EL2520 – Control Theory and Practice
Classical Loop-Shaping

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Abstract

In this report, we consider the classical loop-shaping procedure for control design. We finished two required tasks in the following. The first one is designing a lead-lag controller in order to reach idea phase margin and remove stationary error. The second one is disturbance attenuation. In this task, we reduce overshoot and rising time in the closed-loop control system, as well as decreasing the influence of disturbances.

Basics

A system is modeled by the transfer function (given in [1])

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)} \tag{1}$$

We will design a lead-lag compensator F such that the closed loop system in Figure 1 fulfills the following specification:

- Crossover frequency $\omega_c = 0.4$ rad/s.
- Phase margin $\varphi_m = 30^\circ$.
- No stationary error for a step response.

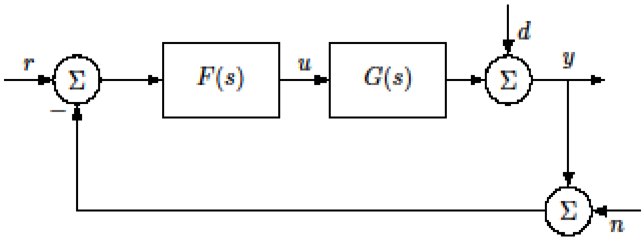

Figure 1: Closed loop block diagram, where F–controller, G–system, r–reference signal, u–control signal.

Exercise 4.1.1

We follow the procedure from [2] to determine the parameters K, β, τ_I, τ_D, and γ in the lead-lag compensator:

$$F(s) = K\frac{(\tau_D s+1)(\tau_I s+1)}{(\beta\tau_D s+1)(\tau_I s+\gamma)} \tag{2}$$

First we compute the error $E(s)$:

$$E(s) = R(s) - Y(s) = R(s) - \frac{F(s)G(s)}{1+F(s)G(s)}R(s) = \frac{1}{1+F(s)G(s)}R(s) \tag{3}$$

since the closed-loop system is required to achieve zero steady-state error for a step response:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s * \frac{1}{1+F(s)G(s)}R(s) = \lim_{s\to 0}\frac{1}{1+F(s)G(s)} = \lim_{s\to 0}\frac{1}{1+3*K/\gamma} \tag{4}$$

If condition $e(\infty) = 0$ is desired, then $K \to \infty\ or\ \gamma = 0$ should be satisfied. Since the gain goes to infinity is unrealistic, we set $\gamma = 0$.

Here we experimentally set $\tau_I = 1$, then the lag controller becomes $F_{lag} = \frac{s+1}{s}$. We can obtain the current phase margin of the lag controller $F_{lag}(s)$ and $G(s)$ using Matlab: $\varphi_c = 130.6°$. To guarantee a phase margin $\varphi_m = 30^\circ$, the phase margin of the lead controller can be calculated in the following way:

$$\varphi_{lead} = 180° - (\varphi_c - \varphi_m) = 180° - 100.6° = 79.4° \tag{5}$$

Then K, β, τ_D can be obtained using following equations (given in [2]):

$$\beta = \frac{1 - sin\varphi_{lead}}{1 + sin\varphi_{lead}} = 0.0086 \tag{6}$$

$$\tau_D = \frac{1}{\omega_c + \sqrt{\beta}} = 26.95 \tag{7}$$

$$K = \frac{1}{|GF_{lead}F_{lag}(j\omega_c)|} = 0.0983 \tag{8}$$

Table 1: Parameters for the lead-lag compensator.

K	β	τ_I	τ_D	γ
0.0983	0.0086	1	26.95	0

The final lead-lag controller is given by eq. (2) with the parameters in Table 1. The bode diagram of original system and compensated system is shown in Figure 2, and the phase margin is kept to be 30 degree at the cross-over frequency 0.4 rad/s.

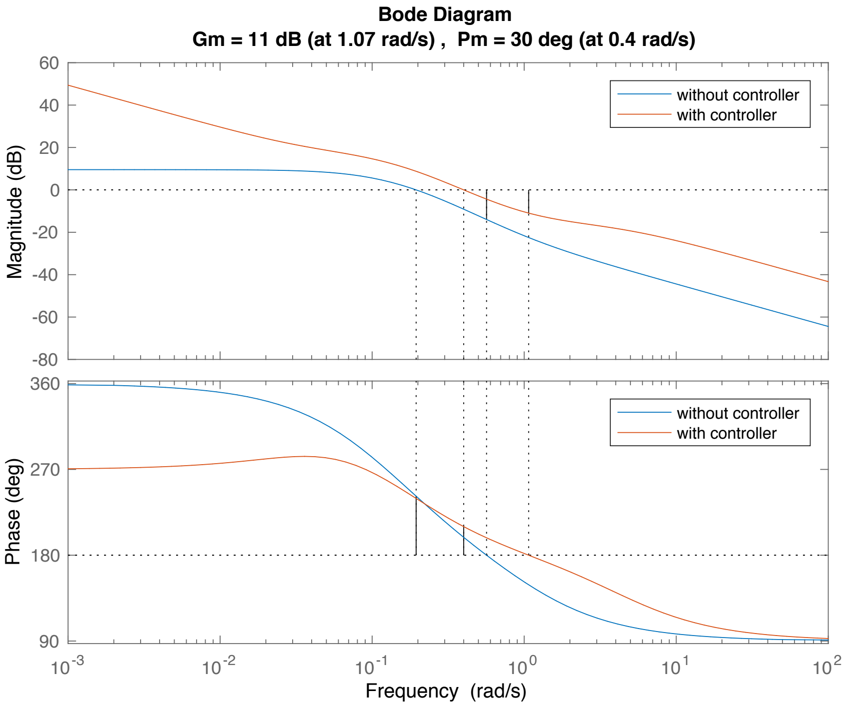

Figure 2: Bode diagram, with the lead-lag controller (in red) and without it (in blue).

Exercise 4.1.2

After constructing the lead-lag controller, several characteristics of the closed-loop system can be obtained with the help of Matlab. The results are listed in Table 2. The rise time and overshoot is determined from the step response in Figure 3.

Table 2: Closed loop system(phase margin=30 degree) characteristics.

ω_B [rad/s]	M_T [dB]	T_r [s]	M [%]
0.7474	5.8247	2.4583	37.7922

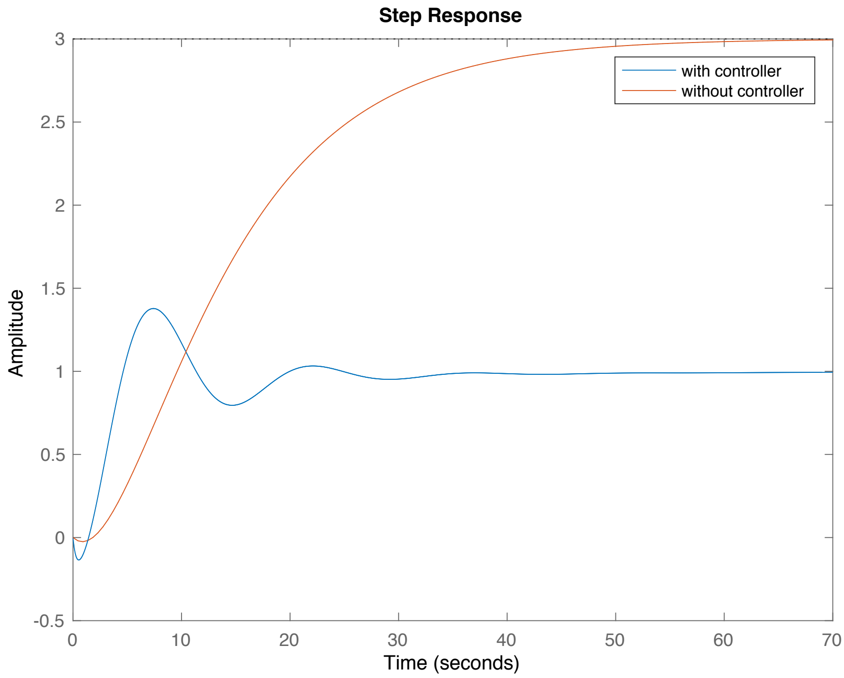

Figure 3: Step response for the closed loop system in Figure 1, with the lead-lag controller (in blue) and without it (in red).

Exercise 4.1.3

Next, we increase the phase margin from 30° to 50° while keeping the cross-over frequency the same. If we keep the lag controller as same as pervious, the phase margin can only reach 31.2 degree, far away from the requirement. So we experimentally change $\tau_I = 3$. Following the same process, we can get the bode diagram in Figure 4 and step response in Figure 5 respectively. The closed loop system characteristics is listed in Table 3.

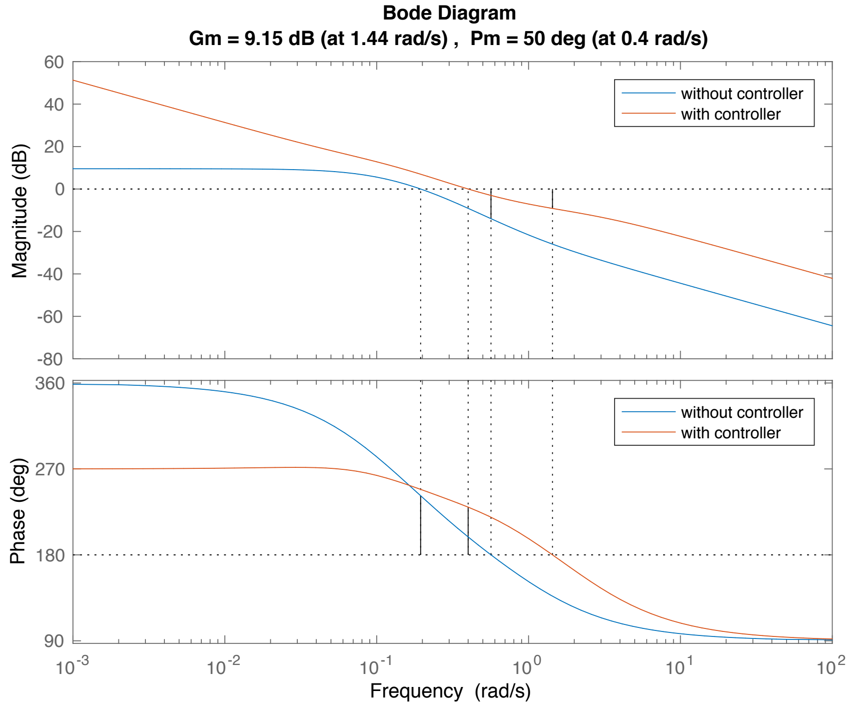

Figure 4: Bode diagram, with the lead-lag controller (in red) and without (in blue).

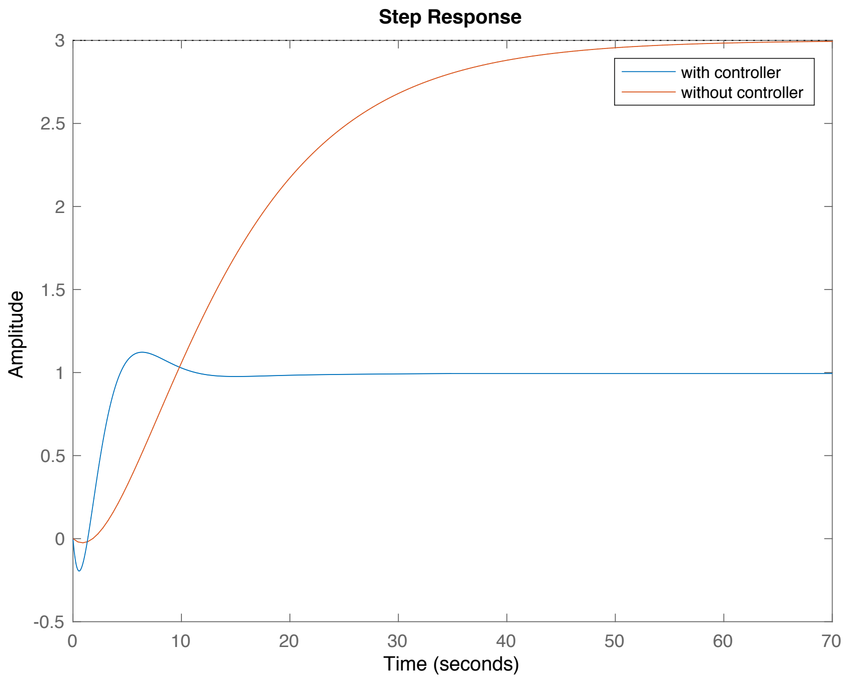

Figure 5: Step response for the closed loop system in Figure 1, with the lead-lag controller (in blue) and without it (in red).

Table 3: Closed loop system (phase margin = 50 degree) characteristics.

ω_B [rad/s]	M_T [dB]	T_r [s]	M [%]
1.0633	1.4590	2.2276	12.253

Disturbance attenuation

The transfer function has been estimated to be

$$G(s) = \frac{20}{(s+1)\left(\left(\frac{s}{20}\right)^2 + \frac{s}{20} + 1\right)} \tag{9}$$

$$G_d(s) = \frac{10}{s+1} \tag{10}$$

In this section, we will design controllers F_y, F_r such that the closed-loop system in Figure 1 fulfills the following specification:

- The rise time for a step change in the reference signal less than 0.2 s and the overshoot is less than 10%.
- For a step in the disturbance, we have $|y(t)| \leq 1 \ \forall t$ and $|y(t)| \leq 0.1$ for $t > 0.5$ s.
- Since the signals are scaled the control signal obeys $|u(t)| \leq 1 \ \forall t$.

The block diagram is shown in Figure 6.

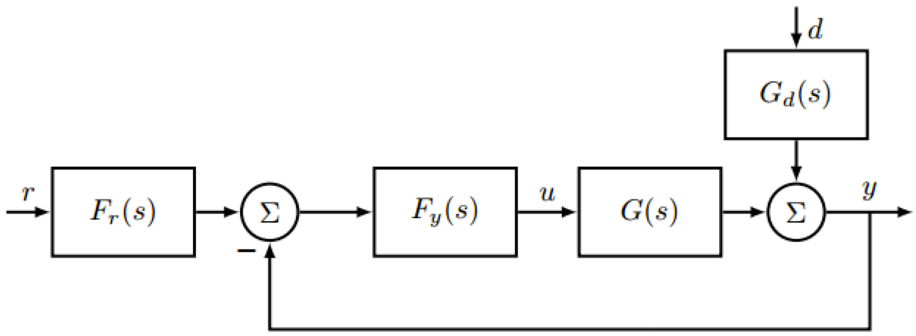

Figure 6: Fr–prefilter, Fy–feedback controller, G–system, Gd–disturbance dynamics,r–reference signal, u–control signal, d–disturbance signal, y–measurement signal.

Exercise 4.2.1

First, we need to find the cross-over frequency ω_c. Since $G_d(j\omega_c) = 1$, as a result we can get $\omega_c = 9.9473\ rad/s$. Therefore, the control action is needed at least in interval $[0, \omega_c]$ in order to attenuate disturbances, otherwise the disturbances will seriously affect the system's performance.

Improper design

We will try improper design in the first place. Under this condition, we have $L = F_y G = \frac{\omega_c}{s}$. So the feedback controller is formed below:

$$F_{y1} = \frac{\omega_c}{s} * \frac{(s+1)\left(\left(\frac{s}{20}\right)^2 + \frac{s}{20} + 1\right)}{20} \tag{11}$$

Proper design

Since the controller in (11) cannot be realized in practice (numerator has a higher order polynomial than denominator, which indicates future information needed), some poles are added to make the controller proper. Here, we choose to add the minimum number, which is two, of poles p_1 *and* p_2. The controller now is shown in eq.(12):

$$F_{y2} = F_{y2} * \frac{p_1 p_2}{(s+p_1)(s+p_2)} = \frac{\omega_c}{s} * \frac{p_1 p_2 (s+1)\left(\left(\frac{s}{20}\right)^2 + \frac{s}{20} + 1\right)}{20(s+p_1)(s+p_2)} \tag{12}$$

The added poles should pose little effect to the system between the frequency interval $[0, \omega_c]$. As a result, p_1 *and* p_2 need to be much larger than ω_c. We test two cases: $p_1 = p_2 = 10\omega_c$ and $p_1 = p_2 = 100\omega_c$. The bode diagram of closed loop system from disturbances to output and corresponding step response are shown in Figure 7 and Figure 8 respectively. From the plot, we find the poles which are placed far away from ω_c could bring a better result, which satisfies our assumption above.

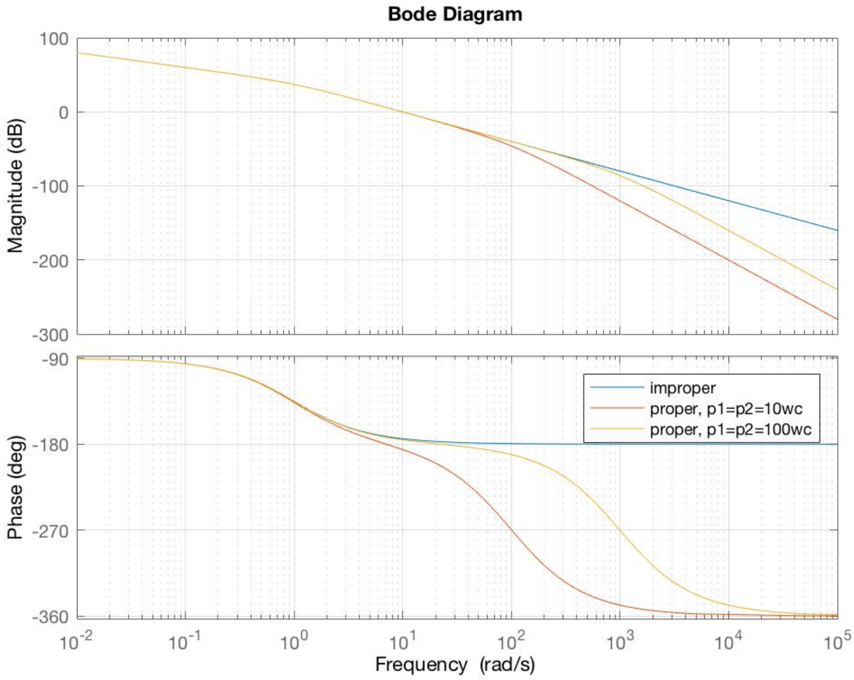

Figure 7: Bode diagram of $F_y G$ in Figure 6, with improper controller (blue) and proper controller1 (red), $p_1 = p_2 = 10\omega_c$; proper controller2 (orange), $p_1 = p_2 = 100\omega_c$.

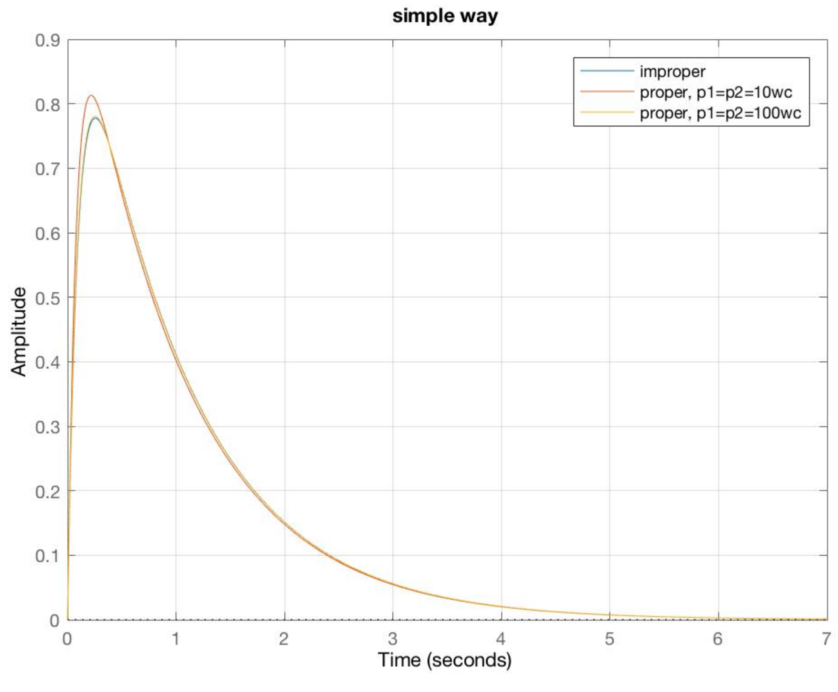

Figure 8: Step response from d to y for the closed loop system in Figure 6, with improper controller (blue) and proper controller1 (red), $p_1 = p_2 = 10\omega_c$; proper controller2(orange), $p_1 = p_2 = 100\omega_c$.

Exercise 4.2.2

Now in order to improve the system's performance, an integral action is introduced in the feedback controller, which is shown in eq.(13).

$$F_y = \frac{s+\omega_I}{s} G^{-1} G_d \tag{13}$$

From several experimental trials, we find $\omega_I = 0.7\omega_c$ gives a good result.

Improper design

Under this condition, the feedback controller becomes:

$$F_{y3} = \frac{s+\omega_I}{s} G^{-1} G_d \tag{14}$$

Proper design

Here, some poles are also needed to make the controller proper. Two poles are applied here: $p_1 = p_2 = 10\omega_I$. The controller is shown in eq.(15). Figure 9 shows the step response from disturbances to output, which satisfies the second requirement ($|y(t)| \leq 0.1$ for $t > 0.5$ s). The disturbance is attenuated much faster than the result in Figure 8.

$$F_{y4} = F_{y3} * \frac{p_1 p_2}{(s+p_1)(s+p_2)} \tag{15}$$

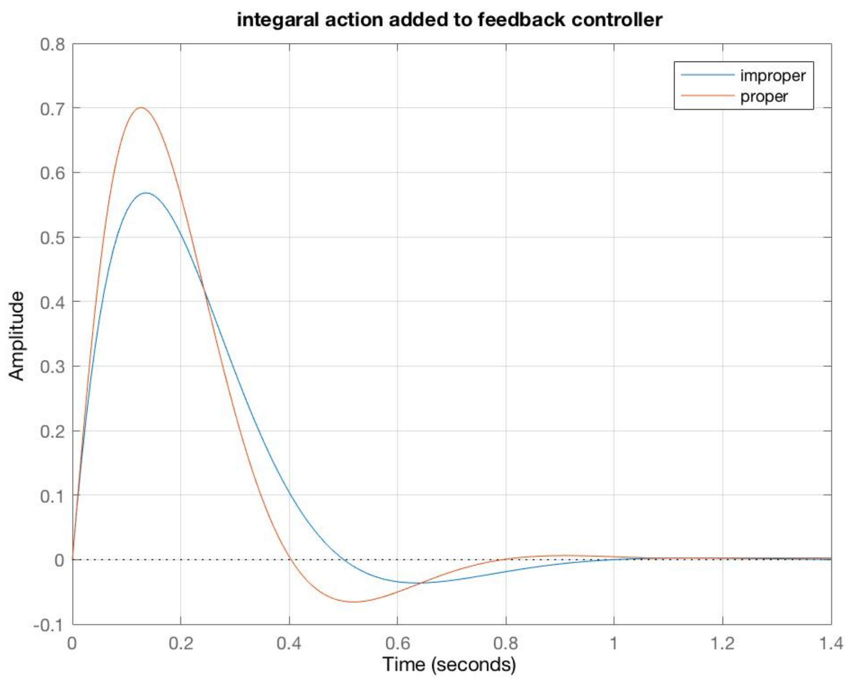

Figure 9: Step response from d to y for the closed loop system in fig. 6, $\omega_I = 0.7\omega_c$, with improper controller (blue) and proper controller (red), $p_1 = p_2 = 10\omega_I$.

Exercise 4.2.3

First, we design a lead controller on the basis of the proper controller above to reduce overshoot. We follow the same process as the Basics (exercise 4.1.1- 4.1.3). since no cross-over frequency and phase margin are given, we could define these parameters by ourselves: phase margin = 36 degree at cross-over frequency. It is better to have cross-over frequency larger than ω_c, so we set $\omega = \omega_c + 10°$. Corresponding parameters are calculated according eq.(5)-(8) and are shown in Table 4.

The bode diagram is presented in Figure 10 and step response is in Figure 11. From Figure 11 we can see, now the resonance peak has been reduced from 0.7 to approximately 0.45.

Table 4: Parameters for the lead controller.

K	β	T_d
2.2539	1.6953	0.0385

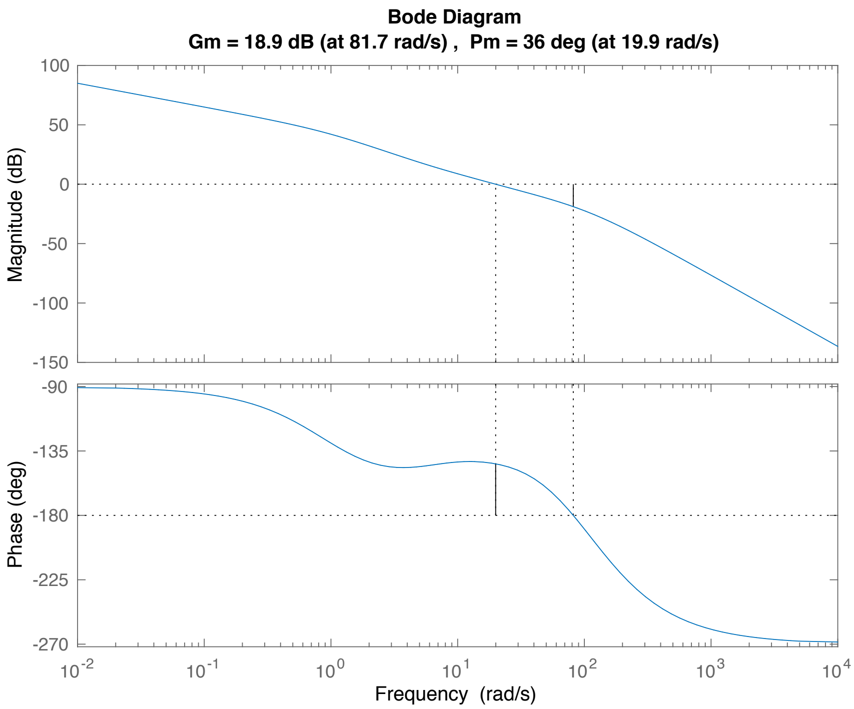

Figure 10: Bode diagram of F_yG with a lead controller in Figure 6. Phase margin is 36 degree at cross-over frequency 19.9 rad/s.

After we are done with the lead controller, we also add a low-pass filter F_r in eq.16. It is kept as simple as possible.

$$F_r = \frac{1}{1 + \tau s} \tag{16}$$

In order to meet the overshoot and rising time requirements, we find when $\tau = 0.12$, the result in Figure 12 is pretty good (overshoot = 0.3157% < 10%, rising time = 0.1350s < 0.2s).

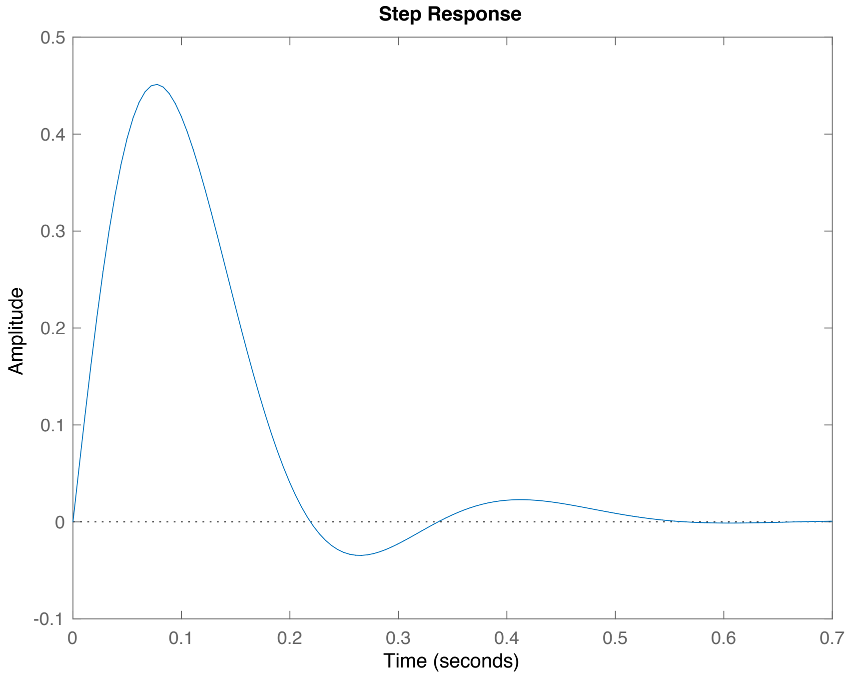

Figure 11: Step response from d to y with a lead compensator for the closed loop system in Figure 6.

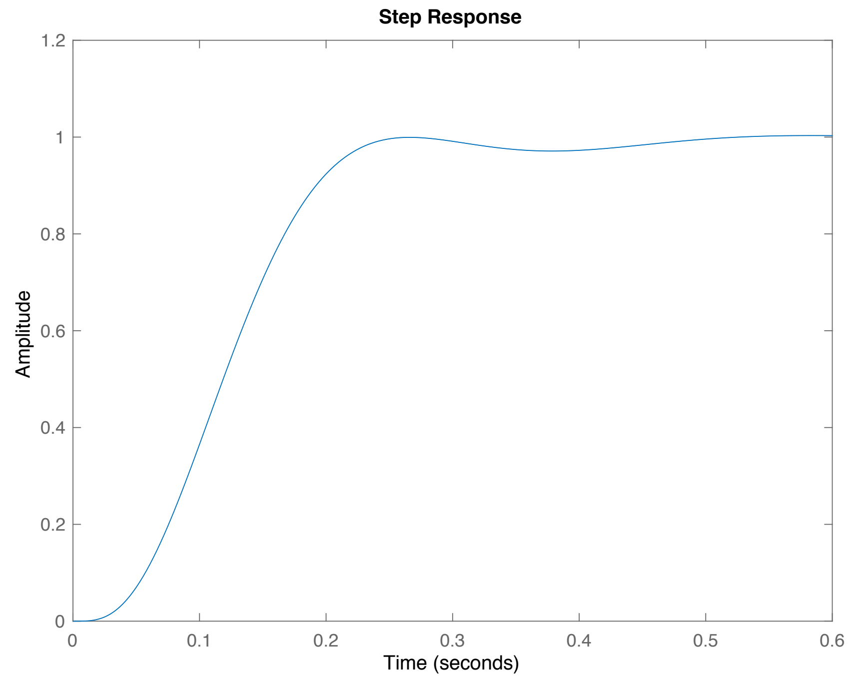

Figure 12: Step response from reference signal to y with a lead compensator for the closed loop system in Figure 6.

Exercise 4.2.4

In Figure 13, the bode diagrams of sensitivity and complementary sensitivity functions are shown:

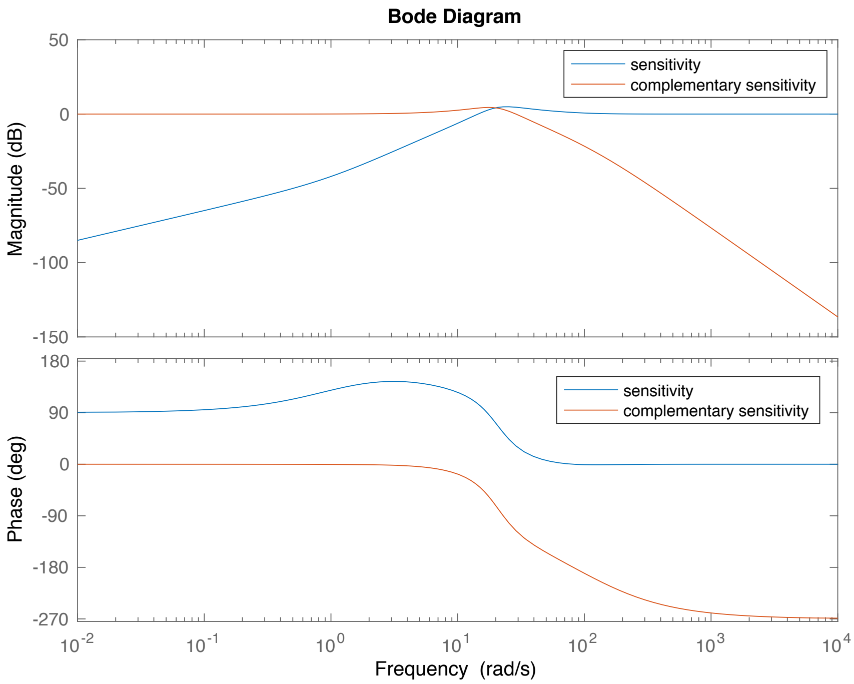

Figure 13: Bode diagram for sensitivity (blue) and complementary sensitivity (orange) of closed loop in Figure 6.

According to Figure 11 and Figure 12, we can validate our system's performance with the predefined requirements:

- The rise time for a step change in the reference signal is 0.1350s < 0.2 s;
- The overshoot for a step change in the reference signal is 0.3157% < 10%;
- For a step in the disturbance, $|y(t)| \leq 1 \ \forall t$ and $|y(t)| \leq 0.1$ for $t > 0.5$ s;
- Since the signals are scaled the control signal obeys $|u(t)| \leq 0.771 \leq 1 \ \forall t$.

The last one can be proved based on the step responses of $F_y F_r S$ and $F_y G_d S$. Since $U = F_y F_r S r - F_y G_d S d$, $|u(t)|$ should be below the maximum absolute value of its step response when either $|r| = 0, |d| = 1$ or $|r| = 1, |d| = 0$. Thus, the control signal is bounded by the step responses of $F_y F_r S$ and $F_y G_d S$, as is shown in Figure 14. Therefore, all the requirements above are satisfied for our system.

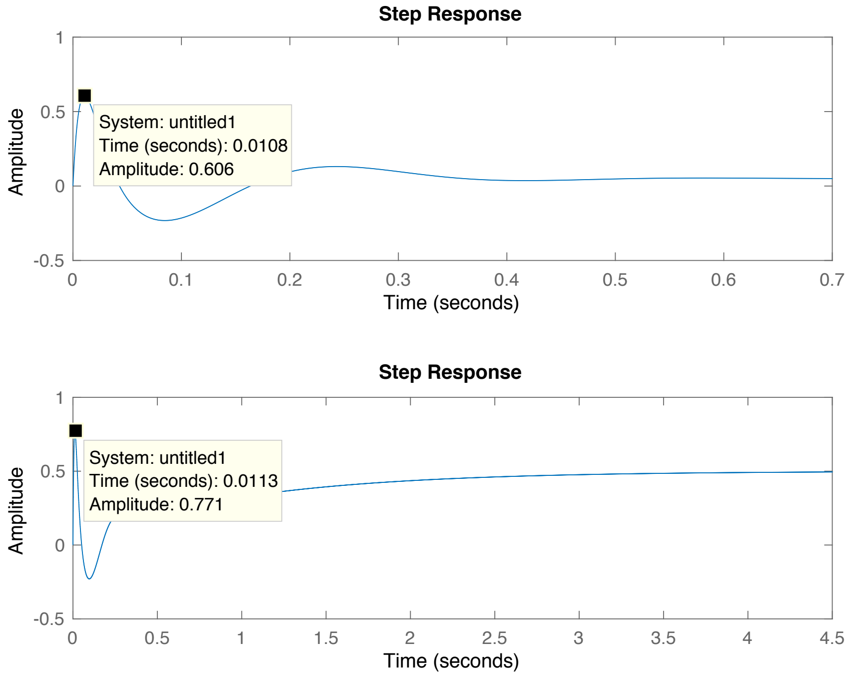

Figure 14: Step response of F_yF_rS (upper) and F_yG_dS (below).

Conclusions

In this report, we succeed in constructing a controller which could both track the reference signal and attenuate disturbances. To achieve this goal, the parameters need to be balanced carefully since there exists a trade-off relation between these specified requirements.

References

[1] EL2520 Control Theory and Practice Advanced Course, Computer Exercise: Classical Loop-Shaping, 2014.
[2] T. Glad and L. Ljung, Reglerteknik, Grundläggande teori, Studentlitteratur, 2006.