HRK Notes

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Contents

Motion

Definition — Vectors are quantities that have both magnitude and direction, and follow a certain set of rules for addition and multiplication.

Unit Vectors — The unit vectors are vectors of length 1, used as the basis for vector space (can be thought of as axis). Common ones are

$$\hat{\mathbf{i}} = \langle 1, 0, 0 \rangle \qquad \hat{\mathbf{j}} = \langle 0, 1, 0 \rangle \qquad \hat{\mathbf{k}} = \langle 0, 0, 1 \rangle$$

Vectors can be expressed in multiple ways; such as with triangle brackets $\langle\rangle$, a combination of unit vectors, or with maginitude and angle. They are often written bolded, and/or with an arrow above, or with a "hat" if its a unit vector.

Dot Product — Given two vectors (generalizes to n-dimensional vectors) $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$ and $\vec{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$, the dot product defines the multiplication of the vectors into a scalar as

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \|\vec{\mathbf{a}}\| \|\vec{\mathbf{b}}\| \cos(\Delta\theta)$$

Cross Product — Given two *3-dimensional* vectors $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$ and $\vec{\mathbf{b}} = \langle b_1, b_2, b_3 \rangle$, the cross product defines the multiplication of the vectors into another 3d vectors as

$$\begin{aligned} \vec{\mathbf{a}} \times \vec{\mathbf{b}} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ &= \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle \end{aligned}$$

and

$$\|\vec{\mathbf{a}} \times \vec{\mathbf{b}}\| = \|\vec{\mathbf{a}}\| \|\vec{\mathbf{b}}\| \sin(\Delta\theta)$$

Variables — To describe motion of an object, the following variables are generally used:

$$\begin{aligned} \vec{\mathbf{x}} &= \text{position} \\ \vec{\mathbf{v}} &= \text{velocity} \\ \vec{\mathbf{a}} &= \text{acceleration} \end{aligned}$$

Relationships —

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\Delta \vec{\mathbf{x}}}{\Delta t} \qquad \vec{\mathbf{v}} = \frac{\mathrm{d}\vec{\mathbf{x}}}{\mathrm{d}\Delta t}$$

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\Delta \vec{\mathbf{v}}}{t} \qquad \vec{\mathbf{a}} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\mathbf{x}}}{\mathrm{d}t^2}$$

Constant Velocity — For a particle moving at constant velocity,

$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{v}}\Delta t$$

Constant Acceleration (Kinematics Equations) — For a particle under constant acceleration,

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$
$$\vec{\mathbf{x}} = \vec{\mathbf{x}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2$$
$$\vec{\mathbf{v}}^2 = \vec{\mathbf{v}}_0^2 + 2\vec{\mathbf{a}}\Delta\vec{\mathbf{x}}$$
$$\vec{\mathbf{x}} = \vec{\mathbf{x}}_0 + \frac{\vec{\mathbf{v}} + \vec{\mathbf{v}}_0}{2}t$$

Note: when a body is freely falling, it is generally assumed $a = g \approx 9.8\,m/s^2$.

Projectile Motion — Using the equations above, we can derive for a projectile launched from ground level at initial speed v_0 and angle θ,

$$t = \frac{2v_0\cos(\theta)}{t}$$
$$H = \frac{v_0^2\sin^2(\theta))}{2t}$$
$$R = \frac{v_0^2\sin(2\theta)}{t}$$

and the trajectory for the projectile as

$$y = \tan(\theta)x - \frac{g}{2v_0^2\cos^2(\theta)}x^2$$

Centripetal Acceleration — In uniform circular motion, only the direction is changing; thus, acceleration is always and only pointing to axis of rotation

$$a_c = \frac{v^2}{r}$$

Definition — Reference Frames are a frame in which we look at the position, speed, and accelerations of an object. A special type of frame, an *inertial reference frame*, is one in which the reference frame is not accelerating; this means that all accelerating objects have the same acceleration in inertial reference frames.

Force and Newton's Laws

Newton's Laws of Motion — All these laws apply within an inertial reference frame, and are the basis of Classical Mechanics.

1. A body remains at rest, or in motion at a constant speed in a straight line, unless acted upon by an external force.
2. The net force acting upon an object is equal to the mass of the object times its acceleration ($\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$) – this law is sometimes other known as the net force is equal to the object's change in momentum.
3. For every action, there is a equal and opposite reaction.

Dynamical Analysis — While analyzing problems with Newton's second law, the following steps are generally needed:

1. Choose a suitable inertial reference frame, with orientation and positive directions for all involved axis.
2. For each object, draw a free body diagram (all the forces, with correct labels, acting on that body expressed as a particle, at its center of mass if a solid object)
3. Find the net force for each direction, and apply the equation as stated in Newton's second law to solve

Definition (Tension) — Tension $\vec{\mathbf{T}}$ is the force exerted through an ideal taut string; when pulling an object connected with a string, it is the tension within the string that is pulling the object.

Definition (Normal Force) — Normal force is the force perependicular to the surface, a contact force that is the reason why a book stays on a table even though its weight is pulling it down.

Atwood's Machine — An Atwood's machine is an arrangement in which two masses are hung straight down, connected with a massless string over a massless pulley. Writing down Newton's second law for each mass gives us

$$m_1 a = T_1 - m_1 g$$
$$m_2 a = m_2 g - T_2$$

and since it's the same string over a massless pulley, the string has equal tension throughout ($T_1 = T_2$). Solving this gives the common equations

$$a = \frac{m_2 - m_1}{m_2 + m_1} g \qquad \text{and} \qquad T = \frac{2m_1 m_2}{m_1 + m_2} g$$

Frictional Forces — Frictional forces oppose motion, and result from the collisions between microscopic contact points on two objects. In an ideal world, there will be no friction; however, due to friction, things like perpetual motion machines are impossible, as it's always present.
Static friction occurs when an object is static relative to another;

$$f_s \leq \mu_s N$$

When the force pushing on the object is greater than the maximum value of static friction, the object begins moving and experiences kinetic friction instead.
Kinetic friction occurs when an object is in motion relative to another;

$$f_k = \mu_k N$$

Kinetic friction is always less than or equal to static friction.
Notice that all frictional forces are multiplied by a constant, μ, which depends on a variety of factors; material, temperature, etc. The frictional forces are also proportional to the normal force.

Centripetal Force — Centripetal force is the force required to keep an object in circular motion. It's direction is changing: always radial.

$$F_c = ma_c = \frac{mv^2}{r}$$

NonInertial Frames and Psuedoforces — In noninertial frames, there exist psuedoforces (also called inertial or fictitious forces), which can't be seen by an observer in an inertial frame, but can by an observer in an inertial frame observing the same act. One famous one is called the *Coriolis* force, a force present in a rotating noninertial frame.

Limitations of Newton's Laws — Newton's laws can't be used with, among perhaps other things,

1. particles moving at speeds comparble to the speed of light (very fast)
2. extremely massive objects
3. and objects as small as atoms

Special relativity, General relativity, and Quantum mechanics are used instead for these cases; however, all of these, when taken in the case of our current conditions, reduce to Newton's laws.
Chaos is also something that cannot be fully explored with Newtonian mechanics; seemling chaotic behavior, where one tiny change is amplified, can have hidden order and patterns.

Momentum and Impulse

Momentum Formulas

Theorem (Newton's Second Law)

The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the direction of that force.

Linear Momentum —

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

$$\sum \vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}t} = ma$$

From the equations above, we have that

$$\begin{aligned} \sum \vec{\mathbf{F}} &= \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \implies \mathrm{d}\vec{\mathbf{p}} &= \sum \vec{\mathbf{F}}\,\mathrm{d}t \\ \implies \int_{\vec{\mathbf{p}}_i}^{\vec{\mathbf{p}}_f} \mathrm{d}\vec{\mathbf{p}} &= \int_{t_i}^{t_f} \sum \vec{\mathbf{F}}\,\mathrm{d}t \end{aligned}$$

Impulse and Momentum —

$$\begin{aligned} \vec{\mathbf{J}} &= \int_{t_i}^{t_f} \vec{\mathbf{F}}\,\mathrm{d}t \\ &= \Delta\vec{\mathbf{p}} \\ &= \vec{\mathbf{F}}_{avg}\Delta t \end{aligned}$$

Theorem (Impulse-Momentum Theorem)

The impulse of the net force acting on a particle during a given time interval is equal to the change in momentum of the particle during that interval.

Law of Conservation of Linear Momentum — When the net external force acting on a system is zero, the total linear momentum of the system remains constant.
valid in any inertial reference frame

this is a *spatial* symetrey – experiment done at one location is the same result as experiment done at anotehr location.

Collisions (2 bodies)

Center of Mass (CM) Frame Setting the v as $v'_{1i} = v_{1i} - v$ and $v'_{2i} = v_{2i} - v$, we get that the velocity of the cm frame is

$$v = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Definition — Collisons:

1. <u>Elastic</u>: bodies bounce off each other, with momentum unchanged in magnitude and reversed in direciton
2. <u>Inelastic</u>: bodies rebound with smaller momenta (usually due to internal friction)
3. <u>Completely Inelastic</u>: Bodies stick together after collision; at rest in the cm frame
4. <u>Explosive</u>: With springs/bombs, bodies rebound with momenta larger than initial

Problems

Problem 1 (HRK Chapter 6 Problem 6) — A very flexible uniform chain of mass M and length L is suspended from one end so that it hangs vertically, the lower end just touching the surface of a table. The upper end is suddenly released so that the chain falls onto the table and coils up in a small heap, each link coming to rest the instant it strikes the table. Find the force exerted by the table on the chain at any instant, in terms of the weight of chain already on the table in that moment.

Systems of Particles

When we want to calculate using newtons laws and such with objects, instead of particles, there is a special point in an object called teh *center of mass* which makes the analysis simple, and same as a normal particle.

Two Particle Systems

In two particle systems, the center of mass can be thought of as a sort of "weighted average", as I like to call it.

Center of Mass 2 Particle System — If we let $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$ denote the positions of m_1 and m_2 at a specific instance in time, then the location of the center of mass would just be

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2}$$

We can also write, through the equation above,

$$\begin{aligned}\vec{\mathbf{v}}_{\text{cm}} &= \frac{\mathrm{d}\vec{\mathbf{r}}_{\text{cm}}}{\mathrm{d}t} \\ &= \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}\end{aligned}$$

and

$$\begin{aligned}\vec{\mathbf{a}}_{\text{cm}} &= \frac{\mathrm{d}\vec{\mathbf{v}}_{\text{cm}}}{\mathrm{d}t} \\ &= \frac{m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2}{m_1 + m_2}\end{aligned}$$

Theorem

Much like Newton's second law, we can ignore internal force and write

$$\sum \vec{\mathbf{F}}_{\text{ext}} = (m_1 + m_2)\vec{\mathbf{a}}_{\text{cm}}$$

With the generalization of two particles as multiple particles, teh same concept holds.

Multiple Particle Systems — With n different particles, the following equations describe teh cm relationships.

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + \cdots + m_n\vec{\mathbf{r}}_n}{m_1 + m_2 + \cdots + m_n} = \frac{1}{M}\sum m_n\vec{\mathbf{r}}_n$$

$$\vec{\mathbf{v}}_{\text{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \cdots + m_n\vec{\mathbf{v}}_n}{m_1 + m_2 + \cdots + m_n} = \frac{1}{M}\sum m_n\vec{\mathbf{v}}_n$$

$$\vec{\mathbf{a}}_{\text{cm}} = \frac{m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 + \cdots + m_n\vec{\mathbf{a}}_n}{m_1 + m_2 + \cdots + m_n} = \frac{1}{M}\sum m_n\vec{\mathbf{a}}_n$$

$$\sum \vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{cm}}$$

Summary: The overall translational motion of a system of particles can be analyzed using newton's laws as if all the mass were concentrated at the center of mass and the total external force were applied at that point.

Center of Mass of Solid Objects

Dividing a solid object into tiny elements of mass δm gives us the cm as

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{1}{M}\sum m_n \delta m_n$$

Now, taking the limit as δm goes to 0 turns this into an integral:

CM Of Solids — The center of mass of a solid object is

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{1}{M}\int \vec{\mathbf{r}}\,\mathrm{d}m$$

Given a density function $\rho = \frac{\mathrm{d}m}{\mathrm{d}V}$, we can also write (holds for 1 and 2 dimensions too)

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{1}{M}\iiint \vec{\mathbf{r}}\rho\,\mathrm{d}V$$

Conservation of Momentum in a System of Particles

With constant mass, the following holds.

Momentum — The total momentum of a system of particles is equal to the product of the total mass of a system and the velocity of its center of mass.

$$\vec{\mathbf{P}} = M\vec{\mathbf{v}}_{\text{cm}}$$

The same relationships can also be written, such as by takign teh derivative, we have

$$\sum \vec{\mathbf{F}}_{\text{ext}} = \frac{\mathrm{d}\vec{\mathbf{P}}}{\mathrm{d}t}$$

However, if mass is constantly changing, we have to approach the problem differently.
We look at a system

Rotation

Rotational Kinematics

Definition — A rigid body moves in pure rotation if every point of the body moves in a circular path. The centers of these circles must lie on a common straight line called the axis of rotation.

Angular Units —

$$1 \text{ revolution} = 2\pi \text{ radians} = 360^\circ$$

Rotational Variables —

$$\phi = \text{angular position}$$
$$\omega = \text{angular velocity}$$
$$\alpha = \text{angular acceleration}$$

Note. All following angular equations are just the basic motion equations but with position replaced by *angular* position, velocity replaced with *angular* velocity, and acceleration replaced with *angular* acceleration.

Angular Relationships —

$$\omega_{\text{avg}} = \frac{\Delta\phi}{\Delta t} \qquad \vec{\omega}_{\text{inst}} = \frac{\mathrm{d}\phi}{\mathrm{d}t}$$
$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} \qquad \vec{\alpha}_{inst} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}^2\phi}{\mathrm{d}t^2}$$

Finite angular displacements cannot be represented as vector quantities, whereas *infinitesimal angular displacements* can be represented as vectors.

This is due to the transitive property ($\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$) not holding unless the angle changes are infinitesimal, resulting in the equality $\mathrm{d}\phi_1 + \mathrm{d}\phi_2 = \mathrm{d}\phi_2 + \mathrm{d}\phi_1$. This implies that angular velocity and angular acceleration are vectors.

Constant Angular Acceleration —

$$\omega = \omega_0 + \alpha t$$
$$\phi = \phi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha\Delta\phi$$
$$\phi = \phi_0 + \frac{\omega + \omega_0}{2} t$$

Relationships between Linear and Angular Variables — All the tangential linear variables are related the same way to their angular counterparts.

$$\begin{aligned} s &= \phi r \\ v_{\mathrm{T}} &= \omega r \\ a_T &= \alpha r \\ a_{\mathrm{R}} &= \frac{v^2}{r} = \omega^2 r \end{aligned}$$

Vector Relationships —

$$\vec{\mathbf{v}} = v_{\mathrm{T}} \hat{\mathbf{u}}_\phi$$

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\mathrm{T}} + \vec{\mathbf{a}}_{\mathrm{r}}$$

Proof. If we consider the radial and tangential unit vectors, $\hat{\mathbf{u}}_r$ and $\hat{\mathbf{u}}_\phi$, where the radial points outwards along the radius and the tangential points tangent to a circular path.
Now, we can write

$$\begin{aligned} \hat{\mathbf{u}}_r &= \cos(\phi)\hat{\mathbf{i}} + \sin(\phi)\hat{\mathbf{j}} \\ \hat{\mathbf{u}}_\phi &= -\sin(\phi)\hat{\mathbf{i}} + \cos(\phi)\hat{\mathbf{j}} \end{aligned}$$

For the velocity equation, since the velocity only involves the radial component, it is as follows.
For the acceleration, we have that

$$\vec{\mathbf{a}} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\mathrm{d}(v_{\mathrm{T}}\hat{\mathbf{u}}_\phi)}{\mathrm{d}t} = \frac{\mathrm{d}v_{\mathrm{T}}}{\mathrm{d}t}\hat{\mathbf{u}}_\phi + v_{\mathrm{T}}\frac{\mathrm{d}\hat{\mathbf{u}}_\phi}{\mathrm{d}t}$$

and since

$$\begin{aligned} \frac{\mathrm{d}\hat{\mathbf{u}}_\phi}{\mathrm{d}t} &= -\frac{\mathrm{d}(\sin(\phi))}{\mathrm{d}t}\hat{\mathbf{i}} + \frac{\mathrm{d}(\cos(\phi))}{\mathrm{d}t}\hat{\mathbf{j}} \\ &= -\omega[(\cos(\phi))\hat{\mathbf{i}} + (\sin(\phi))\hat{\mathbf{j}}] \\ &= -\omega\hat{\mathbf{u}}_r \end{aligned}$$

we have that

$$\begin{aligned} \vec{\mathbf{a}} &= \frac{\mathrm{d}v_{\mathrm{T}}}{\mathrm{d}t}\hat{\mathbf{u}}_\phi + v_{\mathrm{T}}\frac{\mathrm{d}\hat{\mathbf{u}}_\phi}{\mathrm{d}t} \\ &= a_{\mathrm{T}}\hat{\mathbf{u}}_\phi - v_{\mathrm{T}}\omega\hat{\mathbf{u}}_r \\ &= a_T\hat{\mathbf{u}}_\phi - \frac{v_{\mathrm{T}}^2}{r}\hat{\mathbf{u}}_r \\ &= \vec{\mathbf{a}}_{\mathrm{T}} + \vec{\mathbf{a}}_{\mathrm{R}} \end{aligned}$$

□

Rotational Dynamics

Note: Only considering cases in which the rotational axis is fixed in direction.

Torque —

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

Energy

Work and Kinetic Energy

Energy 2

Energy 3

Gravitation

Newton's Law of Universal Gravitation — The magnitude of the gravitational force that two particles of masses m_1 and m_2 seperated by a distance r exert on each other is

$$F = G\frac{m_1 m_2}{r^2}$$

where G, the gravitational constant, is approximately

$$G = 6.67 \cdot 10^{-11}\mathrm{m}^3/\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$$

Calculating the Value of G
The value of G has so far been approximated using a torsional method; two small lead balls are connected in the center with a light rod, then suspended with a fine fiber. Two large lead balls are placed such that the gravitational force gives a net torque which rotates; when the restoring torque from the fiber rotates the small 'dumbbell' back, this angle of 2θ is measured by observing teh deflection of a beam of light. From the torsional constant and the value of θ, the gravitational force can be determined.

Theorem (Shell Theorem 1)
A uniformly dense spherical sheel attracts an external particle as if all the mass of the sheel were concentrated at its center.

Theorem (Shell Theorem 2)
A uniformly dense spherical sheel exerts no gravitational force on a particle located anywhere inside it.

These two theorems help simplify the gravitational force analysis that are *spherically symmetric*.

Gravitational Force is conservative; this can be proven by showing that

$$\oint \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}} = 0 \qquad \text{where} \qquad \vec{\mathbf{F}} = -\frac{Gm_1 m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

Gravitational Work — The work done from a particle of mass M on a particle of mass m as it moves from a to b is given by

$$W_{ab} = \int_{r_a}^{r_b} \frac{GMm}{r^2}\,\mathrm{d}r = GMm\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = -\Delta U$$

where r is the distance from M to m. This can be written since the force is conservative.

Gravitational Potential Energy — The total potential energy would then be the difference between when potential energy at distance r, to when potential energy is 0, making it

$$U = -\frac{GMm}{r}$$

Escape Speed — The escape speed v of a particle is given by

$$v = \sqrt{\frac{2GM}{R}}$$

as the total final energy (0) has to equal the initial KE summed with the initial Gravitational PE.

Gravitational Work and Energy with Multiple Particles

The total potential energy can be thought of as equal to the work done to assemple the system, starting from infinite seperation. Let's take the case of 3 masses, where there exist mass m_1, m_2, and m_3.
Moving m_1 to its spot takes 0 work; moving m_2 to its spot takes $\frac{Gm_1m_2}{r_{12}}$ amoutn of work, and moving m_2 to its spot takes $\frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}$ amount of work, making the total potential energy equal to

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right)$$

Similarly, to seperate and isolate the system again, it would need a similar amount of energy. This is generalizable to any amount of particles.

Keplers Laws — Keplers Laws of motion, studied from the motion of planet Mars, are

1. *The Law of Orbits*: All planets move in elliptical orbits having the Sun at one focus.
2. *The Law of Areas*: A line joining any planet to the Sun sweeps out equal areas in equal times.
3. *The Law of Periods*: The square of the period of any planet about the Sun is proportional to the cube of the planet's mean distance from the Sun, or

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Let the semimajor axis a of an elliptical orbit denote the furthest the edge of the ellipse is to its center; and let the eccentricity be a number e such that $e \cdot a$ is the distance from one foci to the center.
The maximum distance of the orbiting body from the central body is commonly indicated with the prefix *apo-* (equal to $a(1+e)$), with the minimum distance being *peri-* (equal to $a(1-e)$).

Energy within Orbits — The total energy of a body of mass m orbiting about a body of mass M is given by

$$\begin{aligned} E &= K + U \\ &= \left(\frac{1}{2}m\omega^2 r^2\right) + \left(-\frac{GMm}{r}\right) \\ &= \left(\frac{GMm}{2r}\right) - \frac{GMm}{r} \\ &= -\frac{GMm}{2r} \end{aligned}$$

If enough kinetic energy is given, when $E = 0$ the orbit is parabolic, and when $E > 0$ the orbit is hyperbolic. The equation above is also valid for elliptical orbits, replacing r with a (notice that such means the energy doesn't depend on eccentricity).

Fluids

Fluid Statics

Pressure —

$$\Delta\vec{\mathbf{F}} = p\,\Delta\vec{\mathbf{A}}$$

The unit for pressure, $\mathrm{N/m^2}$, is commonly just denoted by a Pascal (Pa).
Pressure at sea level, or atmospheric pressure, is approximately

$$1\,\mathrm{atm} = 1.01325 \times 10^5\,\mathrm{N/m^2} = 14.7\,\mathrm{lb/in^2}$$

Density — An object with mass m and volume V has a density of

$$\rho = \frac{m}{V} \quad (\mathrm{kg/m^3})$$

The density of water is commonly regarded as $1\,\mathrm{kg/m^3}$.

Bulk Modulus — When we increase the pressure on a material by an amount Δp, its volume will decrease by a fractional amount ΔV, defined by its Bulk Modulus

$$B = -V\left(\frac{\Delta P}{\Delta V}\right)$$

A small Bulk Modulus leads an object to become more compressible, and vice versa.

Hydrostatic Pressure — For a homogenous incompressible fluid (constant density), we can write

$$\mathrm{d}w = g\,\mathrm{d}m = g\rho\,\mathrm{d}V = g\rho A\,\mathrm{d}y$$

for a ring of height $\mathrm{d}y$ in the fluid. As this fluid is static, through Newton's law,

$$\begin{aligned} 0 &= (p + \mathrm{d}p)A - pA - \rho g A\,\mathrm{d}y \\ \implies \frac{\mathrm{d}p}{\mathrm{d}y} &= -\rho g \end{aligned}$$

Solving this differential equation gives us

$$p_2 - p_1 = -\rho g(y_2 - y_1)$$

The quantity ρg is also referred to as the *weight density*, or the weight per unit volume.

Variation of Pressure in the Atmosphere — At great distances, the pressure in gasses becomes significant. It is a reasonable assumption that their desnity ρ varies proportionally to hte pressure as they are compressible.
note: look at ideal gas law in ch22

$$\begin{aligned} \frac{\mathrm{d}p}{\mathrm{d}y} &= -\rho g = -\rho_0\left(\frac{p}{p_0}\right)g \\ \implies p &= p_0 e^{-(g\rho_0/p_0)h} \end{aligned}$$

Pascal's Principle

Pressure is applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

Note that this principle is used as leverage in applications such as the hydraulic lever; in which a small force applied to a small area on one side corresponds with a larger force (albeit to a larger area) on another side.

Archimedes' Principle

A body wholly or partially immersed in a fluid is buoyed up by a force equal in magnitude to the weight of the fluid displaced by the body.

Definition — A buoyant force, caused by the pressure difference bewteen the bottom of an object and the top, can be regarded as acting at the center of gravity of the fluid displaced by the submerged part of the floating object. This point is known as the *center of buoyancy*.

Pressure Measurements

There are two types of pressure measurements, *absolute pressure* and *gauge pressure*.

Absolute pressure, or actual pressure at a point, could be measured with a mercury barometer (used due to its large density) which creates a pressure difference between a negligible pressure and the pressure onto a dish – commonly used to measure atmospheric pressure.

Gauge pressure is the pressure difference between one end and another. Commonly, one end is held at atmospheric pressure, and the other end in a tank with the pressure p which we seek to find.o

Surface Tension — later

Fluid Dynamics

There are general characteristics of fluid flow:

- Fluid flow can be steady or nonsteady
- Fluid flow can be compressible or incompressible
- Fluid flow can be viscous or nonviscous
- Fluid flow can be rotational or irrotational

An ideal fluid (what is focused on here) is steady, incompressible, nonviscous, and irrotational.

Definition — Since, in steady flow, the velocity $\vec{\mathbf{v}}$ at a given point P is constant in time, the motion of every particle passing through P follows the same path, called a *streamline*.

Definition — We can select a finite number of streamlines to form a sort of "bundle", called a *tube of flow* – since streamlines can't cross over, this tube of flow behaves somewhat like a pipe.

Mass Flux — Mass Flux is the mass of fluid per unit time passing through any cross section, defined by

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \rho A v$$

for a cross section of mass A_1 where fluid of speed v_1 and density $\rho$is flowing through

Equations of Continuity — Considering a tube of flow, which has no sources (where fluid enters), or sinks(where fluid leaves), mass flux will be equal throughout the tube, giving the equation

$$\rho A v = \text{ constant}$$

Assuming the fluid is incompressible, we can further write the volume flux as

$$Av = \text{ constant}$$

throughout the tube. These are the *equations of continuity*.

Bernoulli's Equation —

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{ constant (along a streamline)}$$

Notice that Bernoulli's Equation is really just the conservation of energy – to prove the equation, we can use a conjunction of the change in work and the change in energy, converting everything to pressure and density instead of forces and mass, and set them equal.

What exactly are these three types of pressure?
So Bernoulli's equation essentially involves three different types of pressure: *static pressure*, *dynamic pressure*, and *hydrostatic pressure* (in order, with p, $\frac{1}{2}\rho v^2$, and $\rho g y$).

- **Static Pressure** can be thought of as the real pressure a body in the fluid experiences; it is the pressure the fluid exerts as it is moving along a streamline at a constant speed or at rest.
- **Dynamic Pressure** is the pressure caused by the moving part of the liquid; imagine being pushed along by a river, sweeping you away. That's the dynamic pressure doing that.
- **Hydrostatic Pressure** is basically static pressure, and it is the pressure caused by the weight of the column of fluid above. It has minor differences, such as how static pressure could be different from hydrostatic, and so on.

The sum of Static and Dynamic pressure is something called **Stagnation Pressure**; notice how Hydrostatic pressure isn't the "real" pressure, and it is already included in the static pressure, which is why it's not being summed.

Oscillations

Frequency, or number of cycles per unit time, is f; period is time it takes for one complete cycle, and is T.

$$f = \frac{1}{T}$$

Simple Harmonic Oscillator — Assume that a particle is subject to a force $F = -kx$.
The equations of motion are

$$\begin{aligned} x &= x_m \cos(\omega t + \phi) \\ v_x &= \frac{\mathrm{d}x}{\mathrm{d}t} = -\omega x_m \sin(\omega t + \phi) \\ a_x &= \frac{\mathrm{d}v_x}{\mathrm{d}t} = -\omega^2 x_m \cos(\omega t + \phi) \end{aligned}$$

We also have that

$$\omega^2 = \frac{k}{m} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

The PE and KE at a point x will be given as

$$\begin{aligned} U &= \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi) \\ K &= \frac{1}{2}mv_x^2 = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi) \qquad \text{(after substituting } m\omega^2 = k\text{)} \end{aligned}$$

Torsional Oscillator — Assume that a restoring torque $\tau_z = -\kappa\theta$ is acting upon a disk.
The equation for motion is

$$\theta = \theta_m \cos(\omega t + \phi)$$

The oscillation is described as

$$\omega^2 = \frac{\kappa}{I} \qquad T = 2\pi\sqrt{\frac{I}{\kappa}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$$

Notice that it is similar to before, with $x \to \theta$, $k \to \kappa$, and $m \to I$.

Simple Pendulum — Suppose a particle is suspended by a light inextesnible cord.
Writing out the restoring torque and using small angle approximation, we have

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

For angles greater than around 20°, the general equation (after solving the diffeq with Taylor) becomes

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{2^2}\sin^2\left(\frac{\theta_m}{2}\right) + \frac{3^2}{2^2 4^2}\sin^4\left(\frac{\theta_m}{2}\right) + \cdots\right)$$

Physical Pendulum — Suppose a rigid body is mounted such that it can swing in a vertical plane (generalization of simple pendulum).
The period is given by

$$T = 2\pi\sqrt{\frac{I}{mgd}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

Note that the simple pendulum is but a special case of $d = L$ and $I = mL^2$.
The center of oscillation occurs at

$$L = \frac{I}{Md}$$

Definition — Define the *center of oscillation* to be the location for where the resulting simple pendulum with the mass concentrated at this point is the same as the original physical pendulum.

This point has an interesting property, where if an impulse is applied upon this point, there is no force on the pivot (try writing newton's second for both rotational and translational about the center of mass). This is what the "sweet spot" is in a baseball bat, and it's also called the *center of percussion*.

Harmonic and Circular Motion
Simple harmonic motion can be described as the projection of uniform circular motion along a diameter of the circle (aka uniform circular motion viewed sideways). Note that we can find the relationships $v_x = \omega r$ and $a_x = -\omega^2 r$, which is present in both circular and harmonic motion (at the point when $r = x_m$).

Damped Harmonic Motion — With friction forces acting on the oscillator, there occurs a loss in amplitude called *damping*.
Writing Newton's second law gives us (assuming the damping constant b gives the drag force as $-bv_x$)

$$ma = -kx - bv_x$$
$$\implies m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -b\frac{\mathrm{d}x}{\mathrm{d}t} - kx$$

giving the solution for motion as

$$x = x_m e^{-bt/2m}\cos(\omega' t + \phi) \qquad \text{where} \qquad \omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

Notice that if $b = 0$, we have the equation for undamped motion – in most cases, we can estimate $\omega' = \omega$ as the damping force is weak, and if we write the lifetime as $\tau = \frac{2m}{b}$, we have the equation

$$x = x_m e^{-t/\tau}\cos(\omega t + \phi)$$

The mechanical energy decreases exponentially with time, with

$$E = \frac{1}{2}kx_m^2 e^{-2t/\tau}$$

Note that *critical damping*, or when $b = 2\sqrt{km}$, results in $\omega' = 0$ meaning that the motion stops without oscillating, giving the shortest possible lifespan of $\tau = \frac{1}{\omega}$.

Forced Oscillations and Resonance — Consider an oscillator, with damping force ($-bv_x$) present. If a periodic force $F_m \cos(\omega'' t)$ is applied (with frequency ω''), we can write Newton's second law as

$$ma = -kx - bv_x + F_m \cos(\omega'' t)$$
$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -b\frac{\mathrm{d}x}{\mathrm{d}t} - kx + F_m \cos(\omega'' t)$$

and solving this differential equation gives the solution (after the initial chaotic transients finish)

$$x = \frac{F_m}{G}\cos(\omega'' t - \beta)$$

where

$$G = \sqrt{m^2\left(\omega''^2 - \omega^2\right)^2 + b^2\omega''^2} \qquad \text{and} \qquad \beta = \cos^{-1}\left(\frac{b\omega''}{G}\right)$$

The motion in the "steady state" after the periodic force is applied has a frequency equal to that of the driving force; the amplitude is dependent upon both the damping magnitude and the difference between the driving frequency ω'' and the natural frequency ω.

At *resonance*, $\omega'' = \omega$ (occasionally there's other definitions). For large dampings, the amplitude increases more rapidly as this condition is approached, than with less dampings (which is useful in application such as radios, which want accurate frequencies).

It should be noted that in the steady state, the energy provided by the damping force is matched with that of the damping force; it is oscillating with constant amplitude.

Two-Body Oscillations — When two objects are oscillating together, such as if connected with a spring, we can express it with a single body having a reduced mass

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

Proof. To find this, we first assume that m_1 is at position x_1 and m_2 is at position x_2 – this means that the length of the spring stretch is $x = (x_1 - x_2) - L$. We can then write (from hooke's law) newton's second law

$$m_1\frac{\mathrm{d}^2 x_1}{\mathrm{d}t^2} = -kx$$
$$m_2\frac{\mathrm{d}^2 x_2}{\mathrm{d}t^2} = kx$$

We can then derive that

$$\frac{m_1 m_2}{m_1 + m_2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}(x + L) = -kx$$

If we substitute

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

we can see that the equation is identical to that of SHM; thus, this system with reduced mass m has the same equations of motion as that of SHM (except it is *relative* motion in this case, which means $x = x_1 - x_2$). □

Waves

Wave Motion

A wave transports energy and momentum from one location to another without the actual particles making the journey.

Classification of Waves — Waves can have a variety of characteristics. A few of them include

1. Direction of Motion

   There include *transverse* waves, which move perpendicular to the motion of the particles, and *longitudinal* waves, which move along the direction of the motion of the particles. Waves, such as on the surface of water, could be a combination of both of these.

2. Number of Dimensions

   Waves could propogate in one, two, or three dimensions.

3. Periodicity

   Periodicity includes how the particles of the medium move in time; examples being one singular pulse, or a train of waves. An example of this is when each particle undergoes shm, in something claled a *harmonic* wave.

4. Shape of Wavefronts

   A *wavefront* is a surface composing of all the points after a wave is formed. A *ray* is a line normal to the wavefronts, indicating the direction of the motion of these waves. There are many types of waves, including *plane* waves and *spherical* wave, which define the shape of the wavefront.

Waves can be described with the function $f(x, t)$, describing the dependence upon both position and time.

Sinusoidal Waves — A transverse wave having a waveform with a sinusoidal shape can be represented with the equation

$$y(x,t) = y_{\mathrm{m}} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right)$$

where

$$y_{\mathrm{m}} = \textit{ Amplitude} \qquad \lambda = \textit{ Wavelength} \qquad T = \textit{Period}$$

Another form of this equation is

$$y(x,t) = y_{\mathrm{m}} \sin(kx - \omega t))$$

where

$$k = \textit{Angular Wave Number} \qquad \omega = \textit{Angular Frequency}$$

Unincluded in the equations, common wave characteristics also include

$$f = \textit{Frequency} \qquad v = \textit{Wave Speed}$$

Relationships — Note the resemblance to oscillation and uniform circular motion.

$$f = \frac{1}{T}$$

$$k = \frac{2pi}{\lambda} \qquad \omega = \frac{2\pi}{T} = 2\pi f$$

$$v = \lambda f = \frac{\lambda}{T} = \frac{\omega}{k}$$

Sinusoidal Wave Speed on a Stretched String — The speed of this wave depends upon the tension applied and the mass density. Suppose a pulse is traveling at speed v on a string with a force $\vec{\mathbf{F}}$ applied on and mass desnity μ.
If we look at a tiny section ∂l on a pulse, and draw a tiny angle theta on both sides, we can write (through centripetal force)

$$(\mu\, \delta l)\frac{v^2}{r} = (\delta m)a_y = 2F\sin(\theta) = F\frac{\delta l}{R}$$

Solving for v, we get

$$v = \sqrt{\frac{F}{\mu}}$$

These equations only hold for small amplitudes, where θ is tiny.

Definition — *Phase Speed* can be used to describe waves that preserve their shape as traveling, or purely sinsoidal waves. In other cases, the speed of the wave must be described with another speed, or *Group Speed*.

Definition — A pulse, if changes while traveling, is said to *disperse*. Most waves are dispersive; however, waves such as sound waves are approximately nondispersive, and light in a vaccume is perfeclty nondispersive.

The Wave Equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

describes the motion of a particle on a transverse wave in a long string under tension. While proving it is just an fbd analysis, what exactly does this equation mean?

As the particle is moving up and down, let's look at the meaning of the symbols within the equation. The lhs of the equation, $\frac{\partial^2 y}{\partial x^2}$, describes the concavity of the string around the particle, and the partial on the rhs of the equation, $\frac{\partial^2 y}{\partial t^2}$, describes the acceleration of the particle. Thus, this equation proportionally relates the concavity of the string around the particle to the particles acceleration; as the concvaity is positive, the particle will accelerate upwards (positive) and vice versa. Furthermore, the concavity and acceleration are directly proportional and relating to wave speed.

From this wave equation, it can also be shown (see proof1 or proof2) that the only solution to such a equation is when $y(x,t) = f(x \pm vt)$, or essentially the general equation for a traveling nonchanging wave.

Energy in Wave Motion — The energy of a tiny element of the string can be written as

$$\mathrm{d}K = \mathrm{d}U = \frac{1}{2}\mu\omega^2 y_{\mathrm{m}}^2 v \cos^2(kx - \omega t)$$

Both kinetic and potential energy is maximized when the element is at its maximum displacement, and minimized midway across its oscillating motion. Notice that the total energy

$$\mathrm{d}E = \mathrm{d}K + \mathrm{d}U$$

is nonconstant; neighboring elements are providing work onto this tiny element.
From this equation, the total energy associated with a wavelength is

$$K_\lambda = U_\lambda = \frac{1}{4}\mu\omega^2 y_{\mathrm{m}}^2 \lambda$$

$$E_\lambda = K_\lambda + U_\lambda = \frac{1}{2}\mu\omega^2 y_{\mathrm{m}}^2 \lambda$$

Power — Power, the rate at which mechanical energy is transmitted along the string, is

$$P = \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{1}{2}\mu\omega^2 y_{\mathrm{m}}^2 v$$

Notice that $P \propto f^2, y_m^2$.

Definition — The *intensity* of a wave is defined as the average power per unit area transmitted across an area A perpendicular to the direction in which the wave is traveling, or

$$I = \frac{P_{\mathrm{av}}}{A}$$

What happens when two waves "collide"?
Such as in an orchestra, there are many different sound waves reaching our ears; we can single certain ones out, while listening to all of them at the same time. This illustrates the *principle of superposition*, which states that when several waves combine at a point, the displacement of any particle at any given time is simply the sum of the displacements that each individual wave acting alone would give it. In other words, let $y_1(x,t)$ and $y_2(x,t)$ be two waves traveling; then the

$$y(x,t) = y_1 + y_2$$

would describe the displacement as those two waves combine. Later on, after seperation, the wave/pulse will move as though nothing had happened.
This leads to the fact that any periodic motion, no matter how random it may be, can be represented as a combination of simple harmonic motions (Fourier). This can be described with a Fourier series; or, if the motion is not periodic, it can be described with a Fourier integral.
The superposition principle does have limits, however – if one of the waves exceeds the elastic limit of the medium, then this principle will not hold.

Definition — *Constructive Interference* occurs when the amplitude of the resultant wave is equal to the sum of the individual amplitudes.

Definition — *Destructive Interference* occurs when the amplitude of the resultant wave is the difference between the individual amplitudes.

Definition — *Standing Waves*, a result of a combination of waves moving in opposite directions, move straight up in down. There are *nodes* (points along the standing wave which contain 0 displacement) and *antinodes* (points which move up and down at maximum displacement).
Note that energy is not transported along the string; instead, it is being shifted between elastic potential energy and kinetic energy.

Wave Transferring Between Different Mediums
Lets first look at two extreme cases, where waves are full on reflected. If a string is tied with both ends fixed, *a transverse wave undergoes a phase change of* $180°$ *(inverts).* If a string is tied with one free end, *a transverse wave is reflected without change of phase.*

However, in real situations, waves are reflected with loss of intensity (i.e looking at a window and seeing light transmitted through the glass and reflected back). Taking two strings tied together as an example, if a wave moves through a string with a smaller mass density to one with a greater mass density, the first case above happens, albeit with a smaller resulting amplitude. The opposite would result in the second case above happening, with a smaller resulting amplitude.

Another portion of the energy from the initial wave, that didn't get reflected back, would result in teh transmitted wave. The transmitted wave will have the same frequency as the original; however, the wave travels slower in the denser string (see eq above), and the wavelength is smaller too (see eq).

Standing Wave Resonances — Any system with standing waves has *natural frequencies*, which are called the *harmonics*. The *fundamental harmonic*, or the first harmonic, occurs when the standing wave satisfies $\lambda = 2L$, where L is the distance between two fixed points that the string in between is connected to. The *n-th harmonic* satisfies

$$n = \frac{2L}{\lambda} = \frac{2Lf}{v} = 2Lf\sqrt{\frac{\mu}{F}}$$

At natural freq, these standing waves can oscillate without any driving force or damping force.

Resonance occurs when a driving force, such as a motor or a persons hand, oscillates a tensioned string such that no work from the standing wave produced by the string is done onto the persons hand. Under no damping, these *resonant frequencies* are the same as the natural frequencies; however, then the amplitude would keep increasing until the elastic limit is reached. In real life circumstances, the resonant frequencies is a tiny bit less due to damping.

Sound Waves

Definition — Sound waves are a type of *mechanical wave*, which is a wave made of oscillating matter, transferring energy only through a medium.

Referring to *Sound Waves*, typically specifically refers to longitudinal waves in the frequency range of 20 Hz to 20,000 Hz.

Definition — Variations in the desnity of air are caused by oscillations from traveling sound waves. The regions of high desnity are called *compressions*, and the regions of low desnity are called *rarefactions*.

Sound Wave Characteristics — A sound wave can be described in terms of the variation of density,

$$\rho = \rho_0 + \Delta\rho(x, t)$$

or the variation of pressure (more common)

$$p = p_0 + \Delta p(x, t)$$

Using the bulk modulus B of the medium, the density and pressure amplitudes can be related with

$$\Delta p_{\mathrm{m}} = B\left(\frac{\Delta\rho_{\mathrm{m}}}{\rho_0}\right)$$

Note that when using the Bulk Modulus for the medium, we must use the *adiabatic*, and not the *isothermal*, bulk modulus (two different numbers).
The *adiabatic* one portrays how as the medium is compressed, the temperature will increase; however, the increased temperature won't have time to spread to the cooler rarefactions, thus there is no heat transfer. The *isothermal* one, however, shows when heat can flow, thus leading to a common temperature.

Displacements — Letting the displacement of a single element be represented by the function s, the change in density is

$$\Delta\rho(x, t) = -\rho_0 \frac{\partial s}{\partial x}$$

and for sinusoidal waves, we can integrate to obtain

$$s = s_{\mathrm{m}} \cos(kx - \omega t) \qquad s_{\mathrm{m}} = \frac{\Delta\rho_{\mathrm{m}}}{k\rho_0} = \frac{\Delta p_{\mathrm{m}}}{kB}$$

By the same note, the velocity function is given by

$$u = u_{\mathrm{m}} \sin(kx - \omega t) \qquad u_{\mathrm{m}} = \frac{\omega\Delta\rho_{\mathrm{m}}}{k\rho_0} = v\frac{\Delta\rho_{\mathrm{m}}}{\rho_0} = v\frac{\Delta p_{\mathrm{m}}}{B}$$

Note that with multiple waves, this description of waves using displacements becomes difficult; superinposing pressures, which add as scalars, instead of displacements which are vectors, are more accurate, and it is the pressure difference that is detected.

Speed of Sound —

$$v = \sqrt{\frac{B}{\rho_0}}$$

The Bulk Modulus can be written as γp_0 in gas mediums, where γ is a constant called the *specific heat ratio*, often between 1.3 and 1.7. The speed of sound in air at 0° C is 331 m/s; it increases as temperature rises.

Power and Intensity — Power and Intensity can be found through pressure and the velocity of the sound wave, and is given by

$$P_{\text{av}} = \frac{A(\Delta p_{\text{m}})^2}{2\rho v} \qquad I = \frac{(\Delta p_{\text{m}})^2}{2\rho v}$$

Human hearing responds logarithmically to the intensity, so a new *sound level* is introduced:

$$SL = 10 \log \frac{I}{I_0} \text{ dB} \qquad I_0 = 10^{-12} \text{ (typical threshold for human hearing)}$$

Sound Wave Interference — If multiple sounds interfere with each other, then the total pressure disturbance at a certain point is the sum of teh individual pressure disturbances.
However, if we think about the *phase difference*, we find that at a specific snapshot in time, the difference is dependent upon the distance;

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda}$$

Note that this means that for a t

Standing Longtitudal waves —

Appendix