Assignment 1

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Download all python codes from

https://github.com/tyagio/AI1103/tree/main/assignment1/codes

and latex-tikz codes from

https://github.com/tyagio/AI1103/tree/main/assignment1/assignment1.tex

1 Problem

Suppose that two cards are drawn at random from a deck of cards.Let X be the number of aces obtained.Then the value of E(X) is

1) 37/221
2) 5/13
3) 1/13
4) 2/13

2 Solution

Total number of cards =52 with 4 aces,48 non-ace's and we need to select 2 cards so X can be 0 ,1 or 2

Let $A \in \{0,1\}$ represent the random variable, where 0 represents first card being an non ace, 1 represents first card being ace.
Let $B \in \{0,1\}$ represent the random variable, where 0 represents second card being an non-ace, 1 represents second card being ace
$\Pr(A=0)$=48/52=12/13
$\Pr(A=1)$=4/52=1/13

Case 1: $X=0$

$$\implies \Pr(X=0) = \Pr(A=0,B=0)$$
$$= \Pr(A=0)\times\Pr(B=0|A=0)$$

there 51 cards and 4 aces left for 2nd draw

$$\Pr(X=0) = \frac{48}{52}\times\frac{47}{51} = 188/221 \tag{2.0.1}$$

Case 2: $X=1$

$$\implies \Pr(X=1) = \Pr(A=1,B=0)+\Pr(A=0,B=1)$$

$$\text{In}\Pr(A=1,B=0) = \Pr(A=1)\times\Pr(B=0|A=1)$$

there 51 cards and 3 aces left for 2nd draw

$$\Pr(A=1,B=0) = \frac{4}{52}\times\frac{48}{51} = 16/221$$

$$\text{In}\Pr(A=0,B=1) = \Pr(A=0)\times\Pr(B=1|A=0)$$

there 51 cards and 4 aces left for 2nd draw

$$\Pr(A=0,B=1) = \frac{48}{52}\times\frac{4}{51} = 16/221$$
$$\implies \Pr(X=1) = \frac{16}{221}+\frac{16}{221} = \frac{32}{221} \tag{2.0.2}$$

Case 3: $X=2$

$$\implies \Pr(X=2) = \Pr(A=1,B=1)$$
$$= \Pr(A=1)\times\Pr(B=1|A=1)$$

there 51 cards and 3 aces left for 2nd draw

$$\Pr(X=2) = \frac{4}{52}\times\frac{3}{51} = 1/221 \tag{2.0.3}$$

Now we know that E(X) denotes the average or expectation value which means that E(X) is the weighted average of all values X can take,each value being weighted by the probability of that particular event/value of X occurring
i.e E(X) is given by

$$E(X) = \sum_{i=0}^{2} X\times\Pr(X)$$

X	0	1	2
$\Pr(X)$	188/221	32/221	1/221
$X\times\Pr(X)$	0	32/221	2/221

$$\implies E(X) = \tfrac{32+2}{221} = \tfrac{2}{13}$$

Final answer E(x) = 2/13 or option 4