Assignment 3

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1 Problem

Let $X_1, X_2...$ be a sequence of independent and identically distributed random variable with

$$\Pr(X_1 = -1) = \Pr(X_1 = 1) = 1/2 \qquad (1.0.1)$$

Suppose for the standard normal random variable Z,

$$\Pr(-0.1 \le Z \le 0.1) = 0.08. \qquad (1.0.2)$$

If $S_n = \sum_{i=1}^{n^2} X_i$, then $\lim_{n\to\infty} \Pr\left(S_n > \dfrac{n}{10}\right) =$

1) 0.42
2) 0.46
3) 0.50
4) 0.54

2 Solution

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{2}, & \text{if n= 1 or n=-1} \\ 0, & \text{otherwise} \end{cases}$$

$$\implies \mu = E(X_i) = 1/2(1-1) = 0 \qquad (2.0.1)$$

$$\implies \sigma^2 = E(X_i^2) - \mu^2 = \frac{1}{2}(1+1) - 0 = 1 \quad (2.0.2)$$

Using Central Limit Theorem, we can say that for a series of random and identical variables X_i with the

Mean $= \mu$ and variance $= \sigma^2$ where i $\in$ 1,2...n

$$\text{Let } \overline{X_n} \equiv \frac{\sum_{i=1}^{n} X_i}{n} \qquad (2.0.3)$$

$$\text{Then } \lim_{n\to\infty} \sqrt{n}(\overline{X_n} - \mu) = N(0, \sigma^2) \qquad (2.0.4)$$

$$\implies \lim_{n\to\infty} \frac{S_n}{n} = N(0, 1) \qquad (2.0.5)$$

$$\implies S_n = nN(0, 1) \qquad (2.0.6)$$

$$\implies \lim_{n\to\infty} \Pr\left(nN(0, 1) > \frac{n}{10}\right) \qquad (2.0.7)$$

$$\implies \lim_{n\to\infty} \Pr\left(N(0, 1) > \frac{1}{10}\right) = Q(0.1) \qquad (2.0.8)$$

Now using (1.0.2)

$$\implies Q(0.1) + (1 - Q(-0.1)) + 0.08 = 1 \quad (2.0.9)$$

Now as $N(0, 1)$ symmetric about 0

$$\implies 2 \times Q(0.1) + 0.08 = 1 \qquad (2.0.10)$$

$$\implies Q(0.1) = 0.46 \qquad (2.0.11)$$

$$\implies \lim_{n\to\infty} \Pr\left(S_n > \frac{n}{10}\right) = 0.46 \qquad (2.0.12)$$

Hence final solution is option 2) or 0.46