# Computer Exercise 3
# EL2520 Control Theory and Practice


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## Suppression of disturbances

The weight is

$$W_S(s) = \frac{1}{\left(s + 0.4 + i\sqrt{(100\pi)^2 - 0.4^2}\right) \cdot \left(s + 0.4 - i\sqrt{(100\pi)^2 - 0.4^2}\right)}$$

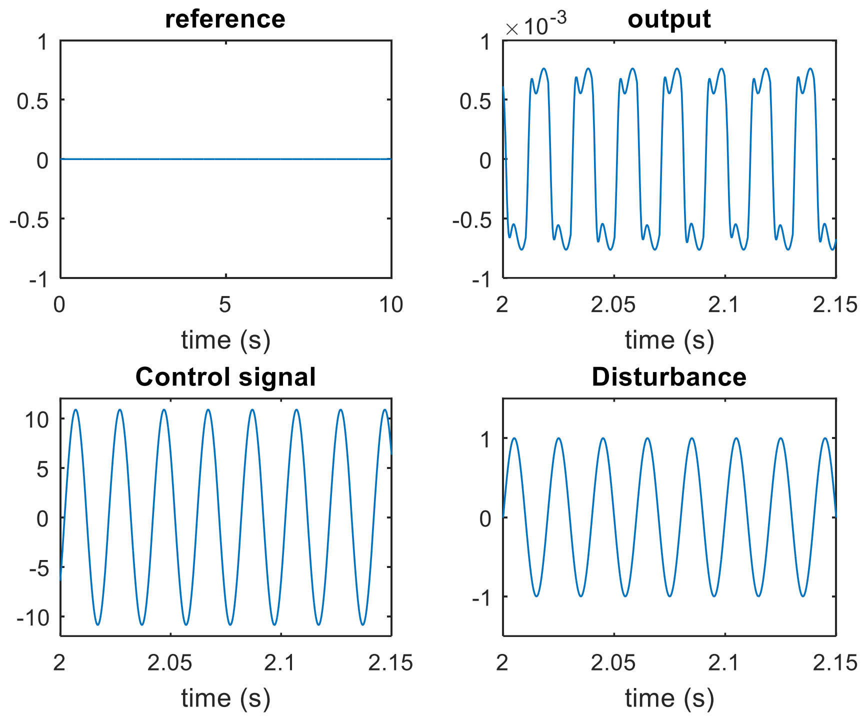


**Fig. 1: Simulation results with system G, using Ws**

According to Fig. 1, the output oscillation has an amplitude of $7.63\times10^{-4}$, and the disturbance amplitude is set to be 1. Therefore, the disturbance is damped by a rate of $7.63\times10^{-4}$.

If the controller is replaced by a P-controller, in order to get the same rate, the amplification of the P-controller needs to be chosen carefully. Using matlab, we can get $|G(i\omega)|_{\omega=100\pi} = 0.092$. Thus,

$$|S| \approx |FG|^{-1} = (0.092|F|)^{-1} = 7.63\times10^{-4} \Rightarrow |F| = 1.4246\times10^{4}$$

The approximate amplification of the P-controller should be $1.4246\times10^4$. The disadvantage of using such a P-controller with a very large amplitude can be boiled down to a huge controlling cost in practice. Also, the P-controller will attenuate disturbances at all frequencies, not only at 50 Hz.

## Robustness

According to $G_0(s)$, we can obtain $\Delta G(s) = -\frac{3}{s+2}$. Based on the small gain theorem,

$$|T(i\omega)\cdot\Delta G(i\omega)| < 1 \Leftrightarrow |T(i\omega)| < |\Delta G^{-1}(i\omega)|$$

To satisfy the requirement that the individual transfer functions should have $H_\infty$ norm which is less than $\gamma$,

$$|T(i\omega)| \le \gamma|W_T^{-1}(i\omega)|$$

Combining the two conditions above together, the weight $W_T$ should be chosen given the following condition:

$$|W_T(i\omega)| > \gamma|\Delta G(i\omega)|$$

The weights are

$$W_S(s) = \frac{1}{\left(s+0.4+i\sqrt{(100\pi)^2-0.4^2}\right)\cdot\left(s+0.4-i\sqrt{(100\pi)^2-0.4^2}\right)}$$

$$W_T(s) = 10^{-4}\cdot\frac{3}{s+2}$$

Fig. 2 shows the bode graph of the complementary sensitivity $T$ and the model uncertainty $1/\Delta G(s)$. The figure proves that the small gain theorem is fulfilled in this case.

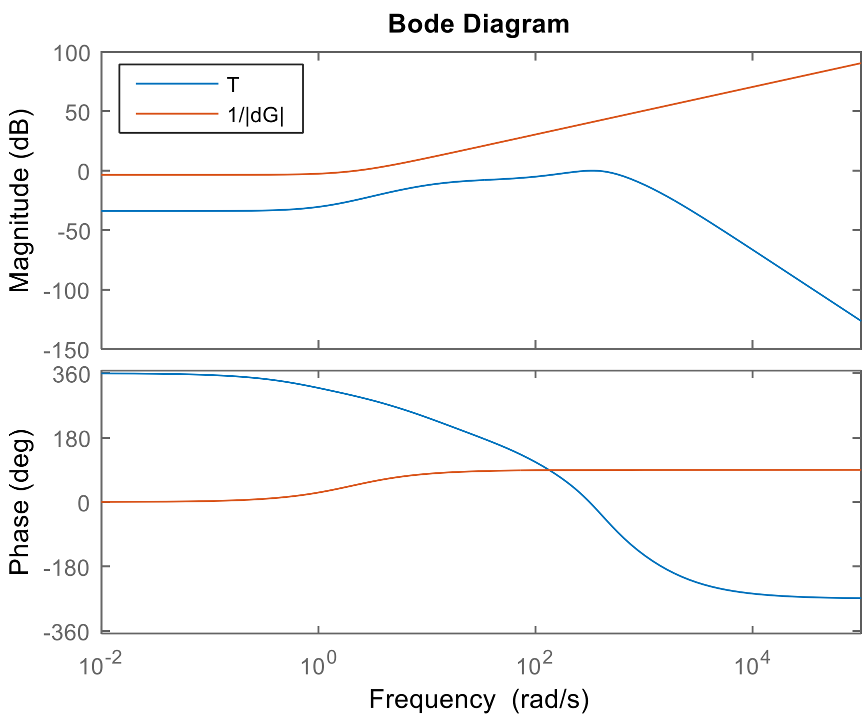


Fig. 2: Bode diagram showing the small gain theorem is satisfied

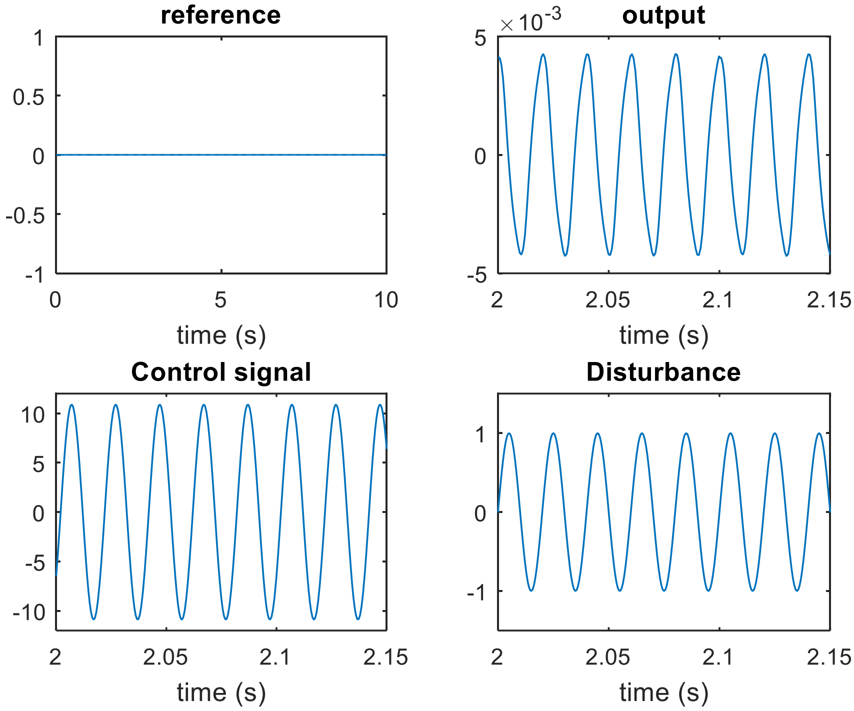

Fig. 3: Simulation results with G0, using Ws and Wt

Comparing the results in Fig. 3 to that of the previous simulation, the rate between the output oscillation and the disturbance amplitude increases to $4.3{\times}10^{-3}$, given $\gamma = 10^{-4}$. Besides, the output becomes more sinusoidal-like.

## Control signal

The weights are

$$W_S(s) = \frac{1}{\left(s + 0.4 + i\sqrt{(100\pi)^2 - 0.4^2}\right) \cdot \left(s + 0.4 - i\sqrt{(100\pi)^2 - 0.4^2}\right)}$$

$$W_T(s) = 10^{-4} \cdot \frac{3}{s+2}$$

$$W_U(s) = \frac{0.5}{s+2}$$

As is shown in Fig. 4, the amplitude of control signal is reduced to half compared to the previous one, which meets the specification perfectly. Meanwhile, we can notice that the output oscillation is badly magnified to an unacceptable value of 0.54 compared to the previous simulation results.

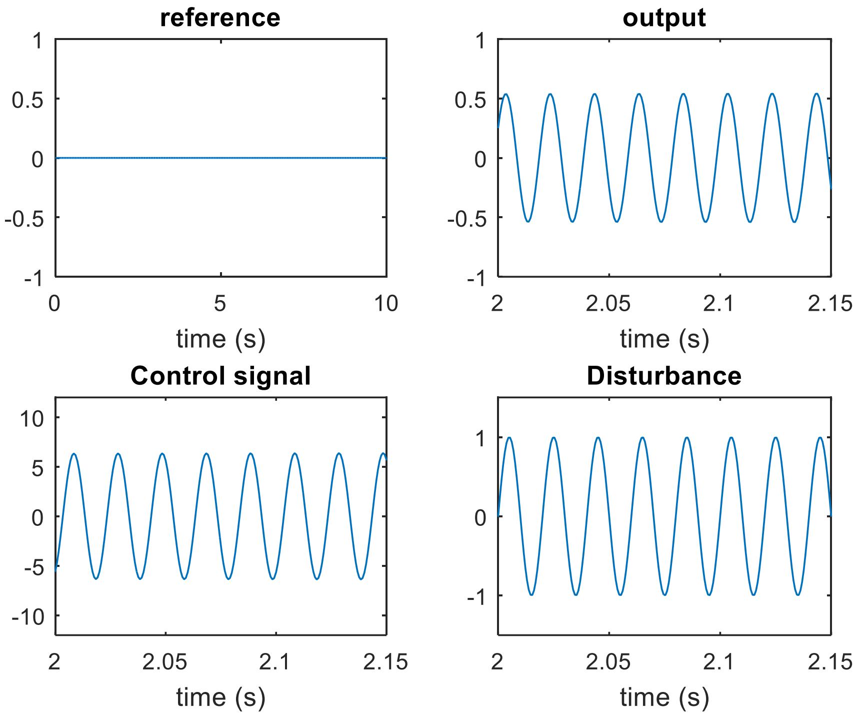


**Fig. 4: Simulation results with G0, using Ws, Wt and Wu**