AMATH 353: Homework 1

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April 3, 2018

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Part 1.)

a.) To create a wave moving left with speed one, I used the equation

$$u(x,t) = exp(-x - ct)^2) \tag{1}$$

with $c = -1$. The following figure displays its movement to the left over time with a speed of 1.

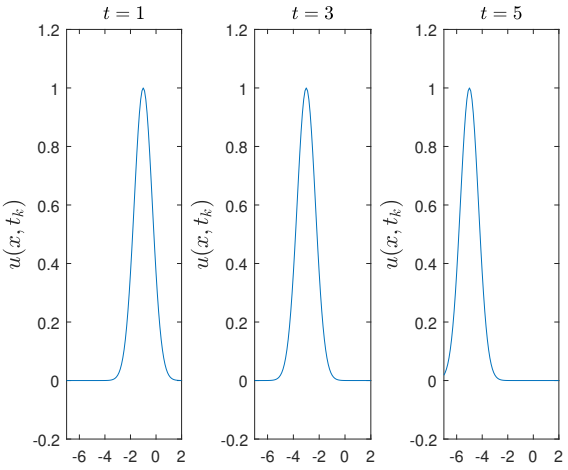

Figure 1: Moving left with speed 1.

b.) and c.) Parts b.) and c.) were executed with the following values for c in the same equation used on part a.), which I have accompanied with visualizations. Note that the 'speed' is displayed by the identical figures and increasing or decreasing x-axes.

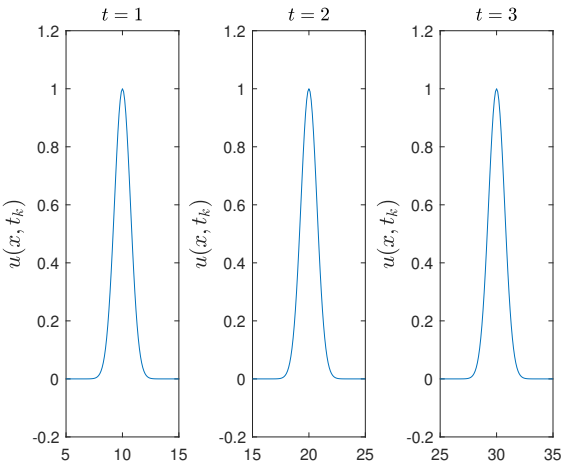

Figure 2: $c = 10$, moving right with speed 10.

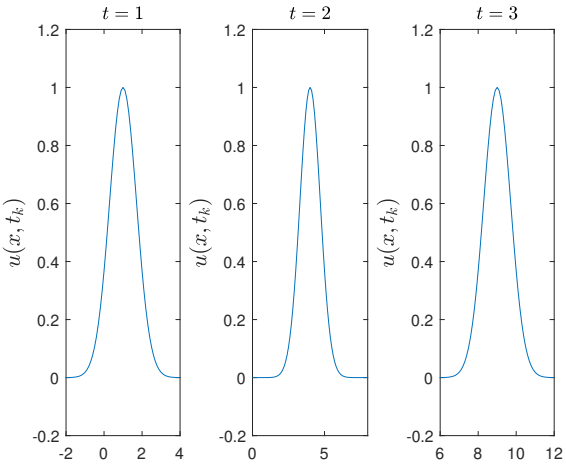

Figure 3: $c = t$, moving right with speed t^2.

d.) Part d.) modifies the equation in a.) to implement decreasing amplitude inversely proportional to t.

$$u(x,t) = \frac{1}{t} exp(-x - ct)^2) \tag{2}$$

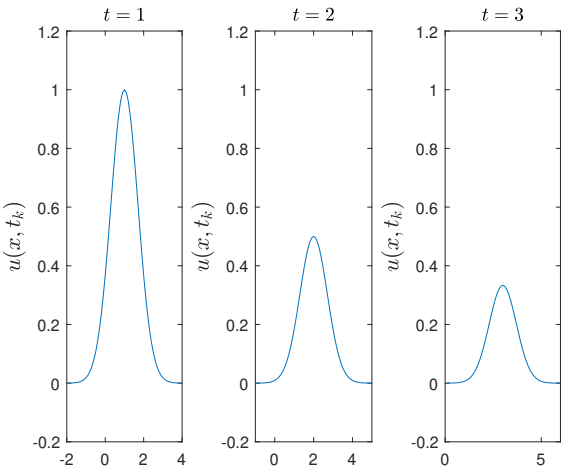

Figure 4: $c = 10$, moving right with speed 1 and amplitude inversely proportional to t.

Part 2.)

a.) $v_{tt} - v_{xxx} = 0$

b.) Let $\alpha = \beta = 1$

By linear differential operators,

$$\begin{aligned} u_{3,tt} &= u_{3,xxx} \\ (u_1 + u_2)_{tt} &= (u_1 + u_2)_{xxx} \\ u_{1,tt} + u_{2,tt} &= u_{1,xxx} + u_{2,xxx} \\ u_{1,tt} + u_{2,tt} - u_{1,xxx} - u_{2,xxx} &= 0 \\ u_{1,tt} - u_{1,xxx} + u_{2,tt} - u_{2,xxx}) &= 0 \end{aligned} \tag{3}$$

Thus as hoped after plugging in u_3 we arrive at the sum of two homogeneous equations of the solutions u_1 and u_2.