# Automata Midterm 2

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# 1 Context-free languages

## 1.1 Pumping lemma for context-free languages

If $A$ is a context-free language, then $\exists p$, the pumping length. If $s$ is a long string in A ($s \in A$ and $|s| > p$), then $s$ may be divided into five pieces $s = uvxyz$ s.t.

- $\forall i \geq 0, uv^i xy^i z \in A$
- $|vy| > 0$ (note, $v$ or $y$ can be the empty string, but not both)
- $|vxy| \leq p$

If a string of terminals is longer than the number of rules in Chomsky Normal form (informally), we know that a variable is repeated, and we can replace the variable inside with the one outside recursively. Pumping once, we get $uvvxyyz$.

## 1.2 Showing nonCFLarity

Assume, FSOC, that $A$ is a CFL. Let $p$ be the pumping length given by PL for CFLs. Consider a string $s = uvxyz$ (e.g. $s = a^p b^p c^p \in A$) with at least length $p$. State some property about what $v$ and $y$ could be (e.g. we know that $xyz$ cannot contain more than two different types of symbols). Pump up, so the number of occurrences of at least one of the symbols is not changing, meaning that the number of occurrences of one type of symbol is still $p$, while the number of occurences of the other symbol is now $p + k$ for some $k$.

## 1.3 Closure

- Regular operators: union, concatenation, kleene star, intersection, complement
- Context free operators: union, concatenation, kleene star
- Turing decidable: union, intersection, concatenation, complement, and kleene star
- Turing recognizable: union, intersection, concatenation, and kleene star

# 2 Turing Machines

## 2.1 Church Turing Thesis

- Algorithms and turing machines are basically equivalent.

## 2.2 Showing Turing Decidable

- Guaranteed to halt on accept/reject.
- By given, want, construction, correctness. Make sure that our machine terminates.

## 2.3 Showing Turing Recognizable

- Guaranteed to accept, but may loop.
- By given, want, construction, correctness.
- Make sure that inner loops terminates in finite steps, but outer loop can be infinite. E.g. use shortlex ordering up to a certain amount of steps.

## 2.4 Co-Turing Recognizable

- The complement is Turing Recognizable.
- If Co-TR and TR, then TD.