AMATH 353: Homework 12
Due May, 18 2018
ID: 1064712

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Part 1 Assuming $u(x,t) = u(x(t),t)$, by the chain rule we have $\frac{d}{dt}u = u_t + u_x\frac{dx}{dt}$. Given that we are solving $u_t + 2u_x = 0$, if we assume $\frac{dx}{dt} = 2$ we get the following:

$$\frac{d}{dt}(u(x(t),t)) = u_t + 2u_x = 0 \tag{1}$$

This gives us the two ODEs:

$$\boxed{\begin{aligned}\frac{dx}{dt} &= 2\\ \frac{du}{dt} &= 0\end{aligned}} \tag{2}$$

Solving the first ODE by separation of variables, we get the following equation for the characteristic curves, which are shown in the plot below:

$$x(t) = 2t + x_0 \tag{3}$$

Out[183]=

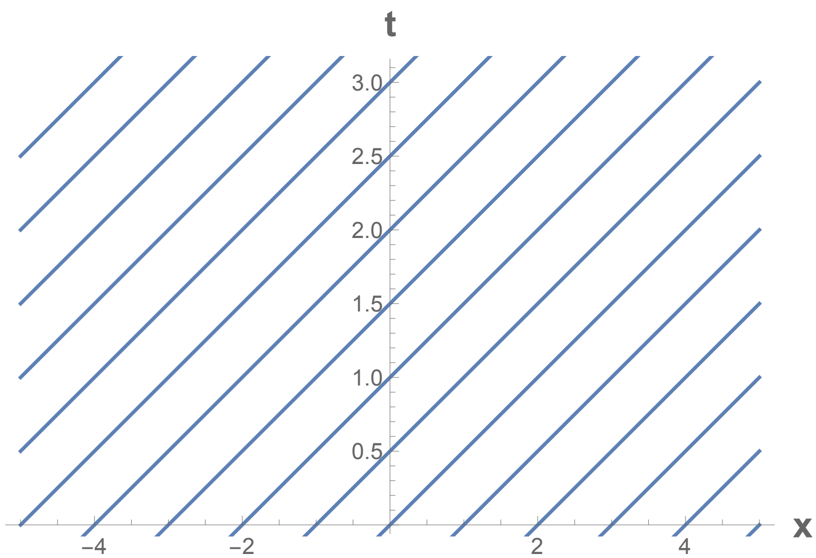

Figure 1: $t = \frac{x - x_0}{2}$

Solving the second ODE we simply get a constant:

$$\begin{aligned} \int \frac{du}{dt} &= \int 0 dt \\ u(x(t), t) &= A \end{aligned} \tag{4}$$

Making use of the initial condition, $u(x, 0) = e^{-x^2}$ we have the following:

$$\begin{aligned} u(x(t), 0) = u(x_0, 0) = u_0(x_0) &= A \\ u_0(x_0) &= e^{-x_0^2} \\ A &= e^{-x_0^2} \end{aligned} \tag{5}$$

$$\boxed{u(x(t), t) = e^{-x_0^2} \qquad (6)}$$

This means that along any given characteristic line $u(x, t) = u(x(t), t)$ we have u constant at a value determined by the initial value of that particular characteristic.

Now using an example of a point $(3, 4)$ we plug it into the characteristic curve and determine it's x_0 value, then determine the value of u_0 at that point, and thus u along that entire characteristic curve:

$$\begin{aligned} x_0 &= x - 2t \\ x_0 &= 3 - 4(4) = -5 \end{aligned} \tag{7}$$

$$\boxed{u_0(-5) = e^{-(-5)^2} = e^{-25}} \tag{8}$$

We can arrive at the same value generally by plugging the x_0 equation into the equation derived above for $u(x(t), t)$, then plugging $(3, 4)$ into that:

$$\begin{aligned} x_0 &= x - 2t \\ u(x(t), t) &= e^{-x_0^2} \\ u(x, t) &= e^{-(x-2t)^2} \end{aligned} \tag{9}$$

$$\boxed{e^{-(3-2(4)^2} = e^{-25}} \tag{10}$$

bla