ACTL2131 1.2 - Univariate Distributions

Tadhg Xu-Glassop

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Today

Today's lesson is quite easy! We will present the majority of the distributions in the formula book, including their use cases.

Although it is provided in the formula book, I **strongly** encourage you to derive μ, σ^2, and the MGF for most of the distributions and go from PDF to CDF as practice.

Distributions

We have seen how we use distributions, including f, F and moments to characterise random variables.

The most common distributions are in your formula book. A skill in this course is to identify the distribution and use what you have to solve the problem.

If X is distributed as G, we say $X \sim G(\tilde{\theta})$, where $\tilde{\theta}$ is a vector of parameters. For example, $X \sim \mathcal{N}(\mu, \sigma^2)$ is a normal distribution, with parameters μ and σ^2.

Discrete

Bernoulli

- Success or failure

Binomial

- Sum of Bernoulli / repeated Bernoulli trials

Geometric

- Number of trials until first success
- Memoryless property

Negative Binomial

- Number of trials until r successes.

Poisson

- Counting number of events in a period of time.

Continuous

Exponential

- Positive valued, memoryless property.

Gamma

- Positive valued, sum of exponential.

Continuous (cont.)

Normal

- Supports $\mathbb{R}$, super common.
- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $(X - \mu)/\sigma \sim \mathcal{N}(0, 1)$, denoted Z.
- You can find probabilities for Z in the formula book.
    - You are given $\Phi(z)$ in your formulae, which returns the area *left* of z in the standard normal.
    - Since Z is symmetrical, $\Phi(-z) = 1 - \Phi(z)$.

Lognormal

- Fat tailed, positive valued.
- Defining property: if $Y \sim \mathrm{LN}(\mu, \sigma^2)$, then $\log Y \sim \mathcal{N}(\mu, \sigma^2)$

Continuous (cont.)

Beta

- Supports $[0, 1]$ - used to model proportions or probabilities.

Uniform

- Everything in an interval $[a, b]$ has the same probability!

Tutorial Questions

1.2.4, 1.2.5, 1.2.8