AMATH 353: Homework 4
Due April, 13 2018
ID: 1064712

Trent Yarosevich

April 13, 2018

Instructor: Jeremy Upsal

Part 1.)

With some slight rearrangement we have:

$$u(x,t) = \frac{3}{1+\cos^2(7(x-\frac{5}{7}t)+2)} \tag{1}$$

This is a traveling wave solution with $c = \frac{5}{7}$ and f, suppose some $f(z)$, as follows:

$$f(z) = \frac{3}{1+\cos^2(7z+2)} \tag{2}$$

Part 2.)

a.) From linearized Burgers: $u_t + au_x = du_{xx}$ we get

$$\begin{aligned} -iwu + aiku &= d(ik)^2 u \\ -iw + ai &= d(ik)^2 \\ -w + ak &= dik^2 \\ w &= ak - dik^2 \end{aligned} \tag{3}$$

a.) is FALSE, this $c_p(k)$ is not real for real k values.
b.) is FALSE because the dispersion relation IS constant in respect to k, i.e. $a - dik$.

b.) From Schrödinger's equation: $iu_t + u_{xx} = 0$ we get

$$\begin{aligned} i(-iw)u + (ik)^2 u &= 0 \\ i(-iw) + (ik)^2 &= 0 \\ -i^2 w + i^2 k^2 &= 0 \\ w &= k^2 \end{aligned} \tag{4}$$

a.) is TRUE because there are no complex numbers in the dispersion relation (and thus the phase velocity).
b.) is FALSE because $c_p(k) = \frac{w}{k} = k$, so the dispersion relation IS constant with respect to k.

c.) From the wave equation $u_{tt} = au_{xx}$ we get:

$$\begin{aligned} (-iw)^2 u &= a(ik)^2 u \\ -(iw)^2 &= a(ik)^2 \\ -i^2 w^2 &= ai^2 k^2 \\ w^2 &= -ak^2 \\ w &= \sqrt{-ak^2} \\ w &= \pm i\sqrt{a}k \end{aligned} \tag{5}$$

a.) is FALSE because the dispersion relation (and phase velocity) contains a complex number.
b.) is TRUE because $c_p(k) = \pm i\sqrt{a}$, and is not dependent on k, so the equation is non-dispersive.