Appendix F

Number Systems

binary	0, 1
octal	0, 1, 2, 3, 4, 5, 6, 7
decimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
hexdecimal	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Number Systems

Computers use binary numbers internally because storage devices like memory and disk are made to store 0s and 1s. A number or a text inside a computer is stored as a sequence of 0s and 1s. Each 0 and 1 is called a *bit*, short for *bi*nary digi*t*. The binary number system has two digits, 0 and 1.

Binary numbers are not intuitive, since we use decimal numbers in our daily life. When you write a number like 20 in a program, it is assumed to be a decimal number. Internally, computer software is used to convert decimal numbers into binary numbers, and vice versa.

Number Systems, cont.

The digits in the decimal number system are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. A decimal number is represented using a sequence of one or more of these digits. The value that each digit in the sequence represents depends on its position. A position in a sequence has a value that is an integral power of 10. For example, the digits 7, 4, 2, and 3 in decimal number 7423 represent 7000, 400, 20, and 3, respectively, as shown below:

7	4	2	3
10^3	10^2	10^1	10^0

$$= 7 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$$

$$= 7000 + 400 + 20 + 3 = 7423$$

The decimal number system has ten digits and the position values are integral powers of 10. We say that 10 is the *base* or *radix* of the decimal number system. Similarly, the base of the binary number system is 2 since the binary number system has two digits and the base of the hex number system is 16 since the hex number system has sixteen digits.

Number Systems, cont.

Binary numbers tend to be very long and cumbersome. Hexadecimal numbers are often used to abbreviate binary numbers. The hexadecimal number system has 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. The letters A, B, C, D, E, and F correspond to the decimal numbers 10, 11, 12, 13, 14, and 15.

Binary Numbers => Decimals

Given a binary number $b_n b_{n-1} b_{n-2} \ldots b_2 b_1 b_0$
the equivalent decimal value is

$$b_n \times 2^n + b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \ldots + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

`10` in binary $\quad 1 \times 2^1 + 0 \quad = \quad$ `2` in decimal

`1000` in binary $\quad 1 \times 2^3 + 0 \times 2^2 + 0 \times 2 + 0 \quad = \quad$ `8` in decimal

`10101011` in binary $\quad 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 1 \quad = \quad$ `171` in decimal

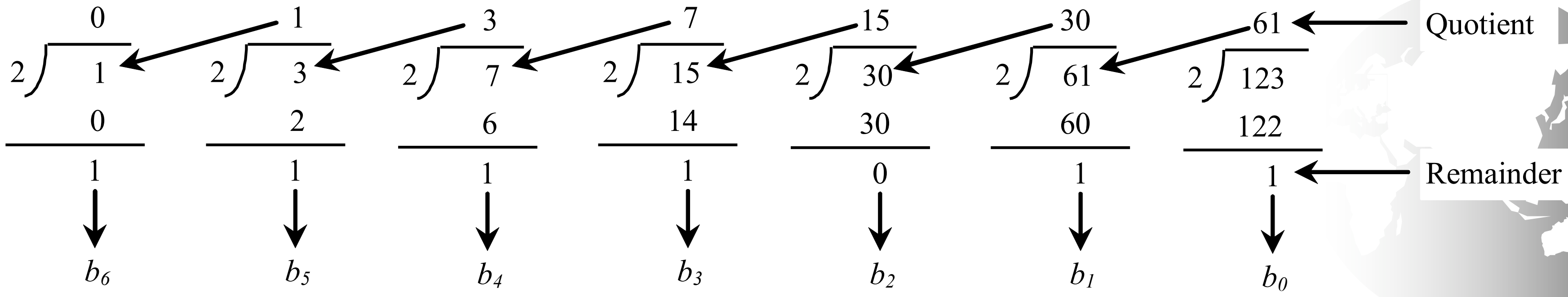
Decimals => Binary

To convert a decimal number d to a binary number is to find the binary digits.. $b_n, b_{n-1}, b_{n-2}, \ldots, b_2, b_1, b_0$ such that

$$d = b_n \times 2^n + b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \ldots + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

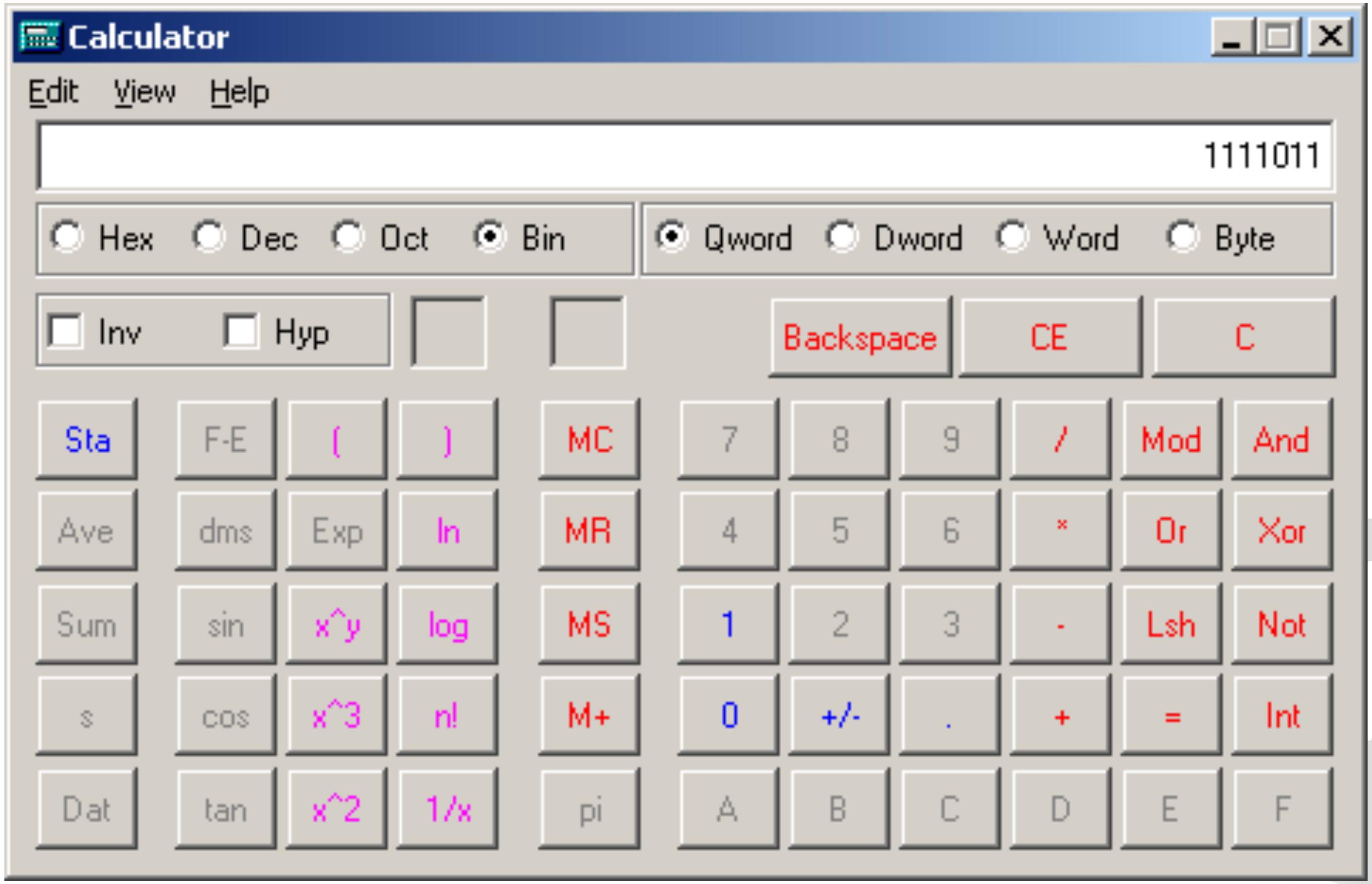
These numbers can be found by successively dividing d by 2 until the quotient is 0. The remainders are $b_n, b_{n-1}, b_{n-2}, \ldots, b_2, b_1, b_0$

For example, the decimal number 123 is 1111011 in binary. The conversion is conducted as follows:



Windows Calculator

The Windows Calculator is a useful tool for performing number conversions. To run it, choose *Programs*, *Accessories*, and *Calculator* from the Start button.



Hexadecimals => Decimals

The hexadecimal number system has sixteen digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. The letters A, B, C, D, E, and F correspond to the decimal numbers 10, 11, 12, 13, 14, and 15. Given a hexadecimal number $h_n h_{n-1} h_{n-2} \ldots h_2 h_1 h_0$

The equivalent decimal value is

$$h_n \times 16^n + h_{n-1} \times 16^{n-1} + h_{n-2} \times 16^{n-2} + \ldots + h_2 \times 16^2 + h_1 \times 16^1 + h_0 \times 16^0$$

`7F` in hex $7 \times 16^1 + 15$ = `127` in decimal

`FFFF` in hex $15 \times 16^3 + 15 \times 16^2 + 15 \times 16 + 15$ = `65535` in decimal

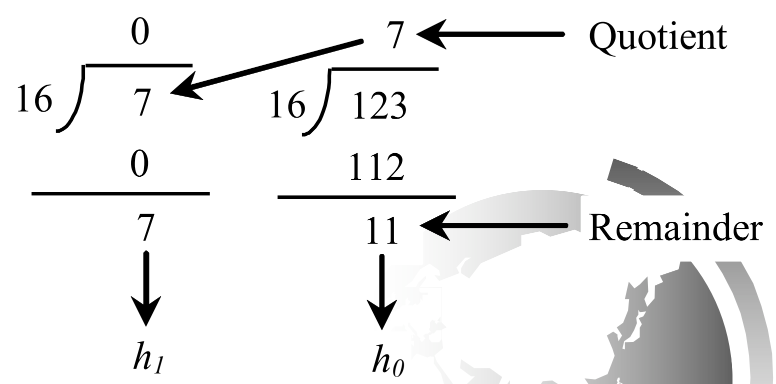
Decimals => Hexadecimal

To convert a decimal number d to a hexadecimal number is to find the hexadecimal digits $h_n, h_{n-1}, h_{n-2}, \ldots, h_2, h_1, h_0$ such that

$$d = h_n \times 16^n + h_{n-1} \times 16^{n-1} + h_{n-2} \times 16^{n-2} + \ldots + h_2 \times 16^2 + h_1 \times 16^1 + h_0 \times 16^0$$

These numbers can be found by successively dividing d by 16 until the quotient is 0. The remainders are $h_0, h_1, h_2, \ldots, h_{n-2}, h_{n-1}, h_n$

For example, the decimal number 123 is 7B in hexadecimal. The conversion is conducted as follows:



Hexadecimal ⇔ Binary

Binary	Hex	Decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	A	10
1011	B	11
1100	C	12
1101	D	13
1110	E	14
1111	F	15

To convert a hexadecimal number to a binary number, simply convert each digit in the hexadecimal number into a four-digit binary number.

To convert a binary number to a hexadecimal, convert every four binary digits from right to left in the binary number into a hexadecimal number. For example,

1 1 1 0 0 0 1 1 0 1

3 8 D