ACTL3142 Week 5 - Generalised Linear Models

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Exponential Family

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A random variable Y comes from the *exponential family* if its density has the form,

$$f_Y(y) = \exp\left[\frac{y\theta - b(\theta)}{\psi} + c(y;\psi)\right].$$

ψ is a scale factor while b and c specify the *type* of distribution. θ actually specifies scale/location of the resulting distribution.

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We also have

$$\mathbb{E}[Y] = b'(\theta) = \mu;$$
$$\mathrm{Var}(Y) = \psi b''(\theta) = V(\theta).$$

Generalised Linear Models

Generalised Linear Models

Assume $Y|X \sim G$, where G is in the *exponential family*. Then, assume $g(\mu_X) = \eta = \mathbf{x}_i\beta$. So,

$$g(\mu_X) = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p.$$

G is the stochastic component, g is the link function, and η is the systematic component.

If we take $g(\mu) := \theta(\mu)$, then g is said to be the 'canonical link.'

(Scaled) Deviance

Scaled Deviance Test for Nested Models

Let D_1 and D_2 be the scaled deviance of $\mathcal{M}_1$ with p predictors and $\mathcal{M}_2$ with $p+q$ predictors. Then under the hypothesis,

$$\mathcal{H}_0 : \beta_{p+1} = \beta_{p+2} \cdots = \beta_{p+q} = 0,$$

we have

$$D_1 - D_2 \sim \chi^2_q.$$

Under $\alpha = 5\%$, we have the critical value of χ^2_q is $2q$.

Pearson / Deviance Residuals

We can extend residuals from MLR to GLM's with Pearson / Deviance residuals. Specifically,

$$r_i^P = \frac{y_i - \hat{\mu}_i}{\sqrt{V(\hat{\mu}_i)}}.$$

Note the z-score resemblance, and thus we expect the residuals to be approximately normal if our GLM is a good fit - very similar to what we're used to from MLR.