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Assignment 1

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Download all python codes from

https://github.com/tyagio/AI1103/tree/main/assignment1/codes

and latex-tikz codes from

https://github.com/tyagio/AI1103/tree/main/assignment1/assignment1.tex

1 Problem

Suppose that two cards are drawn at random from a deck of cards.Let X be the number of aces obtained.Then the value of E(X) is

- 1) 37/221
- 2) 5/13
- 3) 1/13
- 4) 2/13

2 Solution

Total number of cards =52 with 4 aces,48 non-ace's and we need to select 2 cards so X can be 0 ,1 or 2

Case 1: X = 0

$$Pr(X = 0) = \frac{\text{ways of selecting 2 non-ace's}}{\text{total ways of selecting 2 cards}}$$

$$Pr(X = 0) = \frac{\binom{48}{2}}{\binom{52}{2}} = \frac{24 \times 47}{26 \times 51}$$

Case 2: X = 1

$$Pr(X = 1) = \frac{\text{ways of selecting 1 ace and 1 non-ace}}{\text{total ways of selecting 2 cards}}$$

$$Pr(X = 1) = \frac{\binom{4}{1} \times \binom{48}{1}}{\binom{52}{2}} = \frac{48 \times 4}{26 \times 51}$$

Case 3: X = 2

$$Pr(X = 2) = \frac{\text{ways of selecting 2 aces}}{\text{total ways of selecting 2 cards}}$$

$$Pr(X = 2) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{26 \times 51}$$

Now we know that E(X) denotes the average or expectation value which means that E(X) is the weighted average of all values X can take, each value being weighted by the probability of that particular event/value of X occurring

i.e E(X) is given by

$$E(X) = \frac{\sum_{i=0}^2 X \times P(X)}{\sum_{i=0}^2 P(X)}$$

But as summation of P(X) is 1, we get

$$E(X) = \sum_{i=0}^{2} X \times P(X)$$

X	0	1	2
P(X)	$\frac{24 \times 47}{26 \times 51}$	$\frac{48 \times 4}{26 \times 51}$	$\frac{6}{26 \times 51}$
$\mathbf{X} \times \mathbf{P}(\mathbf{X})$	0	$\frac{48 \times 4}{26 \times 51}$	$\frac{12}{26 \times 51}$

$$\implies E(X) = \frac{48 \times 4}{26 \times 51} + \frac{12}{26 \times 51} = \frac{192 + 12}{26 \times 51} = \frac{2}{13}$$

Final answer E(x) = 2/13 or option 4