Assignment 3

Ojjas Tyagi - CS20BTECH11060

Download all python codes from

```
https://github.com/tyagio/AI1103/tree/main/
    assignment3/codes
```

and latex-tikz codes from

```
https://github.com/tyagio/AI1103/tree/main/
    assignment3/assignment3.tex
```

1 Problem

Let $X_1, X_2...$ be a sequence of independent and identically distributed random variable with

$$\Pr(X_1 = -1) = \Pr(X_1 = 1) = 1/2 \tag{1.0.1}$$

Suppose for the standard normal random variable Z,$\Pr(-0.1 \le Z \le 0.1)$=0.08 .
If $S_n = \sum_{i=1}^{n^2} X_i$, then $\lim_{n\to\infty} \Pr\left(S_n > \frac{n}{10}\right) =$

1) 0.42
2) 0.46
3) 0.50
4) 0.54

2 Solution

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{2}, & \text{if n= 1 or n=-1} \\ 0, & \text{otherwise} \end{cases}$$

$$\implies \mu = E(X_i) = 1/2(1-1) = 0 \tag{2.0.1}$$

$$\implies \sigma^2 = E(X_i^2) - \mu^2 = \frac{1}{2}(1+1) - 0 = 1 \tag{2.0.2}$$

Using Central Limit Theorem,we can say that for a series of random and identical variables X_i with Mean = μ and variance = σ^2 where i $\in 1, 2...n$

$$\text{Let } \overline{X_n} \equiv \frac{\sum_{i=1}^{n} X_i}{n} \tag{2.0.3}$$

$$\text{Then} \lim_{n\to\infty} \sqrt{n}(\overline{X_n} - \mu) = N(0, \sigma^2) \tag{2.0.4}$$

$$\implies \lim_{n\to\infty} \frac{S_n}{n} = N(0, 1) \tag{2.0.5}$$

Using value given in question

$$\implies \lim_{n\to\infty} \Pr\left(\frac{-1}{10} \le \frac{S_n}{n} \le \frac{1}{10}\right) = 0.08 \tag{2.0.6}$$

The probability distribution of S_n is symmetric about 0,hence $\Pr\left(S_n > \frac{n}{10}\right) = \Pr\left(S_n < \frac{-n}{10}\right)$

$$\implies \lim_{n\to\infty} \Pr\left(S_n > \frac{n}{10}\right) \times 2 + \tag{2.0.7}$$

$$\lim_{n\to\infty} \Pr\left(\frac{-n}{10} \le S_n \le \frac{n}{10}\right) = 1$$

$$\implies \lim_{n\to\infty} \Pr\left(S_n > \frac{n}{10}\right) = 0.92/2 = 0.46 \tag{2.0.8}$$

Hence final solution is option 2) or 0.46