Calc Final

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Equations

Graph

$$\{(\mathrm{x}, y) \in \mathbb{R}^{n+1} \mid f(\mathrm{x}) = y\}$$

Level set

$$\{(\mathrm{x}) \in \mathbb{R}^{n} \mid f(\mathrm{x}) = c\}$$

Open ball

$$B_r(\bar{\mathrm{x}}) = \{(\mathrm{x}) \mid ((\mathrm{x} - \bar{\mathrm{x}}) < r)\}$$

Derivative, gradient

$$Df(\mathrm{x}) = \nabla f(\mathrm{x})^T = [\partial f_i / \partial x_j]$$

Directional derivative

$$D_{\mathrm{u}} f = \nabla f \cdot \mathrm{u}$$

Steepest descent

$$-\nabla F$$

Chain rule

$$D(f \circ g)(\mathrm{x}) = Df(g(\mathrm{x})) Dg(\mathrm{x})$$

Taylor

$$f(\bar{\mathrm{x}}) \approx f(\bar{\mathrm{x}}) + \nabla f(\bar{\mathrm{x}})(\mathrm{x} - \bar{\mathrm{x}}) + \frac{1}{2}(\mathrm{x} - \bar{\mathrm{x}})^T \nabla^2 f(\bar{\mathrm{x}})(\mathrm{x} - \bar{\mathrm{x}})$$

Lagrange

$$\nabla f = \sum_i \lambda_i \nabla g_i$$

$$g_i = 0$$

Fubinis

$$\int_a^b \int_c^d f(x, y) dy dx = \int_c^d \int_a^b f(x, y) dx dy$$

Simple region

$$\int_R f(x, y) dA = \int_a^b \int_{\varphi_1(x)}^{\varphi_2(x)} f(x, y) dy dx$$

Cavaliers

$$V(S) = \int_a^b A(x) dx = \int_a^b \int_c^d f(x, y) dy dx$$

Change of variables

$$\iint_D f(x, y) dx dy = \iint_{D^*} f(x(u, v), y(u, v))) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv$$

$$\iint_D f(x, y) dx dy = \iint_{D^*} f(x(u, v), y(u, v))) \left| \frac{\partial(x, y)}{\partial(u, v)} \right|^{-1} du dv$$

Flow line

$$\forall t \in \mathbb{R},\ \mathrm{c}'(t) = F(\mathrm{c}(t))$$

Divergence

$$\nabla \cdot F$$

Curl

$$\nabla \times F$$

Curl field has no divergence

$$div(curl(F)) = \nabla \cdot (\nabla \times F) = 0$$

Irrotational

$$\nabla \times F = 0$$

Conservative

$$F = \nabla f$$

Gradient fields are irrotational

$$\nabla \times \nabla f = \nabla \times F = 0$$

Scalar line integral

$$\int_{\mathrm{c}} f ds = \int_{\mathrm{c}} f(\mathrm{c}(t))||\mathrm{c}'(t)||dt$$

Vector line integral (work)

$$\int_{\mathrm{c}} \mathrm{F} \cdot d\mathrm{s} = \int_a^b \mathrm{F}(\mathrm{c}(t)) \cdot \mathrm{c}'(t)dt = \int_c (\mathrm{F} \cdot \mathrm{T})ds$$

$$\int_{\mathrm{c}} \mathrm{F} \cdot d\mathrm{s} = f(c(b)) - f(c(a))$$

Scalar surface integral

$$\iint_S f(x, y, z)dS = \iint_D f(\Phi(u, v))||T_u \times T_v||dudv$$

Vector surface integral (flux)

$$\iint_S \mathrm{F}(x, y, z)d\mathrm{S} = \iint_D \mathrm{F}(\Phi(u, v)) \cdot (T_u \times T_v)dudv$$

Arc length

$$\int_{\mathrm{c}} ds = \int_{\mathrm{c}} ||\mathrm{c}'(t)||dt$$

Surface area

$$\iint_S dS = \iint_D ||T_u \times T_v||dudv$$

Green (scalar curl)

$$\int_{\partial D} \mathrm{F} \cdot d\mathrm{s} = \iint_D (\nabla \times \mathrm{F}) \cdot \mathrm{k} \; dA$$

Stokes (curl)

$$\int_{\partial S} \mathrm{F} \cdot d\mathrm{s} = \iint_S (\nabla \times \mathrm{F}) \cdot d\mathrm{S}$$

Gauss (divergence)

$$\iint_{\partial D} \mathrm{F} \cdot d\mathrm{S} = \iiint_D (\nabla \cdot \mathrm{F})dV$$

Theorems

Conservative

- F is conservative (iff $F = \nabla f$)
- For c a simple closed curve, $\int_c F \cdot d\mathrm{s} = 0$
- For any 2 oriented simple curves with the same endpoints, $\int_{c_1} F \cdot d\mathrm{s} = \int_{c_2} F \cdot d\mathrm{s}$
- $\nabla \times F = 0$

Epsilon delta definition of a limit

$$\lim_{x \to x_0} f(x) = L$$

$$\forall \, \varepsilon > 0, \exists \, \delta > 0 \text{ s.t. } 0 < |x - x_0| < \delta \implies |f(x) - L| < \varepsilon$$

- Choose $\varepsilon > 0$
- Plug in x_0

Definition of a derivative

$$\lim_{h \to 0} \frac{f(\mathrm{x} + h\mathrm{e}_i) - f(\mathrm{x})}{h}$$

Second derivative test

- $D > 0$ local min or max
  - $d^2f/dx^2 < 0$ local max
  - $d^2f/dx^2 < 0$ local min
- $D < 0$ saddle
- $D = 0$ need higher level tests