Q1

$$r_A = \left[I(y;x_1),\ I(y;x_2|x_1) \right]^T$$

$$r_B = \left[I(y;x_1|x_2),\ I(y;x_2) \right]^T$$

Q2

$$r_A = \Big[\log_2 \det\left(\underset{\sim}{I}_m + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_1 \underset{\sim}{Q}_1 \underset{\sim}{H}_1^H + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_2 \underset{\sim}{Q}_2 \underset{\sim}{H}_2^H\right) - \log_2 \det\left(\underset{\sim}{I}_m + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_2 \underset{\sim}{Q}_2 \underset{\sim}{H}_2^H\right),\ \log_2 \det\left(\underset{\sim}{I}_m + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_2 \underset{\sim}{Q}_2 \underset{\sim}{H}_2^H\right) \Big]^T$$

$$r_B = \Big[\log_2 \det\left(\underset{\sim}{I}_m + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_1 \underset{\sim}{Q}_1 \underset{\sim}{H}_1^H + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_2 \underset{\sim}{Q}_2 \underset{\sim}{H}_2^H\right) - \log_2 \det\left(\underset{\sim}{I}_m + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_1 \underset{\sim}{Q}_1 \underset{\sim}{H}_1^H\right),\ \log_2 \det\left(\underset{\sim}{I}_m + \underset{\sim}{C}_n^{-1} \underset{\sim}{H}_1 \underset{\sim}{Q}_1 \underset{\sim}{H}_1^H\right) \Big]^T$$

$P_{TX} = -10\text{dB} \Rightarrow 0.1$

$$r_A = \begin{pmatrix} 0{,}2110 \\ 0{,}2321 \end{pmatrix} \qquad r_B = \begin{pmatrix} 0{,}2232 \\ 0{,}2199 \end{pmatrix}$$

$P_{TX} = 0\text{dB} \Rightarrow 1$

$$r_A = \begin{pmatrix} 0{,}9266 \\ 1{,}5401 \end{pmatrix} \qquad r_B = \begin{pmatrix} 1{,}4982 \\ 0{,}9685 \end{pmatrix}$$

$P_{TX} = 10\text{dB} \Rightarrow 10$

$$r_A = \begin{pmatrix} 1{,}9407 \\ 5{,}1836 \end{pmatrix} \qquad r_B = \begin{pmatrix} 5{,}1049 \\ 2{,}0194 \end{pmatrix}$$

Q3

Figure attached.

Q4

$$Q_{1,single} = \begin{pmatrix} 0{,}2336 + 0i & 0{,}4108 - 0{,}1015i \\ 0{,}4108 + 0{,}1015i & 0{,}7664 + 0i \end{pmatrix}$$

$$C_1 = 1{,}5647$$

$$Q_{2,single} = \begin{pmatrix} 0{,}4804 + 0i & 0{,}4374 - 0{,}2414i \\ 0{,}4374 + 0{,}2414i & 0{,}5196 + 0i \end{pmatrix}$$

$$C_2 = 1{,}6549$$

Q5.

$$\underset{\sim}{X}_j = \underset{\sim}{H}_j^H \left(\underset{\sim}{H}_k \underset{\sim}{Q}_k \underset{\sim}{H}_k^H + \underset{\sim}{C}_n \right)^{-1} \underset{\sim}{H}_j$$

Q6.

$k=1$, $Q_{2,\,single} = \begin{pmatrix} 0{,}7543 & 0{,}3712 - 0{,}2179i \\ 0{,}3712 + 0{,}2179i & 0{,}2457 \end{pmatrix}$, $R_2 = 1{,}0472$

$k=2$, $Q_{1,\,single} = \begin{pmatrix} 0{,}0206 & 0{,}1320 - 0{,}0528i \\ 0{,}1320 + 0{,}0528i & 0{,}9794 \end{pmatrix}$ $R_1 = 0{,}9657$

Q7. $I(x_1, x_2; y) = I(x_1; y) + I(y; x_2 | x_1)$

(update of Q_1)

$= I(x_2; y) + I(y; x_1 | x_2)$

(update of Q_2)

Q8. $C_{sum} = 2,6823$

$$Q_1 = \begin{pmatrix} 0,0128 & 0,1022 - 0,0469\,i \\ 0,1022 + 0,0469\,i & 0,9872 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 0,7845 & 0,3537 - 0,2096\,i \\ 0,3537 + 0,2096\,i & 0,2155 \end{pmatrix}$$

It took 6 iterations.

Q9 No, they do not adjoin.

Q10 $w_2 > w_1 \Rightarrow$ decode x_1 first

$$C_{wsum} = \max_{Q_1, Q_2} w_1 I(x_1; y) + w_2 I(y; x_2 | x_1)$$

$$\text{s.t. } Q_i \succeq 0, \quad tr(Q_i) \le P_i, \ i = 1,2$$

Q11.

$$R_{wsum}(\mathbf{Q}_1, \mathbf{Q}_2) = (w_i - w_j) \log_2 \det(\mathbf{I}_m + \mathbf{C}_n^{-1} \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H)$$

$$+ w_j \log_2 \det(\mathbf{I}_m + \mathbf{C}_n^{-1} \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H + \mathbf{C}_n^{-1} \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H)$$

$$= (W_i - w_j) \frac{1}{\ln 2} \ln \det(\mathbf{I}_m + \mathbf{H}_i \mathbf{Q}_i \mathbf{H}_i^H) +$$

$$w_j \frac{1}{\ln 2} \ln \det(\mathbf{I}_m + \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H + \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H)$$

$$V_i^{(n)} = \frac{\partial}{\partial Q_i^T} R_{wsum}(Q_1, Q_2) \Big|_{Q_1 = Q_1^{(n)},\ Q_2 = Q_2^{(n)}}$$

For $w_1 > w_2$

$$V_1^{(n)} = (w_1 - w_2) \frac{1}{\ln 2} \mathbf{H}_1^H (\mathbf{I}_m + \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H)^{-1} \mathbf{H}_1$$

$$+ W_2 \frac{1}{\ln 2} \mathbf{H}_1^H (\mathbf{I}_m + \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H + \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H)^{-1} \mathbf{H}_1$$

$$V_2^{(n)} = W_2 \frac{1}{\ln 2} \mathbf{H}_2^H (\mathbf{I}_m + \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^H + \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^H)^{-1} \mathbf{H}_2$$

For $w_2 > w_1$:

$$V_2^{(n)} = (w_2 - w_1)\frac{1}{\ln 2}\underset{\sim}{H}_2^H\left(\underset{\sim}{I}_m + \underset{\sim}{H}_2\underset{\sim}{Q}_2\underset{\sim}{H}_2^H\right)^{-1}\underset{\sim}{H}_2$$

$$+ w_1\frac{1}{\ln 2}\underset{\sim}{H}_2^H\left(\underset{\sim}{I}_m + \underset{\sim}{H}_1\underset{\sim}{Q}_1\underset{\sim}{H}_1^H + \underset{\sim}{H}_2\underset{\sim}{Q}_2\underset{\sim}{H}_2^H\right)^{-1}\underset{\sim}{H}_2$$

$$V_1^{(n)} = w_1\frac{1}{\ln 2}\underset{\sim}{H}_1^H\left(\underset{\sim}{I}_m + \underset{\sim}{H}_2\underset{\sim}{Q}_2\underset{\sim}{H}_2^H + \underset{\sim}{H}_1\underset{\sim}{Q}_1\underset{\sim}{H}_1^H\right)^{-1}\underset{\sim}{H}_1$$