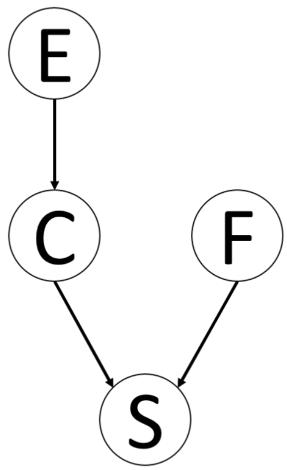
Bayes Net

Suppose John can be very sick (S) that can be caused by two different reasons: getting the seasonal flu and catching a cold (F and C). John thinks that he sometimes catches a cold because he works so hard in his final project that he is exhausted (E) and feels sick. The Bayes' Net and corresponding conditional probability tables are shown below.

For each question, write down the symbolic answer using combinations of P(...) probabilities that are given to you below, e.g., "P(+e) x P(-f)", where "+e" means "E is True" and "-f" means "F is False"). You do not need to calculate the numeric answer. You will use marks if you do not fully develop your answer so that it only uses the probabilities given below.

$P(E)$	
$+e$	0.1
$-e$	0.9

$P(C\|E)$		
$+e$	$+c$	1.0
$+e$	$-c$	0.0
$-e$	$+c$	0.1
$-e$	$-c$	0.9



$P(F)$	
$+f$	0.4
$-f$	0.6

$P(S\|C,F)$			
$+c$	$+f$	$+s$	1.0
$+c$	$+f$	$-s$	0.0
$+c$	$-f$	$+s$	0.9
$+c$	$-f$	$-s$	0.1
$-c$	$+f$	$+s$	0.8
$-c$	$+f$	$-s$	0.2
$-c$	$-f$	$+s$	0.1
$-c$	$-f$	$-s$	0.9

(1) [3%] Please write down the joint probability $P(+s,+c,+f,+e)$.
FORMULA IS OK, NO NEED FOR FINAL NUMBER

$$P(+s,+c,+f,+e) = P(+e)P(+f)P(+c|+e)P(+s|+c,+f) = 0.1 \times 0.4 \times 1.0 \times 1.0 = 0.04$$

so here, P(+e)P(+f)P(+c|+e)P(+s|+c,+f) is ok. FORMULA MUST ONLY USE the P(...) that are in the tables above.

(2) [3%] What is the probability of catching a cold? (Hint: Do not refer to that John feels sick)

$$P(+c) = P(+c|+e)P(+e) + P(+c|-e)P(-e) = 1.0 \times 0.1 + 0.1 \times 0.9 = 0.19$$

(3) [3%] What is the probability that John catches a cold given that he got the seasonal flu?

$$P(+c|+f) = P(+c) = 0.19$$

(4) [3%] What is the probability of working so hard to get a cold $P(+e \mid +c)$?

$$P(+e|+c) = \frac{P(+e,+c)}{P(+c|+e) + P(+c|-e)} = \frac{P(+e)P(+c|+e)}{P(+c|+e) + P(+c|-e)} = \frac{0.1 \times 1.0}{0.1 \times 1.0 + 0.9 \times 0.09} = 0.5263$$

(5) [3%] What is the probability $P(+e \mid +f)$?

$$P(+e|+f) = P(+e) = 0.1$$

(6) [5%] What is the the conditional probability $P(+c \mid +s,+f)$?

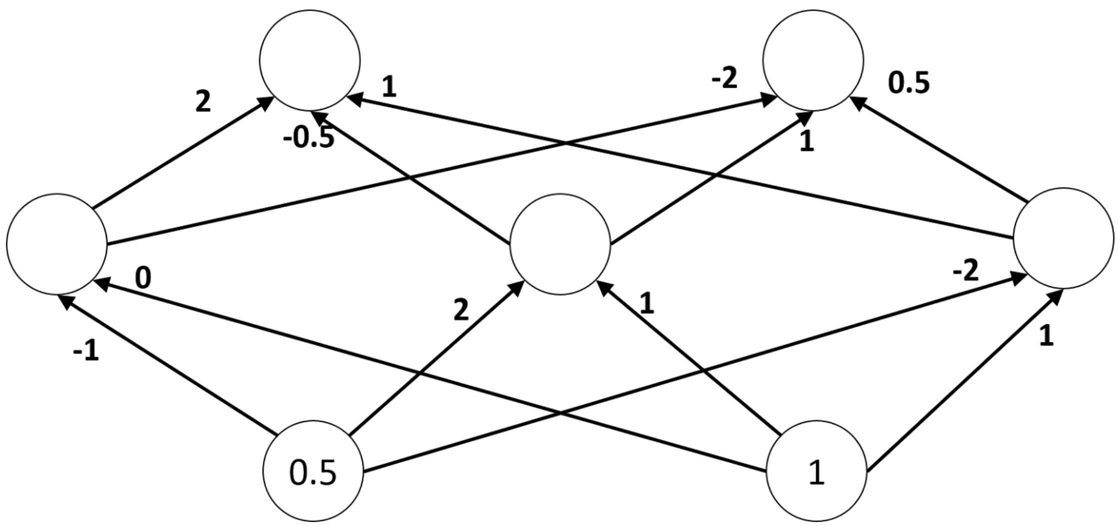
$$P(+c|+s,+f) = \frac{P(+c,+s,+f)}{P(+c,+s,+f) + P(-c,+s,+f)} = \frac{P(+c)P(+f)P(+s|+c,+f)}{P(+c)P(+f)P(+s|+c,+f) + P(-c)P(+f)P(+s|-c,+f)}$$

$$= \frac{0.19 \times 0.4 \times 1.0}{0.19 \times 0.4 \times 1.0 + 0.81 \times 0.4 \times 0.8} = \frac{0.076}{0.076 + 0.2592} = 0.2267$$

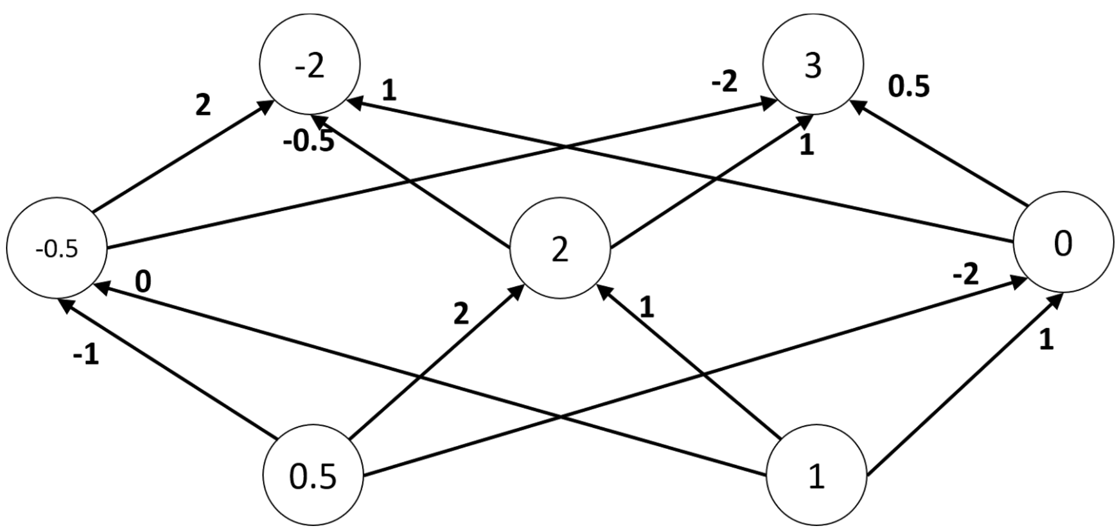
Neural Networks

The following is a network of linear neurons, that is, neurons whose output is identical to their net input. The numbers in the circles indicate the output of a neuron, and the numbers at connections indicate the value of the corresponding weight.

7.1. [5%] Compute the output of the hidden-layer and the output-layer neurons for the given input (0.5, 1) and enter those values into the corresponding circles.



Answer:



7.2. [5%] What is the output of the network for the input (1, 2), i.e. the left input neuron having the value 1 and the right one having the value 2? Do you have to do all the network computations once again in order to answer this question? Explain why you do or do not have to do this.

Answer:

It's (-4, 6). We do not have to do all the computations, because every neuron computes a **linear** function on its inputs, which means that the entire network computes a **linear** function. For such a function, if we double the input, we simply double the output as well.

1. **Decision Tree Learning**
1. I(P) = I (1216 , 416) = -34 log(34) - 14 log(14) = 0.81 (not wanted)
2. a) weather: sunny → 6 yes , 0 no   entropy = -1 * log(1) - 0*log(0) = 0
   weather: cloudy → 3 yes , 3 no   entropy = -0.5*log(0.5) – 0.5*log(0.5) = 1
   b) IG (choresToDo) = I (Hike) – remainder (choresToDo) =
   0.81 – [12*I(46, 26) + 12*I(56, 16)] = 0.81 – [0.5*0.92 + 0.5*0.65] = 0.025
   IG(weather) = I (Hike) – remainder(weather) =
   0.81 – [12*I(1,0) + 12*I(0.5,0.5)] = 0.81 – 0.5 = 0.31
   c) weather is the better choice. Because the information gain (IG) from it is more.

A [5%] You would like to know how useful it is for you to learn to take electric guitar lessons, so you would like to find out what the probability of getting hired by a heavy metal band is given that you took electric guitar lessons. What other probabilities would you need to know? (give a formula to show how these additional probabilities are used).

We're looking for P(hired by h-m band | can play e-guitar)

We need to know P(hired by h-m band), P(can play e-guitar), P(can play e-guitar | hired by h-m band)

P(hired by h-m band | can play e-guitar) = (P(can play e-guitar | hired by h-m band) P(hired by h-m band)) / P(can play e-guitar)

B [5%] (1) We have made a machine to classify whether a room smells fresh or unpleasant based on parts per million (PPM) of methane gas molecules that are in the air. A robot has taken a sample from room which has 100 PPM. From prior sampling, we have derived the following probabilities:

(a) P(100 PPM|Unpleasant) = 0.4
(b) P(100 PPM|Fresh) = 0.3
(c) P(Unpleasant) = 0.2
(d) P(Fresh) = 0.9
(e) P(100 PPM) = 0.2

Show using a Bayesian classifier whether the room's air should be classified as Fresh or Unpleasant. Be sure to show your work.

$$P(Unpleasant|100PPM) = \frac{P(100\ PPM|Unpleasant)P(Unpleasant)}{P(100\ PPM)}$$

$$P(Fresh|100PPM) = \frac{P(100\ PPM|Fresh)P(Fresh)}{P(100\ PPM)}$$

$$P(Unpleasant|100\ PPM) = \frac{0.4x0.2}{2} = 0.04$$

$P(Fresh|100\ PPM) = \frac{0.3x0.9}{0.2} = 1.35$ (*)

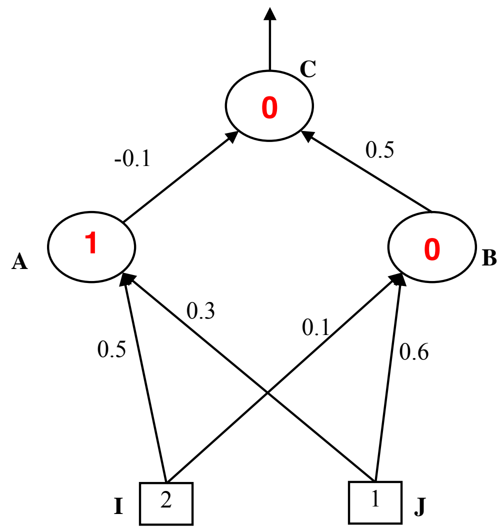
$$P(Fresh|100\ PPM) > P(Unpleasant|100\ PPM)$$

The air in the room is more likely to be fresh.

(*) NOTES: 1. Anyone who applies the Bayes rule correctly and gets 1.35 should get full points.
2. If in addition to that, they have indicated why is this an issue, give an additional 5 points bonus.
3. If they had 1.35, but erased it due to being >1, give full points.

5.1 [8%] Consider the neural network shown below, where I, J are inputs and A, B, C are perceptrons. For each perceptron assume it's firing is based on the threshold rule discussed in class with Threshold = 1 i.e.,

$$f(x) = \begin{cases} 1, & if\ x \geq 1 \\ 0, & otherwise \end{cases}$$



a) [3%] Write the ***output*** for each perceptron in the circles in the above figure.

NOTE: Students may have written the net value (e.g. 1.3) instead of the activation value, in that case give *0* points since the activation function was clearly mentioned.

b) [5%] Given the expected output to be 1, find the (a) error value, and the update weights for (b) AC and (c) BC

(a) **[1%]** Error = |Expected – Observed| = |1 – 0| = 1 (If student have correct formula but incorrect answer, give 0.5 points)

(b) **[2%]** AC_new = AC + Alpha * Error * A_Output = -0.1 + 1 * 1 = 0.9 ***(NOTE: Since the exact learning rate was not given, any answer* greater than -0.1 is acceptable if the student writes the formula or gives correct reasoning in words)**

(c) **[2%]** BC_new = BC + Alpha * Error * B_Output = 0.5 + 1 * 0 = 0.5 (Note: Given B's output is 0, the only acceptable answer is 0.5)

5.2 [12%] You are building a set of perceptrons to mimic logic circuits. For each perceptron assume it's firing is based on the threshold rule discussed in class i.e.

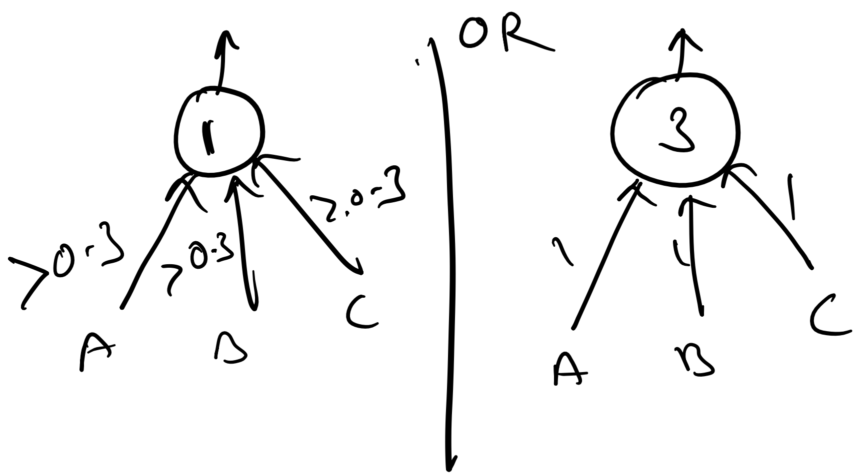
$$f(x) = \begin{cases} 1, & if\ x \geq 1 \\ 0, & otherwise \end{cases}$$

Assume all inputs have value either 0 or 1. Show how perceptrons can be used to mimic the following logic sentences. For each expression, (a) draw the perceptrons, (b) show the weights (c) write the threshold.

NOTE: (1) Looks like there was confusion regarding whether to fix threshold = 1 for all preceptrons or to allow any threshold value. We will accept either answer.

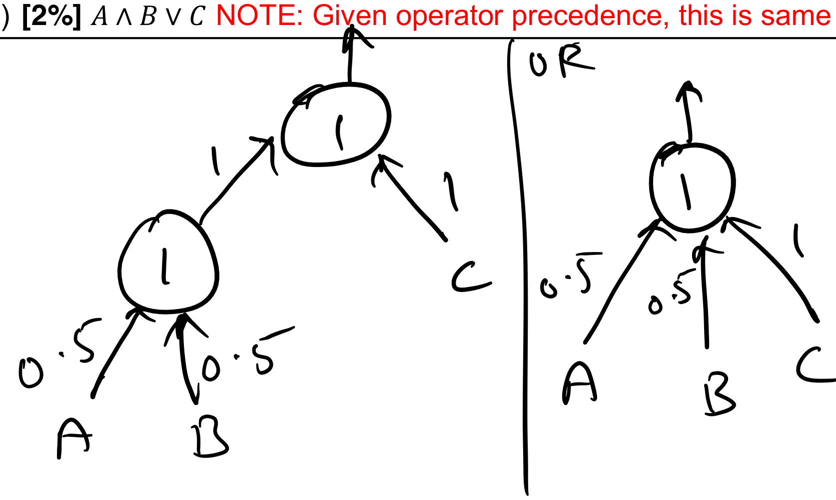
(2) The actual diagrams may differ. verify the solution using the truth table

a) **[2%]** $A \wedge B \wedge C$



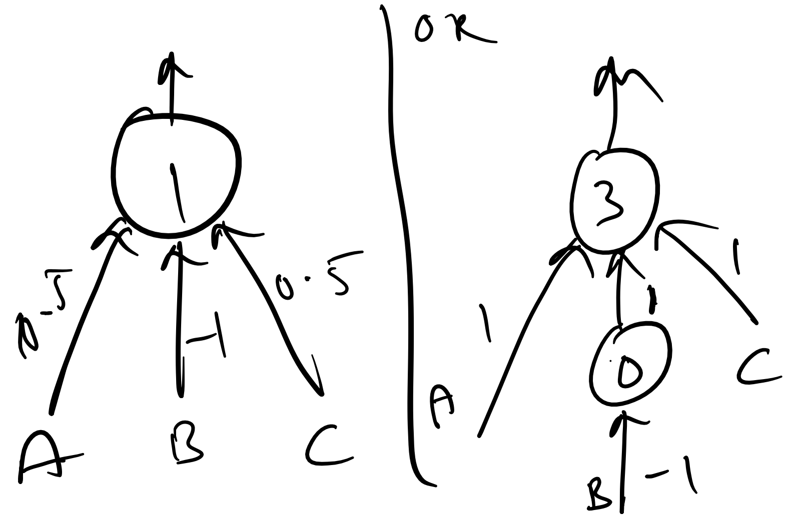
A	B	C	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

b) **[2%]** $A \wedge B \vee C$ NOTE: Given operator precedence, this is same as $(A \wedge B) \vee C$



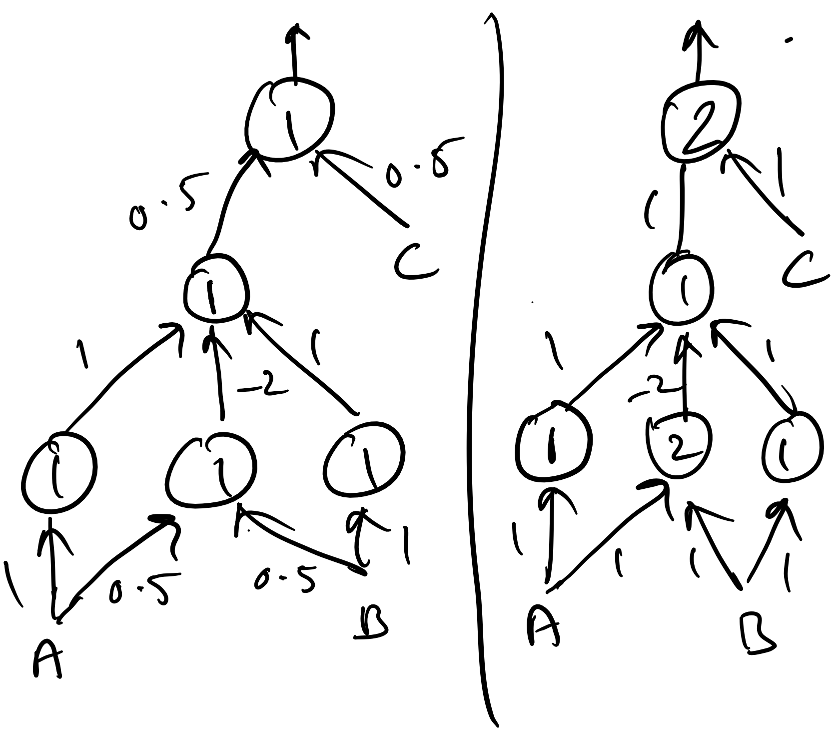
A	B	C	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

c) **[3%]** $A \wedge \neg B \wedge C$



A	B	C	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

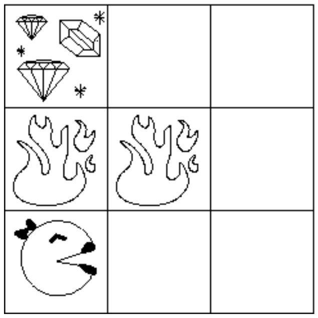
d) **[5%]** $A \oplus B \wedge C$



A	B	C	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

b. MDP - Treasure Hunting

While Pacman is out collecting all the dots from mediumClassic, Ms. Pacman takes some time to go treasure hunting in the Gridworld island. Ever prepared, she has a map that shows where all the hazards are, and where the treasure is. From any unmarked square, Ms. Pacman can take the standard actions (N, S, E, W), but she is surefooted enough that her actions always succeed (i.e. there is no movement noise). If she lands in a hazard (H) square or a treasure (T) square, her only action is to call for an airlift (X), which takes her to the terminal ‘Done’ state; this results in a reward of -64 if she’s escaping a hazard, but +128 if she’s running off with the treasure. There is no “living reward.”



1. What are the optimal values, V^* of each state in the above grid if $\gamma = 0.5$?

128	64	32
-64	-64	16
2	4	8

2. What are the Q-values for the last square on the second row (i.e., the one without fire)?

$Q((1,2), N) = 16$
$Q((1,2), W) = -32$
$Q((1,2), S) = 4$

3. What’s the optimal policy?

X	W	W
X	X	N
E	E	N

It turns out that Ms. Pacman's map is mostly correct, but some of the fire pits may have fizzled out and become regular squares! Thus, when she starts Q-learning, she observes the following episodes:
[(0, 0), N, 0, (0, 1), N, 0, (0, 2), X, 128, Done]
[(0, 0), N, 0, (0, 1), N, 0, (0, 2), X, 128, Done]
[(0, 0), N, 0, (0, 1), E, 0, (1, 1), X, -64, Done]

(f) What are Ms. Pacman's Q-values after observing these episodes? Assume that she initialized her Q-values all to 0 (you only have to write the Q-values that aren't 0) and used a learning rate of 1.0.

$Q((0,2), X) = 128$
$Q((0,1), N) = 64$
$Q((0,0), N) = 32$
$Q((1,1), X) = -64$
$Q((0,1), E) = 0$

(g) In most cases, a learning rate of 1.0 will result in a failure to converge. Why is it safe for Ms. Pacman to use a learning rate of 1.0?

Ms. Pacman's actions are deterministic, so she does not need to average over possible outcomes of a particular action.

(h) Based on your knowledge about the structure of the maze and the episodes Ms. Pacman observed, what are the *true* optimal values of each state?

128	64	32
64	-64	16
32	16	8