

Assignment 1

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Download all python codes from

<https://github.com/tyagio/AI1103/tree/main/assignment1/codes>

and latex-tikz codes from

<https://github.com/tyagio/AI1103/tree/main/assignment1/assignment1.tex>

1 PROBLEM

Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of $E(X)$ is

- 1) $37/221$
- 2) $5/13$
- 3) $1/13$
- 4) $2/13$

2 SOLUTION

Total number of cards = 52 with 4 aces, 48 non-ace's and we need to select 2 cards so X can be 0, 1 or 2

Case 1: $X = 0$

$$Pr(X = 0) = \frac{\text{ways of selecting 2 non-ace's}}{\text{total ways of selecting 2 cards}}$$

$$Pr(X = 0) = \frac{\binom{48}{2}}{\binom{52}{2}} = \frac{24 \times 47}{26 \times 51}$$

Case 2: $X = 1$

$$Pr(X = 1) = \frac{\text{ways of selecting 1 ace and 1 non-ace}}{\text{total ways of selecting 2 cards}}$$

$$Pr(X = 1) = \frac{\binom{4}{1} \times \binom{48}{1}}{\binom{52}{2}} = \frac{48 \times 4}{26 \times 51}$$

Case 3: $X = 2$

$$Pr(X = 2) = \frac{\text{ways of selecting 2 aces}}{\text{total ways of selecting 2 cards}}$$

$$Pr(X = 2) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{26 \times 51}$$

now we know that $E(X)$ is given by

$$E(X) = \sum_{i=0}^2 X \times P(X)$$

X	0	1	2
P(X)	$\frac{24 \times 47}{26 \times 51}$	$\frac{48 \times 4}{26 \times 51}$	$\frac{6}{26 \times 51}$
X.P(X)	0	$\frac{48 \times 4}{26 \times 51}$	$\frac{12}{26 \times 51}$

$$\Rightarrow E(X) = \frac{48 \times 4}{26 \times 51} + \frac{12}{26 \times 51} = \frac{192 + 12}{26 \times 51} = \frac{2}{13}$$

Final answer $E(x) = 2/13$ or option 4