ACTL2131 1.2 - Univariate Distributions

Tadhg Xu-Glassop

2025T1

Today

Today's lesson is quite easy! We will present the majority of the distributions in the formula book, including their use cases.

Although it is provided in the formula book, I **strongly** encourage you to derive μ, σ^2, and the MGF for most of the distributions and go from PDF to CDF as practice.

Distributions

We have seen how we use distributions, including f, F and moments to characterise random variables.

The most common distributions are in your formula book. A skill in this course is to identify the distribution and use what you have to solve the problem.

If X is distributed as G, we say $X \sim G(\tilde{\theta})$, where $\tilde{\theta}$ is a vector of parameters. For example, $X \sim \mathcal{N}(\mu, \sigma^2)$ is a normal distribution, with parameters μ and σ^2.

Discrete

Bernoulli

- Success or failure

Binomial

- Sum of Bernoulli / repeated Bernoulli trials

Geometric

- Number of trials until first success
- Memoryless property

Negative Binomial

- Number of trials until r successes.

Poisson

- Counting number of events in a period of time.

Continuous

Exponential

- Positive valued, memoryless property.

Gamma

- Positive valued, sum of exponential.

Normal

- Supports $\mathbb{R}$, super common.
- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $(X - \mu)/\sigma \sim \mathcal{N}(0, 1)$, denoted Z.
- You can find probabilities for Z in the formula book.

Lognormal

- Fat tailed, positive valued.
- Defining property: if $Y \sim \mathrm{LN}(\mu, \sigma^2)$, then $\log Y \sim \mathcal{N}(\mu, \sigma^2)$

Continuous (cont.)

Beta

- Supports $[0, 1]$ - used to model proportions or probabilities.

Uniform

- Everything in an interval $[a, b]$ has the same probability!

Tutorial Questions

1.2.4, 1.2.5, 1.2.8