AMATH 353: Homework 3
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Part 1.)

a.) By $u(x,t) = f(x - ct)$ and $z = x - ct$ the PDE (Klein-Gordon) $u_{tt} = au_{xx} - bu$ becomes:

$$c^2 f''(z) = af''(z) + bf(z) \tag{1}$$

b.) Rewriting the ODE as $(c^2 - a)f''(z) + bf(z) = 0$ we arrive at a trivial solution when $c^2 = a$, or when $c = \pm\sqrt{a}$. This value of c results in the second order derivative being zero, yielding the trivial solution $bf(z) = 0$ or $f(z) = 0$.

Accordingly, we get non-trivial solutions both when $c^2 > a$ and when $c^2 < a$.

c.) If we let $A^2 = \frac{b}{c^2 - a}$ we then have the equation

$$f''(z) + A^2 f(z) = 0 \tag{2}$$

which we then solve using the characteristic polynomial for both cases of non-trivial solutions. When $c^2 > a$ it follows that A^2 will be positive, resulting in:

$$r^2 + A^2 = 0 \tag{3}$$

and by quadratic formula we find the roots

$$r = \pm Ai \tag{4}$$

which gives us a solution

$$f(z) = C_1 cos(Ax) + C_2 sin(Ax) \tag{5}$$

This then results in an oscillatory solution for values of $c > a^2$

$$u(x,t) = C_1 cos(\sqrt{\frac{b}{c^2 - a}}(x - ct)) + C_2 sin(\sqrt{\frac{b}{c^2 - a}}(x - ct)) \tag{6}$$

d.) For values of $c < a^2$ we take the same approach, but A^2 is now negative, resulting in the following with the characteristic polynomial:

$$\begin{aligned} r^2 - A^2 &= 0 \\ (r - A)(r + A) &= 0 \\ r &= \pm A \end{aligned} \tag{7}$$

resulting in a solution of

$$f(z) = C_1 e^{Az} + C_2 e^{-Az} \tag{8}$$

If we then let $C_1 = 0$ we arrive at a solution that decays as x goes to ∞. Substituting back in for Z:

$$u(x, t) = C_2 e^{\sqrt{\frac{b}{c^2 - a}}(x - ct)} \tag{9}$$