

Assignment 1

Ojjas Tyagi - CS20BTECH11060

Download all python codes from

<https://github.com/tyagio/AI1103/tree/main/assignment1/codes>

and latex-tikz codes from

<https://github.com/tyagio/AI1103/tree/main/assignment1/assignment1.tex>

1 PROBLEM

Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of $E(X)$ is

- 1) $37/221$
- 2) $5/13$
- 3) $1/13$
- 4) $2/13$

2 SOLUTION

Total number of cards = 52 with 4 aces, 48 non-ace's and we need to select 2 cards so X can be 0, 1 or 2. Let A denote case when first card drawn is an ace i.e. $\Pr(A) = 1/13$.

Let B denote case when second card drawn is an ace (no replacements). A and B not independent.

Case 1: $X = 0$

$$\Rightarrow \Pr(X = 0) = \Pr(A'B') = \Pr(A') \times \Pr(B'|A')$$

there 51 cards and 4 aces left for 2nd draw

$$\Pr(X = 0) = \frac{48}{52} \times \frac{47}{51} = 188/221 \quad (2.0.1)$$

Case 2: $X = 1$

$$\Rightarrow \Pr(X = 1) = \Pr(AB') + \Pr(A'B)$$

$$\text{In } \Pr(AB') = \Pr(A) \times \Pr(B'|A)$$

there 51 cards and 3 aces left for 2nd draw

$$\Pr(AB') = \frac{4}{52} \times \frac{48}{51} = 16/221$$

$$\text{In } \Pr(A'B) = \Pr(A') \times \Pr(B|A')$$

there 51 cards and 4 aces left for 2nd draw

$$\Pr(A'B) = \frac{48}{52} \times \frac{4}{51} = 16/221$$

$$\Rightarrow \Pr(X = 1) = \frac{16}{221} + \frac{16}{221} = \frac{32}{221} \quad (2.0.2)$$

Case 3: $X = 2$

$$\Rightarrow \Pr(X = 2) = \Pr(AB) = \Pr(A) \times \Pr(B|A)$$

there 51 cards and 3 aces left for 2nd draw

$$\Pr(X = 2) = \frac{4}{52} \times \frac{3}{51} = 1/221 \quad (2.0.3)$$

Now we know that $E(X)$ denotes the average or expectation value which means that $E(X)$ is the weighted average of all values X can take, each value being weighted by the probability of that particular event/value of X occurring.

i.e. $E(X)$ is given by

$$E(X) = \sum_{i=0}^2 X \times \Pr(X)$$

X	0	1	2
$\Pr(X)$	188/221	32/221	1/221
$X \times \Pr(X)$	0	32/221	2/221

$$\Rightarrow E(X) = \frac{32+2}{221} = \frac{2}{13}$$

Final answer $E(x) = 2/13$ or option 4