Chapter 26 AVL Trees



Objectives

- **◆** To know what an AVL tree is (§26.1).
- ★ To understand how to rebalance a tree using the LL rotation, LR rotation, RR rotation, and RL rotation (§26.2).
- → To know how to design the AVLTree class (§26.3).
- **◆** To insert elements into an AVL tree (§26.4).
- → To implement node rebalancing (§26.5).
- **→** To delete elements from an AVL tree (§26.6).
- → To implement the AVLTree class (§26.7).
- **◆** To test the AVLTree class (§26.8).
- ◆ To analyze the complexity of search, insert, and delete operations in AVL trees (§26.9).

Why AVL Tree?

The search, insertion, and deletion time for a binary tree is dependent on the height of the tree. In the worst case, the height is O(n). If a tree is *perfectly* balanced, i.e., a complete binary tree, its height is. Can we maintain a perfectly balanced tree? Yes. But it will be costly to do so. The compromise is to maintain a well-balanced tree, i.e., the heights of two subtrees for every node are about the same.

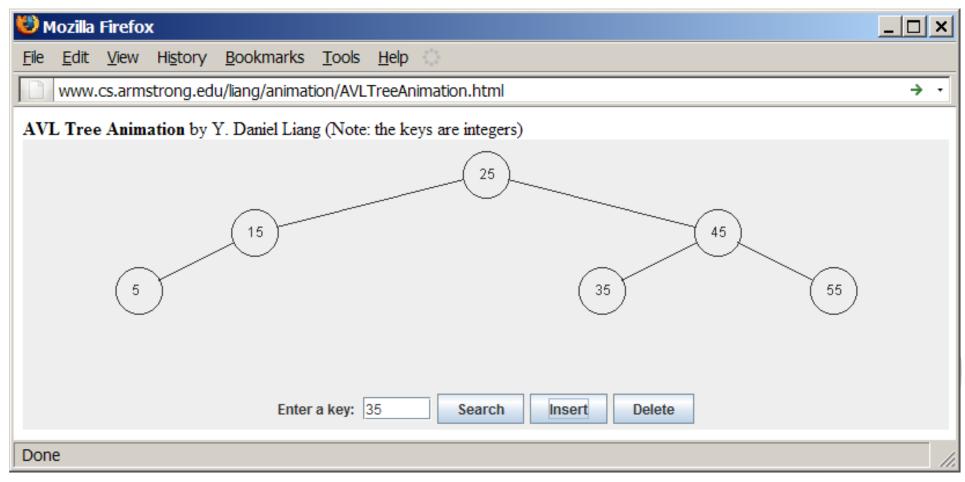
What is an AVL Tree?

AVL trees are well-balanced. AVL trees were invented by two Russian computer scientists G. M. Adelson-Velsky and E. M. Landis in 1962. In an AVL tree, the difference between the heights of two subtrees for every node is 0 or 1. It can be shown that the maximum height of an AVL tree is O(logn).

AVL Tree Animation

https://liveexample.pearsoncmg.com/dsanimation/AVLTree.html





Balance Factor/Left-Heavy/Right-Heavy

The process for inserting or deleting an element in an AVL tree is the same as in a regular binary search tree. The difference is that you may have to rebalance the tree after an insertion or deletion operation. The balance factor of a node is the height of its right subtree minus the height of its left subtree. A node is said to be balanced if its balance factor is -1, 0, or 1. A node is said to be *left-heavy* if its balance factor is -1. A node is said to be right-heavy if its balance factor is +1.

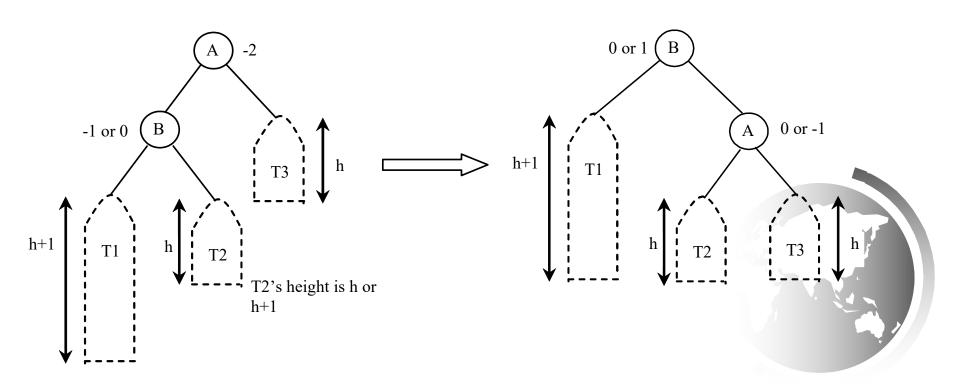
Balancing Trees

If a node is not balanced after an insertion or deletion operation, you need to rebalance it. The process of rebalancing a node is called a *rotation*. There are four possible rotations.



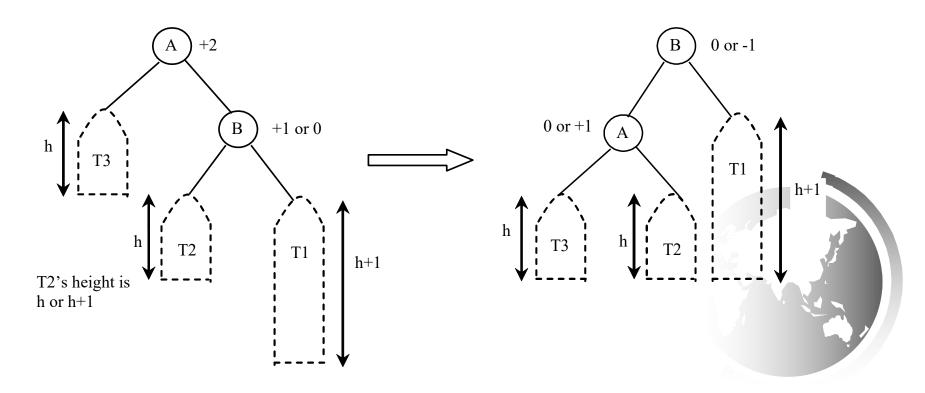
LL imbalance and LL rotation

LL Rotation: An *LL imbalance* occurs at a node A such that A has a balance factor -2 and a left child B with a balance factor -1 or 0. This type of imbalance can be fixed by performing a single right rotation at A.



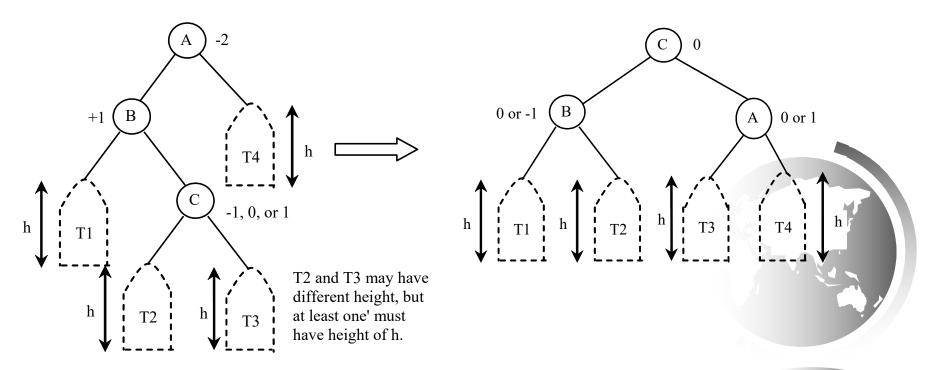
RR imbalance and RR rotation

RR Rotation: An *RR imbalance* occurs at a node A such that A has a balance factor +2 and a right child B with a balance factor +1 or 0. This type of imbalance can be fixed by performing a single left rotation at A.



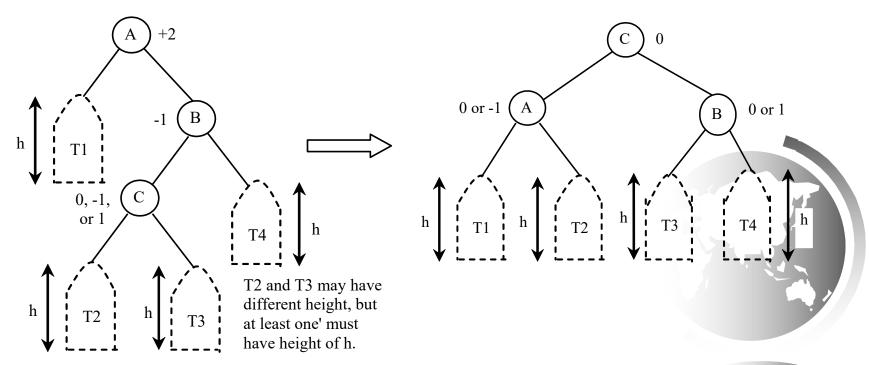
LR imbalance and LR rotation

LR Rotation: An *LR imbalance* occurs at a node A such that A has a balance factor -2 and a left child B with a balance factor +1. Assume B's right child is C. This type of imbalance can be fixed by performing a double rotation (first a single left rotation at B and then a single right rotation at A).



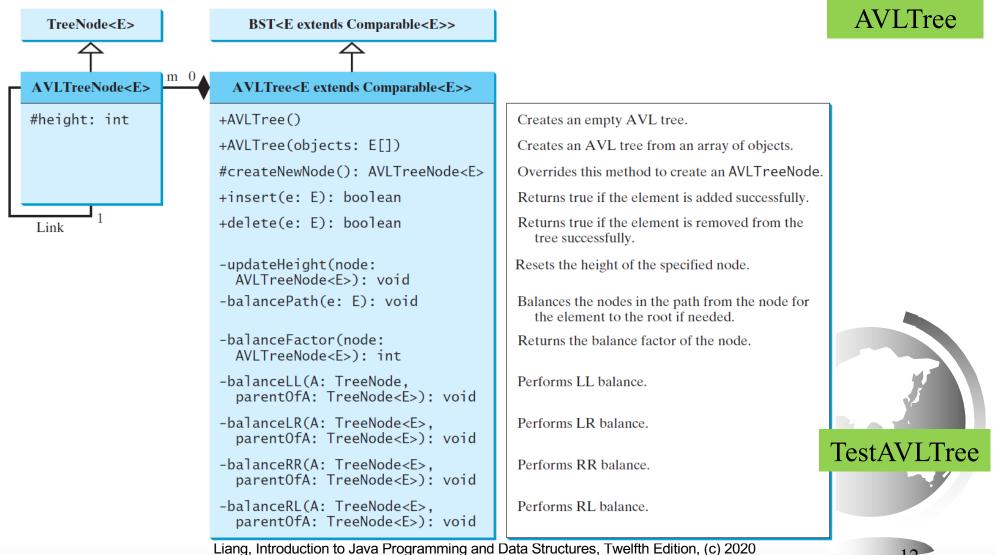
RL imbalance and RL rotation

RL Rotation: An *RL imbalance* occurs at a node A such that A has a balance factor +2 and a right child B with a balance factor -1. Assume B's left child is C. This type of imbalance can be fixed by performing a double rotation (first a single right rotation at B and then a single left rotation at A).



Designing Classes for AVL Trees

An AVL tree is a binary tree. So you can define the AVLTree class to extend the BST class.



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