

AMATH 353: Homework 2

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In order to change the range of t to some $\tau > 0$, I used the following function:

$$\tau(t) = t - T \tag{1}$$

By plugging this new variable into u and using the chain rule we get:

$$\begin{aligned} \hat{u}_{tt} &= \frac{d}{dt} \left(\frac{du}{dt} \right) \\ \hat{u}_{tt} &= \frac{d}{dt} \left(\frac{du}{d\tau} \frac{d\tau}{dt} \right) \end{aligned} \tag{2}$$

Because $\frac{d\tau}{dt} = 1$ we then get:

$$\hat{u}_{tt} = \frac{d}{dt} \left(\frac{du}{d\tau} \right) \tag{3}$$

We then do the same procedure again:

$$\begin{aligned} \hat{u}_{tt} &= \frac{d^2 u}{d\tau} \frac{d\tau}{dt} \\ \hat{u}_{tt} &= \frac{d^2 u}{d\tau} = u_{\tau\tau} \end{aligned} \tag{4}$$

The fact that there is no change in the PDE is to be expected since we are merely re-locating the function to the left, with no alterations to its rate of change. The values for the boundary conditions are adjusted accordingly to $\hat{u}(a, h_1(\tau)) = u(a, t - T)$ and $\hat{u}(b, h_2(\tau)) = u(b, t - T)$.