Assignment 3

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Download all python codes from

```
https://github.com/tyagio/AI1103/tree/main/
    assignment3/codes
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and latex-tikz codes from

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https://github.com/tyagio/AI1103/tree/main/
    assignment3/assignment3.tex
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1 Problem

Let X1,X2... be a sequence of independent and identically distributed random variable with

$$\Pr(X_1 = -1) = \Pr(X_1 = 1) = 1/2 \tag{1.0.1}$$

Suppose for the standard normal random variable Z,$\Pr(-0.1 \le Z \le 0.1)$=0.08 .
If $S_n = \sum_{i=1}^{n^2} X_i$, then $\lim_{x\to\infty} \Pr\left(S_n > \frac{n}{10}\right) =$

1) 0.42
2) 0.46
3) 0.50
4) 0.54

2 Solution

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{2}, & \text{if n= 1 or n=-1} \\ 0, & \text{otherwise} \end{cases}$$

the Z transform for X_i is given by

$$P_{X_i}(z) = \frac{1}{2}(z + z^{-1}) \tag{2.0.1}$$

$$\implies P_{S_n}(z) = \prod_{i=1}^{n^2} P_{X_i}(z) \tag{2.0.2}$$

$$\implies P_{S_n}(z) = \frac{1}{2^{n^2}} \sum_{i=1}^{n^2} \binom{n^2}{i} z^{n^2-i} z^{-i} \tag{2.0.3}$$

Now we can say that to find the the probability that $-n/10 \le S_n \le n/10$ is the same as finding the sum of all terms in $P_{S_n}(z)$ where coefficient of z is between -n/10 and n/10,WOLG we can say

$$\Pr\left(\frac{-n}{10} \le S_n \le \frac{n}{10}\right) = \frac{1}{2^{n^2}} \sum_{i=n^2/2-n/20}^{n^2/2+n/20} \binom{n^2}{i} \tag{2.0.4}$$

Using the approximation

$$\binom{n}{k} p^k q^{n-k} \simeq \frac{1}{\sqrt{2\pi npq}} \exp(-(k-np)^2)/(2npq)) \tag{2.0.5}$$

In our case $p = q = 1/2$

This holds for large n and hence as our n tends to infinity we can transform the summation into an integral which is given by

$$\frac{\sqrt{2}}{\sqrt{\pi n^2}} \int_{n^2/2-n/20}^{n^2/2+n/20} \exp(-(k - \frac{n^2}{2})^2)/(\frac{n^2}{2}))dk \tag{2.0.6}$$

$$\implies \frac{\sqrt{2}}{n\sqrt{\pi}} \int_{-n/20}^{n/20} \exp(-2k^2/n^2)dk \tag{2.0.7}$$

$$\implies \frac{1}{\sqrt{2\pi}} \int_{-1/10}^{1/10} \exp(-k^2/2)dk \tag{2.0.8}$$

This is just $G(0.1) - G(-0.1)$ which is given to us in the question as 0.08

$$\implies \Pr\left(\frac{-n}{10} \le S_n \le \frac{n}{10}\right) = 0.08 \tag{2.0.9}$$

(variable Z corresponds to standard normal random variable or $N(0,1)$.)
The probability distribution of S_n is symmetric about 0,hence $\Pr\left(S_n > \frac{n}{10}\right) = \Pr\left(S_n < \frac{-n}{10}\right)$

$$\implies 2 \times \Pr\left(S_n > \frac{n}{10}\right) + \Pr\left(\frac{-n}{10} \le S_n \le \frac{n}{10}\right) = 1 \tag{2.0.10}$$

$$\implies \Pr\left(S_n > \frac{n}{10}\right) = 0.92/2 = 0.46 \tag{2.0.11}$$

Hence final solution is options 2) or 0.46