Performance Analysis of Digital Communications over Rician-TWDP Channels

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Introduction

Contents

This paper presentation explores the conceptual behavior of digital communication for the composite Rician-Two Wave Diffuse Power (TWDP) fading-shadowing model. Here, small scale fading is attributed by the Rician channel, whereas shadowing effects are modeled by TWDP. We explore various statistical characteristics such as PDF,CDF and Outage Probability.

Function Definitions

Gamma Function $\Gamma(x)$

Gamma function is defined as

$$\Gamma(x) = \int_0^\infty t^{x-1} \exp(-t)dt \tag{1}$$

$$\Gamma(n) = (n-1)! \quad \text{when n is a positive integer}$$

Gen. Hypergeometric Function ${}_pF_q$

A generalized hypergeometric function ${}_pF_q$ is given by

$${}_pF_q(a_1, ..., a_p; b_1, ...b_q; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n ...(a_p)_n}{(b_1)_n ...(b_q)_n} \times \frac{z^n}{n!} \tag{2}$$

where $(x)_n$ stands for pochhammer symbol i.e

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)(x+2)...(x+n-1)$$

Modified Bessel function of first kind and zero order I_0

Bessel differential equation which is given by

$$x^2y'' + xy' - (x^2 + n^2) = 0$$

which has the linearly independent solutions $I_n(x)$ and $K_n(x)$ which are the modified bessel functions of the first and second kind respectively,where n is the order

when n is real $I_n(x)$ can be computed as

$$I_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{x^2}{4}\right)^k}{\Gamma(n+k+1)k!} \tag{3}$$

Lower Incomplete Gamma Function $\gamma(s, x)$

$$\gamma(s, x) = \int_0^x t^{s-1} \exp(-t)dt \tag{4}$$

Some useful terms

Types of components

1. **Specular/LOS component** singular term with fixed magnitude and random phase which has direct line of sight with receiver is known as Specular component
2. **Diffused component** Non specular component ,made of many waves each carrying power negligible to total average power,given by zero mean gaussian variables

Processes in wireless communication

1. **Shadowing** Occurs when there is large obstacle between signal and receiver
2. **Smallscale Fading** Power fluctuation for very short duration due to rapid interference of radio signals

Both of these are shown by fading models

System Model

Rician Multipath Fading

In Rician case there is only 1 LOS component along with diffused component.PDF of fading is given by

$$f_R(r) = \frac{r}{P_1} \exp\left(-\frac{r^2}{2P_1} - K_1\right) I_0\left(r\sqrt{\frac{2K_1}{P_1}}\right) \tag{5}$$

here
R denotes magnitude of received complex envelope/fading amplitude,relates to amplitude of received signal
K_1 is Rician K factor = $A^2/2P_1$ where
A is magnitude of specular component and
P_1 is Mean squared voltage of diffused component
I_0 is modified bessel function of first kind and zero order

TWDP shadowing

Two waves with diffuse power fading consist of two direct specular components along with diffused components.The shadowing follows the TWDP distribution whose closely approximate PDF is given as

$$f_Y(y) = \frac{y}{2P_2} \sum_{j=0}^{1} \exp\left(\frac{-y^2}{2P_2}\right) \sum_{i=1}^{L} a_i \exp(-P_{2ij}) I_0\left(y\sqrt{\frac{2P_{2ij}}{P_2}}\right) \tag{6}$$

here
P_2 is mean squared voltage of diffused components
K_2 is TWDP K factor $=(A_1^2 + A_2^2)/2P_2$ where
A_1 and A_2 are magnitudes of the two specular components
$P_{2ij} = K_2\left(1 + (-1)^j \Delta \cos\left(\pi\left(\frac{i-1}{2L-1}\right)\right)\right)$
Δ is relative strength of 2 LOS components $=\frac{2A_1A_2}{A_1^2+A_2^2}$
L is order of approx TWDP and should be $\geq \frac{1}{2}K\Delta$

Order	a_1	a_2	a_3	a_4	a_5
1	1				
2	$\frac{1}{4}$	$\frac{3}{4}$			
3	$\frac{19}{144}$	$\frac{25}{48}$	$\frac{25}{72}$		
4	$\frac{751}{8640}$	$\frac{3577}{8640}$	$\frac{49}{320}$	$\frac{2989}{8640}$	
5	$\frac{2857}{44800}$	$\frac{15741}{44800}$	$\frac{27}{1120}$	$\frac{1209}{2800}$	$\frac{2889}{22400}$

Figure: Values of a_i

Statistical Characteristics

Composite PDF

We studied the Rician fading channel in which the dominant line-of-sight component is subjected to TWDP shadowing. Using the concept of shadowing, the signal envelope, R, at receiver in a shadowed Rician-TWDP channel can be obtained by solving the conditional probability given as:

$$f_R(r) = \int_0^\infty f_{(R|Y)}(r|y) f_Y(y) dy \tag{7}$$

where

$$f_{(R|Y)}(r|y) = \frac{r}{P_1} \exp\left(-\frac{r^2}{2P_1} - K_1 y^2\right) I_0\left(yr\sqrt{\frac{2K_1}{P_1}}\right) \tag{8}$$

Now using eqns (6),(7) and (8) and , we get

$$f_R(r) = \frac{r}{2P_1P_2}\exp\left(\frac{-r^2}{2P_1}\right)\sum_{j=0}^{1}\sum_{i=1}^{L} a_i \exp(-P_{2ij})\times \tag{9}$$

$$\int_0^\infty y\exp\left(-y^2\left(K_1+\frac{1}{2P_2}\right)\right) I_0\left(yr\sqrt{\frac{2K_1}{P_1}}\right) I_0\left(y\sqrt{\frac{2P_{2ij}}{P_2}}\right)dy$$

now using (3) and the following identity

$$\int_0^\infty x^{\mu-0.5}\exp(-\alpha x) I_{2\nu}(2\beta\sqrt{x})dx \tag{10}$$

$$= \frac{\Gamma(\mu+\nu+0.5)}{\Gamma(1+2\nu)}\frac{\exp(\frac{\beta^2}{2\alpha})}{\beta\alpha^\mu} M_{-\mu,\nu}(\frac{\beta^2}{\alpha})$$

where $M_{k,m}(z)$ is defined as

$$M_{k,m}(z) = \exp(-0.5z)z^{m+0.5}{}_1F_1(0.5+m-k;1+2m;z) \tag{11}$$

Using (3),(10) and (11) we get

$$f_R(r) = \frac{r}{2P_1(2K_1P_2+1)} \exp\left(\frac{-r^2}{2P_1}\right) \sum_{i=1}^{L} a_i \sum_{j=0}^{1} \exp(-P_{2ij}) \times \tag{12}$$
$$\sum_{v=0}^{\infty} \frac{\Gamma(v+1)}{(v!)^2} \left(\frac{P_{2ij}}{2K_1P_2+1}\right)^v {}_1F_1\left(v+1; 1; \frac{K_1P_2r^2}{P_1(1+2K_1P_2)}\right)$$

Instantaneous SNR PDF derivation f_γ

When this signal is passed into the channel, it is added with Gaussian Noise. As a result, the received instantaneous signal to noise power ratio is raised by R^2,If we define instantaneous signal to noise power ratio per symbol as, $\gamma = \frac{R^2 E_s}{N_0}$,then the pdf of the instantaneous SNR is given by using (12) and $f_\gamma(\gamma) = \frac{f_R(\sqrt{\Omega\gamma/\bar{\gamma}})}{2\sqrt{\gamma\bar{\gamma}/\Omega}}$, we get

$$f_\gamma(\gamma) = \frac{\Omega}{4P_1(2K_1P_2)\bar{\gamma}} \sum_{i=1}^{L} a_i \sum_{j=0}^{1} \exp(-P_{2ij} - \frac{\gamma\Omega}{2P_1\bar{\gamma}}) \times \tag{13}$$

$$\sum_{v=0}^{\infty} \frac{\Gamma(v+1)}{(v!)^2} \left(\frac{P_{2ij}}{2K_1P_2+1}\right)^v {}_1F_1\left(v+1;1;\frac{K_1P_2\gamma\Omega}{P_1(1+2K_1P_2)\bar{\gamma}}\right)$$

where $\bar{\gamma}$ is the average SNR and $\Omega = E[R^2]$
E_s is the energy per symbol
N_0 is one-sided power spectral density of AWGN

CDF derivation

CDF is given by $F_R(r) = \int_0^r f_R(a)da$, now using (2),(12) and the following idenitity

$$\int_0^u x^m \exp(-\beta x^n)dx = \frac{\gamma(v, \beta u^n)}{n\beta^v} \quad v = \frac{m+1}{n} \tag{14}$$

Now we get CDF to be

$$F_R(r) = \frac{1}{2(2K_1P_2+1)} \sum_{i=1}^{L} a_i \sum_{j=0}^{1} \exp(-P_{2ij}) \times \sum_{v=0}^{\infty} \frac{1}{(v!)^2} \left(\frac{P_{2ij}}{2K_1P_2+1}\right)^v \sum_{s=0}^{\infty} \frac{\Gamma(v+s+1)}{s!\,\Gamma(s+1)} \gamma\left(s+1, \frac{r^2}{2P_1}\right) \tag{15}$$

n-moment derivation

We can find the n^{th} moment by

$$E[R^n] = \int_0^\infty r^n f_R(r) dr \tag{16}$$

Using (12) and identity given below

$$\int_0^\infty e^{-st} t^{b-1} {}_1F_1(a; c; kt) dt = \Gamma(b) s^{-b} {}_2F_1(a, b; c; \frac{k}{s}) \tag{17}$$

now we get

$$E[R^n] = \Gamma(\frac{n}{2} + 1)(2P_1)^{\frac{n}{2}} W^{(n)}(v, K_1, a_i, P_{2ij}, P_2) \tag{18}$$

$$W^{(n)}(v, K_1, a_i, P_{2ij}, P_2) = \frac{1}{2(2K_1P_2 + 1)} \sum_{i=1}^{L} a_i \sum_{j=0}^{1} \exp(-P_{2ij}) \times$$

$$\sum_{v=0}^{\infty} \frac{\Gamma(v+1)}{(v!)^2} \left(\frac{P_{2ij}}{2K_1P_2 + 1}\right)^v {}_2F_1\left(v+1, \frac{n}{2} + 1; 1; \frac{2K_1P_2}{2K_1P_2 + 1}\right)$$

Mean

Given by $E[R]$,using n=1 in (18) we get

$$E[R] = \sqrt{\frac{\pi P_1}{2}} W^{(1)}(v, K_1, a_i, P_{2ij}, P_2) \tag{19}$$

Second moment

Given by $E[R^2]$,using n=2 in (18) we get

$$E[R] = 2P_1 W^{(2)}(v, K_1, a_i, P_{2ij}, P_2) \tag{20}$$

Variance

Given by $\sigma_R^2 = E[R^2] - (E[R])^2$,using (20) and (19) we get

$$\sigma_R^2 = 2P_1 W^{(2)}(v, K_1, a_i, P_{2ij}, P_2) - \frac{\pi P_1}{2}(W^{(1)}(v, K_1, a_i, P_{2ij}, P_2))^2 \tag{21}$$

Amount of Fading

It defines the extreme level of fading parameters.Given by

$$AF = \frac{var[R^2]}{(E[R^2])^2} = \frac{E[R^4] - (E[R^2])^2}{(E[R^2])^2}$$
$$\implies AF = \frac{2W^{(4)}(v, K_1, a_i, P_{2ij}, P_2) - (W^{(2)}(v, K_1, a_i, P_{2ij}, P_2))^2}{(W^{(2)}(v, K_1, a_i, P_{2ij}, P_2))^2} \quad (22)$$

Outage Probability

Defined as

$$P_{out} = P_r(\gamma \leq \gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(\gamma) d\gamma = F_\gamma(\gamma_{th}) \quad (23)$$

Now using (13) and (14) we can say

$$P_{out} = \frac{1}{2(2K_1 P_2 + 1)} \sum_{i=1}^{L} a_i \sum_{j=0}^{1} \exp(-P_{2ij}) \times$$
$$\sum_{v=0}^{\infty} \frac{1}{(v!)^2} \left(\frac{P_{2ij}}{2K_1 P_2 + 1} \right)^v \sum_{s=0}^{\infty} \frac{\Gamma(v+s+1)}{s!\,\Gamma(s+1)} \gamma\left(s+1, \frac{\gamma_{th}\Omega}{2P_1\bar{\gamma}} \right) \tag{24}$$

Results and Discussion

Graph 1

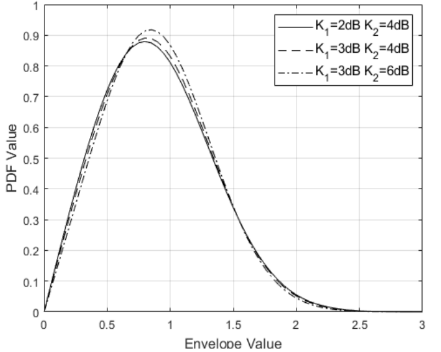

Fig.1: PDF of the model with various K_1, K_2, and $\Delta = 0.1$ values.

In Fig.1 for fixed Δ= 0.1, as Rician factor K_1 is increased from K_1=2dB to K_1=3dB, there is a significant rise in PDF peak due to a drop in shadowing for fixed TWDP factor K_2=4dB. Similar effect seen when TWDP factor K_2 is varied from K_2=4dB to K_2=6dB at fixed K_1=3dB. i.e more LOS component leads to stronger signals

Graph 2

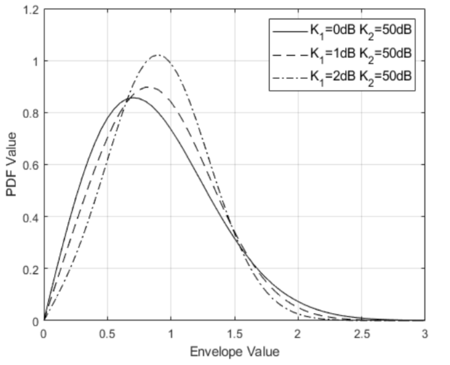

Fig.2: PDF of the model for a special case at $\Delta = 0$

The curves resemble the special case of Rayleigh and Rician distribution when K_2 made very high (shadowing disappears completely) and $\Delta = 0$, as shown in Fig. 2. At K_1 =0, it behaves as Rayleigh(no LOS component) and for other values it acts as Rician.

Graph3

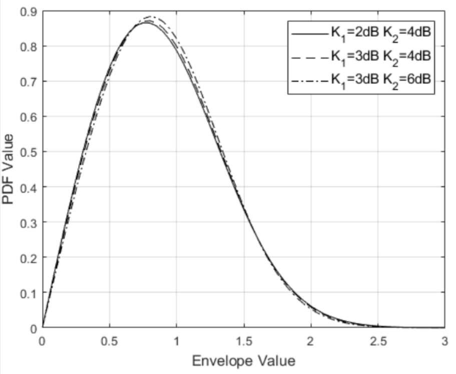

Fig.3: PDF of the model for parameters K_1, K_2, and $\Delta = 0.5$.

In Fig. 3 the PDF is plotted for $\Delta = 0.5$ value and it is clearly observed in comparison to Graph 1 that as Δ value increases, the peak density level decreases gradually. i.e One LOS component is better than two.

Graph 4

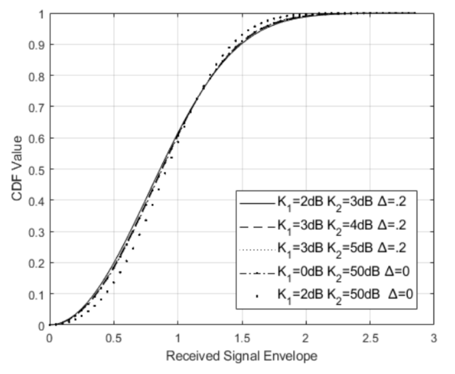

Figure: Fig.4: CDF of the model for parameters K_1, K_2, and Δ values.

The CDF of the Rician TWDP shadowed fading model with Δ=0.2 value is shown in Fig.4 ,we can see that the pure rician case $(K_1 = 0dB, K_2 = 50dB, \Delta = 0)$ performs better than the rest

Graph 5

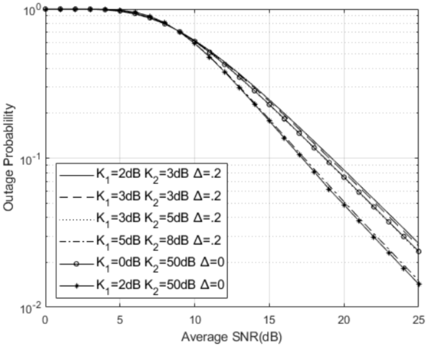

Fig.5: Outage Probability versus Average SNR

Fig.5 shows the graph of outage probability w.r.t average SNR at fixed γ_{th}=10dB and Δ =0.2.this shows that OP curve decreases quicker for higher values of K_1 and K_2.Special cases also shown with pure rician being best performing curve.