

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(b) Determine the nature of the stationary point. [3]

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(c) Find the equation of the curve. [4]

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Topical Question No: 2

11 It is given that a curve has equation $y = k(3x - k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

The function f has a stationary value at $x = a$ and is defined by

$$f(x) = 4(3x - 4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

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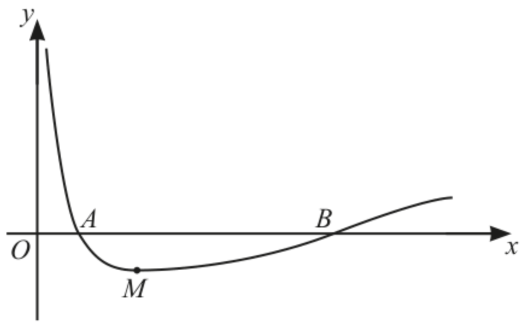

The diagram shows the curve with equation $y = 2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1$ for $x > 0$. The curve crosses the x-axis at points A and B and has a minimum point M.

(a) Find the exact coordinates of M. [4]

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(b) Find the area of the region bounded by the curve and the line segment AB. [7]

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9 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{x^4} - \frac{5}{x^3}$. It is given that the curve has a stationary point at $\left(\frac{1}{2}, 9\right)$.

(a) Use the expression for $\frac{d^2y}{dx^2}$ to determine whether the stationary point is a maximum or a minimum point. [2]

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(b) Find the equation of the curve. [7]

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Topical Question No: 5

9 The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

Topical Question No: 6

10 The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each of the stationary points. [2]

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11 The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

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(b) Find the equation of the curve. [4]

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(c) Find $\frac{d^2y}{dx^2}$. [2]

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Topical Question No: 8

10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. [1]

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(b) Find the equation of the curve. [5]

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(c) Show that the curve has no other stationary points. [3]

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11 The equation of a curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

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(b) Determine the nature of this stationary point. [2]

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(c) Find the equation of the curve. [3]

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5 The equation of a curve is $y = 2x^2 - \dfrac{1}{2x} + 3$.

(a) Find the coordinates of the stationary point. [3]

(b) Determine the nature of the stationary point. [2]

(c) For positive values of x, determine whether the curve shows a function that is increasing, decreasing or neither. Give a reason for your answer. [2]