Final Exam

CSCI 561 Fall 2016: Artificial Intelligence

Student ID: | | | | | | | | | | |

Last Name: ____

First Name: ____

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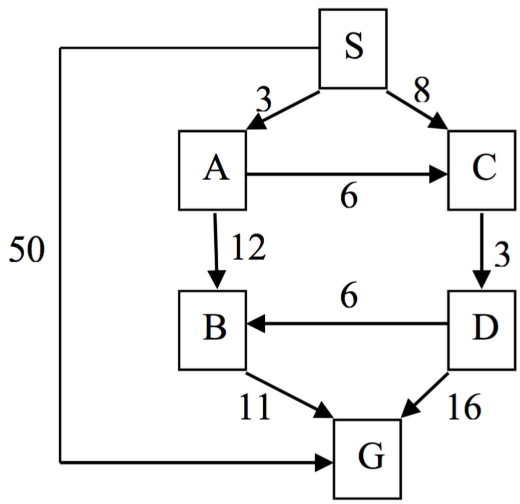
Instructions:

1. Date: **12/12/2016 from 2:00 pm – 4:00 pm**
2. Maximum credits/points/percentage for this exam: 100
3. The percentages for each question are indicated in square brackets [ ] near the question.
4. **No books** (or any other material) are allowed.
5. **Write down your name, student ID and USC email address.**
6. **Your exam will be scanned and uploaded online.**
7. **Write within the boxes provided for your answers.**
8. **Do NOT write on the 2D barcode.**
9. **The back of the pages will not be graded. You may use it for scratch paper.**
10. No questions during the exam. **If something is unclear to you, write that in your exam.**
11. **Be brief: a few words are often enough if they are precise and use the correct vocabulary studied in class.**
12. When finished, raise completed exam sheets until approached by proctor.
13. **Adhere to the Academic Integrity code.**

Problems	**100 Percent total**
1- Search	10
2- Bayesian networks	20
3- Decision trees	10
4- Markov decision processes	20
5- Neural networks	10
6- Bayes theorem	10
7- FOL resolution proof	20

1. [10%] Search

Consider the following search problem where **S**
is the start state and **G** satisfies the goal test.
Arcs are labeled with the cost of traversing them:



The heuristic estimates of the distance to G are:						
from:	S	A	B	C	D	G
distance:	22	20	8	12	10	0

For each of the following search strategies, indicate which goal state is reached (if any) and list, in order, all the states of the nodes popped off of the OPEN queue, and the cost of the path found by the strategy to reach the goal state from S. When all else is equal, nodes should be removed from OPEN in alphabetical order.

Please apply the "clean and robust" algorithm studied in class for loop detection.

Note how the arcs in the figure are oriented, which means that you can only go from one state to another in the direction of the arrow.

a) [5%] Uniform cost Search **G SACDBG 26**

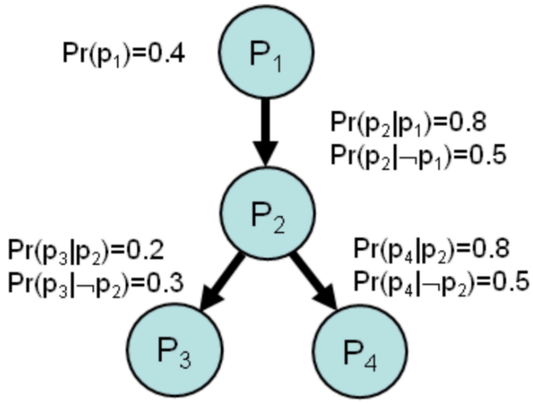
Goal state reached: ________ States popped off OPEN: ________________________________ Path Cost _____

b) [5%] A* search **G SCDABG 26**

Goal state reached: ________ States popped off OPEN: ________________________________ Path Cost _____

2. [20%] Bayesian Networks

Consider the following Bayesian Network:



Derive symbolic and numerical expressions for the following probabilities given the network, using **inference by enumeration**. Please first write symbolic expressions (e.g., Pr(p1) x Pr(p3|p2) + …) and then use the above probabilities values to write numerical expressions (e.g., 0.4 x 0.2 + …). You need not compute the final numerical result, a correct numerical expression (with sums and products of numerical values) is sufficient to gain full credits. You will lose marks if either the symbolic expression or the numerical expression is missing.

A. [10%] Compute Pr(¬p3):

$$
\begin{aligned}
Pr(\neg p_3) &= \sum_{P_1,P_2,P_4} Pr(P_1,P_2,\neg p_3,P_4) = \sum_{P_1,P_2,P_4} Pr(P_1)Pr(P_2|P_1)Pr(\neg p_3|P_2)Pr(P_4|P_2) = \\
&= Pr(p_1)Pr(p_2|p_1)Pr(\neg p_3|p_2)Pr(p_4|p_2) + Pr(p_1)Pr(p_2|p_1)Pr(\neg p_3|p_2)Pr(\neg p_4|p_2)+ \\
&+ Pr(p_1)Pr(\neg p_2|p_1)Pr(\neg p_3|\neg p_2)Pr(p_4|\neg p_2) + Pr(p_1)Pr(\neg p_2|p_1)Pr(\neg p_3|\neg p_2)Pr(\neg p_4|\neg p_2)+ \\
&+ Pr(\neg p_1)Pr(p_2|\neg p_1)Pr(\neg p_3|p_2)Pr(p_4|p_2) + Pr(\neg p_1)Pr(p_2|\neg p_1)Pr(\neg p_3|p_2)Pr(\neg p_4|p_2)+ \\
&+ Pr(\neg p_1)Pr(\neg p_2|\neg p_1)Pr(\neg p_3|\neg p_2)Pr(p_4|\neg p_2) + Pr(\neg p_1)Pr(\neg p_2|\neg p_1)Pr(\neg p_3|\neg p_2)Pr(\neg p_4|\neg p_2) = \\
&= .4\times.8\times.8\times.8 + .4\times.8\times.8\times.2 + .4\times.2\times.7\times.5 + .4\times.2\times.7\times.5+ \\
&+ .6\times.5\times.8\times.8 + .6\times.5\times.8\times.2 + .6\times.5\times.7\times.5 + .6\times.5\times.7\times.5 =
\end{aligned}
$$

No need to compute full numerical answer, see instructions above

B. [10%] Compute Pr(p1|¬p3, p4):

$$
\begin{aligned}
Pr(p_1|\neg p_3,p_4) &= \frac{Pr(p_1,\neg p_3,p_4)}{Pr(\neg p_3,p_4)} = \frac{.2328}{.5298} = \mathbf{.4394} \\
Pr(p_1,\neg p_3,p_4) &= \sum_{P_2} Pr(p_1,P_2,\neg p_3,p_4) = \sum_{P_2} Pr(p_1)Pr(P_2|p_1)Pr(\neg p_3|P_2)Pr(p_4|P_2) = \\
&= Pr(p_1)Pr(p_2|p_1)Pr(\neg p_3|p_2)Pr(p_4|p_2) + Pr(p_1)Pr(\neg p_2|p_1)Pr(\neg p_3|\neg p_2)Pr(p_4|\neg p_2) = \\
&= .4\times.8\times.8\times.8 + .4\times.2\times.7\times.5 = .2048 + .028 = \mathbf{.2328} \\
Pr(\neg p_3,p_4) &= Pr(p_1,\neg p_3,p_4) + Pr(\neg p_1,\neg p_3,p_4) = .2328 + .297 = \mathbf{.5298} \\
Pr(\neg p_1,\neg p_3,p_4) &= \sum_{P_2} Pr(\neg p_1,P_2,\neg p_3,p_4) = \sum_{P_2} Pr(\neg p_1)Pr(P_2|\neg p_1)Pr(\neg p_3|P_2)Pr(p_4|P_2) = \\
&= Pr(\neg p_1)Pr(p_2|\neg p_1)Pr(\neg p_3|p_2)Pr(p_4|p_2) + Pr(\neg p_1)Pr(\neg p_2|\neg p_1)Pr(\neg p_3|\neg p_2)Pr(p_4|\neg p_2) = \\
&= .6\times.5\times.8\times.8 + .6\times.5\times.7\times.5 = .192 + .105 = \mathbf{.297}
\end{aligned}
$$

No need to compute full numerical answer, see instructions above

3. [10%] Decision trees

Given the following decision tree, show
how the new examples in the table would be
classified, by filling in the last column in the table.
If an example cannot be classified, enter *UNKNOWN* in the last column.
You receive 2% for each correct answer.

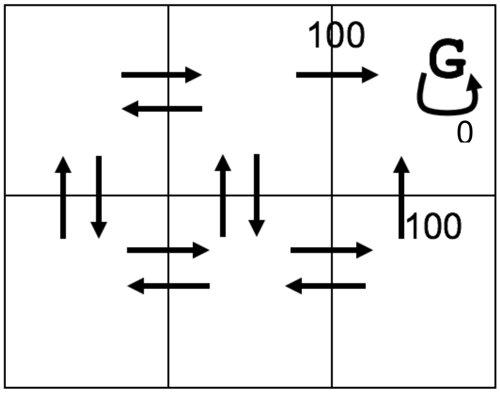
```
                   Color
                     |
              -------------------
      Blue|       Red|     Green|
          |          |          |
        Width       NO      Height
     -----------            ---------
Thin|        Fat|    Short|      Tall|
    |           |         |          |
   NO          YES       NO         YES
```

Example	Color	Height	Width	Class
A	Red	Short	Thin	NO
B	Blue	Tall	Fat	YES
C	Green	Short	Fat	NO
D	Green	Tall	Thin	YES
E	Blue	Short	Thin	NO

(space below available for rough work)

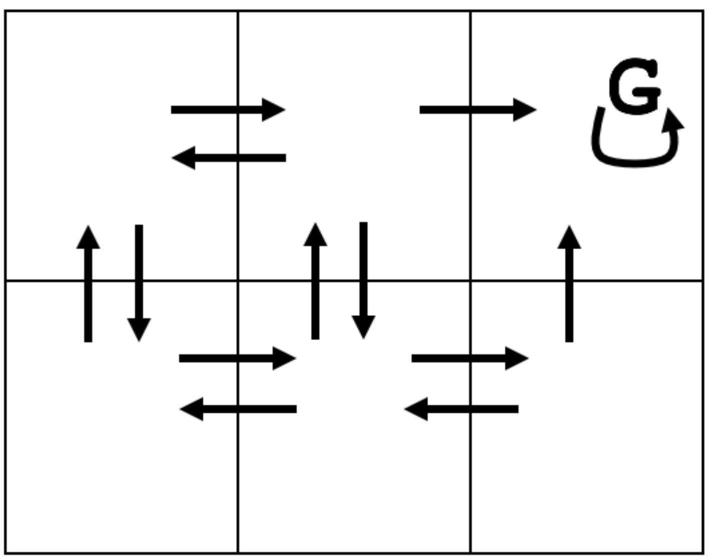
4. [20%] Markov Decision Processes

Consider the following MDP problem:



Assume that the value function is initialized to 0 in every cell. Assume a discount factor γ=0.5 and assume that the immediate reward associated with actions is 1 everywhere except for: a) the two actions that lead to G, whose immediate reward is 100, and b) the action from G to G, whose immediate reward is 0, as shown above. Assume that the actions always succeed.

Please fill in the values computed by the value iteration algorithm, at convergence, in the cells below:



answer: **51 100 0**

26.5 51 100

5. [10%] Neural Networks

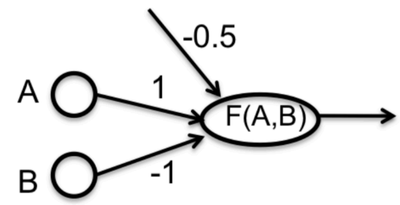
Can you represent the following boolean function f(A, B) with a single artificial neuron? If yes, show the weights and threshold. If not, explain why not in 1-2 sentences.

A	B	f(A,B)
1	1	0
0	0	0
1	0	1
0	1	0

Possible solution, assuming a threshold at 0

Other solutions possible with different weights and threshold values, please check against truth table

$$F(A,B) = 1\{A - B - 0.5 > 0\}$$



6. [10%] Bayes theorem

I don't have a car. I come to work either by bike or by bus. If I take the bus, there is a 10% chance that I am late. If I take the bike, there is a 2% chance that I am late. I take the bike 4 days out of 5. Today I was late. **What is the probability that I took the bus?**

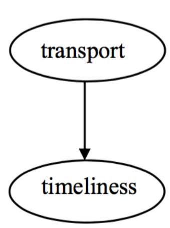
A [2%] Write down and explain the formula used in Bayes' theorem for this problem.

$$P(\text{bus} \mid \text{late}) = \frac{P(\text{late AND bus})}{P(\text{late})} = \frac{P(\text{late} \mid \text{bus}) * P(\text{bus})}{P(\text{late} \mid \text{bus}) * P(\text{bus}) + P(\text{late} \mid \text{bike}) * P(\text{bike})}$$

B. [3%] Use Bayes' theorem to calculate the probability that I took the bus today.

$$P(\text{bus} \mid \text{late}) = \frac{0.1 * 0.2}{0.1*0.2 + 0.02*0.8} = \frac{2}{2+1.6} = 20/36 = 5/9.$$

C. [5%] Model the situation as a Bayesian network with 2 nodes, and give the conditional probability tables for both nodes.



	bus	bike
	0.2	0.8

transport	late	on time
bus	0.1	0.9
bike	0.02	0.98

7. [20%] FOL Resolution Proof

Given: 1. $\forall_{S1,S2}$ subset$(S1, S2) \Leftrightarrow [\forall_X$ member$(X, S1) \Rightarrow$ member$(X, S2)]$.
Prove: H. $\forall_{S1,S2,S3}$ [subset$(S1, S2) \wedge$ subset$(S2, S3)] \Rightarrow$ subset$(S1, S3)$.

a. [9%] Convert sentence 1 and the negation of sentence H to CNF:

1a. $\neg$subset$(S1, S2) \vee \neg$member$(X, S1) \vee$ member$(X, S2)$.
1b. member(sk0$(S1, S2)$,$S1$) $\vee$ subset$(S1, S2)$.
1c. $\neg$member(sk0$(S1, S2)$,$S2$) $\vee$ subset$(S1, S2)$.
2a. subset(sk1,sk2).
2b. subset(sk2,sk3).
2c. $\neg$subset(sk1,sk3).

b. [11%] Draw your resolution proof. Only use the resolution inference rule, as you will lose points if you use any other rule. Please clearly show which sentences are resolved and what results. If unification is used at any step, please show the substitution, or you will lose points for each missing substitution.

From 2a and 1a, infer	3. $\neg$member(X,sk1) $\vee$ member(X,sk2).
From 2b and 1a, infer	4. $\neg$member(X,sk2) $\vee$ member(X,sk3).
From 3 and 4, infer	5. $\neg$member(X,sk1) $\vee$ member(X,sk3).
From 2c and 1b infer	6. member(sk0(sk1,sk3),sk1).
From 2c and 1c infer	7. $\neg$member(sk0(sk1,sk3),sk3).
From 6 and 5 infer	8. member(sk0(sk1,sk3), sk3).
From 7 and 8 infer	9. The empty clause.