Unit 3 Equation Sheet

Terry Yu

2021

§1 Prerequisite Concepts

Vectors — For two vectors $\vec{\mathbf{x}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ and $\vec{\mathbf{y}} = d\hat{\mathbf{i}} + e\hat{\mathbf{j}} + f\hat{\mathbf{k}}$,
Vector Addition/Subtraction:

$$\vec{\mathbf{x}} + \vec{\mathbf{y}} = (a+d)\,\hat{\mathbf{i}} + (b+e)\,\hat{\mathbf{j}} + (c+f)\,\hat{\mathbf{k}}$$

Dot Product
there are two ways to find the dot product (where $|\vec{\mathbf{x}}| = \sqrt{a^2+b^2+c^2}$, or the magnitude of x, and so on):

$$\vec{\mathbf{x}} \cdot \vec{\mathbf{y}} = ad + be + fc = |\vec{\mathbf{x}}||\vec{\mathbf{y}}|\cos(\theta)$$

Conservative Forces — Basically her explanation in class was that conservative forces can be reversible, such as pushing a carton, or gravity.
The more rigorous definition is that the work done by conservative forces is the same regardless of whatever path taken; friction is an example of a non conservative force, as the work done by friction depends on how long your path is (not all paths result in the same work), whereas gravity always does teh same work (mgh) regardless of how you get from one point to another.

§2 Work (pt 1) & Power

Work (in Joules (J)) — For constant forces,

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos(\theta)$$

where θ is the angle between the vectors $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$.
When $\vec{\mathbf{F}}$ is nonconstant, we have

$$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{x}}$$

We can also get work from power, with

$$W = \int_{t_i}^{t_f} P_{inst}\,\mathrm{d}t$$

More on work below in Work Energy Thm.

Power (in Watts (W)) — There is both instantaneous and avg power.

$$P_{av} = \frac{W}{t} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}_{av} = \frac{\Delta E}{\Delta t}$$

$$P_{inst} = \frac{\mathrm{d}W}{\mathrm{d}t} = \vec{\mathbf{F}} \cdot \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = \frac{\mathrm{d}E}{\mathrm{d}t}$$

§3 Work (pt 2) & Energy

Energy —

$$\text{Kinetic Energy} = K = \frac{1}{2}mv^2$$
$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$\text{Gravitational PE} = U_g = mgh$$
$$\Delta U_g = mgh_f - mgh_i = mg(h_f - h_i)$$

$$\text{Elastic PE} = U_s = \frac{1}{2}kx^2$$
$$\Delta U_s = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = \frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

$$\text{Thermal Change} = \Delta E_{\text{th}} = f_k d$$

$$E_{\text{mech}} = K + U$$
$$\Delta E_{\text{mech}} = \Delta K + \Delta U$$

Work Energy Thm —

$$\begin{aligned} W_{\text{net}} &= \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= -\Delta U \end{aligned}$$

Conservation of Mechanical Energy — When all the forces acting on a closed system are conservative, mechanical energy is conserved at any time.

$$\begin{aligned} &\Delta E_{\text{mech}} = 0 \\ \implies &E_{\text{initial mech}} = E_{\text{final mech}} \\ \implies &K_i + U_i = K_f + U_f \end{aligned}$$

Conservation of Energy — Conservation of energy is always true, unlike conservation of just mechanical energy. This gives

$$W = \Delta E = \Delta E_{\text{mech}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$$

Note that this can be very useful when work is known; if the net force acting on the system is 0/system is isolated, $W = 0$.
In the case where $W = 0$, it is a closed system (no net/external forces), and thus

$$E_{\text{final mech}} = E_{\text{initial mech}} - \Delta E_{th} - \Delta E_{int}$$

Note: I dont think we really covered internal energy this unit so dw abt that.

Potential Energy Curves —

$$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$$

As $E_{\text{mech}} = K + U$,

$$U = E_{\text{mech}} - K$$

Equilibrium: When the slope $= 0$ (thus $F = \frac{\mathrm{d}U}{\mathrm{d}x} = 0$).
Turning Point: When $K = 0$, or $E_{\text{mech}} = U$.