99-101

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1.1

§87

$$g_{00} = 1 + \frac{2\phi}{c^2}, g_{0\alpha} = 0, g_{\alpha\beta} = \delta_{\alpha\beta} \tag{1}$$

$$u^\alpha = 0, u^0 = u_0 = 1$$

$$\mu = \sum_a m\delta(r - r_a)$$

$$T_i^k = \mu c^2 u_i u^k \tag{2}$$

$$T_0^0 = \mu c^2 \tag{3}$$

$$T = T_i^i = T_0^0 = \mu c^2 \tag{4}$$

$$R_i^k = \frac{8\pi k}{c^4}(T_i^k - \frac{1}{2}\delta_i^k T) \tag{5}$$

R_0^0

$$R_0^0 = \frac{8\pi k}{c^4}(T_0^0 - \frac{1}{2}T) \tag{6}$$

$$= \frac{4\pi k}{c^2}\mu \tag{7}$$

$i \neq k$ R_i^k

$$R_i^k = 0 \tag{8}$$

$$(92.7)$$

$$R_{ik} = \frac{\partial \Gamma_{ik}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^k} + \Gamma_{ik}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{km}^l \tag{9}$$

$$\phi$$

$$R_{ik} = \frac{\partial \Gamma^l_{ik}}{\partial x^l} - \frac{\partial \Gamma^l_{il}}{\partial x^k} \tag{10}$$

$$= \frac{1}{2} g^{lt} \left(\frac{\partial}{\partial x^l} (\frac{\partial g_{kt}}{\partial x^i} + \frac{\partial g_{it}}{\partial x^k} - \frac{\partial g_{ik}}{\partial x^t}) - \frac{\partial}{\partial x^k} (\frac{\partial g_{lt}}{\partial x^i}) \right) \tag{11}$$

$$= \frac{1}{2} g^{lt} \left(\frac{\partial^2 g_{kt}}{\partial x^l \partial x^i} + \frac{\partial^2 g_{it}}{\partial x^l \partial x^k} - \frac{\partial^2 g_{ik}}{\partial x^l \partial x^t} - \frac{\partial^2 g_{lt}}{\partial x^k \partial x^i} \right) \tag{12}$$

$$R_{00} = \frac{1}{2} g^{lt} \left(\frac{\partial^2 g_{0t}}{\partial x^l \partial x^0} + \frac{\partial^2 g_{0t}}{\partial x^l \partial x^0} - \frac{\partial^2 g_{00}}{\partial x^l \partial x^t} - \frac{\partial^2 g_{lt}}{\partial x^0 \partial x^0} \right) \tag{13}$$

$$\sim \frac{1}{2} g^{\alpha\beta} (-\frac{\partial^2 g_{00}}{\partial x^\alpha \partial x^\beta}) \tag{14}$$

$$= \frac{1}{2} \frac{\partial^2 g_{00}}{\partial x^{\alpha 2}} \tag{15}$$

$$= \frac{1}{c^2} \frac{\partial^2 \phi}{\partial x^{\alpha 2}} \tag{16}$$

$$R^0_0 = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial x^{\alpha 2}} = \frac{4\pi k}{c^2} \mu \tag{17}$$

$$\Delta \phi = 4\pi k \mu \tag{18}$$

$$\phi = -k \int \frac{\mu dV}{R} \tag{19}$$

$$m, m'$$

$$\phi = -\frac{km}{R} \tag{20}$$

$$F = -m' \frac{\partial \phi}{\partial R} \tag{21}$$

$$= -m \frac{kmm'}{R^2} \tag{22}$$

$$U = \frac{1}{2} \int \mu \phi dV \tag{23}$$

1.2

$$0$$

$$R_0 \qquad\qquad R_0$$

$$\phi = -k \int dV \left(\frac{\mu}{R_0} + \frac{1}{2} \mu x_\alpha x_\beta \frac{\partial^2}{\partial X_\alpha \partial X_\beta} (\frac{1}{R_0}) + \cdots \right) \tag{24}$$

$$\Delta \frac{1}{R_0} = \delta_{\alpha\beta} x_\alpha x_\beta \frac{\partial^2}{\partial X_\alpha \partial X_\beta} (\frac{1}{R_0}) = 0 \tag{25}$$

$\phi^{(2)}$ $\quad x_\alpha x_\beta$

$$\phi^{(2)} = -k \int dV \mu (x_\alpha x_\beta - \frac{1}{3} r^2 \delta_{\alpha\beta}) \frac{\partial^2}{\partial X_\alpha \partial X_\beta} (\frac{1}{R_0}) \tag{26}$$

$$= -k \int dV \frac{1}{6} D_{\alpha\beta} \frac{\partial^2}{\partial X_\alpha \partial X_\beta} (\frac{1}{R_0}) \tag{27}$$

$$D_{\alpha\beta} = \int dV \mu (3 x_\alpha x_\beta - r^2 \delta_{\alpha\beta}) \tag{28}$$

$$J_{\alpha\beta} = \int \mu (r^2 \delta_{\alpha\beta} - x_\alpha x_\beta) \tag{29}$$

$$J_{\gamma\gamma} = \int 2\mu r^2 dV \tag{30}$$

$$D_{\alpha\beta} = J_{\gamma\gamma} \delta_{\alpha\beta} - 3 J_{\alpha\beta} \tag{31}$$

1.3

μ

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{x^2} = 1 \tag{32}$$

$U = \frac{1}{2} \int \mu \phi dV$

1.4

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$$ds^2 = cdt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{33}$$

$$dt = dr = 0$$

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \equiv r^2 d\Omega^2 \tag{34}$$

$$dl^2 = f(r,t) d\Omega^2 \tag{35}$$

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$$4\pi r'^2 \qquad\qquad 4\pi f(r,t) \qquad\qquad f(r,t) \equiv r'^2$$

$$(r,t) \qquad (r',t')$$

$$(r',t') \qquad (r,t) \qquad\qquad \text{r} \qquad\qquad \text{t=}$$

$$ds^2 = l(r,t)dt^2 - r^2 d\Omega^2 + h(r,t)dr^2 + a(r,t)drdt \tag{36}$$

$$d\phi, d\theta \qquad\qquad \phi \to -\phi \quad \theta \to \pi - \theta \qquad\qquad a(r,t)$$

$$t \to -t \qquad\qquad (r',t')$$

$$ds^2 = l(r,t)dt^2 - r^2 d\Omega^2 + h(r,t)dr^2 \tag{37}$$
$$= e^\nu c^2 dt^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) - e^\lambda dr^2 \tag{38}$$

$$0$$

$$g_{00} = e^\nu \ , g_{11} = -e^\lambda \ , g_{22} = -r^2 \ , g_{33} = -r^2\sin^2\theta \tag{39}$$
$$g^{00} = e^{-\nu} \ , g^{11} = -e^{-\lambda} \ , g^{22} = -r^{-2} \ , g^{33} = -r^{-2}\sin^{-2}\theta \tag{40}$$
$$\tag{41}$$

$$\Gamma^i_{kl} = \tfrac{1}{2}g^{it}(\tfrac{\partial g_{kt}}{\partial x^l} + \tfrac{\partial g_{tl}}{\partial x^k} - \tfrac{\partial g_{kl}}{\partial x^t}) \qquad\qquad 0$$

$$\Gamma^0_{00} = \frac{1}{2}g^{0t}(\frac{\partial g_{0t}}{\partial x^0} + \frac{\partial g_{t0}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^t}) = \frac{1}{2}g^{00}(\frac{\partial g_{00}}{\partial x^0} + \frac{\partial g_{00}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^0}) = \frac{\dot\nu}{2} \tag{42}$$

$$\Gamma^1_{11} = \frac{1}{2}g^{1t}(\frac{\partial g_{1t}}{\partial x^1} + \frac{\partial g_{t1}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^t}) = \frac{1}{2}g^{11}(\frac{\partial g_{11}}{\partial x^1} + \frac{\partial g_{11}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^1}) = \frac{\lambda'}{2} \tag{43}$$

$$\Gamma^0_{10} = \frac{1}{2}g^{0t}(\frac{\partial g_{1t}}{\partial x^0} + \frac{\partial g_{t0}}{\partial x^1} - \frac{\partial g_{10}}{\partial x^t}) = \frac{1}{2}g^{00}(\frac{\partial g_{10}}{\partial x^0} + \frac{\partial g_{00}}{\partial x^1} - \frac{\partial g_{10}}{\partial x^0}) = \frac{\nu'}{2} \tag{44}$$

$$\Gamma^0_{11} = \frac{1}{2}g^{0t}(\frac{\partial g_{1t}}{\partial x^1} + \frac{\partial g_{t1}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^t}) = \frac{1}{2}g^{00}(\frac{\partial g_{10}}{\partial x^1} + \frac{\partial g_{01}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^0}) = \frac{\dot\lambda}{2}e^{\lambda-\nu} \tag{45}$$

$$\Gamma^1_{10} = \frac{1}{2}g^{1t}(\frac{\partial g_{1t}}{\partial x^0} + \frac{\partial g_{t0}}{\partial x^1} - \frac{\partial g_{10}}{\partial x^t}) = \frac{1}{2}g^{11}(\frac{\partial g_{11}}{\partial x^0} + \frac{\partial g_{10}}{\partial x^1} - \frac{\partial g_{10}}{\partial x^1}) = \frac{\dot\lambda}{2} \tag{46}$$

$$\Gamma^1_{00} = \frac{1}{2}g^{1t}(\frac{\partial g_{0t}}{\partial x^0} + \frac{\partial g_{t0}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^t}) = \frac{1}{2}g^{11}(\frac{\partial g_{01}}{\partial x^0} + \frac{\partial g_{10}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^1}) = \frac{\nu'}{2}e^{\nu-\lambda} \tag{47}$$

$$\Gamma^1_{22} = \frac{1}{2}g^{1t}(\frac{\partial g_{2t}}{\partial x^2} + \frac{\partial g_{t2}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^t}) = \frac{1}{2}g^{11}(\frac{\partial g_{21}}{\partial x^2} + \frac{\partial g_{12}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^1}) = -re^{-\lambda} \tag{48}$$

$$\Gamma^1_{33} = \frac{1}{2}g^{1t}(\frac{\partial g_{3t}}{\partial x^3} + \frac{\partial g_{t3}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^t}) = \frac{1}{2}g^{11}(\frac{\partial g_{31}}{\partial x^3} + \frac{\partial g_{13}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^1}) = -r\sin^2\theta e^{-\lambda} \tag{49}$$

$$\Gamma^2_{33} = \frac{1}{2}g^{2t}(\frac{\partial g_{3t}}{\partial x^3} + \frac{\partial g_{t3}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^t}) = \frac{1}{2}g^{22}(\frac{\partial g_{32}}{\partial x^3} + \frac{\partial g_{23}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^2}) = -\sin\theta\cos\theta \tag{50}$$

$$\Gamma^3_{23} = \frac{1}{2}g^{3t}(\frac{\partial g_{2t}}{\partial x^3} + \frac{\partial g_{t3}}{\partial x^2} - \frac{\partial g_{23}}{\partial x^t}) = \frac{1}{2}g^{33}(\frac{\partial g_{23}}{\partial x^3} + \frac{\partial g_{33}}{\partial x^2} - \frac{\partial g_{23}}{\partial x^3}) = -\cot\theta \tag{51}$$

$$\Gamma^2_{12} = \frac{1}{2}g^{2t}(\frac{\partial g_{1t}}{\partial x^2} + \frac{\partial g_{t2}}{\partial x^1} - \frac{\partial g_{12}}{\partial x^t}) = \frac{1}{2}g^{22}(\frac{\partial g_{12}}{\partial x^2} + \frac{\partial g_{22}}{\partial x^1} - \frac{\partial g_{12}}{\partial x^2}) = \frac{1}{r} \tag{52}$$

$$\Gamma^3_{13} = \frac{1}{2}g^{3t}(\frac{\partial g_{1t}}{\partial x^3} + \frac{\partial g_{t3}}{\partial x^1} - \frac{\partial g_{13}}{\partial x^t}) = \frac{1}{2}g^{33}(\frac{\partial g_{13}}{\partial x^3} + \frac{\partial g_{33}}{\partial x^1} - \frac{\partial g_{13}}{\partial x^3}) = \frac{1}{r} \tag{53}$$

$$\tag{54}$$

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$$R_{ik} = \frac{\partial \Gamma_{ik}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^k} + \Gamma_{ik}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{km}^l \tag{55}$$

$$R_{00} = \frac{\partial \Gamma_{00}^l}{\partial x^l} - \frac{\partial \Gamma_{0l}^l}{\partial x^0} + \Gamma_{00}^l \Gamma_{lm}^m - \Gamma_{0l}^m \Gamma_{0m}^l \tag{56}$$

$$= \frac{\partial \Gamma_{00}^l}{\partial x^l} + \frac{\partial \Gamma_{0l}^l}{\partial x^0} + \Gamma_{00}^l \Gamma_{lm}^m \tag{57}$$

$$\frac{\partial \Gamma_{00}^l}{\partial x^l} = \frac{\partial \Gamma_{00}^0}{\partial x^0} + \frac{\partial \Gamma_{00}^1}{\partial x^1} \tag{58}$$

$$= \frac{\partial \frac{\dot{\nu}}{2}}{\partial x^0} + \frac{\partial \frac{\nu'}{2} e^{\nu - \lambda}}{\partial x^1} \tag{59}$$

$$= \frac{\partial \frac{\dot{\nu}}{2}}{\partial x^0} + \frac{\nu'}{2} \frac{\partial e^{\nu - \lambda}}{\partial x^1} + e^{\nu - \lambda} \frac{\partial \frac{\nu'}{2}}{\partial x^1} \tag{60}$$

$$= \frac{\partial \frac{\dot{\nu}}{2}}{\partial x^0} + \frac{\nu'}{2} (\nu' - \lambda') e^{\nu - \lambda} + e^{\nu - \lambda} \frac{\nu''}{2} \tag{61}$$

$$\frac{\partial \Gamma_{0l}^l}{\partial x^0} = \frac{\partial \Gamma_{00}^0}{\partial x^0} + \frac{\partial \Gamma_{01}^1}{\partial x^0} \tag{62}$$

$$\tag{63}$$

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