Assignment 1

Ojjas Tyagi - CS20BTECH11060

Download all python codes from

```
https://github.com/tyagio/AI1103/tree/main/
    assignment1/codes
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and latex-tikz codes from

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https://github.com/tyagio/AI1103/tree/main/
    assignment1/assignment1.tex
```

1 Problem

Suppose that two cards are drawn at random from a deck of cards.Let X be the number of aces obtained.Then the value of E(X) is

1) 37/221
2) 5/13
3) 1/13
4) 2/13

2 Solution

Total number of cards =52 with 4 aces,48 non-ace's and we need to select 2 cards so X can be 0 ,1 or 2

Let $A \in \{0,1\}$ represent the random variable, where 0 represents first card being an non ace, 1 represents first card being ace.
Let $B \in \{0,1\}$ represent the random variable, where 0 represents second card being an non-ace, 1 represents second card being ace

$\Pr(A=0)$	48/52	$\Pr(A=1)$	4/52
$\Pr(B=0\mid A=0)$	47/51	$\Pr(B=0\mid A=1)$	48/51
$\Pr(B=1\mid A=0)$	4/51	$\Pr(B=1\mid A=1)$	3/51

if A=1 then 3 aces left and if A=0 then
4 aces left in remaining 51 cards

Case 1: $X = 0$

$$\begin{gathered} \implies \Pr(X=0) = \Pr(A=0, B=0) \\ = \Pr(A=0) \times \Pr(B=0\mid A=0) \\ \Pr(X=0) = \frac{48}{52} \times \frac{47}{51} = 188/221 \end{gathered} \tag{2.0.1}$$

Case 2: $X = 1$

$$\begin{gathered} \implies \Pr(X=1) = \Pr(A=1, B=0) + \Pr(A=0, B=1) \\ \Pr(A=1, B=0) = \Pr(A=1) \times \Pr(B=0\mid A=1) \\ \Pr(A=1, B=0) = \frac{4}{52} \times \frac{48}{51} = 16/221 \\ \Pr(A=0, B=1) = \Pr(A=0) \times \Pr(B=1\mid A=0) \\ \Pr(A=0, B=1) = \frac{48}{52} \times \frac{4}{51} = 16/221 \\ \implies \Pr(X=1) = \frac{16}{221} + \frac{16}{221} = \frac{32}{221} \end{gathered} \tag{2.0.2}$$

Case 3: $X = 2$

$$\begin{gathered} \implies \Pr(X=2) = \Pr(A=1, B=1) \\ = \Pr(A=1) \times \Pr(B=1\mid A=1) \\ \Pr(X=2) = \frac{4}{52} \times \frac{3}{51} = 1/221 \end{gathered} \tag{2.0.3}$$

Now we know that E(X) denotes the average or expectation value which means that E(X) is the weighted average of all values X can take,each value being weighted by the probability of that particular event/value of X occurring
i.e E(X) is given by

$$E(X) = \sum_{i=0}^{2} X \times \Pr(X)$$

X	0	1	2
$\Pr(X)$	188/221	32/221	1/221
$X\times$ $\Pr(X)$	0	32/221	2/221

$$\implies E(X) = \frac{32+2}{221} = \frac{2}{13}$$

Final answer E(x) = 2/13 or option 4