Project 2 (Group member : Jianqing Li (alone))

Q1)

$r_A = [I(y;x_1),\ I(y;x_2|x_1)]^T$

$r_B = [I(y;x_1|x_2),\ I(y;x_2)]^T$

Q2)

-10 dB :	$r_A = [0.1225\ \ 0.2808]^T$	$r_B = [0.1398\ \ 0.2635]^T$
0dB :	$r_A = [0.6359\ \ 1.6549]^T$	$r_B = [1.1060\ \ 1.1848]^T$
10 dB :	$r_A = [2.2464\ \ 4.4912]^T$	$r_B = [4.7024\ \ 2.0352]^T$

Q3) see project file

Q4)

$Q_1 = \begin{bmatrix} 0.2336 & 0.4108 - 0.1015i \\ 0.4108 + 0.1015i & 0.7664 \end{bmatrix}$, $C_1 = 1.5647$ (1)

$Q_2 = \begin{bmatrix} 0.4804 & 0.4374 - 0.2414i \\ 0.4374 + 0.2414i & 0.5196 \end{bmatrix}$, $C_2 = 1.6549$ (2)

Q5)

$X_j = H_j^H (C_n + H_k Q_k H_k^H)^{-1} H_j$

Q6)

① user 1 employs single-user capacity, user 2 use unconditioned covariace

① $Q_2 = \begin{bmatrix} 0.8428 & 0.3111 - 0.1889i \\ 0.3111 + 0.1889i & 0.1572 \end{bmatrix}$, $C_2 = 1.0338$

② user 2 employs single user capacity, user 1 use unconditioned covariace

$Q_1 = \begin{bmatrix} 0.0017 & 0.273 - 0.0314i \\ 0.0273 + 0.0314i & 0.9983 \end{bmatrix}$, $C_1 = 0.9644$ (2)

Q7)

$$I(X_1, X_2; Y) = \underbrace{I(X_1; Y)}_{\text{update of } Q_1} + I(X_2; Y \mid X_1)$$

$$= I(X_1; Y \mid X_2) + \underbrace{I(X_2; Y)}_{\text{update of } Q_2}$$

Q8)

10 iterations

Q9)

They do not adjoin at the capacity region boundary, since the iterative waterfilling can achieve the optimal capacity (the joint mutual information), the others are just suboptimal and can only reach some lower rate in comparison. The optimal Q_1, Q_2 can not be acquired by fixing one as single user transmission, must be found by iterative procedure.

Q10) $w_2 > w_1 \Rightarrow$ decode X_1 first

$$I(X_1, X_2; Y) = \underbrace{I(X_1; Y)}_{R_1} + \underbrace{I(X_2; Y \mid X_1)}_{R_2}$$

$$C_{\text{wsum}} = \max_{Q_1, Q_2 \geq 0} w_1 I(X_1; Y) + w_2 I(X_2; Y \mid X_1) \quad \text{s.t. } \operatorname{tr}(Q_i) \leq P_i, \; i = 1, 2$$

for $w_2 > w_1$

Q11)

- $P_1 = P_2 = -10\,dB$ is closest to a rectangular shape
- $P_1 = P_2 = +10\,dB$ is closest to the triangle.