Time Series Analysis

Topics

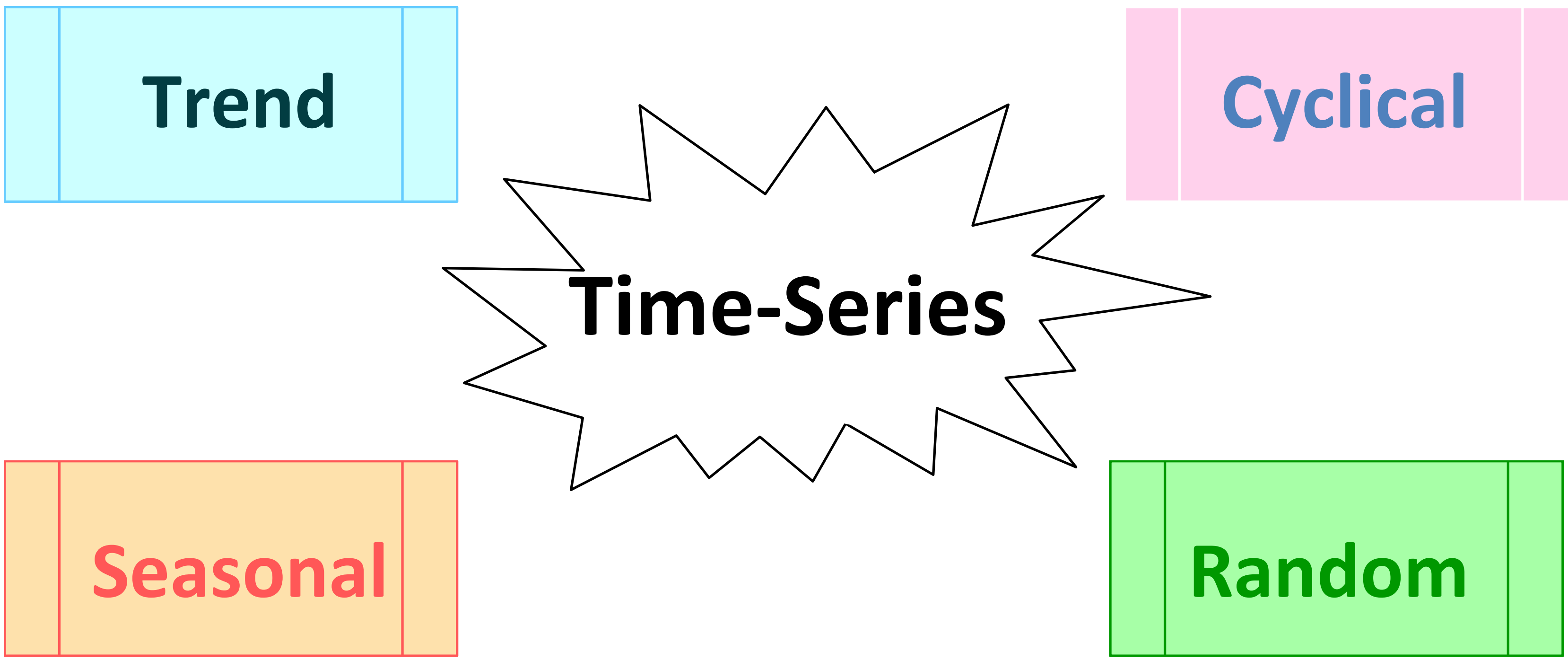
- Component Factors of the Time-Series Model
- Smoothing of Data Series
  - Moving Averages
  - Exponential Smoothing
- Least Square Trend Fitting and Forecasting
  - Linear, Quadratic and Exponential Models
- Autoregressive Models
- Choosing Appropriate Models
- Monthly or Quarterly Data

What Is Time-Series

- A Quantitative Forecasting Method to Predict Future Values
- Numerical Data Obtained at Regular Time Intervals
- Projections Based on Past and Present Observations
- Example:

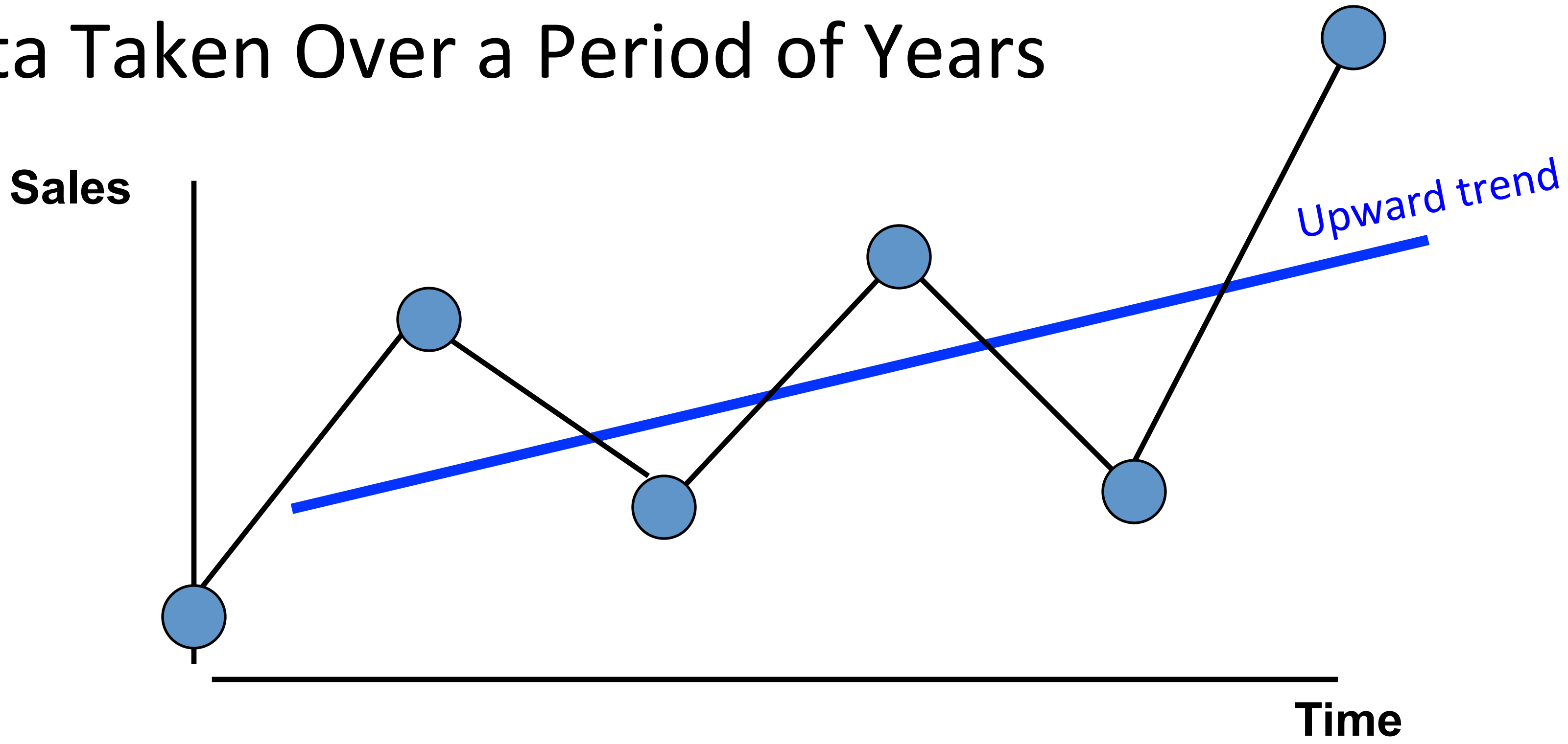
Year:	1994	1995	1996	1997	1998
Sales:	75.3	74.2	78.5	79.7	80.2

Time-Series Components



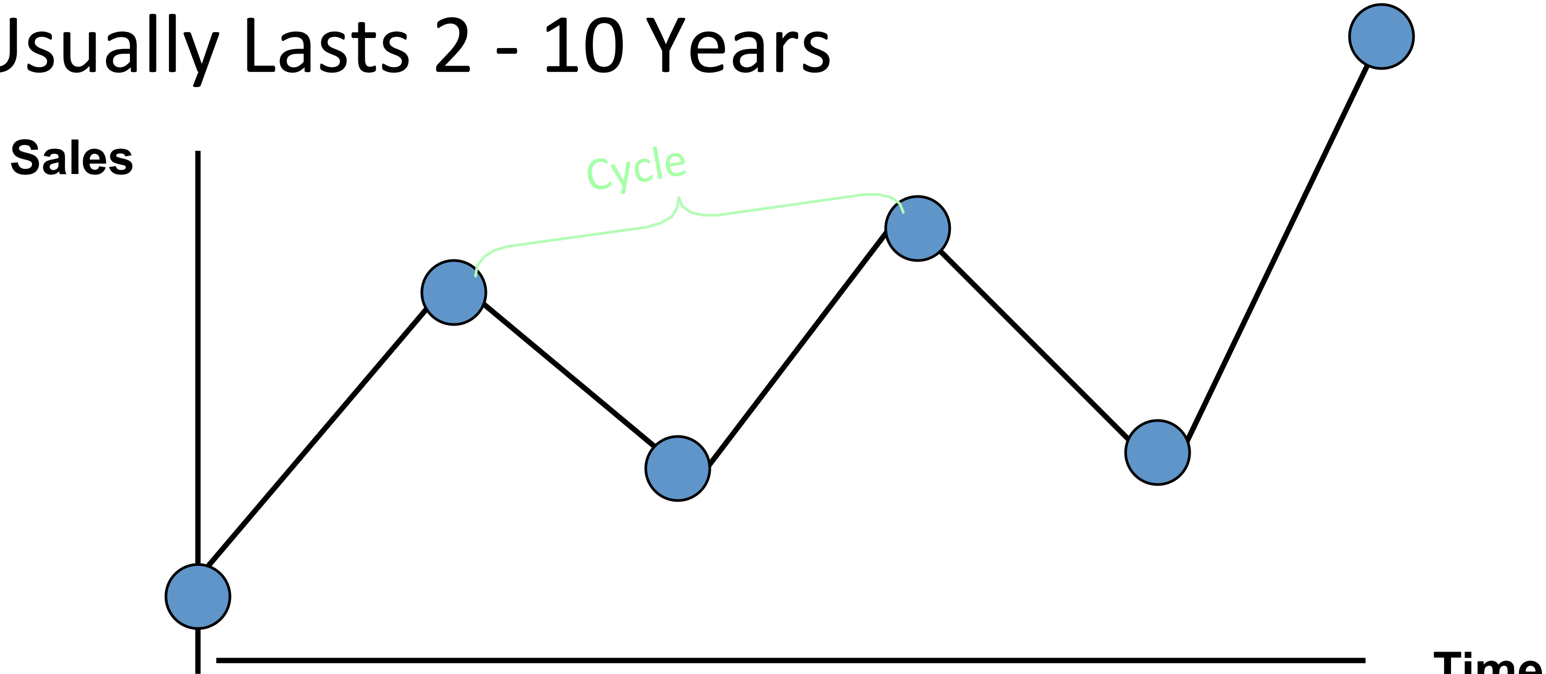
Trend Component

- Overall Upward or Downward Movement
- Data Taken Over a Period of Years



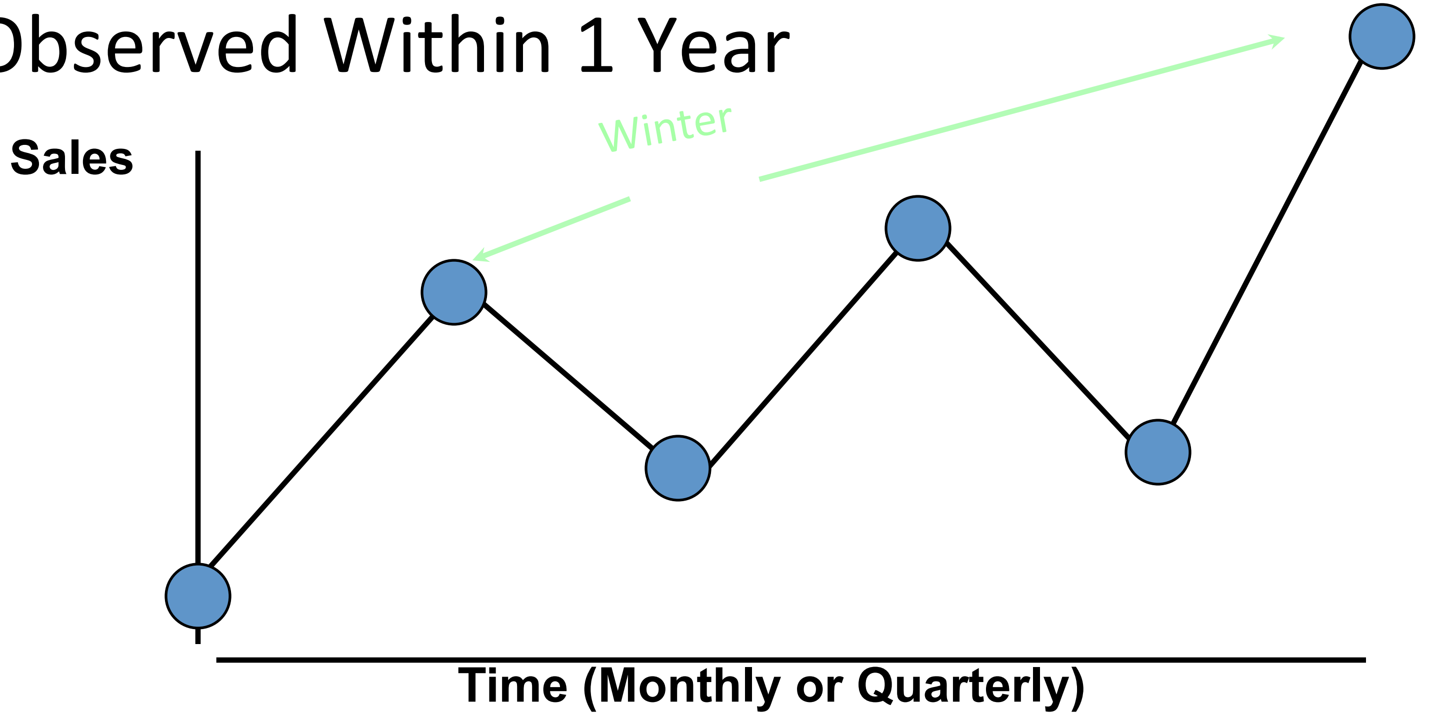
Cyclical Component

- Upward or Downward Swings
- May Vary in Length
- Usually Lasts 2 - 10 Years



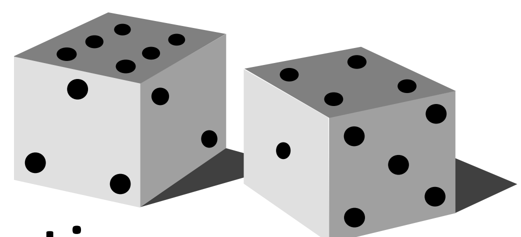
Seasonal Component

- Upward or Downward Swings
- Regular Patterns
- Observed Within 1 Year



Random or Irregular Component

- Erratic, Nonsystematic, Random, ‘Residual’ Fluctuations
- Due to Random Variations of
  - Nature
  - Accidents
- Short Duration and Non-repeating



Multiplicative Time-Series Model

- Used Primarily for Forecasting
- Observed Value in Time Series is the product of Components
- For Annual Data:

$$Y_i = T_i \times C_i \times I_i$$

- For Quarterly or Monthly Data:

$$Y_i = T_i \times S_i \times C_i \times I_i$$

T_i = *Trend*

C_i = *Cyclical*

I_i = *Irregular*

S_i = *Seasonal*

Moving Averages

- Used for Smoothing
- Series of Arithmetic Means Over Time
- Result Dependent Upon Choice of *L*, Length of Period for Computing Means
- For Annual Time-Series, *L* Should be Odd
- Example: 3-year Moving Average
  - First Average: $MA(3) = \dfrac{Y_1 + Y_2 + Y_3}{3}$
  - Second Average: $MA(3) = \dfrac{Y_2 + Y_3 + Y_4}{3}$

Moving Average Example

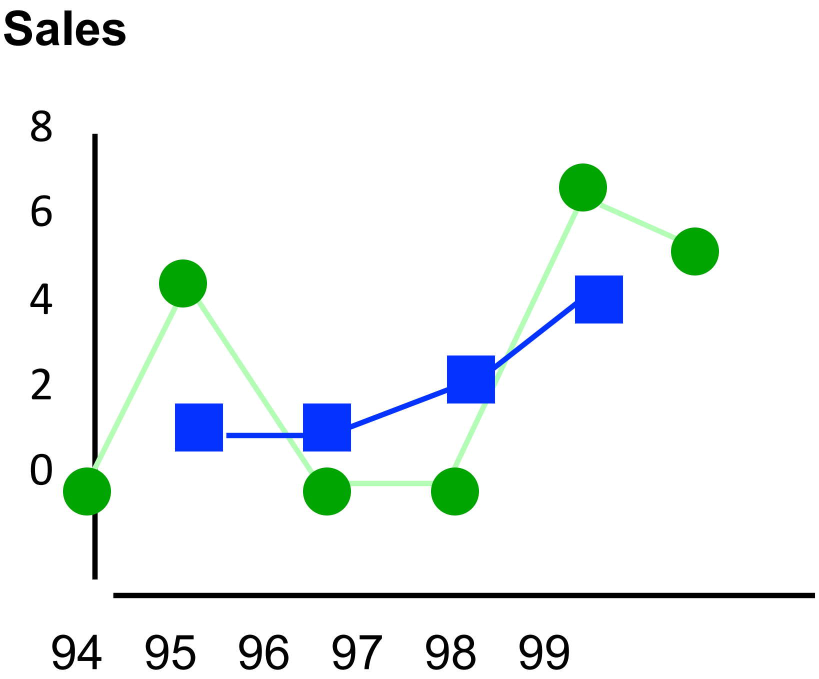
John is a building contractor with a record of a total of 24 single family homes constructed over a 6 year period.

Provide John with a Moving Average Graph.

Year	Units	Moving Ave
1994	2	NA
1995	5	3
1996	2	3
1997	2	3.67
1998	7	5
1999	6	NA

Moving Average Example Solution

Year	Response	Moving Ave
1994	2	NA
1995	5	3
1996	2	3
1997	2	3.67
1998	7	5
1999	6	NA



Exponential Smoothing

- Weighted Moving Average
  - Weights Decline Exponentially
  - Most Recent Observation Weighted Most
- Used for Smoothing and Short Term Forecasting
- Weights Are:
  - Subjectively Chosen
  - Ranges from 0 to 1
    - Close to 0 for Smoothing
    - Close to 1 for Forecasting

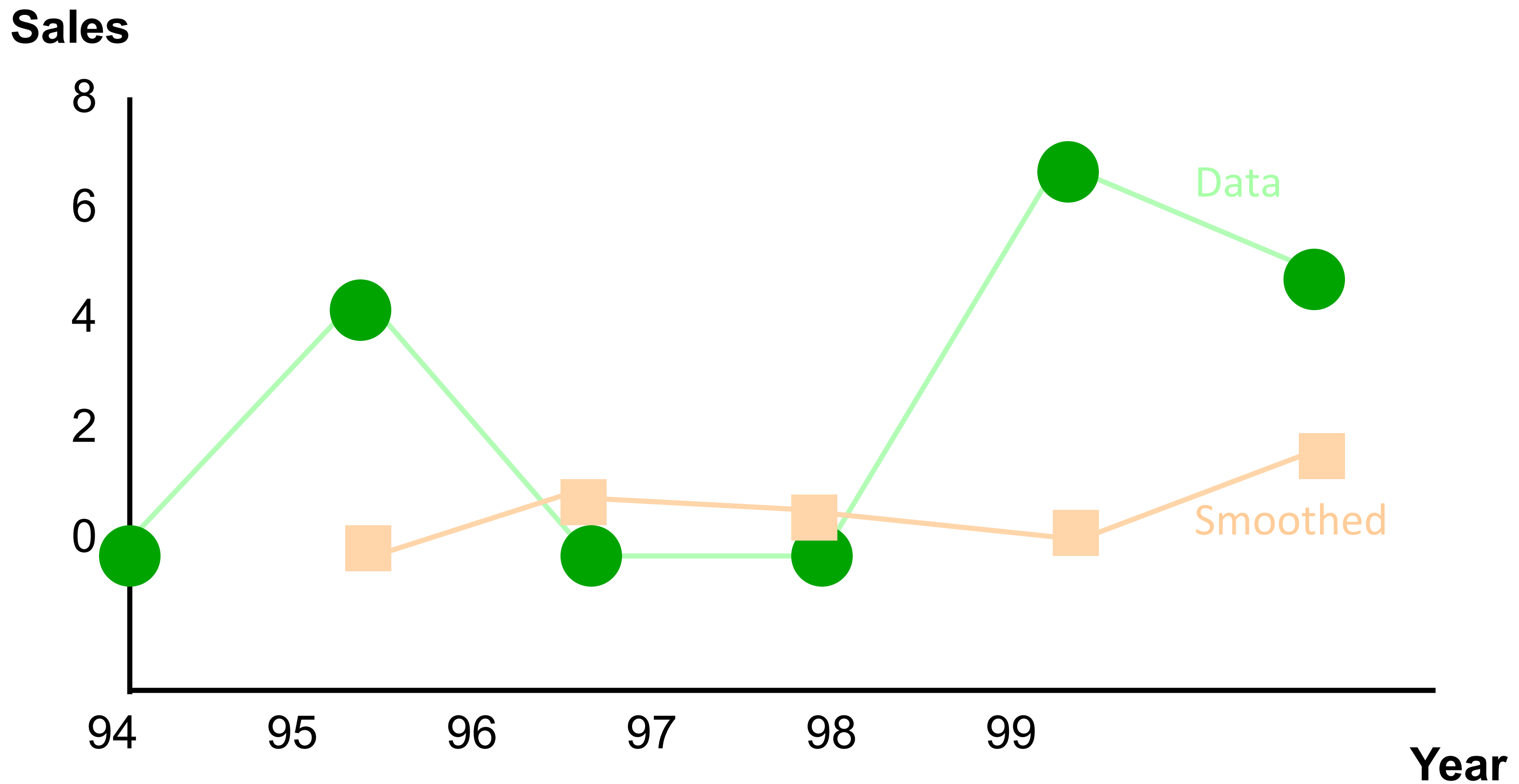
Exponential Weight: Example

$$Y_{N+1} = \alpha X_N + (1-\alpha) Y_N$$

	A	B	C	D
1		**Alpha =**	**0.2**	
2				
3	**n**	**X**	**Y**	**formula/comment**
4	0	0.66	-	
5	1	0.58	0.66	*= B4 { first point }*
6	2	0.39	0.64	'=C$1*B5+(1-C$1)*C5
7	3	0.7	0.59	'=C$1*B6+(1-C$1)*C6
8	4	0.05	0.62	'=C$1*B7+(1-C$1)*C7
9	5	0.99	0.5	'=C$1*B8+(1-C$1)*C8
10	6	0.79	0.6	'=C$1*B9+(1-C$1)*C9

$$Y_1 = X_0$$
$$Y_2 = \alpha X_1 + (1-\alpha) Y_1 \ = \ \alpha X_1 + (1-\alpha) X_0$$
$$Y_3 = \alpha X_2 + (1-\alpha) Y_2 \ = \ \alpha X_2 + (1-\alpha)(\alpha X_1 + (1-\alpha) X_0) \ = \ \alpha X_2 + \alpha(1-\alpha) X_1 + (1-\alpha)^2 X_0$$
$$Y_4 = \alpha X_3 + (1-\alpha) Y_3 \ = \ \dots \ = \ \alpha X_3 + \alpha(1-\alpha) X_2 + \alpha(1-\alpha)^2 X_1 + (1-\alpha)^3 X_0$$

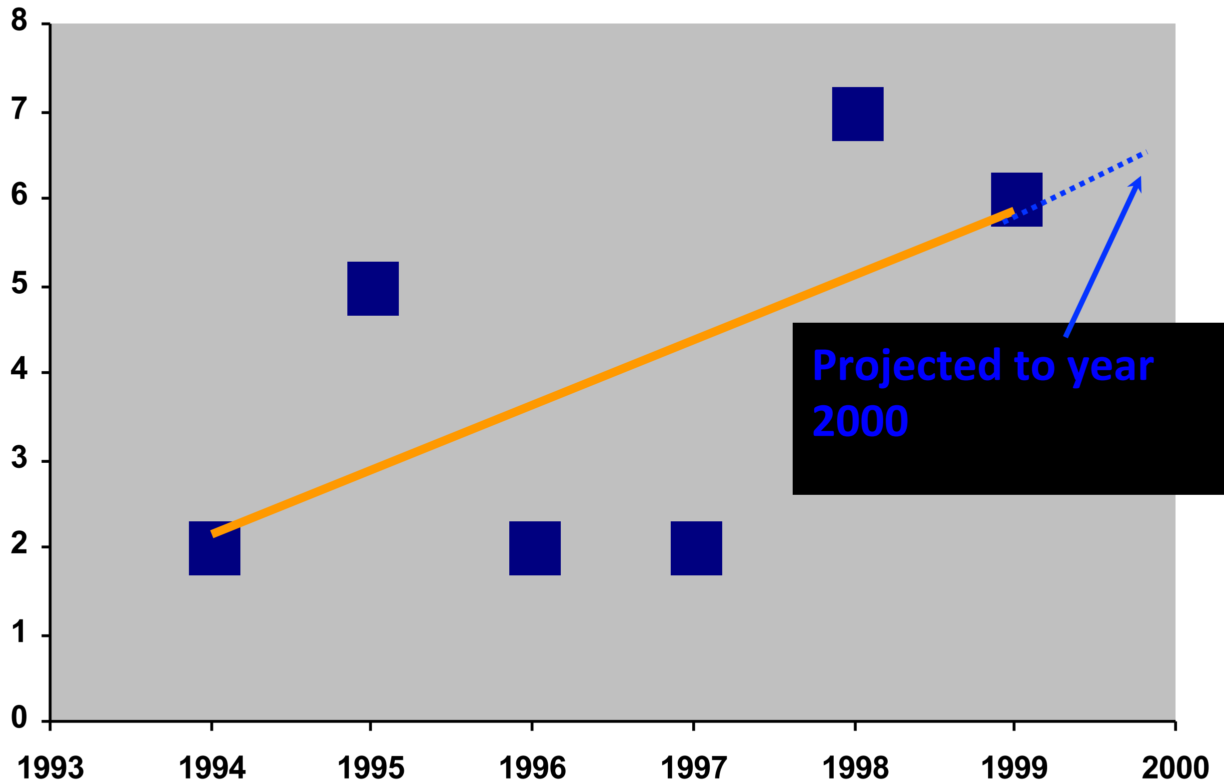
Exponential Weight: Example Graph



The Linear Trend Model

Year	Coded	Sales
94	0	2
95	1	5
96	2	2
97	3	2
98	4	7
99	5	6

$$\hat{Y}_i = b_0 + b_1 X_i = 2.143 + .743\, X_i$$



Excel Output

	Coefficients
Intercept	2.14285714
X Variable	0.74285714

The Quadratic Trend Model

Year	Coded	Sales
94	0	2
95	1	5
96	2	2
97	3	2
98	4	7
99	5	6

$$\hat{Y}_i = b_0 + b_1 X_i + b_2 X_i^2$$

	Coefficients
Intercept	2.85714286
X Variable 1	-0.3285714
X Variable 2	0.21428571

Excel Output

$$\hat{Y}_i = 2.857 - 0.33 X_i + .214 X_i^2$$

Autogregressive Modeling

- Used for Forecasting
- Takes Advantage of Autocorrelation
  - 1st order - correlation between consecutive values
  - 2nd order - correlation between values 2 periods apart
- Autoregressive Model for *pth* order:

Random Error

$$Y_i = A_0 + A_1 Y_{i-1} + A_2 Y_{i-2} + \bullet\bullet\bullet + A_p Y_{i-p} + \delta_i$$

Autoregressive Model: Example

The Office Concept Corp. has acquired a number of office units (in thousands of square feet) over the last 8 years. Develop the 2nd order Autoregressive models.

Year	Units
92	4
93	3
94	2
95	3
96	2
97	2
98	4
99	6

Autoregressive Model: Example Solution

- Develop the 2nd order table
- Use Excel to run a regression model

Year	Y_i	Y_{i-1}	Y_{i-2}
92	4	---	---
93	3	4	---
94	2	3	4
95	3	2	3
96	2	3	2
97	2	2	3
98	4	2	2
99	6	4	2

Excel Output

	Coefficients
Intercept	3.5
X Variable 1	0.8125
X Variable 2	-0.9375

$$Y_i = 3.5 + .8125\,Y_{i-1} - .9375\,Y_{i-2}$$

Autoregressive Model Example: Forecasting

Use the 2nd order model to forecast number of units for 2000:

$$Y_i = 3.5 + .8125\,Y_{i-1} - .9375\,Y_{i-2}$$

$$\begin{aligned} Y_{2000} &= 3.5 + .8125\,Y_{1999} - .9375\,Y_{1998} \\ &= 3.5 + .8125 \times 6 - .9375 \times 4 \\ &= 4.625 \end{aligned}$$

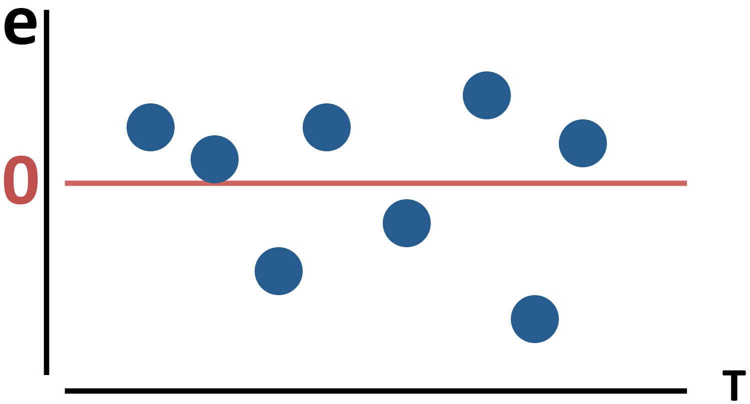
Autoregressive Modeling Steps

- 1. Choose p: Note that df = n - $2p$ - 1
- 2. Form a series of "lag predictor" variables
- Y_{i-1} , Y_{i-2} , … Y_{i-p}
- 3. Use Excel to run regression model using all p variables
- 4. Test significance of AR_p

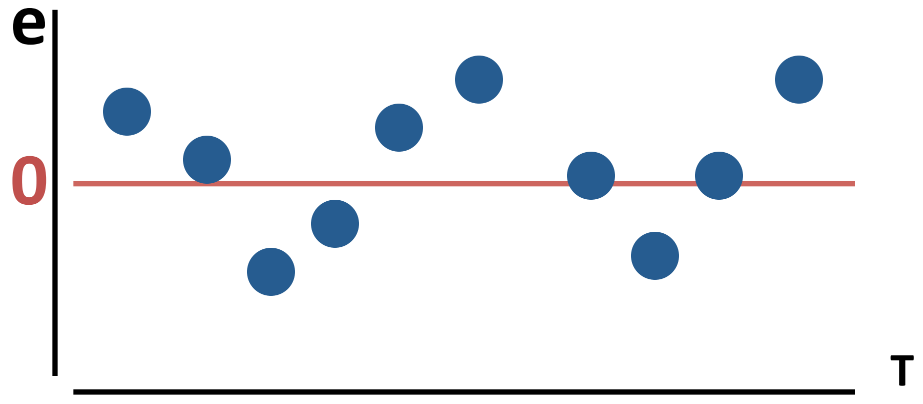
Selecting A Forecasting Model

- Perform A Residual Analysis
  - Look for pattern or direction
- Measure Sum Square Errors - SSE (residual errors)
- Measure Residual Errors Using MAD
- Use Simplest Model
  - Principle of Parsimony

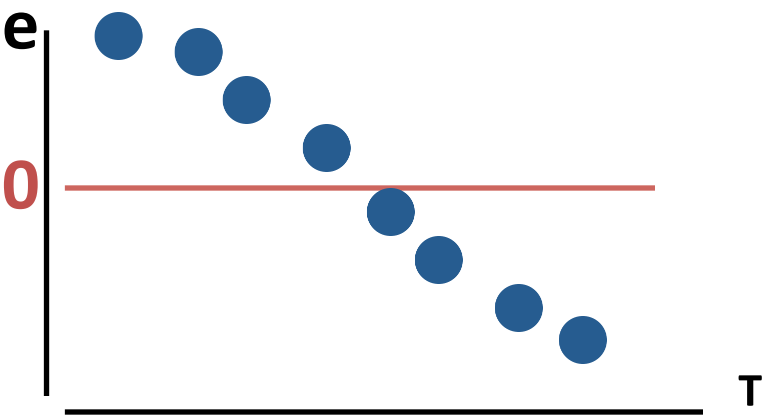
Residual Analysis



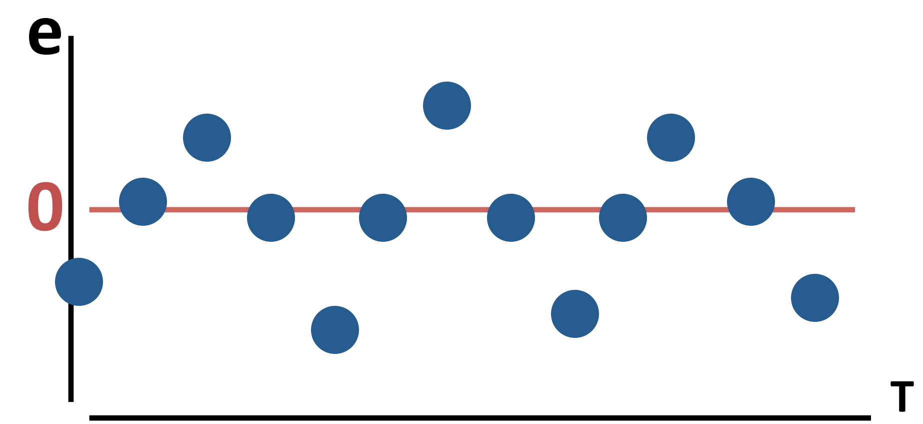
Random errors



Cyclical effects not accounted for



Trend not accounted for



Seasonal effects not accounted for

Measuring Errors

- Sum Square Error (SSE)

$$SSE = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

- Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n}\left|Y_i - \hat{Y}_i\right|}{n}$$

Further Study

- Introduction to AR
- https://www.youtube.com/watch?v=AN0a58F6cxA

- Time Series Forecasting Using Neural Network and Statistic Models

- https://www.youtube.com/watch?v=i40Road82No