Optimization using the Matlab Optimization Toolbox
Assignment 1: nonlinear programming with two variables

4DM20 Engineering Optimization
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Objectives

- Learn to make contour plots of two variable optimization problems using Matlab
- Get acquainted with optimization terminology such as design variables, objective function, constraints, constraint activity, dominance, and well boundedness.
- Learn the importance of normalizing your optimization problems.
- Practice with Matlab's constrained minimizer `fmincon`.

Course material

- Chapter 1 and Sect 4.4 of Papalambros and Wilde [1]
- Matlab optimization toolbox [2].

Normalization Problem

Consider an optimization problem of the form

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) = (\mathbf{x}-\mathbf{c})^\top A(\mathbf{x}-\mathbf{c}) \\ \text{s.t.} \quad & \mathbf{x}, \mathbf{c} \in \mathbb{R}^2, \end{aligned}$$

with $A = \begin{bmatrix} 3 \cdot 10^{-12} & 10^{-6} \\ 10^{-6} & 1 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 2 \cdot 10^{6} \\ 1 \end{bmatrix}$.

Questions

a) Find the optimum analytically.

b) Open the Exercise `ComputerAssignment1.mlx` and follow the instructions to find the optimum numerically.

Design problem

Consider the three bar truss visualized in Figure 1. The truss structure is to be designed for minimum mass. The bar lengths and the topology of the structure is fixed (bar 2 has a scaled length of 1) The cross sectional areas of the bars are to be determined. For reasons of symmetry the cross sectional areas of bars 1 and 3 are chosen equal.

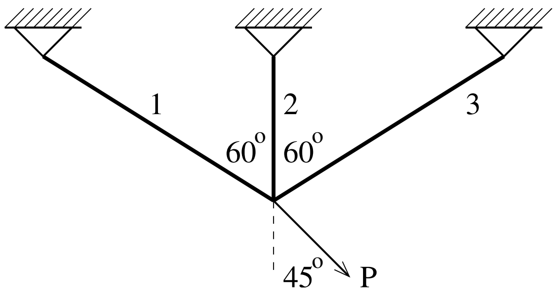

Figure 1: Three-bar truss

The truss structure is subject to a load P. The maximum allowed tensile and compression stress equals σ_0. If we define the normalized design variables to be,

$$x_1 = \frac{A_1\sigma_0}{P}, \qquad x_2 = \frac{A_2\sigma_0}{P}$$

the dimensionless stresses in the three bars are respectively,

$$\begin{aligned}
s_1(\mathbf{x}) &= \frac{\sigma_1}{\sigma_0} = \frac{1}{2}\sqrt{2}\left(\frac{\sqrt{3}}{3x_1} + \frac{1}{x_1+4x_2}\right)\\
s_2(\mathbf{x}) &= \frac{\sigma_2}{\sigma_0} = \frac{2\sqrt{2}}{x_1+4x_2}\\
s_3(\mathbf{x}) &= \frac{\sigma_3}{\sigma_0} = \frac{1}{2}\sqrt{2}\left(\frac{-\sqrt{3}}{3x_1} + \frac{1}{x_1+4x_2}\right),
\end{aligned}$$

with $\mathbf{x} = [x_1, x_2]^T$. The optimization problem then becomes:

$$\begin{aligned}
\min_{\mathbf{x}} \quad & F(\mathbf{x}) = 4x_1 + x_2\\
\text{s.t.} \quad & g_1(\mathbf{x}) = s_1 - 1 \leq 0\\
& g_2(\mathbf{x}) = s_2 - 1 \leq 0\\
& g_3(\mathbf{x}) = s_3 - 1 \leq 0\\
& g_4(\mathbf{x}) = -s_1 - 1 \leq 0\\
& g_5(\mathbf{x}) = -s_2 - 1 \leq 0\\
& g_6(\mathbf{x}) = -s_3 - 1 \leq 0\\
& \chi : x_1, x_2 > 0
\end{aligned}$$

Questions

Open the Exercise `ComputerAssignment1.mlx` and answer the following questions.

a) Use the Matlab file to plot the contour plot of the optimization problem. Explain how the Matlab code works.

b) Identify in the plot the objective function contour lines and the constraint contour lines $g_i(\mathbf{x}) = 0$.

c) Identify in the plot the feasible domain and the location of the minimizer. Which constraints are active? Which constraints are dominated?

d) Verify the convexity of the objective function and the constraints (Hint: see Exercise 4 of Exercises and Study Questions 1). Is the optimization problem convex over χ? And how about the convexity of feasible domain spanned by the active constraints?

e) Solve the three-bar truss design problem using `fmincon`. Compare the outcome with the plot. Which constraints are active at the minimizer?

References

[1] Papalambros, P.Y. and Wilde, D.J., *Principles of optimal design: modeling and computation*, 2nd edition, Cambridge University Press, 2000.

[2] Matlab Optimization Toolbox, The Mathworks.