ASSIGNMENT 4DM70 2024-2025

Problem 1: Network synchronization (50 points)

You will investigate synchronization in networks of Hindmarsh-Rose (HR) model neurons. This HR model neuron, which models the spiking or bursting activity of a neuron, is given by the following set of differential equations:

(1a) $$\dot{y}_i(t) = 100\big(- y_i^3(t) + 3y_i(t) - 8 + 5z_{i,1}(t) - z_{i,2}(t) + E + u_i(t)\big)$$

(1b) $$\dot{z}_{i,1}(t) = 100(-y_i^2(t) - 2y_i(t) - z_{i,1}(t))$$

(1c) $$\dot{z}_{i,2}(t) = 0.5(4y_i(t) + 4.472 - z_{i,2}(t))$$

with parameter E. We fix $E = 3.3$, for which the HR neuron is operating in a so-called chaotic bursting mode. Output $y_i(t)$ represents the membrane potential of the neuron, and $z_{i,1}(t)$ and $z_{i,2}(t)$ are internal variables that are related to ionic (potassium and sodium) currents.

Problem 1.1: HR model properties

You have to show that the HR model neuron (1) is strictly semi-passive and that it has exponentially convergent dynamics. Recall that these properties ensure that synchronization in a network of HR model neurons will happen if the interaction parameters (i.e., network structure including interaction weights, overall coupling strength, time-delay) are chosen properly.

Questions:

a. (4 pts) Show, using the Demidovich condition (see slides *Network Synchronization Part I*), that the internal $(z_{i,1}, z_{i,2})$-dynamics of the HR model neuron (1) are exponentially convergent.

b. (10 pts) Use the radially unbounded and positive definite storage function

$$S(y_i, z_{i,1}, z_{i,2}) = \frac{1}{200}y_i^2 + \frac{1}{80}z_{i,1}^2 + \frac{1}{4}z_{i,2}^2$$

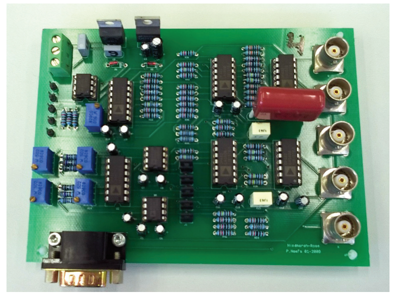
to show that HR model neuron (1) is strictly semipassive from input u_i to output y_i. Hint: you may use that $y_i^4 \pm \frac{5}{2}y_i^2 z_{i,1} + \frac{5}{2}z_{i,1}^2 > \frac{1}{4}(y_i^4 + z_{i,1}^2)$.

Problem 1.2: Network synchronization: theory and experiments

You will use the experimental synchronization setup that is found in the DSD Motion Control laboratory (GEM-N 0.750). This experimental setup consists of a number of electronic HR model neurons, shown in Figure 1, a coupling interface and data acquisition devices. The dynamics of the electronic HR model neurons are accurately described by the differential equations (1).

The (electronic) HR model neurons interact via coupling of the form

$$u_i(t) = \sigma \sum_{j \in \mathcal{N}_i} w_{ij}[y_j(t - \tau) - y_i(t - \tau)]$$



(A)

FIGURE 1. The electronic HR model neuron

with positive constant σ being the coupling strength, non-negative constant τ denotes the time-delay, and positive constants w_{ij} are the interaction weights. In the experiments, the interaction between the HR model neurons is established via the coupling interface. You can define any network structure, up to eight neurons, by specifying the (weighted) adjacency matrix A. The interactions weights w_{ij} can be chosen as integers ranging from 0 to 5. (Obviously, 0 means there is no interaction.) Furthermore you can set the coupling strength in the range from 0 to 10 and the time-delay can be set from 0 [ms] to 10 [ms] with increments of 0.1 [ms].

The synchronization results presented in the lecture slides are derived under the assumption that the systems (here: HR model neurons) have identical dynamics. However, because of imperfections in the electronic components on the circuits, and noise, we can not expect the HR circuits to be completely identical. As a result, for suitable values of the coupling strength σ and time-delay τ, we may therefore only expect that the difference in outputs of the electronic HR model neurons becomes sufficiently small. This motivates a definition of *practical synchronization*. We say that two HR model neurons i and j *practically synchronize* if, after some transient time T,

$$|y_i(t) - y_j(t)| \leq 0.25$$

holds for all $t \geq T$.

Questions synchronization and scaling:

a. (4 pts) **[Experiment]** Couple two electronic coupled HR neurons with $w_{12} = w_{21} = 1$. For each $\tau = 0, 1, \ldots, 5$, determine a lower-bound and upper-bound for the coupling strength such that the two coupled electronic HR model neurons practically synchronize. Note that the maximum value of the coupling strength you can select is 10; In case the systems remain practically synchronous for $\sigma = 10$ you take 10 as upper-bound.

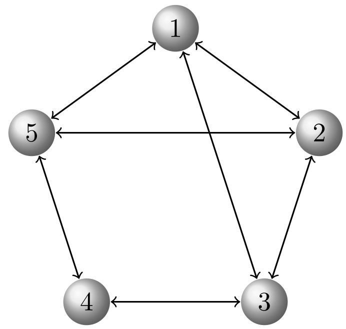


FIGURE 2. The network of Problems 1.2b and 1.2c. All edges have weight 1

b. (6 pts) [**Theory & Experiment**] Consider the network shown in Figure 2 with corresponding adjacency matrix:

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

- Use your results of problem 1.2a. to predict the values of σ and τ for which this network of electronic HR model neurons practically synchronizes. In particular, determine the lower-bounds and upper-bounds of the coupling strength for $\tau = 0, 1, 2, \ldots$.
- Verify your predictions with the experimental setup and discuss your results

c. (6 pts) [**Theory & Experiment**]Add one edge of weight 1 to the network depicted in Figure 2 to improve synchronization in the sense that, for the given values of τ, the intervals of σ for which there is practical synchronization get as large as possible. Explain how you did determine the edge to be added. Validate your result with an experiment.

Questions partial/cluster synchronization The remaining questions (d.-g.) are about the network of eight coupled neurons defined by the adjacency matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

For $\tau = 0$, the whole network of electronic HR model neurons practically synchronizes for $\sigma \geq 2.75$ (approximately). (You may verify this, but no points are awarded for the verification.)

d. (2 pts) **[Experiment]** At $\sigma = 1.5$ and $\tau = 0$, thus before the network is practically synchronized for $\tau = 0$, there is a mode of partial practical synchronization. Describe this mode of partial practical synchronization and give a corresponding permutation matrix.

e. (2 pts) **[Experiment]** Take $\sigma = 3.5$ and increase the time-delay starting from $\tau = 0$ with increments of 0.1. For some value of τ you find that the practical synchronization is lost yet the network exhibits partial practical synchronization. Specify the value of the time-delay at the transition from practical synchronization to partial practical synchronization, describe the observed mode of partial practical synchronization, and give the corresponding permutation matrix.

f.(8 pts) **[Theory]** Explain your results of problems 1.2d and 1.2e. using the theory of partial synchronization.

g.(8 pts) **[Experiment & Theory]** Take $\tau = 0$. Add at most two edges of weight 1 to the network such that, for some value of the coupling strength lower than required for full practical synchronization, you get practical partial synchronization of the form:

- neurons 1 and 2 practically synchronize, and
- neurons 6 and 7 practically synchronize, and
- neurons 5 and 8 practically synchronize, and
- both neurons 3 and 4 do not practically synchronize (with any other neuron).

Explain your approach and do an experimental validation.

Problem 2: Multi-Robot Coordination (50pts)

In the second part of the assignment, you will study a safe multi-robot coordination task involving the formation of an equilateral triangle using three unicycle robots and Voronoi coverage control. You may use the MATLAB files available on Canvas.

Consider a group of three unicycle robots whose equations of motion, for the ith robot, where $i = 1, 2, 3$, are given by

$$\dot{x}_i = v_i \cos\theta_i,$$
$$\dot{y}_i = v_i \sin\theta_i,$$
$$\dot{\theta}_i = \omega_i,$$

where $x_i, y_i \in \mathbb{R}$ denote the x- and y-coordinates of the robot position, $\theta_i \in [0, 2\pi)$ is the robot's counterclockwise orientation angle with respect to the x-axis, and v_i and ω_i are, respectively, the linear and angular speed control inputs. Suppose that the robots are disk-shaped with a body radius of 0.2 m and their positions and orientations are known by all robots.

In this problem, you are asked to design and implement a safe Voronoi coverage control approach for multi-robot coordination. Specifically, consider a 5 m × 5 m square workspace with a Gaussian mixture event distribution of the form $\phi(\mathrm{p}) = \sum_{j=1}^{3} e^{-\|\mathrm{p}-\mathrm{p}_j\|^2}$, where p_1, p_2, and p_3 are fixed 2D points in the workspace, each spaced 1 meter apart. Your task is to design a Voronoi-based coordination strategy that asymptotically drives three unicycles from any initial collision-free configuration in the workspace to an optimal Voronoi coverage configuration, while ensuring collision avoidance between the robots.

Please answer the following questions clearly and concisely.

a) *[10pts] (Controller Design)* Design control (i.e., linear and angular speed) inputs for all three unicycle robots to perform safe Voronoi coverage control. Please clearly and precisely describe your coverage objective function and collision avoidance functions, and explain how they are used in the control design.

b) *[15pts] (Stability Analysis)* Show that your control design ensures collision avoidance and asymptotically stable.

c) *[15pts] (Numerical Simulations)* Demonstrate your multi-robot coordination algorithm in at least three different MATLAB simulations and reflect on your findings and observations. Please provide an intuitive description of the emerging group behavior and the role of the event distribution parameters p_1, p_2, and p_3.

d) *[10pts] (Experimental Demonstration)* Test your multi-robot coordination algorithm in at least two different experiments using TurtleBots in the ME Robotics lab and reflect on your results and observations.

Figure 3. TurtleBots: Experimental Platforms for Multi-Robot Coordination