Assignment 1

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Download all python codes from

https://github.com/tyagio/AI1103/tree/main/assignment1/codes

and latex-tikz codes from

https://github.com/tyagio/AI1103/tree/main/assignment1/assignment1.tex

1 Problem

Suppose that two cards are drawn at random from a deck of cards.Let X be the number of aces obtained.Then the value of E(X) is

1) 37/221
2) 5/13
3) 1/13
4) 2/13

2 Solution

Total number of cards =52 with 4 aces,48 non-ace's and we need to select 2 cards so X can be 0 ,1 or 2
Let A denote case when first card drawn is an ace i.e $\Pr(A)$=1/13
Let B denote case when second card drawn is an ace(no replacements).A and B not independent

Case 1: $X = 0$

$$\implies \Pr(X=0) = \Pr(A'B') = \Pr(A') \times \Pr(B'|A')$$

there 51 cards and 4 aces left for 2nd draw

$$\Pr(X=0) = \frac{48}{52} \times \frac{47}{51} = 188/221 \tag{2.0.1}$$

Case 2: $X = 1$

$$\implies \Pr(X=1) = \Pr(AB') + \Pr(A'B)$$

$$\text{In } \Pr(AB') = \Pr(A) \times \Pr(B'|A)$$

there 51 cards and 3 aces left for 2nd draw

$$\Pr(AB') = \frac{4}{52} \times \frac{48}{51} = 16/221$$

$$\text{In } \Pr(A'B) = \Pr(A') \times \Pr(B|A')$$

there 51 cards and 4 aces left for 2nd draw

$$\Pr(A'B) = \frac{48}{52} \times \frac{4}{51} = 16/221$$

$$\implies \Pr(X=1) = \frac{16}{221} + \frac{16}{221} = \frac{32}{221} \tag{2.0.2}$$

Case 3: $X = 2$

$$\implies \Pr(X=2) = \Pr(AB) = \Pr(A) \times \Pr(B|A)$$

there 51 cards and 3 aces left for 2nd draw

$$\Pr(X=2) = \frac{4}{52} \times \frac{3}{51} = 1/221 \tag{2.0.3}$$

Now we know that E(X) denotes the average or expectation value which means that E(X) is the weighted average of all values X can take,each value being weighted by the probability of that particular event/value of X occurring
i.e E(X) is given by

$$E(X) = \sum_{i=0}^{2} X \times P(X)$$

X	0	1	2
$\Pr(X)$	188/221	32/221	1/221
$X\times \Pr(X)$	0	32/221	2/221

$$\implies E(X) = \tfrac{32+2}{221} = \tfrac{2}{13}$$

Final answer E(x) = 2/13 or option 4