Equation Booklet

University of New South Wales
School of Risk and Actuarial Studies

February 24, 2025

Contents

1 Mathematical Methods

1.1 Series

Exponential function

$$\exp(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Natural log function

$$\log(1+x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \quad (-1 < x \leq 1)$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + b^n$$

where n is a positive integer

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!} x^2 + \frac{p(p-1)(p-2)}{3!} x^3 + \cdots \quad (-1 < x < 1)$$

1.2 Calculus

Taylor series (one variable)

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \cdots$$

Taylor series (two variables)

$$f(x+h, y+k) = f(x,y) + hf'_x(x,y) + kf'_y(x,y) + \frac{1}{2!}\left(h^2 f''_{xx}(x,y) + 2hk f''_{xy}(x,y) + k^2 f''_{yy}(x,y)\right) + \cdots$$

Integration by parts

$$\int_a^b u \frac{dv}{dx}\, dx = [uv]_a^b - \int_a^b v \frac{du}{dx}\, dx$$

Double integrals (changing the order of integration)

$$\int_a^b \left(\int_a^x f(x,y)\, dy \right) dx = \int_a^b \left(\int_y^b f(x,y)\, dx \right) dy \quad \text{or}$$

$$\int_a^b dx \int_a^x dy f(x,y) = \int_a^b dy \int_y^b dx f(x,y)$$

The domain of integration here is the set of values (x, y) for which $a \leq y \leq x \leq b$.

Differentiating an integral

$$\frac{d}{dy}\int_{a(y)}^{b(y)} f(x,y)\,dx = b'(y)f\left[b(y),y\right] - a'(y)f\left[a(y),y\right] + \int_{a(y)}^{b(y)} \frac{\partial f(x,y)}{\partial y}\,dx$$

1.3 Solving Equations

Newton-Raphson method

If x is a sufficiently good approximation to a root of the equation $f(x) = 0$ then (provided convergence occurs) a better approximation is

$$x^* = x - \frac{f(x)}{f'(x)}$$

Integrating factors

The integrating factor for solving the differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is:

$$\exp\left(\int P(x)\,dx\right)$$

Second-order difference equations

The general solution of the difference equation

$$ax_{n+2} + bx_{n+1} + cx_n = 0 \text{ is:}$$

if $b^2 - 4ac > 0$:

$$x_n = A\lambda_1^n + B\lambda_2^n \quad \text{(distinct real roots, } \lambda_1 \neq \lambda_2\text{)}$$

if $b^2 - 4ac = 0$:

$$x_n = (A + Bn)\lambda^n \quad \text{(equal real roots, } \lambda_1 = \lambda_2 = \lambda\text{)}$$

if $b^2 - 4ac < 0$:

$$x_n = r^n(A\cos n\theta + B\sin n\theta) \quad \text{(complex roots, } \lambda_1 = \overline{\lambda_2} = re^{i\theta}\text{)}$$

where λ_1 and λ_2 are the roots of the quadratic equation

$$a\lambda^2 + b\lambda + c = 0$$

1.4 Gamma Function

Definition

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt, \quad x > 0$$

Properties

$$\Gamma(x) = (x-1)\Gamma(x-1)$$
$$\Gamma(n) = (n-1)!, \quad n = 1, 2, 3, \ldots$$
$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Basic Probability Formulas

Conditional Probability

Let A and B be two events. The conditional probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

Independent Events

Events$A$ and B be are said to be independent if the occurrence of A does not change the probability of B.

$$P(A|B) = P(A)$$
$$P(A \cap B) = P(A) \times P(B)$$

Bayes' Formula

Let $A_1, A_2, \ldots, A_n$ be a collection of mutually exclusive and exhaustive events with $P(A_i) \neq 0$, $i = 1, 2, \ldots, n$.

For any event B such that $P(B) \neq 0$:

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)}, \quad i = 1, 2, \ldots, n$$

2 Statistical Distributions

2.1 Notation

PMF	= Probability function, $pmf(x)$
PDF	= Probability density function, $f(x)$
DF	= Distribution function, $F(x)$
PGF	= Probability generating function, $G(s)$
MGF	= Moment generating function, $M(t)$

Note. Where formulae have been omitted below, this indicates that (a) there is no simple formula or (b) the function does not have a finite value or (c) the function equals zero.

2.2 Discrete Distributions

Binomial distribution

Parameters:	n, p $\quad (n =$ positive integer, $0 < p < 1$ with $q = 1 - p)$
PMF:	$pmf(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots, n$
DF:	The distribution function is tabulated in the statistical tables section.
PGF:	$G(s) = (q + ps)^n$
MGF:	$M(t) = (q + pe^t)^n$
Moments:	$E(X) = np, \quad \mathrm{var}(X) = npq$
Coefficient of skewness:	$\frac{q-p}{\sqrt{npq}}$

Bernoulli distribution

The Bernoulli distribution is the same as the binomial distribution with parameter $n = 1$.

Poisson distribution

Parameter:	$\mu \, (\mu > 0)$
PMF:	$pmf(x) = \frac{e^{-\mu}\mu^x}{x!}, \quad x = 0, 1, 2, \ldots$
DF:	The distribution function is tabulated in the statistical tables section.
PGF:	$G(s) = e^{\mu(s-1)}$
MGF:	$M(t) = e^{\mu(e^t-1)}$
Moments:	$E(X) = \mu, \quad \mathrm{var}(X) = \mu$
Coefficient of skewness:	$\frac{1}{\sqrt{\mu}}$

Negative binomial distribution – Type 1

Parameters: $k,\ p \quad (k = \text{positive integer},\ 0 < p < 1 \text{ with } q = 1 - p)$

PMF: $pmf(x) = \binom{x-1}{k-1} p^k q^{x-k}, \quad x = k, k+1, k+2, \ldots$

PGF: $G(s) = \left(\frac{ps}{1-qs}\right)^k$

MGF: $M(t) = \left(\frac{pe^t}{1-qe^t}\right)^k$

Moments: $E(X) = \frac{k}{p}, \quad \text{var}(X) = \frac{kq}{p^2}$

Coefficient of skewness: $\frac{2-p}{\sqrt{kq}}$

Negative binomial distribution – Type 2

Parameters: $k,\ p \quad (k > 0,\ 0 < p < 1 \text{ with } q = 1 - p)$

PMF: $pmf(x) = \frac{\Gamma(k+x)}{\Gamma(x+1)\Gamma(k)} p^k q^x, \quad x = 0, 1, 2, \ldots$

PGF: $G(s) = \left(\frac{p}{1-qs}\right)^k$

MGF: $M(t) = \left(\frac{p}{1-qe^t}\right)^k$

Moments: $E(X) = \frac{kq}{p}, \quad \text{var}(X) = \frac{kq}{p^2}$

Coefficient of skewness: $\frac{2-p}{\sqrt{kq}}$

Geometric distribution

The geometric distribution is the same as the negative binomial distribution with parameter $k = 1$.

Uniform distribution (discrete)

Parameters: $a,\ b,\ h \quad (a < b,\ h > 0,\ b - a \text{ is a multiple of } h)$

PMF: $pmf(x) = \frac{h}{b-a+h}, \quad x = a, a+h, a+2h, \ldots, b-h, b$

PGF: $G(s) = \frac{h}{b-a+h}\left(\frac{s^{b+h}-s^a}{s^h-1}\right)$

MGF: $M(t) = \frac{h}{b-a+h}\left(\frac{e^{(b+h)t}-e^{at}}{e^{ht}-1}\right)$

Moments: $E(X) = \frac{1}{2}(a+b), \quad \text{var}(X) = \frac{1}{12}(b-a)(b-a+2h)$

2.3 Continuous Distributions

Standard normal distribution – $N(0,1)$

Parameters:	none
PDF:	$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \quad -\infty < x < \infty$
DF:	The distribution function is tabulated in the statistical tables section.
MGF:	$M(t) = e^{\frac{1}{2}t^2}$
Moments:	$E(X) = 0, \quad \text{var}(X) = 1$
	$E(X^r) = \frac{1}{2^{r/2}} \frac{\Gamma(1+r)}{\Gamma\left(1+\frac{r}{2}\right)}, \quad r = 2, 4, 6, \ldots$

Normal (Gaussian) distribution – $N(\mu, \sigma^2)$

Parameters:	μ, $\sigma^2 \quad (\sigma > 0)$
PDF:	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), \quad -\infty < x < \infty$
MGF:	$M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$
Moments:	$E(X) = \mu, \quad \text{var}(X) = \sigma^2$

Exponential distribution

Parameter:	$\lambda \quad (\lambda > 0)$
PDF:	$f(x) = \lambda e^{-\lambda x}, \quad x > 0$
DF:	$F(x) = 1 - e^{-\lambda x}$
MGF:	$M(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}, \quad t < \lambda$
Moments:	$E(X) = \frac{1}{\lambda}, \quad \text{var}(X) = \frac{1}{\lambda^2}$
	$E(X^r) = \frac{\Gamma(1+r)}{\lambda^r}, \quad r = 1, 2, 3, \ldots$
Coefficient of skewness:	2

Gamma distribution

Parameters:	$\alpha,\, \lambda \quad (\alpha > 0,\, \lambda > 0)$
PDF:	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \quad x > 0$
DF:	When 2α is an integer, probabilities for the gamma distribution can be fou
	$2\lambda X \sim \chi^2_{2\alpha}$
MGF:	$M(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha}, \quad t < \lambda$
Moments:	$E(X) = \frac{\alpha}{\lambda}, \quad \mathrm{var}(X) = \frac{\alpha}{\lambda^2}$
	$E(X^r) = \frac{\Gamma(\alpha+r)}{\Gamma(\alpha)\lambda^r}, \quad r = 1, 2, 3, \ldots$
Coefficient of skewness:	$\frac{2}{\sqrt{\alpha}}$

Chi-square distribution $- \chi^2_\nu$

The chi-square distribution with ν degrees of freedom is the same as the gamma distribution with parameters $\alpha = \frac{\nu}{2}$ and $\lambda = \frac{1}{2}$.

The distribution function for the chi-square distribution is tabulated in the statistical tables section.

Uniform distribution (continuous) $- U(a, b)$

Parameters:	$a,\, b \quad (a < b)$
PDF:	$f(x) = \frac{1}{b-a}, \quad a < x < b$
DF:	$F(x) = \frac{x-a}{b-a}$
MGF:	$M(t) = \frac{1}{(b-a)t}\left(e^{bt} - e^{at}\right)$
Moments:	$E(X) = \frac{1}{2}(a+b), \quad \mathrm{var}(X) = \frac{1}{12}(b-a)^2$
	$E(X^r) = \frac{1}{(b-a)r+1}\left(b^{r+1} - a^{r+1}\right), \quad r = 1, 2, 3, \ldots$

Beta distribution

Parameters:	$\alpha,\, \beta \quad (\alpha > 0,\, \beta > 0)$
PDF:	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$
Moments:	$E(X) = \frac{\alpha}{\alpha+\beta}, \quad \mathrm{var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
	$E(X^r) = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\alpha+\beta+r)}, \quad r = 1, 2, 3, \ldots$
Coefficient of skewness:	$\frac{2(\beta-\alpha)}{(\alpha+\beta+2)}\sqrt{\frac{\alpha+\beta+1}{\alpha\beta}}$

Lognormal distribution

Parameters:	$\mu,\ \sigma^2 \quad (\sigma > 0)$
PDF:	$f(x) = \frac{1}{\sigma\sqrt{2\pi}x} \exp\left(-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2\right), \quad x > 0$
Moments:	$E(X) = e^{\mu + \frac{1}{2}\sigma^2}, \quad \mathrm{var}(X) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$
	$E(X^r) = e^{r\mu + \frac{1}{2}r^2\sigma^2}, \quad r = 1, 2, 3, \ldots$
Coefficient of skewness:	$\left(e^{\sigma^2} + 2\right)\sqrt{e^{\sigma^2} - 1}$

Pareto distribution (two parameter version)

Parameters:	$\alpha,\ \lambda \quad (\alpha > 0,\ \lambda > 0)$
PDF:	$f(x) = \frac{\alpha\lambda^\alpha}{(x+\lambda)^{\alpha+1}}, \quad x > 0$
DF:	$F(x) = 1 - \left(\frac{\lambda}{\lambda+x}\right)^\alpha$
Moments:	$E(X) = \frac{\lambda}{\alpha-1}, (\alpha > 1) \quad \mathrm{var}(X) = \frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)} \quad (\alpha > 2)$
	$E(X^r) = \frac{\Gamma(\alpha-r)\Gamma(1+r)}{\Gamma(\alpha)}\lambda^r, \quad r = 1, 2, 3, \ldots, r < \alpha$
Coefficient of skewness:	$\frac{2(1+\alpha)}{(\alpha-3)}\sqrt{\frac{\alpha-2}{\alpha}} \quad (\alpha > 3)$

Pareto distribution (three parameter version)

Parameters:	$\alpha,\ \lambda,\ k \quad (\alpha > 0,\ \lambda > 0,\ k > 0)$
PDF:	$f(x) = \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)\Gamma(k)}\frac{\lambda^\alpha x^{k-1}}{(\lambda+x)^{\alpha+k}}, \quad x > 0$
Moments:	$E(X) = \frac{k\lambda}{\alpha-1} \quad (\alpha > 1), \quad \mathrm{var}(X) = \frac{k\lambda^2(\alpha+k-1)}{(\alpha-1)^2(\alpha-2)} \quad (\alpha > 2)$
	$E(X^r) = \frac{\Gamma(\alpha-r)\Gamma(k+r)}{\Gamma(\alpha)\Gamma(k)}\lambda^r, \quad r = 1, 2, 3, \ldots, r < \alpha$

Weibull distribution

Parameters:	$c,\ \gamma \quad (c > 0,\ \gamma > 0)$
PDF:	$f(x) = c\gamma x^{\gamma-1}e^{-cx^\gamma}, \quad x > 0$
DF:	$F(x) = 1 - e^{-cx^\gamma}$
Moments:	$E(X^r) = \left(\frac{1}{c}\right)^{r/\gamma}\Gamma\left(1 + \frac{r}{\gamma}\right), \quad r = 1, 2, 3, \ldots$

Burr distribution

Parameters:	$\alpha,\ \lambda,\ \gamma \quad (\alpha > 0,\ \lambda > 0,\ \gamma > 0)$
PDF:	$f(x) = \frac{\alpha\gamma\lambda^\alpha x^{\gamma-1}}{(\lambda+x^\gamma)^{\alpha+1}}, \quad x > 0$
DF:	$F(x) = 1 - \left(\frac{\lambda}{\lambda+x}\right)^\alpha$
Moments:	$E(X^r) = \Gamma(\alpha - \frac{r}{\gamma})\Gamma(1+\frac{r}{\gamma})\frac{\lambda^{\frac{r}{\gamma}}}{\Gamma(\alpha)}, \quad r = 1, 2, 3, \ldots, r < \alpha\gamma$

2.4 Compound Distributions

Conditional expectation and variance

$$E(Y) = E[E(Y \mid X)]$$
$$\mathrm{var}(Y) = \mathrm{var}(E(Y \mid X)) + E[\mathrm{var}(Y \mid X)]$$

Moments of a compound distribution

If $X_1, X_2, \ldots$ are IID random variables with MGF $M_X(t)$ and N is an independent nonnegative integer-valued random variable, then $S = X_1 + \cdots + X_N$ (with $S = 0$ when $N = 0$) has the following properties:

Mean:	$E(S) = E(N)E(X)$
Variance:	$\mathrm{var}(S) = E(N)\mathrm{var}(X) + \mathrm{var}(N)[E(X)]^2$
MGF:	$M_S(t) = M_N[\log M_X(t)]$

Compound Poisson distribution

Mean:	λm_1
Variance:	λm_2
Third central moment:	λm_3

where $\lambda = E(N)$ and $m_r = E(X^r)$.

Recursive formulae for integer-valued distributions

(a,b,0) class of distributions

Let $g_r = P(S = r)$, $r = 0, 1, 2, \ldots$ and $f_j = P(X = j)$, $j = 1, 2, 3, \ldots$.

If $p_r = P(N = r)$, where $p_r = \left(a + \frac{b}{r}\right) p_{r-1}$, $r = 1, 2, 3, \ldots$, then

$$g_0 = p_0$$
$$g_r = \sum_{j=1}^{r} \left(a + \frac{bj}{r}\right) f_j g_{r-j}, \quad r = 1, 2, 3, \ldots$$

Compound Poisson distribution

If N has a Poisson distribution with mean λ, then $a = 0$ and $b = \lambda$, and

$$g_0 = e^{-\lambda}$$
$$g_r = \frac{\lambda}{r} \sum_{j=1}^{r} f_j j g_{r-j}, \quad r = 1, 2, 3, \ldots$$

2.5 Truncated Moments

Normal distribution

If $f(x)$ is the PDF of the $N(\mu, \sigma^2)$ distribution, then

$$\int_L^U x f(x)\, dx = \mu[\Phi(U') - \Phi(L')] - \sigma[\phi(U') - \phi(L')]$$

where $L' = \frac{L-\mu}{\sigma}$ and $U' = \frac{U-\mu}{\sigma}$.

Lognormal distribution

If $f(x)$ is the PDF of the lognormal distribution with parameters μ and σ^2, then

$$\int_L^U x^k f(x)\, dx = e^{k\mu + \frac{1}{2}k^2\sigma^2}[\Phi(U_k) - \Phi(L_k)]$$

where $L_k = \frac{\log L - \mu}{\sigma} - k\sigma$ and $U_k = \frac{\log U - \mu}{\sigma} - k\sigma$.

3 Statistical Methods

3.1 Sample Mean and Variance

The random sample $(x_1, x_2, \ldots, x_n)$ has the following sample moments:

Sample mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Sample variance:

$$s^2 = \frac{1}{n-1} \left[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2 \right]$$

3.2 Parametric Inference (Normal Model)

One sample

For a single sample of size n under the normal model $X \sim N(\mu, \sigma^2)$:

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \quad \text{and} \quad \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

Two samples

For two independent samples of sizes m and n under the normal models $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$:

$$\frac{S_X^2/\sigma_X^2}{S_Y^2/\sigma_Y^2} \sim F_{m-1,n-1}$$

Under the additional assumption that $\sigma_X^2 = \sigma_Y^2$:

$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{S_P\sqrt{\frac{1}{m} + \frac{1}{n}}} \sim t_{m+n-2}$$

where $S_P^2 = \frac{1}{m+n-2}\left[(m-1)S_X^2 + (n-1)S_Y^2\right]$ is the pooled sample variance.

3.3 Maximum Likelihood Estimators

Asymptotic distribution

If $\hat{\theta}$ is the maximum likelihood estimator of a parameter θ based on a sample $\underline{X}$, then $\hat{\theta}$ is asymptotically normally distributed with mean θ and variance equal to the Cramér-Rao lower bound

$$\mathrm{CRLB}(\theta) = \frac{-1}{\left[E\left(\frac{\partial^2}{\partial\theta^2}\log L(\theta, \underline{X})\right)\right]}$$

Likelihood ratio test

$$-2(\ell_p - \ell_{p+q}) = -2\log\left(\frac{\max\limits_{H_0} L}{\max\limits_{H_0 \cup H_1} L}\right) \sim \chi^2_q \quad \text{approximately (under } H_0\text{)}$$

where $\ell_p = \max\limits_{H_0} \log L$ is the maximum log-likelihood for the model under H_0 (in which there are p free parameters) and $\ell_{p+q} = \max\limits_{H_0 \cup H_1} \log L$ is the maximum log-likelihood for the model under $H_0 \cup H_1$ (in which there are $p+q$ free parameters).

3.4 Linear Regression Model with Normal Errors

Simple Linear Regression Model

$$Y = \beta_0 + \beta_1 X + \epsilon$$

Intermediate calculations

$$s_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$
$$s_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$$
$$s_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$

Parameter estimates

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}, \quad \hat{\beta} = \frac{s_{xy}}{s_{xx}}$$
$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \frac{1}{n-2}\left(s_{yy} - \frac{s_{xy}^2}{s_{xx}}\right)$$

Distribution of $\hat{\beta}$

$$\frac{\hat{\beta} - \beta}{\sqrt{\hat{\sigma}^2 / s_{xx}}} \sim t_{n-2}$$

Variance of predicted mean response

$$\mathrm{var}(\hat{\alpha} + \hat{\beta} x_0) = \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{s_{xx}}\right)\hat{\sigma}^2$$

An additional σ^2 must be added to obtain the variance of the predicted individual response.

Multiple Linear Regression Model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

If we denote $\mathbf{x}_i$ to be the i'th row of X,

$$y_i = \mathbf{x}_i \beta.$$

Minimise the residuals sum of squared (RSS)

$$\mathrm{RSS} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}\left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \cdots - \hat{\beta}_p x_{ip}\right)^2$$

$$= (Y - X\beta)^\top (Y - X\beta) = \sum_{i=1}^{n} \hat{\epsilon}_i^2.$$

If $(X^\top X)^{-1}$ exists, it can be shown that the solution is given by:

$$\hat{\beta} = (X^\top X)^{-1} X^\top Y.$$

The corresponding vector of fitted (or predicted) values is

$$\hat{Y} = X\hat{\beta}.$$

Sum of squares relationship

$$\underbrace{\sum_{i=1}^{n}(y_i - \overline{y})^2}_{\text{TSS}} = \underbrace{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}_{\text{RSS}} + \underbrace{\sum_{i=1}^{n}(\hat{y}_i - \overline{y})^2}_{\text{SSM}},$$

the corresponding F-test to check if the multiple linear regression model is significantly better than just predicting the mean $\overline{Y}$.

$$H_0 : \beta_1 = \cdots = \beta_p = 0$$

$$H_a : \text{at least one } \beta_j \text{ is non-zero}$$

$$\text{F-statistic} = \frac{(\text{TSS} - \text{RSS})/p}{\text{RSS}/(n - p - 1)} \sim F_{p,n-p-1}$$

3.5 Analysis of Variance

Single factor normal model

$$Y_{ij} \sim N(\mu + \tau_i, \sigma^2), \quad i = 1, 2, \ldots, k, \quad j = 1, 2, \ldots, n_i$$

where $n = \sum_{i=1}^{k} n_i$, with $\sum_{i=1}^{k} n_i \tau_i = 0$.

Intermediate calculations (sums of squares)

Total:

$$SS_T = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_{..})^2 = \sum_{i=1}^{k}\sum_{j=1}^{n_i} Y_{ij}^2 - \frac{Y_{..}^2}{n}$$

Between treatments:

$$SS_B = \sum_{i=1}^{k} n_i(\bar{Y}_{i.} - \bar{Y}_{..})^2 = \sum_{i=1}^{k} \frac{Y_{i.}^2}{n_i} - \frac{Y_{..}^2}{n}$$

Residual:

$$SS_R = SS_T - SS_B$$

Variance estimate

$$\hat{\sigma}^2 = \frac{SS_R}{n-k}$$

Statistical test

Under the appropriate null hypothesis:

$$\frac{SS_B/(k-1)}{SS_R/(n-k)} \sim F_{k-1,n-k}$$

3.6 Generalised Linear Models

Exponential dispersion family

For a random variable Y from the exponential family, with natural parameter θ and scale parameter ϕ:

$$f_Y(y;\theta,\phi) = \exp\left[\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right]$$

Mean:

$$E(Y) = b'(\theta)$$

Variance:

$$\text{var}(Y) = a(\phi)b''(\theta)$$

Canonical link functions

$$\begin{aligned} \textit{Binomial:} \quad & g(\mu) = \log\frac{\mu}{1-\mu} \\ \textit{Poisson:} \quad & g(\mu) = \log\mu \\ \textit{Normal:} \quad & g(\mu) = \mu \\ \textit{Gamma:} \quad & g(\mu) = \frac{1}{\mu} \end{aligned}$$

Model selection criteria for GLMs

The scaled deviance is used for comparing nested models. It has (approximately) a chi-squared distribution with degrees of freedom equal to the number of observations minus the number of estimated parameters.

$$\frac{D(y,\hat{\mu})}{\psi} \to \chi^2_{n-(p+1)} \quad \text{when} \quad n \to \infty$$

Model 1: $\eta = \beta_0 + \beta_1 x_1 + \cdots \beta_q x_q$ (q parameters, with scaled deviance D_1)
Model 2: $\eta = \beta_0 + \beta_1 x_1 + \cdots \beta_q x_q + \beta_{q+1} x_{q+1} + \cdots \beta_p x_p$ (p parameters, $p > q$, with scaled deviance D_2)

$$H_0 : \beta_{q+1} = \cdots = \beta_p = 0$$
$$H_a : \text{at least one } \beta_j \text{ is non-zero}$$

Since
$$P\left[\chi^2(\nu) > 2\nu\right] \approx 5\%,$$
the following rule of thumb can be used as an approximation:
$$\text{Model 2 is preferred if } D_1 - D_2 > 2(p-q).$$

3.7 Bayesian Methods

Relationship between posterior and prior distributions

$$\textit{Posterior} \propto \textit{Prior} \times \textit{Likelihood}$$

The posterior distribution $f(\theta \mid \underline{x})$ for the parameter θ is related to the prior distribution $f(\theta)$ via the likelihood function $f(\underline{x} \mid \theta)$:

$$f(\theta \mid \underline{x}) \propto f(\theta) \times f(\underline{x} \mid \theta)$$

Normal / normal model

If $\underline{x}$ is a random sample of size n from a $N(\mu, \sigma^2)$ distribution, where σ^2 is known, and the prior distribution for the parameter μ is $N(\mu_0, \sigma_0^2)$, then the posterior distribution for μ is:

$$\mu \mid \underline{x} \sim N(\mu_*, \sigma_*^2)$$

where
$$\mu_* = \left(\frac{n\bar{x}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}\right) \Bigg/ \left(\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}\right)$$
and
$$\sigma_*^2 = \left(\frac{1}{\sigma^2} + \frac{1}{\sigma_0^2}\right)^{-1}$$

3.8 Empirical Bayes Credibility – Model 1

Data requirements

$$\{X_{ij}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n\}$$

X_{ij} represents the aggregate claims in the jth year from the ith risk.

Intermediate calculations

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}, \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} \bar{X}_i$$

Parameter estimation

Quantity	Estimator
$E[m(\theta)]$	$\bar{X}$
$E[S^2(\theta)]$	$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2\right)$
$\mathrm{var}[m(\theta)]$	$\frac{1}{N-1}\sum_{i=1}^{N}(\bar{X}_i-\bar{X})^2-\frac{1}{Nn}\sum_{i=1}^{N}\left(\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2\right)$

Credibility factor

$$Z=\frac{n}{n+\frac{E[S^2(\theta)]}{\mathrm{var}[m(\theta)]}}$$

3.9 Empirical Bayes Credibility – Model 2

Data requirements

$$\{X_{ij}, i=1,2,\ldots,N, j=1,2,\ldots,n\},\quad \{\bar{P}_{ij}, i=1,2,\ldots,N, j=1,2,\ldots,n\}$$

X_{ij} represents the aggregate claims in the jth year from the ith risk; $\bar{P}_{ij}$ is the corresponding risk volume.

Intermediate calculations

$$\bar{P}_i=\sum_{j=1}^{n}\bar{P}_{ij},\quad \bar{P}=\sum_{i=1}^{N}\bar{P}_i,\quad \bar{P}^*=\frac{1}{Nn-1}\sum_{i=1}^{N}\bar{p}_i\left(1-\frac{\bar{P}_i}{\bar{P}}\right)$$

$$X_{ij}^*=\frac{X_{ij}}{\bar{P}_{ij}},\quad \bar{X}_i=\frac{\sum_{j=1}^{n}\bar{P}_{ij}X_{ij}^*}{\bar{P}_i},\quad \bar{X}=\sum_{i=1}^{N}\sum_{j=1}^{n}\frac{\bar{P}_{ij}X_{ij}^*}{\bar{P}}$$

Parameter estimation

Quantity	Estimator
$E[m(\theta)]$	$\bar{X}$
$E[S^2(\theta)]$	$\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{n-1}\sum_{j=1}^{n}P_{ij}(X_{ij}^*-\bar{X}_i)^2\right)$
$\mathrm{var}[m(\theta)]$	$\frac{1}{\bar{P}^*}\left(\frac{1}{Nn-1}\sum_{i=1}^{N}\sum_{j=1}^{n}P_{ij}(\bar{X}_i-\bar{X})^2-\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{n-1}\sum_{j=1}^{n}P_{ij}(X_{ij}^*-\bar{X}_i)^2\right)\right)$

Credibility factor

$$Z_i=\frac{\sum_{j=1}^{n}P_{ij}}{\sum_{j=1}^{n}P_{ij}+\frac{E[S^2(\theta)]}{\mathrm{var}[m(\theta)]}}$$

4 Compound Interest

Increasing/decreasing annuity functions

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}, \quad (D\ddot{a})_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

Accumulation factor for variable interest rates

$$A(t_1, t_2) = \exp\left(\int_{t_1}^{t_2} \delta(t)\, dt\right)$$

5 Survival Models

5.1 Mortality "Laws"

Survival probabilities

$${}_tp_x = \exp\left(-\int_0^t \mu_{x+s}\, ds\right)$$

Gompertz' Law

$$\mu_x = Bc^x, \quad {}_tp_x = g^{c^x\left(C^t - 1\right)} \quad \text{where} \quad g = e^{-B/logc}$$

Makeham's Law

$$\mu_x = A + Bc^x, \quad {}_tp_x = s^t g^{c^x\left(C^t - 1\right)} \quad \text{where} \quad s = e^{-A}$$

Gompertz-Makeham formula

The Gompertz-Makeham graduation formula, denoted by GM(r, s), states that

$$\mu_x = poly_1(t) + \exp[poly_2(t)]$$

where t is a linear function of x and $poly_1(t)$ and $poly_2(t)$ are polynomials of degree r and s, respectively.

5.2 Empirical Estimation

Greenwood's formula for the variance of the Kaplan-Meier estimator

$$\text{var}[\hat{F}(t)] = \left[1 - \hat{F}(t)\right]^2 \sum_{t_i \le t} \frac{d_j}{n_j(n_j - d_j)}$$

Variance of the Nelson-Aalen estimate of the integrated hazard

$$\mathrm{var}[\hat{\Lambda}_t] = \sum_{t_j \le t} \frac{d_j(n_j - d_j)}{n_j^3}$$

5.3 Mortality Assumptions

Balducci assumption

$${}_{1-t}q_{x+t} = (1-t)q_x \quad (x \text{ is an integer, } 0 \le t \le 1)$$

5.4 General Markov Model

Kolmogorov forward differential equation

$$\frac{\partial}{\partial t}{}_tp_x^{gh} = \sum_{j \ne h} \left({}_tp_x^{gj}\mu_{x+t}^{jh} - {}_tp_x^{gh}\mu_{x+t}^{hj} \right)$$

5.5 Graduation Tests

Grouping of signs test

If there are n_1 positive signs and n_2 negative signs and G denotes the observed number of positive runs, then:

$$P(G=t) = \binom{n_1-1}{t-1}\binom{n_2+1}{t} \Big/ \binom{n_1+n_2}{n1}$$

and, approximately,

$$G \sim N\left(\frac{n_1(n_2+1)}{n_1+n_2}, \frac{(n_1n_2)^2}{(n_1+n_2)^3}\right)$$

Critical values for the grouping of signs test are tabulated in the statistical tables section for small values of n_1 and n_2. For larger values of n_1 and n_2 the normal approximation can be used.

Serial correlation test

$$r_j = \frac{1}{m-j}\sum_{i=1}^{m-j}(z_i - \bar{z})(z_{i+j} - \bar{z}) \Bigg/ \frac{1}{m}\sum_{i=1}^{m}(z_i - \bar{z})^2$$

where $\bar{z} = \frac{1}{m}\sum_{i=1}^{m} z_i$

$$r_j \times \sqrt{m} \sim N(0,1) \quad \text{approximately}$$

Variance adjustment factor

$$r_x = \sum_i i^2 \pi_i \bigg/ \sum_i i \pi_i$$

where π_i is the proportion of lives at age x who have exactly i policies.

5.6 Multiple Decrement Tables

For a multiple decrement table with three decrements α, β and γ, each uniform over the year of age $(x, x+1)$ in its single decrement table, then

$$(aq)_x^\alpha = q_x^\alpha \left[1 - \frac{1}{2}(q_x^\beta + q_x^\gamma) + \frac{1}{3} q_x^\beta q_x^\gamma\right]$$

5.7 Population Projection Models

Logistic model

$$\frac{1}{P(t)} \frac{dP(t)}{dt} = \rho - kP(t) \quad \text{has general solution} \quad P(t) = \frac{\rho}{C\rho e^{-\rho t} + k}$$

where C is a constant.

6 Annuities and Assurances

6.1 Approximations for Non Annual Annuities

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m}$$

$$\ddot{a}_{x:\overline{n}|}^{(m)} \approx \ddot{a}_{x:\overline{n}|} - \frac{m-1}{2m}\left(1 - \frac{D_{x+n}}{D_x}\right)$$

6.2 Moments of Annuities and Assurances

Let K_x and T_x denote the curtate and complete future lifetimes (respectively) of a life aged exactly x.

Whole life assurances

$$E[v^{K_x+1}] = A_x, \quad \text{var}[v^{K_x+1}] = {}^2A_x - (A_x)^2$$

$$E[v^{T_x}] = \overline{A}_x, \quad \text{var}[v^{T_x}] = {}^2\overline{A}_x - (\overline{A}_x)^2$$

Similar relationships hold for endowment assurances (with status $_{\ldots x:\overline{n}|}$), pure endowments (with status $_{\ldots x:\overset{1}{\overline{n}|}}$), term assurances (with status $_{\ldots \overset{1}{x}:\overline{n}|}$) and deferred whole life assurances (with status $_{m|\cdots x}$).

Whole life annuities

$$E\left[\ddot{a}_{\overline{K_x+1}|}\right] = \ddot{a}_x, \quad \text{var}\left[\ddot{a}_{\overline{K_x+1}|}\right] = \frac{{}^2A_x - (A_x)^2}{d^2}$$

$$E\left[\bar{a}_{\overline{T_x}|}\right] = \bar{a}_x, \quad \text{var}\left[\bar{a}_{\overline{T_x}|}\right] = \frac{{}^2\overline{A}_x - (\overline{A}_x)^2}{\delta^2}$$

Similar relationships hold for temporary annuities (with status $_{\ldots x:\overline{n}|}$).

6.3 Premiums and Reserves

Premium conversion relationship between annuities and assurances

$$A_x = 1 - d\ddot{a}_x, \quad \overline{A}_x = 1 - \delta\overline{a}_x$$

Similar relationships hold for endowment assurance policies (with status $_{\ldots x:\overline{n}|}$).

Net premium reserve

$${}_tV_x = 1 - \frac{\ddot{a}_{x+1}}{\ddot{a}_x}, \quad {}_t\overline{V}_x = 1 - \frac{\overline{a}_x + t}{\overline{a}_x}$$

Similar formulae hold for endowment assurance policies (with statuses $\ldots_{x:\overline{n}|}$ and $\ldots_{x+t:\overline{n-t}|}$).

6.4 Thiele's Differential Equation

Whole life assurance

$$\frac{\partial}{\partial t}{}_t\overline{V}_x = \delta_t\overline{V}_x + \overline{P}_x - (1 - {}_t\overline{V}_x)\mu_{x+t}$$

Similar formulae hold for other types of policies.

Multiple state model

$$\frac{\partial}{\partial t}{}_tV_x^j = \delta_tV_x^j + b_{x+t}^j - \sum_{k \neq j} \mu_{x+t}^{jk}(b_{x+t}^{jk} + {}_tV_x^k - {}_tV_x^j)$$

7 Stochastic Processes

7.1 Markov "Jump" Processes

Kolmogorov differential equations

Forward equation:

$$\frac{\partial}{\partial t}p_{ij}(s,t) = \sum_{k \in S} p_{ik}(s,t)\sigma_{kj}(t)$$

Backward equation:

$$\frac{\partial}{\partial s} p_{ij}(s,t) = -\sum_{k\in S} \sigma_{ik}(s) p_{kj}(s,t)$$

where $\sigma_{ij}(t)$ is the transition rate from state i to state j ($j \neq i$) at time t, and $\sigma_{ii} = -\sum_{j\neq i} \sigma_{ij}$.

Expected time to reach a subsequent state k

$$m_i = \frac{1}{\lambda_i} + \sum_{j\neq i, j\neq k} \frac{\sigma_{ij}}{\lambda_i} m_j, \quad \text{where} \quad \lambda_i = \sum_{j\neq i} \sigma_{ij}$$

7.2 Brownian Motion and Related Processes

Martingales for standard Brownian motion

If $\{B_t, t \geq 0\}$ is a standard Brownian motion, then the following processes are martingales:

$$B_t, \quad B_t^2 - t \quad \text{and} \quad \exp(\lambda B_t - \frac{1}{2}\lambda^2 t)$$

Distribution of the maximum value

$$P\left(\max_{0\leq s\leq t}(B_s + \mu s) > y\right) = \Phi\left(\frac{-y+\mu t}{\sqrt{t}}\right) + e^{2\mu y}\Phi\left(\frac{-y-\mu t}{\sqrt{t}}\right), \quad y > 0$$

Hitting times

If $\tau_y = \min\{s : B_s + \mu s = y\}$ where $\mu > 0$ and $y < 0$, then

$$E[e^{-\lambda\tau_y}] = e^{y(\mu+\sqrt{\mu^2+2\lambda})}, \quad \lambda > 0$$

Ornstein-Uhlenbeck process

$$dX_t = -\gamma X_t dt + \sigma dB_t, \quad \gamma > 0$$

7.3 Monte Carlo Methods

Box-Muller formulae

If U_1 and U_2 are independent random variables from the $U(0,1)$ distribution then

$$Z_1 = \sqrt{-2\log U_1}\cos(2\pi U_2) \quad \text{and} \quad Z_2 = \sqrt{-2\log U_1}\sin(2\pi U_2)$$

are independent standard normal variables.

Polar method

If V_1 and V_2 are independent random variables from the $U(-1,1)$ distribution and $S = V_1^2 + V_2^2$ then, conditional on $0 < S \leq 1$,

$$Z_1 = V_1\sqrt{\frac{-2\log S}{S}} \quad \text{and} \quad Z_2 = V_2\sqrt{\frac{-2\log S}{S}}$$

are independent standard normal variables.

Pseudorandom values from the $U(0,1)$ distribution and the $N(0,1)$ distribution are included in the statistical tables section.

8 Time Series

8.1 Time Series - Time Domain

Sample autocovariance and autocorrelation function

Autocovariance:

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}(x_t - \hat{\mu})(x_{t-k} - \hat{\mu}), \quad \text{where} \quad \hat{\mu} = \frac{1}{n}\sum_{t=1}^{n} x_t$$

Autocorrelation:

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0}$$

Autocorrelation function for ARMA(1,1)

For the process $X_t = \alpha X_{t-1} + e_t + \beta e_{t-1}$:

$$\rho_k = \frac{(1+\beta\alpha)(\alpha+\beta)}{(1+\beta^2+2\alpha\beta)}\alpha^{k-1}, \quad k = 1, 2, 3, \ldots$$

Partial autocorrelation function

$$\phi_1 = \rho_1, \quad \phi_2 = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$$

$$\phi_k = \frac{\det \mathbf{P}_k^*}{\det \mathbf{P}_k}, \quad k = 2, 3, \ldots,$$

where

$$\mathbf{P}_k = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{k-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{pmatrix}$$

and $\mathbf{P}_k^*$ equals $\mathbf{P}_k$ but with the last column replaced with $(\rho_1, \rho_2, \rho_3, \ldots, \rho_{k-1})^\top$.

Partial autocorrelation function for MA(1)

For the process $X_t = \mu + e_t + \beta e_{t-1}$:

$$\phi_k = \frac{(-1)^{k+1}(1-\beta^2)\beta^k}{(1-\beta^{2(k+1)})}, \quad k = 1, 2, 3, \ldots$$

8.2 Time Series - Frequency Domain

Spectral density function

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-ik\omega} \gamma_k, \quad -\pi < \omega < \pi$$

Inversion formula

$$\gamma_k = \int_{-\pi}^{\pi} e^{ik\omega} f(\omega) d\omega$$

Spectral density function for ARMA(p,q)

The spectral density function of the process $\phi(B)\,(X_t - \mu) = \theta(B)e_t$, where $\text{var}(e_t) = \sigma^2$, is

$$f(\omega) = \frac{\sigma^2}{2\pi} \frac{\theta(e^{-i\omega})\theta(e^{i\omega})}{\phi(e^{-i\omega})\phi(e^{i\omega})}$$

Linear filters

For the linear filter $Y_t = \sum_{k=-\infty}^{\infty} a_k Y_{t-k}$:

$$f_Y(\omega) = |A(\omega)|^2 f_X(\omega),$$

where $A(\omega) = \sum_{k=-\infty}^{\infty} e^{-i\omega k} a_k$ is the transfer function for the filter.

8.3 Time Series - Box-Jenkins Methodology

Ljung and Box "portmanteau" test of the residuals for an ARMA(p,q) model

$$n(n+2) \sum_{k=1}^{m} \frac{r_k^2}{n-k} \sim \chi^2_{m-(p+q)}$$

where r_k $(k = 1, 2, \ldots, m)$ is the estimated value of the k-th autocorrelation coefficient of the residuals and n is the number of data values used in the $ARMA(p,q)$ series.

Turning point test

In a sequence of n independent random variables the number of turning points T is such that:

$$E(T) = \frac{2}{3}(n-2) \quad \text{and} \quad \text{var}(T) = \frac{16n-29}{90}$$

9 Economic Models

9.1 Utility Theory

Utility functions

Exponential: $U(w) = -e^{-aw}, \quad a > 0$

Logarithmic: $U(w) = \log w$

Power: $U(w) = \frac{w^\gamma - 1}{\gamma}, \quad \gamma \neq 0$

Quadratic: $U(w) = w + dw^2, \quad d < 0$

Measures of risk aversion

Absolute risk aversion: $A(w) = -\frac{U''(w)}{U'(w)}$

Relative risk aversion: $R(w) = wA(w)$

9.2 Capital Asset Pricing Model (CAPM)

Security market line

$$E_i - r = \beta_i(E_M - r) \quad \text{where} \quad \beta_i = \frac{\text{cov}(R_i, R_M)}{\text{var}(R_M)}$$

Capital market line (for efficient portfolios)

$$E_P - r = (E_M - r)\frac{\sigma_P}{\sigma_M}$$

9.3 Interest Rate Models

Spot rates and forward rates for zero-coupon bonds

Let $P(\tau)$ be the price at time 0 of a zero-coupon bond that pays 1 unit at time τ.

Let $s(\tau)$ be the spot rate for the period $(0, \tau)$.

Let $f(\tau)$ be the instantaneous forward rate at time 0 for time τ.

Spot rate

$$P(\tau) = e^{-\tau s(\tau)} \quad \text{or} \quad s(\tau) = -\frac{1}{\tau}\log P(\tau)$$

Instantaneous forward rate

$$P(\tau) = \exp\left(-\int_0^\tau f(s)ds\right) \quad \text{or} \quad f(\tau) = -\frac{d}{d\tau}\log P(\tau)$$

Vasicek model
Instantaneous forward rate

$$f(\tau) = e^{-\alpha\tau}R + (1 - e^{-\alpha\tau})L + \frac{\beta}{\alpha}e^{-\alpha\tau}\left(1 - e^{-\alpha\tau}\right)$$

Price of a zero-coupon bond

$$P(\tau) = \exp\left[-D(\tau)R - (t - D(\tau))L - \frac{\beta}{2}D(\tau)^2\right]$$

where

$$D(\tau) = \frac{1 - e^{-\alpha\tau}}{\alpha}$$

10 Financial Derivatives

Note. In this section, q denotes the (continuously-payable) dividend rate.

10.1 Price of a Forward or Futures Contract

For an asset with fixed income of present value I:

$$F = (S_0 - I)e^{rT}$$

For an asset with dividends:

$$F = S_0 e^{(r-q)T}$$

10.2 Binomial Pricing ("Tree") Model

Risk-neutral probabilities

$$\textbf{Up-step probability} = \frac{e^{r\Delta t} - d}{u - d}$$

$$\text{where} \quad u = e^{\sigma\sqrt{\Delta t} + q\Delta t} \quad \text{and} \quad d = e^{-\sigma\sqrt{\Delta t} + q\Delta t}$$

10.3 Stochastic Differential Equations

Generalised Wiener process

$$dx = adt + bdz$$

where a and b are constant and dz is the increment for a Wiener process (standard Brownian motion).

Ito process

$$dx = a(x,t)dt + b(x,t)dz$$

Ito's lemma for a function $G(x,t)$

$$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}bdz$$

Models for the short rate r_t

$$\begin{aligned}
\text{Ho-Lee:} &\quad dr = \theta(t)dt + \sigma dz \\
\text{Hull-White:} &\quad dr = [\theta(t) - ar]dt + \sigma dz \\
\text{Vasicek:} &\quad dr = a(b-r)dt + \sigma dz \\
\text{Cox-Ingersoll-Ross:} &\quad dr = a(b-r)dt + \sigma\sqrt{r}dz
\end{aligned}$$

10.4 Black-Scholes Formulae for European Options

Geometric Brownian motion model for a stock price S_t

$$dS_t = S_t(\mu dt + \sigma dz)$$

Black-Scholes partial differential equation

$$\frac{\partial f}{\partial t} + (r-q)S_t\frac{\partial f}{\partial S_t} + \frac{1}{2}\sigma^2 S_t^2\frac{\partial^2 f}{\partial S_t^2} = rf$$

Garman-Kohlhagen formulae for the price of call and put options

Call: $c_t = S_t e^{-q(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$

Put: $p_t = Ke^{-r(T-t)}\Phi(-d_2) - S_t e^{-q(T-t)}\Phi(-d_1)$

where

$$d_1 = \frac{\log(S_t/K) + (r - q + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\log(S_t/K) + (r - q - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

10.5 Put-Call Parity Relationship

$$c_t + Ke^{-r(T-t)} = p_t + S_t e^{-q(T-t)}$$

Portfolio Theory

- For an N-asset portfolio, the Mean-Variance optimization problem is:

$$\min_w \frac{1}{2} w^\top \Sigma w$$

subject to

$$w^\top 1 = 1,$$
$$w^\top z = \mu.$$

where μ is known and fixed, and 1 is a vector of ones. Optimal weights on the MVS for risky asset only portfolio can be represented as:

$$w = \lambda \Sigma^{-1} 1 + \gamma \Sigma^{-1} z.$$

In manipulating the optimization problem, you may find the following definitions useful:

$$A = 1^\top \Sigma^{-1} 1,$$
$$B = 1^\top \Sigma^{-1} z = z^\top \Sigma^{-1} 1,$$
$$C = z^\top \Sigma^{-1} z,$$
$$\Delta = AC - B^2.$$

- Optimization problem associated with the one-fund theorem, that is, N-risky asset portfolio plus a risk-free asset can be represented as:

$$\min_w \frac{1}{2} w^\top \Sigma w$$

subject to

$$(z - r_f 1)^\top w = \mu - r_f.$$

The corresponding solution of the weights vector is

$$w = \gamma \Sigma^{-1} (z - r_f \mathbf{1}),$$

with γ being a scalar.

Asset Pricing

- The Security Market Line for asset i can be represented as

$$z_i = r_f + \beta_i (z_M - r_f),$$

where

$$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2}.$$

- Total risk of any security i can be represented as

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\epsilon i}^2.$$

- Under Single Factor Modeling (SFM), the respective mean, variance, and covariances of returns of any security i can be represented as

$$E(r_i) = \alpha_i + \beta_i \mu_f, \quad \sigma_i^2 = \beta_i^2 \sigma_f^2 + \sigma_{\epsilon i}^2, \quad \sigma_{i,j} = \beta_i \beta_j \sigma_f^2.$$

- The single factor Arbitrage Pricing Theory (APT) expression can be represented as

$$E(r_i) = \lambda_0 + b_{i1}\lambda_1, \quad \text{for } i = 1, 2, \ldots, N.$$

Option bounds

Option	Lower Bound	Upper Bound
European call	$c_t \geq S_t - Ke^{-r(T-t)}$	$c_t \leq S_t$
European put	$p_t \geq Ke^{-r(T-t)} - S_t$	$p_t \leq Ke^{-r(T-t)}$
American call	$C_t \geq S_t - Ke^{-r(T-t)}$	$C_t \leq S_t$
American put	$P_t \geq K - S_t$	$P_t \leq K$

Contingent claim trading strategy

For a contingent claim with payoff X at time T:

$$\begin{aligned}
\phi_{\text{now}} &= f_{\text{up}} - f_{\text{down}}, \\
\psi_{\text{now}} &= \frac{1}{B(\text{now})} e^{-r\delta t}(f_{\text{up}} - \phi_{\text{sup}}), \\
f_{\text{now}} &= \phi_{\text{now}} s_{\text{now}} + \psi_{\text{now}} B(\text{now}), \\
q_{\text{now}} &= \frac{s_{\text{now}} e^{r\delta t} - s_{\text{down}}}{s_{\text{up}} - s_{\text{down}}}, \\
V_{\text{now}} &= E_Q\left[\frac{B(0)}{B(T)} X\right].
\end{aligned}$$

Martingale process

A process $M(\cdot)$ is a martingale with regards to the measure Q and filtration $\{F\}$ if:

$$E_Q[M(u) \mid F(t)] = M(t), \quad \forall t \leq u$$

Binomial Martingale Representation Theorem

The Binomial Martingale Representation Theorem says that there exists a previsible process $\phi(\cdot)$ such that:

$$Y(t) = Y(0) + \sum_{k=1}^{t} \phi(k)(Z(k) - Z(k-1)).$$

Stochastic Processes

- A Brownian motion process:

$$W_n(t) \sim N(0, t) \quad \text{as } n \to \infty.$$

- Consider a stochastic process $X(t)$ with

$$dX(t) = \sigma(X(t))dW(t) + \mu(X(t))dt,$$

  and f is a deterministic twice continuously differentiable function $f(X(t))$. Application of Ito's Lemma yields:

$$df(X(t)) = \frac{\partial f}{\partial x} dX(t) + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(dX(t))^2.$$

- If $W(\cdot)$ is a P Brownian motion, and a pre-visible process $\gamma(\cdot)$, then there exist a measure Q such that:

  1. Q is equivalent to P.
  2. $$\frac{dQ}{dP} = e^{-\int_0^T \gamma(t)dW(t) - \frac{1}{2}\int_0^T \gamma^2(t)dt}.$$
  3. $$W_Q(t) = W(t) + \int_0^t \gamma(s)ds,$$

     is a Q Brownian motion.

11 Tables

11.1 Probabilities for the Standard Normal distribution

The distribution function is denoted by $\Phi(x)$, and the probability density function is denoted by $\phi(x)$.

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt$$

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.50000	0.40	0.65542	0.80	0.78814	1.20	0.88493	1.60	0.94520
0.01	0.50399	0.41	0.65910	0.81	0.79103	1.21	0.88686	1.61	0.94630
0.02	0.50798	0.42	0.66276	0.82	0.79389	1.22	0.88877	1.62	0.94738
0.03	0.51197	0.43	0.66640	0.83	0.79673	1.23	0.89065	1.63	0.94845
0.04	0.51595	0.44	0.67003	0.84	0.79955	1.24	0.89251	1.64	0.94950
0.05	0.51994	0.45	0.67364	0.85	0.80234	1.25	0.89435	1.65	0.95053
0.06	0.52392	0.46	0.67724	0.86	0.80511	1.26	0.89617	1.66	0.95154
0.07	0.52790	0.47	0.68082	0.87	0.80785	1.27	0.89796	1.67	0.95254
0.08	0.53188	0.48	0.68439	0.88	0.81057	1.28	0.89973	1.68	0.95352
0.09	0.53586	0.49	0.68793	0.89	0.81327	1.29	0.90147	1.69	0.95449
0.10	0.53983	0.50	0.69146	0.90	0.81594	1.30	0.90320	1.70	0.95543
0.11	0.54380	0.51	0.69497	0.91	0.81859	1.31	0.90490	1.71	0.95637
0.12	0.54776	0.52	0.69847	0.92	0.82121	1.32	0.90658	1.72	0.95728
0.13	0.55172	0.53	0.70194	0.93	0.82381	1.33	0.90824	1.73	0.95818
0.14	0.55567	0.54	0.70540	0.94	0.82639	1.34	0.90988	1.74	0.95907
0.15	0.55962	0.55	0.70884	0.95	0.82894	1.35	0.91149	1.75	0.95994
0.16	0.56356	0.56	0.71226	0.96	0.83147	1.36	0.91309	1.76	0.96080
0.17	0.56749	0.57	0.71566	0.97	0.83398	1.37	0.91466	1.77	0.96164
0.18	0.57142	0.58	0.71904	0.98	0.83646	1.38	0.91621	1.78	0.96246
0.19	0.57535	0.59	0.72240	0.99	0.83891	1.39	0.91774	1.79	0.96327
0.20	0.57926	0.60	0.72575	1.00	0.84134	1.40	0.91924	1.80	0.96407
0.21	0.58317	0.61	0.72907	1.01	0.84375	1.41	0.92073	1.81	0.96485
0.22	0.58706	0.62	0.73237	1.02	0.84614	1.42	0.92220	1.82	0.96562
0.23	0.59095	0.63	0.73565	1.03	0.84849	1.43	0.92364	1.83	0.96638
0.24	0.59483	0.64	0.73891	1.04	0.85083	1.44	0.92507	1.84	0.96712
0.25	0.59871	0.65	0.74215	1.05	0.85314	1.45	0.92647	1.85	0.96784
0.26	0.60257	0.66	0.74537	1.06	0.85543	1.46	0.92785	1.86	0.96856
0.27	0.60642	0.67	0.74857	1.07	0.85769	1.47	0.92922	1.87	0.96926
0.28	0.61026	0.68	0.75175	1.08	0.85993	1.48	0.93056	1.88	0.96995
0.29	0.61409	0.69	0.75490	1.09	0.86214	1.49	0.93189	1.89	0.97062
0.30	0.61791	0.70	0.75804	1.10	0.86433	1.50	0.93319	1.90	0.97128
0.31	0.62172	0.71	0.76115	1.11	0.86650	1.51	0.93448	1.91	0.97193
0.32	0.62552	0.72	0.76424	1.12	0.86864	1.52	0.93574	1.92	0.97257
0.33	0.62930	0.73	0.76730	1.13	0.87076	1.53	0.93699	1.93	0.97320
0.34	0.63307	0.74	0.77035	1.14	0.87286	1.54	0.93822	1.94	0.97381
0.35	0.63683	0.75	0.77337	1.15	0.87493	1.55	0.93943	1.95	0.97441
0.36	0.64058	0.76	0.77637	1.16	0.87698	1.56	0.94062	1.96	0.97500
0.37	0.64431	0.77	0.77935	1.17	0.87900	1.57	0.94179	1.97	0.97558
0.38	0.64803	0.78	0.78230	1.18	0.88100	1.58	0.94295	1.98	0.97615
0.39	0.65173	0.79	0.78524	1.19	0.88298	1.59	0.94408	1.99	0.97670
0.40	0.65542	0.80	0.78814	1.20	0.88493	1.60	0.94520	2.00	0.97725

Probabilities for the Standard Normal distribution

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2.00	0.97725	2.40	0.99180	2.80	0.99744	3.20	0.99931	3.60	0.99984	4.00	0.99997
2.01	0.97778	2.41	0.99202	2.81	0.99752	3.21	0.99934	3.61	0.99985	4.01	0.99997
2.02	0.97831	2.42	0.99224	2.82	0.99760	3.22	0.99936	3.62	0.99985	4.02	0.99997
2.03	0.97882	2.43	0.99245	2.83	0.99767	3.23	0.99938	3.63	0.99986	4.03	0.99997
2.04	0.97932	2.44	0.99266	2.84	0.99774	3.24	0.99940	3.64	0.99986	4.04	0.99997
2.05	0.97982	2.45	0.99286	2.85	0.99781	3.25	0.99942	3.65	0.99987	4.05	0.99997
2.06	0.98030	2.46	0.99305	2.86	0.99788	3.26	0.99944	3.66	0.99987	4.06	0.99998
2.07	0.98077	2.47	0.99324	2.87	0.99795	3.27	0.99946	3.67	0.99988	4.07	0.99998
2.08	0.98124	2.48	0.99343	2.88	0.99801	3.28	0.99948	3.68	0.99988	4.08	0.99998
2.09	0.98169	2.49	0.99361	2.89	0.99807	3.29	0.99950	3.69	0.99989	4.09	0.99998
2.10	0.98214	2.50	0.99379	2.90	0.99813	3.30	0.99952	3.70	0.99989	4.10	0.99998
2.11	0.98257	2.51	0.99396	2.91	0.99819	3.31	0.99953	3.71	0.99990	4.11	0.99998
2.12	0.98300	2.52	0.99413	2.92	0.99825	3.32	0.99955	3.72	0.99990	4.12	0.99998
2.13	0.98341	2.53	0.99430	2.93	0.99831	3.33	0.99957	3.73	0.99990	4.13	0.99998
2.14	0.98382	2.54	0.99446	2.94	0.99836	3.34	0.99958	3.74	0.99991	4.14	0.99998
2.15	0.98422	2.55	0.99461	2.95	0.99841	3.35	0.99960	3.75	0.99991	4.15	0.99998
2.16	0.98461	2.56	0.99477	2.96	0.99846	3.36	0.99961	3.76	0.99992	4.16	0.99998
2.17	0.98500	2.57	0.99492	2.97	0.99851	3.37	0.99962	3.77	0.99992	4.17	0.99998
2.18	0.98537	2.58	0.99506	2.98	0.99856	3.38	0.99964	3.78	0.99992	4.18	0.99999
2.19	0.98574	2.59	0.99520	2.99	0.99861	3.39	0.99965	3.79	0.99992	4.19	0.99999
2.20	0.98610	2.60	0.99534	3.00	0.99865	3.40	0.99966	3.80	0.99993	4.20	0.99999
2.21	0.98645	2.61	0.99547	3.01	0.99869	3.41	0.99968	3.81	0.99993	4.21	0.99999
2.22	0.98679	2.62	0.99560	3.02	0.99874	3.42	0.99969	3.82	0.99993	4.22	0.99999
2.23	0.98713	2.63	0.99573	3.03	0.99878	3.43	0.99970	3.83	0.99994	4.23	0.99999
2.24	0.98745	2.64	0.99585	3.04	0.99882	3.44	0.99971	3.84	0.99994	4.24	0.99999
2.25	0.98778	2.65	0.99598	3.05	0.99886	3.45	0.99972	3.85	0.99994	4.25	0.99999
2.26	0.98809	2.66	0.99609	3.06	0.99889	3.46	0.99973	3.86	0.99994	4.26	0.99999
2.27	0.98840	2.67	0.99621	3.07	0.99893	3.47	0.99974	3.87	0.99995	4.27	0.99999
2.28	0.98870	2.68	0.99632	3.08	0.99896	3.48	0.99975	3.88	0.99995	4.28	0.99999
2.29	0.98899	2.69	0.99643	3.09	0.99900	3.49	0.99976	3.89	0.99995	4.29	0.99999
2.30	0.98928	2.70	0.99653	3.10	0.99903	3.50	0.99977	3.90	0.99995	4.30	0.99999
2.31	0.98956	2.71	0.99664	3.11	0.99906	3.51	0.99978	3.91	0.99995	4.31	0.99999
2.32	0.98983	2.72	0.99674	3.12	0.99910	3.52	0.99978	3.92	0.99996	4.32	0.99999
2.33	0.99010	2.73	0.99683	3.13	0.99913	3.53	0.99979	3.93	0.99996	4.33	0.99999
2.34	0.99036	2.74	0.99693	3.14	0.99916	3.54	0.99980	3.94	0.99996	4.34	0.99999
2.35	0.99061	2.75	0.99702	3.15	0.99918	3.55	0.99981	3.95	0.99996	4.35	0.99999
2.36	0.99086	2.76	0.99711	3.16	0.99921	3.56	0.99981	3.96	0.99996	4.36	0.99999
2.37	0.99111	2.77	0.99720	3.17	0.99924	3.57	0.99982	3.97	0.99996	4.37	0.99999
2.38	0.99134	2.78	0.99728	3.18	0.99926	3.58	0.99983	3.98	0.99997	4.38	0.99999
2.39	0.99158	2.79	0.99736	3.19	0.99929	3.59	0.99983	3.99	0.99997	4.39	0.99999
2.40	0.99180	2.80	0.99744	3.20	0.99931	3.60	0.99984	4.00	0.99997	4.40	0.99999

11.2 Percentage Points for the Standard Normal distribution

The table gives percentage points x defined by the equation.

$$P = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$$

P	x	P	x	P	x	P	x	P	x	P	x
50%	0.0000	5.0%	1.6449	3.0%	1.8808	2.0%	2.0537	1.0%	2.3263	0.10%	3.0902
45%	0.1257	4.8%	1.6646	2.9%	1.8957	1.9%	2.0749	0.9%	2.3656	0.09%	3.1214
40%	0.2533	4.6%	1.6849	2.8%	1.9110	1.8%	2.0969	0.8%	2.4089	0.08%	3.1559
35%	0.3853	4.4%	1.7060	2.7%	1.9268	1.7%	2.1201	0.7%	2.4573	0.07%	3.1947
30%	0.5244	4.2%	1.7279	2.6%	1.9431	1.6%	2.1444	0.6%	2.5121	0.06%	3.2389
25%	0.6745	4.0%	1.7507	2.5%	1.9600	1.5%	2.1701	0.5%	2.5758	0.05%	3.2905
20%	0.8416	3.8%	1.7744	2.4%	1.9774	1.4%	2.1973	0.4%	2.6521	0.01%	3.7190
15%	1.0364	3.6%	1.7991	2.3%	1.9954	1.3%	2.2262	0.3%	2.7478	0.005%	3.8906
10%	1.2816	3.4%	1.8250	2.2%	2.0141	1.2%	2.2571	0.2%	2.8782	0.001%	4.2649
5%	1.6449	3.2%	1.8522	2.1%	2.0335	1.1%	2.2904	0.1%	3.0902	0.0005%	4.4172

11.3 Percentage Points for the t distribution

This table gives percentage points x defined by the equation

$$P = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \int_x^\infty \frac{dt}{\left(1+\frac{t^2}{\nu}\right)^{(\nu+1)/2}}$$

The limiting distribution of t as ν tends to infinity is the standard normal distribution. When ν is large, interpolation in ν should be harmonic.

$P =$ ν	40%	30%	25%	20%	15%	10%	5%	2.5%	1%	0.5%	0.1%	0.05%
1	0.3249	0.7265	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

11.4 Probabilities for the χ^2 distribution

The function tabulated is:

$$F_\nu(x) = \frac{1}{2^{\nu/2}\Gamma\left(\frac{\nu}{2}\right)} \int_0^x t^{\nu/2-1} e^{-t/2} dt$$

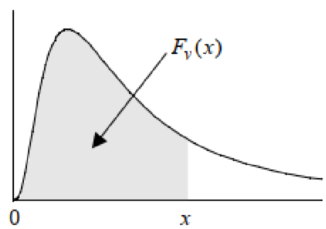

Figure 1: χ^2 distribution

(The above shape applies for $\nu \geq 3$ only. When $\nu < 3$ the mode is at the origin.)

ν	1		1		2		2		3		3
x		x		x		x		x		x	
0.0	0.0000	4.0	0.9545	0.0	0.0000	4.0	0.8647	0.0	0.0000	4.0	0.7385
0.1	0.2482	4.1	0.9571	0.1	0.0488	4.1	0.8713	0.1	0.0082	4.2	0.7593
0.2	0.3453	4.2	0.9596	0.2	0.0952	4.2	0.8775	0.2	0.0224	4.4	0.7786
0.3	0.4161	4.3	0.9619	0.3	0.1393	4.3	0.8835	0.3	0.0400	4.6	0.7965
0.4	0.4729	4.4	0.9641	0.4	0.1813	4.4	0.8892	0.4	0.0598	4.8	0.8130
0.5	0.5205	4.5	0.9661	0.5	0.2212	4.5	0.8946	0.5	0.0811	5.0	0.8282
0.6	0.5614	4.6	0.9680	0.6	0.2592	4.6	0.8997	0.6	0.1036	5.2	0.8423
0.7	0.5972	4.7	0.9698	0.7	0.2953	4.7	0.9046	0.7	0.1268	5.4	0.8553
0.8	0.6289	4.8	0.9715	0.8	0.3297	4.8	0.9093	0.8	0.1505	5.6	0.8672
0.9	0.6572	4.9	0.9731	0.9	0.3624	4.9	0.9137	0.9	0.1746	5.8	0.8782
1.0	0.6827	5.0	0.9747	1.0	0.3935	5.0	0.9179	1.0	0.1987	6.0	0.8884
1.1	0.7057	5.1	0.9761	1.1	0.4231	5.1	0.9219	1.1	0.2229	6.2	0.8977
1.2	0.7267	5.2	0.9774	1.2	0.4512	5.2	0.9257	1.2	0.2470	6.4	0.9063
1.3	0.7458	5.3	0.9787	1.3	0.4780	5.3	0.9293	1.3	0.2709	6.6	0.9142
1.4	0.7633	5.4	0.9799	1.4	0.5034	5.4	0.9328	1.4	0.2945	6.8	0.9214
1.5	0.7793	5.5	0.9810	1.5	0.5276	5.5	0.9361	1.5	0.3177	7.0	0.9281
1.6	0.7941	5.6	0.9820	1.6	0.5507	5.6	0.9392	1.6	0.3406	7.2	0.9342
1.7	0.8077	5.7	0.9830	1.7	0.5726	5.7	0.9422	1.7	0.3631	7.4	0.9398
1.8	0.8203	5.8	0.9840	1.8	0.5934	5.8	0.9450	1.8	0.3851	7.6	0.9450
1.9	0.8319	5.9	0.9849	1.9	0.6133	5.9	0.9477	1.9	0.4066	7.8	0.9497
2.0	0.8427	6.0	0.9857	2.0	0.6321	6.0	0.9502	2.0	0.4276	8.0	0.9540
2.1	0.8527	6.1	0.9865	2.1	0.6501	6.2	0.9550	2.1	0.4481	8.2	0.9579
2.2	0.8620	6.2	0.9872	2.2	0.6671	6.4	0.9592	2.2	0.4681	8.4	0.9616
2.3	0.8706	6.3	0.9879	2.3	0.6834	6.6	0.9631	2.3	0.4875	8.6	0.9649
2.4	0.8787	6.4	0.9886	2.4	0.6988	6.8	0.9666	2.4	0.5064	8.8	0.9679
2.5	0.8862	6.5	0.9892	2.5	0.7135	7.0	0.9698	2.5	0.5247	9.0	0.9707
2.6	0.8931	6.6	0.9898	2.6	0.7275	7.2	0.9727	2.6	0.5425	9.2	0.9733
2.7	0.8997	6.7	0.9904	2.7	0.7408	7.4	0.9753	2.7	0.5598	9.4	0.9756
2.8	0.9057	6.8	0.9909	2.8	0.7534	7.6	0.9776	2.8	0.5765	9.6	0.9777
2.9	0.9114	6.9	0.9914	2.9	0.7654	7.8	0.9798	2.9	0.5927	9.8	0.9797
3.0	0.9167	7.0	0.9918	3.0	0.7769	8.0	0.9817	3.0	0.6084	10.0	0.9814
3.1	0.9217	7.1	0.9923	3.1	0.7878	8.2	0.9834	3.1	0.6235	10.2	0.9831
3.2	0.9264	7.2	0.9927	3.2	0.7981	8.4	0.9850	3.2	0.6382	10.4	0.9845
3.3	0.9307	7.3	0.9931	3.3	0.8080	8.6	0.9864	3.3	0.6524	10.6	0.9859
3.4	0.9348	7.4	0.9935	3.4	0.8173	8.8	0.9877	3.4	0.6660	10.8	0.9871
3.5	0.9386	7.5	0.9938	3.5	0.8262	9.0	0.9889	3.5	0.6792	11.0	0.9883
3.6	0.9422	7.6	0.9942	3.6	0.8347	9.2	0.9899	3.6	0.6920	11.2	0.9893
3.7	0.9456	7.7	0.9945	3.7	0.8428	9.4	0.9909	3.7	0.7043	11.4	0.9903
3.8	0.9487	7.8	0.9948	3.8	0.8504	9.6	0.9918	3.8	0.7161	11.6	0.9911
3.9	0.9517	7.9	0.9951	3.9	0.8577	9.8	0.9926	3.9	0.7275	11.8	0.9919
4.0	0.9545	8.0	0.9953	4.0	0.8647	10.0	0.9933	4.0	0.7385	12.0	0.9926

Probabilities for the χ^2 distribution

ν x	4	5	6	7	8	9	10	11	12	13	14
0.5	0.0265	0.0079	0.0022	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0	0.0902	0.0374	0.0144	0.0052	0.0018	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000
1.5	0.1734	0.0869	0.0405	0.0177	0.0073	0.0029	0.0011	0.0004	0.0001	0.0000	0.0000
2.0	0.2642	0.1509	0.0803	0.0402	0.0190	0.0085	0.0037	0.0015	0.0006	0.0002	0.0001
2.5	0.3554	0.2235	0.1315	0.0729	0.0383	0.0191	0.0091	0.0042	0.0018	0.0008	0.0003
3.0	0.4422	0.3000	0.1912	0.1150	0.0656	0.0357	0.0186	0.0093	0.0045	0.0021	0.0009
3.5	0.5221	0.3766	0.2560	0.1648	0.1008	0.0589	0.0329	0.0177	0.0091	0.0046	0.0022
4.0	0.5940	0.4506	0.3233	0.2202	0.1429	0.0886	0.0527	0.0301	0.0166	0.0088	0.0045
4.5	0.6575	0.5201	0.3907	0.2793	0.1906	0.1245	0.0780	0.0471	0.0274	0.0154	0.0084
5.0	0.7127	0.5841	0.4562	0.3400	0.2424	0.1657	0.1088	0.0688	0.0420	0.0248	0.0142
5.5	0.7603	0.6421	0.5185	0.4008	0.2970	0.2113	0.1446	0.0954	0.0608	0.0375	0.0224
6.0	0.8009	0.6938	0.5768	0.4603	0.3528	0.2601	0.1847	0.1266	0.0839	0.0538	0.0335
6.5	0.8352	0.7394	0.6304	0.5173	0.4086	0.3110	0.2283	0.1620	0.1112	0.0739	0.0477
7.0	0.8641	0.7794	0.6792	0.5711	0.4634	0.3629	0.2746	0.2009	0.1424	0.0978	0.0653
7.5	0.8883	0.8140	0.7229	0.6213	0.5162	0.4148	0.3225	0.2427	0.1771	0.1254	0.0863
8.0	0.9084	0.8438	0.7619	0.6674	0.5665	0.4659	0.3712	0.2867	0.2149	0.1564	0.1107
8.5	0.9251	0.8693	0.7963	0.7094	0.6138	0.5154	0.4199	0.3321	0.2551	0.1904	0.1383
9.0	0.9389	0.8909	0.8264	0.7473	0.6577	0.5627	0.4679	0.3781	0.2971	0.2271	0.1689
9.5	0.9503	0.9093	0.8527	0.7813	0.6981	0.6075	0.5146	0.4242	0.3403	0.2658	0.2022
10.0	0.9596	0.9248	0.8753	0.8114	0.7350	0.6495	0.5595	0.4696	0.3840	0.3061	0.2378
10.5	0.9672	0.9378	0.8949	0.8380	0.7683	0.6885	0.6022	0.5140	0.4278	0.3474	0.2752
11.0	0.9734	0.9486	0.9116	0.8614	0.7983	0.7243	0.6425	0.5567	0.4711	0.3892	0.3140
11.5	0.9785	0.9577	0.9259	0.8818	0.8251	0.7570	0.6801	0.5976	0.5134	0.4310	0.3536
12.0	0.9826	0.9652	0.9380	0.8994	0.8488	0.7867	0.7149	0.6364	0.5543	0.4724	0.3937
12.5	0.9860	0.9715	0.9483	0.9147	0.8697	0.8134	0.7470	0.6727	0.5936	0.5129	0.4338
13.0	0.9887	0.9766	0.9570	0.9279	0.8882	0.8374	0.7763	0.7067	0.6310	0.5522	0.4735
13.5	0.9909	0.9809	0.9643	0.9392	0.9042	0.8587	0.8030	0.7381	0.6662	0.5900	0.5124
14.0	0.9927	0.9844	0.9704	0.9488	0.9182	0.8777	0.8270	0.7670	0.6993	0.6262	0.5503
14.5	0.9941	0.9873	0.9755	0.9570	0.9304	0.8944	0.8486	0.7935	.7301	0.6604	0.5868
15.0	0.9953	0.9896	0.9797	0.9640	0.9409	0.9091	0.8679	0.8175	0.7586	0.6926	0.6218
15.5	0.9962	0.9916	0.9833	0.9699	0.9499	0.9219	0.8851	0.8393	0.7848	0.7228	0.6551
16.0	0.9970	0.9932	0.9862	0.9749	0.9576	0.9331	0.9004	0.8589	0.8088	0.7509	0.6866
16.5	0.9976	0.9944	0.9887	0.9791	0.9642	0.9429	0.9138	0.8764	0.8306	0.7768	0.7162
17.0	0.9981	0.9955	0.9907	0.9826	0.9699	0.9513	0.9256	0.8921	0.8504	0.8007	0.7438
17.5	0.9985	0.9964	0.9924	0.9856	0.9747	0.9586	0.9360	0.9061	0.8683	0.8226	0.7695
18.0	0.9988	0.9971	0.9938	0.9880	0.9788	0.9648	0.9450	0.9184	0.8843	0.8425	0.7932
18.5	0.9990	0.9976	0.9949	0.9901	0.9822	0.9702	0.9529	0.9293	0.8987	0.8606	0.8151
19.0	0.9992	0.9981	0.9958	0.9918	0.9851	0.9748	0.9597	0.9389	0.9115	0.8769	0.8351
19.5	0.9994	0.9984	0.9966	0.9932	0.9876	0.9787	0.9656	0.9473	0.9228	0.8916	0.8533
20	0.9995	0.9988	0.9972	0.9944	0.9897	0.9821	0.9707	0.9547	0.9329	0.9048	0.8699
21	0.9997	0.9992	0.9982	0.9962	0.9929	0.9873	0.9789	0.9666	0.9496	0.9271	0.8984
22	0.9998	0.9995	0.9988	0.9975	0.9951	0.9911	0.9849	0.9756	0.9625	0.9446	0.9214
23	0.9999	0.9997	0.9992	0.9983	0.9966	0.9938	0.9893	0.9823	0.9723	0.9583	0.9397
24	0.9999	0.9998	0.9995	0.9989	0.9977	0.9957	0.9924	0.9873	0.9797	0.9689	0.9542
25	0.9999	0.9999	0.9997	0.9992	0.9984	0.9970	0.9947	0.9909	0.9852	0.9769	0.9654
26	1.0000	0.9999	0.9998	0.9995	0.9989	0.9980	0.9963	0.9935	0.9893	0.9830	0.9741
27	1.0000	0.9999	0.9999	0.9997	0.9993	0.9986	0.9974	0.9954	0.9923	0.9876	0.9807
28	1.0000	1.0000	0.9999	0.9998	0.9995	0.9990	0.9982	0.9968	0.9945	0.9910	0.9858
29	1.0000	1.0000	0.9999	0.9999	0.9997	0.9994	0.9988	0.9977	0.9961	0.9935	0.9895
30	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9972	0.9953	0.9924

Probabilities for the χ^2 distribution

$\nu =$ x	15	16	17	18	19	20	21	22	23	24	25
3	0.0004	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0079	0.0042	0.0022	0.0011	0.0006	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000
6	0.0203	0.0119	0.0068	0.0038	0.0021	0.0011	0.0006	0.0003	0.0001	0.0001	0.0000
7	0.0424	0.0267	0.0165	0.0099	0.0058	0.0033	0.0019	0.0010	0.0005	0.0003	0.0001
8	0.0762	0.0511	0.0335	0.0214	0.0133	0.0081	0.0049	0.0028	0.0016	0.0009	0.0005
9	0.1225	0.0866	0.0597	0.0403	0.0265	0.0171	0.0108	0.0067	0.0040	0.0024	0.0014
10	0.1803	0.1334	0.0964	0.0681	0.0471	0.0318	0.0211	0.0137	0.0087	0.0055	0.0033
11	0.2474	0.1905	0.1434	0.1056	0.0762	0.0538	0.0372	0.0253	0.0168	0.0110	0.0071
12	0.3210	0.2560	0.1999	0.1528	0.1144	0.0839	0.0604	0.0426	0.0295	0.0201	0.0134
13	0.3977	0.3272	0.2638	0.2084	0.1614	0.1226	0.0914	0.0668	0.0480	0.0339	0.0235
14	0.4745	0.4013	0.3329	0.2709	0.2163	0.1695	0.1304	0.0985	0.0731	0.0533	0.0383
15	0.5486	0.4754	0.4045	0.3380	0.2774	0.2236	0.1770	0.1378	0.1054	0.0792	0.0586
16	0.6179	0.5470	0.4762	0.4075	0.3427	0.2834	0.2303	0.1841	0.1447	0.1119	0.0852
17	0.6811	0.6144	0.5456	0.4769	0.4101	0.3470	0.2889	0.2366	0.1907	0.1513	0.1182
18	0.7373	0.6761	0.6112	0.5443	0.4776	0.4126	0.3510	0.2940	0.2425	0.1970	0.1576
19	0.7863	0.7313	0.6715	0.6082	0.5432	0.4782	0.4149	0.3547	0.2988	0.2480	0.2029
20	0.8281	0.7798	0.7258	0.6672	0.6054	0.5421	0.4787	0.4170	0.3581	0.3032	0.2532
21	0.8632	0.8215	0.7737	0.7206	0.6632	0.6029	0.5411	0.4793	0.4189	0.3613	0.3074
22	0.8922	0.8568	0.8153	0.7680	0.7157	0.6595	0.6005	0.5401	0.4797	0.4207	0.3643
23	0.9159	0.8863	0.8507	0.8094	0.7627	0.7112	0.6560	0.5983	0.5392	0.4802	0.4224
24	0.9349	0.9105	0.8806	0.8450	0.8038	0.7576	0.7069	0.6528	0.5962	0.5384	0.4806
25	0.9501	0.9302	0.9053	0.8751	0.8395	0.7986	0.7528	0.7029	0.6497	0.5942	0.5376
26	0.9620	0.9460	0.9255	0.9002	0.8698	0.8342	0.7936	0.7483	0.6991	0.6468	0.5924
27	0.9713	0.9585	0.9419	0.9210	0.8953	0.8647	0.8291	0.7888	0.7440	0.6955	0.6441
28	0.9784	0.9684	0.9551	0.9379	0.9166	0.8906	0.8598	0.8243	0.7842	0.7400	0.6921
29	0.9839	0.9761	0.9655	0.9516	0.9340	0.9122	0.8860	0.8551	0.8197	0.7799	0.7361
30	0.9881	0.9820	0.9737	0.9626	0.9482	0.9301	0.9080	0.8815	0.8506	0.8152	0.7757
31	0.9912	0.9865	0.9800	0.9712	0.9596	0.9448	0.9263	0.9039	0.8772	0.8462	0.8110
32	0.9936	0.9900	0.9850	0.9780	0.9687	0.9567	0.9414	0.9226	0.8999	0.8730	0.8420
33	0.9953	0.9926	0.9887	0.9833	0.9760	0.9663	0.9538	0.9381	0.9189	0.8959	0.8689
34	0.9966	0.9946	0.9916	0.9874	0.9816	0.9739	0.9638	0.9509	0.9348	0.9153	0.8921
35	0.9975	0.9960	0.9938	0.9905	0.9860	0.9799	0.9718	0.9613	0.9480	0.9316	0.9118
36	0.9982	0.9971	0.9954	0.9929	0.9894	0.9846	0.9781	0.9696	0.9587	0.9451	0.9284
37	0.9987	0.9979	0.9966	0.9948	0.9921	0.9883	0.9832	0.9763	0.9675	0.9562	0.9423
38	0.9991	0.9985	0.9975	0.9961	0.9941	0.9911	0.9871	0.9817	0.9745	0.9653	0.9537
39	0.9994	0.9989	0.9982	0.9972	0.9956	0.9933	0.9902	0.9859	0.9802	0.9727	0.9632
40	0.9995	0.9992	0.9987	0.9979	0.9967	0.9950	0.9926	0.9892	0.9846	0.9786	0.9708
41	0.9997	0.9994	0.9991	0.9985	0.9976	0.9963	0.9944	0.9918	0.9882	0.9833	0.9770
42	0.9998	0.9996	0.9993	0.9989	0.9982	0.9972	0.9958	0.9937	0.9909	0.9871	0.9820
43	0.9998	0.9997	0.9995	0.9992	0.9987	0.9980	0.9969	0.9953	0.9931	0.9901	0.9860
44	0.9999	0.9998	0.9997	0.9994	0.9991	0.9985	0.9977	0.9965	0.9947	0.9924	0.9892
45	0.9999	0.9999	0.9998	0.9996	0.9993	0.9989	0.9983	0.9973	0.9960	0.9942	0.9916
46	0.9999	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9980	0.9970	0.9956	0.9936
47	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9985	0.9978	0.9967	0.9951
48	1.0000	1.0000	0.9999	0.9998	0.9997	0.9996	0.9993	0.9989	0.9983	0.9975	0.9963
49	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9992	0.9988	0.9981	0.9972
50	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9986	0.9979

11.5 Percentage Points for the χ^2 distribution

$P =$ ν	99.95%	99.9%	99.5%	99%	97.5%	95%	90%	80%	70%	60%
1	3.927E–07	1.571E–06	3.927E–05	1.571E–04	9.821E–04	0.003932	0.01579	0.06418	0.1485	0.2750
2	0.001000	0.002001	0.01003	0.02010	0.05064	0.1026	0.2107	0.4463	0.7133	1.022
3	0.01528	0.02430	0.07172	0.1148	0.2158	0.3518	0.5844	1.005	1.424	1.869
4	0.06392	0.09080	0.2070	0.2971	0.4844	0.7107	1.064	1.649	2.195	2.753
5	0.1581	0.2102	0.4118	0.5543	0.8312	1.145	1.610	2.343	3.000	3.656
6	0.2994	0.3810	0.6757	0.8721	1.237	1.635	2.204	3.070	3.828	4.570
7	0.4849	0.5985	0.9893	1.239	1.690	2.167	2.833	3.822	4.671	5.493
8	0.7104	0.8571	1.344	1.647	2.180	2.733	3.490	4.594	5.527	6.423
9	0.9718	1.152	1.735	2.088	2.700	3.325	4.168	5.380	6.393	7.357
10	1.265	1.479	2.156	2.558	3.247	3.940	4.865	6.179	7.267	8.295
11	1.587	1.834	2.603	3.053	3.816	4.575	5.578	6.989	8.148	9.237
12	1.935	2.214	3.074	3.571	4.404	5.226	6.304	7.807	9.034	10.18
13	2.305	2.617	3.565	4.107	5.009	5.892	7.041	8.634	9.926	11.13
14	2.697	3.041	4.075	4.660	5.629	6.571	7.790	9.467	10.82	12.08
15	3.107	3.483	4.601	5.229	6.262	7.261	8.547	10.31	11.72	13.03
16	3.536	3.942	5.142	5.812	6.908	7.962	9.312	11.15	12.62	13.98
17	3.980	4.416	5.697	6.408	7.564	8.672	10.09	12.00	13.53	14.94
18	4.439	4.905	6.265	7.015	8.231	9.390	10.86	12.86	14.44	15.89
19	4.913	5.407	6.844	7.633	8.907	10.12	11.65	13.72	15.35	16.85
20	5.398	5.921	7.434	8.260	9.591	10.85	12.44	14.58	16.27	17.81
21	5.895	6.447	8.034	8.897	10.28	11.59	13.24	15.44	17.18	18.77
22	6.404	6.983	8.643	9.542	10.98	12.34	14.04	16.31	18.10	19.73
23	6.924	7.529	9.260	10.20	11.69	13.09	14.85	17.19	19.02	20.69
24	7.453	8.085	9.886	10.86	12.40	13.85	15.66	18.06	19.94	21.65
25	7.991	8.649	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62
26	8.537	9.222	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58
27	9.093	9.803	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54
28	9.656	10.39	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44
32	11.98	12.81	15.13	16.36	18.29	20.07	22.27	25.15	27.37	29.38
34	13.18	14.06	16.50	17.79	19.81	21.66	23.95	26.94	29.24	31.31
36	14.40	15.32	17.89	19.23	21.34	23.27	25.64	28.73	31.12	33.25
38	15.64	16.61	19.29	20.69	22.88	24.88	27.34	30.54	32.99	35.19
40	16.91	17.92	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13
50	23.46	24.67	27.99	29.71	32.36	34.76	37.69	41.45	44.31	46.86
60	30.34	31.74	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62
70	37.47	39.04	43.28	45.44	48.76	51.74	55.33	59.90	63.35	66.40
80	44.79	46.52	51.17	53.54	57.15	60.39	64.28	69.21	72.92	76.19
90	52.28	54.16	59.20	61.75	65.65	69.13	73.29	78.56	82.51	85.99
100	59.89	61.92	67.33	70.06	74.22	77.93	82.36	87.95	92.13	95.81

$P =$ ν	50%	40%	30%	20%	10%	5%	2.5%	1%	0.5%	0.1%	0.05%
1	0.4549	0.7083	1.074	1.642	2.706	3.841	5.024	6.635	7.879	10.83	12.12
2	1.386	1.833	2.408	3.219	4.605	5.991	7.378	9.210	10.60	13.82	15.20
3	2.366	2.946	3.665	4.642	6.251	7.815	9.348	11.34	12.84	16.27	17.73
4	3.357	4.045	4.878	5.989	7.779	9.488	11.14	13.28	14.86	18.47	20.00
5	4.351	5.132	6.064	7.289	9.236	11.07	12.83	15.09	16.75	20.51	22.11
6	5.348	6.211	7.231	8.558	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	6.346	7.283	8.383	9.803	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	7.344	8.351	9.524	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	8.343	9.414	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	9.342	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	10.34	11.53	12.90	14.63	17.28	19.68	21.92	24.73	26.76	31.26	33.14
12	11.34	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	12.34	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	13.34	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	14.34	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.72
16	15.34	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	16.34	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	17.34	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	18.34	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	19.34	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	20.34	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	21.34	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	22.34	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	23.34	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	24.34	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	25.34	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	26.34	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.65	55.48	57.86
28	27.34	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	28.34	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73
30	29.34	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16
32	31.34	33.38	35.66	38.47	42.58	46.19	49.48	53.49	56.33	62.49	64.99
34	33.34	35.44	37.80	40.68	44.90	48.60	51.97	56.06	58.96	65.25	67.80
36	35.34	37.50	39.92	42.88	47.21	51.00	54.44	58.62	61.58	67.98	70.59
38	37.34	39.56	42.05	45.08	49.51	53.38	56.90	61.16	64.18	70.70	73.35
40	39.34	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.10
50	49.33	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56
60	59.33	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.7
70	69.33	72.36	75.69	79.71	85.53	90.53	95.02	100.4	104.2	112.3	115.6
80	79.33	82.57	86.12	90.41	96.58	101.9	106.6	112.3	116.3	124.8	128.3
90	89.33	92.76	96.52	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8
100	99.33	102.9	106.9	111.7	118.5	124.3	129.6	135.8	140.2	149.4	153.2

11.6 Percentage Points for the F distribution

The function tabulated is x defined for the specified percentage points P by the equation

$$P = \frac{\Gamma\left(\frac{v_1+v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)} v_1^{v_1/2} v_2^{v_2/2} \int_x^\infty \frac{t^{v_1/2-1}}{(v_2+v_1 t)^{(v_1+v_2)/2}} dt$$

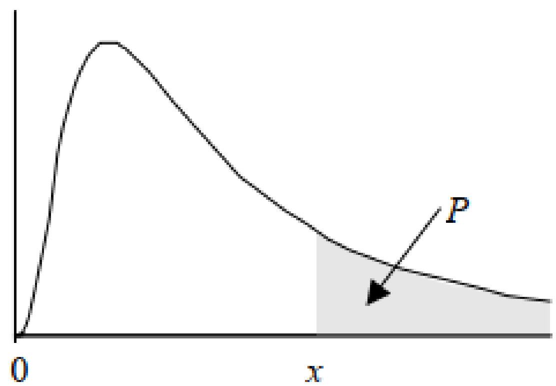

Figure 2: F Distribution

The above shape applies only for $v_1 \geq 3$. When $v_1 < 3$, the mode is at the origin.

10% points for the F distribution

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	24	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	62.00	63.33
2	8.526	9.000	9.162	9.243	9.293	9.326	9.349	9.367	9.381	9.392	9.408	9.450	9.491
3	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.240	5.230	5.216	5.176	5.134
4	4.545	4.325	4.191	4.107	4.051	4.010	3.979	3.955	3.936	3.920	3.896	3.831	3.761
5	4.060	3.780	3.619	3.520	3.453	3.405	3.368	3.339	3.316	3.297	3.268	3.191	3.105
6	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.958	2.937	2.905	2.818	2.722
7	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.725	2.703	2.668	2.575	2.471
8	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.561	2.538	2.502	2.404	2.293
9	3.360	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.440	2.416	2.379	2.277	2.159
10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.347	2.323	2.284	2.178	2.055
11	3.225	2.860	2.660	2.536	2.451	2.389	2.342	2.304	2.274	2.248	2.209	2.100	1.972
12	3.177	2.807	2.606	2.480	2.394	2.331	2.283	2.245	2.214	2.188	2.147	2.036	1.904
13	3.136	2.763	2.560	2.434	2.347	2.283	2.234	2.195	2.164	2.138	2.097	1.983	1.846
14	3.102	2.726	2.522	2.395	2.307	2.243	2.193	2.154	2.122	2.095	2.054	1.938	1.797
15	3.073	2.695	2.490	2.361	2.273	2.208	2.158	2.119	2.086	2.059	2.017	1.899	1.755
16	3.048	2.668	2.462	2.333	2.244	2.178	2.128	2.088	2.055	2.028	1.985	1.866	1.718
17	3.026	2.645	2.437	2.308	2.218	2.152	2.102	2.061	2.028	2.001	1.958	1.836	1.686
18	3.007	2.624	2.416	2.286	2.196	2.130	2.079	2.038	2.005	1.977	1.933	1.810	1.657
19	2.990	2.606	2.397	2.266	2.176	2.109	2.058	2.017	1.984	1.956	1.912	1.787	1.631
20	2.975	2.589	2.380	2.249	2.158	2.091	2.040	1.999	1.965	1.937	1.892	1.767	1.607
21	2.961	2.575	2.365	2.233	2.142	2.075	2.023	1.982	1.948	1.920	1.875	1.748	1.586
22	2.949	2.561	2.351	2.219	2.128	2.060	2.008	1.967	1.933	1.904	1.859	1.731	1.567
23	2.937	2.549	2.339	2.207	2.115	2.047	1.995	1.953	1.919	1.890	1.845	1.716	1.549
24	2.927	2.538	2.327	2.195	2.103	2.035	1.983	1.941	1.906	1.877	1.832	1.702	1.533
25	2.918	2.528	2.317	2.184	2.092	2.024	1.971	1.929	1.895	1.866	1.820	1.689	1.518
26	2.909	2.519	2.307	2.174	2.082	2.014	1.961	1.919	1.884	1.855	1.809	1.677	1.504
27	2.901	2.511	2.299	2.165	2.073	2.005	1.952	1.909	1.874	1.845	1.799	1.666	1.491
28	2.894	2.503	2.291	2.157	2.064	1.996	1.943	1.900	1.865	1.836	1.790	1.656	1.478
29	2.887	2.495	2.283	2.149	2.057	1.988	1.935	1.892	1.857	1.827	1.781	1.647	1.467
30	2.881	2.489	2.276	2.142	2.049	1.980	1.927	1.884	1.849	1.819	1.773	1.638	1.456
32	2.869	2.477	2.263	2.129	2.036	1.967	1.913	1.870	1.835	1.805	1.758	1.622	1.437
34	2.859	2.466	2.252	2.118	2.024	1.955	1.901	1.858	1.822	1.793	1.745	1.608	1.420
36	2.850	2.456	2.243	2.108	2.014	1.945	1.891	1.847	1.811	1.781	1.734	1.595	1.404
38	2.842	2.448	2.234	2.099	2.005	1.935	1.881	1.838	1.802	1.772	1.724	1.584	1.390
40	2.835	2.440	2.226	2.091	1.997	1.927	1.873	1.829	1.793	1.763	1.715	1.574	1.377
60	2.791	2.393	2.177	2.041	1.946	1.875	1.819	1.775	1.738	1.707	1.657	1.511	1.292
120	2.748	2.347	2.130	1.992	1.896	1.824	1.767	1.722	1.684	1.652	1.601	1.447	1.193
∞	2.706	2.303	2.084	1.945	1.847	1.774	1.717	1.670	1.632	1.599	1.546	1.383	1.000

5% points for the F distribution

$v_1 =$ v_2	1	2	3	4	5	6	7	8	9	10	12	24	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	249.1	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.45	19.50
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.785	8.745	8.638	8.527
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.688	3.581	3.500	3.438	3.388	3.347	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.609	2.405
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.082	1.843
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321	2.250	2.054	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.028	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.005	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	1.984	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.165	1.964	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220	2.148	1.946	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	1.930	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	1.915	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.104	1.901	1.638
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.092	1.887	1.622
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244	2.189	2.142	2.070	1.864	1.594
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225	2.170	2.123	2.050	1.843	1.569
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209	2.153	2.106	2.033	1.824	1.547
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194	2.138	2.091	2.017	1.808	1.527
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077	2.003	1.793	1.509
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993	1.917	1.700	1.389
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.959	1.910	1.834	1.608	1.254
∞	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.752	1.517	1.000

2.5% points for the F distribution

$\nu_1 =$	1	2	3	4	5	6	7	8	9	10	12	24	∞
ν_2													
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.6	963.3	968.6	976.7	997.3	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.46	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.12	13.90
4	12.22	10.65	9.979	9.605	9.364	9.197	9.074	8.980	8.905	8.844	8.751	8.511	8.257
5	10.01	8.434	7.764	7.388	7.146	6.978	6.853	6.757	6.681	6.619	6.525	6.278	6.015
6	8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.523	5.461	5.366	5.117	4.849
7	8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.823	4.761	4.666	4.415	4.142
8	7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.357	4.295	4.200	3.947	3.670
9	7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	4.026	3.964	3.868	3.614	3.333
10	6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.779	3.717	3.621	3.365	3.080
11	6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.588	3.526	3.430	3.173	2.883
12	6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.436	3.374	3.277	3.019	2.725
13	6.414	4.965	4.347	3.996	3.767	3.604	3.483	3.388	3.312	3.250	3.153	2.893	2.596
14	6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	3.209	3.147	3.050	2.789	2.487
15	6.200	4.765	4.153	3.804	3.576	3.415	3.293	3.199	3.123	3.060	2.963	2.701	2.395
16	6.115	4.687	4.077	3.729	3.502	3.341	3.219	3.125	3.049	2.986	2.889	2.625	2.316
17	6.042	4.619	4.011	3.665	3.438	3.277	3.156	3.061	2.985	2.922	2.825	2.560	2.248
18	5.978	4.560	3.954	3.608	3.382	3.221	3.100	3.005	2.929	2.866	2.769	2.503	2.187
19	5.922	4.508	3.903	3.559	3.333	3.172	3.051	2.956	2.880	2.817	2.720	2.452	2.133
20	5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.837	2.774	2.676	2.408	2.085
21	5.827	4.420	3.819	3.475	3.250	3.090	2.969	2.874	2.798	2.735	2.637	2.368	2.042
22	5.786	4.383	3.783	3.440	3.215	3.055	2.934	2.839	2.763	2.700	2.602	2.332	2.003
23	5.750	4.349	3.750	3.408	3.183	3.023	2.902	2.808	2.731	2.668	2.570	2.299	1.968
24	5.717	4.319	3.721	3.379	3.155	2.995	2.874	2.779	2.703	2.640	2.541	2.269	1.935
25	5.686	4.291	3.694	3.353	3.129	2.969	2.848	2.753	2.677	2.613	2.515	2.242	1.906
26	5.659	4.265	3.670	3.329	3.105	2.945	2.824	2.729	2.653	2.590	2.491	2.217	1.878
27	5.633	4.242	3.647	3.307	3.083	2.923	2.802	2.707	2.631	2.568	2.469	2.195	1.853
28	5.610	4.221	3.626	3.286	3.063	2.903	2.782	2.687	2.611	2.547	2.448	2.174	1.829
29	5.588	4.201	3.607	3.267	3.044	2.884	2.763	2.669	2.592	2.529	2.430	2.154	1.807
30	5.568	4.182	3.589	3.250	3.026	2.867	2.746	2.651	2.575	2.511	2.412	2.136	1.787
32	5.531	4.149	3.557	3.218	2.995	2.836	2.715	2.620	2.543	2.480	2.381	2.103	1.750
34	5.499	4.120	3.529	3.191	2.968	2.808	2.688	2.593	2.516	2.453	2.353	2.075	1.717
36	5.471	4.094	3.505	3.167	2.944	2.785	2.664	2.569	2.492	2.429	2.329	2.049	1.687
38	5.446	4.071	3.483	3.145	2.923	2.763	2.643	2.548	2.471	2.407	2.307	2.027	1.661
40	5.424	4.051	3.463	3.126	2.904	2.744	2.624	2.529	2.452	2.388	2.288	2.007	1.637
60	5.286	3.925	3.343	3.008	2.786	2.627	2.507	2.412	2.334	2.270	2.169	1.882	1.482
120	5.152	3.805	3.227	2.894	2.674	2.515	2.395	2.299	2.222	2.157	2.055	1.760	1.311
∞	5.024	3.689	3.116	2.786	2.567	2.408	2.288	2.192	2.114	2.048	1.945	1.640	1.000

1% points for the F distribution

$v_1 =$	1	2	3	4	5	6	7	8	9	10	12	24	∞
v_2													
1	4052	4999	5403	5625	5764	5859	5928	5981	6022	6056	6107	6234	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.38	99.39	99.40	99.42	99.46	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.60	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	13.93	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.888	9.466	9.021
6	13.75	10.92	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874	7.718	7.313	6.880
7	12.25	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620	6.469	6.074	5.650
8	11.26	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.667	5.279	4.859
9	10.56	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.111	4.729	4.311
10	10.04	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849	4.706	4.327	3.909
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.397	4.021	3.603
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296	4.155	3.780	3.361
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100	3.960	3.587	3.165
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939	3.800	3.427	3.004
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.666	3.294	2.869
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.890	3.780	3.691	3.553	3.181	2.753
17	8.400	6.112	5.185	4.669	4.336	4.101	3.927	3.791	3.682	3.593	3.455	3.083	2.653
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.371	2.999	2.566
19	8.185	5.926	5.010	4.500	4.171	3.939	3.765	3.631	3.523	3.434	3.297	2.925	2.489
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.231	2.859	2.421
21	8.017	5.780	4.874	4.369	4.042	3.812	3.640	3.506	3.398	3.310	3.173	2.801	2.360
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	3.121	2.749	2.306
23	7.881	5.664	4.765	4.264	3.939	3.710	3.539	3.406	3.299	3.211	3.074	2.702	2.256
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	3.032	2.659	2.211
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	2.993	2.620	2.170
26	7.721	5.526	4.637	4.140	3.818	3.591	3.421	3.288	3.182	3.094	2.958	2.585	2.132
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062	2.926	2.552	2.097
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.120	3.032	2.896	2.522	2.064
29	7.598	5.420	4.538	4.045	3.725	3.499	3.330	3.198	3.092	3.005	2.868	2.495	2.034
30	7.562	5.390	4.510	4.018	3.699	3.473	3.305	3.173	3.067	2.979	2.843	2.469	2.006
32	7.499	5.336	4.459	3.969	3.652	3.427	3.258	3.127	3.021	2.934	2.798	2.423	1.956
34	7.444	5.289	4.416	3.927	3.611	3.386	3.218	3.087	2.981	2.894	2.758	2.383	1.911
36	7.396	5.248	4.377	3.890	3.574	3.351	3.183	3.052	2.946	2.859	2.723	2.347	1.872
38	7.353	5.211	4.343	3.858	3.542	3.319	3.152	3.021	2.915	2.828	2.692	2.316	1.837
40	7.314	5.178	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.665	2.288	1.805
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.718	2.632	2.496	2.115	1.601
120	6.851	4.787	3.949	3.480	3.174	2.956	2.792	2.663	2.559	2.472	2.336	1.950	1.381
∞	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321	2.185	1.791	1.000

11.7 Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	= x
μ															μ
0.05	0.95123	0.99879	0.99998												0.05
0.10	0.90484	0.99532	0.99985												0.10
0.15	0.86071	0.98981	0.99950	0.99998											0.15
0.20	0.81873	0.98248	0.99885	0.99994											0.20
0.30	0.74082	0.96306	0.99640	0.99973	0.99998					All 1.00000					0.30
0.40	0.67032	0.93845	0.99207	0.99922	0.99994										0.40
0.50	0.60653	0.90980	0.98561	0.99825	0.99983	0.99999									0.50
0.60	0.54881	0.87810	0.97688	0.99664	0.99961	0.99996									0.60
0.70	0.49659	0.84420	0.96586	0.99425	0.99921	0.99991	0.99999								0.70
0.80	0.44933	0.80879	0.95258	0.99092	0.99859	0.99982	0.99998								0.80
0.90	0.40657	0.77248	0.93714	0.98654	0.99766	0.99966	0.99996								0.90
1.00	0.36788	0.73576	0.91970	0.98101	0.99634	0.99941	0.99992	0.99999							1.00
1.10	0.33287	0.69903	0.90042	0.97426	0.99456	0.99903	0.99985	0.99998							1.10
1.20	0.30119	0.66263	0.87949	0.96623	0.99225	0.99850	0.99975	0.99996							1.20
1.30	0.27253	0.62682	0.85711	0.95690	0.98934	0.99777	0.99960	0.99994	0.99999						1.30
1.40	0.24660	0.59183	0.83350	0.94627	0.98575	0.99680	0.99938	0.99989	0.99998						1.40
1.50	0.22313	0.55783	0.80885	0.93436	0.98142	0.99554	0.99907	0.99983	0.99997						1.50
1.60	0.20190	0.52493	0.78336	0.92119	0.97632	0.99396	0.99866	0.99974	0.99995	0.99999					1.60
1.70	0.18268	0.49325	0.75722	0.90681	0.97039	0.99200	0.99812	0.99961	0.99993	0.99999					1.70
1.80	0.16530	0.46284	0.73062	0.89129	0.96359	0.98962	0.99743	0.99944	0.99989	0.99998					1.80
1.90	0.14957	0.43375	0.70372	0.87470	0.95592	0.98678	0.99655	0.99921	0.99984	0.99997	0.99999				1.90
2.00	0.13534	0.40601	0.67668	0.85712	0.94735	0.98344	0.99547	0.99890	0.99976	0.99995	0.99999				2.00
2.10	0.12246	0.37961	0.64963	0.83864	0.93787	0.97955	0.99414	0.99851	0.99966	0.99993	0.99999				2.10
2.20	0.11080	0.35457	0.62271	0.81935	0.92750	0.97509	0.99254	0.99802	0.99953	0.99990	0.99998				2.20
2.30	0.10026	0.33085	0.59604	0.79935	0.91625	0.97002	0.99064	0.99741	0.99936	0.99986	0.99997	0.99999			2.30
2.40	0.09072	0.30844	0.56971	0.77872	0.90413	0.96433	0.98841	0.99666	0.99914	0.99980	0.99996	0.99999			2.40
2.50	0.08208	0.28730	0.54381	0.75758	0.89118	0.95798	0.98581	0.99575	0.99886	0.99972	0.99994	0.99999			2.50
2.60	0.07427	0.26738	0.51843	0.73600	0.87742	0.95096	0.98283	0.99467	0.99851	0.99962	0.99991	0.99998			2.60
2.70	0.06721	0.24866	0.49362	0.71409	0.86291	0.94327	0.97943	0.99338	0.99809	0.99950	0.99988	0.99997	0.99999		2.70

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{i=0}^{x} \frac{e^{-\mu}\mu^i}{i!}$$

$\mu\backslash x$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	$\mu\backslash x$
2.80	0.06081	0.23108	0.46945	0.69194	0.84768	0.93489	0.97559	0.99187	0.99757	0.99934	0.99984	0.99996	0.99999	1.00000	2.80
2.90	0.05502	0.21459	0.44596	0.66962	0.83178	0.92583	0.97128	0.99012	0.99694	0.99914	0.99978	0.99995	0.99999	1.00000	2.90
3.00	0.04979	0.19915	0.42319	0.64723	0.81526	0.91608	0.96649	0.98810	0.99620	0.99890	0.99971	0.99993	0.99998	1.00000	3.00
3.10	0.04505	0.18470	0.40116	0.62484	0.79819	0.90567	0.96120	0.98579	0.99532	0.99860	0.99962	0.99990	0.99998	1.00000	3.10
3.20	0.04076	0.17120	0.37990	0.60252	0.78061	0.89459	0.95538	0.98317	0.99429	0.99824	0.99950	0.99987	0.99997	0.99999	3.20
3.30	0.03688	0.15860	0.35943	0.58034	0.76259	0.88288	0.94903	0.98022	0.99309	0.99781	0.99936	0.99983	0.99996	0.99999	3.30
3.40	0.03337	0.14684	0.33974	0.55836	0.74418	0.87054	0.94215	0.97693	0.99171	0.99729	0.99919	0.99978	0.99994	0.99999	3.40
3.50	0.03020	0.13589	0.32085	0.53663	0.72544	0.85761	0.93471	0.97326	0.99013	0.99669	0.99898	0.99971	0.99992	0.99998	3.50
3.60	0.02732	0.12569	0.30275	0.51522	0.70644	0.84412	0.92673	0.96921	0.98833	0.99598	0.99873	0.99963	0.99990	0.99997	3.60
3.70	0.02472	0.11620	0.28543	0.49415	0.68722	0.83009	0.91819	0.96476	0.98630	0.99515	0.99843	0.99953	0.99987	0.99997	3.70
3.80	0.02237	0.10738	0.26890	0.47348	0.66784	0.81556	0.90911	0.95989	0.98402	0.99420	0.99807	0.99941	0.99983	0.99996	3.80
3.90	0.02024	0.09919	0.25313	0.45325	0.64837	0.80056	0.89948	0.95460	0.98147	0.99311	0.99765	0.99926	0.99978	0.99994	3.90
4.00	0.01832	0.09158	0.23810	0.43347	0.62884	0.78513	0.88933	0.94887	0.97864	0.99187	0.99716	0.99908	0.99973	0.99992	4.00
4.10	0.01657	0.08452	0.22381	0.41418	0.60931	0.76931	0.87865	0.94269	0.97551	0.99046	0.99659	0.99887	0.99966	0.99990	4.10
4.20	0.01500	0.07798	0.21024	0.39540	0.58983	0.75314	0.86746	0.93606	0.97207	0.98887	0.99593	0.99863	0.99957	0.99987	4.20
4.30	0.01357	0.07191	0.19735	0.37715	0.57044	0.73666	0.85579	0.92897	0.96830	0.98709	0.99518	0.99833	0.99947	0.99984	4.30
4.40	0.01228	0.06630	0.18514	0.35945	0.55118	0.71991	0.84365	0.92142	0.96420	0.98511	0.99431	0.99799	0.99934	0.99980	4.40
4.50	0.01111	0.06110	0.17358	0.34230	0.53210	0.70293	0.83105	0.91341	0.95974	0.98291	0.99333	0.99760	0.99919	0.99975	4.50
4.60	0.01005	0.05629	0.16264	0.32571	0.51323	0.68576	0.81803	0.90495	0.95493	0.98047	0.99222	0.99714	0.99902	0.99969	4.60
4.70	0.00910	0.05184	0.15230	0.30968	0.49461	0.66844	0.80461	0.89603	0.94974	0.97779	0.99098	0.99661	0.99882	0.99961	4.70
4.80	0.00823	0.04773	0.14254	0.29423	0.47626	0.65101	0.79080	0.88667	0.94418	0.97486	0.98958	0.99601	0.99858	0.99953	4.80
4.90	0.00745	0.04393	0.13333	0.27934	0.45821	0.63350	0.77665	0.87686	0.93824	0.97166	0.98803	0.99532	0.99830	0.99942	4.90
5.00	0.00674	0.04043	0.12465	0.26503	0.44049	0.61596	0.76218	0.86663	0.93191	0.96817	0.98630	0.99455	0.99798	0.99930	5.00
5.10	0.00610	0.03719	0.11648	0.25127	0.42313	0.59842	0.74742	0.85598	0.92518	0.96440	0.98440	0.99367	0.99761	0.99916	5.10
5.20	0.00552	0.03420	0.10879	0.23807	0.40613	0.58091	0.73239	0.84492	0.91806	0.96033	0.98230	0.99269	0.99719	0.99899	5.20

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	0	1	2	3	4	5	6	7	8	9	10	11	12	13	= x μ
5.30	0.00499	0.03145	0.10155	0.22541	0.38952	0.56347	0.71713	0.83348	0.91055	0.95594	0.98000	0.99159	0.99671	0.99880	5.30
5.40	0.00452	0.02891	0.09476	0.21329	0.37331	0.54613	0.70167	0.82166	0.90265	0.95125	0.97749	0.99037	0.99617	0.99857	5.40
5.50	0.00409	0.02656	0.08838	0.20170	0.35752	0.52892	0.68604	0.80949	0.89436	0.94622	0.97475	0.98901	0.99555	0.99831	5.50
5.60	0.00370	0.02441	0.08239	0.19062	0.34215	0.51186	0.67026	0.79698	0.88568	0.94087	0.97178	0.98751	0.99486	0.99802	5.60
5.70	0.00335	0.02242	0.07677	0.18005	0.32721	0.49498	0.65437	0.78415	0.87662	0.93518	0.96856	0.98586	0.99408	0.99768	5.70
5.80	0.00303	0.02059	0.07151	0.16996	0.31272	0.47831	0.63839	0.77103	0.86719	0.92916	0.96510	0.98405	0.99321	0.99730	5.80
5.90	0.00274	0.01890	0.06658	0.16035	0.29866	0.46187	0.62236	0.75763	0.85739	0.92279	0.96137	0.98207	0.99224	0.99686	5.90
6.00	0.00248	0.01735	0.06197	0.15120	0.28506	0.44568	0.60630	0.74398	0.84724	0.91608	0.95738	0.97991	0.99117	0.99637	6.00
6.10	0.00224	0.01592	0.05765	0.14250	0.27189	0.42975	0.59024	0.73010	0.83674	0.90902	0.95311	0.97756	0.98999	0.99582	6.10
6.20	0.00203	0.01461	0.05362	0.13423	0.25918	0.41411	0.57421	0.71602	0.82591	0.90162	0.94856	0.97502	0.98868	0.99520	6.20
6.30	0.00184	0.01341	0.04985	0.12637	0.24690	0.39877	0.55823	0.70175	0.81477	0.89388	0.94372	0.97227	0.98725	0.99451	6.30
6.40	0.00166	0.01230	0.04632	0.11892	0.23507	0.38374	0.54233	0.68732	0.80331	0.88580	0.93859	0.96930	0.98568	0.99375	6.40
6.50	0.00150	0.01128	0.04304	0.11185	0.22367	0.36904	0.52652	0.67276	0.79157	0.87738	0.93316	0.96612	0.98397	0.99290	6.50
6.60	0.00136	0.01034	0.03997	0.10515	0.21270	0.35467	0.51084	0.65808	0.77956	0.86864	0.92743	0.96271	0.98211	0.99196	6.60
6.70	0.00123	0.00948	0.03711	0.09881	0.20216	0.34065	0.49530	0.64332	0.76728	0.85957	0.92140	0.95906	0.98009	0.99093	6.70
6.80	0.00111	0.00869	0.03444	0.09281	0.19203	0.32698	0.47992	0.62849	0.75477	0.85018	0.91507	0.95517	0.97790	0.98979	6.80
6.90	0.00101	0.00796	0.03195	0.08713	0.18231	0.31366	0.46472	0.61361	0.74203	0.84049	0.90843	0.95104	0.97554	0.98855	6.90
7.00	0.00091	0.00730	0.02964	0.08177	0.17299	0.30071	0.44971	0.59871	0.72909	0.83050	0.90148	0.94665	0.97300	0.98719	7.00
7.25	0.00071	0.00586	0.02452	0.06963	0.15138	0.26992	0.41316	0.56152	0.69596	0.80427	0.88279	0.93454	0.96581	0.98324	7.25
7.50	0.00055	0.00470	0.02026	0.05915	0.13206	0.24144	0.37815	0.52464	0.66197	0.77641	0.86224	0.92076	0.95733	0.97844	7.50
7.75	0.00043	0.00377	0.01670	0.05012	0.11487	0.21522	0.34485	0.48837	0.62740	0.74712	0.83990	0.90527	0.94749	0.97266	7.75
8.00	0.00034	0.00302	0.01375	0.04238	0.09963	0.19124	0.31337	0.45296	0.59255	0.71662	0.81589	0.88808	0.93620	0.96582	8.00
8.25	0.00026	0.00242	0.01131	0.03576	0.08619	0.16939	0.28380	0.41864	0.55770	0.68516	0.79032	0.86919	0.92341	0.95782	8.25
8.50	0.00020	0.00193	0.00928	0.03011	0.07436	0.14960	0.25618	0.38560	0.52311	0.65297	0.76336	0.84866	0.90908	0.94859	8.50
8.75	0.00016	0.00154	0.00761	0.02530	0.06401	0.13174	0.23051	0.35398	0.48902	0.62031	0.73519	0.82657	0.89320	0.93805	8.75

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	0	1	2	3	4	5	6	7	8	9	10	11	12	13	= x μ
9.00	0.00012	0.00123	0.00623	0.02123	0.05496	0.11569	0.20678	0.32390	0.45565	0.58741	0.70599	0.80301	0.87577	0.92615	9.00
9.25	0.00010	0.00099	0.00510	0.01777	0.04709	0.10133	0.18495	0.29544	0.42320	0.55451	0.67597	0.77810	0.85683	0.91285	9.25
9.50	0.00007	0.00079	0.00416	0.01486	0.04026	0.08853	0.16495	0.26866	0.39182	0.52183	0.64533	0.75199	0.83643	0.89814	9.50
9.75	0.00006	0.00063	0.00340	0.01240	0.03435	0.07716	0.14671	0.24359	0.36166	0.48957	0.61428	0.72483	0.81464	0.88200	9.75
10.00	0.00005	0.00050	0.00277	0.01034	0.02925	0.06709	0.13014	0.22022	0.33282	0.45793	0.58304	0.69678	0.79156	0.86446	10.00
10.25	0.00004	0.00040	0.00226	0.00860	0.02486	0.05820	0.11515	0.19854	0.30538	0.42707	0.55179	0.66802	0.76729	0.84556	10.25
10.50	0.00003	0.00032	0.00183	0.00715	0.02109	0.05038	0.10163	0.17851	0.27941	0.39713	0.52074	0.63873	0.74196	0.82535	10.50
10.75	0.00002	0.00025	0.00149	0.00593	0.01786	0.04352	0.08949	0.16008	0.25494	0.36825	0.49005	0.60908	0.71572	0.80390	10.75
11.00	0.00002	0.00020	0.00121	0.00492	0.01510	0.03752	0.07861	0.14319	0.23199	0.34051	0.45989	0.57927	0.68870	0.78129	11.00
11.25	0.00001	0.00016	0.00098	0.00407	0.01275	0.03228	0.06891	0.12777	0.21054	0.31401	0.43041	0.54945	0.66105	0.75763	11.25
11.50	0.00001	0.00013	0.00080	0.00336	0.01075	0.02773	0.06027	0.11373	0.19059	0.28879	0.40173	0.51980	0.63295	0.73304	11.50
11.75	0.00001	0.00010	0.00065	0.00278	0.00904	0.02377	0.05260	0.10101	0.17210	0.26492	0.37397	0.49047	0.60453	0.70763	11.75
12.00	0.00001	0.00008	0.00052	0.00229	0.00760	0.02034	0.04582	0.08950	0.15503	0.24239	0.34723	0.46160	0.57597	0.68154	12.00
12.25	0.00000	0.00006	0.00042	0.00189	0.00638	0.01738	0.03984	0.07914	0.13932	0.22123	0.32158	0.43332	0.54740	0.65489	12.25
12.50	0.00000	0.00005	0.00034	0.00155	0.00535	0.01482	0.03457	0.06983	0.12492	0.20143	0.29707	0.40576	0.51898	0.62784	12.50
12.75	0.00000	0.00004	0.00028	0.00128	0.00447	0.01262	0.02994	0.06148	0.11175	0.18297	0.27377	0.37901	0.49083	0.60051	12.75
13.00	0.00000	0.00003	0.00022	0.00105	0.00374	0.01073	0.02589	0.05403	0.09976	0.16581	0.25168	0.35316	0.46310	0.57304	13.00
13.25	0.00000	0.00003	0.00018	0.00086	0.00312	0.00911	0.02234	0.04739	0.08886	0.14993	0.23083	0.32829	0.43590	0.54558	13.25
13.50	0.00000	0.00002	0.00014	0.00071	0.00260	0.00773	0.01925	0.04148	0.07900	0.13526	0.21123	0.30445	0.40933	0.51825	13.50
13.75	0.00000	0.00002	0.00012	0.00058	0.00217	0.00654	0.01656	0.03625	0.07008	0.12177	0.19285	0.28169	0.38349	0.49116	13.75
14.00	0.00000	0.00001	0.00009	0.00047	0.00181	0.00553	0.01423	0.03162	0.06206	0.10940	0.17568	0.26004	0.35846	0.46445	14.00
14.25	0.00000	0.00001	0.00008	0.00039	0.00150	0.00467	0.01220	0.02753	0.05484	0.09808	0.15970	0.23952	0.33430	0.43820	14.25
14.50	0.00000	0.00001	0.00006	0.00032	0.00125	0.00394	0.01045	0.02394	0.04838	0.08776	0.14486	0.22013	0.31108	0.41253	14.50
14.75	0.00000	0.00001	0.00005	0.00026	0.00103	0.00332	0.00894	0.02077	0.04260	0.07837	0.13113	0.20188	0.28884	0.38751	14.75
15.00	0.00000	0.00000	0.00004	0.00021	0.00086	0.00279	0.00763	0.01800	0.03745	0.06985	0.11846	0.18475	0.26761	0.36322	15.00

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	0	1	2	3	4	5	6	7	8	9	10	11	12	13	= x μ
15.50			0.00003	0.00014	0.00059	0.00197	0.00554	0.01346	0.02879	0.05519	0.09612	0.15378	0.22827	0.31708	15.50
16.00			0.00002	0.00009	0.00040	0.00138	0.00401	0.01000	0.02199	0.04330	0.07740	0.12699	0.19312	0.27451	16.00
16.50			0.00001	0.00006	0.00027	0.00097	0.00288	0.00739	0.01669	0.03374	0.06187	0.10407	0.16210	0.23574	16.50
17.00			0.00001	0.00004	0.00018	0.00067	0.00206	0.00543	0.01260	0.02612	0.04912	0.08467	0.13502	0.20087	17.00
17.50			0.00000	0.00003	0.00012	0.00047	0.00147	0.00397	0.00945	0.02010	0.03875	0.06840	0.11165	0.16987	17.50
18.00				0.00002	0.00008	0.00032	0.00104	0.00289	0.00706	0.01538	0.03037	0.05489	0.09167	0.14260	18.00
18.50				0.00001	0.00006	0.00022	0.00074	0.00210	0.00524	0.01170	0.02366	0.04376	0.07475	0.11886	18.50
19.00				0.00001	0.00004	0.00015	0.00052	0.00151	0.00387	0.00886	0.01832	0.03467	0.06056	0.09840	19.00
19.50					0.00003	0.00011	0.00036	0.00109	0.00285	0.00667	0.01411	0.02731	0.04875	0.08092	19.50
20.00					0.00002	0.00007	0.00026	0.00078	0.00209	0.00500	0.01081	0.02139	0.03901	0.06613	20.00
20.50					0.00001	0.00005	0.00018	0.00056	0.00152	0.00373	0.00824	0.01666	0.03103	0.05371	20.50
21.00		All 0.00000			0.00001	0.00003	0.00012	0.00039	0.00111	0.00277	0.00625	0.01290	0.02455	0.04336	21.00
21.50						0.00002	0.00009	0.00028	0.00080	0.00204	0.00472	0.00995	0.01931	0.03481	21.50
22.00						0.00002	0.00006	0.00020	0.00058	0.00150	0.00355	0.00763	0.01512	0.02778	22.00
22.50						0.00001	0.00004	0.00014	0.00041	0.00110	0.00265	0.00583	0.01177	0.02206	22.50
23.00						0.00001	0.00003	0.00010	0.00030	0.00081	0.00198	0.00443	0.00912	0.01743	23.00
23.50							0.00002	0.00007	0.00021	0.00059	0.00147	0.00335	0.00704	0.01370	23.50
24.00							0.00001	0.00005	0.00015	0.00043	0.00108	0.00252	0.00540	0.01072	24.00
24.50							0.00001	0.00003	0.00011	0.00031	0.00080	0.00189	0.00413	0.00834	24.50
25.00							0.00001	0.00002	0.00008	0.00022	0.00059	0.00142	0.00314	0.00647	25.00

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	14	15	16	17	18	19	20	21	22	23	24	25	= x μ
3.30													3.30
3.40													3.40
3.50													3.50
3.60	0.99999												3.60
3.70	0.99999												3.70
3.80	0.99999						All 1.0000						3.80
3.90	0.99999												3.90
4.00	0.99998												4.00
4.10	0.99997	0.99999											4.10
4.20	0.99997	0.99999											4.20
4.30	0.99996	0.99999											4.30
4.40	0.99994	0.99998											4.40
4.50	0.99993	0.99998	0.99999										4.50
4.60	0.99991	0.99997	0.99999										4.60
4.70	0.99988	0.99997	0.99999										4.70
4.80	0.99985	0.99996	0.99999										4.80
4.90	0.99982	0.99995	0.99998										4.90
5.00	0.99977	0.99993	0.99998	0.99999									5.00
5.10	0.99972	0.99991	0.99997	0.99999									5.10
5.20	0.99966	0.99989	0.99997	0.99999									5.20
5.30	0.99959	0.99987	0.99996	0.99999									5.30
5.40	0.99950	0.99984	0.99995	0.99999									5.40
5.50	0.99940	0.99980	0.99994	0.99998	0.99999								5.50
5.60	0.99928	0.99976	0.99992	0.99998	0.99999								5.60
5.70	0.99915	0.99970	0.99990	0.99997	0.99999								5.70
5.80	0.99899	0.99964	0.99988	0.99996	0.99999								5.80
5.90	0.99881	0.99957	0.99986	0.99995	0.99999								5.90
6.00	0.99860	0.99949	0.99983	0.99994	0.99998	0.99999							6.00
6.10	0.99836	0.99939	0.99979	0.99993	0.99998	0.99999							6.10
6.20	0.99809	0.99928	0.99975	0.99991	0.99997	0.99999							6.20

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	14	15	16	17	18	19	20	21	22	23	24	25	= x μ
6.30	0.99778	0.99916	0.99970	0.99990	0.99997	0.99999							6.30
6.40	0.99744	0.99901	0.99964	0.99987	0.99996	0.99999							6.40
6.50	0.99704	0.99884	0.99957	0.99985	0.99995	0.99998							6.50
6.60	0.99661	0.99865	0.99949	0.99982	0.99994	0.99998	0.99999						6.60
6.70	0.99611	0.99843	0.99940	0.99978	0.99993	0.99998	0.99999				All 1.00000		6.70
6.80	0.99557	0.99818	0.99930	0.99974	0.99991	0.99997	0.99999						6.80
6.90	0.99496	0.99791	0.99918	0.99969	0.99989	0.99996	0.99999						6.90
7.00	0.99428	0.99759	0.99904	0.99964	0.99987	0.99996	0.99999						7.00
7.25	0.99227	0.99664	0.99862	0.99946	0.99980	0.99993	0.99998	0.99999					7.25
7.50	0.98974	0.99539	0.99804	0.99921	0.99970	0.99989	0.99996	0.99999					7.50
7.75	0.98659	0.99379	0.99728	0.99887	0.99955	0.99983	0.99994	0.99998	0.99999				7.75
8.00	0.98274	0.99177	0.99628	0.99841	0.99935	0.99975	0.99991	0.99997	0.99999				8.00
8.25	0.97810	0.98925	0.99500	0.99779	0.99907	0.99963	0.99986	0.99995	0.99998	0.99999			8.25
8.50	0.97257	0.98617	0.99339	0.99700	0.99870	0.99947	0.99979	0.99992	0.99997	0.99999			8.50
8.75	0.96608	0.98243	0.99137	0.99597	0.99821	0.99924	0.99969	0.99988	0.99996	0.99998	0.99999		8.75
9.00	0.95853	0.97796	0.98889	0.99468	0.99757	0.99894	0.99956	0.99983	0.99993	0.99998	0.99999		9.00
9.25	0.94986	0.97269	0.98588	0.99306	0.99675	0.99855	0.99938	0.99975	0.99990	0.99996	0.99999		9.25
9.50	0.94001	0.96653	0.98227	0.99107	0.99572	0.99804	0.99914	0.99964	0.99985	0.99994	0.99998	0.99999	9.50
9.75	0.92891	0.95941	0.97799	0.98864	0.99442	0.99738	0.99882	0.99949	0.99979	0.99992	0.99997	0.99999	9.75
10.00	0.91654	0.95126	0.97296	0.98572	0.99281	0.99655	0.99841	0.99930	0.99970	0.99988	0.99995	0.99998	10.00
10.25	0.90287	0.94203	0.96712	0.98224	0.99085	0.99550	0.99788	0.99905	0.99959	0.99983	0.99993	0.99997	10.25
10.50	0.88789	0.93167	0.96039	0.97814	0.98849	0.99421	0.99721	0.99871	0.99943	0.99976	0.99990	0.99996	10.50
10.75	0.87160	0.92013	0.95273	0.97335	0.98566	0.99263	0.99637	0.99829	0.99922	0.99966	0.99986	0.99994	10.75
11.00	0.85404	0.90740	0.94408	0.96781	0.98231	0.99071	0.99533	0.99775	0.99896	0.99954	0.99980	0.99992	11.00
11.25	0.83524	0.89345	0.93438	0.96146	0.97839	0.98841	0.99405	0.99707	0.99861	0.99937	0.99972	0.99988	11.25
11.50	0.81526	0.87829	0.92360	0.95425	0.97383	0.98568	0.99250	0.99623	0.99818	0.99915	0.99962	0.99984	11.50
11.75	0.79416	0.86194	0.91172	0.94612	0.96858	0.98247	0.99063	0.99519	0.99763	0.99888	0.99949	0.99977	11.75
12.00	0.77202	0.84442	0.89871	0.93703	0.96258	0.97872	0.98840	0.99393	0.99695	0.99853	0.99931	0.99969	12.00
12.25	0.74895	0.82576	0.88457	0.92695	0.95579	0.97438	0.98577	0.99242	0.99612	0.99809	0.99909	0.99958	12.25
12.50	0.72503	0.80603	0.86931	0.91584	0.94815	0.96941	0.98269	0.99060	0.99509	0.99754	0.99881	0.99944	12.50

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	14	15	16	17	18	19	20	21	22	23	24	25	= x μ
12.75	0.70039	0.78529	0.85294	0.90368	0.93962	0.96374	0.97911	0.98845	0.99386	0.99686	0.99845	0.99926	12.75
13.00	0.67513	0.76361	0.83549	0.89046	0.93017	0.95733	0.97499	0.98592	0.99238	0.99603	0.99801	0.99903	13.00
13.25	0.64938	0.74108	0.81701	0.87619	0.91976	0.95014	0.97027	0.98297	0.99062	0.99502	0.99746	0.99875	13.25
13.50	0.62327	0.71779	0.79755	0.86088	0.90838	0.94213	0.96491	0.97955	0.98854	0.99382	0.99678	0.99838	13.50
13.75	0.59691	0.69385	0.77716	0.84454	0.89601	0.93326	0.95886	0.97563	0.98611	0.99238	0.99597	0.99794	13.75
14.00	0.57044	0.66936	0.75592	0.82720	0.88264	0.92350	0.95209	0.97116	0.98329	0.99067	0.99498	0.99739	14.00
14.25	0.54396	0.64443	0.73391	0.80891	0.86829	0.91282	0.94455	0.96608	0.98003	0.98867	0.99380	0.99673	14.25
14.50	0.51760	0.61916	0.71121	0.78972	0.85296	0.90122	0.93622	0.96038	0.97630	0.98634	0.99241	0.99592	14.50
14.75	0.49146	0.59368	0.68791	0.76968	0.83668	0.88869	0.92705	0.95399	0.97206	0.98364	0.99076	0.99496	14.75
15.00	0.46565	0.56809	0.66412	0.74886	0.81947	0.87522	0.91703	0.94689	0.96726	0.98054	0.98884	0.99382	15.00
15.50	0.41541	0.51701	0.61544	0.70518	0.78246	0.84551	0.89437	0.93043	0.95584	0.97296	0.98402	0.99087	15.50
16.00	0.36753	0.46674	0.56596	0.65934	0.74235	0.81225	0.86817	0.91077	0.94176	0.96331	0.97768	0.98688	16.00
16.50	0.32254	0.41802	0.51648	0.61205	0.69965	0.77572	0.83848	0.88780	0.92478	0.95131	0.96955	0.98159	16.50
17.00	0.28083	0.37145	0.46774	0.56402	0.65496	0.73632	0.80548	0.86147	0.90473	0.93670	0.95935	0.97476	17.00
17.50	0.24264	0.32754	0.42040	0.51600	0.60893	0.69453	0.76943	0.83185	0.88150	0.91928	0.94682	0.96611	17.50
18.00	0.20808	0.28665	0.37505	0.46865	0.56224	0.65092	0.73072	0.79912	0.85509	0.89889	0.93174	0.95539	18.00
18.50	0.17714	0.24903	0.33214	0.42259	0.51555	0.60607	0.68979	0.76355	0.82558	0.87547	0.91392	0.94238	18.50
19.00	0.14975	0.21479	0.29203	0.37836	0.46948	0.56061	0.64717	0.72550	0.79314	0.84902	0.89325	0.92687	19.00
19.50	0.12573	0.18398	0.25497	0.33639	0.42461	0.51514	0.60342	0.68538	0.75804	0.81963	0.86968	0.90872	19.50
20.00	0.10486	0.15651	0.22107	0.29703	0.38142	0.47026	0.55909	0.64370	0.72061	0.78749	0.84323	0.88782	20.00
20.50	0.08690	0.13227	0.19040	0.26050	0.34034	0.42648	0.51477	0.60095	0.68127	0.75285	0.81399	0.86413	20.50
21.00	0.07157	0.11107	0.16292	0.22696	0.30168	0.38426	0.47097	0.55769	0.64046	0.71603	0.78216	0.83770	21.00
21.50	0.05860	0.09269	0.13852	0.19647	0.26568	0.34401	0.42821	0.51442	0.59866	0.67741	0.74796	0.80863	21.50
22.00	0.04769	0.07689	0.11704	0.16900	0.23250	0.30603	0.38691	0.47164	0.55638	0.63742	0.71172	0.77710	22.00
22.50	0.03860	0.06341	0.09830	0.14447	0.20219	0.27054	0.34744	0.42983	0.51409	0.59652	0.67379	0.74334	22.50
23.00	0.03107	0.05200	0.08208	0.12277	0.17477	0.23771	0.31010	0.38938	0.47227	0.55515	0.63458	0.70766	23.00
23.50	0.02488	0.04241	0.06814	0.10372	0.15017	0.20761	0.27512	0.35065	0.43134	0.51378	0.59451	0.67039	23.50
24.00	0.01983	0.03440	0.05626	0.08713	0.12828	0.18026	0.24264	0.31393	0.39170	0.47285	0.55400	0.63191	24.00
24.50	0.01572	0.02776	0.04620	0.07278	0.10896	0.15561	0.21276	0.27943	0.35367	0.43276	0.51350	0.59262	24.50
25.00	0.01240	0.02229	0.03775	0.06048	0.09204	0.13357	0.18549	0.24730	0.31753	0.39388	0.47340	0.55292	25.00

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	26	27	28	29	30	31	32	33	34	35	36	37	= x μ
9.00													9.00
9.25													9.25
9.50													9.50
9.75													9.75
10.00	0.99999					All 1.00000							10.00
10.25	0.99999												10.25
10.50	0.99999	0.99999											10.50
10.75	0.99998	0.99999											10.75
11.00	0.99997	0.99999											11.00
11.25	0.99995	0.99998	0.99999										11.25
11.50	0.99993	0.99997	0.99999										11.50
11.75	0.99990	0.99996	0.99998	0.99999									11.75
12.00	0.99987	0.99994	0.99998	0.99999									12.00
12.25	0.99982	0.99992	0.99997	0.99999	0.99999								12.25
12.50	0.99975	0.99989	0.99995	0.99998	0.99999								12.50
12.75	0.99966	0.99985	0.99994	0.99997	0.99999								12.75
13.00	0.99955	0.99980	0.99991	0.99996	0.99998	0.99999							13.00
13.25	0.99940	0.99972	0.99988	0.99995	0.99998	0.99999							13.25
13.50	0.99922	0.99963	0.99983	0.99993	0.99997	0.99999	0.99999						13.50
13.75	0.99898	0.99951	0.99978	0.99990	0.99996	0.99998	0.99999						13.75
14.00	0.99869	0.99936	0.99970	0.99986	0.99994	0.99997	0.99999						14.00
14.25	0.99833	0.99918	0.99961	0.99982	0.99992	0.99996	0.99998	0.99999					14.25
14.50	0.99789	0.99894	0.99948	0.99976	0.99989	0.99995	0.99998	0.99999					14.50
14.75	0.99734	0.99865	0.99933	0.99968	0.99985	0.99993	0.99997	0.99999	0.99999				14.75
15.00	0.99669	0.99828	0.99914	0.99958	0.99980	0.99991	0.99996	0.99998	0.99999				15.00
15.50	0.99496	0.99731	0.99861	0.99930	0.99966	0.99984	0.99993	0.99997	0.99999	0.99999			15.50
16.00	0.99254	0.99589	0.99781	0.99887	0.99943	0.99972	0.99987	0.99994	0.99997	0.99999			16.00
16.50	0.98923	0.99390	0.99665	0.99822	0.99908	0.99954	0.99978	0.99989	0.99995	0.99998	0.99999		16.50
17.00	0.98483	0.99117	0.99502	0.99727	0.99855	0.99925	0.99963	0.99982	0.99991	0.99996	0.99998	0.99999	17.00
17.50	0.97908	0.98750	0.99275	0.99593	0.99778	0.99882	0.99939	0.99970	0.99985	0.99993	0.99997	0.99999	17.50

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	26	27	28	29	30	31	32	33	34	35	36	37	= x μ
18.00	0.97177	0.98268	0.98970	0.99406	0.99667	0.99819	0.99904	0.99951	0.99975	0.99988	0.99994	0.99997	18.00
18.50	0.96263	0.97650	0.98567	0.99152	0.99512	0.99728	0.99852	0.99922	0.99960	0.99980	0.99990	0.99995	18.50
19.00	0.95144	0.96873	0.98046	0.98815	0.99302	0.99600	0.99777	0.99879	0.99936	0.99967	0.99984	0.99992	19.00
19.50	0.93800	0.95914	0.97387	0.98377	0.99021	0.99425	0.99672	0.99818	0.99902	0.99948	0.99973	0.99987	19.50
20.00	0.92211	0.94752	0.96567	0.97818	0.98653	0.99191	0.99527	0.99731	0.99851	0.99920	0.99958	0.99978	20.00
20.50	0.90366	0.93368	0.95565	0.97119	0.98180	0.98882	0.99332	0.99611	0.99780	0.99878	0.99934	0.99966	20.50
21.00	0.88257	0.91746	0.94363	0.96258	0.97585	0.98483	0.99073	0.99448	0.99680	0.99819	0.99900	0.99946	21.00
21.50	0.85880	0.89875	0.92943	0.95217	0.96847	0.97978	0.98737	0.99232	0.99545	0.99737	0.99852	0.99919	21.50
22.00	0.83242	0.87750	0.91291	0.93978	0.95949	0.97347	0.98308	0.98949	0.99364	0.99624	0.99784	0.99879	22.00
22.50	0.80353	0.85368	0.89399	0.92526	0.94871	0.96573	0.97770	0.98586	0.99126	0.99473	0.99690	0.99822	22.50
23.00	0.77230	0.82737	0.87260	0.90848	0.93598	0.95639	0.97106	0.98128	0.98819	0.99274	0.99564	0.99745	23.00
23.50	0.73897	0.79866	0.84876	0.88936	0.92117	0.94527	0.96298	0.97559	0.98430	0.99015	0.99397	0.99640	23.50
24.00	0.70382	0.76774	0.82253	0.86788	0.90415	0.93224	0.95330	0.96862	0.97943	0.98684	0.99179	0.99499	24.00
24.50	0.66717	0.73483	0.79402	0.84403	0.88487	0.91715	0.94187	0.96021	0.97343	0.98269	0.98899	0.99316	24.50
25.00	0.62939	0.70019	0.76340	0.81790	0.86331	0.89993	0.92854	0.95022	0.96616	0.97754	0.98545	0.99079	25.00

Probabilities for the Poisson distribution

$$P(X \leq x) = \sum_{t=0}^{x} \frac{e^{-\mu}\mu^t}{t!}$$

x = μ	38	39	40	41	42	43	44	45	46	47	48	49	= x μ
15.50													15.50
16.00													16.00
16.50													16.50
17.00													17.00
17.50	0.99999					All1.00000							17.50
18.00	0.99999	0.99999											18.00
18.50	0.99998	0.99999											18.50
19.00	0.99996	0.99998	0.99999										19.00
19.50	0.99993	0.99997	0.99999	0.99999									19.50
20.00	0.99989	0.99995	0.99997	0.99999	0.99999								20.00
20.50	0.99982	0.99991	0.99996	0.99998	0.99999								20.50
21.00	0.99972	0.99986	0.99993	0.99996	0.99998	0.99999							21.00
21.50	0.99956	0.99977	0.99988	0.99994	0.99997	0.99999	0.99999						21.50
22.00	0.99933	0.99964	0.99981	0.99990	0.99995	0.99998	0.99999	0.99999					22.00
22.50	0.99900	0.99945	0.99971	0.99985	0.99992	0.99996	0.99998	0.99999					22.50
23.00	0.99854	0.99918	0.99955	0.99976	0.99988	0.99994	0.99997	0.99998	0.99999				23.00
23.50	0.99790	0.99880	0.99933	0.99963	0.99981	0.99990	0.99995	0.99997	0.99999	0.99999			23.50
24.00	0.99702	0.99827	0.99901	0.99945	0.99970	0.99984	0.99992	0.99996	0.99998	0.99999	0.99999		24.00
24.50	0.99585	0.99754	0.99857	0.99919	0.99955	0.99976	0.99987	0.99993	0.99997	0.99998	0.99999		24.50
25.00	0.99430	0.99656	0.99796	0.99882	0.99933	0.99963	0.99980	0.99989	0.99994	0.99997	0.99999	0.99999	25.00

11.8 Probabilities for the Binomial distribution

The function tabulated is $P(X \le x) = \sum_{t=0}^{x} \binom{n}{t} p^t q^{n-t}$

n	p = / x	0.01	0.05	0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9	0.95	0.99
2	0	0.9801	0.9025	0.8100	0.6400	0.5625	0.4900	0.3600	0.2500	0.1600	0.0900	0.0625	0.0400	0.0100	0.0025	0.0001
2	1	0.9999	0.9975	0.9900	0.9600	0.9375	0.9100	0.8400	0.7500	0.6400	0.5100	0.4375	0.3600	0.1900	0.0975	0.0199
3	0	0.9703	0.8574	0.7290	0.5120	0.4219	0.3430	0.2160	0.1250	0.0640	0.0270	0.0156	0.0080	0.0010	0.0001	0.0000
3	1	0.9997	0.9928	0.9720	0.8960	0.8438	0.7840	0.6480	0.5000	0.3520	0.2160	0.1563	0.1040	0.0280	0.0073	0.0003
3	2	1.0000	0.9999	0.9990	0.9920	0.9844	0.9730	0.9360	0.8750	0.7840	0.6570	0.5781	0.4880	0.2710	0.1426	0.0297
4	0	0.9606	0.8145	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0039	0.0016	0.0001	0.0000	0.0000
4	1	0.9994	0.9860	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0508	0.0272	0.0037	0.0005	0.0000
4	2	1.0000	0.9995	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.2617	0.1808	0.0523	0.0140	0.0006
4	3	1.0000	1.0000	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.6836	0.5904	0.3439	0.1855	0.0394
5	0	0.9510	0.7738	0.5905	0.3277	0.2373	0.1681	0.0778	0.0313	0.0102	0.0024	0.0010	0.0003	0.0000	0.0000	0.0000
5	1	0.9990	0.9774	0.9185	0.7373	0.6328	0.5282	0.3370	0.1875	0.0870	0.0308	0.0156	0.0067	0.0005	0.0000	0.0000
5	2	1.0000	0.9988	0.9914	0.9421	0.8965	0.8369	0.6826	0.5000	0.3174	0.1631	0.1035	0.0579	0.0086	0.0012	0.0000
5	3	1.0000	1.0000	0.9995	0.9933	0.9844	0.9692	0.9130	0.8125	0.6630	0.4718	0.3672	0.2627	0.0815	0.0226	0.0010
5	4	1.0000	1.0000	1.0000	0.9997	0.9990	0.9976	0.9898	0.9688	0.9222	0.8319	0.7627	0.6723	0.4095	0.2262	0.0490
6	0	0.9415	0.7351	0.5314	0.2621	0.1780	0.1176	0.0467	0.0156	0.0041	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
6	1	0.9985	0.9672	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.0410	0.0109	0.0046	0.0016	0.0001	0.0000	0.0000
6	2	1.0000	0.9978	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.0376	0.0170	0.0013	0.0001	0.0000
6	3	1.0000	0.9999	0.9987	0.9830	0.9624	0.9295	0.8208	0.6563	0.4557	0.2557	0.1694	0.0989	0.0159	0.0022	0.0000
6	4	1.0000	1.0000	0.9999	0.9984	0.9954	0.9891	0.9590	0.8906	0.7667	0.5798	0.4661	0.3446	0.1143	0.0328	0.0015
6	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.8220	0.7379	0.4686	0.2649	0.0585
7	0	0.9321	0.6983	0.4783	0.2097	0.1335	0.0824	0.0280	0.0078	0.0016	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
7	1	0.9980	0.9556	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0013	0.0004	0.0000	0.0000	0.0000
7	2	1.0000	0.9962	0.9743	0.8520	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0129	0.0047	0.0002	0.0000	0.0000
7	3	1.0000	0.9998	0.9973	0.9667	0.9294	0.8740	0.7102	0.5000	0.2898	0.1260	0.0706	0.0333	0.0027	0.0002	0.0000
7	4	1.0000	1.0000	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.2436	0.1480	0.0257	0.0038	0.0000
7	5	1.0000	1.0000	1.0000	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.5551	0.4233	0.1497	0.0444	0.0020
7	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9984	0.9922	0.9720	0.9176	0.8665	0.7903	0.5217	0.3017	0.0679

Probabilities for the Binomial distribution

The function tabulated is $P(X \leq x) = \sum_{t=0}^{x} \binom{n}{t} p^t q^{n-t}$

n	p = x	0.01	0.05	0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9	0.95	0.99
8	0	0.9227	0.6634	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
8	1	0.9973	0.9428	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000
8	2	0.9999	0.9942	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0042	0.0012	0.0000	0.0000	0.0000
8	3	1.0000	0.9996	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0273	0.0104	0.0004	0.0000	0.0000
8	4	1.0000	1.0000	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.1138	0.0563	0.0050	0.0004	0.0000
8	5	1.0000	1.0000	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.3215	0.2031	0.0381	0.0058	0.0001
8	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.6329	0.4967	0.1869	0.0572	0.0027
8	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8999	0.8322	0.5695	0.3366	0.0773
9	0	0.9135	0.6302	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1	0.9966	0.9288	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
9	2	0.9999	0.9916	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0013	0.0003	0.0000	0.0000	0.0000
9	3	1.0000	0.9994	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0100	0.0031	0.0001	0.0000	0.0000
9	4	1.0000	1.0000	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0489	0.0196	0.0009	0.0000	0.0000
9	5	1.0000	1.0000	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.1657	0.0856	0.0083	0.0006	0.0000
9	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.3993	0.2618	0.0530	0.0084	0.0001
9	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.6997	0.5638	0.2252	0.0712	0.0034
9	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.9249	0.8658	0.6126	0.3698	0.0865
10	0	0.9044	0.5987	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1	0.9957	0.9139	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
10	2	0.9999	0.9885	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000
10	3	1.0000	0.9990	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0035	0.0009	0.0000	0.0000	0.0000
10	4	1.0000	0.9999	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0197	0.0064	0.0001	0.0000	0.0000
10	5	1.0000	1.0000	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0781	0.0328	0.0016	0.0001	0.0000
10	6	1.0000	1.0000	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.2241	0.1209	0.0128	0.0010	0.0000
10	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.4744	0.3222	0.0702	0.0115	0.0001
10	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.7560	0.6242	0.2639	0.0861	0.0043
10	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9990	0.9940	0.9718	0.9437	0.8926	0.6513	0.4013	0.0956

Probabilities for the Binomial distribution

The function tabulated is $P(X \leq x) = \sum_{t=0}^{x} \binom{n}{t} p^t q^{n-t}$

n	p = x	0.01	0.05	0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9	0.95	0.99
12	0	0.8864	0.5404	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1	0.9938	0.8816	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	2	0.9998	0.9804	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
12	3	1.0000	0.9978	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000
12	4	1.0000	0.9998	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0028	0.0006	0.0000	0.0000	0.0000
12	5	1.0000	1.0000	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0143	0.0039	0.0001	0.0000	0.0000
12	6	1.0000	1.0000	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0544	0.0194	0.0005	0.0000	0.0000
12	7	1.0000	1.0000	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.1576	0.0726	0.0043	0.0002	0.0000
12	8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.3512	0.2054	0.0256	0.0022	0.0000
12	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.6093	0.4417	0.1109	0.0196	0.0002
12	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9968	0.9804	0.9150	0.8416	0.7251	0.3410	0.1184	0.0062
12	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9978	0.9862	0.9683	0.9313	0.7176	0.4596	0.1136
20	0	0.8179	0.3585	0.1216	0.0115	0.0032	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	1	0.9831	0.7358	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	2	0.9990	0.9245	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	3	1.0000	0.9841	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	4	1.0000	0.9974	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	5	1.0000	0.9997	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	6	1.0000	1.0000	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
20	7	1.0000	1.0000	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
20	8	1.0000	1.0000	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
20	9	1.0000	1.0000	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
20	10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0139	0.0026	0.0000	0.0000	0.0000
20	11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9435	0.7483	0.4044	0.1133	0.0409	0.0100	0.0001	0.0000	0.0000
20	12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9790	0.8684	0.5841	0.2277	0.1018	0.0321	0.0004	0.0000	0.0000
20	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9935	0.9423	0.7500	0.3920	0.2142	0.0867	0.0024	0.0000	0.0000
20	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9984	0.9793	0.8744	0.5836	0.3828	0.1958	0.0113	0.0003	0.0000
20	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9941	0.9490	0.7625	0.5852	0.3704	0.0432	0.0026	0.0000
20	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9987	0.9840	0.8929	0.7748	0.5886	0.1330	0.0159	0.0000
20	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9964	0.9645	0.9087	0.7939	0.3231	0.0755	0.0010
20	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9924	0.9757	0.9308	0.6083	0.2642	0.0169
20	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9992	0.9968	0.9885	0.8784	0.6415	0.1821

11.9 Critical values for the Grouping of Signs test

											n2															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	1																									
	2																									
	3									1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	4					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2
	5				1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2
	6			1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3
	7			1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
	8			1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	4	4
	9			1	1	1	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4
	10		1	1	1	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4
n1	11		1	1	1	2	2	2	2	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5
	12		1	1	1	2	2	2	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5
	13		1	1	1	2	2	2	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5	5	6	6
	14		1	1	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	5	6	6	6	6
	15		1	1	2	2	2	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6	6	6	6	6
	16		1	1	2	2	2	3	3	3	4	4	4	5	5	5	5	5	6	6	6	6	6	6	7	7
	17		1	1	2	2	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7	7
	18		1	1	2	2	3	3	3	4	4	4	5	5	5	5	6	6	6	6	6	7	7	7	7	7
	19		1	1	2	2	3	3	3	4	4	4	5	5	5	6	6	6	6	6	7	7	7	7	7	8
	20		1	1	2	2	3	3	4	4	4	5	5	5	5	6	6	6	6	7	7	7	7	7	8	8
	21		1	1	2	2	3	3	4	4	4	5	5	5	6	6	6	6	7	7	7	7	8	8	8	8
	22		1	1	2	2	3	3	4	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	8	8
	23		1	1	2	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	8	8	9
	24		1	2	2	3	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8	8	9	9
	25		1	2	2	3	3	4	4	4	5	5	6	6	6	7	7	7	7	8	8	8	8	9	9	9

The table shows the greatest integer x for which $\sum_{t=1}^{x} \binom{n_1 - 1}{t - 1} \binom{n_2 + 1}{t} / \binom{n_1 + n_2}{n_1} < 0.05$

11.10 Pseudorandom values from U(0,1)

1	2	3	4	5	6	7	8	9	10
0.587	0.155	0.999	0.122	0.659	0.975	0.059	0.567	0.651	0.686
0.030	0.447	0.048	0.201	0.931	0.071	0.033	0.388	0.849	0.033
0.048	0.224	0.359	0.463	0.710	0.861	0.972	0.543	0.550	0.248
0.593	0.478	0.929	0.301	0.688	0.750	0.211	0.911	0.479	0.046
0.165	0.113	0.695	0.513	0.711	0.402	0.121	0.843	0.951	0.229
0.788	0.493	0.329	0.160	0.708	0.309	0.878	0.650	0.279	0.617
0.714	0.980	0.946	0.530	0.973	0.440	0.728	0.652	0.303	0.398
0.265	0.320	0.065	0.573	0.708	0.682	0.014	0.128	0.113	0.938
0.712	0.524	0.747	0.136	0.004	0.165	0.070	0.431	0.201	0.965
0.630	0.933	0.863	0.802	0.642	0.625	0.244	0.961	0.458	0.127
0.569	0.813	0.341	0.055	0.483	0.756	0.186	0.273	0.443	0.618
0.766	0.449	0.026	0.276	0.977	0.410	0.102	0.695	0.487	0.640
0.638	0.335	0.466	0.808	0.907	0.162	0.355	0.333	0.529	0.390
0.984	0.575	0.300	0.836	0.276	0.638	0.674	0.625	0.885	0.451
0.721	0.857	0.303	0.076	0.124	0.688	0.455	0.536	0.842	0.533
0.028	0.271	0.245	0.290	0.534	0.924	0.093	0.724	0.651	0.422
0.726	0.399	0.474	0.221	0.898	0.838	0.723	0.139	0.219	0.711
0.218	0.240	0.036	0.206	0.582	0.203	0.676	0.371	0.791	0.069
0.792	0.704	0.959	0.615	0.440	0.311	0.994	0.785	0.041	0.737
0.656	0.285	0.886	0.954	0.846	0.595	0.215	0.484	0.158	0.435

Pseudorandom values from N(0,1)

1	2	3	4	5	6	7	8	9	10
−0.603	0.825	1.166	1.880	1.261	2.542	0.312	0.611	0.286	0.223
1.469	0.282	−1.250	−1.176	−0.064	0.860	−1.505	−0.828	−0.965	−0.166
−2.199	0.169	0.278	0.580	−0.875	0.373	−0.132	−0.153	−1.322	2.340
1.863	−1.302	0.260	−1.023	0.114	−0.904	0.500	−0.255	0.283	0.291
0.076	0.373	−0.448	0.998	0.149	1.987	−0.405	0.324	0.112	−1.367
−0.667	−0.589	0.080	1.007	1.548	1.204	1.886	−0.080	0.341	−0.808
0.495	−1.693	0.647	0.172	1.143	−1.519	−2.557	1.351	−0.466	0.494
−0.161	0.990	−1.348	2.047	0.167	0.599	−0.530	1.244	0.278	0.627
1.105	0.851	−1.012	0.891	0.256	0.297	1.267	−0.053	−1.776	1.392
0.800	−0.867	0.229	−0.534	−0.602	1.685	−1.210	−0.986	0.979	0.810
−0.738	0.765	−2.068	−0.660	2.704	0.161	0.790	−0.284	−1.041	−0.852
−0.489	−0.250	−0.917	−2.549	−1.879	0.156	−1.451	−0.158	−2.252	−0.309
0.170	−1.623	0.442	−0.253	−0.786	−0.468	0.435	1.544	−1.014	−1.187
−1.301	−0.901	0.810	−0.244	0.524	−0.622	−0.785	−0.949	−0.923	0.510
0.059	−1.489	0.235	−0.230	1.262	0.751	−0.377	0.631	0.520	1.508
0.599	0.196	−1.785	−0.899	−1.347	−0.227	1.027	0.704	1.943	−0.902
0.329	−1.008	0.834	1.079	−0.101	−0.322	−0.315	−0.254	−0.711	−0.285
−0.229	0.446	0.086	0.024	0.555	−0.360	0.111	0.589	−0.325	−0.056
−0.987	−0.214	0.925	−0.656	1.991	1.030	−0.961	−0.078	1.023	−0.070
0.805	−0.359	−1.179	0.324	−0.208	−0.632	1.170	−0.432	0.716	−1.801

12 Compound Interest Tables

12.1 Compound Interest

$\frac{1}{2}\%$

i	0.005000
i^2	0.004994
i^4	0.004991
i^{12}	0.004989
δ	0.004988
$(1+i)^{1/2}$	1.002497
$(1+i)^{1/4}$	1.001248
$(1+i)^{1/12}$	1.000416
v	0.995025
$v^{1/2}$	0.997509
$v^{1/4}$	0.998754
$v^{1/12}$	0.999584
d	0.004975
d^2	0.004981
d^4	0.004984
d^{12}	0.004987
i/i^2	1.001248
i/i^4	1.001873
i/i^{12}	1.002290
i/δ	1.002498
i/d^2	1.003748
i/d^4	1.003123
i/d^{12}	1.002706

n	$(1+i)^n$	v^n	$s_{\overline{n}\rvert}$	$a_{\overline{n}\rvert}$	$(Ia)_{\overline{n}\rvert}$	$(Da)_{\overline{n}\rvert}$	n
1	1.00500	0.99502	1.0000	0.9950	0.9950	0.9950	1
2	1.01003	0.99007	2.0050	1.9851	2.9752	2.9801	2
3	1.01508	0.98515	3.0150	2.9702	5.9306	5.9504	3
4	1.02015	0.98025	4.0301	3.9505	9.8516	9.9009	4
5	1.02525	0.97537	5.0503	4.9259	14.7285	14.8267	5
6	1.03038	0.97052	6.0755	5.8964	20.5516	20.7231	6
7	1.03553	0.96569	7.1059	6.8621	27.3114	27.5852	7
8	1.04071	0.96089	8.1414	7.8230	34.9985	35.4082	8
9	1.04591	0.95610	9.1821	8.7791	43.6034	44.1872	9
10	1.05114	0.95135	10.2280	9.7304	53.1169	53.9176	10
11	1.05640	0.94661	11.2792	10.6770	63.5297	64.5947	11
12	1.06168	0.94191	12.3356	11.6189	74.8325	76.2136	12
13	1.06699	0.93722	13.3972	12.5562	87.0164	88.7697	13
14	1.07232	0.93256	14.4642	13.4887	100.0722	102.2584	14
15	1.07768	0.92792	15.5365	14.4166	113.9909	116.6751	15
16	1.08307	0.92330	16.6142	15.3399	128.7637	132.0150	16
17	1.08849	0.91871	17.6973	16.2586	144.3817	148.2736	17
18	1.09393	0.91414	18.7858	17.1728	160.8362	165.4464	18
19	1.09940	0.90959	19.8797	18.0824	178.1184	183.5288	19
20	1.10490	0.90506	20.9791	18.9874	196.2196	202.5162	20
21	1.11042	0.90056	22.0840	19.8880	215.1314	222.4041	21
22	1.11597	0.89608	23.1944	20.7841	234.8451	243.1882	22
23	1.12155	0.89162	24.3104	21.6757	255.3524	264.8639	23
24	1.12716	0.88719	25.4320	22.5629	276.6449	287.4268	24
25	1.13280	0.88277	26.5591	23.4456	298.7142	310.8724	25
26	1.13846	0.87838	27.6919	24.3240	321.5521	335.1964	26
27	1.14415	0.87401	28.8304	25.1980	345.1503	360.3944	27
28	1.14987	0.86966	29.9745	26.0677	369.5009	386.4621	28
29	1.15562	0.86533	31.1244	26.9330	394.5956	413.3952	29
30	1.16140	0.86103	32.2800	27.7941	420.4265	441.1892	30
31	1.16721	0.85675	33.4414	28.6508	446.9856	469.8400	31
32	1.17304	0.85248	34.6086	29.5033	474.2651	499.3433	32
33	1.17891	0.84824	35.7817	30.3515	502.2571	529.6948	33
34	1.18480	0.84402	36.9606	31.1955	530.9538	560.8904	34
35	1.19073	0.83982	38.1454	32.0354	560.3476	592.9257	35
36	1.19668	0.83564	39.3361	32.8710	590.4308	625.7968	36
37	1.20266	0.83149	40.5328	33.7025	621.1959	659.4993	37
38	1.20868	0.82735	41.7354	34.5299	652.6352	694.0291	38
39	1.21472	0.82323	42.9441	35.3531	684.7414	729.3822	39
40	1.22079	0.81914	44.1588	36.1722	717.5069	765.5544	40
41	1.22690	0.81506	45.3796	36.9873	750.9245	802.5417	41
42	1.23303	0.81101	46.6065	37.7983	784.9869	840.3400	42
43	1.23920	0.80697	47.8396	38.6053	819.6867	878.9453	43
44	1.24539	0.80296	49.0788	39.4082	855.0169	918.3535	44
45	1.25162	0.79896	50.3242	40.2072	890.9703	958.5607	45
46	1.25788	0.79499	51.5758	41.0022	927.5398	999.5629	46
47	1.26417	0.79103	52.8337	41.7932	964.71841	041.3561	47
48	1.27049	0.78710	54.0978	42.5803	1002.4991	1083.9364	48
49	1.27684	0.78318	55.3683	43.3635	1040.8751	1127.2999	49
50	1.28323	0.77929	56.6452	44.1428	1079.8394	1171.4427	50
60	1.34885	0.74137	69.7700	51.7256	1500.3714	1654.8878	60
70	1.41783	0.70530	83.5661	58.9394	1972.5822	2212.1165	70
80	1.49034	0.67099	98.0677	65.8023	2490.4478	2839.5389	80
90	1.56655	0.63834	113.3109	72.3313	3048.4082	3533.7401	90
100	1.64667	0.60729	129.3337	78.5426	3641.3361	4291.4710	100

Compound Interest

1%

i	0.010 000
i^{2}	0.009 975
i^{4}	0.009 963
i^{12}	0.009 954
δ	0.009 950
$(1+i)^{1/2}$	1.004 988
$(1+i)^{1/4}$	1.002 491
$(1+i)^{1/12}$	1.000 830
v	0.990 099
$v^{1/2}$	0.995 037
$v^{1/4}$	0.997 516
$v^{1/12}$	0.999 171
d	0.009 901
d^{2}	0.009 926
d^{4}	0.009 938
d^{12}	0.009 946
i/i^{2}	1.002 494
i/i^{4}	1.003 742
i/i^{12}	1.004 575
i/δ	1.004 992
i/d^{2}	1.007 494
i/d^{4}	1.006 242
i/d^{12}	1.005 408

n	$(1+i)^n$	v^n	$s_{\overline{n\rvert}}$	$a_{\overline{n\rvert}}$	$(Ia)_{\overline{n\rvert}}$	$(Da)_{\overline{n\rvert}}$	n
1	1.01000	0.99010	1.0000	0.9901	0.9901	0.9901	1
2	1.02010	0.98030	2.0100	1.9704	2.9507	2.9605	2
3	1.03030	0.97059	3.0301	2.9410	5.8625	5.9015	3
4	1.04060	0.96098	4.0604	3.9020	9.7064	9.8034	4
5	1.05101	0.95147	5.1010	4.8534	14.4637	14.6569	5
6	1.06152	0.94205	6.1520	5.7955	20.1160	20.4524	6
7	1.07214	0.93272	7.2135	6.7282	26.6450	27.1805	7
8	1.08286	0.92348	8.2857	7.6517	34.0329	34.8322	8
9	1.09369	0.91434	9.3685	8.5660	42.2619	43.3982	9
10	1.10462	0.90529	10.4622	9.4713	51.3148	52.8695	10
11	1.11567	0.89632	11.5668	10.3676	61.1744	63.2372	11
12	1.12683	0.88745	12.6825	11.2551	71.8238	74.4923	12
13	1.13809	0.87866	13.8093	12.1337	83.2464	86.6260	13
14	1.14947	0.86996	14.9474	13.0037	95.4258	99.6297	14
15	1.16097	0.86135	16.0969	13.8651	108.3461	113.4947	15
16	1.17258	0.85282	17.2579	14.7179	121.9912	128.2126	16
17	1.18430	0.84438	18.4304	15.5623	136.3456	143.7749	17
18	1.19615	0.83602	19.6147	16.3983	151.3940	160.1731	18
19	1.20811	0.82774	20.8109	17.2260	167.1210	177.3992	19
20	1.22019	0.81954	22.0190	18.0456	183.5119	195.4447	20
21	1.23239	0.81143	23.2392	18.8570	200.5519	214.3017	21
22	1.24472	0.80340	24.4716	19.6604	218.2267	233.9621	22
23	1.25716	0.79544	25.7163	20.4558	236.5218	254.4179	23
24	1.26973	0.78757	26.9735	21.2434	255.4234	275.6613	24
25	1.28243	0.77977	28.2432	22.0232	274.9176	297.6844	25
26	1.29526	0.77205	29.5256	22.7952	294.9909	320.4796	26
27	1.30821	0.76440	30.8209	23.5596	315.6298	344.0392	27
28	1.32129	0.75684	32.1291	24.3164	336.8212	368.3557	28
29	1.33450	0.74934	33.4504	25.0658	358.5521	393.4215	29
30	1.34785	0.74192	34.7849	25.8077	380.8098	419.2292	30
31	1.36133	0.73458	36.1327	26.5423	403.5817	445.7715	31
32	1.37494	0.72730	37.4941	27.2696	426.8554	473.0411	32
33	1.38869	0.72010	38.8690	27.9897	450.6188	501.0307	33
34	1.40258	0.71297	40.2577	28.7027	474.8599	529.7334	34
35	1.41660	0.70591	41.6603	29.4086	499.5669	559.1420	35
36	1.43077	0.69892	43.0769	30.1075	524.7282	589.2495	36
37	1.44508	0.69200	44.5076	30.7995	550.3324	620.0490	37
38	1.45953	0.68515	45.9527	31.4847	576.3682	651.5337	38
39	1.47412	0.67837	47.4123	32.1630	602.8246	683.6967	39
40	1.48886	0.67165	48.8864	32.8347	629.6907	716.5314	40
41	1.50375	0.66500	50.3752	33.4997	656.9559	750.0311	41
42	1.51879	0.65842	51.8790	34.1581	684.6095	784.1892	42
43	1.53398	0.65190	53.3978	34.8100	712.6412	818.9992	43
44	1.54932	0.64545	54.9318	35.4555	741.0408	854.4546	44
45	1.56481	0.63905	56.4811	36.0945	769.7982	890.5492	45
46	1.58046	0.63273	58.0459	36.7272	798.9037	927.2764	46
47	1.59626	0.62646	59.6263	37.3537	828.3475	964.6301	47
48	1.61223	0.62026	61.2226	37.9740	858.1200	1002.6041	48
49	1.62835	0.61412	62.8348	38.5881	888.2118	1041.1921	49
50	1.64463	0.60804	64.4632	39.1961	918.6137	1080.3882	50
60	1.81670	0.55045	81.6697	44.9550	1237.7612	1504.4962	60
70	2.00676	0.49831	100.6763	50.1685	1578.8160	1983.1486	70
80	2.21672	0.45112	121.6715	54.8882	1934.7653	2511.1794	80
90	2.44863	0.40839	144.8633	59.1609	2299.7284	3083.9119	90
100	2.70481	0.36971	170.4814	63.0289	2668.8046	3697.1121	100

Compound Interest

$1\frac{1}{2}$%

i	0.015 00
i^{2}	0.014 944
i^{4}	0.014 916
i^{12}	0.014 898
δ	0.014 889
$(1+i)^{1/2}$	1.007 472
$(1+i)^{1/4}$	1.003 729
$(1+i)^{1/12}$	1.001 241
v	0.985 222
$v^{1/2}$	0.992 583
$v^{1/4}$	0.996 285
$v^{1/12}$	0.998 760
d	0.014 778
d^{2}	0.014 833
d^{4}	0.014 861
d^{12}	0.014 879
i/i^{2}	1.003 736
i/i^{4}	1.005 608
i/i^{12}	1.006 857
i/δ	1.007 481
i/d^{2}	1.011 236
i/d^{4}	1.009 358
i/d^{12}	1.008 107

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.01500	0.98522	1.0000	0.9852	0.9852	0.9852	1
2	1.03023	0.97066	2.0150	1.9559	2.9265	2.9411	2
3	1.04568	0.95632	3.0452	2.9122	5.7955	5.8533	3
4	1.06136	0.94218	4.0909	3.8544	9.5642	9.7077	4
5	1.07728	0.92826	5.1523	4.7826	14.2055	14.4903	5
6	1.09344	0.91454	6.2296	5.6972	19.6928	20.1875	6
7	1.10984	0.90103	7.3230	6.5982	26.0000	26.7857	7
8	1.12649	0.88771	8.4328	7.4859	33.1017	34.2717	8
9	1.14339	0.87459	9.5593	8.3605	40.9730	42.6322	9
10	1.16054	0.86167	10.7027	9.2222	49.5897	51.8544	10
11	1.17795	0.84893	11.8633	10.0711	58.9279	61.9255	11
12	1.19562	0.83639	13.0412	10.9075	68.9646	72.8330	12
13	1.21355	0.82403	14.2368	11.7315	79.6769	84.5645	13
14	1.23176	0.81185	15.4504	12.5434	91.0428	97.1079	14
15	1.25023	0.79985	16.6821	13.3432	103.0406	110.4511	15
16	1.26899	0.78803	17.9324	14.1313	115.6491	124.5824	16
17	1.28802	0.77639	19.2014	14.9076	128.8476	139.4900	17
18	1.30734	0.76491	20.4894	15.6726	142.6160	155.1626	18
19	1.32695	0.75361	21.7967	16.4262	156.9346	171.5888	19
20	1.34686	0.74247	23.1237	17.1686	171.7840	188.7574	20
21	1.36706	0.73150	24.4705	17.9001	187.1455	206.6576	21
22	1.38756	0.72069	25.8376	18.6208	203.0006	225.2784	22
23	1.40838	0.71004	27.2251	19.3309	219.3314	244.6092	23
24	1.42950	0.69954	28.6335	20.0304	236.1205	264.6396	24
25	1.45095	0.68921	30.0630	20.7196	253.3506	285.3593	25
26	1.47271	0.67902	31.5140	21.3986	271.0052	306.7579	26
27	1.49480	0.66899	32.9867	22.0676	289.0678	328.8255	27
28	1.51722	0.65910	34.4815	22.7267	307.5226	351.5522	28
29	1.53998	0.64936	35.9987	23.3761	326.3540	374.9283	29
30	1.56308	0.63976	37.5387	24.0158	345.5468	398.9441	30
31	1.58653	0.63031	39.1018	24.6461	365.0864	423.5903	31
32	1.61032	0.62099	40.6883	25.2671	384.9582	448.8574	32
33	1.63448	0.61182	42.2986	25.8790	405.1481	474.7364	33
34	1.65900	0.60277	43.9331	26.4817	425.6424	501.2181	34
35	1.68388	0.59387	45.5921	27.0756	446.4277	528.2937	35
36	1.70914	0.58509	47.2760	27.6607	467.4909	555.9544	36
37	1.73478	0.57644	48.9851	28.2371	488.8193	584.1915	37
38	1.76080	0.56792	50.7199	28.8051	510.4005	612.9966	38
39	1.78721	0.55953	52.4807	29.3646	532.2222	642.3611	39
40	1.81402	0.55126	54.2679	29.9158	554.2727	672.2770	40
41	1.84123	0.54312	56.0819	30.4590	576.5404	702.7359	41
42	1.86885	0.53509	57.9231	30.9941	599.0142	733.7300	42
43	1.89688	0.52718	59.7920	31.5212	621.6830	765.2512	43
44	1.92533	0.51939	61.6889	32.0406	644.5361	797.2919	44
45	1.95421	0.51171	63.6142	32.5523	667.5633	829.8442	45
46	1.98353	0.50415	65.5684	33.0565	690.7543	862.9007	46
47	2.01328	0.49670	67.5519	33.5532	714.0993	896.4539	47
48	2.04348	0.48936	69.5652	34.0426	737.5887	930.4964	48
49	2.07413	0.48213	71.6087	34.5247	761.2131	965.0211	49
50	2.10524	0.47500	73.6828	34.9997	784.9633	1000.0208	50
60	2.44322	0.40930	96.2147	39.3803	1027.5477	1374.6487	60
70	2.83546	0.35268	122.3638	43.1549	1274.3207	1789.6752	70
80	3.29066	0.30389	152.7109	46.4073	1519.4814	2239.5118	80
90	3.81895	0.26185	187.9299	49.2099	1758.7537	2719.3430	90
100	4.43205	0.22563	228.8030	51.6247	1989.0753	3225.0198	100

2%

i	0.020 000
i^{2}	0.019 901
i^{4}	0.019 852
i^{12}	0.019 819
δ	0.019 803
$(1+i)^{1/2}$	1.009 950
$(1+i)^{1/4}$	1.004 963
$(1+i)^{1/12}$	1.001 652
v	0.980 392
$v^{1/2}$	0.990 148
$v^{1/4}$	0.995 062
$v^{1/12}$	0.998 351
d	0.019 608
d^{2}	0.019 705
d^{4}	0.019 754
d^{12}	0.019 786
i/i^{2}	1.004 975
i/i^{4}	1.007 469
i/i^{12}	1.009 134
i/δ	1.009 967
i/d^{2}	1.014 975
i/d^{4}	1.012 469
i/d^{12}	1.010 801

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.02000	0.98039	1.0000	0.9804	0.9804	0.9804	1
2	1.04040	0.96117	2.0200	1.9416	2.9027	2.9220	2
3	1.06121	0.94232	3.0604	2.8839	5.7297	5.8058	3
4	1.08243	0.92385	4.1216	3.8077	9.4251	9.6136	4
5	1.10408	0.90573	5.2040	4.7135	13.9537	14.3270	5
6	1.12616	0.88797	6.3081	5.6014	19.2816	19.9285	6
7	1.14869	0.87056	7.4343	6.4720	25.3755	26.4004	7
8	1.17166	0.85349	8.5830	7.3255	32.2034	33.7259	8
9	1.19509	0.83676	9.7546	8.1622	39.7342	41.8882	9
10	1.21899	0.82035	10.9497	8.9826	47.9377	50.8707	10
11	1.24337	0.80426	12.1687	9.7868	56.7846	60.6576	11
12	1.26824	0.78849	13.4121	10.5753	66.2465	71.2329	12
13	1.29361	0.77303	14.6803	11.3484	76.2959	82.5813	13
14	1.31948	0.75788	15.9739	12.1062	86.9062	94.6876	14
15	1.34587	0.74301	17.2934	12.8493	98.0514	107.5368	15
16	1.37279	0.72845	18.6393	13.5777	109.7065	121.1145	16
17	1.40024	0.71416	20.0121	14.2919	121.8473	135.4064	17
18	1.42825	0.70016	21.4123	14.9920	134.4502	150.3984	18
19	1.45681	0.68643	22.8406	15.6785	147.4923	166.0769	19
20	1.48595	0.67297	24.2974	16.3514	160.9518	182.4283	20
21	1.51567	0.65978	25.7833	17.0112	174.8071	199.4395	21
22	1.54598	0.64684	27.2990	17.6580	189.0375	217.0976	22
23	1.57690	0.63416	28.8450	18.2922	203.6231	235.3898	23
24	1.60844	0.62172	30.4219	18.9139	218.5444	254.3037	24
25	1.64061	0.60953	32.0303	19.5235	233.7827	273.8272	25
26	1.67342	0.59758	33.6709	20.1210	249.3198	293.9482	26
27	1.70689	0.58586	35.3443	20.7069	265.1380	314.6551	27
28	1.74102	0.57437	37.0512	21.2813	281.2205	335.9364	28
29	1.77584	0.56311	38.7922	21.8444	297.5508	357.7808	29
30	1.81136	0.55207	40.5681	22.3965	314.1129	380.1772	30
31	1.84759	0.54125	42.3794	22.9377	330.8915	403.1149	31
32	1.88454	0.53063	44.2270	23.4683	347.8718	426.5833	32
33	1.92223	0.52023	46.1116	23.9886	365.0393	450.5718	33
34	1.96068	0.51003	48.0338	24.4986	382.3803	475.0704	34
35	1.99989	0.50003	49.9945	24.9986	399.8813	500.0690	35
36	2.03989	0.49022	51.9944	25.4888	417.5293	525.5579	36
37	2.08069	0.48061	54.0343	25.9695	435.3119	551.5273	37
38	2.12230	0.47119	56.1149	26.4406	453.2170	577.9680	38
39	2.16474	0.46195	58.2372	26.9026	471.2330	604.8706	39
40	2.20804	0.45289	60.4020	27.3555	489.3486	632.2260	40
41	2.25220	0.44401	62.6100	27.7995	507.5530	660.0255	41
42	2.29724	0.43530	64.8622	28.2348	525.8358	688.2603	42
43	2.34319	0.42677	67.1595	28.6616	544.1869	716.9219	43
44	2.39005	0.41840	69.5027	29.0800	562.5965	746.0018	44
45	2.43785	0.41020	71.8927	29.4902	581.0553	775.4920	45
46	2.48661	0.40215	74.3306	29.8923	599.5544	805.3843	46
47	2.53634	0.39427	76.8172	30.2866	618.0850	835.6709	47
48	2.58707	0.38654	79.3535	30.6731	636.6388	866.3440	48
49	2.63881	0.37896	81.9406	31.0521	655.2078	897.3961	49
50	2.69159	0.37153	84.5794	31.4236	673.7842	928.8197	50
60	3.28103	0.30478	114.0515	34.7609	858.4584	1261.9557	60
70	3.99956	0.25003	149.9779	37.4986	1037.3329	1625.0690	70
80	4.87544	0.20511	193.7720	39.7445	1206.5313	2012.7743	80
90	5.94313	0.16826	247.1567	41.5869	1363.7570	2420.6535	90
100	7.24465	0.13803	312.2323	43.0984	1507.8511	2845.0824	100

$2\frac{1}{2}$%

i	0.025 000
i^{2}	0.024 846
i^{4}	0.024 769
i^{12}	0.024 718
δ	0.024 693
$(1+i)^{1/2}$	1.012 423
$(1+i)^{1/4}$	1.006 192
$(1+i)^{1/12}$	1.002 060
v	0.975 610
$v^{1/2}$	0.987 730
$v^{1/4}$	0.993 846
$v^{1/12}$	0.997 944
d	0.024 390
d^{2}	0.024 541
d^{4}	0.024 617
d^{12}	0.024 667
i/i^{2}	1.006 211
i/i^{4}	1.009 327
i/i^{12}	1.011 407
i/δ	1.012 449
i/d^{2}	1.018 711
i/d^{4}	1.015 577
i/d^{12}	1.013 491

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.02500	0.97561	1.0000	0.9756	0.9756	0.9756	1
2	1.05063	0.95181	2.0250	1.9274	2.8792	2.9030	2
3	1.07689	0.92860	3.0756	2.8560	5.6650	5.7591	3
4	1.10381	0.90595	4.1525	3.7620	9.2888	9.5210	4
5	1.13141	0.88385	5.2563	4.6458	13.7081	14.1669	5
6	1.15969	0.86230	6.3877	5.5081	18.8819	19.6750	6
7	1.18869	0.84127	7.5474	6.3494	24.7707	26.0244	7
8	1.21840	0.82075	8.7361	7.1701	31.3367	33.1945	8
9	1.24886	0.80073	9.9545	7.9709	38.5433	41.1654	9
10	1.28008	0.78120	11.2034	8.7521	46.3553	49.9174	10
11	1.31209	0.76214	12.4835	9.5142	54.7389	59.4317	11
12	1.34489	0.74356	13.7956	10.2578	63.6615	69.6894	12
13	1.37851	0.72542	15.1404	10.9832	73.0920	80.6726	13
14	1.41297	0.70773	16.5190	11.6909	83.0002	92.3635	14
15	1.44830	0.69047	17.9319	12.3814	93.3572	104.7449	15
16	1.48451	0.67362	19.3802	13.0550	104.1352	117.7999	16
17	1.52162	0.65720	20.8647	13.7122	115.3075	131.5121	17
18	1.55966	0.64117	22.3863	14.3534	126.8485	145.8655	18
19	1.59865	0.62553	23.9460	14.9789	138.7335	160.8443	19
20	1.63862	0.61027	25.5447	15.5892	150.9389	176.4335	20
21	1.67958	0.59539	27.1833	16.1845	163.4420	192.6181	21
22	1.72157	0.58086	28.8629	16.7654	176.2210	209.3835	22
23	1.76461	0.56670	30.5844	17.3321	189.2551	226.7156	23
24	1.80873	0.55288	32.3490	17.8850	202.5241	244.6006	24
25	1.85394	0.53939	34.1578	18.4244	216.0088	263.0249	25
26	1.90029	0.52623	36.0117	18.9506	229.6909	281.9756	26
27	1.94780	0.51340	37.9120	19.4640	243.5527	301.4396	27
28	1.99650	0.50088	39.8598	19.9649	257.5773	321.4045	28
29	2.04641	0.48866	41.8563	20.4535	271.7485	341.8580	29
30	2.09757	0.47674	43.9027	20.9303	286.0508	362.7883	30
31	2.15001	0.46511	46.0003	21.3954	300.4693	384.1837	31
32	2.20376	0.45377	48.1503	21.8492	314.9900	406.0329	32
33	2.25885	0.44270	50.3540	22.2919	329.5992	428.3248	33
34	2.31532	0.43191	52.6129	22.7238	344.2840	451.0485	34
35	2.37321	0.42137	54.9282	23.1452	359.0320	474.1937	35
36	2.43254	0.41109	57.3014	23.5563	373.8313	497.7500	36
37	2.49335	0.40107	59.7339	23.9573	388.6708	521.7073	37
38	2.55568	0.39128	62.2273	24.3486	403.5396	546.0559	38
39	2.61957	0.38174	64.7830	24.7303	418.4276	570.7862	39
40	2.68506	0.37243	67.4026	25.1028	433.3248	595.8890	40
41	2.75219	0.36335	70.0876	25.4661	448.2220	621.3551	41
42	2.82100	0.35448	72.8398	25.8206	463.1104	647.1757	42
43	2.89152	0.34584	75.6608	26.1664	477.9814	673.3422	43
44	2.96381	0.33740	78.5523	26.5038	492.8272	699.8460	44
45	3.03790	0.32917	81.5161	26.8330	507.6401	726.6790	45
46	3.11385	0.32115	84.5540	27.1542	522.4128	753.8332	46
47	3.19170	0.31331	87.6679	27.4675	537.1385	781.3007	47
48	3.27149	0.30567	90.8596	27.7732	551.8107	809.0739	48
49	3.35328	0.29822	94.1311	28.0714	566.4233	837.1452	49
50	3.43711	0.29094	97.4843	28.3623	580.9704	865.5075	50
60	4.39979	0.22728	135.9916	30.9087	721.7743	1163.6537	60
70	5.63210	0.17755	185.2841	32.8979	851.6621	1484.0857	70
80	7.20957	0.13870	248.3827	34.4518	968.6699	1821.9273	80
90	9.22886	0.10836	329.1543	35.6658	1072.2157	2173.3693	90
100	11.81372	0.08465	432.5487	36.6141	1162.5888	2535.4358	100

3%

i	0.030 000
i^{2}	0.029 778
i^{4}	0.029 668
i^{12}	0.029 595
δ	0.029 559
$(1+i)^{1/2}$	1.014 889
$(1+i)^{1/4}$	1.007 417
$(1+i)^{1/12}$	1.002 466
v	0.970 874
$v^{1/2}$	0.985 329
$v^{1/4}$	0.992 638
$v^{1/12}$	0.997 540
d	0.029 126
d^{2}	0.029 341
d^{4}	0.029 450
d^{12}	0.029 522
i/i^{2}	1.007 445
i/i^{4}	1.011 181
i/i^{12}	1.013 677
i/δ	1.014 926
i/d^{2}	1.022 445
i/d^{4}	1.018 681
i/d^{12}	1.016 177

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.03000	0.97087	1.0000	0.9709	0.9709	0.9709	1
2	1.06090	0.94260	2.0300	1.9135	2.8561	2.8843	2
3	1.09273	0.91514	3.0909	2.8286	5.6015	5.7130	3
4	1.12551	0.88849	4.1836	3.7171	9.1554	9.4301	4
5	1.15927	0.86261	5.3091	4.5797	13.4685	14.0098	5
6	1.19405	0.83748	6.4684	5.4172	18.4934	19.4270	6
7	1.22987	0.81309	7.6625	6.2303	24.1850	25.6572	7
8	1.26677	0.78941	8.8923	7.0197	30.5003	32.6769	8
9	1.30477	0.76642	10.1591	7.7861	37.3981	40.4630	9
10	1.34392	0.74409	11.4639	8.5302	44.8390	48.9932	10
11	1.38423	0.72242	12.8078	9.2526	52.7856	58.2459	11
12	1.42576	0.70138	14.1920	9.9540	61.2022	68.1999	12
13	1.46853	0.68095	15.6178	10.6350	70.0546	78.8348	13
14	1.51259	0.66112	17.0863	11.2961	79.3102	90.1309	14
15	1.55797	0.64186	18.5989	11.9379	88.9381	102.0688	15
16	1.60471	0.62317	20.1569	12.5611	98.9088	114.6299	16
17	1.65285	0.60502	21.7616	13.1661	109.1941	127.7961	17
18	1.70243	0.58739	23.4144	13.7535	119.7672	141.5496	18
19	1.75351	0.57029	25.1169	14.3238	130.6026	155.8734	19
20	1.80611	0.55368	26.8704	14.8775	141.6761	170.7508	20
21	1.86029	0.53755	28.6765	15.4150	152.9647	186.1659	21
22	1.91610	0.52189	30.5368	15.9369	164.4463	202.1028	22
23	1.97359	0.50669	32.4529	16.4436	176.1002	218.5464	23
24	2.03279	0.49193	34.4265	16.9355	187.9066	235.4819	24
25	2.09378	0.47761	36.4593	17.4131	199.8468	252.8951	25
26	2.15659	0.46369	38.5530	17.8768	211.9028	270.7719	26
27	2.22129	0.45019	40.7096	18.3270	224.0579	289.0990	27
28	2.28793	0.43708	42.9309	18.7641	236.2961	307.8631	28
29	2.35657	0.42435	45.2189	19.1885	248.6021	327.0515	29
30	2.42726	0.41199	47.5754	19.6004	260.9617	346.6520	30
31	2.50008	0.39999	50.0027	20.0004	273.3613	366.6524	31
32	2.57508	0.38834	52.5028	20.3888	285.7881	387.0411	32
33	2.65234	0.37703	55.0778	20.7658	298.2300	407.8069	33
34	2.73191	0.36604	57.7302	21.1318	310.6755	428.9388	34
35	2.81386	0.35538	60.4621	21.4872	323.1139	450.4260	35
36	2.89828	0.34503	63.2759	21.8323	335.5351	472.2583	36
37	2.98523	0.33498	66.1742	22.1672	347.9295	494.4255	37
38	3.07478	0.32523	69.1594	22.4925	360.2881	516.9179	38
39	3.16703	0.31575	72.2342	22.8082	372.6024	539.7262	39
40	3.26204	0.30656	75.4013	23.1148	384.8647	562.8409	40
41	3.35990	0.29763	78.6633	23.4124	397.0675	586.2533	41
42	3.46070	0.28896	82.0232	23.7014	409.2038	609.9547	42
43	3.56452	0.28054	85.4839	23.9819	421.2671	633.9366	43
44	3.67145	0.27237	89.0484	24.2543	433.2515	658.1909	44
45	3.78160	0.26444	92.7199	24.5187	445.1512	682.7096	45
46	3.89504	0.25674	96.5015	24.7754	456.9611	707.4850	46
47	4.01190	0.24926	100.3965	25.0247	468.6762	732.5097	47
48	4.13225	0.24200	104.4084	25.2667	480.2922	757.7764	48
49	4.25622	0.23495	108.5406	25.5017	491.8047	783.2781	49
50	4.38391	0.22811	112.7969	25.7298	503.2101	809.0079	50
60	5.89160	0.16973	163.0534	27.6756	610.7282	1077.4812	60
70	7.91782	0.12630	230.5941	29.1234	705.2103	1362.5526	70
80	10.64089	0.09398	321.3630	30.2008	786.2873	1659.9746	80
90	14.30047	0.06993	443.3489	31.0024	854.6326	196.5864	90
100	19.21863	0.05203	607.2877	31.5989	911.4530	2280.0365	100

4%

i	0.040 000
i^{2}	0.039 608
i^{4}	0.039 414
i^{12}	0.039 285
δ	0.039 221
$(1+i)^{1/2}$	1.019 804
$(1+i)^{1/4}$	1.009 853
$(1+i)^{1/12}$	1.003 274
v	0.961 538
$v^{1/2}$	0.980 581
$v^{1/4}$	0.990 243
$v^{1/12}$	0.996 737
d	0.038 462
d^{2}	0.038 839
d^{4}	0.039 029
d^{12}	0.039 157
i/i^{2}	1.009 902
i/i^{4}	1.014 877
i/i^{12}	1.018 204
i/δ	1.019 869
i/d^{2}	1.029 902
i/d^{4}	1.024 877
i/d^{12}	1.021 537

n	$(1+i)^n$	v^n	$s_{\overline{n\|}}$	$a_{\overline{n\|}}$	$(Ia)_{\overline{n\|}}$	$(Da)_{\overline{n\|}}$	n
1	1.04000	0.96154	1.0000	0.9615	0.9615	0.9615	1
2	1.08160	0.92456	2.0400	1.8861	2.8107	2.8476	2
3	1.12486	0.88900	3.1216	2.7751	5.4776	5.6227	3
4	1.16986	0.85480	4.2465	3.6299	8.8969	9.2526	4
5	1.21665	0.82193	5.4163	4.4518	13.0065	13.7044	5
6	1.26532	0.79031	6.6330	5.2421	17.7484	18.9466	6
7	1.31593	0.75992	7.8983	6.0021	23.0678	24.9486	7
8	1.36857	0.73069	9.2142	6.7327	28.9133	31.6814	8
9	1.42331	0.70259	10.5828	7.4353	35.2366	39.1167	9
10	1.48024	0.67556	12.0061	8.1109	41.9922	47.2276	10
11	1.53945	0.64958	13.4864	8.7605	49.1376	55.9881	11
12	1.60103	0.62460	15.0258	9.3851	56.6328	65.3732	12
13	1.66507	0.60057	16.6268	9.9856	64.4403	75.3588	13
14	1.73168	0.57748	18.2919	10.5631	72.5249	85.9219	14
15	1.80094	0.55526	20.0236	11.1184	80.8539	97.0403	15
16	1.87298	0.53391	21.8245	11.6523	89.3964	108.6926	16
17	1.94790	0.51337	23.6975	12.1657	98.1238	120.8583	17
18	2.02582	0.49363	25.6454	12.6593	107.0091	133.5176	18
19	2.10685	0.47464	27.6712	13.1339	116.0273	146.6515	19
20	2.19112	0.45639	29.7781	13.5903	125.1550	160.2418	20
21	2.27877	0.43883	31.9692	14.0292	134.3705	174.2710	21
22	2.36992	0.42196	34.2480	14.4511	143.6535	188.7221	22
23	2.46472	0.40573	36.6179	14.8568	152.9852	203.5790	23
24	2.56330	0.39012	39.0826	15.2470	162.3482	218.8259	24
25	2.66584	0.37512	41.6459	15.6221	171.7261	234.4480	25
26	2.77247	0.36069	44.3117	15.9828	181.1040	250.4308	26
27	2.88337	0.34682	47.0842	16.3296	190.4680	266.7604	27
28	2.99870	0.33348	49.9676	16.6631	199.8054	283.4234	28
29	3.11865	0.32065	52.9663	16.9837	209.1043	300.4071	29
30	3.24340	0.30832	56.0849	17.2920	218.3539	317.6992	30
31	3.37313	0.29646	59.3283	17.5885	227.5441	335.2877	31
32	3.50806	0.28506	62.7015	17.8736	236.6660	353.1612	32
33	3.64838	0.27409	66.2095	18.1476	245.7111	371.3089	33
34	3.79432	0.26355	69.8579	18.4112	254.6719	389.7201	34
35	3.94609	0.25342	73.6522	18.6646	263.5414	408.3847	35
36	4.10393	0.24367	77.5983	18.9083	272.3135	427.2930	36
37	4.26809	0.23430	81.7022	19.1426	280.9825	446.4355	37
38	4.43881	0.22529	85.9703	19.3679	289.5433	465.8034	38
39	4.61637	0.21662	90.4091	19.5845	297.9915	485.3879	39
40	4.80102	0.20829	95.0255	19.7928	306.3231	505.1807	40
41	4.99306	0.20028	99.8265	19.9931	314.5345	525.1737	41
42	5.19278	0.19257	104.8196	20.1856	322.6226	545.3593	42
43	5.40050	0.18517	110.0124	20.3708	330.5849	565.7301	43
44	5.61652	0.17805	115.4129	20.5488	338.4189	586.2790	44
45	5.84118	0.17120	121.0294	20.7200	346.1228	606.9990	45
46	6.07482	0.16461	126.8706	20.8847	353.6951	627.8837	46
47	6.31782	0.15828	132.9454	21.0429	361.1343	648.9266	47
48	6.57053	0.15219	139.2632	21.1951	368.4397	670.1217	48
49	6.83335	0.14634	145.8337	21.3415	375.6104	691.4632	49
50	7.10668	0.14071	152.6671	21.4822	382.6460	712.9454	50
60	10.51963	0.09506	237.9907	22.6235	445.6201	934.4128	60
70	15.57162	0.06422	364.2905	23.3945	495.8734	1165.1371	70
80	23.04980	0.04338	551.2450	23.9154	535.0315	1402.1152	80
90	34.11933	0.02931	827.9833	24.2673	565.0042	1643.3181	90
100	50.50495	0.01980	1237.6237	24.5050	587.6299	1887.3750	100

Compound Interest

5%

i	0.050 000
i^{2}	0.049 390
i^{4}	0.049 089
i^{12}	0.048 889
δ	0.048 790
$(1+i)^{1/2}$	1.024 695
$(1+i)^{1/4}$	1.012 272
$(1+i)^{1/12}$	1.004 074
v	0.952 381
$v^{1/2}$	0.975 900
$v^{1/4}$	0.987 877
$v^{1/12}$	0.995 942
d	0.047 619
d^{2}	0.048 200
d^{4}	0.048 494
d^{12}	0.048 691
i/i^{2}	1.012 348
i/i^{4}	1.018 559
i/i^{12}	1.022 715
i/δ	1.024 797
i/d^{2}	1.037 348
i/d^{4}	1.031 059
i/d^{12}	1.026 881

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.05000	0.95238	1.0000	0.9524	0.9524	0.9524	1
2	1.10250	0.90703	2.0500	1.8594	2.7664	2.8118	2
3	1.15763	0.86384	3.1525	2.7232	5.3580	5.5350	3
4	1.21551	0.82270	4.3101	3.5460	8.6488	9.0810	4
5	1.27628	0.78353	5.5256	4.3295	12.5664	13.4105	5
6	1.34010	0.74622	6.8019	5.0757	17.0437	18.4862	6
7	1.40710	0.71068	8.1420	5.7864	22.0185	24.2725	7
8	1.47746	0.67684	9.5491	6.4632	27.4332	30.7357	8
9	1.55133	0.64461	11.0266	7.1078	33.2347	37.8436	9
10	1.62889	0.61391	12.5779	7.7217	39.3738	45.5653	10
11	1.71034	0.58468	14.2068	8.3064	45.8053	53.8717	11
12	1.79586	0.55684	15.9171	8.8633	52.4873	62.7350	12
13	1.88565	0.53032	17.7130	9.3936	59.3815	72.1285	13
14	1.97993	0.50507	19.5986	9.8986	66.4524	82.0272	14
15	2.07893	0.48102	21.5786	10.3797	73.6677	92.4068	15
16	2.18287	0.45811	23.6575	10.8378	80.9975	103.2446	16
17	2.29202	0.43630	25.8404	11.2741	88.4145	114.5187	17
18	2.40662	0.41552	28.1324	11.6896	95.8939	126.2083	18
19	2.52695	0.39573	30.5390	12.0853	103.4128	138.2936	19
20	2.65330	0.37689	33.0660	12.4622	110.9506	150.7558	20
21	2.78596	0.35894	35.7193	12.8212	118.4884	163.5769	21
22	2.92526	0.34185	38.5052	13.1630	126.0091	176.7399	22
23	3.07152	0.32557	41.4305	13.4886	133.4973	190.2285	23
24	3.22510	0.31007	44.5020	13.7986	140.9389	204.0272	24
25	3.38635	0.29530	47.7271	14.0939	148.3215	218.1211	25
26	3.55567	0.28124	51.1135	14.3752	155.6337	232.4963	26
27	3.73346	0.26785	54.6691	14.6430	162.8656	247.1393	27
28	3.92013	0.25509	58.4026	14.8981	170.0082	262.0375	28
29	4.11614	0.24295	62.3227	15.1411	177.0537	277.1785	29
30	4.32194	0.23138	66.4388	15.3725	183.9950	292.5510	30
31	4.53804	0.22036	70.7608	15.5928	190.8261	308.1438	31
32	4.76494	0.20987	75.2988	15.8027	197.5419	323.9465	32
33	5.00319	0.19987	80.0638	16.0025	204.1377	339.9490	33
34	5.25335	0.19035	85.0670	16.1929	210.6097	356.1419	34
35	5.51602	0.18129	90.3203	16.3742	216.9549	372.5161	35
36	5.79182	0.17266	95.8363	16.5469	223.1705	389.0630	36
37	6.08141	0.16444	101.6281	16.7113	229.2547	405.7743	37
38	6.38548	0.15661	107.7095	16.8679	235.2057	422.6421	38
39	6.70475	0.14915	114.0950	17.0170	241.0224	439.6592	39
40	7.03999	0.14205	120.7998	17.1591	246.7043	456.8183	40
41	7.39199	0.13528	127.8398	17.2944	252.2508	474.1126	41
42	7.76159	0.12884	135.2318	17.4232	257.6621	491.5358	42
43	8.14967	0.12270	142.9933	17.5459	262.9384	509.0818	43
44	8.55715	0.11686	151.1430	17.6628	268.0803	526.7445	44
45	8.98501	0.11130	159.7002	17.7741	273.0886	544.5186	45
46	9.43426	0.10600	168.6852	17.8801	277.9645	562.3987	46
47	9.90597	0.10095	178.1194	17.9810	282.7091	580.3797	47
48	10.40127	0.09614	188.0254	18.0772	287.3239	598.4568	48
49	10.92133	0.09156	198.4267	18.1687	291.8105	616.6256	49
50	11.46740	0.08720	209.3480	18.2559	296.1707	634.8815	50
60	18.67919	0.05354	353.5837	18.9293	333.2725	821.4142	60
70	30.42643	0.03287	588.5285	19.3427	360.1836	1013.1465	70
80	49.56144	0.02018	971.2288	19.5965	379.2425	1208.0708	80
90	80.73037	0.01239	1594.6073	19.7523	392.5011	1404.9548	90
100	131.50126	0.00760	2610.0252	19.8479	401.5971	1603.0418	100

Compound Interest

6%

i	0.060 000
i^{2}	0.059 126
i^{4}	0.058 695
i^{12}	0.058 411
δ	0.058 269
$(1+i)^{1/2}$	1.029 563
$(1+i)^{1/4}$	1.014 674
$(1+i)^{1/12}$	1.004 868
v	0.943 396
$v^{1/2}$	0.971 286
$v^{1/4}$	0.985 538
$v^{1/12}$	0.995 156
d	0.056 604
d^{2}	0.057 428
d^{4}	0.057 847
d^{12}	0.058 128
i/i^{2}	1.014 782
i/i^{4}	1.022 227
i/i^{12}	1.027 211
i/δ	1.029 709
i/d^{2}	1.044 782
i/d^{4}	1.037 227
i/d^{12}	1.032 211

n	$(1+i)^n$	v^n	$s_{\overline{n\|}}$	$a_{\overline{n\|}}$	$(Ia)_{\overline{n\|}}$	$(Da)_{\overline{n\|}}$	n
1	1.06000	0.94340	1.0000	0.9434	0.9434	0.9434	1
2	1.12360	0.89000	2.0600	1.8334	2.7234	2.7768	2
3	1.19102	0.83962	3.1836	2.6730	5.2422	5.4498	3
4	1.26248	0.79209	4.3746	3.4651	8.4106	8.9149	4
5	1.33823	0.74726	5.6371	4.2124	12.1469	13.1273	5
6	1.41852	0.70496	6.9753	4.9173	16.3767	18.0446	6
7	1.50363	0.66506	8.3938	5.5824	21.0321	23.6270	7
8	1.59385	0.62741	9.8975	6.2098	26.0514	29.8368	8
9	1.68948	0.59190	11.4913	6.8017	31.3785	36.6385	9
10	1.79085	0.55839	13.1808	7.3601	36.9624	43.9985	10
11	1.89830	0.52679	14.9716	7.8869	42.7571	51.8854	11
12	2.01220	0.49697	16.8699	8.3838	48.7207	60.2693	12
13	2.13293	0.46884	18.8821	8.8527	54.8156	69.1220	13
14	2.26090	0.44230	21.0151	9.2950	61.0078	78.4169	14
15	2.39656	0.41727	23.2760	9.7122	67.2668	88.1292	15
16	2.54035	0.39365	25.6725	10.1059	73.5651	98.2351	16
17	2.69277	0.37136	28.2129	10.4773	79.8783	108.7123	17
18	2.85434	0.35034	30.9057	10.8276	86.1845	119.5399	18
19	3.02560	0.33051	33.7600	11.1581	92.4643	130.6981	19
20	3.20714	0.31180	36.7856	11.4699	98.7004	142.1680	20
21	3.39956	0.29416	39.9927	11.7641	104.8776	153.9321	21
22	3.60354	0.27751	43.3923	12.0416	110.9827	165.9736	22
23	3.81975	0.26180	46.9958	12.3034	117.0041	178.2770	23
24	4.04893	0.24698	50.8156	12.5504	122.9316	190.8274	24
25	4.29187	0.23300	54.8645	12.7834	128.7565	203.6107	25
26	4.54938	0.21981	59.1564	13.0032	134.4716	216.6139	26
27	4.82235	0.20737	63.7058	13.2105	140.0705	229.8244	27
28	5.11169	0.19563	68.5281	13.4062	145.5482	243.2306	28
29	5.41839	0.18456	73.6398	13.5907	150.9003	256.8213	29
30	5.74349	0.17411	79.0582	13.7648	156.1236	270.5861	30
31	6.08810	0.16425	84.8017	13.9291	161.2155	284.5152	31
32	6.45339	0.15496	90.8898	14.0840	166.1742	298.5993	32
33	6.84059	0.14619	97.3432	14.2302	170.9983	312.8295	33
34	7.25103	0.13791	104.1838	14.3681	175.6873	327.1976	34
35	7.68609	0.13011	111.4348	14.4982	180.2410	341.6959	35
36	8.14725	0.12274	119.1209	14.6210	184.6596	356.3169	36
37	8.63609	0.11579	127.2681	14.7368	188.9440	371.0537	37
38	9.15425	0.10924	135.9042	14.8460	193.0951	385.8997	38
39	9.70351	0.10306	145.0585	14.9491	197.1142	400.8488	39
40	10.28572	0.09722	154.7620	15.0463	201.0031	415.8951	40
41	10.90286	0.09172	165.0477	15.1380	204.7636	431.0331	41
42	11.55703	0.08653	175.9505	15.2245	208.3978	446.2576	42
43	12.25045	0.08163	187.5076	15.3062	211.9078	461.5638	43
44	12.98548	0.07701	199.7580	15.3832	215.2962	476.9470	44
45	13.76461	0.07265	212.7435	15.4558	218.5655	492.4028	45
46	14.59049	0.06854	226.5081	15.5244	221.7182	507.9272	46
47	15.46592	0.06466	241.0986	15.5890	224.7572	523.5162	47
48	16.39387	0.06100	256.5645	15.6500	227.6851	539.1662	48
49	17.37750	0.05755	272.9584	15.7076	230.5048	554.8738	49
50	18.42015	0.05429	290.3359	15.7619	233.2192	570.6357	50
60	32.98769	0.03031	533.1282	16.1614	255.2042	730.6429	60
70	59.07593	0.01693	967.9322	16.3845	269.7117	893.5909	70
80	105.79599	0.00945	1746.5999	16.5091	279.0584	1058.1812	80
90	189.46451	0.00528	3141.0752	16.5787	284.9733	1223.6883	90
100	339.30208	0.00295	5638.3681	16.6175	288.6646	1389.7076	100

Compound Interest

7%

i	0.070 000
i^{2}	0.068 816
i^{4}	0.068 234
i^{12}	0.067 850
δ	0.067 659
$(1+i)^{1/2}$	1.034 408
$(1+i)^{1/4}$	1.017 059
$(1+i)^{1/12}$	1.005 654
v	0.934 579
$v^{1/2}$	0.966 736
$v^{1/4}$	0.983 228
$v^{1/12}$	0.994 378
d	0.065 421
d^{2}	0.066 527
d^{4}	0.067 090
d^{12}	0.067 468
i/i^{2}	1.017 204
i/i^{4}	1.025 880
i/i^{12}	1.031 691
i/δ	1.034 605
i/d^{2}	1.052 204
i/d^{4}	1.043 380
i/d^{12}	1.037 525

n	$(1+i)^n$	v^n	$s_{\overline{n\vert}}$	$a_{\overline{n\vert}}$	$(Ia)_{\overline{n\vert}}$	$(Da)_{\overline{n\vert}}$	n
1	1.07000	0.93458	1.0000	0.9346	0.9346	0.9346	1
2	1.14490	0.87344	2.0700	1.8080	2.6815	2.7426	2
3	1.22504	0.81630	3.2149	2.6243	5.1304	5.3669	3
4	1.31080	0.76290	4.4399	3.3872	8.1819	8.7541	4
5	1.40255	0.71299	5.7507	4.1002	11.7469	12.8543	5
6	1.50073	0.66634	7.1533	4.7665	15.7449	17.6209	6
7	1.60578	0.62275	8.6540	5.3893	20.1042	23.0102	7
8	1.71819	0.58201	10.2598	5.9713	24.7602	28.9814	8
9	1.83846	0.54393	11.9780	6.5152	29.6556	35.4967	9
10	1.96715	0.50835	13.8164	7.0236	34.7391	42.5203	10
11	2.10485	0.47509	15.7836	7.4987	39.9652	50.0189	11
12	2.25219	0.44401	17.8885	7.9427	45.2933	57.9616	12
13	2.40985	0.41496	20.1406	8.3577	50.6878	66.3193	13
14	2.57853	0.38782	22.5505	8.7455	56.1173	75.0647	14
15	2.75903	0.36245	25.1290	9.1079	61.5540	84.1727	15
16	2.95216	0.33873	27.8881	9.4466	66.9737	93.6193	16
17	3.15882	0.31657	30.8402	9.7632	72.3555	103.3825	17
18	3.37993	0.29586	33.9990	10.0591	77.6810	113.4416	18
19	3.61653	0.27651	37.3790	10.3356	82.9347	123.7772	19
20	3.86968	0.25842	40.9955	10.5940	88.1031	134.3712	20
21	4.14056	0.24151	44.8652	10.8355	93.1748	145.2068	21
22	4.43040	0.22571	49.0057	11.0612	98.1405	156.2680	22
23	4.74053	0.21095	53.4361	11.2722	102.9923	167.5402	23
24	5.07237	0.19715	58.1767	11.4693	107.7238	179.0095	24
25	5.42743	0.18425	63.2490	11.6536	112.3301	190.6631	25
26	5.80735	0.17220	68.6765	11.8258	116.8071	202.4889	26
27	6.21387	0.16093	74.4838	11.9867	121.1523	214.4756	27
28	6.64884	0.15040	80.6977	12.1371	125.3635	226.6127	28
29	7.11426	0.14056	87.3465	12.2777	129.4399	238.8904	29
30	7.61226	0.13137	94.4608	12.4090	133.3809	251.2994	30
31	8.14511	0.12277	102.0730	12.5318	137.1868	263.8312	31
32	8.71527	0.11474	110.2182	12.6466	140.8585	276.4778	32
33	9.32534	0.10723	118.9334	12.7538	144.3973	289.2316	33
34	9.97811	0.10022	128.2588	12.8540	147.8047	302.0856	34
35	10.67658	0.09366	138.2369	12.9477	151.0829	315.0333	35
36	11.42394	0.08754	148.9135	13.0352	154.2342	328.0685	36
37	12.22362	0.08181	160.3374	13.1170	157.2612	341.1855	37
38	13.07927	0.07646	172.5610	13.1935	160.1665	354.3790	38
39	13.99482	0.07146	185.6403	13.2649	162.9533	367.6439	39
40	14.97446	0.06678	199.6351	13.3317	165.6245	380.9756	40
41	16.02267	0.06241	214.6096	13.3941	168.1833	394.3697	41
42	17.14426	0.05833	230.6322	13.4524	170.6331	407.8222	42
43	18.34435	0.05451	247.7765	13.5070	172.9772	421.3291	43
44	19.62846	0.05095	266.1209	13.5579	175.2188	434.8870	44
45	21.00245	0.04761	285.7493	13.6055	177.3614	448.4925	45
46	22.47262	0.04450	306.7518	13.6500	179.4084	462.1426	46
47	24.04571	0.04159	329.2244	13.6916	181.3630	475.8342	47
48	25.72891	0.03887	353.2701	13.7305	183.2286	489.5647	48
49	27.52993	0.03632	378.9990	13.7668	185.0085	503.3314	49
50	29.45703	0.03395	406.5289	13.8007	186.7059	517.1322	50
60	57.94643	0.01726	813.5204	14.0392	199.8069	656.5831	60
70	113.98939	0.00877	1614.1342	14.1604	207.6789	797.7087	70
80	224.23439	0.00446	3189.0627	14.2220	212.2968	939.6856	80
90	441.10298	0.00227	6287.1854	14.2533	214.9575	1082.0953	90
100	867.71633	0.00115	12381.6618	14.2693	216.4693	1224.7250	100

Compound Interest

8%

i	0.080 000
i^{2}	0.078 461
i^{4}	0.077 706
i^{12}	0.077 208
δ	0.076 961
$(1+i)^{1/2}$	1.039 230
$(1+i)^{1/4}$	1.019 427
$(1+i)^{1/12}$	1.006 434
v	0.925 926
$v^{1/2}$	0.962 250
$v^{1/4}$	0.980 944
$v^{1/12}$	0.993 607
d	0.074 074
d^{2}	0.075 499
d^{4}	0.076 225
d^{12}	0.076 715
i/i^{2}	1.019 615
i/i^{4}	1.029 519
i/i^{12}	1.036 157
i/δ	1.039 487
i/d^{2}	1.059 615
i/d^{4}	1.049 519
i/d^{12}	1.042 824

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.08000	0.92593	1.0000	0.9259	0.9259	0.9259	1
2	1.16640	0.85734	2.0800	1.7833	2.6406	2.7092	2
3	1.25971	0.79383	3.2464	2.5771	5.0221	5.2863	3
4	1.36049	0.73503	4.5061	3.3121	7.9622	8.5984	4
5	1.46933	0.68058	5.8666	3.9927	11.3651	12.5911	5
6	1.58687	0.63017	7.3359	4.6229	15.1462	17.2140	6
7	1.71382	0.58349	8.9228	5.2064	19.2306	22.4204	7
8	1.85093	0.54027	10.6366	5.7466	23.5527	28.1670	8
9	1.99900	0.50025	12.4876	6.2469	28.0550	34.4139	9
10	2.15892	0.46319	14.4866	6.7101	32.6869	41.1240	10
11	2.33164	0.42888	16.6455	7.1390	37.4046	48.2629	11
12	2.51817	0.39711	18.9771	7.5361	42.1700	55.7990	12
13	2.71962	0.36770	21.4953	7.9038	46.9501	63.7028	13
14	2.93719	0.34046	24.2149	8.2442	51.7165	71.9470	14
15	3.17217	0.31524	27.1521	8.5595	56.4451	80.5065	15
16	3.42594	0.29189	30.3243	8.8514	61.1154	89.3579	16
17	3.70002	0.27027	33.7502	9.1216	65.7100	98.4795	17
18	3.99602	0.25025	37.4502	9.3719	70.2144	107.8514	18
19	4.31570	0.23171	41.4463	9.6036	74.6170	117.4550	19
20	4.66096	0.21455	45.7620	9.8181	78.9079	127.2732	20
21	5.03383	0.19866	50.4229	10.0168	83.0797	137.2900	21
22	5.43654	0.18394	55.4568	10.2007	87.1264	147.4907	22
23	5.87146	0.17032	60.8933	10.3711	91.0437	157.8618	23
24	6.34118	0.15770	66.7648	10.5288	94.8284	168.3905	24
25	6.84848	0.14602	73.1059	10.6748	98.4789	179.0653	25
26	7.39635	0.13520	79.9544	10.8100	101.9941	189.8753	26
27	7.98806	0.12519	87.3508	10.9352	105.3742	200.8104	27
28	8.62711	0.11591	95.3388	11.0511	108.6198	211.8615	28
29	9.31727	0.10733	103.9659	11.1584	111.7323	223.0199	29
30	10.06266	0.09938	113.2832	11.2578	114.7136	234.2777	30
31	10.86767	0.09202	123.3459	11.3498	117.5661	245.6275	31
32	11.73708	0.08520	134.2135	11.4350	120.2925	257.0625	32
33	12.67605	0.07889	145.9506	11.5139	122.8958	268.5764	33
34	13.69013	0.07305	158.6267	11.5869	125.3793	280.1633	34
35	14.78534	0.06763	172.3168	11.6546	127.7466	291.8179	35
36	15.96817	0.06262	187.1021	11.7172	130.0010	303.5351	36
37	17.24563	0.05799	203.0703	11.7752	132.1465	315.3103	37
38	18.62528	0.05369	220.3159	11.8289	134.1868	327.1391	38
39	20.11530	0.04971	238.9412	11.8786	136.1256	339.0177	39
40	21.72452	0.04603	259.0565	11.9246	137.9668	350.9423	40
41	23.46248	0.04262	280.7810	11.9672	139.7143	362.9096	41
42	25.33948	0.03946	304.2435	12.0067	141.3718	374.9163	42
43	27.36664	0.03654	329.5830	12.0432	142.9430	386.9595	43
44	29.55597	0.03383	356.9496	12.0771	144.4317	399.0366	44
45	31.92045	0.03133	386.5056	12.1084	145.8415	411.1450	45
46	34.47409	0.02901	418.4261	12.1374	147.1758	423.2824	46
47	37.23201	0.02686	452.9002	12.1643	148.4382	435.4467	47
48	40.21057	0.02487	490.1322	12.1891	149.6319	447.6358	48
49	43.42742	0.02303	530.3427	12.2122	150.7602	459.8480	49
50	46.90161	0.02132	573.7702	12.2335	151.8263	472.0814	50
60	101.25706	0.00988	1253.2133	12.3766	159.6766	595.2931	60
70	218.60641	0.00457	2720.0801	12.4428	163.9754	719.4648	70
80	471.95483	0.00212	5886.9354	12.4735	166.2736	844.0811	80
90	1018.91509	0.00098	12723.9386	12.4877	167.4803	968.9033	90
100	2199.76126	0.00045	27484.5157	12.4943	168.1050	1093.8210	100

Compound Interest

9%

i	0.090 000
$i^{(2)}$	0.088 061
$i^{(4)}$	0.087 113
$i^{(12)}$	0.086 488
δ	0.086 178
$(1+i)^{1/2}$	1.044 031
$(1+i)^{1/4}$	1.021 778
$(1+i)^{1/12}$	1.007 207
v	0.917 431
$v^{1/2}$	0.957 826
$v^{1/4}$	0.978 686
$v^{1/12}$	0.992 844
d	0.082 569
$d^{(2)}$	0.084 347
$d^{(4)}$	0.085 256
$d^{(12)}$	0.085 869
$i/i^{(2)}$	1.022 015
$i/i^{(4)}$	1.033 144
$i/i^{(12)}$	1.040 608
i/δ	1.044 354
$i/d^{(2)}$	1.067 015
$i/d^{(4)}$	1.055 644
$i/d^{(12)}$	1.048 108

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.09000	0.91743	1.0000	0.9174	0.9174	0.9174	1
2	1.18810	0.84168	2.0900	1.7591	2.6008	2.6765	2
3	1.29503	0.77218	3.2781	2.5313	4.9173	5.2078	3
4	1.41158	0.70843	4.5731	3.2397	7.7510	8.4476	4
5	1.53862	0.64993	5.9847	3.8897	11.0007	12.3372	5
6	1.67710	0.59627	7.5233	4.4859	14.5783	16.8231	6
7	1.82804	0.54703	9.2004	5.0330	18.4075	21.8561	7
8	1.99256	0.50187	11.0285	5.5348	22.4225	27.3909	8
9	2.17189	0.46043	13.0210	5.9952	26.5663	33.3861	9
10	2.36736	0.42241	15.1929	6.4177	30.7904	39.8038	10
11	2.58043	0.38753	17.5603	6.8052	35.0533	46.6090	11
12	2.81266	0.35553	20.1407	7.1607	39.3197	53.7697	12
13	3.06580	0.32618	22.9534	7.4869	43.5600	61.2566	13
14	3.34173	0.29925	26.0192	7.7862	47.7495	69.0428	14
15	3.64248	0.27454	29.3609	8.0607	51.8676	77.1035	15
16	3.97031	0.25187	33.0034	8.3126	55.8975	85.4160	16
17	4.32763	0.23107	36.9737	8.5436	59.8257	93.9597	17
18	4.71712	0.21199	41.3013	8.7556	63.6416	102.7153	18
19	5.14166	0.19449	46.0185	8.9501	67.3369	111.6654	19
20	5.60441	0.17843	51.1601	9.1285	70.9055	120.7939	20
21	6.10881	0.16370	56.7645	9.2922	74.3432	130.0862	21
22	6.65860	0.15018	62.8733	9.4424	77.6472	139.5286	22
23	7.25787	0.13778	69.5319	9.5802	80.8162	149.1088	23
24	7.91108	0.12640	76.7898	9.7066	83.8499	158.8154	24
25	8.62308	0.11597	84.7009	9.8226	86.7491	168.6380	25
26	9.39916	0.10639	93.3240	9.9290	89.5153	178.5670	26
27	10.24508	0.09761	102.7231	10.0266	92.1507	188.5936	27
28	11.16714	0.08955	112.9682	10.1161	94.6580	198.7097	28
29	12.17218	0.08215	124.1354	10.1983	97.0405	208.9080	29
30	13.26768	0.07537	136.3075	10.2737	99.3017	219.1816	30
31	14.46177	0.06915	149.5752	10.3428	101.4452	229.5244	31
32	15.76333	0.06344	164.0370	10.4062	103.4753	239.9307	32
33	17.18203	0.05820	179.8003	10.4644	105.3959	250.3951	33
34	18.72841	0.05339	196.9823	10.5178	107.2113	260.9129	34
35	20.41397	0.04899	215.7108	10.5668	108.9258	271.4798	35
36	22.25123	0.04494	236.1247	10.6118	110.5437	282.0915	36
37	24.25384	0.04123	258.3759	10.6530	112.0692	292.7445	37
38	26.43668	0.03783	282.6298	10.6908	113.5066	303.4353	38
39	28.81598	0.03470	309.0665	10.7255	114.8600	314.1609	39
40	31.40942	0.03184	337.8824	10.7574	116.1335	324.9182	40
41	34.23627	0.02921	369.2919	10.7866	117.3311	335.7048	41
42	37.31753	0.02680	403.5281	10.8134	118.4566	346.5182	42
43	40.67611	0.02458	440.8457	10.8380	119.5137	357.3561	43
44	44.33696	0.02255	481.5218	10.8605	120.5061	368.2166	44
45	48.32729	0.02069	525.8587	10.8812	121.4373	379.0978	45
46	52.67674	0.01898	574.1860	10.9002	122.3105	389.9980	46
47	57.41765	0.01742	626.8628	10.9176	123.1291	400.9156	47
48	62.58524	0.01598	684.2804	10.9336	123.8960	411.8492	48
49	68.21791	0.01466	746.8656	10.9482	124.6143	422.7974	49
50	74.35752	0.01345	815.0836	10.9617	125.2867	433.7591	50
60	176.03129	0.00568	1944.7921	11.0480	130.0162	543.9112	60
70	416.73009	0.00240	4619.2232	11.0844	132.3786	654.6172	70
80	986.55167	0.00101	10950.5741	11.0998	133.5305	765.5572	80
90	2335.52658	0.00043	25939.1842	11.1064	134.0821	876.5961	90
100	5529.04079	0.00018	61422.6755	11.1091	134.3426	987.6766	100

Compound Interest

10%

i	0.100 000
$i^{(2)}$	0.097 618
$i^{(4)}$	0.096 455
$i^{(12)}$	0.095 690
δ	0.095 310
$(1+i)^{1/2}$	1.048 809
$(1+i)^{1/4}$	1.024 114
$(1+i)^{1/12}$	1.007 974
v	0.909 091
$v^{1/2}$	0.953 463
$v^{1/4}$	0.976 454
$v^{1/12}$	0.992 089
d	0.090 909
$d^{(2)}$	0.093 075
$d^{(4)}$	0.094 184
$d^{(12)}$	0.094 933
$i/i^{(2)}$	1.024 404
$i/i^{(4)}$	1.036 756
$i/i^{(12)}$	1.045 045
i/δ	1.049 206
$i/d^{(2)}$	1.074 404
$i/d^{(4)}$	1.061 756
$i/d^{(12)}$	1.053 378

n	$(1+i)^n$	v^n	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	$(Ia)_{\overline{n}\vert}$	$(Da)_{\overline{n}\vert}$	n
1	1.10000	0.90909	1.0000	0.9091	0.9091	0.9091	1
2	1.21000	0.82645	2.1000	1.7355	2.5620	2.6446	2
3	1.33100	0.75131	3.3100	2.4869	4.8159	5.1315	3
4	1.46410	0.68301	4.6410	3.1699	7.5480	8.3013	4
5	1.61051	0.62092	6.1051	3.7908	10.6526	12.0921	5
6	1.77156	0.56447	7.7156	4.3553	14.0394	16.4474	6
7	1.94872	0.51316	9.4872	4.8684	17.6315	21.3158	7
8	2.14359	0.46651	11.4359	5.3349	21.3636	26.6507	8
9	2.35795	0.42410	13.5795	5.7590	25.1805	32.4098	9
10	2.5934	0.38554	15.9374	6.1446	29.0359	38.5543	10
11	2.8532	0.35049	18.5312	6.4951	32.8913	45.0494	11
12	3.1383	0.31863	21.3843	6.8137	36.7149	51.8631	12
13	3.4527	0.28966	24.5227	7.1034	40.4805	58.9664	13
14	3.7970	0.26333	27.9750	7.3667	44.1672	66.3331	14
15	4.1775	0.23939	31.7725	7.6061	47.7581	73.9392	15
16	4.5947	0.21763	35.9497	7.8237	51.2401	81.7629	16
17	5.0547	0.19784	40.5447	8.0216	54.6035	89.7845	17
18	5.5592	0.17986	45.5992	8.2014	57.8410	97.9859	18
19	6.1151	0.16351	51.1591	8.3649	60.9476	106.3508	19
20	6.7270	0.14864	57.2750	8.5136	63.9205	114.8644	20
21	7.4005	0.13513	64.0025	8.6487	66.7582	123.5131	21
22	8.1407	0.12285	71.4027	8.7715	69.4608	132.2846	22
23	8.9540	0.11168	79.5430	8.8832	72.0294	141.1678	23
24	9.8493	0.10153	88.4973	8.9847	74.4660	150.1526	24
25	10.83471	0.09230	98.3471	9.0770	76.7734	159.2296	25
26	11.91818	0.08391	109.1818	9.1609	78.9550	168.3905	26
27	13.10999	0.07628	121.0999	9.2372	81.0145	177.6278	27
28	14.42099	0.06934	134.2099	9.3066	82.9561	186.9343	28
29	15.86309	0.06304	148.6309	9.3696	84.7842	196.3039	29
30	17.44940	0.05731	164.4940	9.4269	86.5035	205.7309	30
31	19.19434	0.05210	181.9434	9.4790	88.1186	215.2099	31
32	21.11378	0.04736	201.1378	9.5264	89.6342	224.7362	32
33	23.22515	0.04306	222.2515	9.5694	91.0550	234.3057	33
34	25.54767	0.03914	245.4767	9.6086	92.3859	243.9143	34
35	28.10244	0.03558	271.0244	9.6442	93.6313	253.5584	35
36	30.91268	0.03235	299.1268	9.6765	94.7959	263.2349	36
37	34.00395	0.02941	330.0395	9.7059	95.8840	272.9408	37
38	37.40434	0.02673	364.0434	9.7327	96.8999	282.6735	38
39	41.14478	0.02430	401.4478	9.7570	97.8478	292.4304	39
40	45.25926	0.02209	442.5926	9.7791	98.7316	302.2095	40
41	49.78518	0.02009	487.8518	9.7991	99.5551	312.0086	41
42	54.76370	0.01826	537.6370	9.8174	100.3221	321.8260	42
43	60.24007	0.01660	592.4007	9.8340	101.0359	331.6600	43
44	66.26408	0.01509	652.6408	9.8491	101.6999	341.5091	44
45	72.89048	0.01372	718.9048	9.8628	102.3172	351.3719	45
46	80.17953	0.01247	791.7953	9.8753	102.8910	361.2472	46
47	88.19749	0.01134	871.9749	9.8866	103.4238	371.1338	47
48	97.01723	0.01031	960.1723	9.8969	103.9186	381.0307	48
49	106.71896	0.00937	1057.1896	9.9063	104.3778	390.9370	49
50	117.39085	0.00852	1163.9085	9.9148	104.8037	400.8519	50
60	304.48164	0.00328	3034.8164	9.9672	107.6682	500.3284	60
70	789.74696	0.00127	7887.4696	9.9873	108.9744	600.1266	70
80	2048.40021	0.00049	20474.0021	9.9951	109.5558	700.0488	80
90	5313.02261	0.00019	53120.2261	9.9981	109.8099	800.0188	90
100	13780.61234	0.00007	137796.1234	9.9993	109.9195	900.0073	100

Compound Interest

12%

i	0.120 000
i^2	0.116 601
i^4	0.114 949
i^{12}	0.113 866
δ	0.113 329
$(1+i)^{1/2}$	1.058 301
$(1+i)^{1/4}$	1.028 737
$(1+i)^{1/12}$	1.009 489
v	0.892 857
$v^{1/2}$	0.944 911
$v^{1/4}$	0.972 065
$v^{1/12}$	0.990 600
d	0.107 143
d^2	0.110 178
d^4	0.111 738
d^{12}	0.112 795
i/i^2	1.029 150
i/i^4	1.043 938
i/i^{12}	1.053 875
i/δ	1.058 867
i/d^2	1.089 150
i/d^4	1.073 938
i/d^{12}	1.063 875

n	$(1+i)^n$	v^n	$s_{\overline{n\vert}}$	$a_{\overline{n\vert}}$	$(Ia)_{\overline{n\vert}}$	$(Da)_{\overline{n\vert}}$	n
1	1.12000	0.89286	1.0000	0.8929	0.8929	0.8929	1
2	1.25440	0.79719	2.1200	1.6901	2.4872	2.5829	2
3	1.40493	0.71178	3.3744	2.4018	4.6226	4.9847	3
4	1.57352	0.63552	4.7793	3.0373	7.1647	8.0221	4
5	1.76234	0.56743	6.3528	3.6048	10.0018	11.6269	5
6	1.97382	0.50663	8.1152	4.1114	13.0416	15.7383	6
7	2.21068	0.45235	10.0890	4.5638	16.2080	20.3020	7
8	2.47596	0.40388	12.2997	4.9676	19.4391	25.2697	8
9	2.77308	0.36061	14.7757	5.3282	22.6846	30.5979	9
10	3.10585	0.32197	17.5487	5.6502	25.9043	36.2481	10
11	3.47855	0.28748	20.6546	5.9377	29.0665	42.1858	11
12	3.89598	0.25668	24.1331	6.1944	32.1467	48.3802	12
13	4.36349	0.22917	28.0291	6.4235	35.1259	54.8038	13
14	4.88711	0.20462	32.3926	6.6282	37.9906	61.4319	14
15	5.47357	0.18270	37.2797	6.8109	40.7310	68.2428	15
16	6.13039	0.16312	42.7533	6.9740	43.3410	75.2168	16
17	6.86604	0.14564	48.8837	7.1196	45.8169	82.3364	17
18	7.68997	0.13004	55.7497	7.2497	48.1576	89.5861	18
19	8.61276	0.11611	63.4397	7.3658	50.3637	96.9519	19
20	9.64629	0.10367	72.0524	7.4694	52.4370	104.4213	20
21	10.80385	0.09256	81.6987	7.5620	54.3808	111.9833	21
22	12.10031	0.08264	92.5026	7.6446	56.1989	119.6280	22
23	13.55235	0.07379	104.6029	7.7184	57.8960	127.3464	23
24	15.17863	0.06588	118.1552	7.7843	59.4772	135.1307	24
25	17.00006	0.05882	133.3339	7.8431	60.9478	142.9738	25
26	19.04007	0.05252	150.3339	7.8957	62.3133	150.8695	26
27	21.32488	0.04689	169.3740	7.9426	63.5794	158.8121	27
28	23.88387	0.04187	190.6989	7.9844	64.7518	166.7965	28
29	26.74993	0.03738	214.5828	8.0218	65.8359	174.8183	29
30	29.95992	0.03338	241.3327	8.0552	66.8372	182.8735	30
31	33.55511	0.02980	271.2926	8.0850	67.7611	190.9585	31
32	37.58173	0.02661	304.8477	8.1116	68.6126	199.0700	32
33	42.09153	0.02376	342.4294	8.1354	69.3966	207.2054	33
34	47.14252	0.02121	384.5210	8.1566	70.1178	215.3620	34
35	52.79962	0.01894	431.6635	8.1755	70.7807	223.5375	35
36	59.13557	0.01691	484.4631	8.1924	71.3894	231.7299	36
37	66.23184	0.01510	543.5987	8.2075	71.9481	239.9374	37
38	74.17966	0.01348	609.8305	8.2210	72.4604	248.1584	38
39	83.08122	0.01204	684.0102	8.2330	72.9298	256.3914	39
40	93.05097	0.01075	767.0914	8.2438	73.3596	264.6352	40
41	104.21709	0.00960	860.1424	8.2534	73.7531	272.8886	41
42	116.72314	0.00857	964.3595	8.2619	74.1129	281.1505	42
43	130.72991	0.00765	1081.0826	8.2696	74.4418	289.4201	43
44	146.41750	0.00683	1211.8125	8.2764	74.7423	297.6965	44
45	163.98760	0.00610	1358.2300	8.2825	75.0167	305.9790	45
46	183.66612	0.00544	1522.2176	8.2880	75.2672	314.2670	46
47	205.70605	0.00486	1705.8838	8.2928	75.4957	322.5598	47
48	230.39078	0.00434	1911.5898	8.2972	75.7040	330.8570	48
49	258.03767	0.00388	2141.9806	8.3010	75.8939	339.1580	49
50	289.00219	0.00346	2400.0182	8.3045	76.0669	347.4625	50
60	897.59693	0.00111	7471.6411	8.3240	77.1341	430.6329	60
70	2787.79983	0.00036	23223.3319	8.3303	77.5406	513.9138	70
80	8658.48310	0.00012	72145.6925	8.3324	77.6918	597.2302	80
90	26891.93422	0.00004	224091.1185	8.3330	77.7470	680.5581	90
100	83522.26573	0.00001	696010.5477	8.3332	77.7669	763.8897	100

Compound Interest

15%

i	0.150 000
$i^{(2)}$	0.144 761
$i^{(4)}$	0.142 232
$i^{(12)}$	0.140 579
δ	0.139 762
$(1+i)^{1/2}$	1.072 381
$(1+i)^{1/4}$	1.035 558
$(1+i)^{1/12}$	1.011 715
v	0.869 565
$v^{1/2}$	0.932 505
$v^{1/4}$	0.965 663
$v^{1/12}$	0.988 421
d	0.130 435
$d^{(2)}$	0.134 990
$d^{(4)}$	0.137 348
$d^{(12)}$	0.138 951
$i/i^{(2)}$	1.036 190
$i/i^{(4)}$	1.054 613
$i/i^{(12)}$	1.067 016
i/δ	1.073 254
$i/d^{(2)}$	1.111 190
$i/d^{(4)}$	1.092 113
$i/d^{(12)}$	1.079 516

n	$(1+i)^n$	v^n	$s_{\overline{n\vert}}$	$a_{\overline{n\vert}}$	$(Ia)_{\overline{n\vert}}$	$(Da)_{\overline{n\vert}}$	n
1	1.15000	0.86957	1.0000	0.8696	0.8696	0.8696	1
2	1.32250	0.75614	2.1500	1.6257	2.3819	2.4953	2
3	1.52088	0.65752	3.4725	2.2832	4.3544	4.7785	3
4	1.74901	0.57175	4.9934	2.8550	6.6414	7.6335	4
5	2.01136	0.49718	6.7424	3.3522	9.1273	10.9856	5
6	2.31306	0.43233	8.7537	3.7845	11.7213	14.7701	6
7	2.66002	0.37594	11.0668	4.1604	14.3528	18.9305	7
8	3.05902	0.32690	13.7268	4.4873	16.9680	23.4179	8
9	3.51788	0.28426	16.7858	4.7716	19.5264	28.1894	9
10	4.04556	0.24718	20.3037	5.0188	21.9982	33.2082	10
11	4.65239	0.21494	24.3493	5.2337	24.3626	38.4419	11
12	5.35025	0.18691	29.0017	5.4206	26.6055	43.8625	12
13	6.15279	0.16253	34.3519	5.5831	28.7184	49.4457	13
14	7.07571	0.14133	40.5047	5.7245	30.6970	55.1702	14
15	8.13706	0.12289	47.5804	5.8474	32.5404	61.0175	15
16	9.35762	0.10686	55.7175	5.9542	34.2502	66.9718	16
17	10.76126	0.09293	65.0751	6.0472	35.8300	73.0189	17
18	12.37545	0.08081	75.8364	6.1280	37.2845	79.1469	18
19	14.23177	0.07027	88.2118	6.1982	38.6195	85.3451	19
20	16.36654	0.06110	102.4436	6.2593	39.8415	91.6045	20
21	18.82152	0.05313	118.8101	6.3125	40.9572	97.9169	21
22	21.64475	0.04620	137.6316	6.3587	41.9737	104.2756	22
23	24.89146	0.04017	159.2764	6.3988	42.8977	110.6744	23
24	28.62518	0.03493	184.1678	6.4338	43.7361	117.1082	24
25	32.91895	0.03038	212.7930	6.4641	44.4955	123.5723	25
26	37.85680	0.02642	245.7120	6.4906	45.1823	130.0629	26
27	43.53531	0.02297	283.5688	6.5135	45.8025	136.5764	27
28	50.06561	0.01997	327.1041	6.5335	46.3618	143.1099	28
29	57.57545	0.01737	377.1697	6.5509	46.8655	149.6608	29
30	66.21177	0.01510	434.7451	6.5660	47.3186	156.2268	30
31	76.14354	0.01313	500.9569	6.5791	47.7257	162.8059	31
32	87.56507	0.01142	577.1005	6.5905	48.0911	169.3964	32
33	100.69983	0.00993	664.6655	6.6005	48.4188	175.9969	33
34	115.80480	0.00864	765.3654	6.6091	48.7124	182.6060	34
35	133.17552	0.00751	881.1702	6.6166	48.9752	189.2226	35
36	153.15185	0.00653	1014.3457	6.6231	49.2103	195.8458	36
37	176.12463	0.00568	1167.4975	6.6288	49.4204	202.4746	37
38	202.54332	0.00494	1343.6222	6.6338	49.6080	209.1083	38
39	232.92482	0.00429	1546.1655	6.6380	49.7754	215.7464	39
40	267.86355	0.00373	1779.0903	6.6418	49.9248	222.3881	40
41	308.04308	0.00325	2046.9539	6.6450	50.0579	229.0332	41
42	354.24954	0.00282	2354.9969	6.6478	50.1764	235.6810	42
43	407.38697	0.00245	2709.2465	6.6503	50.2820	242.3313	43
44	468.49502	0.00213	3116.6334	6.6524	50.3759	248.9838	44
45	538.76927	0.00186	3585.1285	6.6543	50.4594	255.6380	45
46	619.58466	0.00161	4123.8977	6.6559	50.5337	262.2940	46
47	712.52236	0.00140	4743.4824	6.6573	50.5996	268.9513	47
48	819.40071	0.00122	5456.0047	6.6585	50.6582	275.6098	48
49	942.31082	0.00106	6275.4055	6.6596	50.7102	282.2694	49
50	1083.65744	0.00092	7217.7163	6.6605	50.7563	288.9299	50

Compound Interest

20%

i	0.200 000
i^{2}	0.190 890
i^{4}	0.186 541
i^{12}	0.183 714
δ	0.182 322
$(1+i)^{1/2}$	1.095 445
$(1+i)^{1/4}$	1.046 635
$(1+i)^{1/12}$	1.015 309
v	0.833 333
$v^{1/2}$	0.912 871
$v^{1/4}$	0.955 443
$v^{1/12}$	0.984 921
d	0.166 667
d^{2}	0.174 258
d^{4}	0.178 229
d^{12}	0.180 943
i/i^{2}	1.047 723
i/i^{4}	1.072 153
i/i^{12}	1.088 651
i/δ	1.096 963
i/d^{2}	1.147 723
i/d^{4}	1.122 153
i/d^{12}	1.105 317

n	$(1+i)^n$	v^n	$s_{\overline{n\|}}$	$a_{\overline{n\|}}$	$(Ia)_{\overline{n\|}}$	$(Da)_{\overline{n\|}}$	n
1	1.20000	0.83333	1.0000	0.8333	0.8333	0.8333	1
2	1.44000	0.69444	2.2000	1.5278	2.2222	2.3611	2
3	1.72800	0.57870	3.6400	2.1065	3.9583	4.4676	3
4	2.07360	0.48225	5.3680	2.5887	5.8873	7.0563	4
5	2.48832	0.40188	7.4416	2.9906	7.8967	10.0469	5
6	2.98598	0.33490	9.9299	3.3255	9.9061	13.3724	6
7	3.58318	0.27908	12.9159	3.6046	11.8597	16.9770	7
8	4.29982	0.23257	16.4991	3.8372	13.7202	20.8142	8
9	5.15978	0.19381	20.7989	4.0310	15.4645	24.8452	9
10	6.19174	0.16151	25.9587	4.1925	17.0796	29.0376	10
11	7.43008	0.13459	32.1504	4.3271	18.5600	33.3647	11
12	8.91610	0.11216	39.5805	4.4392	19.9059	37.8039	12
13	10.69932	0.09346	48.4966	4.5327	21.1209	42.3366	13
14	12.83918	0.07789	59.1959	4.6106	22.2113	46.9472	14
15	15.40702	0.06491	72.0351	4.6755	23.1849	51.6226	15
16	18.48843	0.05409	87.4421	4.7296	24.0503	56.3522	16
17	22.18611	0.04507	105.9306	4.7746	24.8166	61.1268	17
18	26.62333	0.03756	128.1167	4.8122	25.4927	65.9390	18
19	31.94800	0.03130	154.7400	4.8435	26.0874	70.7825	19
20	38.33760	0.02608	186.6880	4.8696	26.6091	75.6521	20
21	46.00512	0.02174	225.0256	4.8913	27.0655	80.5434	21
22	55.20614	0.01811	271.0307	4.9094	27.4641	85.4528	22
23	66.24737	0.01509	326.2369	4.9245	27.8112	90.3774	23
24	79.49685	0.01258	392.4842	4.9371	28.1131	95.3145	24
25	95.39622	0.01048	471.9811	4.9476	28.3752	100.2621	25
26	114.47546	0.00874	567.3773	4.9563	28.6023	105.2184	26
27	137.37055	0.00728	681.8528	4.9636	28.7989	110.1820	27
28	164.84466	0.00607	819.2233	4.9697	28.9687	115.1517	28
29	197.81359	0.00506	984.0680	4.9747	29.1153	120.1264	29
30	237.37631	0.00421	1181.8816	4.9789	29.2417	125.1053	30
31	284.85158	0.00351	1419.2579	4.9824	29.3505	130.0878	31
32	341.82189	0.00293	1704.1095	4.9854	29.4442	135.0731	32
33	410.18627	0.00244	2045.9314	4.9878	29.5246	140.0609	33
34	492.22352	0.00203	2456.1176	4.9898	29.5937	145.0508	34
35	590.66823	0.00169	2948.3411	4.9915	29.6529	150.0423	35
36	708.80187	0.00141	3539.0094	4.9929	29.7037	155.0353	36
37	850.56225	0.00118	4247.8112	4.9941	29.7472	160.0294	37
38	1020.67470	0.00098	5098.3735	4.9951	29.7845	165.0245	38
39	1224.80964	0.00082	6119.0482	4.9959	29.8163	170.0204	39
40	1469.77157	0.00068	7343.8578	4.9966	29.8435	175.0170	40
41	1763.72588	0.00057	8813.6294	4.9972	29.8668	180.0142	41
42	2116.47106	0.00047	10577.3553	4.9976	29.8866	185.0118	42
43	2539.76527	0.00039	12693.8263	4.9980	29.9035	190.0098	43
44	3047.71832	0.00033	15233.5916	4.9984	29.9180	195.0082	44
45	3657.26199	0.00027	18281.3099	4.9986	29.9303	200.0068	45
46	4388.71439	0.00023	21938.5719	4.9989	29.9408	205.0057	46
47	5266.45726	0.00019	26327.2863	4.9991	29.9497	210.0047	47
48	6319.74872	0.00016	31593.7436	4.9992	29.9573	215.0040	48
49	7583.69846	0.00013	37913.4923	4.9993	29.9637	220.0033	49
50	9100.43815	0.00011	45497.1908	4.9995	29.9692	225.0027	50

Compound Interest

25%

i	0.250 000
i^{2}	0.236 068
i^{4}	0.229 485
i^{12}	0.225 231
δ	0.223 144
$(1+i)^{1/2}$	1.118 034
$(1+i)^{1/4}$	1.057 371
$(1+i)^{1/12}$	1.018 769
v	0.800 000
$v^{1/2}$	0.894 427
$v^{1/4}$	0.945 742
$v^{1/12}$	0.981 577
d	0.200 000
d^{2}	0.211 146
d^{4}	0.217 034
d^{12}	0.221 082
i/i^{2}	1.059 017
i/i^{4}	1.089 396
i/i^{12}	1.109 971
i/δ	1.120 355
i/d^{2}	1.184 017
i/d^{4}	1.151 896
i/d^{12}	1.130 804

n	$(1+i)^n$	v^n	$s_{\overline{n\rvert}}$	$a_{\overline{n\rvert}}$	$(Ia)_{\overline{n\rvert}}$	$(Da)_{\overline{n\rvert}}$	n
1	1.25000	0.80000	1.0000	0.8000	0.8000	0.8000	1
2	1.56250	0.64000	2.2500	1.4400	2.0800	2.2400	2
3	1.95313	0.51200	3.8125	1.9520	3.6160	4.1920	3
4	2.44141	0.40960	5.7656	2.3616	5.2544	6.5536	4
5	3.05176	0.32768	8.2070	2.6893	6.8928	9.2429	5
6	3.81470	0.26214	11.2588	2.9514	8.4657	12.1943	6
7	4.76837	0.20972	15.0735	3.1611	9.9337	15.3554	7
8	5.96046	0.16777	19.8419	3.3289	11.2758	18.6844	8
9	7.45058	0.13422	25.8023	3.4631	12.4838	22.1475	9
10	9.31323	0.10737	33.2529	3.5705	13.5575	25.7180	10
11	11.64153	0.08590	42.5661	3.6564	14.5024	29.3744	11
12	14.55192	0.06872	54.2077	3.7251	15.3271	33.0995	12
13	18.18989	0.05498	68.7596	3.7801	16.0418	36.8796	13
14	22.73737	0.04398	86.9495	3.8241	16.6575	40.7037	14
15	28.42171	0.03518	109.6868	3.8593	17.1853	44.5629	15
16	35.52714	0.02815	138.1085	3.8874	17.6356	48.4504	16
17	44.40892	0.02252	173.6357	3.9099	18.0184	52.3603	17
18	55.51115	0.01801	218.0446	3.9279	18.3427	56.2882	18
19	69.38894	0.01441	273.5558	3.9424	18.6165	60.2306	19
20	86.73617	0.01153	342.9447	3.9539	18.8471	64.1845	20
21	108.42022	0.00922	429.6809	3.9631	19.0408	68.1476	21
22	135.52527	0.00738	538.1011	3.9705	19.2031	72.1181	22
23	169.40659	0.00590	673.6264	3.9764	19.3389	76.0944	23
24	211.75824	0.00472	843.0329	3.9811	19.4522	80.0756	24
25	264.69780	0.00378	1054.7912	3.9849	19.5467	84.0604	25
26	330.87225	0.00302	1319.4890	3.9879	19.6252	88.0484	26
27	413.59031	0.00242	1650.3612	3.9903	19.6905	92.0387	27
28	516.98788	0.00193	2063.9515	3.9923	19.7447	96.0309	28
29	646.23485	0.00155	2580.9394	3.9938	19.7896	100.0248	29
30	807.79357	0.00124	3227.1743	3.9950	19.8267	104.0198	30
31	1009.74196	0.00099	4034.9678	3.9960	19.8574	108.0158	31
32	1262.17745	0.00079	5044.7098	3.9968	19.8827	112.0127	32
33	1577.72181	0.00063	6306.8872	3.9975	19.9037	116.0101	33
34	1972.15226	0.00051	7884.6091	3.9980	19.9209	120.0081	34
35	2465.19033	0.00041	9856.7613	3.9984	19.9351	124.0065	35
36	3081.48791	0.00032	12321.9516	3.9987	19.9468	128.0052	36
37	3851.85989	0.00026	15403.4396	3.9990	19.9564	132.0042	37
38	4814.82486	0.00021	19255.2994	3.9992	19.9643	136.0033	38
39	6018.53108	0.00017	24070.1243	3.9993	19.9708	140.0027	39
40	7523.16385	0.00013	30088.6554	3.9995	19.9761	144.0021	40
41	9403.95481	0.00011	37611.8192	3.9996	19.9804	148.0017	41
42	11754.94351	0.00009	47015.7740	3.9997	19.9840	152.0014	42
43	14693.67939	0.00007	58770.7175	3.9997	19.9869	156.0011	43
44	18367.09923	0.00005	73464.3969	3.9998	19.9893	160.0009	44
45	22958.87404	0.00004	91831.4962	3.9998	19.9913	164.0007	45
46	28698.59255	0.00003	114790.3702	3.9999	19.9929	168.0006	46
47	35873.24069	0.00003	143488.9627	3.9999	19.9942	172.0004	47
48	44841.55086	0.00002	179362.2034	3.9999	19.9953	176.0004	48
49	56051.93857	0.00002	224203.7543	3.9999	19.9961	180.0003	49
50	70064.92322	0.00001	280255.6929	3.9999	19.9969	184.0002	50

13 Population Mortality Tables: ELT15 (Males) and ELT15 (Females)

This table is based on the mortality of the population of England and Wales during the years 1990, 1991, and 1992. Full details are given in English Life Tables No. 15 published by The Stationery Office.

Note that no μ_0 values have been included because of the difficulty of calculating reasonable estimates from observed data.

ELT14(Males)

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
0	100000	814	0.00814		73.413	0
1	99186	62	0.00062	0.00080	73.019	1
2	99124	38	0.00038	0.00043	72.064	2
3	99086	30	0.00030	0.00033	71.091	3
4	99056	24	0.00024	0.00027	70.113	4
5	99032	22	0.00022	0.00023	69.130	5
6	99010	20	0.00020	0.00021	68.145	6
7	98990	18	0.00019	0.00019	67.158	7
8	98972	19	0.00018	0.00018	66.171	8
9	98953	18	0.00018	0.00018	65.183	9
10	98935	18	0.00018	0.00018	64.195	10
11	98917	18	0.00018	0.00018	63.206	11
12	98899	19	0.00019	0.00019	62.218	12
13	98880	23	0.00023	0.00021	61.230	13
14	98857	29	0.00029	0.00026	60.244	14
15	98828	39	0.00040	0.00034	59.261	15
16	98789	52	0.00052	0.00045	58.285	16
17	98737	74	0.00075	0.00064	57.315	17
18	98663	86	0.00087	0.00083	56.358	18
19	98577	81	0.00083	0.00085	55.406	19
20	98496	83	0.00084	0.00083	54.452	20
21	98 413	85	0.00086	0.00085	53.497	21
22	98328	87	0.00089	0.00088	52.543	22
23	98241	87	0.00089	0.00089	51.589	23
24	98154	87	0.00088	0.00089	50.635	24
25	98067	84	0.00086	0.00087	49.679	25
26	97983	83	0.00085	0.00085	48.721	26
27	97900	83	0.00085	0.00084	47.762	27
28	97817	85	0.00087	0.00086	46.802	28
29	97732	87	0.00090	0.00088	45.842	29
30	97645	89	0.00091	0.00090	44.883	30
31	97556	91	0.00094	0.00092	43.923	31
32	97465	95	0.00097	0.00096	42.964	32
33	97370	97	0.00099	0.00098	42.005	33
34	97273	103	0.00106	0.00102	41.046	34
35	97170	113	0.00116	0.00111	40.090	35
36	97057	124	0.00127	0.00122	39.136	36
37	96933	133	0.00138	0.00133	38.185	37
38	96800	145	0.00149	0.00144	37.237	38
39	96655	155	0.00160	0.00155	36.292	39
40	96500	166	0.00172	0.00166	35.349	40
41	96334	179	0.00186	0.00179	34.409	41
42	96155	194	0.00201	0.00193	33.473	42
43	95961	210	0.00219	0.00210	32.539	43
44	95751	230	0.00240	0.00229	31.609	44
45	95521	255	0.00266	0.00253	30.684	45
46	95266	283	0.00297	0.00281	29.765	46
47	94983	315	0.00332	0.00314	28.852	47
48	94668	352	0.00371	0.00352	27.947	48
49	94316	391	0.00415	0.00393	27.049	49
50	93925	436	0.00464	0.00440	26.159	50
51	93489	485	0.00519	0.00492	25.279	51
52	93004	537	0.00577	0.00549	24.408	52
53	92467	594	0.00642	0.00610	23.547	53
54	91873	656	0.00714	0.00679	22.696	54

ELT14(Males)

x	l_x	d_x	q_x	μ_x	e_x°	x
55	91217	727	0.00797	0.00757	21.856	55
56	90490	806	0.00890	0.00845	21.027	56
57	89684	892	0.00995	0.00945	20.211	57
58	88792	987	0.01112	0.01057	19.409	58
59	87805	1091	0.01243	0.01182	18.622	59
60	86714	1207	0.01392	0.01323	17.850	60
61	85507	1334	0.01560	0.01483	17.095	61
62	84173	1472	0.01749	0.01664	16.357	62
63	82701	1625	0.01965	0.01870	15.640	63
64	81076	1783	0.02199	0.02101	14.943	64
65	79293	1940	0.02447	0.02348	14.267	65
66	77353	2097	0.02711	0.02610	13.612	66
67	75256	2255	0.02997	0.02893	12.978	67
68	73001	2403	0.03292	0.03192	12.363	68
69	70598	2543	0.03602	0.03505	11.767	69
70	68055	2674	0.03930	0.03833	11.187	70
71	65381	2819	0.04311	0.04198	10.624	71
72	62562	2969	0.04745	0.04626	10.080	72
73	59593	3109	0.05217	0.05105	9.557	73
74	56484	3218	0.05697	0.05609	9.056	74
75	53266	3301	0.06197	0.06123	8.572	75
76	49965	3386	0.06777	0.06694	8.106	76
77	46579	3455	0.07418	0.07352	7.658	77
78	43124	3494	0.08101	0.08068	7.232	78
79	39630	3502	0.08838	0.08840	6.825	79
80	36128	3474	0.09616	0.09675	6.438	80
81	32654	3400	0.10411	0.10544	6.070	81
82	29254	3300	0.11279	0.11464	5.718	82
83	25954	3175	0.12235	0.12491	5.382	83
84	22779	3023	0.13270	0.13627	5.063	84
85	19756	2839	0.14372	0.14857	4.762	85
86	16917	2637	0.15585	0.16208	4.478	86
87	14280	2406	0.16848	0.17689	4.213	87
88	11874	2144	0.18061	0.19190	3.968	88
89	9730	1873	0.19246	0.20647	3.734	89
90	7857	1608	0.20465	0.22114	3.508	90
91	6249	1369	0.21911	0.23754	3.285	91
92	4880	1154	0.23655	0.25793	3.071	92
93	3726	953	0.25575	0.28226	2.872	93
94	2773	762	0.27483	0.30837	2.693	94
95	2011	590	0.29311	0.33424	2.531	95
96	1421	442	0.31104	0.35974	2.383	96
97	979	322	0.32919	0.38579	2.244	97
98	657	229	0.34783	0.41313	2.114	98
99	428	157	0.36712	0.44216	1.991	99
100	271	105	0.38705	0.47312	1.874	100
101	166	68	0.40760	0.50609	1.764	101
102	98	42	0.42870	0.54117	1.660	102
103	56	25	0.45030	0.57832	1.562	103
104	31	15	0.47428	0.61901	1.468	104
105	16	8	0.49634	0.66418	1.384	105
106	8	4	0.51841	0.70630	1.306	106
107	4	2	0.54041	0.75111	1.234	107
108	2	1	0.56225	0.79741	1.166	108
109	1	1	0.58385	0.84499	1.104	109

ELT15(Females)

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
0	100000	632	0.00632		78.956	0
1	99368	55	0.00055	0.00073	78.462	1
2	99313	30	0.00030	0.00035	77.505	2
3	99283	22	0.00022	0.00025	76.528	3
4	99261	18	0.00018	0.00020	75.545	4
5	99243	15	0.00016	0.00017	74.559	5
6	99228	15	0.00015	0.00015	73.570	6
7	99213	14	0.00014	0.00014	72.581	7
8	99199	14	0.00014	0.00014	71.591	8
9	99185	13	0.00013	0.00014	70.601	9
10	99172	13	0.00013	0.00013	69.610	10
11	99159	14	0.00014	0.00014	68.620	11
12	99145	14	0.00014	0.00014	67.629	12
13	99131	15	0.00015	0.00014	66.638	13
14	99116	18	0.00018	0.00017	65.649	14
15	99098	21	0.00022	0.00020	64.660	15
16	99077	26	0.00026	0.00024	63.674	16
17	99051	31	0.00031	0.00029	62.691	17
18	99020	31	0.00031	0.00031	61.710	18
19	98989	32	0.00032	0.00032	60.729	19
20	98957	31	0.00031	0.00032	59.748	20
21	98926	32	0.00032	0.00032	58.767	21
22	98894	32	0.00033	0.00032	57.786	22
23	98862	33	0.00033	0.00033	56.805	23
24	98829	32	0.00033	0.00033	55.823	24
25	98797	34	0.00034	0.00033	54.842	25
26	98763	34	0.00035	0.00034	53.860	26
27	98729	35	0.00036	0.00035	52.878	27
28	98694	38	0.00038	0.00037	51.897	28
29	98656	39	0.00040	0.00039	50.917	29
30	98617	43	0.00043	0.00042	49.937	30
31	98574	46	0.00047	0.00045	48.958	31
32	98528	51	0.00052	0.00050	47.981	32
33	98477	57	0.00057	0.00054	47.006	33
34	98420	61	0.00063	0.00060	46.032	34
35	98359	68	0.00069	0.00066	45.061	35
36	98291	74	0.00075	0.00072	44.092	36
37	98217	81	0.00082	0.00079	43.124	37
38	98136	88	0.00090	0.00086	42.160	38
39	98048	96	0.00098	0.00094	41.197	39
40	97952	105	0.00107	0.00102	40.237	40
41	97847	114	0.00117	0.00112	39.279	41
42	97733	126	0.00129	0.00123	38.325	42
43	97607	138	0.00142	0.00135	37.374	43
44	97469	154	0.00158	0.00149	36.426	44
45	97315	173	0.00177	0.00167	35.483	45
46	97142	192	0.00198	0.00187	34.545	46
47	96950	212	0.00219	0.00208	33.612	47
48	96738	234	0.00241	0.00230	32.685	48
49	96504	257	0.00266	0.00253	31.763	49
50	96247	283	0.00294	0.00280	30.846	50
51	95964	312	0.00326	0.00310	29.936	51
52	95652	342	0.00357	0.00342	29.032	52
53	95310	372	0.00390	0.00374	28.134	53
54	94938	406	0.00428	0.00408	27.242	54

ELT15(Females)

x	l_x	d_x	q_x	μ_x	$\mathring{e}_x$	x
55	94532	450	0.00475	0.00451	26.357	55
56	94082	499	0.00531	0.00503	25.481	56
57	93583	554	0.00592	0.00562	24.614	57
58	93029	614	0.00660	0.00626	23.757	58
59	92415	683	0.00739	0.00700	22.912	59
60	91732	761	0.00830	0.00786	22.079	60
61	90971	839	0.00922	0.00880	21.259	61
62	90132	915	0.01015	0.00972	20.452	62
63	89217	1007	0.01129	0.01074	19.657	63
64	88210	1117	0.01266	0.01203	18.875	64
65	87093	1218	0.01399	0.01342	18.111	65
66	85875	1308	0.01523	0.01470	17.361	66
67	84567	1417	0.01676	0.01609	16.621	67
68	83150	1533	0.01844	0.01774	15.896	68
69	81617	1647	0.02017	0.01949	15.185	69
70	79970	1751	0.02190	0.02123	14.487	70
71	78219	1876	0.02399	0.02311	13.800	71
72	76343	2056	0.02693	0.02569	13.127	72
73	74287	2239	0.03014	0.02897	12.476	73
74	72048	2366	0.03284	0.03203	11.848	74
75	69682	2487	0.03569	0.03480	11.234	75
76	67195	2634	0.03919	0.03803	10.631	76
77	64561	2812	0.04356	0.04214	10.044	77
78	61749	2984	0.04833	0.04694	9.478	78
79	58765	3158	0.05373	0.05228	8.934	79
80	55607	3314	0.05961	0.05827	8.413	80
81	52293	3435	0.06568	0.06464	7.914	81
82	48858	3526	0.07216	0.07131	7.435	82
83	45332	3596	0.07933	0.07861	6.974	83
84	41736	3655	0.08757	0.08691	6.532	84
85	38081	3706	0.09731	0.09674	6.111	85
86	34375	3724	0.10833	0.10841	5.715	86
87	30651	3634	0.11859	0.12052	5.349	87
88	27017	3475	0.12860	0.13174	5.002	88
89	23542	3330	0.14146	0.14462	4.667	89
90	20212	3143	0.15550	0.16053	4.354	90
91	17069	2903	0.17006	0.17751	4.065	91
92	14166	2631	0.18573	0.19573	3.797	92
93	11535	2321	0.20126	0.21498	3.551	93
94	9214	2008	0.21790	0.23490	3.322	94
95	7206	1702	0.23619	0.25732	3.112	95
96	5504	1395	0.25344	0.28114	2.925	96
97	4109	1102	0.26820	0.30267	2.754	97
98	3007	853	0.28352	0.32241	2.588	98
99	2154	653	0.30331	0.34628	2.422	99
100	1501	488	0.32489	0.37671	2.269	100
101	1013	350	0.34562	0.40887	2.133	101
102	663	240	0.36186	0.43769	2.011	102
103	423	161	0.37992	0.46273	1.887	103
104	262	105	0.40045	0.49300	1.758	104
105	157	68	0.43618	0.53729	1.621	105
106	89	41	0.45994	0.59908	1.518	106
107	48	23	0.48389	0.63785	1.425	107
108	25	13	0.50791	0.68388	1.338	108
109	12	6	0.53190	0.73191	1.257	109
110	6	3	0.55574	0.78181	1.183	110
111	3	2	0.57932	0.83337	1.114	111
112	1	1	0.60255	0.88629	1.050	112

14 Assured Lives Mortality Tables

AM92

This table is based on the mortality of assured male lives in the UK during the years 1991, 1992, 1993, and 1994. Full details are given in *C.M.I.R. 17*.

Due to potential rounding errors at high ages, the commutation functions (D_x, N_x, S_x, C_x, M_x and R_x) are tabulated here to age 110 only.

AM92

x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	x
17	9997.8091		10000.0000	17
18	9991.8904	9993.5400	9994.0000	18
19	9986.0351	9987.6338	9988.0636	19
20	9980.2432	9981.7911	9982.2006	20
21	9974.5046	9976.0016	9976.3909	21
22	9968.8391	9970.2654	9970.6346	22
23	9963.1967	9964.5824	9964.9313	23
24	9957.5775	9958.9225	9959.2613	24
25	9951.9913	9953.2858	9953.6144	25
26	9946.3982	9947.6622	9947.9807	26
27	9940.7984	9942.0218	9942.3402	27
28	9935.1818	9936.3549	9936.6730	28
29	9929.5088	9930.6613	9930.9694	29
30	9923.7497	9924.8916	9925.2094	30
31	9917.9145	9919.0260	9919.3535	31
32	9911.9538	9913.0547	9913.3821	32
33	9905.8282	9906.9285	9907.2655	33
34	9899.4984	9900.6078	9900.9645	34
35	9892.9151	9894.0536	9894.4299	35
36	9886.0395	9887.2069	9887.6126	36
37	9878.8128	9880.0288	9880.4540	37
38	9871.1665	9872.4508	9872.8954	38
39	9863.0227	9864.4047	9864.8688	39
40	9854.3036	9855.7931	9856.2863	40
41	9844.9025	9846.5384	9847.0510	41
42	9834.7030	9836.5245	9837.0661	42
43	9823.5994	9825.6354	9826.2060	43
44	9811.4473	9813.7463	9814.3359	44
45	9798.0837	9800.6939	9801.3123	45
46	9783.3371	9786.3162	9786.9534	46
47	9766.9983	9770.4231	9771.0789	47
48	9748.8603	9752.7874	9753.4714	48
49	9728.6499	9733.1938	9733.8865	49
50	9706.0977	9711.3524	9712.0728	50
51	9680.8990	9686.9669	9687.7149	51
52	9652.6965	9659.7075	9660.5021	52
53	9621.1006	9629.2115	9630.0522	53
54	9585.6916	9595.0563	9595.9715	54
55	9545.9929	9556.8003	9557.8179	55
56	9501.4839	9513.9375	9515.1040	56
57	9451.5938	9465.9293	9467.2906	57
58	9395.6971	9412.1712	9413.8004	58
59	9333.1284	9352.0165	9354.0040	59
60	9263.1422	9284.7641	9287.2164	60
61	9184.9687	9209.6568	9212.7143	61
62	9097.7405	9125.8818	9129.7170	62
63	9000.5884	9032.5642	9037.3973	63
64	8892.5741	8928.8177	8934.8771	64

AM92

x	$l_{[x]}$	$l_{[x-1]+1}$	l_x	x
65	8772.7359	8813.6881	8821.2612	65
66	8640.0481	8686.2016	8695.6199	66
67	8493.5187	8545.3532	8557.0118	67
68	8332.1396	8390.1611	8404.4916	68
69	8154.9318	8219.6390	8237.1329	69
70	7960.9776	8032.8606	8054.0544	70
71	7749.4659	7828.9686	7854.4508	71
72	7519.7027	7607.2400	7637.6208	72
73	7271.1461	7367.0828	7403.0084	73
74	7003.5216	7108.1052	7150.2401	74
75	6716.8231	6830.1844	6879.1673	75
76	6411.3459	6533.5008	6589.9258	76
77	6087.8084	6218.5759	6282.9803	77
78	5747.3624	5886.3628	5959.1680	78
79	5391.6400	5538.2791	5619.7577	79
80	5022.7931	5176.2224	5266.4604	80
81	4643.5129	4802.6290	4901.4789	81
82	4257.0056	4420.4525	4527.4960	82
83	3866.9884	4033.1467	4147.6708	83
84	3477.5929	3644.6327	3765.5998	84
85	3093.2863	3259.1862	3385.2479	85
86	2718.7128	2881.3467	3010.8395	86
87	2358.5299	2515.7310	2646.7416	87
88	2017.2298	2166.8805	2297.2976	88
89	1698.9089	1839.0458	1966.6499	89
90	1407.0550	1535.9801	1658.5545	90
91		1260.7354	1376.1906	91
92			1121.9889	92
93			897.5025	93
94			703.3242	94
95			539.0643	95
96			403.4023	96
97			294.2061	97
98			208.7060	98
99			143.7120	99
100			95.8476	100
101			61.7733	101
102			38.3796	102
103			22.9284	103
104			13.1359	104
105			7.1968	105
106			3.7596	106
107			1.8669	107
108			0.8784	108
109			0.3903	109
110			0.1632	110
111			0.0640	111
112			0.0234	112
113			0.0080	113
114			0.0025	114
115			0.0007	115
116			0.0002	116
117			0.0000	117
118			0.0000	118
119			0.0000	119
120			0.0000	120

AM92

x	$d_{[x]}$	$d_{[x-1]+1}$	d_x	x
17	4.2691		6.0000	17
18	4.2565	5.4765	5.9364	18
19	4.2441	5.4333	5.8630	19
20	4.2416	5.4001	5.8096	20
21	4.2392	5.3671	5.7564	21
22	4.2567	5.3341	5.7032	22
23	4.2742	5.3211	5.6700	23
24	4.2917	5.3081	5.6469	24
25	4.3291	5.3051	5.6337	25
26	4.3764	5.3220	5.6405	26
27	4.4435	5.3488	5.6671	27
28	4.5205	5.3855	5.7037	28
29	4.6172	5.4519	5.7600	29
30	4.7237	5.5381	5.8559	30
31	4.8598	5.6439	5.9715	31
32	5.0254	5.7892	6.1166	32
33	5.2204	5.9640	6.3010	33
34	5.4447	6.1780	6.5346	34
35	5.7082	6.4410	6.8173	35
36	6.0107	6.7530	7.1586	36
37	6.3620	7.1334	7.5585	37
38	6.7617	7.5820	8.0267	38
39	7.2296	8.1184	8.5824	39
40	7.7652	8.7421	9.2353	40
41	8.3780	9.4724	9.9849	41
42	9.0676	10.3185	10.8601	42
43	9.8531	11.2995	11.8701	43
44	10.7533	12.4340	13.0236	44
45	11.7675	13.7406	14.3589	45
46	12.9140	15.2373	15.8744	46
47	14.2110	16.9517	17.6075	47
48	15.6664	18.9009	19.5850	48
49	17.2975	21.1210	21.8136	49
50	19.1307	23.6374	24.3579	50
51	21.1915	26.4648	27.2128	51
52	23.4850	29.6553	30.4499	52
53	26.0443	33.2400	34.0808	53
54	28.8913	37.2384	38.1536	54
55	32.0554	41.6963	42.7139	55
56	35.5546	46.6468	47.8134	56
57	39.4226	52.1289	53.4902	57
58	43.6806	58.1672	59.7965	58
59	48.3643	64.8001	66.7876	59
60	53.4854	72.0498	74.5020	60
61	59.0869	79.9398	82.9973	61
62	65.1762	88.4846	92.3197	62
63	71.7707	97.6872	102.5202	63
64	78.8860	107.5565	113.6159	64

AM92

x	$d_{[x]}$	$d_{[x-1]+1}$	d_x	x
65	86.5343	118.0682	125.6412	65
66	94.6949	129.1899	138.6082	66
67	103.3576	140.8616	152.5202	67
68	112.5005	153.0281	167.3586	68
69	122.0712	165.5846	183.0785	69
70	132.0089	178.4098	199.6036	70
71	142.2259	191.3478	216.8300	71
72	152.6199	204.2316	234.6124	72
73	163.0409	216.8427	252.7683	73
74	173.3372	228.9379	271.0728	74
75	183.3223	240.2586	289.2415	75
76	192.7699	250.5206	306.9456	76
77	201.4456	259.4079	323.8122	77
78	209.0833	266.6051	339.4104	78
79	215.4176	271.8187	353.2973	79
80	220.1641	274.7435	364.9815	80
81	223.0604	275.1330	373.9828	81
82	223.8589	272.7817	379.8252	82
83	222.3557	267.5468	382.0710	83
84	218.4067	259.3849	380.3519	84
85	211.9396	248.3467	374.4084	85
86	202.9818	234.6050	364.0978	86
87	191.6494	218.4334	349.4440	87
88	178.1839	200.2306	330.6478	88
89	162.9288	180.4913	308.0954	89
90	146.3197	159.7895	282.3639	90
91		138.7464	254.2017	91
92			224.4864	92
93			194.1783	93
94			164.2600	94
95			135.6620	95
96			109.1962	96
97			85.5001	97
98			64.9940	98
99			47.8644	99
100			34.0743	100
101			23.3937	101
102			15.4512	102
103			9.7925	103
104			5.9391	104
105			3.4373	105
106			1.8927	106
107			.9885	107
108			.4881	108
109			.2271	109
110			.0992	110
111			.0405	111
112			.0154	112
113			.0055	113
114			.0018	114
115			.0005	115
116			.0001	116
117			.0000	117
118			.0000	118
119			.0000	119
120			.0000	120

AM92

x	$q_{[x]}$	$q_{[x-1]+1}$	q_x	x
17	.000427		.000600	17
18	.000426	.000548	.000594	18
19	.000425	.000544	.000587	19
20	.000425	.000541	.000582	20
21	.000425	.000538	.000577	21
22	.000427	.000535	.000572	22
23	.000429	.000534	.000569	23
24	.000431	.000533	.000567	24
25	.000435	.000533	.000566	25
26	.000440	.000535	.000567	26
27	.000447	.000538	.000570	27
28	.000455	.000542	.000574	28
29	.000465	.000549	.000580	29
30	.000476	.000558	.000590	30
31	.000490	.000569	.000602	31
32	.000507	.000584	.000617	32
33	.000527	.000602	.000636	33
34	.000550	.000624	.000660	34
35	.000577	.000651	.000689	35
36	.000608	.000683	.000724	36
37	.000644	.000722	.000765	37
38	.000685	.000768	.000813	38
39	.000733	.000823	.000870	39
40	.000788	.000887	.000937	40
41	.000851	.000962	.001014	41
42	.000922	.001049	.001104	42
43	.001003	.001150	.001208	43
44	.001096	.001267	.001327	44
45	.001201	.001402	.001465	45
46	.001320	.001557	.001622	46
47	.001455	.001735	.001802	47
48	.001607	.001938	.002008	48
49	.001778	.002170	.002241	49
50	.001971	.002434	.002508	50
51	.002189	.002732	.002809	51
52	.002433	.003070	.003152	52
53	.002707	.003452	.003539	53
54	.003014	.003881	.003976	54
55	.003358	.004363	.004469	55
56	.003742	.004903	.005025	56
57	.004171	.005507	.005650	57
58	.004649	.006180	.006352	58
59	.005182	.006929	.007140	59
60	.005774	.007760	.008022	60
61	.006433	.008680	.009009	61
62	.007164	.009696	.010112	62
63	.007974	.010815	.011344	63
64	.008871	.012046	.012716	64

AM92

x	$q_{[x]}$	$q_{[x-1]+1}$	q_x	x
65	.009864	.013396	.014243	65
66	.010960	.014873	.015940	66
67	.012169	.016484	.017824	67
68	.013502	.018239	.019913	68
69	.014969	.020145	.022226	69
70	.016582	.022210	.024783	70
71	.018353	.024441	.027606	71
72	.020296	.026847	.030718	72
73	.022423	.029434	.034144	73
74	.024750	.032208	.037911	74
75	.027293	.035176	.042046	75
76	.030067	.038344	.046578	76
77	.033090	.041715	.051538	77
78	.036379	.045292	.056956	78
79	.039954	.049080	.062867	79
80	.043833	.053078	.069303	80
81	.048037	.057288	.076300	81
82	.052586	.061709	.083893	82
83	.057501	.066337	.092117	83
84	.062804	.071169	.101007	84
85	.068516	.076199	.110600	85
86	.074661	.081422	.120929	86
87	.081258	.086827	.132028	87
88	.088331	.092405	.143929	88
89	.095902	.098144	.156660	89
90	.103990	.104031	.170247	90
91		.110052	.184714	91
92			.200079	92
93			.216354	93
94			.233548	94
95			.251662	95
96			.270688	96
97			.290613	97
98			.311414	98
99			.333058	99
100			.355505	100
101			.378702	101
102			.402588	102
103			.427090	103
104			.452127	104
105			.477608	105
106			.503432	106
107			.529493	107
108			.555674	108
109			.581857	109
110			.607918	110
111			.633731	111
112			.659171	112
113			.684114	113
114			.708442	114
115			.732042	115
116			.754809	116
117			.776648	117
118			.797477	118
119			.817225	119
120			1.000000	120

AM92

x	$\mu_{[x]}$	$\mu_{[x-1]+1}$	μ_x	x
17	0.000367		0.000603	17
18	0.000367	0.000488	0.000597	18
19	0.000367	0.000485	0.000591	19
20	0.000369	0.000483	0.000585	20
21	0.000370	0.000482	0.000580	21
22	0.000374	0.000480	0.000574	22
23	0.000377	0.000481	0.000570	23
24	0.000380	0.000481	0.000568	24
25	0.000385	0.000482	0.000566	25
26	0.000391	0.000485	0.000566	26
27	0.000400	0.000489	0.000568	27
28	0.000408	0.000495	0.000572	28
29	0.000419	0.000502	0.000577	29
30	0.000430	0.000512	0.000585	30
31	0.000443	0.000523	0.000596	31
32	0.000460	0.000537	0.000609	32
33	0.000479	0.000555	0.000626	33
34	0.000500	0.000576	0.000647	34
35	0.000524	0.000601	0.000674	35
36	0.000551	0.000630	0.000706	36
37	0.000582	0.000665	0.000744	37
38	0.000616	0.000706	0.000788	38
39	0.000656	0.000754	0.000840	39
40	0.000701	0.000810	0.000902	40
41	0.000752	0.000875	0.000974	41
42	0.000808	0.000950	0.001057	42
43	0.000871	0.001037	0.001154	43
44	0.000943	0.001136	0.001265	44
45	0.001023	0.001250	0.001394	45
46	0.001113	0.001380	0.001541	46
47	0.001214	0.001529	0.001709	47
48	0.001326	0.001698	0.001902	48
49	0.001451	0.001890	0.002122	49
50	0.001592	0.002108	0.002372	50
51	0.001750	0.002354	0.002656	51
52	0.001925	0.002633	0.002978	52
53	0.002122	0.002947	0.003343	53
54	0.002342	0.003300	0.003756	54
55	0.002588	0.003696	0.004221	55
56	0.002862	0.004139	0.004747	56
57	0.003170	0.004636	0.005340	57
58	0.003513	0.005189	0.006005	58
59	0.003898	0.005806	0.006754	59
60	0.004327	0.006493	0.007593	60
61	0.004809	0.007254	0.008533	61
62	0.005348	0.008099	0.009586	62
63	0.005949	0.009032	0.010763	63
64	0.006623	0.010063	0.012078	64

AM92

x	$\mu_{[x]}$	$\mu_{[x-1]+1}$	μ_x	x
65	0.007377	0.011199	0.013544	65
66	0.008220	0.012449	0.015176	66
67	0.009162	0.013821	0.016993	67
68	0.010216	0.015326	0.019012	68
69	0.011393	0.016972	0.021255	69
70	0.012709	0.018771	0.023741	70
71	0.014178	0.020733	0.026496	71
72	0.015819	0.022869	0.029543	72
73	0.017648	0.025190	0.032912	73
74	0.019687	0.027708	0.036631	74
75	0.021959	0.030436	0.040732	75
76	0.024487	0.033385	0.045251	76
77	0.027300	0.036569	0.050223	77
78	0.030423	0.040000	0.055689	78
79	0.033892	0.043691	0.061689	79
80	0.037737	0.047656	0.068271	80
81	0.041996	0.051909	0.075481	81
82	0.046709	0.056462	0.083372	82
83	0.051916	0.061329	0.091999	83
84	0.057665	0.066524	0.101417	84
85	0.064000	0.072061	0.111691	85
86	0.070978	0.077952	0.122884	86
87	0.078646	0.084213	0.135066	87
88	0.087067	0.090853	0.148309	88
89	0.096302	0.097889	0.162691	89
90	0.106409	0.105333	0.178289	90
91		0.113198	0.195190	91
92			0.213482	92
93			0.233257	93
94			0.254610	94
95			0.277645	95
96			0.302462	96
97			0.329170	97
98			0.357882	98
99			0.388711	99
100			0.421777	100
101			0.457202	101
102			0.495111	102
103			0.535631	103
104			0.578890	104
105			0.625023	105
106			0.674162	106
107			0.726443	107
108			0.782002	108
109			0.840973	109
110			0.903494	110
111			0.969700	111
112			1.039723	112
113			1.113695	113
114			1.191744	114
115			1.274000	115
116			1.360581	116
117			1.451603	117
118			1.547178	118
119			1.647417	119
120			2.000000	120

AM92

x	$e_{[x]}$	$e_{[x-1]+1}$	e_x	x
17	61.353		61.339	17
18	60.389	60.379	60.376	18
19	59.424	59.414	59.412	19
20	58.458	58.449	58.447	20
21	57.492	57.483	57.481	21
22	56.524	56.516	56.514	22
23	55.556	55.548	55.546	23
24	54.587	54.580	54.578	24
25	53.618	53.611	53.609	25
26	52.648	52.641	52.639	26
27	51.677	51.671	51.669	27
28	50.706	50.700	50.699	28
29	49.735	49.729	49.728	29
30	48.764	48.758	48.757	30
31	47.792	47.787	47.785	31
32	46.821	46.816	46.814	32
33	45.850	45.845	45.843	33
34	44.879	44.874	44.872	34
35	43.909	43.904	43.902	35
36	42.939	42.934	42.932	36
37	41.970	41.965	41.963	37
38	41.003	40.997	40.995	38
39	40.036	40.031	40.029	39
40	39.071	39.066	39.064	40
41	38.108	38.102	38.100	41
42	37.148	37.141	37.139	42
43	36.189	36.182	36.180	43
44	35.234	35.226	35.224	44
45	34.282	34.273	34.271	45
46	33.333	33.323	33.321	46
47	32.388	32.377	32.375	47
48	31.448	31.436	31.433	48
49	30.513	30.499	30.497	49
50	29.583	29.567	29.565	50
51	28.660	28.642	28.639	51
52	27.742	27.722	27.720	52
53	26.833	26.810	26.808	53
54	25.931	25.905	25.903	54
55	25.037	25.009	25.006	55
56	24.153	24.122	24.119	56
57	23.279	23.244	23.240	57
58	22.415	22.376	22.373	58
59	21.563	21.520	21.516	59
60	20.724	20.676	20.670	60
61	19.897	19.844	19.837	61
62	19.084	19.026	19.018	62
63	18.286	18.222	18.212	63
64	17.503	17.433	17.421	64

AM92

x	$e_{[x]}$	$e_{[x-1]+1}$	e_x	x
65	16.736	16.660	16.645	65
66	15.987	15.903	15.886	66
67	15.255	15.164	15.143	67
68	14.541	14.443	14.418	68
69	13.847	13.740	13.711	69
70	13.172	13.057	13.023	70
71	12.517	12.394	12.354	71
72	11.883	11.751	11.704	72
73	11.270	11.129	11.075	73
74	10.679	10.529	10.467	74
75	10.110	9.950	9.879	75
76	9.562	9.393	9.313	76
77	9.037	8.859	8.768	77
78	8.534	8.346	8.244	78
79	8.053	7.856	7.742	79
80	7.594	7.388	7.261	80
81	7.157	6.942	6.802	81
82	6.741	6.518	6.364	82
83	6.347	6.116	5.947	83
84	5.974	5.734	5.550	84
85	5.620	5.374	5.174	85
86	5.287	5.034	4.817	86
87	4.972	4.713	4.480	87
88	4.676	4.412	4.161	88
89	4.397	4.129	3.861	89
90	4.136	3.864	3.578	90
91		3.616	3.312	91
92			3.063	92
93			2.829	93
94			2.610	94
95			2.405	95
96			2.214	96
97			2.035	97
98			1.869	98
99			1.715	99
100			1.571	100
101			1.437	101
102			1.314	102
103			1.199	103
104			1.093	104
105			0.994	105
106			0.904	106
107			0.820	107
108			0.743	108
109			0.672	109
110			0.606	110
111			0.546	111
112			0.491	112
113			0.440	113
114			0.394	114
115			0.352	115
116			0.313	116
117			0.277	117
118			0.240	118
119			0.183	119
120			0.000	120

AM92

4%

x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	x
17	5132.61		5133.73	17
18	4932.28	4933.09	4933.32	18
19	4739.80	4740.55	4740.76	19
20	4554.85	4555.56	4555.75	20
21	4377.15	4377.80	4377.98	21
22	4206.41	4207.01	4207.16	22
23	4042.33	4042.89	4043.04	23
24	3884.66	3885.19	3885.32	24
25	3733.16	3733.64	3733.77	25
26	3587.56	3588.01	3588.13	26
27	3447.63	3448.06	3448.17	27
28	3313.16	3313.55	3313.66	28
29	3183.91	3184.28	3184.38	29
30	3059.68	3060.03	3060.13	30
31	2940.27	2940.60	2940.69	31
32	2825.48	2825.79	2825.89	32
33	2715.13	2715.43	2715.52	33
34	2609.03	2609.33	2609.42	34
35	2507.02	2507.31	2507.40	35
36	2408.92	2409.20	2409.30	36
37	2314.57	2314.86	2314.96	37
38	2223.83	2224.12	2224.22	38
39	2136.53	2136.83	2136.93	39
40	2052.54	2052.85	2052.96	40
41	1971.72	1972.04	1972.15	41
42	1893.92	1894.27	1894.37	42
43	1819.02	1819.40	1819.50	43
44	1746.89	1747.30	1747.41	44
45	1677.42	1677.86	1677.97	45
46	1610.47	1610.96	1611.07	46
47	1545.95	1546.49	1546.59	47
48	1483.73	1484.32	1484.43	48
49	1423.70	1424.37	1424.47	49
50	1365.77	1366.51	1366.61	50
51	1309.83	1310.65	1310.75	51
52	1255.78	1256.70	1256.80	52
53	1203.53	1204.55	1204.65	53
54	1152.98	1154.11	1154.22	54
55	1104.05	1105.30	1105.41	55
56	1056.63	1058.02	1058.15	56
57	1010.66	1012.19	1012.34	57
58	966.04	967.73	967.90	58
59	922.70	924.57	924.76	59
60	880.56	882.61	882.85	60
61	839.55	841.80	842.08	61
62	799.59	802.06	802.40	62
63	760.62	763.33	763.74	63
64	722.59	725.54	726.03	64

AM92

4%

x	$D_{[x]}$	$D_{[x-1]+1}$	D_x	x
65	685.44	688.64	689.23	65
66	649.11	652.57	653.28	66
67	613.56	617.30	618.14	67
68	578.75	582.78	583.77	68
69	544.65	548.97	550.14	69
70	511.25	515.87	517.23	70
71	478.53	483.43	485.01	71
72	446.48	451.68	453.48	72
73	415.12	420.59	422.64	73
74	384.46	390.20	392.51	74
75	354.54	360.52	363.11	75
76	325.40	331.60	334.46	76
77	297.09	303.48	306.62	77
78	269.69	276.21	279.63	78
79	243.27	249.89	253.56	79
80	217.91	224.57	228.48	80
81	193.71	200.35	204.47	81
82	170.75	177.31	181.60	82
83	149.14	155.55	159.97	83
84	128.97	135.16	139.65	84
85	110.30	116.22	120.71	85
86	93.22	98.79	103.23	86
87	77.76	82.94	87.26	87
88	63.95	68.69	72.83	88
89	51.78	56.06	59.95	89
90	41.24	45.02	48.61	90
91		35.53	38.78	91
92			30.40	92
93			23.38	93
94			17.62	94
95			12.99	95
96			9.34	96
97			6.55	97
98			4.47	98
99			2.96	99
100			1.90	100
101			1.18	101
102			.70	102
103			.40	103
104			.22	104
105			.12	105
106			.06	106
107			.03	107
108			.01	108
109			.01	109
110			.00	110

AM92

4%

x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	x
17	119958.58		119959.94	17
18	114824.96	114825.98	114826.20	18
19	109891.73	109892.68	109892.88	19
20	105151.06	105151.94	105152.13	20
21	100595.40	100596.21	100596.38	21
22	96217.50	96218.25	96218.40	22
23	92010.40	92011.10	92011.24	23
24	87967.43	87968.07	87968.21	24
25	84082.16	84082.76	84082.88	25
26	80348.43	80349.00	80349.12	26
27	76760.35	76760.88	76760.99	27
28	73312.22	73312.71	73312.82	28
29	69998.60	69999.06	69999.16	29
30	66814.23	66814.68	66814.78	30
31	63754.13	63754.56	63754.65	31
32	60813.46	60813.87	60813.96	32
33	57987.58	57987.98	57988.07	33
34	55272.07	55272.45	55272.55	34
35	52662.65	52663.03	52663.13	35
36	50155.24	50155.63	50155.73	36
37	47745.94	47746.33	47746.43	37
38	45430.98	45431.37	45431.47	38
39	43206.74	43207.15	43207.25	39
40	41069.80	41070.21	41070.31	40
41	39016.82	39017.25	39017.36	41
42	37044.65	37045.10	37045.21	42
43	35150.25	35150.73	35150.84	43
44	33330.72	33331.23	33331.34	44
45	31583.27	31583.82	31583.93	45
46	29905.26	29905.86	29905.96	46
47	28294.14	28294.79	28294.89	47
48	26747.50	26748.20	26748.30	48
49	25263.01	25263.77	25263.87	49
50	23838.46	23839.30	23839.41	50
51	22471.77	22472.69	22472.79	51
52	21160.92	21161.94	21162.04	52
53	19904.01	19905.14	19905.24	53
54	18699.23	18700.48	18700.59	54
55	17544.87	17546.25	17546.37	55
56	16439.29	16440.82	16440.95	56
57	15380.96	15382.66	15382.81	57
58	14368.41	14370.30	14370.47	58
59	13400.27	13402.37	13402.57	59
60	12475.24	12477.57	12477.80	60
61	11592.08	11594.68	11594.96	61
62	10749.66	10752.54	10752.88	62
63	9946.87	9950.07	9950.48	63
64	9182.71	9186.25	9186.74	64

AM92

x	$N_{[x]}$	$N_{[x-1]+1}$	N_x	x	4%
65	8456.21	8460.12	8460.71	65	
66	7766.46	7770.77	7771.48	66	
67	7112.62	7117.36	7118.20	67	
68	6493.86	6499.06	6500.06	68	
69	5909.43	5915.12	5916.29	69	
70	5358.59	5364.78	5366.14	70	
71	4840.63	4847.34	4848.92	71	
72	4354.86	4362.10	4363.91	72	
73	3900.59	3908.38	3910.43	73	
74	3477.14	3485.47	3487.78	74	
75	3083.84	3092.69	3095.27	75	
76	2719.96	2729.30	2732.16	76	
77	2384.76	2394.56	2397.70	77	
78	2077.47	2087.67	2091.08	78	
79	1797.25	1807.78	1811.45	79	
80	1543.20	1553.98	1557.89	80	
81	1314.35	1325.29	1329.41	81	
82	1109.67	1120.65	1124.94	82	
83	928.03	938.92	943.34	83	
84	768.19	778.88	783.37	84	
85	628.87	639.22	643.72	85	
86	508.67	518.57	523.01	86	
87	406.14	415.45	419.77	87	
88	319.75	328.38	332.51	88	
89	247.93	255.80	259.69	89	
90	189.12	196.15	199.74	90	
91		147.88	151.13	91	
92			112.35	92	
93			81.95	93	
94			58.56	94	
95			40.94	95	
96			27.95	96	
97			18.61	97	
98			12.06	98	
99			7.59	99	
100			4.63	100	
101			2.73	101	
102			1.55	102	
103			.85	103	
104			.45	104	
105			.23	105	
106			.11	106	
107			.05	107	
108			.02	108	
109			.01	109	
110			.00	110	

AM92

4%

x	$S_{[x]}$	$S_{[x-1]+1}$	S_x	x
17	2398085.62		2398087.20	17
18	2278125.81	2278127.03	2278127.26	18
19	2163299.72	2163300.85	2163301.06	19
20	2053406.94	2053407.99	2053408.17	20
21	1948254.91	1948255.88	1948256.05	21
22	1847658.63	1847659.51	1847659.67	22
23	1751440.30	1751441.12	1751441.27	23
24	1659429.12	1659429.89	1659430.03	24
25	1571460.98	1571461.70	1571461.82	25
26	1487378.14	1487378.82	1487378.94	26
27	1407029.07	1407029.71	1407029.82	27
28	1330268.14	1330268.73	1330268.83	28
29	1256955.35	1256955.92	1256956.02	29
30	1186956.21	1186956.76	1186956.85	30
31	1120141.46	1120141.98	1120142.07	31
32	1056386.83	1056387.32	1056387.42	32
33	995572.87	995573.36	995573.46	33
34	937584.81	937585.29	937585.38	34
35	882312.25	882312.74	882312.84	35
36	829649.12	829649.61	829649.71	36
37	779493.40	779493.88	779493.98	37
38	731746.96	731747.45	731747.56	38
39	686315.48	686315.99	686316.09	39
40	643108.22	643108.74	643108.84	40
41	602037.89	602038.43	602038.53	41
42	563020.51	563021.07	563021.17	42
43	525975.27	525975.86	525975.96	43
44	490824.40	490825.02	490825.13	44
45	457493.03	457493.69	457493.79	45
46	425909.06	425909.76	425909.86	46
47	396003.05	396003.80	396003.90	47
48	367708.11	367708.91	367709.01	48
49	340959.74	340960.61	340960.71	49
50	315695.79	315696.73	315696.84	50
51	291856.30	291857.33	291857.43	51
52	269383.41	269384.53	269384.64	52
53	248221.26	248222.49	248222.60	53
54	228315.88	228317.24	228317.35	54
55	209615.14	209616.65	209616.77	55
56	192068.59	192070.27	192070.40	56
57	175627.43	175629.30	175629.44	57
58	160244.38	160246.47	160246.64	58
59	145873.64	145875.97	145876.17	59
60	132470.75	132473.37	132473.60	60
61	119992.59	119995.52	119995.80	61
62	108397.21	108400.50	108400.84	62
63	97643.87	97647.55	97647.96	63
64	87692.86	87696.99	87697.49	64

AM92

x	$S_{[x]}$	$S_{[x-1]+1}$	S_x	x	4%
65	78505.54	78510.15	78510.74	65	
66	70044.17	70049.32	70050.03	66	
67	62271.97	62277.71	62278.55	67	
68	55152.99	55159.35	55160.35	68	
69	48652.08	48659.12	48660.29	69	
70	42734.88	42742.64	42744.01	70	
71	37367.77	37376.29	37377.86	71	
72	32517.84	32527.14	32528.95	72	
73	28152.89	28162.99	28165.04	73	
74	24241.39	24252.30	24254.61	74	
75	20752.53	20764.24	20766.83	75	
76	17656.21	17668.69	17671.56	76	
77	14923.03	14936.25	14939.39	77	
78	12524.40	12538.27	12541.69	78	
79	10432.48	10446.93	10450.60	79	
80	8620.33	8635.24	8639.15	80	
81	7061.91	7077.14	7081.26	81	
82	5732.17	5747.56	5751.85	82	
83	4607.11	4622.49	4626.91	83	
84	3663.90	3679.09	3683.57	84	
85	2880.92	2895.71	2900.21	85	
86	2237.83	2252.05	2256.49	86	
87	1715.71	1729.16	1733.48	87	
88	1297.05	1309.57	1313.71	88	
89	965.85	977.30	981.19	89	
90	707.63	717.91	721.51	90	
91		518.51	521.76	91	
92			370.63	92	
93			258.28	93	
94			176.34	94	
95			117.78	95	
96			76.84	96	
97			48.88	97	
98			30.28	98	
99			18.22	99	
100			10.63	100	
101			6.00	101	
102			3.27	102	
103			1.72	103	
104			.87	104	
105			.42	105	
106			.19	106	
107			.09	107	
108			.04	108	
109			.01	109	
110			.01	110	

AM92

4%	x	$C_{[x]}$	$C_{[x-1]+1}$	C_x	x
	17	2.11		2.96	17
	18	2.02	2.60	2.82	18
	19	1.94	2.48	2.68	19
	20	1.86	2.37	2.55	20
	21	1.79	2.26	2.43	21
	22	1.73	2.16	2.31	22
	23	1.67	2.08	2.21	23
	24	1.61	1.99	2.12	24
	25	1.56	1.91	2.03	25
	26	1.52	1.85	1.96	26
	27	1.48	1.78	1.89	27
	28	1.45	1.73	1.83	28
	29	1.42	1.68	1.78	29
	30	1.40	1.64	1.74	30
	31	1.39	1.61	1.70	31
	32	1.38	1.59	1.68	32
	33	1.38	1.57	1.66	33
	34	1.38	1.57	1.66	34
	35	1.39	1.57	1.66	35
	36	1.41	1.58	1.68	36
	37	1.43	1.61	1.70	37
	38	1.46	1.64	1.74	38
	39	1.51	1.69	1.79	39
	40	1.56	1.75	1.85	40
	41	1.61	1.82	1.92	41
	42	1.68	1.91	2.01	42
	43	1.75	2.01	2.11	43
	44	1.84	2.13	2.23	44
	45	1.94	2.26	2.36	45
	46	2.04	2.41	2.51	46
	47	2.16	2.58	2.68	47
	48	2.29	2.77	2.87	48
	49	2.43	2.97	3.07	49
	50	2.59	3.20	3.30	50
	51	2.76	3.44	3.54	51
	52	2.94	3.71	3.81	52
	53	3.13	4.00	4.10	53
	54	3.34	4.31	4.41	54
	55	3.56	4.64	4.75	55
	56	3.80	4.99	5.11	56
	57	4.05	5.36	5.50	57
	58	4.32	5.75	5.91	58
	59	4.60	6.16	6.35	59
	60	4.89	6.59	6.81	60
	61	5.19	7.03	7.29	61
	62	5.51	7.48	7.80	62
	63	5.83	7.94	8.33	63
	64	6.16	8.40	8.88	64

AM92

4%

x	$C_{[x]}$	$C_{[x-1]+1}$	C_x	x
65	6.50	8.87	9.44	65
66	6.84	9.33	10.01	66
67	7.18	9.78	10.59	67
68	7.51	10.22	11.18	68
69	7.84	10.63	11.76	69
70	8.15	11.02	12.33	70
71	8.44	11.36	12.87	71
72	8.71	11.66	13.39	72
73	8.95	11.90	13.88	73
74	9.15	12.08	14.31	74
75	9.30	12.19	14.68	75
76	9.41	12.23	14.98	76
77	9.45	12.17	15.19	77
78	9.43	12.03	15.31	78
79	9.35	11.79	15.33	79
80	9.18	11.46	15.23	80
81	8.95	11.04	15.00	81
82	8.63	10.52	14.65	82
83	8.25	9.92	14.17	83
84	7.79	9.25	13.56	84
85	7.27	8.52	12.84	85
86	6.69	7.73	12.00	86
87	6.08	6.92	11.08	87
88	5.43	6.10	10.08	88
89	4.78	5.29	9.03	89
90	4.12	4.50	7.96	90
91		3.76	6.89	91
92			5.85	92
93			4.86	93
94			3.96	94
95			3.14	95
96			2.43	96
97			1.83	97
98			1.34	98
99			.95	99
100			.65	100
101			.43	101
102			.27	102
103			.17	103
104			.10	104
105			.05	105
106			.03	106
107			.01	107
108			.01	108
109			.00	109
110			.00	110

AM92

4%

x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	x
17	518.82		519.89	17
18	515.93	516.71	516.93	18
19	513.19	513.91	514.11	19
20	510.58	511.25	511.43	20
21	508.09	508.72	508.88	21
22	505.73	506.31	506.46	22
23	503.47	504.01	504.14	23
24	501.30	501.80	501.93	24
25	499.23	499.69	499.81	25
26	497.23	497.67	497.78	26
27	495.31	495.72	495.82	27
28	493.46	493.83	493.93	28
29	491.66	492.01	492.10	29
30	489.90	490.23	490.33	30
31	488.19	488.50	488.59	31
32	486.50	486.80	486.89	32
33	484.84	485.12	485.21	33
34	483.18	483.46	483.55	34
35	481.53	481.80	481.90	35
36	479.87	480.14	480.24	36
37	478.19	478.46	478.56	37
38	476.48	476.76	476.86	38
39	474.74	475.02	475.12	39
40	472.94	473.23	473.33	40
41	471.07	471.38	471.48	41
42	469.12	469.46	469.56	42
43	467.09	467.44	467.55	43
44	464.94	465.33	465.43	44
45	462.68	463.10	463.20	45
46	460.27	460.74	460.84	46
47	457.71	458.23	458.33	47
48	454.98	455.55	455.65	48
49	452.05	452.68	452.78	49
50	448.91	449.61	449.71	50
51	445.53	446.32	446.42	51
52	441.90	442.78	442.88	52
53	437.99	438.96	439.07	53
54	433.78	434.86	434.97	54
55	429.24	430.44	430.55	55
56	424.35	425.68	425.80	56
57	419.08	420.55	420.69	57
58	413.41	415.03	415.19	58
59	407.30	409.09	409.28	59
60	400.74	402.71	402.93	60
61	393.70	395.85	396.12	61
62	386.14	388.50	388.83	62
63	378.05	380.63	381.02	63
64	369.41	372.22	372.69	64

AM92

4%

x	$M_{[x]}$	$M_{[x-1]+1}$	M_x	x
65	360.20	363.25	363.82	65
66	350.40	353.70	354.38	66
67	339.99	343.56	344.37	67
68	328.98	332.81	333.77	68
69	317.37	321.47	322.59	69
70	305.15	309.53	310.84	70
71	292.35	297.00	298.51	71
72	278.98	283.90	285.64	72
73	265.09	270.27	272.24	73
74	250.72	256.14	258.37	74
75	235.93	241.57	244.06	75
76	220.78	226.63	229.38	76
77	205.37	211.38	214.40	77
78	189.79	195.92	199.20	78
79	174.14	180.36	183.89	79
80	158.56	164.80	168.56	80
81	143.16	149.37	153.34	81
82	128.07	134.21	138.34	82
83	113.45	119.44	123.69	83
84	99.42	105.20	109.52	84
85	86.12	91.63	95.96	85
86	73.65	78.85	83.12	86
87	62.14	66.96	71.11	87
88	51.65	56.06	60.04	88
89	42.25	46.22	49.96	89
90	33.97	37.47	40.93	90
91		29.84	32.97	91
92			26.08	92
93			20.23	93
94			15.37	94
95			11.41	95
96			8.27	96
97			5.84	97
98			4.01	98
99			2.67	99
100			1.72	100
101			1.07	101
102			.64	102
103			.37	103
104			.21	104
105			.11	105
106			.05	106
107			.03	107
108			.01	108
109			.01	109
110			.00	110

AM92

4%

x	$R_{[x]}$	$R_{[x-1]+1}$	R_x	x
17	27724.52		27725.81	17
18	27204.73	27205.71	27205.92	18
19	26687.90	26688.80	26689.00	19
20	26173.87	26174.71	26174.89	20
21	25662.51	25663.29	25663.45	21
22	25153.71	25154.42	25154.57	22
23	24647.32	24647.98	24648.11	23
24	24143.23	24143.85	24143.97	24
25	23641.35	23641.93	23642.04	25
26	23141.58	23142.12	23142.23	26
27	22643.84	22644.35	22644.45	27
28	22148.06	22148.53	22148.63	28
29	21654.16	21654.60	21654.70	29
30	21162.07	21162.50	21162.60	30
31	20671.77	20672.17	20672.27	31
32	20183.20	20183.59	20183.68	32
33	19696.32	19696.70	19696.79	33
34	19211.11	19211.48	19211.57	34
35	18727.56	18727.93	18728.02	35
36	18245.66	18246.03	18246.12	36
37	17765.43	17765.79	17765.89	37
38	17286.86	17287.23	17287.33	38
39	16809.99	16810.38	16810.47	39
40	16334.87	16335.26	16335.36	40
41	15861.52	15861.93	15862.03	41
42	15390.01	15390.45	15390.55	42
43	14920.43	14920.89	14920.99	43
44	14452.85	14453.35	14453.45	44
45	13987.39	13987.91	13988.01	45
46	13524.14	13524.71	13524.81	46
47	13063.26	13063.87	13063.97	47
48	12604.88	12605.55	12605.65	48
49	12149.17	12149.90	12150.00	49
50	11696.32	11697.12	11697.22	50
51	11246.53	11247.41	11247.51	51
52	10800.02	10800.99	10801.09	52
53	10357.04	10358.12	10358.22	53
54	9917.85	9919.05	9919.15	54
55	9482.75	9484.07	9484.19	55
56	9052.04	9053.51	9053.63	56
57	8626.06	8627.69	8627.83	57
58	8205.17	8206.98	8207.14	58
59	7789.75	7791.76	7791.95	59
60	7380.21	7382.44	7382.67	60
61	6976.98	6979.47	6979.73	61
62	6580.53	6583.29	6583.61	62
63	6191.34	6194.39	6194.79	63
64	5809.91	5813.29	5813.76	64

AM92

x	$R_{[x]}$	$R_{[x-1]+1}$	R_x	x	4%
65	5436.77	5440.50	5441.07	65	
66	5072.46	5076.57	5077.25	66	
67	4717.54	4722.06	4722.87	67	
68	4372.60	4377.55	4378.51	68	
69	4038.20	4043.61	4044.74	69	
70	3714.94	3720.83	3722.14	70	
71	3403.41	3409.79	3411.31	71	
72	3104.17	3111.06	3112.79	72	
73	2817.78	2825.19	2827.16	73	
74	2544.78	2552.69	2554.91	74	
75	2285.66	2294.06	2296.55	75	
76	2040.87	2049.74	2052.49	76	
77	1810.80	1820.09	1823.11	77	
78	1595.76	1605.43	1608.71	78	
79	1396.00	1405.97	1409.51	79	
80	1211.64	1221.85	1225.62	80	
81	1042.74	1053.09	1057.05	81	
82	889.21	899.59	903.72	82	
83	750.83	761.13	765.38	83	
84	627.27	637.38	641.69	84	
85	518.06	527.85	532.17	85	
86	422.60	431.95	436.22	86	
87	340.15	348.95	353.10	87	
88	269.86	278.01	281.99	88	
89	210.79	218.21	221.95	89	
90	161.90	168.54	171.99	90	
91		127.94	131.06	91	
92			98.09	92	
93			72.01	93	
94			51.78	94	
95			36.41	95	
96			25.00	96	
97			16.73	97	
98			10.89	98	
99			6.89	99	
100			4.22	100	
101			2.50	101	
102			1.43	102	
103			.79	103	
104			.41	104	
105			.21	105	
106			.10	106	
107			.05	107	
108			.02	108	
109			.01	109	
110			.00	110	

AM92

4%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	$\ddot{a}_x$	A_x	${}^2A_x$	x
17	23.372	0.10108	0.01696	23.367	0.10127	0.01716	17
18	23.280	0.10460	0.01778	23.276	0.10478	0.01797	18
19	23.185	0.10827	0.01867	23.180	0.10844	0.01885	19
20	23.086	0.11210	0.01964	23.081	0.11226	0.01982	20
21	22.982	0.11608	0.02070	22.978	0.11624	0.02086	21
22	22.874	0.12023	0.02184	22.870	0.12038	0.02200	22
23	22.762	0.12455	0.02308	22.758	0.12469	0.02324	23
24	22.645	0.12905	0.02443	22.641	0.12919	0.02458	24
25	22.523	0.13373	0.02589	22.520	0.13386	0.02603	25
26	22.396	0.13860	0.02747	22.393	0.13873	0.02761	26
27	22.265	0.14367	0.02917	22.261	0.14379	0.02931	27
28	22.128	0.14894	0.03102	22.124	0.14906	0.03115	28
29	21.985	0.15442	0.03301	21.982	0.15454	0.03314	29
30	21.837	0.16011	0.03515	21.834	0.16023	0.03528	30
31	21.683	0.16603	0.03747	21.680	0.16615	0.03759	31
32	21.523	0.17218	0.03996	21.520	0.17230	0.04008	32
33	21.357	0.17857	0.04264	21.354	0.17868	0.04276	33
34	21.185	0.18520	0.04552	21.182	0.18531	0.04565	34
35	21.006	0.19207	0.04861	21.003	0.19219	0.04874	35
36	20.821	0.19921	0.05193	20.818	0.19933	0.05207	36
37	20.628	0.20660	0.05549	20.625	0.20672	0.05563	37
38	20.429	0.21426	0.05930	20.426	0.21439	0.05945	38
39	20.223	0.22220	0.06338	20.219	0.22234	0.06354	39
40	20.009	0.23041	0.06775	20.005	0.23056	0.06792	40
41	19.788	0.23891	0.07241	19.784	0.23907	0.07259	41
42	19.560	0.24770	0.07738	19.555	0.24787	0.07758	42
43	19.324	0.25678	0.08267	19.319	0.25696	0.08289	43
44	19.080	0.26615	0.08832	19.075	0.26636	0.08856	44
45	18.829	0.27583	0.09431	18.823	0.27605	0.09458	45
46	18.569	0.28580	0.10068	18.563	0.28605	0.10098	46
47	18.302	0.29607	0.10744	18.295	0.29635	0.10778	47
48	18.027	0.30664	0.11460	18.019	0.30695	0.11498	48
49	17.745	0.31752	0.12217	17.736	0.31786	0.12260	49
50	17.454	0.32868	0.13017	17.444	0.32907	0.13065	50
51	17.156	0.34014	0.13861	17.145	0.34058	0.13915	51
52	16.851	0.35189	0.14749	16.838	0.35238	0.14811	52
53	16.538	0.36392	0.15684	16.524	0.36448	0.15755	53
54	16.218	0.37623	0.16665	16.202	0.37685	0.16745	54
55	15.891	0.38879	0.17693	15.873	0.38950	0.17785	55
56	15.558	0.40161	0.18769	15.537	0.40240	0.18874	56
57	15.219	0.41466	0.19893	15.195	0.41556	0.20012	57
58	14.874	0.42794	0.21064	14.847	0.42896	0.21200	58
59	14.523	0.44143	0.22282	14.493	0.44258	0.22437	59
60	14.167	0.45510	0.23547	14.134	0.45640	0.23723	60
61	13.808	0.46894	0.24857	13.769	0.47041	0.25058	61
62	13.444	0.48292	0.26211	13.401	0.48458	0.26440	62
63	13.077	0.49703	0.27608	13.029	0.49890	0.27868	63
64	12.708	0.51123	0.29046	12.653	0.51333	0.29340	64

Note. ${}^2A_{[x]} = A_{[x]}$ at 8.16% and ${}^2A_x = A_x$ at 8.16%.

AM92

4%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	$\ddot{a}_x$	A_x	${}^2A_x$	x
65	12.337	0.52550	0.30522	12.276	0.52786	0.30855	65
66	11.965	0.53981	0.32033	11.896	0.54246	0.32410	66
67	11.592	0.55414	0.33578	11.515	0.55710	0.34003	67
68	11.221	0.56844	0.35151	11.135	0.57175	0.35630	68
69	10.850	0.58270	0.36751	10.754	0.58638	0.37289	69
70	10.481	0.59687	0.38372	10.375	0.60097	0.38975	70
71	10.116	0.61093	0.40012	9.998	0.61548	0.40686	71
72	9.754	0.62485	0.41665	9.623	0.62988	0.42416	72
73	9.396	0.63860	0.43327	9.252	0.64414	0.44162	73
74	9.044	0.65214	0.44993	8.886	0.65824	0.45919	74
75	8.698	0.66545	0.46659	8.524	0.67214	0.47683	75
76	8.359	0.67851	0.48320	8.169	0.68581	0.49448	76
77	8.027	0.69127	0.49971	7.820	0.69924	0.51210	77
78	7.703	0.70373	0.51609	7.478	0.71238	0.52965	78
79	7.388	0.71585	0.53227	7.144	0.72523	0.54707	79
80	7.082	0.72762	0.54822	6.818	0.73775	0.56432	80
81	6.785	0.73903	0.56390	6.502	0.74993	0.58136	81
82	6.499	0.75005	0.57927	6.194	0.76175	0.59814	82
83	6.222	0.76068	0.59430	5.897	0.77319	0.61461	83
84	5.957	0.77090	0.60895	5.610	0.78425	0.63075	84
85	5.701	0.78072	0.62320	5.333	0.79490	0.64652	85
86	5.457	0.79012	0.63701	5.066	0.80514	0.66188	86
87	5.223	0.79911	0.65038	4.811	0.81498	0.67680	87
88	5.000	0.80769	0.66329	4.566	0.82439	0.69127	88
89	4.788	0.81585	0.67573	4.332	0.83338	0.70525	89
90	4.586	0.82362	0.68768	4.109	0.84196	0.71874	90
91				3.897	0.85012	0.73172	91
92				3.695	0.85787	0.74417	92
93				3.504	0.86522	0.75609	93
94				3.323	0.87218	0.76748	94
95				3.153	0.87875	0.77834	95
96				2.992	0.88494	0.78867	96
97				2.840	0.89077	0.79847	97
98				2.698	0.89625	0.80776	98
99				2.564	0.90139	0.81654	99
100				2.439	0.90621	0.82483	100
101				2.321	0.91071	0.83263	101
102				2.212	0.91492	0.83997	102
103				2.110	0.91885	0.84686	103
104				2.015	0.92251	0.85331	104
105				1.926	0.92591	0.85934	105
106				1.844	0.92907	0.86498	106
107				1.768	0.93201	0.87023	107
108				1.697	0.93472	0.87512	108
109				1.632	0.93724	0.87966	109
110				1.571	0.93956	0.88387	110
111				1.516	0.94170	0.88777	111
112				1.464	0.94367	0.89137	112
113				1.417	0.94549	0.89469	113
114				1.374	0.94715	0.89775	114
115				1.334	0.94868	0.90056	115
116				1.298	0.95008	0.90315	116
117				1.264	0.95139	0.90557	117
118				1.229	0.95273	0.90804	118
119				1.176	0.95478	0.91181	119
120				1.000	0.96154	0.92456	120

Note. ${}^2A_{[x]} = A_{[x]}$ at 8.16% and ${}^2A_x = A_x$ at 8.16%.

AM92

4%

x	$(I\ddot{a})_{[x]}$	$(LA)_{[x]}$	$(I\ddot{a})_x$	$(LA)_x$	x
17	467.226	5.40164	467.124	5.40071	17
18	461.881	5.51565	461.784	5.51473	18
19	456.412	5.63060	456.320	5.62969	19
20	450.817	5.74637	450.729	5.74547	20
21	445.097	5.86284	445.013	5.86195	21
22	439.249	5.97986	439.170	5.97899	22
23	433.275	6.09730	433.200	6.09644	23
24	427.174	6.21501	427.102	6.21415	24
25	420.947	6.33280	420.878	6.33195	25
26	414.593	6.45051	414.528	6.44967	26
27	408.114	6.56794	408.051	6.56710	27
28	401.510	6.68488	401.450	6.68405	28
29	394.783	6.80112	394.726	6.80029	29
30	387.935	6.91644	387.878	6.91559	30
31	380.966	7.03057	380.911	7.02972	31
32	373.879	7.14328	373.825	7.14242	32
33	366.676	7.25428	366.623	7.25340	33
34	359.361	7.36331	359.308	7.36239	34
35	351.937	7.47005	351.883	7.46909	35
36	344.407	7.57421	344.353	7.57320	36
37	336.776	7.67546	336.720	7.67438	37
38	329.048	7.77346	328.991	7.77231	38
39	321.228	7.86788	321.169	7.86663	39
40	313.323	7.95835	313.260	7.95699	40
41	305.337	8.04452	305.271	8.04303	41
42	297.278	8.12602	297.207	8.12435	42
43	289.153	8.20246	289.077	8.20060	43
44	280.970	8.27347	280.888	8.27137	44
45	272.737	8.33865	272.647	8.33628	45
46	264.462	8.39762	264.365	8.39493	46
47	256.156	8.45001	256.049	8.44695	47
48	247.828	8.49542	247.711	8.49193	48
49	239.488	8.53351	239.360	8.52950	49
50	231.149	8.56390	231.007	8.55929	50
51	222.820	8.58624	222.664	8.58095	51
52	214.514	8.60022	214.342	8.59412	52
53	206.244	8.60554	206.053	8.59851	53
54	198.022	8.60190	197.811	8.59381	54
55	189.861	8.58908	189.627	8.57976	55
56	181.774	8.56687	181.516	8.55611	56
57	173.775	8.53508	173.489	8.52268	57
58	165.878	8.49360	165.561	8.47931	58
59	158.094	8.44234	157.744	8.42588	59
60	150.440	8.38128	150.053	8.36234	60
61	142.926	8.31044	142.499	8.28867	61
62	135.566	8.22990	135.096	8.20491	62
63	128.373	8.13981	127.856	8.11117	63
64	121.359	8.04036	120.790	8.00760	64

AM92

4%

x	$(I\ddot{a})_{[x]}$	$(LA)_{[x]}$	$(I\ddot{a})_x$	$(LA)_x$	x
65	114.533	7.93182	113.911	7.89442	65
66	107.909	7.81453	107.228	7.77192	66
67	101.494	7.68886	100.751	7.64043	67
68	95.297	7.55527	94.489	7.50035	68
69	89.327	7.41426	88.450	7.35215	69
70	83.589	7.26640	82.641	7.19635	70
71	78.089	7.11229	77.067	7.03351	71
72	72.832	6.95257	71.732	6.86424	72
73	67.819	6.78795	66.640	6.68922	73
74	63.053	6.61914	61.793	6.50913	74
75	58.534	6.44687	57.192	6.32470	75
76	54.260	6.27192	52.836	6.13669	76
77	50.230	6.09504	48.723	5.94586	77
78	46.440	5.91697	44.851	5.75298	78
79	42.885	5.73848	41.215	5.55883	79
80	39.559	5.56029	37.811	5.36417	80
81	36.457	5.38308	34.633	5.16976	81
82	33.570	5.20753	31.673	4.97631	82
83	30.890	5.03426	28.924	4.78453	83
84	28.410	4.86382	26.378	4.59508	84
85	26.118	4.69675	24.025	4.40856	85
86	24.007	4.53350	21.858	4.22555	86
87	22.065	4.37448	19.866	4.04657	87
88	20.283	4.22003	18.039	3.87208	88
89	18.651	4.07043	16.368	3.70250	89
90	17.159	3.92589	14.843	3.53817	90
91			13.453	3.37939	91
92			12.191	3.22640	92
93			11.045	3.07939	93
94			10.007	2.93848	94
95			9.070	2.80378	95
96			8.223	2.67530	96
97			7.460	2.55306	97
98			6.774	2.43701	98
99			6.156	2.32708	99
100			5.602	2.22316	100
101			5.104	2.12512	101
102			4.659	2.03281	102
103			4.259	1.94607	103
104			3.902	1.86471	104
105			3.582	1.78853	105
106			3.295	1.71734	106
107			3.039	1.65092	107
108			2.811	1.58907	108
109			2.606	1.53158	109
110			2.424	1.47823	110
111			2.261	1.42882	111
112			2.115	1.38315	112
113			1.985	1.34102	113
114			1.869	1.30222	114
115			1.765	1.26654	115
116			1.672	1.23370	116
117			1.584	1.20299	117
118			1.492	1.17157	118
119			1.351	1.12376	119
120			1.000	0.96154	120

AM92

4%	x	$\ddot{a}_{[x]\overline{n}\rvert}$	$A_{[x]\overline{n}\rvert}$	$n = 60 - x$	$\ddot{a}_{x\overline{n}\rvert}$	$A_{x\overline{n}\rvert}$	x
	17	20.941	0.19459	43	20.936	0.19475	17
	18	20.750	0.20190	42	20.746	0.20206	18
	19	20.552	0.20953	41	20.548	0.20968	19
	20	20.346	0.21746	40	20.342	0.21760	20
	21	20.131	0.22572	39	20.128	0.22586	21
	22	19.908	0.23432	38	19.904	0.23445	22
	23	19.675	0.24327	37	19.672	0.24340	23
	24	19.433	0.25259	36	19.430	0.25271	24
	25	19.181	0.26228	35	19.178	0.26240	25
	26	18.918	0.27237	34	18.916	0.27248	26
	27	18.645	0.28287	33	18.643	0.28297	27
	28	18.361	0.29379	32	18.359	0.29389	28
	29	18.066	0.30515	31	18.064	0.30525	29
	30	17.759	0.31697	30	17.756	0.31706	30
	31	17.439	0.32926	29	17.437	0.32935	31
	32	17.107	0.34204	28	17.105	0.34212	32
	33	16.762	0.35533	27	16.759	0.35541	33
	34	16.402	0.36914	26	16.400	0.36923	34
	35	16.029	0.38350	25	16.027	0.38359	35
	36	15.641	0.39843	24	15.639	0.39852	36
	37	15.237	0.41395	23	15.235	0.41403	37
	38	14.818	0.43007	22	14.816	0.43016	38
	39	14.383	0.44682	21	14.380	0.44692	39
	40	13.930	0.46423	20	13.927	0.46433	40
	41	13.460	0.48231	19	13.457	0.48242	41
	42	12.971	0.50110	18	12.969	0.50121	42
	43	12.464	0.52061	17	12.461	0.52073	43
	44	11.937	0.54088	16	11.934	0.54100	44
	45	11.390	0.56193	15	11.386	0.56206	45
	46	10.821	0.58380	14	10.818	0.58393	46
	47	10.231	0.60651	13	10.227	0.60665	47
	48	9.617	0.63010	12	9.613	0.63025	48
	49	8.980	0.65461	11	8.976	0.65477	49
	50	8.318	0.68007	10	8.314	0.68024	50
	51	7.630	0.70654	9	7.625	0.70672	51
	52	6.914	0.73406	8	6.910	0.73424	52
	53	6.170	0.76268	7	6.166	0.76286	53
	54	5.396	0.79246	6	5.391	0.79264	54
	55	4.590	0.82348	5	4.585	0.82365	55
	56	3.749	0.85580	4	3.745	0.85595	56
	57	2.873	0.88952	3	2.870	0.88963	57
	58	1.957	0.92473	2	1.955	0.92479	58
	59	1.000	0.96154	1	1.000	0.96154	59

AM92

x	$\ddot{a}_{[x]\overline{n}\vert}$	$A_{[x]\overline{n}\vert}$	n = 65 - x	$\ddot{a}_{x\overline{n}\vert}$	$A_{x\overline{n}\vert}$	x	4%
17	21.723	0.16448	48	21.719	0.16466	17	
18	21.565	0.17058	47	21.561	0.17074	18	
19	21.400	0.17693	46	21.396	0.17709	19	
20	21.228	0.18354	45	21.224	0.18369	20	
21	21.049	0.19042	44	21.045	0.19057	21	
22	20.863	0.19759	43	20.859	0.19773	22	
23	20.669	0.20505	42	20.665	0.20518	23	
24	20.467	0.21281	41	20.464	0.21294	24	
25	20.257	0.22090	40	20.254	0.22102	25	
26	20.038	0.22931	39	20.035	0.22942	26	
27	19.811	0.23805	38	19.808	0.23817	27	
28	19.574	0.24716	37	19.571	0.24726	28	
29	19.328	0.25662	36	19.325	0.25673	29	
30	19.072	0.26647	35	19.069	0.26657	30	
31	18.806	0.27671	34	18.803	0.27681	31	
32	18.529	0.28735	33	18.526	0.28745	32	
33	18.241	0.29842	32	18.239	0.29852	33	
34	17.942	0.30992	31	17.940	0.31002	34	
35	17.631	0.32187	30	17.629	0.32197	35	
36	17.308	0.33429	29	17.306	0.33439	36	
37	16.973	0.34719	28	16.970	0.34729	37	
38	16.625	0.36059	27	16.622	0.36070	38	
39	16.263	0.37451	26	16.260	0.37462	39	
40	15.887	0.38896	25	15.884	0.38907	40	
41	15.497	0.40395	24	15.494	0.40407	41	
42	15.092	0.41952	23	15.089	0.41965	42	
43	14.672	0.43567	22	14.669	0.43581	43	
44	14.237	0.45243	21	14.233	0.45258	44	
45	13.785	0.46982	20	13.780	0.46998	45	
46	13.316	0.48786	19	13.311	0.48803	46	
47	12.829	0.50656	18	12.824	0.50675	47	
48	12.325	0.52596	17	12.320	0.52617	48	
49	11.802	0.54608	16	11.796	0.54630	49	
50	11.259	0.56695	15	11.253	0.56719	50	
51	10.697	0.58858	14	10.690	0.58884	51	
52	10.113	0.61102	13	10.106	0.61130	52	
53	9.508	0.63430	12	9.500	0.63460	53	
54	8.880	0.65846	11	8.872	0.65878	54	
55	8.228	0.68354	10	8.219	0.68388	55	
56	7.551	0.70958	9	7.542	0.70993	56	
57	6.847	0.73664	8	6.838	0.73701	57	
58	6.115	0.76479	7	6.106	0.76516	58	
59	5.353	0.79410	6	5.344	0.79446	59	
60	4.559	0.82465	5	4.550	0.82499	60	
61	3.730	0.85654	4	3.722	0.85685	61	
62	2.863	0.88990	3	2.857	0.89013	62	
63	1.954	0.92485	2	1.951	0.92498	63	
64	1.000	0.96154	1	1.000	0.96154	64	

AM92

6%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	$^2A_{[x]}$	$\ddot{a}_x$	A_x	^2A_x	x
17	16.977	0.03902	0.00611	16.974	0.03921	0.00630	17
18	16.946	0.04080	0.00630	16.943	0.04099	0.00648	18
19	16.912	0.04270	0.00652	16.909	0.04288	0.00669	19
20	16.877	0.04472	0.00677	16.874	0.04489	0.00693	20
21	16.839	0.04686	0.00705	16.836	0.04703	0.00721	21
22	16.798	0.04914	0.00738	16.796	0.04930	0.00753	22
23	16.756	0.05157	0.00775	16.753	0.05172	0.00790	23
24	16.710	0.05414	0.00816	16.708	0.05428	0.00831	24
25	16.662	0.05686	0.00863	16.660	0.05701	0.00877	25
26	16.611	0.05976	0.00916	16.609	0.05990	0.00930	26
27	16.557	0.06282	0.00975	16.554	0.06296	0.00988	27
28	16.499	0.06607	0.01041	16.497	0.06620	0.01054	28
29	16.439	0.06951	0.01115	16.436	0.06964	0.01128	29
30	16.374	0.07316	0.01197	16.372	0.07328	0.01210	30
31	16.306	0.07701	0.01289	16.304	0.07714	0.01301	31
32	16.234	0.08109	0.01390	16.232	0.08121	0.01403	32
33	16.158	0.08540	0.01503	16.156	0.08552	0.01515	33
34	16.078	0.08995	0.01627	16.075	0.09007	0.01640	34
35	15.993	0.09475	0.01765	15.990	0.09488	0.01778	35
36	15.903	0.09982	0.01916	15.901	0.09995	0.01930	36
37	15.809	0.10516	0.02084	15.806	0.10530	0.02098	37
38	15.709	0.11079	0.02267	15.707	0.11094	0.02282	38
39	15.605	0.11672	0.02469	15.602	0.11688	0.02485	39
40	15.494	0.12296	0.02690	15.491	0.12313	0.02707	40
41	15.378	0.12952	0.02933	15.375	0.12970	0.02951	41
42	15.257	0.13641	0.03198	15.253	0.13660	0.03218	42
43	15.129	0.14365	0.03487	15.125	0.14385	0.03509	43
44	14.995	0.15123	0.03802	14.991	0.15146	0.03826	44
45	14.855	0.15918	0.04145	14.850	0.15943	0.04172	45
46	14.708	0.16750	0.04517	14.703	0.16778	0.04548	46
47	14.554	0.17619	0.04921	14.548	0.17651	0.04956	47
48	14.393	0.18528	0.05359	14.387	0.18563	0.05398	48
49	14.226	0.19476	0.05832	14.219	0.19516	0.05876	49
50	14.051	0.20463	0.06342	14.044	0.20508	0.06392	50
51	13.870	0.21491	0.06892	13.861	0.21542	0.06949	51
52	13.681	0.22560	0.07483	13.671	0.22617	0.07548	52
53	13.485	0.23669	0.08118	13.474	0.23734	0.08192	53
54	13.282	0.24818	0.08797	13.269	0.24892	0.08882	54
55	13.072	0.26008	0.09524	13.057	0.26092	0.09621	55
56	12.855	0.27237	0.10298	12.838	0.27333	0.10409	56
57	12.631	0.28506	0.11123	12.612	0.28614	0.11250	57
58	12.400	0.29812	0.11998	12.378	0.29935	0.12144	58
59	12.163	0.31155	0.12926	12.138	0.31294	0.13093	59
60	11.919	0.32533	0.13907	11.891	0.32692	0.14098	60
61	11.670	0.33945	0.14941	11.638	0.34125	0.15160	61
62	11.415	0.35388	0.16029	11.379	0.35592	0.16280	62
63	11.155	0.36861	0.17171	11.114	0.37091	0.17457	63
64	10.890	0.38360	0.18366	10.844	0.38620	0.18692	64

Note. $^2A_{[x]} = A_{[x]}$ at 12.36% and $^2A_x = A_x$ at 12.36%.

AM92

6%

x	$\ddot{a}_{[x]}$	$A_{[x]}$	${}^2A_{[x]}$	$\ddot{a}_x$	A_x	${}^2A_x$	x
65	10.621	0.39883	0.19614	10.569	0.40177	0.19985	65
66	10.348	0.41427	0.20913	10.289	0.41758	0.21335	66
67	10.072	0.42988	0.22262	10.006	0.43361	0.22740	67
68	9.794	0.44564	0.23658	9.720	0.44982	0.24200	68
69	9.513	0.46150	0.25100	9.431	0.46617	0.25712	69
70	9.232	0.47743	0.26583	9.140	0.48265	0.27274	70
71	8.950	0.49338	0.28106	8.848	0.49919	0.28882	71
72	8.669	0.50933	0.29664	8.555	0.51578	0.30534	72
73	8.388	0.52521	0.31254	8.262	0.53236	0.32226	73
74	8.109	0.54101	0.32870	7.969	0.54890	0.33955	74
75	7.832	0.55667	0.34509	7.679	0.56535	0.35714	75
76	7.559	0.57215	0.36164	7.390	0.58169	0.37501	76
77	7.289	0.58742	0.37833	7.105	0.59786	0.39309	77
78	7.024	0.60244	0.39508	6.822	0.61383	0.41133	78
79	6.763	0.61717	0.41186	6.544	0.62956	0.42969	79
80	6.509	0.63159	0.42860	6.271	0.64501	0.44811	80
81	6.260	0.64566	0.44525	6.004	0.66016	0.46652	81
82	6.018	0.65935	0.46177	5.742	0.67497	0.48488	82
83	5.783	0.67265	0.47811	5.487	0.68942	0.50313	83
84	5.556	0.68553	0.49422	5.239	0.70346	0.52121	84
85	5.336	0.69797	0.51005	4.998	0.71710	0.53907	85
86	5.124	0.70997	0.52557	4.765	0.73029	0.55667	86
87	4.920	0.72150	0.54075	4.540	0.74304	0.57396	87
88	4.724	0.73258	0.55555	4.323	0.75531	0.59088	88
89	4.537	0.74318	0.56994	4.114	0.76711	0.60741	89
90	4.358	0.75332	0.58390	3.914	0.77843	0.62350	90
91				3.723	0.78925	0.63913	91
92				3.541	0.79959	0.65426	92
93				3.367	0.80944	0.66888	93
94				3.201	0.81880	0.68296	94
95				3.044	0.82769	0.69649	95
96				2.896	0.83610	0.70946	96
97				2.755	0.84406	0.72187	97
98				2.622	0.85156	0.73370	98
99				2.498	0.85863	0.74496	99
100				2.380	0.86527	0.75565	100
101				2.270	0.87151	0.76579	101
102				2.167	0.87736	0.77537	102
103				2.070	0.88283	0.78442	103
104				1.980	0.88794	0.79293	104
105				1.895	0.89271	0.80094	105
106				1.817	0.89715	0.80845	106
107				1.744	0.90128	0.81548	107
108				1.676	0.90511	0.82205	108
109				1.614	0.90866	0.82817	109
110				1.556	0.91195	0.83387	110
111				1.502	0.91499	0.83917	111
112				1.452	0.91779	0.84408	112
113				1.407	0.92037	0.84861	113
114				1.365	0.92275	0.85280	114
115				1.326	0.92492	0.85666	115
116				1.291	0.92693	0.86022	116
117				1.258	0.92880	0.86355	117
118				1.224	0.93072	0.86694	118
119				1.172	0.93364	0.87210	119
120				1.000	0.94340	0.89000	120

Note. ${}^2A_{[x]} = A_{[x]}$ at 12.36% and ${}^2A_x = A_x$ at 12.36%.

AM92

6%

x	$(I\ddot{a})_{[x]}$	$(LA)_{[x]}$	$(I\ddot{a})_x$	$(LA)_x$	x
17	268.142	1.79955	268.083	1.79940	17
18	266.392	1.86708	266.336	1.86692	18
19	264.567	1.93681	264.514	1.93664	19
20	262.666	2.00874	262.615	2.00856	20
21	260.687	2.08289	260.638	2.08270	21
22	258.626	2.15925	258.579	2.15906	22
23	256.482	2.23782	256.437	2.23762	23
24	254.253	2.31858	254.210	2.31837	24
25	251.936	2.40151	251.896	2.40129	25
26	249.531	2.48657	249.491	2.48635	26
27	247.034	2.57373	246.996	2.57350	27
28	244.444	2.66293	244.407	2.66270	28
29	241.759	2.75410	241.724	2.75386	29
30	238.978	2.84718	238.943	2.84692	30
31	236.099	2.94206	236.065	2.94180	31
32	233.120	3.03864	233.087	3.03837	32
33	230.041	3.13681	230.008	3.13653	33
34	226.861	3.23643	226.827	3.23613	34
35	223.579	3.33735	223.545	3.33702	35
36	220.194	3.43940	220.159	3.43904	36
37	216.706	3.54239	216.671	3.54200	37
38	213.116	3.64613	213.079	3.64569	38
39	209.424	3.75037	209.385	3.74989	39
40	205.630	3.85489	205.589	3.85435	40
41	201.736	3.95942	201.692	3.95880	41
42	197.744	4.06368	197.696	4.06297	42
43	193.654	4.16736	193.603	4.16655	43
44	189.471	4.27014	189.416	4.26922	44
45	185.197	4.37170	185.136	4.37062	45
46	180.834	4.47166	180.768	4.47041	46
47	176.388	4.56965	176.315	4.56820	47
48	171.863	4.66529	171.783	4.66359	48
49	167.264	4.75818	167.175	4.75618	49
50	162.597	4.84789	162.497	4.84555	50
51	157.867	4.93400	157.757	4.93126	51
52	153.082	5.01609	152.959	5.01287	52
53	148.249	5.09372	148.113	5.08994	53
54	143.376	5.16647	143.224	5.16203	54
55	138.472	5.23389	138.302	5.22868	55
56	133.545	5.29558	133.356	5.28947	56
57	128.605	5.35113	128.394	5.34397	57
58	123.662	5.40016	123.427	5.39176	58
59	118.726	5.44229	118.464	5.43247	59
60	113.808	5.47720	113.516	5.46572	60
61	108.918	5.50457	108.594	5.49118	61
62	104.067	5.52416	103.707	5.50856	62
63	99.267	5.53574	98.868	5.51759	63
64	94.528	5.53913	94.087	5.51808	64

AM92

6%

x	$(I\ddot{a})_{[x]}$	$(LA)_{[x]}$	$(I\ddot{a})_x$	$(LA)_x$	x
65	89.861	5.53421	89.374	5.50985	65
66	85.277	5.52093	84.740	5.49280	66
67	80.785	5.49928	80.196	5.46688	67
68	76.397	5.46931	75.752	5.43209	68
69	72.121	5.43114	71.416	5.38851	69
70	67.965	5.38497	67.198	5.33628	70
71	63.939	5.33101	63.105	5.27560	71
72	60.048	5.26959	59.146	5.20673	72
73	56.300	5.20107	55.326	5.12999	73
74	52.700	5.12586	51.652	5.04577	74
75	49.251	5.04444	48.128	4.95452	75
76	45.958	4.95731	44.758	4.85672	76
77	42.822	4.86504	41.545	4.75291	77
78	39.846	4.76819	38.491	4.64369	78
79	37.028	4.66737	35.596	4.52964	79
80	34.369	4.56320	32.860	4.41142	80
81	31.866	4.45630	30.283	4.28968	81
82	29.517	4.34729	27.861	4.16509	82
83	27.320	4.23678	25.594	4.03831	83
84	25.268	4.12536	23.475	3.91000	84
85	23.359	4.01361	21.503	3.78082	85
86	21.586	3.90205	19.671	3.65139	86
87	19.944	3.79119	17.974	3.52231	87
88	18.426	3.68149	16.406	3.39416	88
89	17.026	3.57336	14.962	3.26746	89
90	15.738	3.46716	13.634	3.14270	90
91			12.417	3.02033	91
92			11.303	2.90075	92
93			10.287	2.78431	93
94			9.361	2.67132	94
95			8.518	2.56202	95
96			7.754	2.45663	96
97			7.061	2.35532	97
98			6.435	2.25821	98
99			5.869	2.16537	99
100			5.358	2.07686	100
101			4.898	1.99270	101
102			4.483	1.91286	102
103			4.111	1.83731	103
104			3.776	1.76598	104
105			3.475	1.69878	105
106			3.205	1.63563	106
107			2.963	1.57639	107
108			2.746	1.52096	108
109			2.551	1.46920	109
110			2.377	1.42096	110
111			2.221	1.37611	111
112			2.081	1.33450	112
113			1.956	1.29598	113
114			1.845	1.26040	114
115			1.744	1.22760	115
116			1.654	1.19734	116
117			1.570	1.16904	117
118			1.481	1.14018	118
119			1.345	1.09631	119
120			1.000	0.94340	120

AM92

6%

x	$\ddot{a}_{[x]\overline{n}\vert}$	$A_{[x]\overline{n}\vert}$	$n = 60 - x$	$\ddot{a}_{x\overline{n}\vert}$	$A_{x\overline{n}\vert}$	x
17	16.076	0.09005	43	16.072	0.09024	17
18	15.990	0.09493	42	15.986	0.09511	18
19	15.898	0.10011	41	15.895	0.10028	19
20	15.801	0.10561	40	15.798	0.10577	20
21	15.698	0.11145	39	15.695	0.11160	21
22	15.588	0.11764	38	15.586	0.11779	22
23	15.472	0.12422	37	15.470	0.12436	23
24	15.349	0.13119	36	15.347	0.13133	24
25	15.218	0.13859	35	15.216	0.13872	25
26	15.080	0.14643	34	15.078	0.14656	26
27	14.933	0.15475	33	14.931	0.15487	27
28	14.777	0.16357	32	14.775	0.16369	28
29	14.612	0.17292	31	14.610	0.17303	29
30	14.437	0.18283	30	14.435	0.18294	30
31	14.251	0.19333	29	14.249	0.19344	31
32	14.054	0.20446	28	14.053	0.20457	32
33	13.846	0.21626	27	13.844	0.21636	33
34	13.625	0.22875	26	13.624	0.22885	34
35	13.392	0.24198	25	13.390	0.24208	35
36	13.144	0.25599	24	13.142	0.25609	36
37	12.882	0.27082	23	12.880	0.27093	37
38	12.605	0.28653	22	12.603	0.28664	38
39	12.311	0.30316	21	12.309	0.30327	39
40	12.000	0.32076	20	11.998	0.32088	40
41	11.671	0.33938	19	11.669	0.33951	41
42	11.323	0.35910	18	11.320	0.35923	42
43	10.954	0.37996	17	10.952	0.38010	43
44	10.564	0.40203	16	10.561	0.40219	44
45	10.151	0.42539	15	10.149	0.42556	45
46	9.715	0.45011	14	9.712	0.45028	46
47	9.253	0.47626	13	9.249	0.47645	47
48	8.764	0.50394	12	8.760	0.50415	48
49	8.246	0.53324	11	8.242	0.53346	49
50	7.698	0.56426	10	7.694	0.56449	50
51	7.118	0.59711	9	7.114	0.59735	51
52	6.503	0.63191	8	6.499	0.63216	52
53	5.851	0.66879	7	5.847	0.66904	53
54	5.160	0.70791	6	5.156	0.70815	54
55	4.427	0.74941	5	4.423	0.74965	55
56	3.648	0.79350	4	3.645	0.79370	56
57	2.820	0.84036	3	2.817	0.84052	57
58	1.939	0.89024	2	1.937	0.89034	58
59	1.000	0.94340	1	1.000	0.94340	59

AM92

6%

x	$\ddot{a}_{[x]\overline{n\rvert}}$	$A_{[x]\overline{n\rvert}}$	$n = 65 - x$	$\ddot{a}_{x\overline{n\rvert}}$	$A_{x\overline{n\rvert}}$	x
17	16.409	0.07121	48	16.405	0.07140	17
18	16.343	0.07495	47	16.339	0.07513	18
19	16.272	0.07892	46	16.269	0.07909	19
20	16.198	0.08313	45	16.195	0.08330	20
21	16.119	0.08761	44	16.116	0.08777	21
22	16.035	0.09236	43	16.032	0.09251	22
23	15.946	0.09740	42	15.943	0.09754	23
24	15.852	0.10274	41	15.849	0.10288	24
25	15.751	0.10842	40	15.749	0.10855	25
26	15.645	0.11443	39	15.643	0.11456	26
27	15.532	0.12081	38	15.530	0.12094	27
28	15.413	0.12758	37	15.411	0.12770	28
29	15.286	0.13475	36	15.284	0.13486	29
30	15.152	0.14234	35	15.150	0.14246	30
31	15.010	0.15039	34	15.008	0.15050	31
32	14.859	0.15892	33	14.857	0.15903	32
33	14.700	0.16795	32	14.698	0.16806	33
34	14.531	0.17751	31	14.529	0.17762	34
35	14.352	0.18763	30	14.350	0.18774	35
36	14.163	0.19833	29	14.161	0.19845	36
37	13.963	0.20967	28	13.960	0.20979	37
38	13.751	0.22165	27	13.749	0.22178	38
39	13.527	0.23433	26	13.525	0.23446	39
40	13.290	0.24774	25	13.288	0.24787	40
41	13.040	0.26191	24	13.037	0.26206	41
42	12.775	0.27689	23	12.772	0.27705	42
43	12.495	0.29272	22	12.492	0.29289	43
44	12.200	0.30944	21	12.197	0.30963	44
45	11.888	0.32711	20	11.884	0.32731	45
46	11.558	0.34578	19	11.554	0.34599	46
47	11.210	0.36549	18	11.206	0.36572	47
48	10.842	0.38630	17	10.837	0.38656	48
49	10.454	0.40828	16	10.449	0.40857	49
50	10.044	0.43150	15	10.038	0.43181	50
51	9.610	0.45602	14	9.604	0.45635	51
52	9.153	0.48191	13	9.146	0.48228	52
53	8.669	0.50927	12	8.662	0.50967	53
54	8.159	0.53819	11	8.151	0.53862	54
55	7.618	0.56877	10	7.610	0.56922	55
56	7.047	0.60112	9	7.038	0.60160	56
57	6.442	0.63536	8	6.433	0.63586	57
58	5.801	0.67165	7	5.792	0.67216	58
59	5.121	0.71015	6	5.112	0.71066	59
60	4.398	0.75104	5	4.390	0.75152	60
61	3.630	0.79454	4	3.622	0.79497	61
62	2.811	0.84090	3	2.805	0.84123	62
63	1.936	0.89042	2	1.933	0.89060	63
64	1.000	0.94340	1	1.000	0.94340	64

15 Pensioner Mortality Tables

15.1 PMA92 and PFA92 (Base tables) and PMA92C20 and PFA92C20 (Projected tables)

The Base tables are based on the mortality of pensioners insured by UK life offices during the years 1991, 1992, 1993, and 1994. Mortality is measured by amounts of annuities held.

The projected tables are projected to the calendar year 2020.
Full details are given in *C.M.I.R. 16 and 17*.

15.2 Projection Formulae

The projected mortality rate applicable in a particular calendar year is calculated using the formula:

$$q_x^{\text{Year}}(\text{projected}) = q_x^{\text{Base}} \times RF(x,t) \quad \text{where} \quad t = \text{Year} - 1992$$

The reduction factor is calculated as:

$$RF(x,t) = \alpha + (1-\alpha)(1-f)^{t/20}$$

The parameters used are:

Age range	(α)	(f)
$x < 60$	0.13	0.55
$60 \leq x \leq 110$	$1 - 0.87\left(\frac{110-x}{50}\right)$	$0.55\left(\frac{110-x}{50}\right) + 0.29\left(\frac{x-60}{50}\right)$
$x > 110$	1	0.29

PMA92Base

x	q_x
50	0.001315
51	0.001519
52	0.001761
53	0.002045
54	0.002379
55	0.002771
56	0.003228
57	0.003759
58	0.004376
59	0.005090
60	0.005914
61	0.006861
62	0.007947
63	0.009189
64	0.010604
65	0.012211
66	0.014032
67	0.016088
68	0.018402
69	0.020998
70	0.023901
71	0.027137
72	0.030732
73	0.034713
74	0.039105
75	0.043935
76	0.049227
77	0.055006
78	0.061292
79	0.068106
80	0.075464
81	0.083379
82	0.091862
83	0.100917
84	0.110544
85	0.120739
86	0.131492
87	0.142786
88	0.154599
89	0.166903
90	0.179664
91	0.192841
92	0.206389
93	0.220257
94	0.234389
95	0.248727
96	0.263206
97	0.277762
98	0.292327
99	0.306832
100	0.321209
101	0.335389
102	0.349305
103	0.362893
104	0.376091
105	0.388838

PMA92Base

x	q_x
50	0.001271
51	0.001456
52	0.001670
53	0.001917
54	0.002200
55	0.002524
56	0.002894
57	0.003317
58	0.003799
59	0.004345
60	0.004965
61	0.005667
62	0.006458
63	0.007350
64	0.008352
65	0.009476
66	0.010734
67	0.012138
68	0.013703
69	0.015442
70	0.017371
71	0.019505
72	0.021861
73	0.024455
74	0.027306
75	0.030432
76	0.033849
77	0.037577
78	0.041632
79	0.046035
80	0.050800
81	0.055946
82	0.061488
83	0.067441
84	0.073817
85	0.080629
86	0.087885
87	0.095594
88	0.103761
89	0.112386
90	0.121470
91	0.131009
92	0.140996
93	0.151420
94	0.162267
95	0.173519
96	0.185155
97	0.197150
98	0.209477
99	0.222103
100	0.234995
101	0.248115
102	0.261424
103	0.274879
104	0.288437
105	0.302054

PMA92C20

x	l_x	d_x	q_x	μ_x	e_x°	x
50	9941.923	5.418	0.000545	0.000507	34.10	50
51	9936.504	6.260	0.000630	0.000585	33.12	51
52	9930.244	7.249	0.000730	0.000677	32.14	52
53	9922.995	8.415	0.000848	0.000786	31.17	53
54	9914.580	9.776	0.000986	0.000914	30.19	54
55	9904.805	11.371	0.001148	0.001063	29.22	55
56	9893.434	13.237	0.001338	0.001239	28.25	56
57	9880.196	15.393	0.001558	0.001444	27.29	57
58	9864.803	17.895	0.001814	0.001681	26.33	58
59	9846.908	20.777	0.002110	0.001957	25.38	59
60	9826.131	24.084	0.002451	0.002266	24.43	60
61	9802.048	28.965	0.002955	0.002685	23.49	61
62	9773.083	34.694	0.003550	0.003241	22.56	62
63	9738.388	41.398	0.004251	0.003889	21.64	63
64	9696.990	49.193	0.005073	0.004651	20.73	64
65	9647.797	58.195	0.006032	0.005543	19.83	65
66	9589.602	68.537	0.007147	0.006583	18.95	66
67	9521.065	80.348	0.008439	0.007792	18.08	67
68	9440.717	93.746	0.009930	0.009191	17.23	68
69	9346.970	108.836	0.011644	0.010806	16.40	69
70	9238.134	125.685	0.013605	0.012661	15.59	70
71	9112.449	144.350	0.015841	0.014783	14.79	71
72	8968.099	164.834	0.018380	0.017204	14.02	72
73	8803.265	187.096	0.021253	0.019956	13.28	73
74	8616.170	211.010	0.024490	0.023072	12.55	74
75	8405.160	236.362	0.028121	0.026587	11.86	75
76	8168.798	262.864	0.032179	0.030537	11.18	76
77	7905.934	290.116	0.036696	0.034962	10.54	77
78	7615.818	317.595	0.041702	0.039899	9.92	78
79	7298.223	344.688	0.047229	0.045390	9.33	79
80	6953.536	370.644	0.053303	0.051473	8.77	80
81	6582.891	394.658	0.059952	0.058188	8.23	81
82	6188.234	415.856	0.067201	0.065576	7.73	82
83	5772.378	433.321	0.075068	0.073676	7.25	83
84	5339.057	446.180	0.083569	0.082522	6.80	84
85	4892.878	453.648	0.092716	0.092149	6.37	85
86	4439.230	455.092	0.102516	0.102590	5.97	86
87	3984.138	450.084	0.112969	0.113873	5.59	87
88	3534.054	438.463	0.124068	0.126023	5.24	88
89	3095.591	420.387	0.135802	0.139060	4.91	89
90	2675.203	396.334	0.148151	0.152998	4.61	90
91	2278.869	367.099	0.161088	0.167846	4.32	91
92	1911.771	333.759	0.174581	0.183606	4.06	92
93	1578.012	297.596	0.188589	0.200273	3.81	93
94	1280.416	260.008	0.203065	0.217836	3.59	94
95	1020.409	222.405	0.217957	0.236273	3.38	95
96	798.003	186.098	0.233205	0.255556	3.18	96
97	611.905	152.209	0.248746	0.275647	3.00	97
98	459.696	121.595	0.264511	0.296499	2.84	98
99	338.101	94.813	0.280429	0.318054	2.68	99
100	243.288	72.117	0.296425	0.340247	2.54	100
101	171.171	53.478	0.312423	0.363002	2.41	101
102	117.693	38.644	0.328344	0.386232	2.29	102
103	79.050	27.202	0.344113	0.409842	2.18	103
104	51.848	18.647	0.359653	0.433729	2.08	104
105	33.200	12.446	0.374887	0.457778	1.99	105

PMA92C20

x	l_x	d_x	q_x	μ_x	e_x°	x
50	9952.697	5.245	0.000527	0.000492	37.08	50
51	9947.452	5.998	0.000603	0.000563	36.10	51
52	9941.454	6.879	0.000692	0.000645	35.12	52
53	9934.574	7.898	0.000795	0.000741	34.15	53
54	9926.676	9.053	0.000912	0.000851	33.17	54
55	9917.623	10.374	0.001046	0.000976	32.20	55
56	9907.249	11.879	0.001199	0.001120	31.24	56
57	9895.370	13.606	0.001375	0.001284	30.27	57
58	9881.764	15.564	0.001575	0.001472	29.31	58
59	9866.200	17.769	0.001801	0.001685	28.36	59
60	9848.431	20.268	0.002058	0.001918	27.41	60
61	9828.163	23.991	0.002441	0.002236	26.46	61
62	9804.173	28.285	0.002885	0.002655	25.53	62
63	9775.888	33.248	0.003401	0.003135	24.60	63
64	9742.640	38.932	0.003996	0.003691	23.68	64
65	9703.708	45.423	0.004681	0.004332	22.78	65
66	9658.285	52.802	0.005467	0.005069	21.88	66
67	9605.483	61.158	0.006367	0.005914	21.00	67
68	9544.325	70.580	0.007395	0.006882	20.13	68
69	9473.745	81.124	0.008563	0.007986	19.28	69
70	9392.621	92.874	0.009888	0.009240	18.44	70
71	9299.747	105.887	0.011386	0.010663	17.62	71
72	9193.860	120.210	0.013075	0.012272	16.81	72
73	9073.650	135.860	0.014973	0.014086	16.03	73
74	8937.791	152.836	0.017100	0.016126	15.27	74
75	8784.955	171.113	0.019478	0.018414	14.52	75
76	8613.841	190.598	0.022127	0.020974	13.80	76
77	8423.243	211.162	0.025069	0.023829	13.10	77
78	8212.080	232.615	0.028326	0.027004	12.42	78
79	7979.465	254.729	0.031923	0.030527	11.77	79
80	7724.737	277.179	0.035882	0.034425	11.14	80
81	7447.558	299.593	0.040227	0.038728	10.54	81
82	7147.965	321.523	0.044981	0.043464	9.96	82
83	6826.442	342.455	0.050166	0.048664	9.41	83
84	6483.987	361.832	0.055804	0.054357	8.88	84
85	6122.154	379.053	0.061915	0.060576	8.37	85
86	5743.101	393.506	0.068518	0.067349	7.89	86
87	5349.595	404.595	0.075631	0.074708	7.43	87
88	4945.000	411.770	0.083270	0.082686	7.00	88
89	4533.230	414.537	0.091444	0.091308	6.59	89
90	4118.693	412.545	0.100164	0.100604	6.20	90
91	3706.149	405.590	0.109437	0.110601	5.84	91
92	3300.559	393.644	0.119266	0.121325	5.49	92
93	2906.914	376.882	0.129650	0.132801	5.17	93
94	2530.033	355.677	0.140582	0.145048	4.87	94
95	2174.356	330.617	0.152053	0.158084	4.58	95
96	1843.738	302.467	0.164051	0.171926	4.32	96
97	1541.271	272.119	0.176555	0.186586	4.07	97
98	1269.152	240.562	0.189545	0.202071	3.84	98
99	1028.591	208.795	0.202991	0.218386	3.62	99
100	819.796	177.783	0.216863	0.235531	3.41	100
101	642.013	148.385	0.231125	0.253502	3.22	101
102	493.627	121.303	0.245737	0.272288	3.05	102
103	372.325	97.048	0.260654	0.291872	2.89	103
104	275.277	75.930	0.275830	0.312234	2.73	104
105	199.347	58.053	0.291217	0.333348	2.59	105

4%	PMA92C20			PFA92C20	
x	$\ddot{a}_x$	^2A_x	x	$\ddot{a}_x$	^2A_x
50	18.843	0.08802	50	19.539	0.07421
51	18.567	0.09471	51	19.291	0.07978
52	18.281	0.10187	52	19.034	0.08574
53	17.985	0.10954	53	18.768	0.09211
54	17.680	0.11773	54	18.494	0.09891
55	17.364	0.12647	55	18.210	0.10616
56	17.038	0.13580	56	17.917	0.11390
57	16.702	0.14574	57	17.615	0.12214
58	16.356	0.15632	58	17.303	0.13091
59	15.999	0.16756	59	16.982	0.14024
60	15.632	0.17950	60	16.652	0.15015
61	15.254	0.19217	61	16.311	0.16068
62	14.868	0.20550	62	15.963	0.17177
63	14.475	0.21950	63	15.606	0.18343
64	14.073	0.23416	64	15.242	0.19566
65	13.666	0.24946	65	14.871	0.20847
66	13.252	0.26538	66	14.494	0.22183
67	12.834	0.28190	67	14.111	0.23576
68	12.412	0.29899	68	13.723	0.25022
69	11.988	0.31660	69	13.330	0.26521
70	11.562	0.33469	70	12.934	0.28069
71	11.136	0.35320	71	12.535	0.29664
72	10.711	0.37208	72	12.135	0.31302
73	10.288	0.39125	73	11.734	0.32980
74	9.870	0.41065	74	11.333	0.34693
75	9.456	0.43021	75	10.933	0.36437
76	9.049	0.44984	76	10.536	0.38207
77	8.649	0.46947	77	10.142	0.39997
78	8.258	0.48903	78	9.752	0.41802
79	7.877	0.50844	79	9.367	0.43616
80	7.506	0.52762	80	8.989	0.45433
81	7.148	0.54650	81	8.618	0.47247
82	6.801	0.56501	82	8.254	0.49053
83	6.468	0.58310	83	7.900	0.50845
84	6.148	0.60071	84	7.555	0.52616
85	5.842	0.61779	85	7.220	0.54363
86	5.551	0.63429	86	6.896	0.56080
87	5.273	0.65019	87	6.582	0.57762
88	5.010	0.66545	88	6.281	0.59405
89	4.762	0.68006	89	5.991	0.61006
90	4.527	0.69399	90	5.713	0.62560
91	4.306	0.70725	91	5.447	0.64066
92	4.098	0.71983	92	5.193	0.65520
93	3.903	0.73174	93	4.951	0.66921
94	3.721	0.74297	94	4.722	0.68268
95	3.551	0.75356	95	4.504	0.69559
96	3.393	0.76350	96	4.297	0.70794
97	3.245	0.77282	97	4.102	0.71973
98	3.109	0.78155	98	3.918	0.73097
99	2.982	0.78969	99	3.744	0.74164
100	2.864	0.79728	100	3.581	0.75177
101	2.755	0.80434	101	3.428	0.76136
102	2.655	0.81089	102	3.284	0.77043
103	2.562	0.81696	103	3.149	0.77899
104	2.477	0.82257	104	3.023	0.78705
105	2.399	0.82774	105	2.905	0.79463

Note. $^2A_x = A_x$ at 8.16%

PMA92C20 and PFA92C20

$\ddot{a}_{xy}$ for male (x) and female (y)

Age difference d ($= y - x$)

4%

x	d-20	-10	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+10	+20 d	x
50	18.746	18.493	18.192	18.110	18.019	17.918	17.808	17.688	17.556	17.413	17.258	17.090	16.909	15.801	12.638	50
51	18.467	18.206	17.894	17.809	17.715	17.612	17.498	17.374	17.238	17.091	16.931	16.758	16.572	15.433	12.232	51
52	18.179	17.908	17.586	17.499	17.402	17.295	17.178	17.050	16.910	16.758	16.594	16.416	16.225	15.057	11.823	52
53	17.881	17.601	17.269	17.178	17.078	16.968	16.848	16.716	16.572	16.415	16.246	16.064	15.867	14.672	11.413	53
54	17.573	17.283	16.941	16.847	16.744	16.631	16.507	16.371	16.223	16.062	15.888	15.701	15.499	14.279	11.004	54
55	17.255	16.955	16.602	16.506	16.400	16.284	16.156	16.016	15.864	15.699	15.521	15.328	15.121	13.880	10.595	55
56	16.926	16.617	16.253	16.155	16.046	15.926	15.795	15.651	15.495	15.326	15.143	14.945	14.733	13.473	10.189	56
57	16.587	16.269	15.894	15.793	15.681	15.558	15.423	15.276	15.116	14.942	14.755	14.553	14.337	13.061	9.786	57
58	16.238	15.910	15.525	15.421	15.306	15.180	15.041	14.891	14.727	14.549	14.357	14.151	13.932	12.644	9.387	58
59	15.879	15.541	15.146	15.039	14.921	14.791	14.650	14.495	14.327	14.145	13.950	13.742	13.520	12.222	8.993	59
60	15.509	15.161	14.756	14.646	14.526	14.393	14.248	14.090	13.918	13.734	13.536	13.325	13.101	11.796	8.605	60
61	15.129	14.772	14.356	14.244	14.121	13.985	13.837	13.675	13.501	13.314	13.114	12.901	12.675	11.368	8.224	61
62	14.740	14.374	13.949	13.834	13.708	13.569	13.418	13.254	13.078	12.888	12.686	12.472	12.245	10.939	7.851	62
63	14.343	13.968	13.533	13.416	13.287	13.145	12.992	12.826	12.648	12.458	12.255	12.039	11.812	10.511	7.487	63
64	13.939	13.555	13.111	12.991	12.859	12.716	12.561	12.394	12.215	12.023	11.819	11.604	11.376	10.085	7.133	64
65	13.529	13.136	12.682	12.560	12.427	12.282	12.126	11.958	11.778	11.586	11.382	11.167	10.940	9.662	6.790	65
66	13.112	12.711	12.248	12.125	11.991	11.845	11.688	11.520	11.339	11.147	10.944	10.729	10.504	9.243	6.457	66
67	12.692	12.282	11.811	11.687	11.552	11.406	11.248	11.080	10.900	10.708	10.506	10.293	10.070	8.830	6.137	67
68	12.267	11.849	11.372	11.247	11.112	10.966	10.808	10.640	10.460	10.270	10.070	9.859	9.639	8.423	5.829	68
69	11.840	11.414	10.933	10.807	10.672	10.526	10.369	10.201	10.023	9.835	9.637	9.429	9.213	8.025	5.533	69
70	11.412	10.978	10.494	10.368	10.233	10.088	9.932	9.766	9.590	9.404	9.209	9.005	8.792	7.636	5.250	70
75	9.295	8.833	8.357	8.238	8.110	7.975	7.831	7.679	7.520	7.355	7.182	7.005	6.822	5.860	4.027	75
80	7.335	6.876	6.441	6.336	6.224	6.107	5.985	5.857	5.725	5.588	5.449	5.306	5.161	4.422	3.108	80
85	5.660	5.235	4.864	4.777	4.687	4.593	4.496	4.396	4.294	4.189	4.084	3.977	3.870	3.340	2.449	85
90	4.339	3.963	3.664	3.597	3.528	3.456	3.384	3.310	3.235	3.160	3.084	3.008	2.933	2.571	1.998	90
95	3.361	3.039	2.808	2.757	2.706	2.654	2.602	2.549	2.496	2.444	2.391	2.339	2.288	2.049	1.708	95
100	2.670	2.400	2.223	2.186	2.149	2.112	2.075	2.038	2.001	1.965	1.930	1.895	1.861	1.708	1.000	100

16 International Actuarial Notation

Reproduced from Bulletin of the Permanent Committee of the International Congress of Actuaries, 46, 207 (1949), Journal of the Institute of Actuaries, 75, 121 (1949) and Transactions of the Faculty of Actuaries, 19, 89 (1949–50).

The existing international actuarial notation was founded on the "Key to the Notation" given in the Institute of Actuaries Text Book, Part II, Life Contingencies by George King (1887), and was adopted by the Second International Actuarial Congress, London, 1898 (Transactions, pp. 618–640) with minor revisions approved by the Third International Congress, Paris, 1900 (Transactions, pp. 622– 651). Further revisions were discussed during 1937–1939, and were introduced by the Institute and the Faculty in 1949 (J.I.A., 75, 121 and T.F.A., 19, 89). These revisions were finally adopted internationally at the Fourteenth International Actuarial Congress, Madrid, 1954 (Bulletin of the Permanent Committee of the International Congress of Actuaries (1949), 46, pp. 207–217).

The general principles on which the system is based are as follows:

To each fundamental symbolic letter are attached signs and letters each having its own signification.

The lower space to the left is reserved for signs indicating the conditions relative to the duration of the operations and to their position with regard to time.

The lower space to the right is reserved for signs indicating the conditions relative to ages and the order of succession of the events.

The upper space to the right is reserved for signs indicating the periodicity of events.

The upper space to the left is free, and in it can be placed signs corresponding to other notions.

In what follows these two conventions are used:

A letter enclosed in brackets, thus (x), denotes "a person aged x".

A letter or number enclosed in a right angle, thus $\overline{n|}$ or $\overline{15|}$, denotes a term-certain of years.

16.1 Fundamental Symbolic Letters

Interest

i = the effective rate of interest, namely, the total interest earned on 1 in a year on the assumption that the actual interest (if receivable otherwise than yearly) is invested forthwith as it becomes due on the same terms as the original principal.

$\nu = (1+i)^{-1}$ = the present value of 1 due one year hence.

$d = 1 - \nu$ = the discount on 1 due one year hence.

$\delta = \log_e(1+i) = -\log_e(1-d)$ = the force of interest or the force of discount.

$$a_{\overline{n}|} = v + v^2 + \ldots + v^n$$

= the value of an annuity-certain of 1 per annum for n years, the payments being made at the end of each year.

$$\ddot{a}_{\overline{n}|} = 1 + v + v^2 + \ldots + v^{n-1}$$

= the value of a similar annuity, the payments being made at the beginning of each year.

$$s_{\overline{n}|} = 1 + (1+i) + (1+i)^2 + \ldots + (1+i)^{n-1}$$

= the amount of an annuity-certain of 1 per annum for n years, the payments being made at the end of each year.

$$\ddot{s}_{\overline{n}|} = (1+i) + (1+i)^2 + \ldots + (1+i)^n$$

= the amount of a similar annuity, the payments being made at the beginning of each year.

The diaeresis or trema (̈) above the letters a and s is used as a symbol of acceleration of payments.

Mortality Tables

l = number living.

d = number dying.

p = probability of living.

q = probability of dying.

μ = force of mortality.

m = central death rate.

a = present value of an annuity.

s = amount of an annuity.

e = expectation of life.

A = present value of an assurance.

E = present value of an endowment.

$$\left.\begin{array}{lcl} P & = & \text{premium per annum.} \\ \pi & = & \text{premium per annum.} \end{array}\right\}$$ P generally refers to net premiums, π to special premiums.

V = policy value.

W = paid-up policy.

The methods of using the foregoing principal letters and their precise meaning when added to by suffixes, etc., follow

The ages of the lives involved are denoted by letters placed as suffixes in the lower space to the right. Thus:

$l_x =$ the number of persons who attain age x according to the mortality table.

$d_x = l_x - l_{x+1}=$ the number of persons who die between ages x and $x + 1$ according to the mortality table.

$p_x =$ the probability that (x) will live 1 year.

$q_x =$ the probability that (x) will die within 1 year.

$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} =$ the force of mortality at age x.

$m_x =$ the central death-rate for the year of age x to $x + 1 = d_x / \int_0^1 l_{x+t} dt$.

$e_x =$ the curtate "expectation of life" (or average after-lifetime) of (x).

In the following it is always to be understood (unless otherwise expressed) that the annual payment of an annuity is 1, that the sum assured in any case is 1, and that the symbols indicate the present values:

$a_x =$ an annuity, first payment at the end of a year, to continue during the life of (x).

$\ddot{a}_x = 1 + a_x =$ an "annuity-due" to continue during the life of (x),
the first payment to be made at once.

$A_x =$ an assurance payable at the end of the year of death of (x).

Note. $e_x = a_x$ at rate of interest $i = 0$.

A letter or number at the lower left corner of the principal symbol denotes the number of years involved in the probability or benefit in question. Thus:

${}_nP_x =$ the probability that (x) will live n years.

${}_nq_x =$ the probability that (x) will die within n years.

Note. When $n = 1$ it is customary to omit it (as shown above) provided no ambiguity is introduced.

> ${}_nE_x = v^n \, nP_x =$ the value of an endowment on (x) payable at the end of n years if (x) be then alive.

If the letter or number comes before a perpendicular bar it shows that a period of deferment is meant. Thus:

> ${}_{n|}q_x =$ the probability that (x) will die in a year, deferred n years; that is, that he will die in the $(n+1)^{\mathrm{th}}$ year.
>
> ${}_{n|}a_x =$ an annuity on (x) deferred n years; that is, that the first payment is to be made at the end of $(n+1)$ years.
>
> ${}_{n|t}a_x$= an intercepted, or deferred, temporary annuity on (x) deferred n years and, after that, to run for t years.

A letter or number in brackets at the upper right corner of the principal symbol shows the number of intervals into which the year is to be divided. Thus:

> $a_x^{(m)} =$ an annuity of (x) payable by m instalments of $1/m$ each throughout the year, the first payment being one of $1/m$ at the end of the first $1/m^{\mathrm{th}}$ of a year.
>
> $\ddot{a}_x^{(m)} =$ a similar annuity but the first payment of $1/m$ is to be made at once, so that $\ddot{a}_x^{(m)} = 1/m + a_x^{(m)}$.
>
> $A_x^{(m)} =$ an assurance payable at the end of that fraction $1/m$ of a year in which (x) dies.

If $m \to \infty$ then instead of writing (∞) a bar is placed over the principal symbol. Thus:

> $\overline{a} =$ a continuous or momently annuity.
>
> $\overline{A}$= an assurance payable at the moment of death.

A small circle placed over the principal symbol shows that the benefit is to be complete. Thus:

> $\mathring{a} =$ a complete annuity.
>
> $\mathring{e} =$ the complete expectation of life.

Note. Some consider that $\bar{e}$ would be as appropriate as $\mathring{e}$. As $e_x = a_x$ at rate of interest $i = 0$, so also the complete expectation of life $= \bar{a}_x$ at rate of interest $i = 0$.

When more than one life is involved, the following rules are observed:

If there are two or more letters or numbers in a suffix without any distinguishing mark, joint lives are intended. Thus:

$$l_{xy} = l_x \times l_y, \quad d_{xy} = l_{xy} - l_{x+1:y+1}.$$

Note. When, for the sake of distinctness, it is desired to separate letters or numbers in a suffix, a colon is placed between them. A colon is used instead of a point or comma to avoid confusion with decimals when numbers are involved.

$a_{xyz} =$ an annuity, first payment at the end of a year, to continue during the joint lives of $(x), (y)$ and (z).
$A_{xyz} =$ an assurance payable at the end of the year of the failure of the joint lives $(x), (y)$ and (z).

In place of a life a term-certain may be involved. Thus:

$a_{x:\overline{n}|} =$ an annuity to continue during the joint duration of the life of (x) and a term of n years certain; that is, a temporary annuity for n years on the life of (x).

$A_{x:\overline{n}|} =$ an assurance payable at the end of the year of death of (x) if he dies within n years, or at the end of n years if (x) be then alive; that is, an endowment assurance for n years.

If a perpendicular bar separates the letters in the suffix, then the status after the bar is to follow the status before the bar. Thus:
$a_{y|x}$ = a reversionary annuity, that is, an annuity on the life of (x) after the death of (y).

$A_{z|xy}$ = an assurance payable on the failure of the joint lives (x) and (y) provided both these lives survive (z).

If a horizontal bar appears above the suffix then survivors of the lives, and not joint lives, are intended. The number of survivors can be denoted by a letter or number over the right end of the bar. If that letter, say r, is not distinguished by any mark, then the meaning is *at least r survivors*; but if it is enclosed in square brackets, $[r]$, then the meaning is *exactly r survivors*. If no letter or number appears over the bar, then unity is supposed and the meaning is *at least one survivor*. Thus:

$a_{\overline{xyz}}$ = an annuity payable so long as at least one of the three lives $(x), (y)$ and (z) is alive.

$a^{2}_{\overline{xyz}}$= an annuity payable so long as at least two of the three lives $(x), (y)$ and (z) are alive.

$p^{[2]}_{\overline{xyz}}$ = probability that exactly two of the three lives $(x), (y)$ and (z) will survive a year.

${}_nq_{\overline{xy}}$= probability that the survivor of the two lives (x) and (y) will die within n years $= nq_x \times nq_y$.

$nA_{\overline{xy}}$ = an assurance payable at the end of the year of death of the survivor of the lives (x) and (y) provided the death occurs within n years.

When numerals are placed above or below the letters of the suffix, they designate the order in which the lives are to fail. The numeral placed over the suffix points out the life whose failure will finally determine the event; and the numerals placed under the suffix indicate the order in which the other lives involved are to fail. Thus:

A^{1}_{xy} = an assurance payable at the end of the year of death of (x) if he dies first of the two lives (x) and (y).

A^{2}_{xyz} = an assurance payable at the end of the year of death of (x) if he dies second of the three lives $(x), (y)$ and (z).

$A^2_{\binom{xyz}{1}}$ = an assurance payable at the end of the year of death of (x) if he dies second of the three lives, (y) having died first

$A_{xy:\frac{z}{3}}$ = an assurance payable at the end of the year of death of the survivor of (x) and (y) if he dies before (z).
$A^1_{x:\overline{n}|}$ = an assurance payable at the end of the year of death of (x)
if he dies within a term of n years.

$$\begin{cases} a_{\overline{y\frac{z}{1}}|x} \\ a_{\overline{\overset{2}{yz}}|x} \end{cases} = \begin{array}{l} \text{an annuity to } (x) \text{ after the failure of the survivor of } (y) \\ \text{and } (z) \text{, provided } (z) \text{ fails before } (y) \end{array}.$$

Note. Sometimes to make quite clear that a joint-life status is involved a symbol $\sqcap$ is placed above the lives included. Thus

$$A^1_{\sqcap xy:\overline{n}|} = \text{a joint-life temporary assurance on } (x) \text{ and } (y).$$

In the case of reversionary annuities, distinction has sometimes to be made between those where the times of year at which payments are to take place are determined at the outset and those where the times depend on the failure of the preceding status. Thus:

$a_{y|x}$ = annuity to (x), first payment at the end of the year of death of (y)
or, on the average, about 6 months after his death.

$\hat{a}_{y|x}$ = annuity to (x), first payment 1 year after the death of (y).

$\mathring{a}_{y|x}$ = complete annuity to (x), first payment 1 year after the death of (y).

16.2 Annual Premiums

The symbol P with the appropriate suffix or suffixes is used in simple cases, where no misunderstanding can occur, to denote the annual premium for a benefit. Thus:

P_x= the annual premium for an assurance payable at the end of the year of death of (x).

$P_{x:\overline{n|}}$ = the annual premium for an endowment assurance on (x) payable after n years or at the end of the year of death of (x) if he dies within n years.

P^1_{xy} =
the annual premium for a contingent assurance payable at the end of the year of death of (x) if he dies before (y).

In all cases it is optional to use the symbol P in conjunction with the principal symbol denoting the benefit. Thus instead of $P_{x:\overline{n|}}$ we may write $P(A_{x:\overline{n|}})$. In the more complicated cases it is necessary to use the two symbols in this way. Suffixes, etc., showing the conditions of the benefit are to be attached to the principal letter, and those showing the condition of payment of the premium are to be attached to the subsidiary symbol P. Thus:

$nP(\overline{A}_x)$= the annual premium payable for n years only for an assurance payable at the moment of death of (x).

$P_{xy}(A_x)$ = the annual premium payable during the joint lives of (x) and (y) for an assurance payable at the end of the year of death of (x).

$nP_{(n|a_x)}(a_x)$= the annual premium payable for n years only for an annuity on (x) deferred n years.

${}_tP^{(m)}(A_{x:\overline{n|}})$ =the annual premium payable for t years only, by m instalments throughout the year, for an endowment assurance for n years on (x) (see below as to $P^{(m)}$).

Notes
1. As a general rule the symbol P could be used without the principal symbol in the case of assurances where the sum assured is payable at the end of the year of death, but if it is payable at other times, or if the benefit is an annuity, then the principal symbol should be used.

2. $P_x^{(m)}$. A point which was not brought out when the international system was adopted is that there are two kinds of premiums payable m times a year, namely those which cease on payment of the installment immediately preceding death and those which

continue to be payable to the end of the year of death. To distinguish the latter, the m is sometimes enclosed in square brackets, thus $P^{[m]}$.

16.3 Policy Values and Paid-up Policies

${}_tV_x$ = the value of an ordinary whole-life assurance on (x) which has been t years in force, the premium then just due being unpaid.

${}_tW_x$ = the paid-up policy the present value of which is ${}_tV_x$.

The symbols V and W may, in simple cases, be used alone, but in the more complicated cases it is necessary to insert the full symbol for the benefit thus:

$${}_tV^{(m)}\left(\overline{A}_{x:\overline{n|}}\right) \text{ (corresponding to } P^{(m)}\left(\overline{A}_{x:\overline{n|}}\right)), {}_t\,V_{n|}a_x.$$

Note. As a general rule V or W can be used as the main symbol if the sum assured is payable at the end of the year of death and the premium is payable periodically throughout the duration of the assurance. If the premium is payable for a limited number of years, say n, the policy value after t years could be written ${}_tV[{}_nP(A)]$, or, if desired, ${}^n_tV(A)$.

In investigations where modified premiums and policy values are in question such modification may be denoted by adding accents to the symbols. Thus, when a premium other than the net premium (a valuation premium) is used in a valuation it may be denoted by P' and the corresponding policy value by V'. Similarly, the office (or commercial) premium may be denoted by P'' and the corresponding paid-up policy by W''.

16.4 Compound Symbols

$$\left.\begin{array}{rcl} (Ia) & = & \text{an annuity} \\ (L\overline{A}) & = & \text{an assurance} \end{array}\right\} \text{commencing at 1 and increasing 1 per annum.}$$

If the whole benefit is to be temporary the symbol of limitation is placed outside the brackets. Thus:

$$(Ia)_{x:\overline{n|}} = \text{a temporary increasing annuity.}$$

$$(LA)^1_{x:\overline{n|}} = \text{a temporary increasing assurance.}$$

If only the increase is to be temporary but the benefit is to continue thereafter, then the symbol of limitation is placed immediately after the symbol I. Thus:

$$\left.\begin{array}{rcl} (I_{\overline{n|}}a)_x & = & \text{a whole-life annuity} \\ (I_{\overline{n|}}A)_x & = & \text{a whole-life assurance} \end{array}\right\}\text{increasing for years and thereafter stationary.}$$

If the benefit is a decreasing one, the corresponding symbol is D. From the nature of the case this decrease must have a limit, as otherwise negative values might be implied. Thus:

$(D_{\overline{n|}}A)^{1}_{x:\overline{n|}} =$ a temporary assurance commencing at n and decreasing by 1 in each successive year.

If the benefit is a varying one the corresponding symbol is ν. Thus:

$$(\nu a) = \text{a varying annuity.}$$

16.5 Commutation Columns

Single Lives

$$D_x = v^x I_x,$$

$$N_x = D_x + D_{x+1} + D_{x+2} + \text{etc.},$$

$$S_x = N_x + N_{x+1} + N_{x+2} + \text{etc.},$$

$$C_x = v^{x+1} d_x,$$

$$M_x = C_x + C_{x+1} + C_{x+2} + \text{etc.},$$

$$R_x = M_x + M_{x+1} + M_{x+2} + \text{etc.}.$$

When it is desired to construct the assurance columns so as to give directly assurances payable at the moment of death, the symbols are distinguished by a bar placed over them. Thus:

$$\overline{C}_x = v^{x+\frac{1}{2}} d_x, \text{ which is an approximation to } \int_0^1 v^{x+t} \mu_{x+t} l_{x+t} dt.$$

$$\overline{M}_x = \overline{C}_x + \overline{C}_{x+1} + \overline{C}_{x+2} + \text{etc.},$$

$$\overline{R}_x = \overline{M}_x + \overline{M}_{x+1} + \overline{M}_{x+2} + \text{etc.}$$

Joint Lives

$$D_{xy} = v^{\frac{1}{2}(x+y)} I_{xy},$$

$$N_{xy} = D_{xy} + D_{x+1:y+1} + D_{x+2:y+2} + \text{etc.},$$

$$C_{xy} = v^{\frac{1}{2}(x+y)+1} d_{xy},$$

$$M_{xy} = C_{xy} + C_{x+1:y+1} + C_{x+2:y+2} + \text{etc.},$$

$$C^1_{xy} = v^{\frac{1}{2}(x+y)+1} d_{xy},$$

$$M^1_{xy} = C^1_{xy} + C^1_{x+1:y+1} + C^1_{x+2:y+2} + \text{etc.}$$

16.6 Selection

If the suffix to a symbol which denotes the age is enclosed in a square bracket it indicates the age at which the life was selected. To this may be added, outside the bracket, the number of years which have elapsed since selection, so that the total suffix denotes the present age. Thus:

$l_{[x]+t}$ = the number in the select life table who were selected at age x and have attained age x + t.

$d_{[x]+t} = l_{[x]+t} - l_{[x]+t+1}$.

$a_{[x]}$ = value of an annuity on a life now aged x and now select.

$a_{[x-n]+n}$ = value of an annuity on a life now aged x and select n years ago at age x - n.

$N_{[x]} = D_{[x]} + D_{[x]+1} + D_{[x]+2} + \dots$

$\ddot{a}_{[x]} = N_{[x]} + D_{[x]} = 1 + a_{[x]}$.

and similarly for other functions.

When Dr. Sprague presented his statement [in 1900] he mentioned that an objection had been raised that the notation in some cases offers the choice of two symbols for the same benefit. For instance, a temporary annuity may be denoted either by ${}_na_x$ or by $a_{x:\overline{n|}}$. This is, he says, a necessary consequence of the principles underlying the system, and neither of the alternative forms could have been suppressed without injury to the symmetry of the system.

17 Sickness Table (Manchester Unity Methodology)

S(MU)

This table was produced using the methodology underlying that of the Manchester Unity Sickness Experience 1893–97. The underlying rates of sickness have, however, been updated to reflect more modern experience, and have been combined with the mortality of English Life Tables No. 15 (Males).

17.1 S(MU) Central rates of sickness (weeks per annum)

Duration of sickness in weeks

Age	0 − 13	13 − 26	26 − 52	52 − 104	>= 104	*All*	*Age*
16	0.3150	0.0048	0.0012	0.0000	0.0000	0.3210	16
17	0.3323	0.0080	0.0044	0.0020	0.0000	0.3467	17
18	0.3482	0.0088	0.0050	0.0039	0.0011	0.3670	18
19	0.3576	0.0097	0.0056	0.0044	0.0030	0.3803	19
20	0.3665	0.0106	0.0063	0.0051	0.0048	0.3933	20
21	0.3749	0.0116	0.0070	0.0058	0.0068	0.4061	21
22	0.3830	0.0127	0.0078	0.0066	0.0089	0.4190	22
23	0.3905	0.0139	0.0087	0.0074	0.0113	0.4318	23
24	0.3977	0.0151	0.0097	0.0084	0.0140	0.4449	24
25	0.4026	0.0164	0.0108	0.0095	0.0170	0.4563	25
26	0.4109	0.0178	0.0119	0.0107	0.0203	0.4716	26
27	0.4171	0.0193	0.0132	0.0120	0.0241	0.4857	27
28	0.4230	0.0209	0.0146	0.0135	0.0284	0.5004	28
29	0.4287	0.0225	0.0161	0.0151	0.0332	0.5156	29
30	0.4344	0.0243	0.0177	0.0169	0.0386	0.5319	30
31	0.4398	0.0262	0.0195	0.0189	0.0448	0.5492	31
32	0.4454	0.0283	0.0215	0.0211	0.0518	0.5681	32
33	0.4510	0.0304	0.0236	0.0236	0.0596	0.5882	33
34	0.4567	0.0328	0.0259	0.0263	0.0686	0.6103	34
35	0.4626	0.0353	0.0284	0.0293	0.0787	0.6343	35
36	0.4688	0.0379	0.0312	0.0327	0.0901	0.6607	36
37	0.4752	0.0408	0.0342	0.0364	0.1031	0.6897	37
38	0.4822	0.0439	0.0376	0.0405	0.1179	0.7221	38
39	0.4898	0.0473	0.0412	0.0452	0.1346	0.7581	39
40	0.4979	0.0509	0.0453	0.0503	0.1536	0.7980	40
41	0.5067	0.0548	0.0497	0.0561	0.1752	0.8425	41
42	0.5163	0.0591	0.0546	0.0625	0.1997	0.8922	42
43	0.5269	0.0638	0.0601	0.0697	0.2277	0.9482	43
44	0.5386	0.0689	0.0661	0.0778	0.2595	1.0109	44
45	0.5514	0.0745	0.0729	0.0869	0.2959	1.0816	45
46	0.5656	0.0806	0.0804	0.0972	0.3374	1.1612	46
47	0.5812	0.0874	0.0888	0.1088	0.3850	1.2512	47
48	0.5986	0.0948	0.0982	0.1220	0.4395	1.3531	48
49	0.6178	0.1031	0.1088	0.1370	0.5020	1.4687	49
50	0.6390	0.1123	0.1207	0.1540	0.5740	1.6000	50
51	0.6626	0.1225	0.1341	0.1734	0.6569	1.7495	51
52	0.6888	0.1339	0.1493	0.1956	0.7527	1.9203	52
53	0.7178	0.1466	0.1666	0.2210	0.8636	2.1156	53
54	0.7499	0.1609	0.1862	0.2503	0.9921	2.3394	54
55	0.7856	0.1769	0.2085	0.2839	1.1416	2.5965	55
56	0.8251	0.1949	0.2340	0.3228	1.3158	2.8926	56
57	0.8691	0.2153	0.2632	0.3677	1.5193	3.2346	57
58	0.9177	0.2382	0.2967	0.4199	1.7578	3.6303	58
59	0.9717	0.2642	0.3351	0.4804	2.0378	4.0892	59
60	1.0311	0.2935	0.3793	0.5508	2.3677	4.6224	60
61	1.0968	0.3268	0.4300	0.6328	2.7574	5.2438	61
62	1.1690	0.3643	0.4884	0.7285	3.2189	5.9691	62
63	1.2478	0.4067	0.5555	0.8400	3.7670	6.8170	63
64	1.3335	0.4543	0.6325	0.9700	4.4198	7.8101	64

17.2 S(MU) Present value of a sickness benefit payable at the rate of 1 per week during sickness of the following durations. All benefits cease at the earlier of death or attainment of age 65.

Duration of sickness in weeks 4%

Age	0 − 13	13 − 26	26 − 52	52 − 104	>= 104	*All*	*Age*
16	10.236	1.113	1.171	1.515	5.786	19.821	16
17	10.329	1.153	1.217	1.576	6.021	20.297	17
18	10.412	1.192	1.262	1.639	6.266	20.771	18
19	10.482	1.232	1.309	1.702	6.522	21.246	19
20	10.546	1.272	1.357	1.767	6.785	21.726	20
21	10.603	1.313	1.406	1.834	7.057	22.213	21
22	10.654	1.355	1.456	1.903	7.339	22.707	22
23	10.699	1.398	1.508	1.974	7.630	23.209	23
24	10.739	1.441	1.560	2.047	7.931	23.718	24
25	10.772	1.484	1.614	2.122	8.241	24.235	25
26	10.802	1.528	1.669	2.199	8.561	24.760	26
27	10.825	1.573	1.725	2.278	8.890	25.291	27
28	10.842	1.617	1.783	2.359	9.229	25.830	28
29	10.853	1.662	1.841	2.442	9.578	26.376	29
30	10.860	1.707	1.899	2.527	9.936	26.929	30
31	10.862	1.752	1.959	2.613	10.303	27.489	31
32	10.858	1.797	2.020	2.701	10.680	28.055	32
33	10.849	1.842	2.080	2.790	11.065	28.626	33
34	10.834	1.887	2.142	2.880	11.458	29.201	34
35	10.813	1.931	2.203	2.972	11.859	29.778	35
36	10.787	1.974	2.265	3.064	12.267	30.358	36
37	10.754	2.017	2.327	3.158	12.682	30.939	37
38	10.715	2.059	2.388	3.251	13.103	31.517	38
39	10.668	2.100	2.449	3.345	13.527	32.089	39
40	10.613	2.139	2.509	3.438	13.953	32.653	40
41	10.548	2.176	2.568	3.531	14.380	33.203	41
42	10.473	2.212	2.625	3.622	14.804	33.735	42
43	10.387	2.245	2.680	3.710	15.223	34.245	43
44	10.288	2.274	2.732	3.796	15.634	34.725	44
45	10.176	2.301	2.780	3.878	16.034	35.169	45
46	10.048	2.323	2.825	3.955	16.418	35.569	46
47	9.904	2.341	2.864	4.026	16.781	35.916	47
48	9.740	2.353	2.898	4.090	17.117	36.199	48
49	9.556	2.360	2.925	4.145	17.419	36.405	49
50	9.348	2.359	2.944	4.189	17.678	36.517	50
51	9.114	2.350	2.952	4.219	17.884	36.520	51
52	8.851	2.331	2.949	4.233	18.025	36.390	52
53	8.554	2.302	2.932	4.228	18.085	36.101	53
54	8.219	2.259	2.899	4.200	18.046	35.624	54
55	7.842	2.202	2.846	4.143	17.888	34.921	55
56	7.417	2.127	2.770	4.053	17.584	33.951	56
57	6.938	2.033	2.667	3.922	17.104	32.663	57
58	6.397	1.915	2.532	3.743	16.409	30.995	58
59	5.786	1.769	2.358	3.506	15.455	28.875	59
60	5.096	1.592	2.140	3.199	14.184	26.211	60
61	4.316	1.378	1.867	2.808	12.528	22.897	61
62	3.433	1.120	1.531	2.316	10.401	18.800	62
63	2.431	0.810	1.118	1.702	7.698	13.759	63
64	1.293	0.441	0.613	0.941	4.286	7.574	64

17.3 Annuity values, allowing for mortality only, on the basis of ELT15 (Males)

4%	x	$\bar{a}_{x:\overline{65-x}\rvert}$
	16	21.231
	17	21.072
	18	20.911
	19	20.746
	20	20.573
	21	20.394
	22	20.208
	23	20.015
	24	19.813
	25	19.604
	26	19.385
	27	19.157
	28	18.920
	29	18.674
	30	18.418
	31	18.152
	32	17.875
	33	17.588
	34	17.289
	35	16.979
	36	16.658
	37	16.326
	38	15.982
	39	15.626
	40	15.256
	41	14.873
	42	14.476
	43	14.064
	44	13.638
	45	13.197
	46	12.740
	47	12.268
	48	11.779
	49	11.274
	50	10.752
	51	10.212
	52	9.653
	53	9.075
	54	8.475
	55	7.854
	56	7.210
	57	6.541
	58	5.846
	59	5.123
	60	4.368
	61	3.580
	62	2.754
	63	1.886
	64	0.970

18 Sickness Table: Inception Rate / Disability Annuity Methodology

S(ID)

This table was produced using an inception rate/disability annuity method based on results presented in *C.M.I.R. 12*. The following are tabulated:

- claim inception rates
- present values of current claim sickness annuities
- present values of annuities payable during sickness for lives currently healthy

The annuities cease at the earliest of:

- death;
- attainment of age 65;
- recovery from sickness.

18.1 S(ID)

Claim inception rates, , $(ia)_{x,d}$, for the given ages x and deferred periods d years. (These rates are central, and (when d = 0) allow for the possibility of falling sick more than once during the year of age from x to x + 1. It was assumed in the construction of this table that all lives are healthy at exact age 30.)

Deferred period in years, d

age, x	0	1	2
30	0.322744		
31	0.318254	0.000521	
32	0.313615	0.000578	0.000294
33	0.308879	0.000641	0.000330
34	0.304097	0.000709	0.000371
35	0.299317	0.000785	0.000416
36	0.294583	0.000869	0.000467
37	0.289937	0.000961	0.000524
38	0.285418	0.001063	0.000588
39	0.281061	0.001176	0.000659
40	0.276901	0.001301	0.000739
41	0.272968	0.001440	0.000829
42	0.269290	0.001594	0.000930
43	0.265896	0.001767	0.001044
44	0.262810	0.001959	0.001172
45	0.260057	0.002175	0.001317
46	0.257659	0.002416	0.001482
47	0.255639	0.002688	0.001669
48	0.254018	0.002994	0.001882
49	0.252816	0.003340	0.002125
50	0.252056	0.003732	0.002403
51	0.251758	0.004177	0.002721
52	0.251943	0.004682	0.003086
53	0.252630	0.005259	0.003507
54	0.253841	0.005918	0.003992
55	0.255594	0.006674	0.004554
56	0.257906	0.007541	0.005205
57	0.260793	0.008539	0.005962
58	0.264262	0.009690	0.006843
59	0.268316	0.011018	0.007873
60	0.272945	0.012554	0.009076
61	0.278123	0.014332	0.010487
62	0.283800	0.016390	0.012141
63	0.289890	0.018772	0.014083
64	0.296263	0.021524	0.016362

S(ID)

Present values of sickness benefit payable continuously at the rate of 1 per annum during sickness of the specified duration. All benefits cease at the earlier of death or attainment of age 65.

6%

CURRENT STATUS = SICK

The table below gives the present value,

$$\overline{a}^{\overline{ss}}_{x,z}$$

of a "current claim" sickness annuity for a sick life now aged x with current duration of sickness z years. (The annuity does not allow for the possibility of future new episodes of sickness.)

Current duration of sickness, z years

Age, x	0	1	2
	0.0333	3.5702	5.4180
	0.0350	3.6604	5.5051
	0.0368	3.7519	5.5915
	0.0388	3.8443	5.6769
	0.0410	3.9375	5.7610
	0.0435	4.0311	5.8432
	0.0462	4.1246	5.9230
	0.0492	4.2178	5.9997
	0.0525	4.3099	6.0728
	0.0562	4.4006	6.1413
	0.0603	4.4889	6.2044
	0.0649	4.5743	6.2612
	0.0699	4.6557	6.3106
	0.0754	4.7321	6.3512
	0.0815	4.8023	6.3819
	0.0883	4.8651	6.4011
	0.0957	4.9189	6.4071
	0.1038	4.9619	6.3981
	0.1126	4.9923	6.3721
	0.1221	5.0080	6.3269
	0.1324	5.0064	6.2599
	0.1433	4.9849	6.1686
	0.1549	4.9405	6.0498
	0.1670	4.8697	5.9004
	0.1793	4.7688	5.7169
	0.1917	4.6337	5.4952
	0.2035	4.4596	5.2312
	0.2144	4.2414	4.9202
	0.2234	3.9733	4.5571
	0.2295	3.6490	4.1363
	0.2312	3.2614	3.6518
	0.2267	2.8029	3.0970
	0.2134	2.2643	2.4648
	0.1875	1.6336	1.7469
	0.1429	0.8925	0.9315

CURRENT STATUS = HEALTHY

The table below gives the present value,

$$\overline{a}^{HS(d/all)}_{x}$$

of sickness benefit payable during sickness of duration at least d years for a life aged x who is currently healthy. (The value allows for the possibility of more than one episode of sickness.)

Deferred period, d years

Age, x	0	1	2
30	0.330580	0.142025	0.111543
31	0.339378	0.148808	0.116826
32	0.348311	0.155754	0.122226
33	0.357354	0.162837	0.127714
34	0.366480	0.170038	0.133274
35	0.375647	0.177324	0.138875
36	0.384822	0.184665	0.144486
37	0.393952	0.192016	0.150067
38	0.402981	0.199327	0.155573
39	0.411815	0.206529	0.160944
40	0.420352	0.213550	0.166111
41	0.428479	0.220304	0.171001
42	0.436077	0.226698	0.175528
43	0.443010	0.232611	0.179594
44	0.449125	0.237925	0.183090
45	0.454221	0.242488	0.185885
46	0.458091	0.246146	0.187843
47	0.460523	0.248719	0.188814
48	0.461260	0.250010	0.188628
49	0.460010	0.249788	0.187096
50	0.456447	0.247810	0.184025
51	0.450241	0.243825	0.179219
52	0.440992	0.237558	0.172462
53	0.428296	0.228736	0.163569
54	0.411745	0.217100	0.152372
55	0.390935	0.202426	0.138768
56	0.365518	0.184575	0.122748
57	0.335193	0.163508	0.104447
58	0.299804	0.139390	0.084219
59	0.259410	0.112669	0.062755
60	0.214401	0.084217	0.041213
61	0.165680	0.055536	0.021441
62	0.114894	0.029046	0.006275
63	0.064864	0.008533	0.000000
64	0.020334	0.000000	0.000000

18.2 Annuity values, allowing for mortality only, on the basis of ELT15 (Males

6%	x	$\overline{a}_{x:\overline{65-x}\rvert}$
	16	15.881
	17	15.813
	18	15.744
	19	15.673
	20	15.597
	21	15.517
	22	15.432
	23	15.342
	24	15.247
	25	15.146
	26	15.038
	27	14.924
	28	14.803
	29	14.674
	30	14.538
	31	14.394
	32	14.242
	33	14.081
	34	13.911
	35	13.731
	36	13.541
	37	13.342
	38	13.131
	39	12.909
	40	12.675
	41	12.428
	42	12.168
	43	11.893
	44	11.604
	45	11.299
	46	10.978
	47	10.640
	48	10.284
	49	9.910
	50	9.516
	51	9.102
	52	8.666
	53	8.207
	54	7.722
	55	7.211
	56	6.671
	57	6.101
	58	5.496
	59	4.856
	60	4.176
	61	3.452
	62	2.679
	63	1.851
	64	0.961

19 Example Pension Scheme Table: PEN

PEN
Service table and relative salary scale

Age x	l_x	w_x	d_x	i_x	r_x	s^*_x	$s_x = (1.02)^x s^*_x$	z_x	z_{x+1}	Age x
16	100000	10000	50			1.000	1.373			16
17	89950	8995	45			1.177	1.648			17
18	80910	8091	41			1.349	1.927			18
19	72778	7278	36			1.513	2.204			19
20	65464	6546	33			1.672	2.485			20
21	58885	5888	24			1.823	2.763			21
22	52973	5296	21			1.970	3.045			22
23	47656	4763	19			2.108	3.324			23
24	42874	4070	17			2.241	3.605			24
25	38787	3487	16			2.366	3.882			25
26	35284	2994	11			2.483	4.155			26
27	32279	2577	10			2.595	4.429			27
28	29692	2221	9			2.707	4.713			28
29	27462	1916	8			2.810	4.991			29
30	25538	1679	8	10		2.914	5.278	4.711	4.852	30
31	23841	1472	10	12		3.004	5.551	4.994	5.133	31
32	22347	1290	9	13		3.095	5.832	5.273	5.413	32
33	21035	1131	8	15		3.181	6.115	5.554	5.693	33
34	19881	989	8	18		3.259	6.389	5.833	5.972	34
35	18866	863	9	21		3.328	6.655	6.112	6.249	35
36	17973	751	11	21		3.392	6.920	6.386	6.520	36
37	17190	650	12	22		3.448	7.175	6.655	6.786	37
38	16506	558	12	25		3.491	7.410	6.916	7.042	38
39	15911	474	13	27		3.522	7.623	7.168	7.285	39
40	15397	413	14	31		3.539	7.814	7.403	7.509	40
41	14939	356	13	34		3.543	7.980	7.616	7.711	41
42	14536	303	14	38		3.539	8.129	7.806	7.890	42
43	14181	254	16	41		3.522	8.252	7.974	8.047	43
44	13870	207	17	44		3.504	8.375	8.120	8.186	44
45	13602	162	18	47		3.487	8.501	8.252	8.314	45
46	13375	120	19	51		3.470	8.628	8.376	8.439	46
47	13185	79	22	55		3.457	8.768	8.502	8.567	47
48	13029	52	26	62		3.440	8.899	8.632	8.699	48
49	12889	26	28	72		3.422	9.031	8.765	8.832	49
50	12763		32	82		3.405	9.165	8.899	8.965	50
51	12649		35	94		3.392	9.313	9.032	9.101	51
52	12520		39	108		3.375	9.451	9.170	9.240	52
53	12373		43	125		3.358	9.591	9.310	9.381	53
54	12205		47	145		3.345	9.745	9.452	9.524	54
55	12013		51	168		3.328	9.889	9.596	9.669	55
56	11794		55	193		3.310	10.034	9.742	9.815	56
57	11546		58	220		3.297	10.195	9.889	9.964	57
58	11268		63	248		3.280	10.344	10.039	10.115	58
59	10957		67	278		3.267	10.510	10.191	10.270	59
60	10612		73	310	3681	3.250	10.663	10.350	10.428	60
61	6548		50	219	516	3.233	10.819	10.506	10.585	61
62	5763		49	223	453	3.220	10.991	10.664	10.744	62
63	5038		48	224	395	3.203	11.151	10.824	10.906	63
64	4371		47	225	342	3.190	11.328	10.987	11.072	64
65	3757				3757			11.157		65

$$z_x = \frac{1}{3}(s_{x-3} + s_{x-2} + s_{x-1}) \text{ and } z_{x+1/2} = \frac{1}{2}(z_x + z_{x+1})$$

PEN

Contribution functions
4%

Age x	$D_x = v^x l_x$	$\overline{D}_x = \frac{1}{2}(D_x + D_{x+1})$	$\overline{N}_x = \sum \overline{D}_x$	${}^s\overline{D}_x = s_x \overline{D}_x$	${}^s\overline{N}_x = \sum^s \overline{D}^s_x$	${}^sD_x = s_x D_x$	Age x
16	53391	49784	413287	68343	1513322	73294	16
17	46178	43059	363503	70948	1444979	76087	17
18	39939	37241	320444	71761	1374031	76959	18
19	34544	32210	283203	70993	1302270	76136	19
20	29877	27859	250992	69232	1231277	74248	20
21	25841	24096	223134	66590	1162045	71410	21
22	22352	20844	199037	63476	1095455	68070	22
23	19335	18031	178193	59929	1031979	64265	23
24	16726	15638	160163	56376	972050	60299	24
25	14550	13638	144525	52947	915673	56486	25
26	12727	11961	130887	49693	862726	52875	26
27	11195	10548	118926	46719	813033	49583	27
28	9902	9354	108378	44082	766314	46664	28
29	8806	8340	99024	41622	722232	43947	29
30	7874	7471	90684	39431	680611	41558	30
31	7068	6719	83213	37296	641180	39232	31
32	6370	6068	76494	35390	603884	37153	32
33	5766	5503	70427	33647	568494	35255	33
34	5240	5010	64924	32011	534848	33477	34
35	4781	4580	59914	30480	502836	31816	35
36	4379	4204	55333	29087	472356	30305	36
37	4028	3873	51130	27788	443269	28897	37
38	3719	3583	47257	26546	415480	27554	38
39	3447	3327	43674	25361	388934	26275	39
40	3207	3099	40347	24219	363573	25059	40
41	2992	2896	37248	23106	339354	23875	41
42	2799	2713	34352	22052	316248	22757	42
43	2626	2548	31640	21023	294196	21668	43
44	2470	2399	29092	20093	273173	20683	44
45	2329	2265	26693	19256	253080	19796	45
46	2202	2144	24428	18502	233824	18997	46
47	2087	2035	22283	17842	215322	18298	47
48	1983	1935	20248	17215	197480	17645	48
49	1886	1841	18314	16627	180265	17034	49
50	1796	1754	16473	16073	163638	16460	50
51	1711	1670	14719	15554	147565	15939	51
52	1629	1588	13049	15011	132011	15394	52
53	1548	1508	11461	14462	117000	14845	53
54	1468	1429	9953	13923	102538	14306	54
55	1389	1350	8524	13354	88615	13739	55
56	1312	1273	7174	12775	75261	13161	56
57	1235	1197	5901	12199	62486	12587	57
58	1159	1121	4704	11595	50287	11984	58
59	1083	1046	3583	10993	38692	11385	59
60	1009	804	2537	8570	27699	10757	60
61	599	553	1733	5978	19129	6475	61
62	507	466	1181	5123	13152	5567	62
63	426	390	715	4354	8028	4748	63
64	355	324	324	3674	3674	4023	64
65	294						65

PEN
Ill health retirement functions
4%

Age x	$\overline{a}^i_{x+\frac{1}{2}}$	$C^i_x = v^{x+\frac{1}{2}} i_x$	$M^i_x = \sum C^i_x$	$\overline{R}^i_x = \sum(M^i_x - \frac{1}{2}C^i_x)$	$C^{ia}_x = C^i_x a^{-i}_{x+1/2}$	$M^{ia}_x = \sum c^{ia}_x$	$\overline{R}^{ia}_x = \sum(M^{ia}_x - \frac{1}{2}C^{ia}_x)$	Age x
16			414	15416		7023	252924	16
17			414	15002		7023	245901	17
18			414	14588		7023	238878	18
19			414	14173		7023	231855	19
20			414	13759		7023	224831	20
21			414	13345		7023	217808	21
22			414	12930		7023	210785	22
23			414	12516		7023	203762	23
24			414	12102		7023	196739	24
25			414	11688		7023	189715	25
26			414	11273		7023	182692	26
27			414	10859		7023	175669	27
28			414	10445		7023	168646	28
29			414	10030		7023	161622	29
30	21.852	3	414	9616	66	7023	154599	30
31	21.720	3	411	9203	76	6957	147609	31
32	21.583	4	408	8794	78	6881	140690	32
33	21.441	4	404	8388	86	6803	133848	33
34	21.294	5	400	7986	99	6717	127088	34
35	21.142	5	395	7588	110	6617	120421	35
36	20.985	5	390	7195	105	6507	113859	36
37	20.822	5	385	6807	105	6402	107404	37
38	20.654	6	380	6425	114	6297	101055	38
39	20.481	6	375	6047	117	6183	94815	39
40	20.302	6	369	5676	129	6065	88691	40
41	20.118	7	363	5310	134	5937	82691	41
42	19.929	7	356	4951	143	5802	76821	42
43	19.734	7	349	4598	147	5659	71091	43
44	19.534	8	341	4253	150	5512	65505	44
45	19.330	8	334	3916	153	5362	60068	45
46	19.120	8	326	3586	157	5210	54782	46
47	18.906	9	317	3265	161	5052	49651	47
48	18.669	9	309	2951	173	4891	44679	48
49	18.407	10	300	2647	190	4718	39875	49
50	18.135	11	289	2353	205	4528	35251	50
51	17.853	12	278	2069	223	4323	30826	51
52	17.561	14	266	1797	242	4100	26615	52
53	17.259	15	252	1538	265	3858	22635	53
54	16.948	17	236	1294	290	3594	18909	54
55	16.625	19	219	1066	317	3304	15461	55
56	16.292	21	200	856	343	2987	12315	56
57	15.949	23	179	667	368	2644	9500	57
58	15.594	25	156	499	390	2276	7040	58
59	15.229	27	131	355	410	1886	4958	59
60	14.855	29	104	238	429	1476	3277	60
61	14.472	20	75	148	284	1047	2016	61
62	14.081	19	56	82	271	763	1111	62
63	13.682	19	36	36	254	492	484	63
64	13.277	18	18	9	238	238	119	64

PEN

Ill health retirement functions

4%

Age x	${}^s\overline{M}_x^{ia} =$ $s_x(M_x^{ia} - \frac{1}{2}C_x^{ia})$	${}^s\overline{R}_x^{ia} =$ $\sum {}^s\overline{M}_x^{ia}$	${}^zC_x^{ia} =$ $z_{x+\frac{1}{2}}C_x^{ia}$	${}^zM_x^{ia} =$ $\sum {}^zC_x^{ia}$	${}^z\overline{R}_x^{ia} =$ $\sum({}^zM_x^{ia} - \frac{1}{2}{}^zC_x^{ia})$	Age x
16	9641	1533946		64061	2399660	16
17	11572	1524304		64061	2335599	17
18	13533	1512732		64061	2271539	18
19	15480	1499199		64061	2207478	19
20	17454	1483720		64061	2143417	20
21	19409	1466266		64061	2079357	21
22	21388	1446858		64061	2015296	22
23	23343	1425470		64061	1951235	23
24	25320	1402126		64061	1887175	24
25	27266	1376807		64061	1823114	25
26	29179	1349541		64061	1759054	26
27	31106	1320361		64061	1694993	27
28	33099	1289255		64061	1630932	28
29	35051	1256156		64061	1566872	29
30	36894	1221105	321	64061	1502811	30
31	38407	1184211	389	63740	1438911	31
32	39906	1145804	425	63351	1375365	32
33	41334	1105898	492	62927	1312226	33
34	42596	1064565	592	62434	1249546	34
35	43671	1021969	689	61843	1187407	35
36	44664	978298	687	61153	1125909	36
37	45554	933634	714	60467	1065099	37
38	46234	888080	803	59753	1004989	38
39	46683	841846	856	58949	945638	39
40	46889	795163	965	58094	887117	40
41	46836	748274	1036	57128	829506	41
42	46587	701438	1128	56093	772895	42
43	46092	654850	1182	54964	717367	43
44	45540	608759	1228	53782	662994	44
45	44936	563219	1268	52554	609826	45
46	44271	518283	1328	51286	557906	46
47	43590	474013	1383	49957	507285	47
48	42754	430422	1503	48575	458019	48
49	41752	387668	1680	47072	410195	49
50	40560	345917	1840	45392	363963	50
51	39222	305356	2026	43553	319490	51
52	37608	266134	2236	41527	276951	52
53	35735	228526	2482	39291	236542	53
54	33607	192792	2760	36809	198492	54
55	31104	159184	3063	34048	163063	55
56	28252	128080	3366	30986	130546	56
57	25081	99828	3666	27620	101243	57
58	21530	74747	3944	23954	75455	58
59	17668	53218	4215	20011	53473	59
60	13449	35550	4476	15795	35570	60
61	9787	22100	3007	11319	22013	61
62	6895	12313	2907	8313	12197	62
63	4070	5418	2770	5405	5338	63
64	1348	1348	2635	2635	1318	64

PEN

Age retirement functions
4%

Age x	$\overline{a}^r_{x+\frac{1}{2}}$ ($\overline{a}^r_{65}$ at 65)	$C^r_x = v^{x+\frac{1}{2}} r_x$ ($V^{65} r_6 5$ at 65)	$M^r_x = \sum C^r_x$	$\overline{R}^r_x = \sum(M^r_x - \frac{1}{2}C^r_x)$	$C^{ra}_x = C^{r-r}_{x^{a+\frac{1}{2}}}$ ($v^{65} r^{65-r} a_{65}$ at 65)	$M^{ra}_x = \sum C^{ra}_x$	$\overline{R}^{ra}_x = \sum(M^{ra}_x - \frac{1}{2}C^{ra}_x)$	Age x
16			782	36449		11915	553630	16
17			782	35667		11915	541715	17
18			782	34885		11915	529800	18
19			782	34103		11915	517885	19
20			782	33321		11915	505970	20
21			782	32539		11915	494055	21
22			782	31757		11915	482140	22
23			782	30975		11915	470225	23
24			782	30193		11915	458310	24
25			782	29411		11915	446395	25
26			782	28629		11915	434479	26
27			782	27847		11915	422564	27
28			782	27065		11915	410649	28
29			782	26284		11915	398734	29
30			782	25502		11915	386819	30
31			782	24720		11915	374904	31
32			782	23938		11915	362989	32
33			782	23156		11915	351074	33
34			782	22374		11915	339159	34
35			782	21592		11915	327244	35
36			782	20810		11915	315328	36
37			782	20028		11915	303413	37
38			782	19246		11915	291498	38
39			782	18464		11915	279583	39
40			782	17682		11915	267668	40
41			782	16900		11915	255753	41
42			782	16118		11915	243838	42
43			782	15336		11915	231923	43
44			782	14554		11915	220008	44
45			782	13773		11915	208093	45
46			782	12991		11915	196177	46
47			782	12209		11915	184262	47
48			782	11427		11915	172347	48
49			782	10645		11915	160432	49
50			782	9863		11915	148517	50
51			782	9081		11915	136602	51
52			782	8299		11915	124687	52
53			782	7517		11915	112772	53
54			782	6735		11915	100857	54
55			782	5953		11915	88942	55
56			782	5171		11915	77027	56
57			782	4389		11915	65111	57
58			782	3607		11915	53196	58
59			782	2825		11915	41281	59
60	16.292	343	782	2043	5590	11915	29366	60
61	15.949	46	439	1433	738	6325	20246	61
62	15.594	39	393	1017	609	5587	14290	62
63	15.229	33	354	644	498	4979	9007	63
64	14.855	27	321	307	405	4480	4278	64
65	13.883	294	294		4075	4075		65

PEN

Age retirement functions

4%

Age x	${}^s\overline{M}_x^{ra} = s_x(M_x^{ra} - \frac{1}{2}C_x^{ra})$	${}^s\overline{R}_x^{ra} = \sum {}^s\overline{M}_x^{ra}$	${}^zC_x^{ra} = z_{x+\frac{1}{2}}C_x^{ra}$ $(Z_{65}C_{65}^{ra} at 65)$	${}^zM_x^{ra} = \sum {}^zC_x^{ra}$	${}^z\overline{R}_x^{ra} = \sum({}^zM_x^{ra} - 1/2{}^zC_x^{ra})$	Age x
16	16357	3801411		128026	5956885	16
17	19632	3785055		128026	5828859	17
18	22959	3765422		128026	5700833	18
19	26262	3742463		128026	5572807	19
20	29610	3716201		128026	5444781	20
21	32927	3686591		128026	5316755	21
22	36285	3653664		128026	5188729	22
23	39602	3617379		128026	5060703	23
24	42955	3577776		128026	4932677	24
25	46258	3534821		128026	4804651	25
26	49504	3488563		128026	4676625	26
27	52773	3439059		128026	4548599	27
28	56153	3386286		128026	4420573	28
29	59465	3330133		128026	4292547	29
30	62887	3270668		128026	4164521	30
31	66137	3207781		128026	4036495	31
32	69493	3141643		128026	3908469	32
33	72857	3072151		128026	3780443	33
34	76127	2999294		128026	3652417	34
35	79293	2923167		128026	3524390	35
36	82450	2843874		128026	3396364	36
37	85488	2761424		128026	3268338	37
38	88288	2675936		128026	3140312	38
39	90832	2587648		128026	3012286	39
40	93102	2496816		128026	2884260	40
41	95080	2403714		128026	2756234	41
42	96863	2308634		128026	2628208	42
43	98319	2211771		128026	2500182	43
44	99795	2113452		128026	2372156	44
45	101290	2013657		128026	2244130	45
46	102805	1912367		128026	2116104	46
47	104470	1809562		128026	1988078	47
48	106028	1705092		128026	1860052	48
49	107607	1599064		128026	1732026	49
50	109206	1491457		128026	1604000	50
51	110967	1382252		128026	1475974	51
52	112611	1271285		128026	1347948	52
53	114276	1158674		128026	1219921	53
54	116113	1044398		128026	1091895	54
55	117825	928285		128026	963869	55
56	119559	810460		128026	835843	56
57	121473	690902		128026	707817	57
58	123255	569428		128026	579791	58
59	125224	446173		128026	451765	59
60	97250	320949	58293	128026	323739	60
61	64439	223699	7807	69733	224859	61
62	58066	159260	6541	61926	159030	62
63	52736	101194	5436	55385	100374	63
64	48458	48458	4482	49949	47708	64
65			45467	45467		65

PEN

Functions for return of contributions, accumulated with interest at 2% p.a., on death

4%

Age x	${}^{j}c_x^d = V^{x+\frac{1}{2}}(1+j)^{x+\frac{1}{2}}d_x$	${}^{j}M_x^d = \sum {}^{j}C_x^d$	${}^{j}\overline{R}_x^d = \sum(\frac{{}^{j}M_x^d - 1/2\,{}^{j}C_x^d}{(1+j)^{x+1/2}})$	${}^{sj}\overline{R}_x^d = \sum S_x(\frac{{}^{j}M_x^d - 1/2\,{}^{j}C_x^d}{(1+j)^{x+1/2}})$	Age$x$
16	36	601	7617	39369	16
17	32	565	7196	38791	17
18	29	533	6808	38152	18
19	25	504	6449	37459	19
20	22	480	6114	36722	20
21	16	457	5802	35946	21
22	14	442	5508	35134	22
23	12	428	5230	34286	23
24	11	416	4965	33405	24
25	10	406	4712	32494	25
26	7	396	4470	31555	26
27	6	389	4238	30590	27
28	5	383	4014	29598	28
29	5	378	3797	28577	29
30	4	374	3588	27531	30
31	5	369	3385	26460	31
32	5	364	3188	25369	32
33	4	359	2998	24262	33
34	4	355	2815	23138	34
35	5	351	2636	22000	35
36	5	346	2464	20852	36
37	6	341	2297	19698	37
38	6	335	2136	18544	38
39	6	329	1981	17396	39
40	6	323	1832	16258	40
41	6	317	1689	15136	41
42	6	311	1551	14035	42
43	7	305	1418	12956	43
44	7	298	1290	11904	44
45	7	291	1168	10882	45
46	8	283	1052	9891	46
47	9	276	940	8930	47
48	10	267	834	8001	48
49	11	257	734	7109	49
50	12	246	640	6256	50
51	13	234	551	5446	51
52	14	221	469	4681	52
53	15	207	394	3965	53
54	16	192	324	3302	54
55	17	176	262	2693	55
56	18	158	206	2142	56
57	19	140	157	1653	57
58	20	121	116	1227	58
59	21	101	81	867	59
60	23	80	53	575	60
61	15	57	32	355	61
62	15	42	18	197	62
63	14	27	8	86	63
64	13	13	2	21	64

PEN

Functions for return of contributions, accumulated with interest at 2% p.a., on withdrawal

4%

Age x	${}^{j}c_x^w = V^{x+\frac{1}{2}}(1+j)^{x+\frac{1}{2}}w_x$	${}^{j}M_x^w = \sum^{j} C_x^w$	${}^{j}\overline{R}_x^w = \sum(\frac{{}^{j}M_x^w - 1/2^{j}C_x^w}{(1+j)^{x+1/2}})$	${}^{s}{}_{j}\overline{R}_x^w = \sum S_x(\frac{{}^{j}M_x^w - 1/2^{j}C_x^w}{(1+j)^{x+1/2}})$	Age$x$
16	7259	55286	230458	622984	16
17	6404	48027	193200	571836	17
18	5649	41624	161503	519609	18
19	4984	35974	134605	467779	19
20	4396	30991	111848	417622	20
21	3878	26594	92662	369943	21
22	3421	22716	76556	325433	22
23	3018	19294	63103	284465	23
24	2529	16277	51935	247347	24
25	2125	13747	42694	214031	25
26	1790	11622	35038	184310	26
27	1511	9832	28691	157939	27
28	1277	8322	23425	134617	28
29	1080	7045	19056	114025	29
30	929	5964	15429	95926	30
31	798	5036	12423	80058	31
32	686	4237	9938	66267	32
33	590	3551	7892	54334	33
34	506	2961	6215	44080	34
35	433	2454	4848	35344	35
36	370	2021	3740	27971	36
37	314	1652	2849	21802	37
38	264	1338	2137	16699	38
39	220	1074	1575	12531	39
40	188	853	1134	9171	40
41	159	665	794	6510	41
42	133	506	536	4454	42
43	109	374	346	2913	43
44	87	264	212	1800	44
45	67	177	120	1034	45
46	49	110	62	538	46
47	31	62	27	242	47
48	20	30	10	85	48
49	10	10	2	17	49