

Assignment 1

Ojjas Tyagi - CS20BTECH11060

Download all python codes from

<https://github.com/tyagio/AI1103/tree/main/assignment1/codes>

and latex-tikz codes from

<https://github.com/tyagio/AI1103/tree/main/assignment1/assignment1.tex>

1 PROBLEM

Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of $E(X)$ is

- 1) $37/221$
- 2) $5/13$
- 3) $1/13$
- 4) $2/13$

2 SOLUTION

Total number of cards = 52 with 4 aces, 48 non-ace's and we need to select 2 cards so X can be 0, 1 or 2

Let $A \in \{0, 1\}$ represent the random variable, where 0 represents first card being a non ace, 1 represents first card being ace.

Let $B \in \{0, 1\}$ represent the random variable, where 0 represents second card being a non-ace, 1 represents second card being ace

$\Pr(A = 0)$	$48/52$	$\Pr(A = 1)$	$4/52$
$\Pr(B = 0 A = 0)$	$47/51$	$\Pr(B = 0 A = 1)$	$48/51$
$\Pr(B = 1 A = 0)$	$4/51$	$\Pr(B = 1 A = 1)$	$3/51$

if $A=1$ then 3 aces left and if $A=0$ then 4 aces left in remaining 51 cards

Case 1: $X = 0$

$$\begin{aligned}\Rightarrow \Pr(X = 0) &= \Pr(A = 0, B = 0) \\ &= \Pr(A = 0) \times \Pr(B = 0|A = 0) \\ \Pr(X = 0) &= \frac{48}{52} \times \frac{47}{51} = 188/221\end{aligned}$$

(2.0.1)

Case 2: $X = 1$

$$\begin{aligned}\Rightarrow \Pr(X = 1) &= \Pr(A = 1, B = 0) + \Pr(A = 0, B = 1) \\ \Pr(A = 1, B = 0) &= \Pr(A = 1) \times \Pr(B = 0|A = 1) \\ \Pr(A = 1, B = 0) &= \frac{4}{52} \times \frac{48}{51} = 16/221 \\ \Pr(A = 0, B = 1) &= \Pr(A = 0) \times \Pr(B = 1|A = 0) \\ \Pr(A = 0, B = 1) &= \frac{48}{52} \times \frac{4}{51} = 16/221 \\ \Rightarrow \Pr(X = 1) &= \frac{16}{221} + \frac{16}{221} = \frac{32}{221}\end{aligned}$$

(2.0.2)

Case 3: $X = 2$

$$\begin{aligned}\Rightarrow \Pr(X = 2) &= \Pr(A = 1, B = 1) \\ &= \Pr(A = 1) \times \Pr(B = 1|A = 1) \\ \Pr(X = 2) &= \frac{4}{52} \times \frac{3}{51} = 1/221\end{aligned}$$

(2.0.3)

Now we know that $E(X)$ denotes the average or expectation value which means that $E(X)$ is the weighted average of all values X can take, each value being weighted by the probability of that particular event/value of X occurring

i.e $E(X)$ is given by

$$E(X) = \sum_{i=0}^2 X \times \Pr(X)$$

X	0	1	2
$\Pr(X)$	$188/221$	$32/221$	$1/221$
$X \times \Pr(X)$	0	$32/221$	$2/221$

$$\Rightarrow E(X) = \frac{32+2}{221} = \frac{2}{13}$$

Final answer $E(x) = 2/13$ or option 4