## 1. [8%] General AI Knowledge and Application.

**True or False: For each of the statements below, fill in the bubble T if the statement is always true; otherwise, fill in the bubble F.**

1). **[1%]** A probability model is completely determined by its full joint probability distribution.

2). **[1%]** In a Bayesian network of $n$ nodes, if node $X_1$ has exactly two parent nodes, $X_p$ and $X_q$, then
$P(X_1|X_p, X_q) = P(X_1|X_2, X_3, X_4, \cdots, X_n)$.

3). **[1%]** Given a sequence of observations over time, the Viterbi algorithm can be used to compute the posterior distribution of the state sequence that led to those observations.

4). **[1%]** Suppose taking action 1 results in winning 200 dollars, while taking action 2 has a 0.01% chance of winning 1 million dollars but a 99.99% chance of winning nothing. Then the principle of Maximum Expected Utility states that an agent is rational if and only if it prefers action 1.

5). **[1%]** The Bellman equation represents the expected utility of the optimal policy in sequential decision making models.

6). **[1%]** In machine learning, validation sets are typically used to deal with the overfitting problem.

7). **[1%]** In feedforward neural network models, back-propagation uses gradient descent to update the weights on each link.

8). **[1%]** A utility-based reinforcement learning agent (which seeks to compute a state-evaluation function) must have a model of the environment in order to optimize its decisions toward the maximum expected utility.

## 2.[16%] Multiple Choice

Each question has <u>one or more</u> correct choices. Check the boxes of all correct choices and leave the boxes of wrong choices blank. There is <u>no partial credit</u> for question in this section, so you must identify the exact set of correct choice(s) (if any) to get the credit of the question.

**1. [2%]** In the following Bayesian network, the Markov blanket of node D includes:
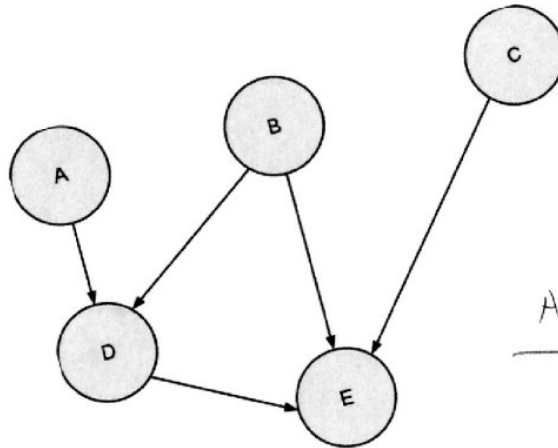


A, B, E, C.

|  |  |  |
|---|---|---|
| ☒ | a | Node A. |
| ☒ | b | Node B. |
| ☒ | c | Node C. |
| ☐ | d | Node D. |
| ☒ | e | Node E. |

**2. [2%]** Which of the following statements about the Gibbs Sampling algorithm, when used to solve Bayesian networks, is/are correct?

- ☒ a  It provides a consistent estimate of the posterior probability of the query variable.
- ☒ b  It is an approximate inference algorithm.
- ☐ c  It is an exact inference algorithm.
- ☒ d  It works by taking a random walk over the non-evidence variables.
- ☐ e  Its time complexity is linear in the number of CPT entries in the network.

**3. [2%]** Two people (A and B) are hunting in the same forest. Each of them can target either a deer or a rabbit, and the payoff matrix is shown below. Which of the following statements correctly describe(s) a Nash equilibrium of this multi-agent environment:

|  | Person A chooses to hunt deer | Person A chooses to hunt rabbit |
|---|---|---|
| Person B chooses to hunt deer | Payoff(A) = 4<br>Payoff(B) = 4 | Payoff(A) = 3<br>Payoff(B) = 1 |
| Person B chooses to hunt rabbit | Payoff(A) = 1<br>Payoff(B) = 3 | Payoff(A) = 2<br>Payoff(B) = 2 |

- ☐ a  Person A chooses to hunt deer, Person B chooses to hunt deer.
- ☐ b  Person A chooses to hunt deer, Person B chooses to hunt rabbit.
- ☐ c  Person A chooses to hunt rabbit, Person B chooses to hunt deer.
- ☒ d  Person A chooses to hunt rabbit, Person B chooses to hunt ra[bbit].
- ☐ e  This game does not admit a pure-strategy Nash equilibrium.

**4. [2%]** Suppose a computer system is learning to classify images into 10 classes. For each input image $x_i$, the system outputs a class label $h_i$ to the user, and receives the correctness of the label $c_i \in \{true, false\}$ as judged by the user, and the training data consists of the tuples $(x_i, h_i, c_i)$ for all images. What type of learning system does this computer system belongs to?

- [ ] a   Unsupervised learning.
- [ ] b   Reinforcement learning.
- [ ] c   Semi-supervised learning.
- [x] d   Supervised learning.

- [ ] e   Transfer learning.

**5.**   **[2%]**    Which of the following statements about the Q-function in Q-learning is/are true?

- [x] a   The Q-function represents the expected immediate reward of taking any possible action in a particular state.
- [x] b   The Q-function represents the expected immediate reward of taking only the best action in a particular state.
- [ ] c   The Q-function represents the expected long-term reward of taking any possible action in a particular state.
- [ ] d   The Q-function represents the expected long-term reward of taking only the best action in a particular state.

(6) **[2%]** Which of the following statements about artificial neural networks is/are true?

☒ a
    The L2 loss function is defined as $\sum_{i}( y_i - f(x_i) )^2$, where $y_i$ is the

    ground-truth value and $f(x_i)$ is the model output, for datum $i$, respectively.

☐ b   Every artificial neural network model has a closed-form (analytical) solution in terms of L2 loss function minimization.

☒ c   Regularization is a technique to iteratively minimize the loss function.

☐ d   L2 regularization tends to result in sparse models.

☐ e   None of the above is correct.

✕

**7. [2%]** Given training data as shown by the following figure (where squares and circles represent two different ground-truth labels, and their centers represent the values of the corresponding data vectors), and given the facts that

i) The straight line through 1 and 3 is parallel with the straight line through 4 and 5;

ii) All angles in the triangle with vertexes 2, 4, and 5 are acute (i.e. less than 90 degrees).

Then, the support vectors identified by the SVM algorithm (**for both classes**) will include:
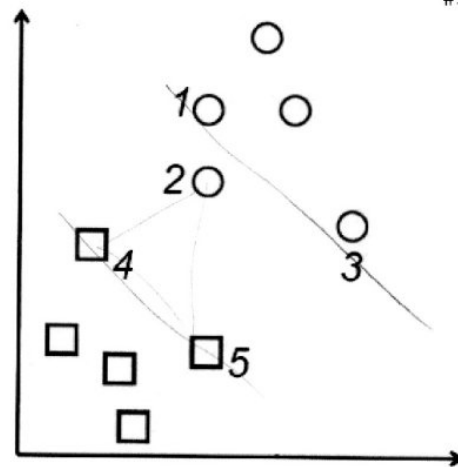
☐ a Vector 1
☒ b Vector 2
☐ c Vector 3
☒ d Vector 4
☒ e Vector 5

**8. [2%]** Which of the following machine learning algorithms use(s) **non-parametric** models?

☐ a Linear regression
☐ b K-nearest neighbor
☒ c Artificial neural network
☒ d Linear classification
☐ e None of above

### 3.[23%] MDPs and RL: Mini-Grids

The following problems take place in various scenarios of a grid-world MDP, illustrated below. In all cases, A is the start state and double-rectangle states are terminal states. From a terminal state, the only action available is Exit, which results in the listed reward and ends the game. From non-exit states, the agent can choose either Left or Right actions, which move the agent in the corresponding direction by 1 square at a time. The only non-zero rewards come from terminal states (exiting the grid); all other squares give zero reward. Throughout this problem, assume that value iteration begins with initial values $V_0(s) = 0$ for all states $s$. First, consider the following mini-grid. For now, the discount is $\gamma = 1$ and legal movement actions will always succeed (and so the state transition function is deterministic).

*(handwritten margin notes: $R(s) = 0$; $10 + \gamma^2 > \gamma$; $\gamma > 0.1$; $\frac{10}{\gamma^2} > \frac{1}{\gamma}$; $\gamma < 10$)*
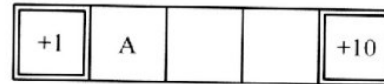
| +1 | A | | | +10 |
|----|---|---|---|-----|

**3A.[1%]** What is the optimal value $V^*(A)$?

> 10  ✓

**3B.[1%]** When running value iteration, remember that we start with $V_0(s) = 0$ for all $s$. What is the first iteration $k$ for which $V_k(A)$ will be non-zero?

> $k = 2$  if in the first iteration agent chooses left.  ✓

**3C.[1%]** What will $V_k(A)$ be when it is first non-zero?

> 1  ✓

**3D.[1%]** After how many iterations $k$ will we have $V_k(A) = V^*(A)$? If they will never become equal, write "never".

> 5  ✗          -1

**3E.[1%]** Now the situation is as before, but $\gamma$ = 0.5. What is the optimal value $V^*(A)$?

a5    ✗

**3F.[2%]** For what range of values $\gamma$ of the discount will it be optimal to go Right from A? Write "all" or "none" if all or no legal values of $\gamma$, respectively, have this property.

$\gamma > 0.1$    ✗

The Left and Right movement actions are now stochastic and fail with probability $f$. When an action fails, the agent stays in place. The Exit action does not fail. For the following mini-grid, the failure probability is $f$ = 0.5. The discount is back to $\gamma$ = 1.

| A | 2.5 | 5 | +10 |

$0.5 \times 10 = 5$

**3G.[1%]** What is the optimal value $V^*(A)$?

2.5    ✗

**3H.[1%]** When running value iteration, what is the smallest value of $k$ for which $V_k(A)$ will be non-zero?

3    ✗

-7

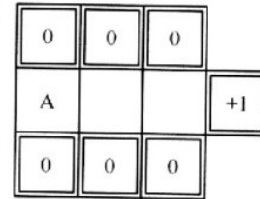**3I.[1%]** What will $V_k(A)$ be when it is first non-zero?

2.5    ✗

**3J.[1%]** After how many iterations $k$ will we have $V_k(A) = V^*(A)$ If they will never become equal, write "Never".

3    ✗

Now consider the following mini-grid, where the actions Left and Right result in a move up or down, with a probability $f/2$ each. Again, the overall failure probability is $f = 0.5$, and $\gamma = 1$. The only action available from the double-walled exit states is Exit.

| 0 | 0 | 0 |
|---|---|---|
| A | | +1 |
| 0 | 0 | 0 |

**3K.[1%]** What is the optimal value $V^*(A)$?

**3L.[1%]** What will be $V_k(A)$ when it is first non-zero?

**3M.[1%]** After how many iterations $k$ will we have $V_k(A)=V^*(A)$? If they will never become equal, write "never".

-3

Finally, consider the following mini-grid (rewards shown on bottom, state names shown on top).

| L<br>+4 | A | R<br>+16 |
|---|---|---|

In this scenario, the discount is $\gamma = 1$. The failure probability is actually $f = 0$, but now we no longer know the details of the MDP ahead of time. We must instead use reinforcement learning to compute the necessary values. We observe the following transition sequence (state X is the end-of-game terminal state):

| s | a | s' | r |
|---|---|---|---|
| A | Right | R | 0 |
| R | Exit | X | 16 |
| A | Left | L | 0 |
| L | Exit | X | 4 |
| A | Right | R | 0 |
| R | Exit | X | 16 |
| A | Left | L | 0 |
| L | Exit | X | 4 |

**3N.[2%]** If these transitions repeated many times and learning rates were appropriately small for convergence, what would temporal difference learning converge to for the value of *A*?

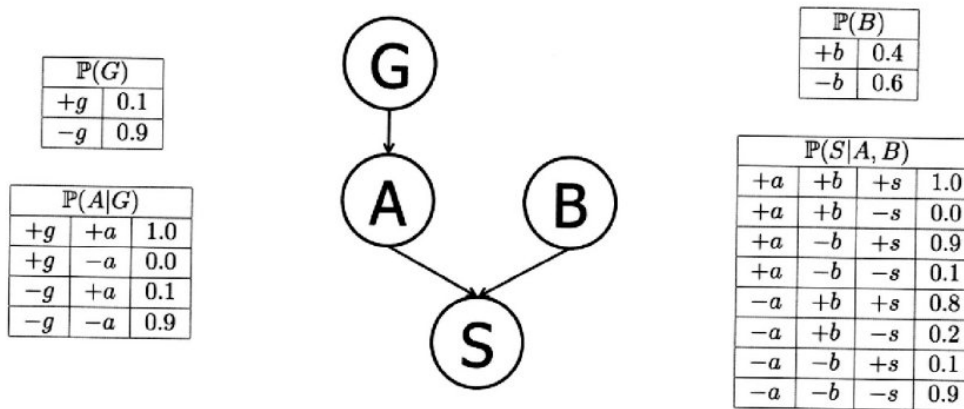**3O.[2%]** After this sequence of transitions, if we use a learning rate of $\alpha = 0.5$, what would Q-learning learn for the Q-value of (*A*, Right)? Remember that $Q(s, a)$ is initialized with 0 for all $(s, a)$.

**3P.[2%]** If these transitions repeated many times and learning rates were appropriately small for convergence, what would Q-learning converge to for the Q-value of (*A*, Right)?

-6

## 4.[18%] Bayesian Network

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding conditional probability tables for this situation are shown below. For each part, you may leave your answer as an arithmetic expression.

$\mathbb{P}(G)$

| | |
|---|---|
| +g | 0.1 |
| −g | 0.9 |

$\mathbb{P}(A|G)$

| | | |
|---|---|---|
| +g | +a | 1.0 |
| +g | −a | 0.0 |
| −g | +a | 0.1 |
| −g | −a | 0.9 |

$\mathbb{P}(B)$

| | |
|---|---|
| +b | 0.4 |
| −b | 0.6 |

$\mathbb{P}(S|A, B)$

| | | | |
|---|---|---|---|
| +a | +b | +s | 1.0 |
| +a | +b | −s | 0.0 |
| +a | −b | +s | 0.9 |
| +a | −b | −s | 0.1 |
| −a | +b | +s | 0.8 |
| −a | +b | −s | 0.2 |
| −a | −b | +s | 0.1 |
| −a | −b | −s | 0.9 |

## 4A.[3%] Compute the following entry from the joint distribution:

P(+g, +a, +b, +s) = ?

$$P(+g, +a, +b, +s) = P(+s|+a, +b) \cdot P(+b) \cdot P(+a|+g) \cdot P(+g)$$

$$= 1 \times 0.4 \times 1 \times 0.1$$

$$= 0.04 \checkmark$$

Score: **8.0**

**4B.[3%]** What is the probability that a patient has disease A?

$$P(+a) = P(+a|+g) \cdot P(+g) + P(+a|-g) \cdot P(-g)$$

✓

**4C.[3%]** What is the probability that a patient who has disease B also has disease A?

$$P(+a|+b) = \frac{P(+a,+b)}{P(+b)} = \frac{P(+b) \cdot \sum_g P(+a|g) \cdot P(g)}{P(+b)}$$

$$= P(+a|+g) \cdot P(+g) + P(+a|-g) \cdot P(-g)$$

✓

**4D.[3%]** What is the probability that a patient who shows symptom S has either disease A or B or both?

$$1 - P(-a, -b|+s)$$

$$= 1 - \frac{P(-a, -b, +s)}{P(+s)}$$

$$= 1 - \frac{P(+s|-a, -b) \cdot P(-b) \cdot \left(P(-a|+g) \cdot P(+g) + P(-a|-g) \cdot P(-g)\right)}{P(+s)}$$

✓

-1: P(S) expansion

**4E.[3%]** What is the probability that a patient who has disease *A* has the disease-carrying gene variation *G*?

$$P(+g \mid +a) = \frac{P(+a \mid +g) \cdot P(+g)}{P(+a)}$$

✓

**4F.[3%]** What is the probability that a patient who has disease *B* has the disease-carrying gene variation *G*?

$$P(+g \mid +b) = P(+g)$$

Since they are conditionally independent.

✓

## 5.[20%] Decision Trees

NASA wants to be able to discriminate between Martians (M) and Humans (H) based on the following characteristics:
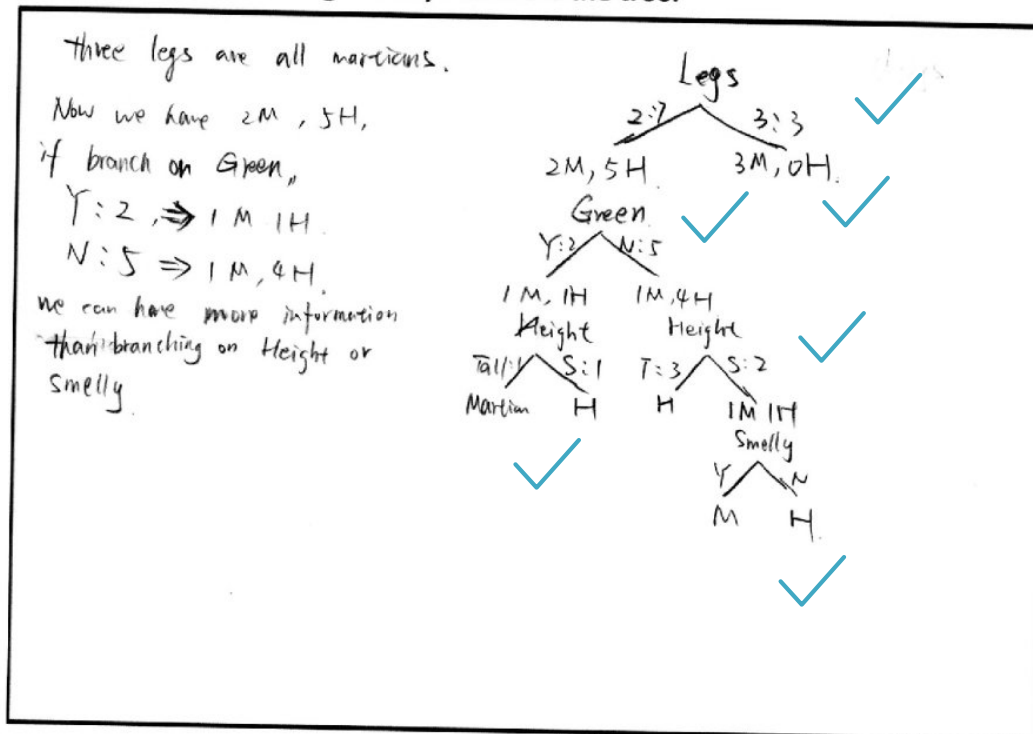
$Green \in \{No, Yes\}$,

$Legs \in \{2, 3\}$,

$Height \in \{Short, Tall\}$,

$Smelly \in \{No, Yes\}$. **Our available training data is in the table:**

| | Species | Green | Legs | Height | Smelly |
|---|---|---|---|---|---|
| 1) | M | N | 3 | S | Y |
| 2) | M | Y | 2 | T | N |
| 3) | M | Y | 3 | T | N |
| 4) | M | N | 2 | S | Y |
| 5) | M | Y | 3 | T | N |
| 6) | H | N | 2 | T | Y |
| 7) | H | N | 2 | S | N |
| 8) | H | N | 2 | T | N |
| 9) | H | Y | 2 | S | N |
| 10) | H | N | 2 | T | Y |

## 5A. [8%] Greedily learn a decision tree by maximizing information gain at each branch (as in the C4.5 algorithm) and draw the tree.

three legs are all martians.

Now we have 2M, 5H,

if branch on Green,

Y: 2, ⟹ 1M 1H.

N: 5 ⟹ 1M, 4H.

we can have more information than branching on Height or Smelly.

Legs
- 2:7 → 2M, 5H
- 3:3 → 3M, 0H

Green
- Y:3 → 1M, 1H
- N:5 → 1M, 4H

Height (under Y:3)
- Tall → Martian
- S:1 → H

Height (under N:5)
- T:3 → H
- S:2 → 1M 1H

Smelly (under S:2)
- Y → M
- N → H

**5B. [6%]** Write the learned concept for Martian as a set of conjunctive rules (e.g., if (Green=Y and Legs=2 and Height=T and Smelly=N), then Martian; else if ... then Martian; ...; else Human).

If (Legs = 3 ) then Martian ✓

If (Legs=2, Green = Y, Height=T ) then Martian ✓

If ( Legs=2, Green = N, Height = S and Smelly = Y) then Martian ✓

**5C. [6%]** The solution of part 5B above uses up to 4 attributes in each conjunction. Find a set of conjunctive rules using only 2 attributes per conjunction that still results in zero error in the training set. Can this simpler hypothesis be represented by a decision tree of depth 2? Justify.

If ( Legs = 3 ) then Martian

If ( Green = Y, Height = T ) then Martian ✓

If ( Height = T, Smelly = Y) then Martian

No, total attributes used are more than two. ✗

## 6.[18%] HMMs

The Avengers have a single key to the infinity stone container (Orb), and they take special care to protect it. Despite their efforts, they still seem to lose the key on a regular basis. On days that Tony Stark has the key, there is a 60% chance that he will forget it next to his Iron Man Armor (Mark III) in his laboratory. When that happens, one of the other three Avengers (Captain America, Thor, Hulk, with each being equally likely) will always find it during their meeting in Tony's lab right afterward. On the days that Captain America has the key, there is a 50% chance that he forgets it at his favorite Starbucks. In those cases, Hulk will always find the key afterward, as the same Starbucks is his favorite, too. Unfortunately, Hulk has a hole in his pocket and, on days when he has the key, he ends up losing it 80% of the time somewhere on SHIELD street. However, Thor takes the same path on his way to Asgard and always finds the key on the street. Thor has a 10% chance to lose the key somewhere in Stark Tower next to the Dummy Robot, but luckily Hulk always finds it in those cases.

The Avengers lose the key at most once per day, around noon (after losing it they become extra careful for the rest of the day), and they always find it the same day in the early afternoon. No Avenger gives up the key voluntarily, so whoever has the key, keeps it until he loses it.

**Score: 5.0**

<u>6A. [8%]</u> Draw the Markov chain capturing the location of the key and fill in the transition probability table. In this table, the entry of row TS (Tony Stark) and column TS corresponds to $P(X_{t+1} = TS \mid X_t = TS)$, the entry of row TS and column CA (Captain America) corresponds to $P(X_{t+1} = CA \mid X_t = TS)$, and so forth.
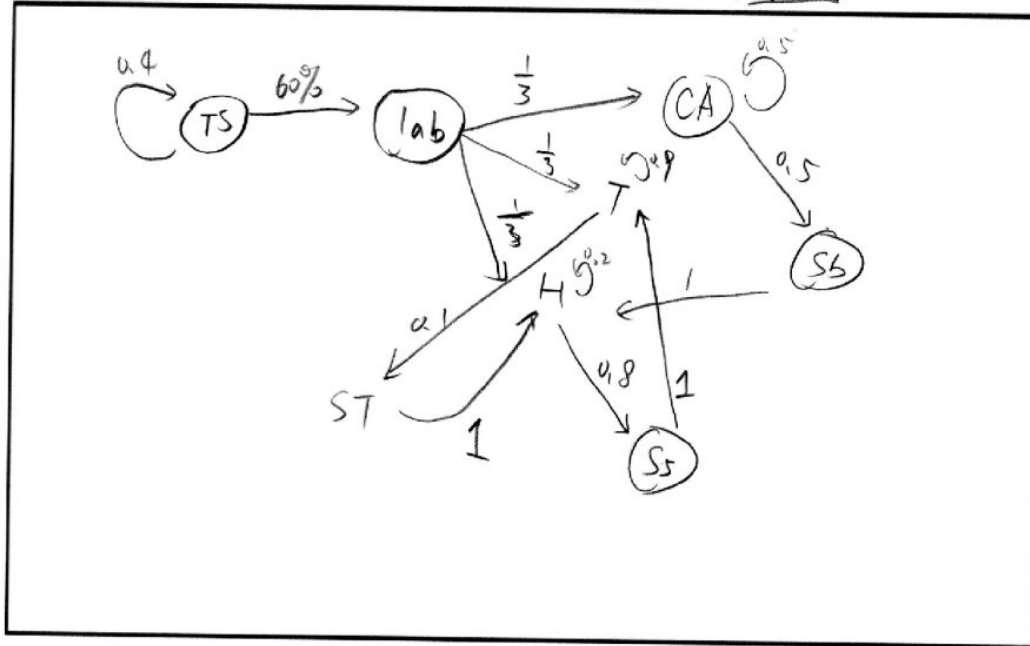


| | $TS_{t+1}$ | $CA_{t+1}$ | $H_{t+1}$ | $T_{t+1}$ |
|---|---|---|---|---|
| $TS_t$ | 0.4 | $0.6 \times \frac{1}{3}$ | ✗ | ✗ |
| $CA_t$ | | 0.5 | ✗ | |
| $H_t$ | | | 0.2 | ✗ |
| $T_t$ | | | ✗ | 0.9 |

Incomplete.Please refer solution.

<u>6B. [4%]</u> **Early Monday morning Agent Phil handed the key to Captain America. (The initial state distribution assigns probability 1 to $X_0 = CA$ and probability 0 to all other states.) Nova Prime is coming to inspect the infinity stone Tuesday at midnight so the Avengers need the key to open the container for the inspection. What is the probability distribution over the Avengers who might have the key at that time? Let $X_0$, $X_{Mon}$ and $X_{Tue}$ be random variables corresponding to who has the key when Agent Phil hands it out, who has the key on Monday evening, and who has the key on Tuesday evening, respectively. Fill in the probabilities in the table below. Please also briefly explain how you get the result for each blank space.**

|     | $P(X_0)$ | $P(X_{Mon})$ | $P(X_{Tue})$ |
|-----|----------|--------------|--------------|
| TS  | 0        |              |              |
| CA  | 1        |              | ✕            |
| H   | 0        |              |              |
| T   | 0        |              |              |

<u>6C. [6%]</u> **Dr. Strange wants to know which Avenger is likely to have the key in the distant future, but he no longer has a Time Stone to answer this question for him. Please help him determine the probability of each Avenger having the key infinitely far into the future.**

✕