EE100

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Contents

§1 Introduction

Simple Terminology —

$$q = \text{charge}$$

$$i = \text{current} = \frac{\mathrm{d}q}{\mathrm{d}t}$$

$$v = \text{voltage} = \frac{\mathrm{d}W}{\mathrm{d}q}$$

$$v \cdot i = \frac{\mathrm{d}W}{\mathrm{d}q} \cdot \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\mathrm{d}w}{\mathrm{d}t} = P$$

$$W = \int P \,\mathrm{d}t$$

From here, we can connect these physics concepts to simpler circuit concepts.

Gauss Law Capacitance — Note that we can use the gauss law to find

$$q = \epsilon \oint \vec{\mathbf{E}}\,\mathrm{d}\vec{\mathbf{A}} = \epsilon E A$$

$$V = E \cdot d \implies q = \epsilon \frac{v}{d} A$$

and after intergrating q to find i

$$i = C\frac{\mathrm{d}v}{\mathrm{d}t}$$

which means that voltage across a capacitator cannot change instananeously

Faradays Law Self Inductance — Using faraday's law, which is

$$V = \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

$$\mathrm{d}B = \frac{\mu i \cos(\alpha)}{4\phi\gamma^2}\,\mathrm{d}l$$

$$\phi = \oint \vec{\mathbf{B}}\,\mathrm{d}\vec{\mathbf{A}} = (\text{huge integral})\, i = Li$$

and through differentiating, we have

$$V = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

Current through inductor cannot change instaneously

Mutual Inductance —

$$V_1 = L_1\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

Transformers

Resistor — Electrons that go through a solid bounce around, loss of energy as bounce around is resistance.

$$\text{current density} = \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$$
$$\frac{L}{A} = \sigma\frac{V}{l}$$
$$v = i\left(\frac{l}{\sigma A}\right) = iR$$

These are all passive elements; don't make electricity by itself.
For these, we can make 3 assumptions:

1. **Linear**
2. **Bilateral**
3. **Lumped**

Linear, which means all these things, such as resistance and inducatnce, increase linearly, which is an assumption because not in real life might be a lil curved. This means anything linear differential equation.
Bilateral element, if

$$V = f(i)$$
$$-V = f(-i)$$

Note that things like diodes aren't bilateral element.
We also assume lump analysis, which means that the current is instaneously chanigng throughout, which is not ture in the real world because current still takes time to travel. In this course too, we assume that there's no interaction between electrical and magnetic fields.In real life, electromagnetic radiation lead to energy losses, which we ignore.

When analyzing circuits, assume directions for voltages/currents first!

Voltage Sources/Current Sources
Ideal voltage source, constant voltage for any current draw, but in real life voltage will drop with higher current draws.
Ideal current source, constant current for any voltage draw, but in real life current will drop with higher current draws.
Dependent or Controlled Sources These are all denoted with a **diamond** in the symbol.
Voltage controlled voltage source, which means that the voltage is controlled. There's also current controlled current source.
Voltage controlled current source, there's a g_m = transconductance, this is a model o fa transistor, and is very useful.
Current Controlled Voltage source.

§2 Circuit Analysis

Theorem (Kirchhoff's Voltage Law (KVL))
The total sum of all voltage changes in a closed-loop circuit is zero.

Theorem (Kirchhoff's Current Law (KCL))
Sum of all currents flowing into a junction of a circuit is equal to the sum of all currents flowing out of a junction.

With KCL, we can divide the circuit up into bigger systems, such as dividing it into two halves.
Following these laws, there cannot have a loop that contains only independent voltage sources, and you can't have two independent current sources connected to each other.
Steps for Kirchoff's Problems Solving for circuit analysis questions:

1. Find loops, define positives and negatives and current directions/loop directions
2. Write down independent KVL and KCL equations
3. Solve Linear System

This will get touched upon in more depth later, on how exactly to write KVL and KCL equations.

Note that all nodes that are connected with just plain wire can be merged together, could make thinking about things simpler.

Equivalent Circuits — Sometimes, circuits can be replaced with equivalents; this can make it easier for analysis.

Series	
Resistors	$R_{\text{eq}} = \sum R_1$
Capacitors	$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$
Inductors	$L_{\text{eq}} = \sum L_i$

Parallel	
Resistors	$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_i}$
Capacitors	$C_{\text{eq}} = \sum C_i$
Inductors	$\frac{1}{L_{\text{eq}}} = \sum \frac{1}{L_i}$

Dividers

Through these, we can see voltage/current dividers, where current is divided inversely propportional to resistances in parallel circuits, and voltages are divided inversely propportional to resistances.

$$V_i = \frac{VR_i}{R_{\text{tot}}}$$

$$I_i = \frac{IR_i}{R_{\text{tot}}}$$

Solving with Kirchhoffs

There are two ways to solve Kirchhoff's questions:

- **Loop Method**: KVL

  This is just writing KVL for a bunch of loops, that produce independent equations, and then solving for current values through that.

  Note that when using this loop method, we can choose loops that don't surround each other, that are sort of in their own "quadrant". We can also simplify circuit diagrams, merging nodes that connect to each other with nothing in between, to make diagrams easier and loops easier to identify.

- **Node Method**: KCL

  This is writing KCL for a bunch of nodes, with voltage drops across all the components with defined voltages on both sides.

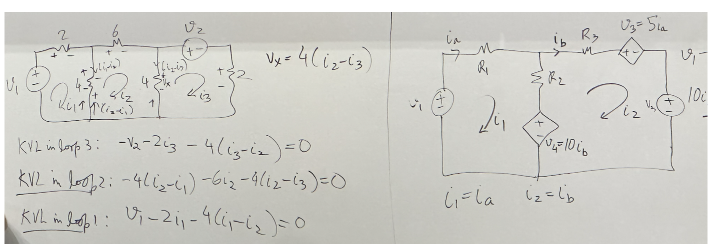
  Just like when doing the loop method, we can simplify and merge nodes that are just shorted with each other – this means that this method is preferred for diagrams that require a lot of loops to solve, but dont have as many nodes.

passive sign convention is when current is flowing across something its ALWYAS first voltage minus second voltage. its a sign convention adopted as common practice.

WRITING KVL

1. Define direction for loops
2. for passive elements like resistors, use KCL to find what the term for the loops are (whether pos or neg), and always write negative – for other elements, the second voltage minus the first voltage will dictate whether its positive or negative
3. write out kvl like that

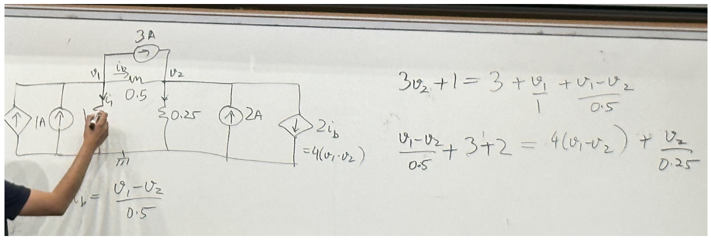
see the following image:

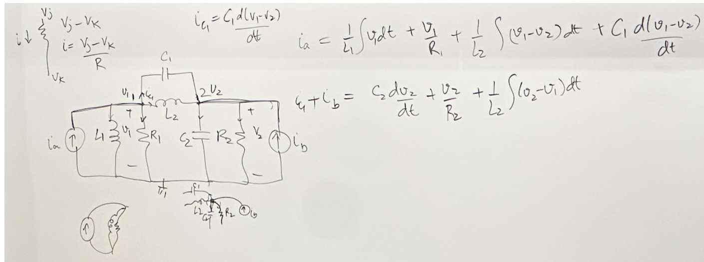


WRITING KCL

1. Define arbitrary v_n values at each node in the circuit
2. Write the currents goign in and out of each non-grounded node to the next node/ground

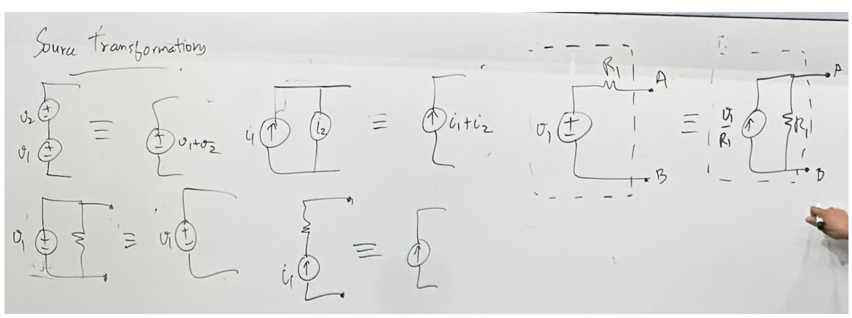
Note that for n amount of nodes, we always ground one, so there will be $n - 1$ amount of equations necessary. The following are examples:





> **Source Transformations** — Common Transformations are, for
>
> - **Two Voltage in Series** is equivalent to one voltage source that's the sum
> - **Two Current in Parallel** is equivalent to one parallel that's the sum
> - **Resistor in parallel with Voltage** can ignore the resistor, because what's happening afterwards doesn't matter wiht the resistance as its in parallel so voltage doesnt change
> - **Resistor in series with Current** as the resistance does nothing to diminish the current, so can be ignored
> - **Voltage in series with Resistor** is equivalent to a current source of $\frac{v}{R}$ in parallel with the resistance

Picture of these visualized:

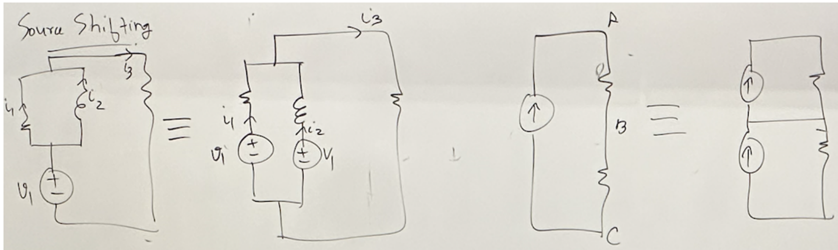


These can simplify circuits to solve for things before using KVL/KCL.

> These can come in very handy, especially when writing KVL and KCL, to convert all the sources to either current sources or voltage sources.

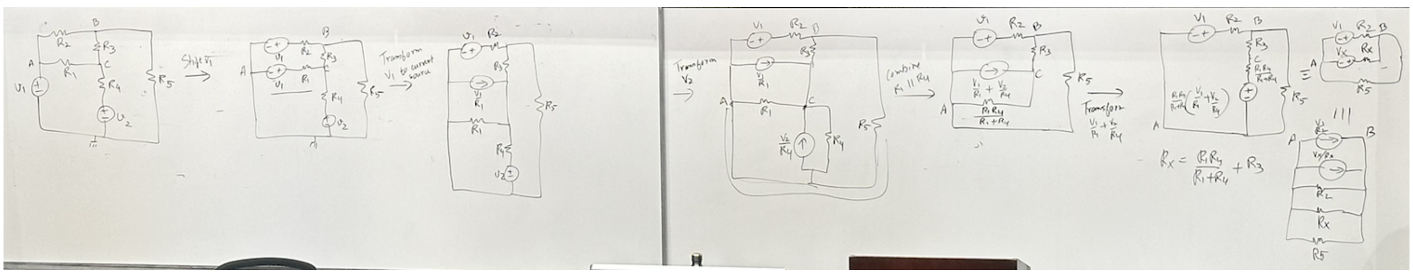
> **Source Shifting** — This is basically shifting a source into seperate parts kind of. These are less commonly used than the source transformations, but can be useful to reposition sources so they can be analyzed easier/transformed easier.

Picture of these:



> **Source Transformations/Shifting**
> The whole point of this whole thing is to get rid of elements, or make things easier to analyze with KCL/KVL. This is the philosophy, solve problems while thinking like this.

Example problem:



Main takeaways from this, voltage + resistor in parallel conver tto current and resistor in series, and try to find things in parallel and series to combine resistor values together. How far you go with this depends on what you're asked to find.

> Note that with source transformations, things like power and current aren't guranteed to be the same INSIDE the source transformation and only stay the same to outside the "black box" which the transformation changed.

Corollaries from Source Transformations

1. Elements in series with a current source or i parallel with a voltage source can be removedwithout affecting rest of circuit
2. Node method, try to convert to current sources
3. Loop method, try to convert to voltage sources

§3 Coupled Inductors

Common case is *transformer*, which transforms one voltage to another voltage.

Dot Convention — Current flowing into one dot induces a + voltage at the other dot. If dots are same side:

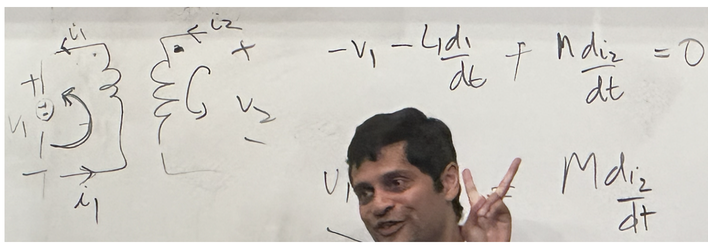
$$v_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

If dots are opposite side:

$$v_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} - M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

L is self inductance, M is mutual inductance.

Every single time you draw two inductors, you have to draw two dots on the circuit – this will mean that the side with the dot is positive voltage. This is showing how based off the current loop direction, voltage and self



inductance oppose so are negative, and mutual inductance increases it.

Transformers — Assumptions:

- $\phi_1 = \phi_2$ (perfect coupling)

Note that because

$$v_1 = N_1\frac{\mathrm{d}\phi_1}{\mathrm{d}t} \qquad v_2 = N_2\frac{\mathrm{d}\phi_2}{\mathrm{d}t}$$

we have that by the assumption,

$$\frac{v_2}{v_1} = \pm\frac{N_2}{N_1} = \pm n \qquad \text{Perfect Transformer}$$

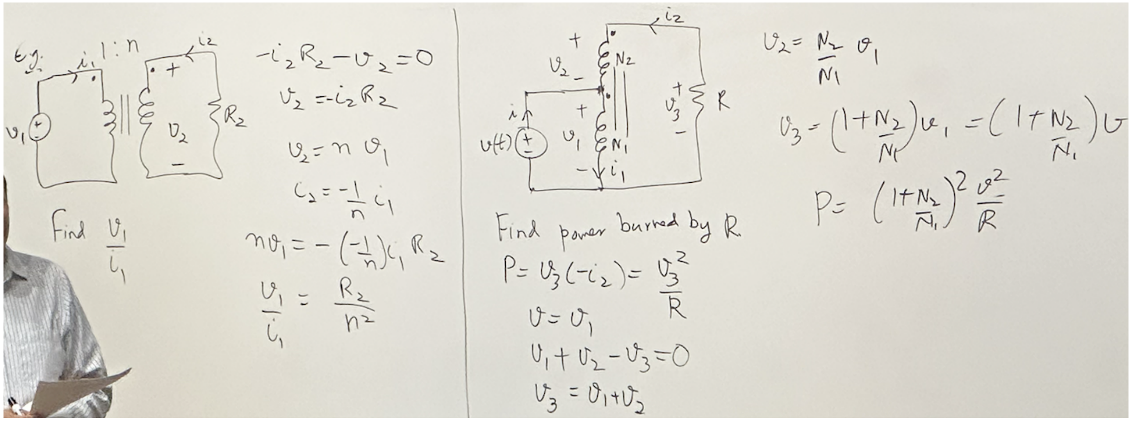
It's positive if i_1, i_2, are both leaving or entering the dot, negative if oen is leaivng and the other is entering. Note that too, because of this assumtpion there is no power loss, so we have

$$v_1 i_1 + v_2 i_2 = 0$$

$$\frac{i_2}{i_1} = \mp\frac{1}{n}$$

If $n > 1$, step up transformer ($v_2 > v_1$), and $n < 1$ is step down transformer.
The symbol for the transformer is the two inductors on both sides, with the two parallel lines in between, and only really have to worry abuot voltage and current relationships, and turns ratio n.

Two examples shown below – note that its just writing out KVL/KCL and the inductance turn ratio relationship.



Note that with the second example, if we use KCL ($i + i_2 = i_1$) and the inductor relationship $i_2 = -\frac{1}{n}i_1$ to solve for the power supplied by the voltage source, we can find that it comes out to be the same thing – this is because **idealized inductors don't have power losses**, so the only power is burned by R.

§4 Linear Functions

A linear function is

$$f(x+y) = f(x) + f(y)$$
$$f(\alpha x) = \alpha f(x)$$

Note that we can treat any circuit as these sort of linear functions, where we can always end up writing it as

$$\begin{aligned} v_1 &= \sum \left(R + L\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{C}\int \mathrm{d}t \right) i_1 \\ v_2 &= \sum \left(R_2 + L_2\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{C_2}\int \mathrm{d}t \right) i_2 \\ \vdots &= \qquad \vdots \\ v_j &= \sum \left(R_{jk} + L_{jk}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{C_{jk}}\int \mathrm{d}t \right) i_k \end{aligned}$$

Note that because of this, we can write this as a matrix where its

$$\begin{pmatrix} v_1 \\ \vdots \\ v_j \end{pmatrix} = \begin{pmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nn} \end{pmatrix} = \begin{pmatrix} i_1 \\ \vdots \\ i_n \end{pmatrix}$$

Then, we can seperate and solve the individual parts – we can seperate as first assuming the rest of the voltages are 0, and sovling for one voltage at a time – shown below (assume Z is the matrix):

$$\begin{aligned} \begin{pmatrix} v_1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} &= Z \begin{pmatrix} i_{11} \\ i_{12} \\ i_{13} \\ \vdots \end{pmatrix} \\ \begin{pmatrix} 0 \\ v_2 \\ 0 \\ \vdots \end{pmatrix} &= Z \begin{pmatrix} i_{21} \\ i_{22} \\ i_{23} \\ \vdots \end{pmatrix} \\ \vdots \quad &= \quad \vdots \end{aligned}$$

And if you add up all them up, then you'll finally get that

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \end{pmatrix} = Z \begin{pmatrix} i_{11} + i_{12} + \cdots \\ i_{12} + i_{22} + \cdots \\ \vdots \end{pmatrix}$$

Where the right matrix will correspond with the final current matrix that you're trying to solve for.
Note that all of this is only made possible if the voltages and currents are linearly independent.

Theorem (Superposition Principle)
Response from all independent sources = sum of responses from sources one at a time. Dependent sources should NOT be zeroed out.

Note that this often leads to big simplifications that cna be done in the circuit – with voltages, you can seperate to voltages outside of the nodes that you're fidning, and inside, and solve like that and add up with superposition.

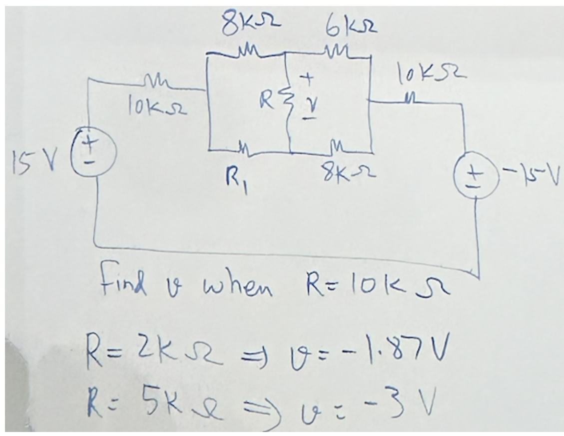
Note that linear elements include **resistors**, **voltages**, **currents**, **capacitators**, **inductors**, but NOT things like power. Superposition would only apply for linear elements.

Note that things like **Homogeneity** which is $f(\alpha x) = \alpha f(x)$ can help too – both of thsee just equate circuits to linear systems of equations.

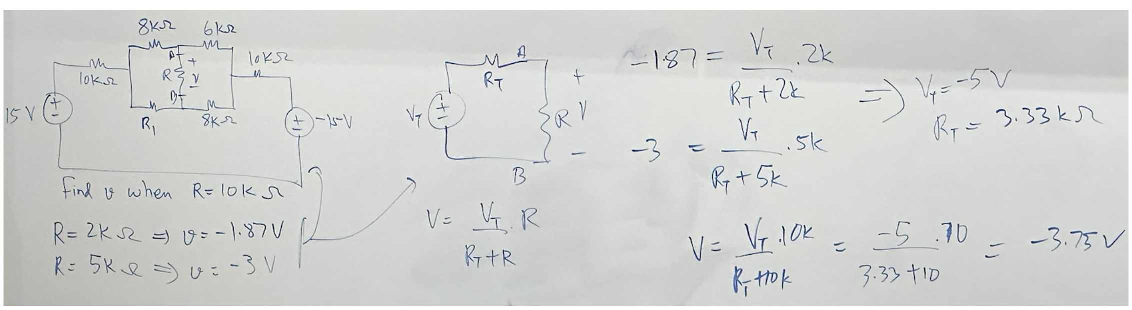
Do not kill the dependent sources.

Special Cases — You can solve things for edge cases, such as making one resistor 0 and also ∞, and then using circuit analysis to analyze.

We can do an example relating to this, which is this problem, which we're given values and want to find the hypothetical at another value: To do this, as shown below, we can draw a simplified equivalent circuit and with



the two values given, solve for the equivalents. With this simplified circuit, we can solve. These are all thermal



resistances that we're finding. TO LOOK INTO MORE
ok i asked him more about thermal resistances, this is what i got

Simplifying
A way to simplify a lot of circuits around an element that we want, is finding the equivalent resistance of everythign around it. How we can do this is

1. opening the circuit up between the two nodes which we awnt to find
2. removing all independent current sources/voltage sources
3. simplifying circuit

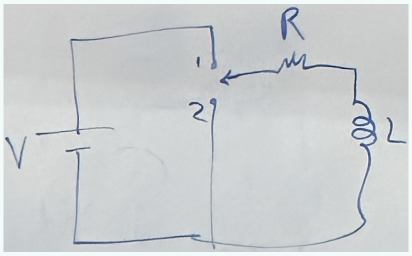
This'll give you the equivalent resistance as if there was only one resistor, the voltage/current sources, and the two things that we want to find. more on derivation of this, but apparently uses superposition principle.

Ok TODO i need to change thermal resistance to thevenin equivalents, i misheard him.

§5 Chapter 5

All this stuff is the stuff after the first midterm.

RL Circuit — This is with a switch to an RL circuit.

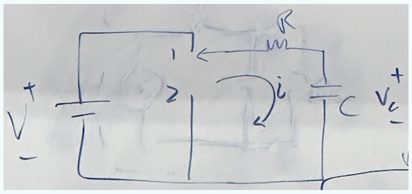


In steady state, L can eb replaced by a short, when switch is in one state, $i(0^-) = i(0^+) = ke^{-\frac{R}{L}t} = \frac{V}{R}e^{-\frac{R}{L}t}$ where $t = 0$, and this is derived through (switch in state 2)

$$0 = iR + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

This means that curent will decay slower is large $\frac{L}{R}$, and vice versa.

RC Circuit — This is with a switch to an RC circuit.



In this, at switch two, we have that in the steady state $i_{ss} = 0$ and $V = iR + V_c = V_C$ in the steady state. Whne the switch is flipped, $V_c(0^-) = V_c(0^+) = V$, and with teh equation

$$\frac{1}{C}\int i\,\mathrm{d}t + iR = 0$$

$$i = Ke^{-\frac{t}{RC}}$$

with KVL

$$V_c(0^+) + i_0 R = 0$$

$$i_0 = -\frac{V}{R} = K$$

which means our final equatin is similar before, being

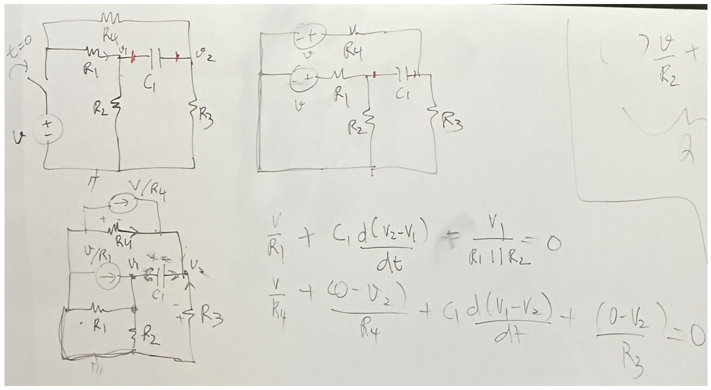
$$i = -\frac{V}{R}e^{-\frac{t}{RC}}$$

The current would decay slower with large RC, and vice versa.

Note that all these are just solving first order homogenous linear differential equations.
break ngl i missed the first 50 minutes of classes, but i think its something about how

$$i = i_{ss} + i_t = i_f + i_n$$

where current is equal to steady state + transient, and also forced resnance + natural **break** Solve this example with capicitators – since we want to use KCL, we wnat to turn the voltage sources into current sources, and then write KCL on the nodes. This is one way to solve:

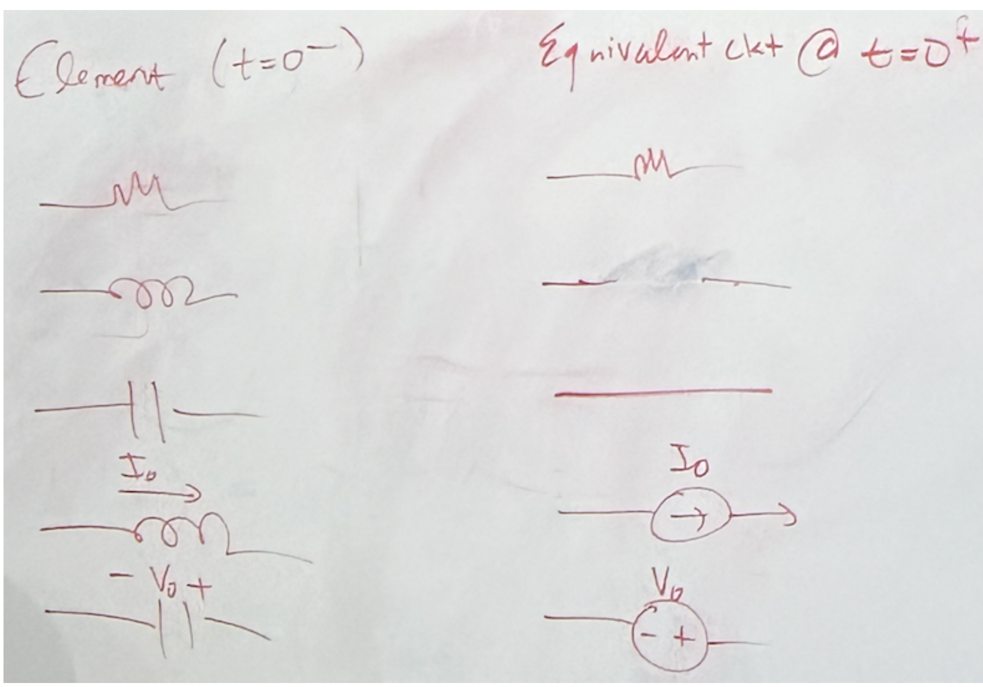


The next way to solve is with thermal equivalents – if we write the equivalent resistance, and then we simplify it it essentially comes out to be a much simpler circuit. oops forgot to take picture

Laws that are true about elements —

1. Resistor can change instantaneously
2. Current through inductor cannot change instantaneously
3. Voltage through capacitator cannot change instantaneously

Note the following elements and what they have (their equivalents) at $t = 0^-$ and $t = 0^+$.



RLC Circuits — In RLC Circuit, note that the equations are

$$V = iR + L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{C}\int i\,\mathrm{d}t$$

$$0 = R\frac{\mathrm{d}i}{\mathrm{d}t} + L\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + \frac{i}{C}$$

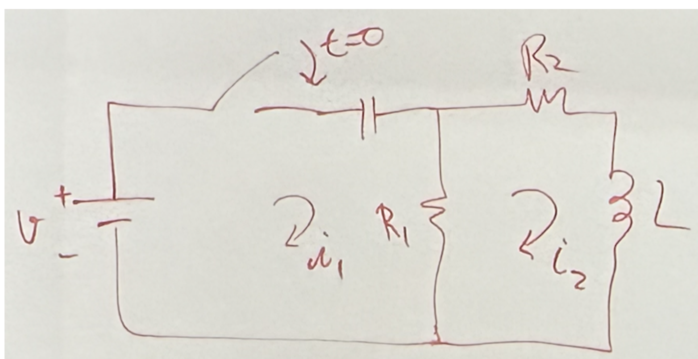
Note that at $t = 0^+$, we have that

$$\begin{aligned} & & & i(0^+) = 0 \\ V_c(0^+) = 0, \quad V = L\frac{\mathrm{d}i}{\mathrm{d}t}\Big|_{0^+} & \implies & & \frac{\mathrm{d}i}{\mathrm{d}t} = \frac{V}{L} \\ \implies \frac{RV}{L} + L\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} + \frac{0}{C} = 0 & \implies & & \frac{\mathrm{d}^2 i}{\mathrm{d}t^2} = -\frac{VR}{L^2} \end{aligned}$$

Evaluating equivalent circuits

Theorem (smthing calculating at infinity time)
Can calculate it only if all sources are constant or their values decay to a const or 0 at $t \to \infty$.

We can see an example of solving an RLC circuit wiht the following example Note that at $t = 0^+$, if the circuit



is closed at $t = 0$, we can make the capacitator a wire and open the inductor, yielding $i_1(0^+) = \frac{v}{R_1}$

We can see that calculating the current at 0^-, 0^+, and ∞ are the important parts of analysis with RL, RC, and RLC circuits. ESP with inductors and capictors, you have to know what's happenign before and after

This is the general concept for all these problems – you find the current/evalutae it at 0^-, and then you use this to evaluate it at 0^+. At 0^-, whether the switch is opoen or closed, it will differ from 0^+ because the capacitor/inductors.

Forcing Functions — Drivng a circuit with a input, it creates a forced response. Note that if we have a forcing function $v(t)$, we'll have

$$v(t) = v_f(t) + v_n(t)$$

where one part is the forced rsponse, and one part is the natural response. Note that the force response will have the same form as the forcing function. For the following forcing functions, the form of response will be the same

Forcing Function	**Form of Response**
V_A	k
$V_A e^{-\omega t}$	$ke^{-\omega t}$
$V_A \cos(\omega t), V_A \sin(\omega t)$	$a \cos(\omega t) + b \sin(\omega t)$