Problems

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Problem 1 (2009 FMA P25) — Two discs are mounted on thin, lightweight rods oriented through their centers and normal to the discs. These axles are constrained to be vertical at all times, and teh discs cna pivot frictionlessly on the rods. The discs have indentical thickness and are made of the same material, but have differeing radii r_1 and r_2. The discs are given angular velocities of magnitudes ω_1 and ω_2, respectivel, and brought inot contact at their edges. After the discs interact via friction it is found that both discs come exactly to a halt. Which of the following must hold? Ignore effects associated with the vertical rods.

(a) $\omega_1 r_1^2 = \omega_2 r_2^2$

(b) ...

Problem 2 (Irodov) — Three points are located at the vertices of an equilateral triangle whose side equals a. They all start moving simultaneously with velocity v constant in modulus, with the first point heading continually for the second, the second for the third, and the third for the first. How soon will the points converge?

Problem 3 (USAPHO) — A uniform pool ball of radius r and mass m begins at rest on a pool table. The ball is given a horizontal impulse of J of fixed magnitude at a distance βr above its center, where $-1 \leq \beta \leq 1$. The coefficient of kinetic friction between the ball and the pool table is μ. You may assume the ball and the table are perfectly rigid. Ignore effects due to deformation. (The moment of inertia about the center of mass of a solid sphere of mass m and radius r is $I_{\text{cm}} = \frac{2}{5} m r^2$).

a) Find an expression for the final speed of the ball as function of J, m, and β.

b) For what value of β does the ball immediately begin to roll without slip ping, regardless of the value of μ ?

Problem 4 (Kalda) — What is the minimum force needed to dislodge a block of mass m resting on an inclined plane of slope angle α, if the coefficient of friction is μ? Note that $0 < \alpha < \mu$.