AMATH 353: Homework 1
Due April, 4 2018
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Part 1.)

a.) To create a wave moving left with speed one, I used the equation

$$u(x,t) = exp(-x - ct)^2) \tag{1}$$

with $c = -1$. The following figure displays its movement to the left over time with a speed of 1.

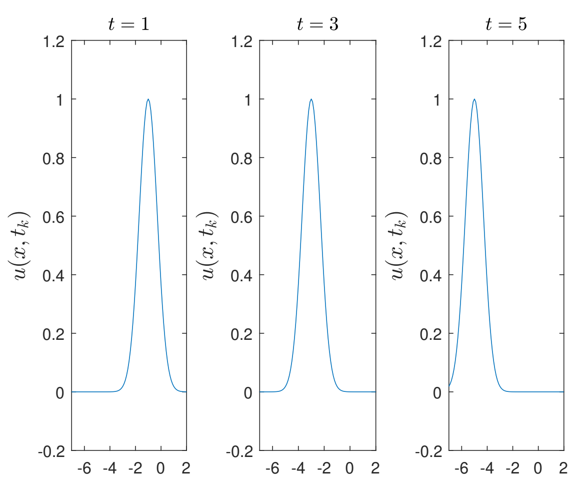

Figure 1: Moving left with speed 1.

b.) and c.) Parts b.) and c.) were executed with the following values for c in the same equation used on part a.), which I have accompanied with visualizations. Note that the 'speed' is displayed by the identical figures and increasing or decreasing x-axes.

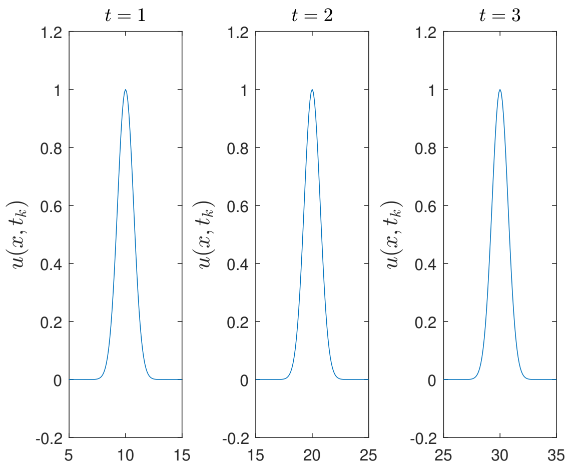

Figure 2: $c = 10$, moving right with speed 10.

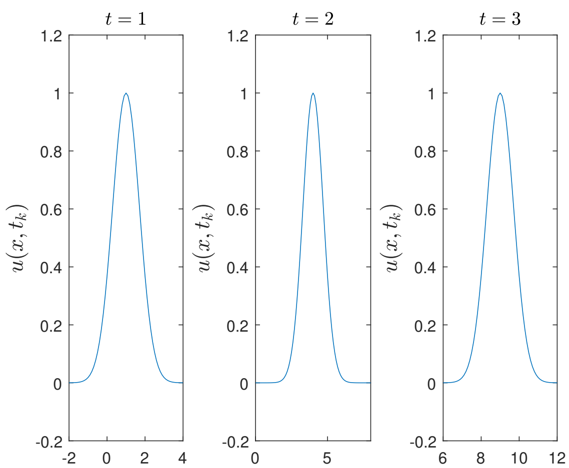

Figure 3: $c = t$, moving right with speed t.

d.) Part d.) modifies the equation in a.) to implement decreasing amplitude inversely proportional to t.

$$u(x,t) = \frac{1}{t} exp(-x - ct)^2) \tag{2}$$

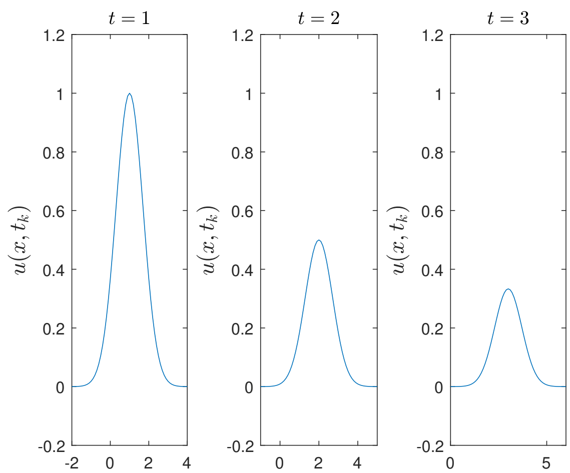

Figure 4: $c = 10$, moving right with speed 1 and amplitude inversely proportional to t.

Part 2.)

a.) $v_{tt} - v_{xxx} = 0$

b.) Let $\alpha = \beta = 1$

By linear differential operators,

$$\begin{aligned} u_{3,tt} &= u_{3,xxx} \\ (u_1 + u_2)_{tt} &= (u_1 + u_2)_{xxx} \\ u_{1,tt} + u_{2,tt} &= u_{1,xxx} + u_{2,xxx} \\ u_{1,tt} + u_{2,tt} - u_{1,xxx} - u_{2,xxx} &= 0 \\ u_{1,tt} - u_{1,xxx} + u_{2,tt} - u_{2,xxx} &= 0 \end{aligned} \tag{3}$$

Thus as hoped after plugging in u_3 we arrive at the sum of two homogeneous equations of the solutions u_1 and u_2.

c.) $v_{tt} + vv_{xxx} = 0$

d.)

$$\begin{aligned} u_{3,tt} + u_3 u_{3,xxx} &= 0 \\ (u_1 + u_2)_{tt} + (u_1 + u_2)(u_1 + u_2)_{xxx} &= 0 \\ u_{1,tt} + u_{2,tt} + u_1 u_{1,xxx} + u_1 u_{2,xxx} + u_2 u_{1,xxx} + u_2 u_{2,xxx} &= 0 \end{aligned} \tag{4}$$

And then, because $v_{tt} + vv_{xxx} = 0$ for both solutions u_1 and u_2, we are left with

$$u_1 u_{2,xxx} + u_2 u_{1,xxx} = 0 \tag{5}$$

which only holds true for certain special cases of u_1 and u_2.

e.) $v_{tt} + kv_t + v_{xxx} = 0$ with some dampening constant k.

f.) Boundary conditions: $v(a,t) = v(b,t) = 0, v_x(a,t) = 1$ for $t > 0$.
Initial values: $v(x, t_0) = f(x)$ and $v_t(x, t_0) = g(x)$.

Part 3.)

a.) Both f_2 and f_3 satisfy periodic boundary conditions.

b.) Only f_1 satisfies decaying boundary conditions as the limit of $|x|$ approaches ∞.

Part 4.) The prompt is essentially asking which equations are both homogeneous and linear. The equations a.), c.) and d.) are linear and homogeneous, and therefore two solutions can be added together, with constants c_1 and c_2 equal to 1, to provide a third solution, in this case u. This does not hold for b.) because the e^{-t} term means it is not homogeneous, resulting in a clearly untrue statement when we plug in u_3 and expand: $2e^{-t} = e^{-t}$. This also does not hold for e.) or f.) because of the non-linear terms $sin(u)$ and uu_x respectively, which lead to results similar to those demonstrated above in 2.d.).