Assignment 6

Ojjas Tyagi - CS20BTECH11060

Download all python codes from

and latex-tikz codes from

1 Problem

Let X and Y be i.i.d random variables uniformly distributed on (0,4).Then $\Pr(X > Y | X < 2Y)$ is

1) 1/3
2) 5/6
3) 1/4
4) 2/3

2 Solution

The PDF is given by

$$f_X(x) = f_Y(x) = \begin{cases} \frac{1}{4}, & \text{if } 0 < x < 4 \\ 0, & \text{otherwise} \end{cases}$$

The CDF is given by

$$F(x) = \int_{-\infty}^{x} f(x)dx$$

$$F_X(x) = F_Y(x) = \begin{cases} 0, & x \le 0 \\ \frac{x}{4}, & \text{if } 0 < x < 4 \\ 1, & x \ge 4 \end{cases}$$

Using definition of conditional probability

$$\Pr(X > Y | X < 2Y) = \frac{\Pr(Y < X < 2Y)}{\Pr(X < 2Y)} \quad (2.0.1)$$

Now finding $\Pr(X < 2Y)$

$$\Pr(X < 2y) = F_X(2y) \quad (2.0.2)$$

$$\implies \Pr(X < 2Y) = \int_{-\infty}^{\infty} f_Y(x) \times F_X(2x)dx \quad (2.0.3)$$

$$\implies \Pr(X < 2Y) = \int_{0}^{2} \frac{x}{8}dx + \int_{2}^{4} \frac{1}{4}dx \quad (2.0.4)$$

$$\implies \Pr(X < 2Y) = \frac{3}{4} = 0.75 \quad (2.0.5)$$

Now to find $\Pr(Y < X < 2Y)$

$$\Pr(y < X < 2y) = F_X(2y) - F_X(y) \quad (2.0.6)$$

$$\implies \Pr(Y < X < 2Y) \quad (2.0.7)$$

$$= \int_{-\infty}^{\infty} f_Y(x)(F_X(2x) - F_X(x))dx$$

$$\implies \int_{0}^{2} \frac{1}{4}\left(\frac{x}{2} - \frac{x}{4}\right)dx + \int_{2}^{4} \frac{1}{4}\left(1 - \frac{x}{4}\right)dx \quad (2.0.8)$$

$$\implies \Pr(Y < X < 2Y) = \frac{1}{4} = 0.25 \quad (2.0.9)$$

Now using (2.0.1),(2.0.5) and (2.0.9)

$$\Pr(X > Y | X < 2Y) = \frac{1/4}{3/4} = \frac{1}{3} \quad (2.0.10)$$

Hence final solution is option 1) or 1/3