# Reasoning about Uncertainty (30pts)

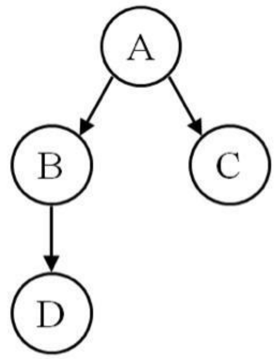
**Bayesian Networks (20pts)** Given this network calculate the following probabilities. Give both the formula and calculations with values. These questions are designed so that they can be answered with a minimum of computation. If you find yourself doing a copious amount of computation for each part, step back and consider whether there is simpler way to deduce the answer.

$P(A)$

| | |
|---|---|
| +a | 0.1 |
| ¬a | 0.9 |

$P(B|A)$

| | | |
|---|---|---|
| +a | +b | 0.5 |
| | ¬b | 0.5 |
| ¬a | +b | 0.8 |
| | ¬b | 0.2 |




$P(C|A)$

| | | |
|---|---|---|
| +a | +c | 0.4 |
| | ¬c | 0.6 |
| ¬a | +c | 0.7 |
| | ¬c | 0.3 |

$P(D|B)$

| | | |
|---|---|---|
| +b | +d | 0.9 |
| | ¬d | 0.1 |
| ¬b | +d | 0.2 |
| | ¬d | 0.8 |

1. [4pts] P(a,¬b,c,¬d)

$= P(a)P(\neg b|a)P(c|a)P(\neg d|\neg b)$ +2pts

$= 0.1 \times 0.5 \times 0.4 \times 0.8 = 0.016$ +2pts

2. [4pts] P(b)

$= \sum_{A=\{a,\neg a\}} P(A)P(b|A)$ +2pts

$= 0.1 \times 0.5 + 0.9 \times 0.8 = 0.77$ +2pts

**Bayesian Networks continued**

3. [4pts] $P(a|b)$

$= P(a)\, P(b|a) \,/\, P(b)$ +2pts

$= 0.1 \text{ x } 0.5 \,/\, .77 = 0.064935$ +2pts

4. [4pts] $P(d|a)$

$= \sum_{B=\{b,\neg b\}} P\,(d|B)p(B|a)$ +2pts

$= 0.9 \times 0.5 + 0.2 \times 0.5 = 0.\ 55$ +2pts

5. [4pts] $P(d|a,c)$

From the conditional independence properties of the graph, $D \perp C|\{A\}$. Hence,
$P(d|a,c) = p(d|a)$ +2pts

$= 0.55$ +2pts

**Probability (10pts)**

Circle **True** for each of the following statements if it is <u>always true</u>.

1. [2pts] [True/False] P(A, B) = P(A)P(B)

2. [2pts] [True/False] P(A|B) = P(A)P(B)

3. [2pts] [True/False] P(A, B) = P(A)P(B) − P(A|B)

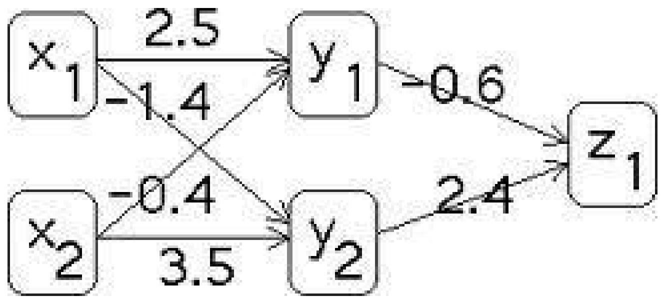
4. [2pts] [True/False] P(A,B,C)=P(A|B,C)P(B|C)P(C)

5. [2pts] [True/False] If P(A|B,C)= P(B|A,C) then P(A|C)=P(B|C)

# Machine Learning (50pts)

## Neural Networks (10pts)

In this network the input layer consists of units X1 and X2. The hidden layer is Y1 and Y2, and the output layer is Z1. Units Y1, Y2 and Z1 all have threshold functions where t>0 for the unit to activate.




1. [1pt] When input X1 = 0 and input X2 = 0, what does the network output?
   0

2. [1pt] When input X1 = 0 and input X2 = 1, what does the network output?

   1
3. [1pt] When input X1 = 1 and input X2 = 0, what does the network output?
   0

4. [1pt] When input X1 = 1 and input X2 = 1, what does the network output?
   1

5. [3pts] Can a weight be changed for the network to compute the Boolean function OR? If yes, which weight and what value should it be changed to?

Yes. +1
-0.6 to number > 0 or -1.4 to number >0 +2

6. [3pts] Can a single weight be changed for the network to compute the Boolean function XOR? If yes, which weight and what value should it be changed to?

No. +3

**Candidate Elimination (20pts)**

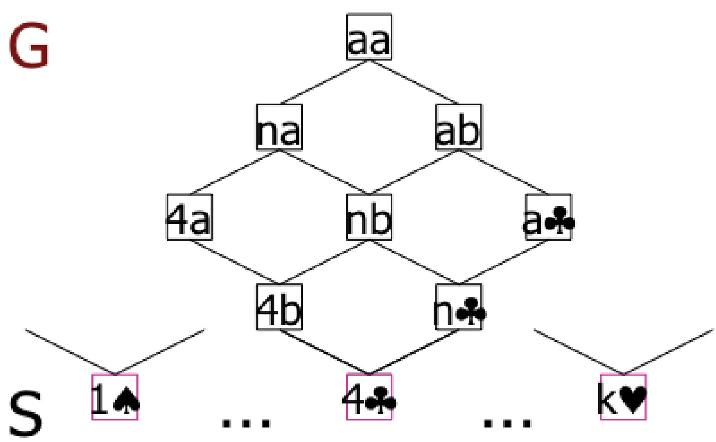
Reward Card Domain: This domain is a deck of 52 cards (13 cards of each suit ª¨© §A subset of the deck are considered "Reward" cards. The goal is given positive and negative examples of Reward cards learn the concept that represents the Reward cards only.

(r=1) v … v (r=10) v (r=J) v (r=Q) v (r=K) **Û** ANY-RANK(r)
(r=1) v … v (r=10) **Û** NUM(r)
(r=J) v (r=Q) v (r=K) **Û** FACE(r)
(s=ª) v (s=§) v (s=¨) v (s=©) **Û** ANY-SUIT(s)
(s=ª) v (s=§) **Û** BLACK(s)
(s=¨) v (s=©) **Û** RED(s)

A hypothesis is represented by rs, with:

- r Î {a, n, f, 1, …, 10, j, q, k}, where a=ANY-RANK, n=NUM, f=FACE
- s Î {a, b, r, §, ª, ¨, ©}, where a=ANY-SUIT, b=BLACK, r=RED

The hypothesis space for the Reward Card domain can be seen as a partial order, where G is the set of most general hypotheses (aa = ANY-RANK, ANY-SUIT), and S is the set of most specific hypotheses (4§Given this subset of the partial order, answer the questions below.




1. [2pts] Given 4♣ is a negative example of a Reward card, list the hypotheses that would classify 4♣ as a true negative.

   1♤  k♡
   -1 for extra incorrect hypotheses

**Candidate Elimination continued**

2. [2pts] Given 7♣ is a positive example of a Reward card, list the hypotheses that would classify 7♣ as a true positive.

aa, ab, na, nb, ac, nc

+1 for 1-5 correct.
+2 for all 6

-1 for extra incorrect hypotheses

3. [2pts] Given 5♡ is a negative example of a Reward card, list the hypotheses that would classify 5♡as a false positive.

aa,+1

na +1

-1 for extra incorrect hypotheses

**Candidate Elimination continued**

4. [2pts] Given 2♠ is a positive example of a Reward card, list the hypotheses that would classify 2♠ as a false negative.

   1♤ k♡ 4♣ 4b n♣ a♣ 4a

   +1 for 1-6 correct.
   +2 for all 7
   -1 for extra incorrect hypotheses

5. [4pts] Given this set of training examples for Reward card, (4♣ is a positive example, 7♣ is a positive example, 5♡ is a negative example, 2♠ is a positive example), list the hypotheses that would correctly classify all examples.

nb,+2

ab +2

   -1 for extra incorrect hypotheses

**Candidate Elimination continued**

6. [4pts] The example j♠ is added to the training set for Reward card. If it is a positive example, list the hypotheses that would correctly classify all examples. How many of the 52 cards are Reward cards?

   ab, +2
   26 cards +2

   -2 for extra incorrect hypotheses

7. [4pts] The example j♠ is added to the training set for Reward card. If it is a negative example, list the hypotheses that would correctly classify all examples. How many of the 52 cards are Reward cards?

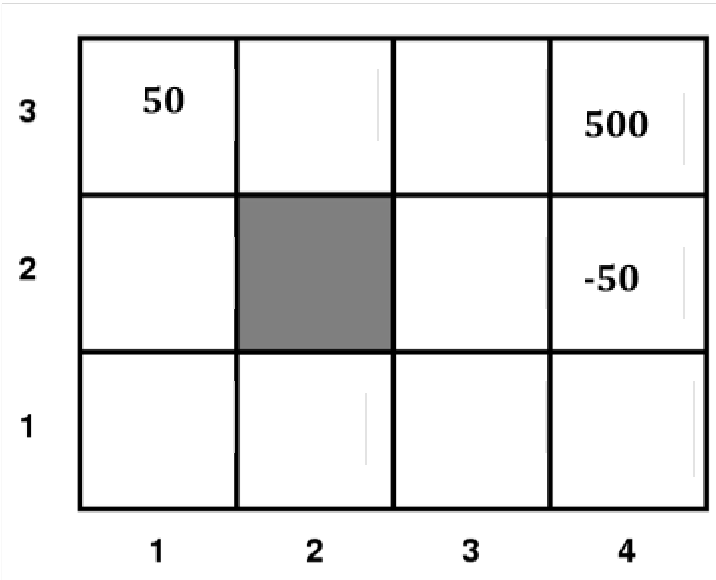
   nb, +2

   20 cards +2

   -2 for extra incorrect hypotheses

**Reinforcement Learning (20pts)**

Given a Gridworld domain, where terminal states (1,3), (4,3), and (4,2) have rewards 50, 500, and -50 respectively, the set of possible actions are {N,E,S,W, or X for terminal states}, the agent moves deterministically, all V and Q values for non terminal states have been initialized to 0, answer the questions below.




Circle the letter that corresponds to the best answer for the question:

1. [4pts] What are the optimal values, $V^*$ of each state in the above grid if $\gamma = 0.5$, c(a)=0, R(s)=0 for non terminal states?

   (Remember $\mathbf{V_{t+1}(s) = R(s) + Max_{a\varepsilon A} \{c(a)+\gamma\Sigma_{s'\varepsilon S} Pr(s'|a,s)\, V_t(s')\}}$ )

   a. $V_{(1,1)}$=15.75, $V_{(1,2)}$=25, $V_{(2,1)}$=31.25, $V_{(2,3)}$=125, $V_{(3,1)}$=62.5, $V_{(3,2)}$=125, $V_{(3,3)}$=250, $V_{(4,1)}$=25
   b. $V_{(1,1)}$=12.5, $V_{(1,2)}$=25, $V_{(2,1)}$=31.25, $V_{(2,3)}$=125, $V_{(3,1)}$=62.5, $V_{(3,2)}$=125, $V_{(3,3)}$=250, $V_{(4,1)}$=31.25
   c. $V_{(1,1)}$=15.75, $V_{(1,2)}$=25, $V_{(2,1)}$=31.25, $V_{(2,3)}$=125, $V_{(3,1)}$=62.5, $V_{(3,2)}$=125, $V_{(3,3)}$=250, $V_{(4,1)}$=31.25
   d. $V_{(1,1)}$=12.5, $V_{(1,2)}$=25, $V_{(2,1)}$=25, $V_{(2,3)}$=25, $V_{(3,1)}$=50, $V_{(3,2)}$=100, $V_{(3,3)}$=250, $V_{(4,1)}$=25
   e. None of the above

E or C correct

Answer: $V_{(1,1)}$=15.625, $V_{(1,2)}$=25, $V_{(2,1)}$=31.25, $V_{(2,3)}$=125, $V_{(3,1)}$=62.5, $V_{(3,2)}$=125, $V_{(3,3)}$=250, $V_{(4,1)}$=31.25

**Reinforcement Learning (continued)**

2. [4pts] What are the Q values of state (3,2) in the above grid if $\gamma = 0.5$, c(a)=0, R(s)=-2 for non terminal states?

   (Remember $\mathbf{Q_{t+1}(a,s) = R(s) + c(a)+\gamma\Sigma_{s'\varepsilon S}\ Pr(s'|a,s)\ Q_t(a's')\}}$ )

   a. $Q_{((3,2),N)}$=120, $Q_{((3,2),E)}$=-27, $Q_{((3,2),S)}$=58
   b. $Q_{((3,2),N)}$=120, $Q_{((3,2),E)}$=-27, $Q_{((3,2),S)}$=27
   c. $Q_{((3,2),N)}$=125, $Q_{((3,2),E)}$=-25, $Q_{((3,2),S)}$=62.5
   d. $Q_{((3,2),N)}$=120, $Q_{((3,2),E)}$=-27, $Q_{((3,2),S)}$=31.5
   e. None of the above

E or B correct

Answer: $Q_{((3,2),N)}$=122, $Q_{((3,2),E)}$=-27, $Q_{((3,2),S)}$=27.5

**For questions 3-5:**

When the agent is not given the map with the reward function, it starts Q-learning with these 3 training examples or episodes/epochs:

E1: (1,1), N, 0, (1,2), N, 0, (1,3), X, 50
E2: (1,1), N, 0, (1,2), N, 0, (1,3), X, 50
E3: (1,1), N, 0, (1,2), S, 0, (1,1), E, 0, (2,1), E, 0, (3,1), E, 0, (4,1), N, 0, (4,2), X, -50

During Q-Learning each example is processed in the order the states are observed, for example in E1 the Q values are calculated for the observed states in this order (1,1), (1,2) (1,3).

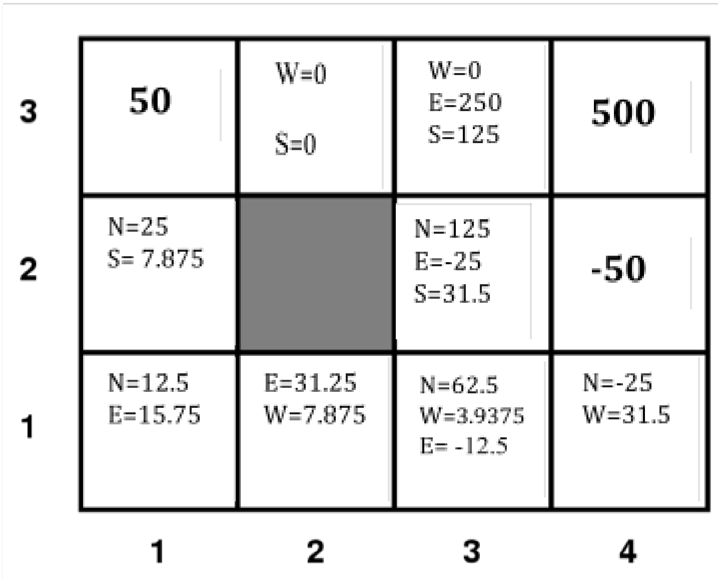
Remember that the Q-Learning equation is:

$$\mathbf{Q(a,s) \square Q(a,s) + \square(R(s) + \gamma Max_{a'\varepsilon A} Q(a',s') - Q(a,s))}$$

3. [4pts] Given □=1 and $\gamma = 0.5$, what are the non-zero Q values after observing all 3 of these examples?

   a. $Q_{((1,1),N)}$=50, $Q_{((1,2),N)}$=50, $Q_{((1,3),X)}$=50, $Q_{((4,2),X)}$=-50
   b. $Q_{((1,1),N)}$=12.5, $Q_{((1,2),N)}$=25, $Q_{((1,3),X)}$=50, $Q_{((4,2),X)}$=-50
   c. $Q_{((1,1),N)}$=12.5, $Q_{((1,2),S)}$=6.25, $Q_{((1,2),N)}$=25, $Q_{((1,3),X)}$=50, $Q_{((4,2),X)}$=-50
   d. $Q_{((1,1),N)}$=12.5, $Q_{((1,2),S)}$=12.5, $Q_{((1,2),N)}$=25, $Q_{((1,3),X)}$=50, $Q_{((4,2),X)}$=-50
   e. None of the above

**Reinforcement Learning (continued)**




4. [4pts] After Q-Learning with many more examples, the Q values are shown in the grid above. What is the optimal policy given these values?
    a. (1,1) N, (1,2) N, (2,1) E, (3,1) N, (4,1) W, (3,2) N, (3,3) E
    b. (1,1) E, (1,2) S, (2,1) E, (3,1) N, (4,1) W, (3,2) N, (3,3) E
    c. (1,1) E, (1,2) N, (2,1) E, (3,1) N, (4,1) W, (3,2) N, (3,3) E
    d. All of the above
    e. None of the above

5. [4pts] If now there is a "living reward" of -5 for all non terminal states (R(s)= -5), so that all new examples would see a -5 instead of 0 at non terminal states. How would this affect the agents policy?
    a. It would prefer actions providing the shortest path to state (4,2)
    b. It would prefer actions providing the shortest path to state (4,3)
    c. It would prefer actions providing the shortest path to state (1,2)
    d. All of the above
    e. None of the above

## Short Answer (20pts)

**Keep your answers brief, one or two sentences.**

1. [4pts] List 2 reasons why machine learning is needed?

Description of 2 of the following:
Unknown environments
Adaptability
Lazy programmers
Autonomous
Human Cognition

See also Lecture 10 slide 4

-2 for additional incorrect answer

2. [4pts] What is Ockham's razor? How is it used in decision tree learning?

Bias for simplest hypothesis +2

Using information gain metric to create smaller decision trees +2

-2 for additional incorrect answer

3. [4pts] What does the size of a hypothesis class have to do with generalization and overfitting?

Many possible answers - this is one example:
Smaller trees allow for more generalization of the concept +2

Larger trees allow for overfitting to the training data +2

-2 for additional incorrect answer

4. [4pts] In reinforcement learning, why can it be useful to sometimes act in a way that is believed to be suboptimal?

To avoid being stuck in a local maxima +2

choosing a suboptimal action may lead to a more rewarding state and improve the agent's policy and performance. +2

-2 for additional incorrect answer

5. [4pts] Choose 2 research topics from Lecture 12 on Communicating, Perceiving, and Acting and describe each topic, including the limitations/pitfalls and future directions.

Choose any 2 research topics from lecture 12 ( See lecture 12 slides 3-6):

Description of research topic +1
Description of its limitations/pitfalls and future directions +1