Probabilistic Graphical Models

HWK#2 Part B

Assigned Tuesday, Feb. 21, 2023

Due: Wed (noon EST), March 8, 2023

Problem 7 (Gaussian Message Passing over Markov Random Field)

Let's consider the belief propagation algorithm for Gaussian pairwise MRF, where the potentials are defined as following:

$$\phi_t(x_t) = exp(-\frac{1}{2}A_{tt}x_t^2 + b_t x_t)$$
$$\phi_{s,t} = exp(-\frac{1}{2}x_s A_{st} x_t)$$

where A is the precision matrix and b is just a parameter. Our goal is to derive the message passed into node x_t, i.e. $m(x_t)$.

Part 1:

To begin with, we need a fact that the product of two Gaussians is a scaled Gaussian. Show:

$$N(x|\mu_1, \lambda_1^{-1}) \times N(x|\mu_2, \lambda_2^{-1}) = CN(x|\mu, \lambda^{-1}),$$

solve for λ and μ, where C is a constant.

Part 2:

Solve for $m(x_t)$.

Hint: First solve for the messages passed into Node t's neighbor nodes from their neighbors excluding t, then use these messaged to represent $m(x_t)$.

You can directly use this result if needed: $\int \exp(-ax^2 + bx)dx = \sqrt{\frac{\pi}{a}}\exp(\frac{b^2}{4a})$

Problem 8

The file ‘diseaseNet.mat’ contains the potentials for a disease bi-partite belief network, with 20 diseases $d_1, \ldots, d_{20}$ and 40 symptoms, $s_1, \ldots, s_{40}$. The disease variables are numbered from 1 to 20 and the symptoms from 21 to 60. Each disease and symptom is a binary variable, and each symptom connects to 3 parent diseases.

1. Using the BRMLtoolbox, construct a junction tree for this distribution and use it to compute all the marginals of the symptoms, i.e., $p(s_i = 1)$. In addition to providing your code, you must display/print the results in your solution paper work.

2. Explain how to compute the marginals $p(s_i = 1)$ (in the previous part) in a way more efficient than using the junction tree formalism. By implementing this method, compare it with the results from the junction tree algorithm from the previous part.

3. Symptoms 1 to 5 are present (state 1), symptoms 6 to 10 not present (state 2), and the rest not known. Compute the marginal $p(d_i = 1|s_{1:10})$ for all diseases.

In all above questions, in addition to providing your codes, you must display/print the results in your solution paper work.