

# Assignment 3

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Download all python codes from

<https://github.com/tyagio/AI1103/tree/main/assignment3/codes>

and latex-tikz codes from

<https://github.com/tyagio/AI1103/tree/main/assignment3/assignment3.tex>

## 1 PROBLEM

Let  $X_1, X_2, \dots$  be a sequence of independent and identically distributed random variable with

$$\Pr(X_1 = -1) = \Pr(X_1 = 1) = 1/2 \quad (1.0.1)$$

Suppose for the standard normal random variable  $Z$ ,

$$\Pr(-0.1 \leq Z \leq 0.1) = 0.08. \quad (1.0.2)$$

$$\text{If } S_n = \sum_{i=1}^n X_i, \text{ then } \lim_{n \rightarrow \infty} \Pr\left(S_n > \frac{n}{10}\right) =$$

- 1) 0.42
- 2) 0.46
- 3) 0.50
- 4) 0.54

## 2 SOLUTION

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{2}, & \text{if } n = 1 \text{ or } n = -1 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow \mu = E(X_i) = 1/2(1 - 1) = 0 \quad (2.0.1)$$

$$\Rightarrow \sigma^2 = E(X_i^2) - \mu^2 = \frac{1}{2}(1 + 1) - 0 = 1 \quad (2.0.2)$$

Using Central Limit Theorem, we can say that for a series of random and identical variables  $X_i$  with the

Mean =  $\mu$  and variance =  $\sigma^2$  where  $i \in 1, 2, \dots, n$

$$\text{Let } \bar{X}_n \equiv \frac{\sum_{i=1}^n X_i}{n} \quad (2.0.3)$$

$$\text{Then } \lim_{n \rightarrow \infty} \sqrt{n}(\bar{X}_n - \mu) = N(0, \sigma^2) \quad (2.0.4)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{S_n}{n} = N(0, 1) \quad (2.0.5)$$

$$\Rightarrow S_n = nN(0, 1) \quad (2.0.6)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \Pr\left(nN(0, 1) > \frac{n}{10}\right) \quad (2.0.7)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \Pr\left(N(0, 1) > \frac{1}{10}\right) = Q(0.1) \quad (2.0.8)$$

Now using (1.0.2)

$$\Rightarrow Q(0.1) + (Q(-\infty) - Q(-0.1)) + 0.08 = 1 \quad (2.0.9)$$

Now as  $N(0, 1)$  symmetric about 0

$$\Rightarrow 2 \times Q(0.1) + 0.08 = 1 \quad (2.0.10)$$

$$\Rightarrow Q(0.1) = 0.46 \quad (2.0.11)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \Pr\left(S_n > \frac{n}{10}\right) = 0.46 \quad (2.0.12)$$

Hence final solution is option 2) or 0.46