$\tan x$ を $\frac{\pi}{4}$ まわりで Taylor 展開

$\tan\frac{\pi}{4} = 1$

$(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$ $\quad \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$(\tan x)'' = \frac{+2\cos x \sin x}{\cos^4 x} = \frac{2\sin x}{\cos^3 x} = 2\tan x \cdot \frac{1}{\cos^2 x} = 2\tan^4 x + 2\tan^2 x$

$(\tan x)'' = 2(1+\tan^2 x) + 6\tan^2 x (1+\tan^2 x)$

$= 2 + 8\tan^2 x + 6\tan^4 x$

$\tan x = 1 + 2\left(x-\frac{\pi}{4}\right) + \frac{4}{2}\left(x-\frac{\pi}{4}\right)^2 + \frac{16}{6}\left(x-\frac{\pi}{4}\right)^3$

$= 1 + 2\left(x-\frac{\pi}{4}\right) + 2\left(x-\frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x-\frac{\pi}{4}\right)^3$