Assignment 3

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Download all python codes from

```
https://github.com/tyagio/AI1103/tree/main/
    assignment3/codes
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and latex-tikz codes from

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https://github.com/tyagio/AI1103/tree/main/
    assignment3/assignment3.tex
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1 Problem

Let $X_1, X_2...$ be a sequence of independent and identically distributed random variable with

$$\Pr(X_1 = -1) = \Pr(X_1 = 1) = 1/2 \tag{1.0.1}$$

Suppose for the standard normal random variable Z,

$$\Pr(-0.1 \leq Z \leq 0.1) = 0.08. \tag{1.0.2}$$

$$\text{If } S_n = \sum_{i=1}^{n^2} X_i, \text{then} \lim_{n\to\infty} \Pr\left(S_n > \frac{n}{10}\right) =$$

1) 0.42
2) 0.46
3) 0.50
4) 0.54

2 Solution

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{2}, & \text{if n= 1 or n=-1} \\ 0, & \text{otherwise} \end{cases}$$

$$\implies \mu = E(X_i) = 1/2(1-1) = 0 \tag{2.0.1}$$

$$\implies \sigma^2 = E(X_i{}^2) - \mu^2 = \frac{1}{2}(1+1) - 0 = 1 \tag{2.0.2}$$

Using Central Limit Theorem,we can say that for a series of random and identical variables X_i with the Mean = μ and variance = σ^2 where i $\in$ 1,2...n

$$\text{Let } \overline{X_n} \equiv \frac{\sum_{i=1}^{n} X_i}{n} \tag{2.0.3}$$

$$\text{Then} \lim_{n\to\infty} \sqrt{n}(\overline{X_n} - \mu) = N(0, \sigma^2) \tag{2.0.4}$$

$$\implies \lim_{n\to\infty} \frac{S_n}{n} = N(0,1) \tag{2.0.5}$$

$$\implies S_n = nN(0,1) \tag{2.0.6}$$

$$\implies \lim_{n\to\infty} \Pr\left(nN(0,1) > \frac{n}{10}\right) \tag{2.0.7}$$

$$\implies \lim_{n\to\infty} \Pr\left(N(0,1) > \frac{1}{10}\right) \tag{2.0.8}$$

$$= Q(0.1) = 0.460172163$$

$$\implies \lim_{n\to\infty} \Pr\left(S_n > \frac{n}{10}\right) \approx 0.46 \tag{2.0.9}$$

Hence final solution is option 2) or 0.46