The LLM Surgeon

Tycho F.A. van der Ouderaa, Markus Nagel, Mart van Baalen, Yuri M. Asano, Tijmen Blankevoort

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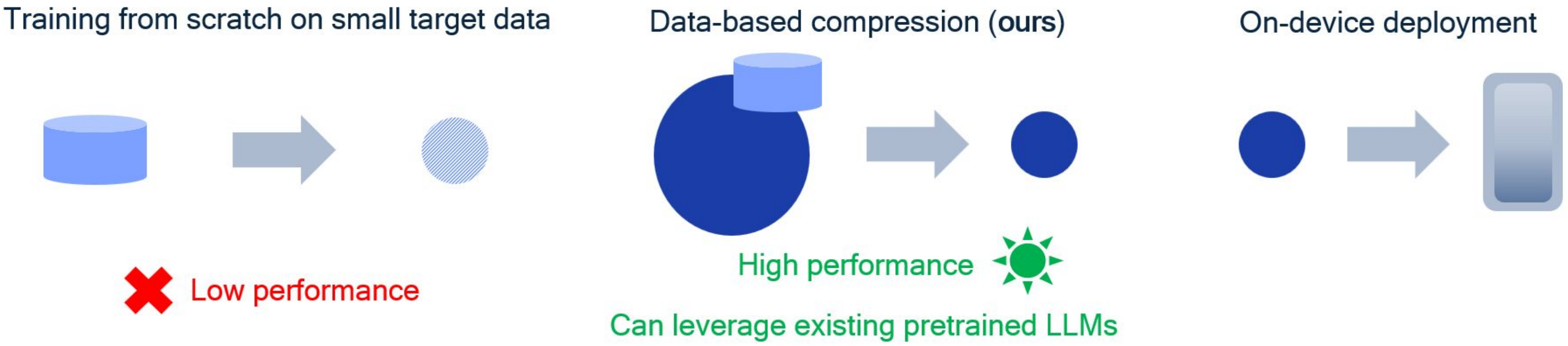
Qualcomm summer internship 2023

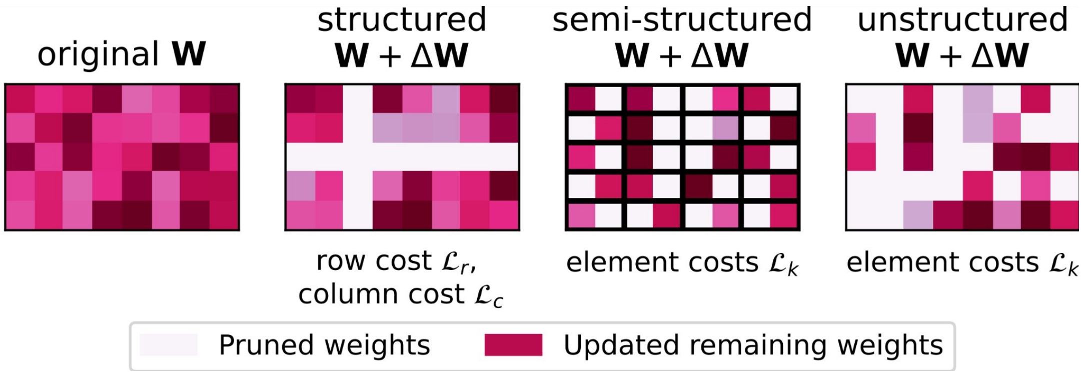
What is this talk about?

- Deploy LLMs within computational, environmental or device-specific constraints.
- We explore data-driven pruning of pretrained models.
- Propose KFAC curvature as scalable quadratic approximation for LLM pruning.



Main achievements

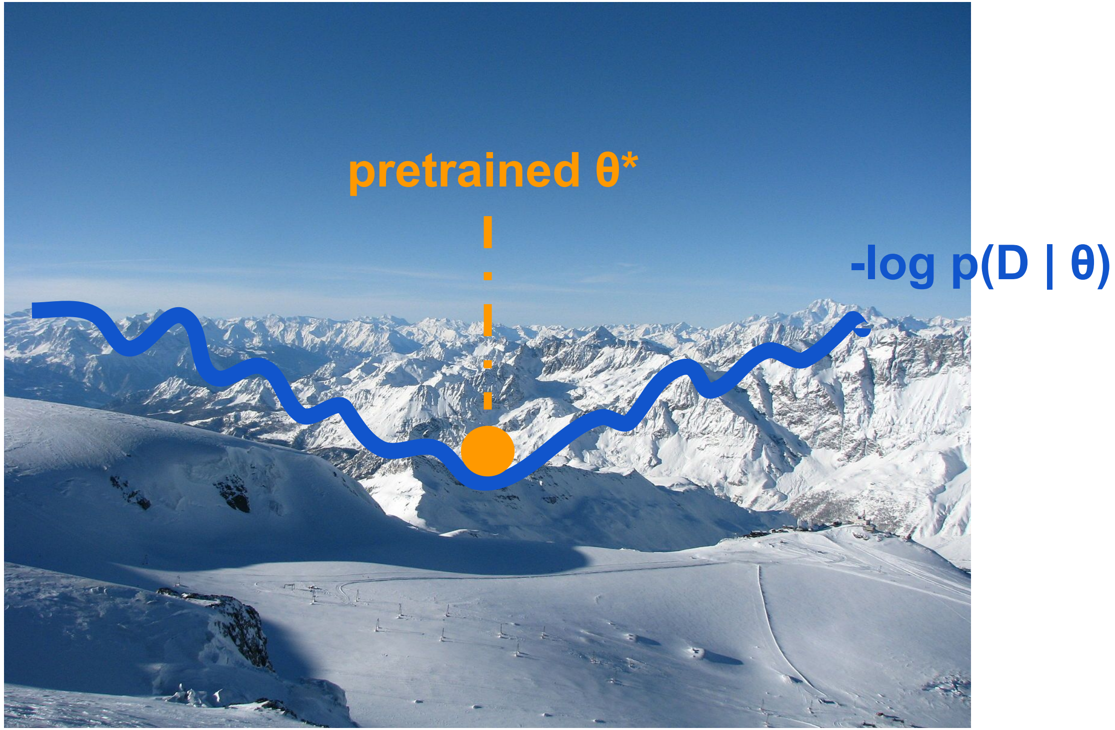
- First to achieve 20-30% structured (!) LLM pruning with performance loss.
- Achieve state-of-the-art results in unstructured and semi-structured pruning.



A tale of a loss surface

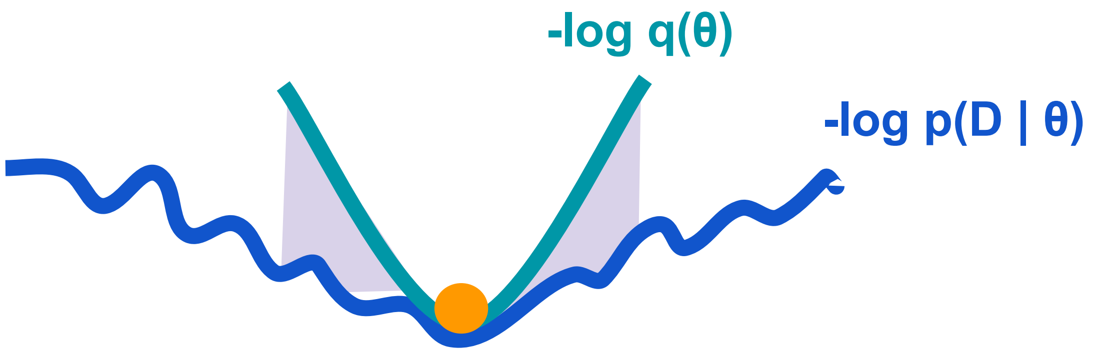
Alps view from Matterhorn Glacier Paradise. (source: Wikipedia)

A tale of a loss surface



Alps view from Matterhorn Glacier Paradise. (source: Wikipedia)

A tale of a loss surface



$$-\log q(\boldsymbol{\theta}) \approx \underbrace{-\log p(\mathcal{D}|\boldsymbol{\theta}^*)}_{\text{pretrained net}} - \underbrace{\cancel{(\boldsymbol{\theta}-\boldsymbol{\theta}^*)^T \nabla \mathcal{L}(\boldsymbol{\theta}^*)}}_{\text{0 at optimum}} - \underbrace{\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}^*)^T \boldsymbol{H}_{\boldsymbol{\theta}^*}(\boldsymbol{\theta}-\boldsymbol{\theta}^*)}_{\text{Hessian/curvature}} + \underbrace{\cancel{O(\theta^4)}}_{\text{higher order terms}}$$

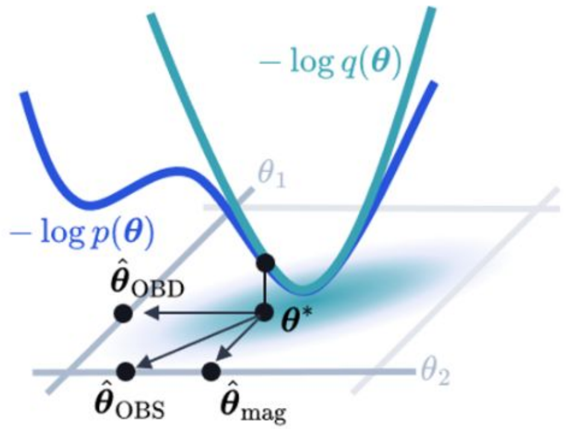
!

Constraint optimization problem

Solve the following quadratic constraint optimization problem (OBS: Hassibi & Stork, 1992)

$$\arg\min_{\Delta\boldsymbol{\theta}} \frac{1}{2}\Delta\boldsymbol{\theta}^T \boldsymbol{F} \Delta\boldsymbol{\theta}$$

$$\text{s.t. } \boldsymbol{e}_k^T \Delta\boldsymbol{\theta} + \boldsymbol{e}_k^T \boldsymbol{\theta} = 0, \forall k \in \mathcal{K}$$



General solution (in LLM context: Kurtic et al. (2022))

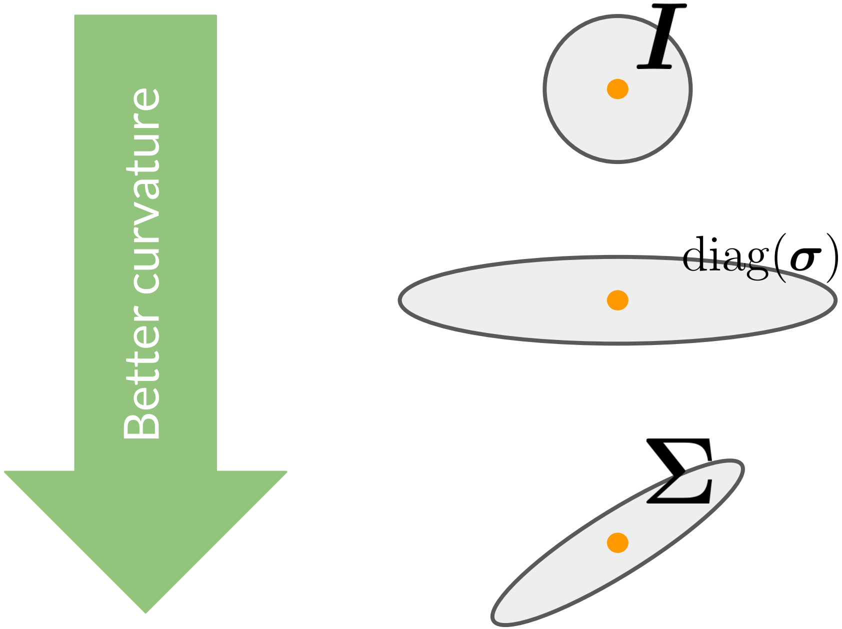
$$\mathcal{L} = \frac{1}{2}(\boldsymbol{E}_K \boldsymbol{\theta}^*)^T \left(\boldsymbol{E}_K \boldsymbol{F}^{-1} \boldsymbol{E}_K^T\right)^{-1} \boldsymbol{E}_K \boldsymbol{\theta}$$

$$\Delta\boldsymbol{\theta} = -\boldsymbol{F}^{-1} \boldsymbol{E}_K^T \left(\boldsymbol{E}_K \boldsymbol{F}^{-1} \boldsymbol{E}_K^T\right)^{-1} \boldsymbol{E}_K \boldsymbol{\theta}$$

Back to the late 1980's...

Approximation

Implied pruning

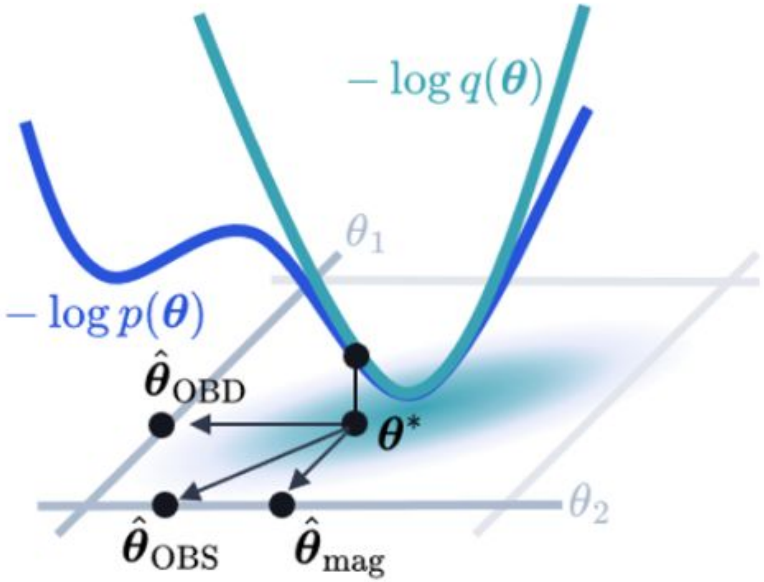


Magnitude pruning

Optimal brain damage (OBD)
(LeCun et al., 1989)

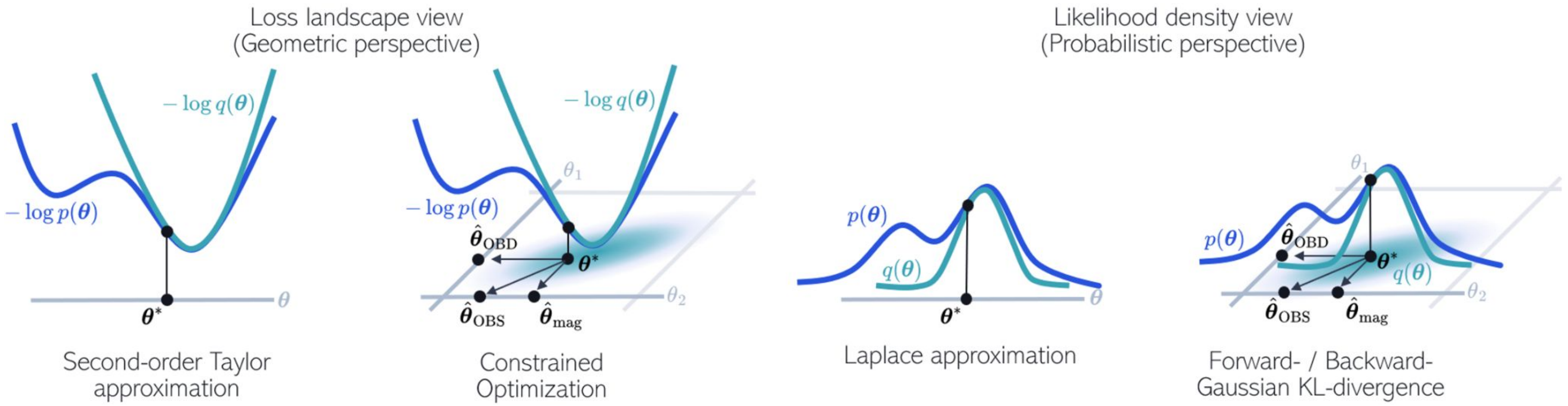
Optimal brain **surgeon** (OBS)
(Hassibi & Stork, 1992)

Constrained optimization



One slide on the probabilistic perspective...

Actually loss is regularised: **$-\log p(D \mid \theta) + \log p(\theta)$** by a log prior.
Prior variance plays critical role in implementation as `damping' term.



- We perform a Laplace approximation of the likelihood or posterior.
- Interactions are correlations. We want to avoid mean-field assumption!

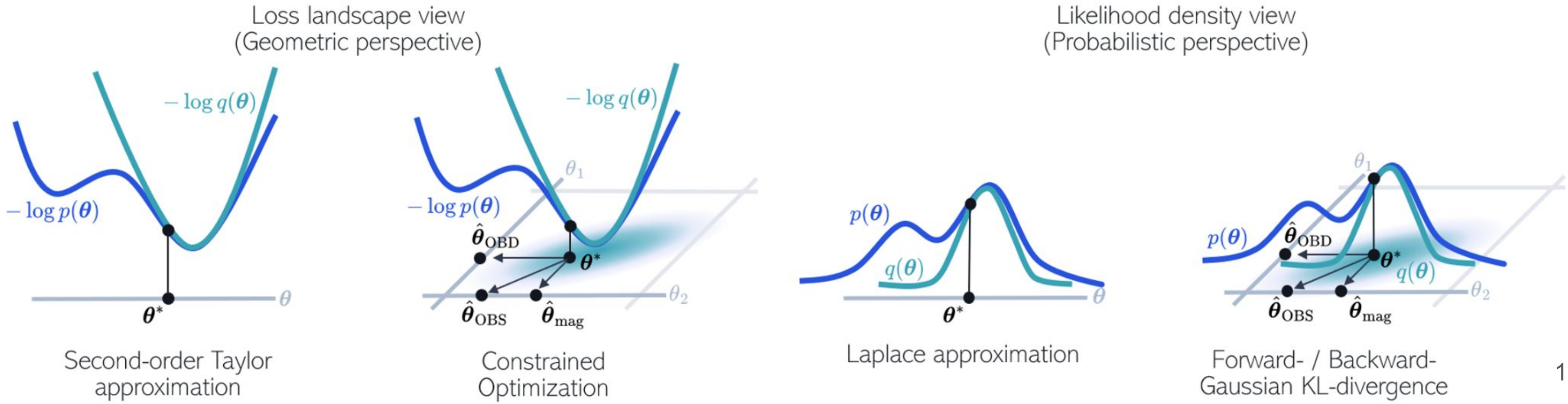
Method

Using surrogate loss:

1. **remove least important weights**

   and then

2. **update remaining weights** in closed-form!



Modern Hessian approximations

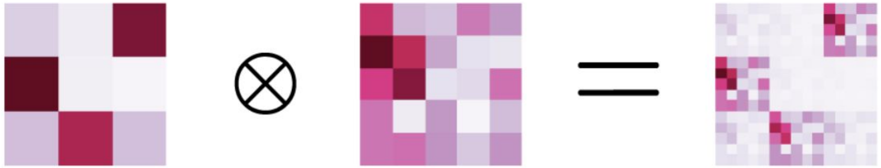
The Hessian of a 13 billion parameter LLM contains 1.69 × 10^20 elements!

13 billion

13 billion

Waaaayyy to big…

Kronecker-factors



The Kronecker product ⊗ operates on two matrices of arbitrary size and results in a block matrix.

Nice way to write factorisations/decompositions for tensors.

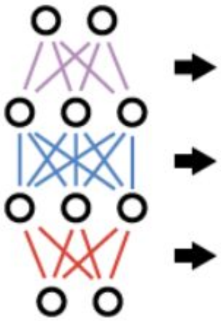
Nothing more than a bit of reshuffling:

```
(A.view(3, 1, 3, 1) * B.view(1, 4, 1, 4)).view(12, 12)
```

Often pops up in factorisations/decompositions of tensors. Keeps math clean.

Modern Hessian approximations

The Hessian of a 13 billion parameter LLM contains 1.69e+20 elements!



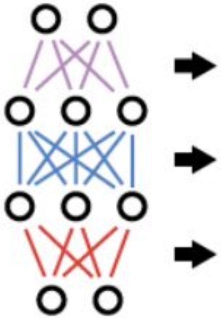
$A_1 \otimes G_1$	$A_1 \otimes G_2$	$A_1 \otimes G_3$
$A_2 \otimes G_1$	$A_2 \otimes G_2$	$A_2 \otimes G_3$
$A_3 \otimes G_1$	$A_3 \otimes G_2$	$A_3 \otimes G_3$

NO approximations yet!

Waaaayyy to big…

What other people do...

Most pruning works ignore `layer-wise' interactions, BUT make it completely `local'.

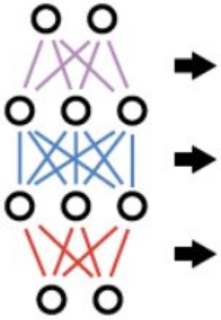


$$\begin{bmatrix} A_1 \otimes I & 0 & 0 \\ 0 & A_2 \otimes I & 0 \\ 0 & 0 & A_3 \otimes I \end{bmatrix}$$

- Very cheap.
- No gradient info.
- Ignores final loss. (equivalent to summing local squared losses on output of each layer)

Or even worse completely diagonal...

Modern Hessian approximations

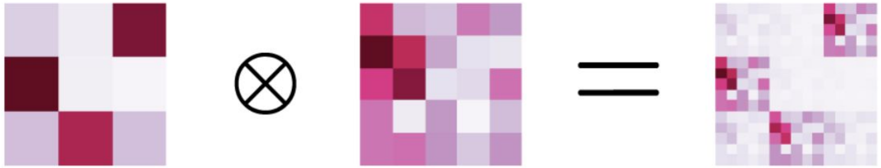


$$\begin{bmatrix} A_1 \otimes G_1 & 0 & 0 \\ 0 & A_2 \otimes G_2 & 0 \\ 0 & 0 & A_3 \otimes G_3 \end{bmatrix}$$

Still quite big…

$$\boldsymbol{F}_l = \sum_{n=1}^{N} \mathbb{E}\left[\underbrace{(\boldsymbol{g}_{l,n}\boldsymbol{g}_{l,n}^T) \otimes (\boldsymbol{a}_{l,n}\boldsymbol{a}_{l,n}^T)}_{RC \times RC}\right]$$

Kronecker-factored approximation



The Kronecker product ⊗ operates on two matrices of arbitrary size and results in a block matrix.

Assume independent input and outputs (KFAC: Martens & Grosse, 2015)

$$\mathbb{E}[\underbrace{\boldsymbol{g}_{l,n}\boldsymbol{g}_{l,n}^T \otimes \boldsymbol{a}_{l,n}\boldsymbol{a}_{l,n}^T}_{\text{Still quite big…}}] \approx \underbrace{\mathbb{E}[\boldsymbol{g}_{l,n}\boldsymbol{g}_{l,n}^T]}_{\text{Great!}} \otimes \mathbb{E}[\boldsymbol{a}_{l,n}\boldsymbol{a}_{l,n}^T]$$

Can be implemented using *hooks:*

During all forward and backward passes, hooks maintain aggregates of activations (aa^T) and gradients (gg^T). Aggregates can be moved to ram, if needed.

Modern Hessian approximations

Constraint optimization problem

Solve the following quadratic constraint optimization problem (OBS: Hassibi & Stork, 1992)

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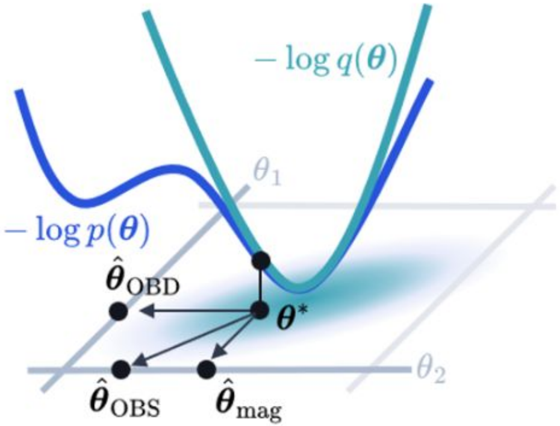


General solution (in LLM context: Kurtic et al. (2022))

$$\mathcal{L} = \frac{1}{2}(\boldsymbol{E}_K \boldsymbol{\theta}^*)^T \left(\boldsymbol{E}_K \boldsymbol{F}^{-1} \boldsymbol{E}_K^T\right)^{-1} \boldsymbol{E}_K \boldsymbol{\theta}$$

$$\Delta\boldsymbol{\theta} = -\boldsymbol{F}^{-1} \boldsymbol{E}_K^T \left(\boldsymbol{E}_K \boldsymbol{F}^{-1} \boldsymbol{E}_K^T\right)^{-1} \boldsymbol{E}_K \boldsymbol{\theta}$$

Paper provides derivations for all structures {*unstructured*, *semi-structured*, *structured*}

Example for structured pruning

1. **Compute removal cost** for each row and column

$$\mathcal{L}_r = \frac{1}{2}\frac{\boldsymbol{\theta}_r^T \boldsymbol{A}\boldsymbol{\theta}_r}{[\boldsymbol{G}^{-1}]_{rr}}, \quad \mathcal{L}_c = \frac{1}{2}\frac{\boldsymbol{\theta}_c^T \boldsymbol{G}\boldsymbol{\theta}_c}{[\boldsymbol{A}^{-1}]_{cc}}$$

2. **Global thresholding** by sorting all costs and selecting op X% for removal
3. **Update** remaining weights using correlated weight updates

$$\Delta \boldsymbol{W} = -\underline{\overline{\boldsymbol{W}}(\boldsymbol{E}_{C'}\boldsymbol{A}^{-1}\boldsymbol{E}_{C'}^T)^{-1}(\boldsymbol{A}^{-1}\boldsymbol{E}_{C'}^T)}$$

$$\Delta \boldsymbol{W} = -\underline{\boldsymbol{G}^{-1}\boldsymbol{E}_{R'}^T(\boldsymbol{E}_{R'}\boldsymbol{G}^{-1}\boldsymbol{E}_{R'}^T)^{-1}\overline{\boldsymbol{W}}}$$

(among new results)

Scales very well (in rows/cols, not in elements!)

4. **Repeat** for multiple shots

Pseudo code

Algorithm 1 LLM Surgeon (*structured*)

Input: initial weights $\boldsymbol{\theta}^0$, target size α, and data $\mathcal{D}$
  For shot t in $[1, 2, \ldots, T]$
    Compute: approximate curvature $\boldsymbol{G}, \boldsymbol{A}$ from data $\mathcal{D}$ ▷ section 3.1
    Compute: costs per row/column $\mathcal{L}_r, \mathcal{L}_c$ from $\boldsymbol{G}, \boldsymbol{A}$ ▷ section 3.2
    Compute: threshold τ using $\mathcal{L}_r$ and $\mathcal{L}_c$ given target size α_t ▷ section 3.3
    Select: rows and columns to remove $\boldsymbol{E}_R, \boldsymbol{E}_C$ based on τ ▷ section 3.3
    Compute: weight update $\Delta\boldsymbol{\theta}^{t-1}$ based on $\boldsymbol{E}_R, \boldsymbol{E}_C$ and $\boldsymbol{G}, \boldsymbol{A}$ ▷ section 3.4
    Update: remaining weights $\boldsymbol{\theta}^t \leftarrow \boldsymbol{\theta}^{t-1} + \Delta\boldsymbol{\theta}^{t-1}$ ▷ section 3.5
    Optionally: $\boldsymbol{\theta}^t \leftarrow$ low-rank update$(\boldsymbol{\theta}^t)$ ▷ section 3.6
  Output: compressed weights $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}^T$

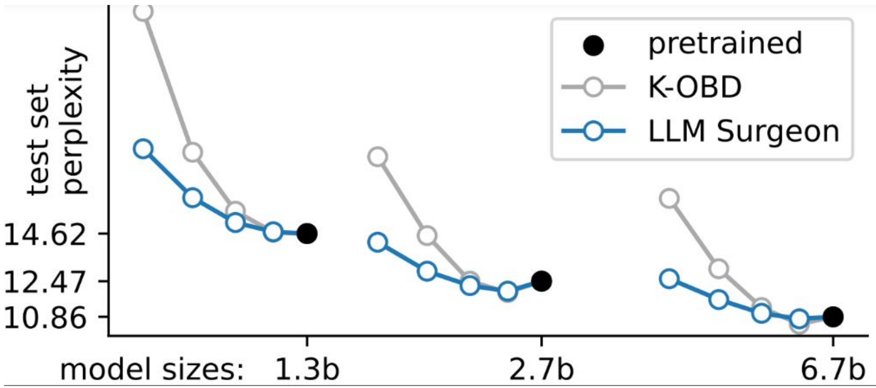
Can be interleaved with first-order LoRA corrections.

Useful trick: absorb in between to allow
increase rank of sum of LoRA updates!

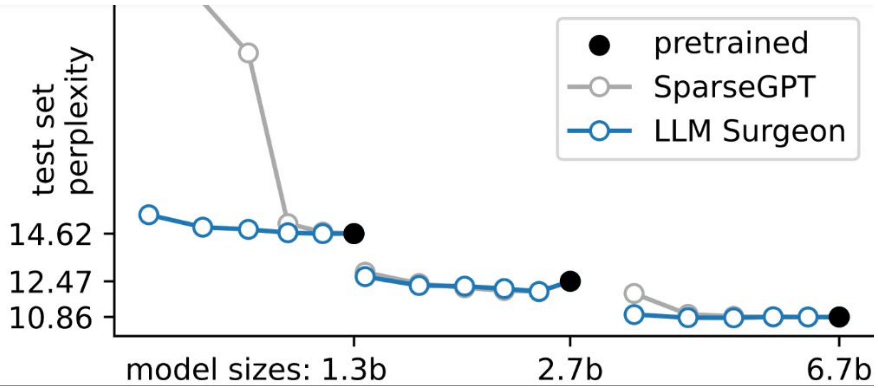
Results

Interpolate model sizes

Structured pruning



Unstructured pruning



Results

Quantitative benchmark

Structured pruning results

Table 1: Structured compression of large language models on wikitext-2 data.

Method	Target size	Test performance (PPL) OPT (125m)	OPT (1.3b)	OPT (2.7b)	OPT (6.7b)	Llama-v2 (7b)
Baseline	100%	**27.65**	**14.62**	**12.47**	**10.86**	**5.12**
Magnitude	90%	767.2	894.4	1229	3464	36746
$\boldsymbol{I} \otimes \boldsymbol{I}$	80%	4685	(1278)	2788	16747	347960
	70%	17970	(3098)	9255	17312	41373
L-OBD	90%	33.3	20.76	17.69	27.20	14259
diag($\boldsymbol{I} \otimes \boldsymbol{A}$)	80%	94.14	1392	3236	7570	15630
multi shot	70%	545.6	2147	7233	7628	21386
K-OBD	90%	27.97	14.68	11.96	10.53	5.48
diag($\boldsymbol{G} \otimes \boldsymbol{A}$)	80%	29.89	15.63	12.47	11.28	9.14
multi shot	70%	36.54	18.29	14.53	13.03	15.43
	60%	47.54	24.65	18.09	16.21	28.03
	50%	75.95	37.68	26.68	25.54	46.64
LLM Surgeon (**ours**)	90%	28.29	14.73	12.00	10.82	5.43
$\boldsymbol{G} \otimes \boldsymbol{A}$	80%	29.37	15.27	12.37	11.22	7.29
within row/col cor. Δ	70%	32.46	16.60	13.16	11.83	10.85
	60%	39.82	19.40	14.79	12.94	16.67
	50%	51.48	23.81	18.01	15.38	25.62
LLM Surgeon (**ours**)	90%	28.01	14.70	12.02	10.77	5.25
$\boldsymbol{G} \otimes \boldsymbol{A}$	80%	28.73	15.12	12.27	11.02	6.18
full cor. Δ	70%	31.82	16.24	12.92	11.64	7.83
	60%	38.47	18.45	14.23	12.58	10.39
	50%	49.78	22.95	17.15	14.90	15.38

Unstructured pruning results

Table 4: Unstructured compression of large language models on wikitext-2 data.

Method	Target size	Test performance (PPL) OPT (125m)	OPT (1.3b)	OPT (2.7b)	OPT (6.7b)	Llama-v2 (7b)
Baseline	100%	**27.65**	**14.62**	**12.47**	**10.86**	**5.12**
Magnitude	90%	27.62	14.69	12.60	10.88	5.18
$\boldsymbol{I} \otimes \boldsymbol{I}$	80%	28.53	15.68	13.18	11.26	5.37
	70%	52.88	140.2	15.22	12.22	6.03
L-OBD	90%	29.70	16.24	14.44	13.43	6.09
diag($\boldsymbol{I} \otimes \boldsymbol{A}$)	80%	32.18	21.92	23.35	39.85	116.2
single shot	70%	49.08	204.7	274.8	810.4	6549
K-OBD	90%	27.64	14.62	12.09	36.89	5.13
$\boldsymbol{G} \otimes \boldsymbol{A}$	80%	27.62	14.37	130220	39928	5.19
single shot	70%	27.92	220.1	23097	19506	5.60
	60%	29.24	13783	10331	33896	9.20
	50%	34.43	7311	10495	91506	118.6
SparseGPT	90%	27.93	14.69	12.00	10.86	5.49
$\boldsymbol{I} \otimes \boldsymbol{A}$	80%	28.18	15.07	12.05	10.86	5.58
	70%	28.93	22.77	12.17	10.89	5.71
	60%	30.20	25.07	12.37	10.98	5.94
	50%	33.17	26.77	12.88	11.92	6.51
LLM Surgeon (**ours**)	90%	27.69	14.62	12.01	10.86	5.13
$\boldsymbol{G}_1 \otimes \boldsymbol{A}_1$	80%	27.83	14.66	12.14	10.87	5.20
full cor. Δ	70%	28.35	14.81	12.25	10.82	5.36
multi shot	60%	28.98	14.91	12.28	10.83	5.66
	50%	30.30	15.47	12.68	10.97	6.08

Semi-structured (2:4) pruning results

Table 3: Semi-structured 2:4 compression for large language models on wikitext-2 data.

Method	$F \approx$	Target size	Test performance (PPL) OPT (125m)	OPT (1.3b)	OPT (2.7b)	OPT (6.7b)
Baseline		100%	**27.65**	**14.62**	**12.47**	**10.86**
Magnitude	$\boldsymbol{I} \otimes \boldsymbol{I}$	50%	342.04	379.57	1106.01	187.29
L-OBD	diag($\boldsymbol{I} \otimes \boldsymbol{A}$)	50%	87.26	44.92	41.40	27.36
K-OBD	diag($\boldsymbol{G} \otimes \boldsymbol{A}$)	50%	68.74	27.22	20.23	15.55
SparseGPT	$\boldsymbol{I} \otimes \boldsymbol{A}$	50%	45.51	29.44	14.92	13.01
LLM Surgeon (**ours**)	$\boldsymbol{G} \otimes \boldsymbol{A}$	50%	44.64	25.10	14.64	12.10

Similar findings for performance on downstream tasks!

Results

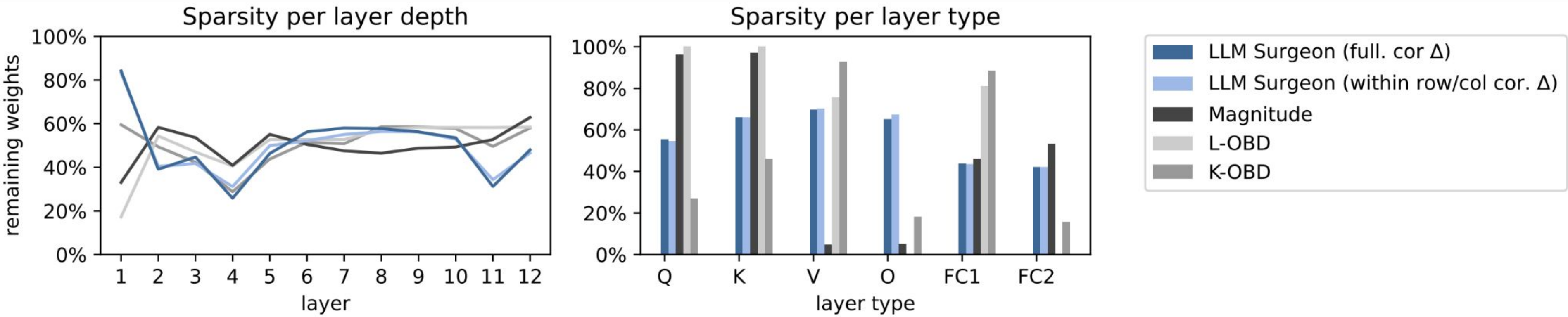
Task-specific compression

Can be used to project existing pretrained models to tailored smaller model.

target	evaluation dataset EN	FR	DE	IT	mask equivalence (%) EN	FR	DE	IT
Pretrained	**27.66**	**22.54**	**24.32**	**27.66**				
EN	**47.46**	172.9	181.1	169.1	1.00	0.74	0.70	0.72
FR	113.4	**28.44**	35.02	34.90	0.74	1.00	0.87	0.90
DE	142.1	35.15	**27.49**	38.49	0.70	0.87	1.00	0.87
IT	123.7	31.85	33.78	**30.58**	0.72	0.90	0.87	1.00

Results

Analysing sparsification



THE LLM SURGEON

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[1]Imperial College London , [2]Qualcomm AI Research[†], [3]QUVA Lab, University of Amsterdam

Qualcomm summer internship 2023

Tycho van der Ouderaa
Twitter/X: `tychovdo`
Email: `tychovdo@gmail.com`
Web: `tychovdo.ai`

Imperial College London

Qualcomm

Beyond independent inputs and outputs

Nearest Kronecker product with Kronecker power iteration

Algorithm 4 Kronecker power method. Finds $\widetilde{\boldsymbol{G}}, \widetilde{\boldsymbol{A}}$ nearest Kronecker product $||\boldsymbol{F} - \widetilde{\boldsymbol{G}} \otimes \widetilde{\boldsymbol{A}}||_F$.

Input: Initialise $\widetilde{\boldsymbol{g}}^0 = \boldsymbol{1}, \widetilde{\boldsymbol{a}}^0 = \boldsymbol{1}$ (or using estimates of previous shot).
Input: Set iterations I (or I=1 if using estimates from previous shot)
Output: $\widetilde{\boldsymbol{G}}, \widetilde{\boldsymbol{A}}$

 for iteration i in $[1, 2, \ldots, I]$ **do**

  Compute: $\widetilde{\boldsymbol{g}}^i = \frac{\mathcal{R}(\widetilde{\boldsymbol{F}})\widetilde{\boldsymbol{a}}^{i-1}}{||\mathcal{R}(\widetilde{\boldsymbol{F}})\widetilde{\boldsymbol{a}}^{i-1}||_2}$, with $\mathcal{R}(\widetilde{\boldsymbol{F}})\widetilde{\boldsymbol{a}}^{i-1} = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{a}_n^T \widetilde{\boldsymbol{A}}^{i-1} \boldsymbol{a}_n \text{vec}(\boldsymbol{g}_n \boldsymbol{g}_n^T)$

  Compute: $\widetilde{\boldsymbol{a}}^i = \frac{\mathcal{R}(\widetilde{\boldsymbol{F}})^T \widetilde{\boldsymbol{g}}^i}{||\mathcal{R}(\widetilde{\boldsymbol{F}})^T \widetilde{\boldsymbol{g}}^i||_2}$, with $\mathcal{R}(\widetilde{\boldsymbol{F}})^T \widetilde{\boldsymbol{g}}^i = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{g}_n^T \widetilde{\boldsymbol{G}}^i \boldsymbol{g}_n \text{vec}(\boldsymbol{a}_n \boldsymbol{a}_n^T)$

  Compute: $\sigma^i = ||\widetilde{\boldsymbol{a}}^i||_2$

 end for

 Return: $\widetilde{\boldsymbol{G}} = \sqrt{\sigma^i}\text{mat}(\widetilde{\boldsymbol{g}}), \widetilde{\boldsymbol{A}} = \sqrt{\sigma^i}\text{mat}(\widetilde{\boldsymbol{a}})$.