Simulation of Brownian Motion – Laboratory nr 1

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1. Introduction

Brownian motion is the random motion of the particles suspended in the fluid, which was first observed by the scottish botanist Robert Brown in 1827 and first explained by Albert Einstein in 1905. This explanation of Brownian motion served as convincing evidence that atoms and molecules exist, and was further verified experimentally by Jean Perrin in 1908. Perrin was awarded the Nobel Prize in Physics in 1926 "for his work on the discontinuous structure of matter". The direction of the force of atomic bombardment is constantly changing, and at different times the particle is hit more on one side than another, leading to the seemingly random nature of the motion.

2. Execution of the exercise

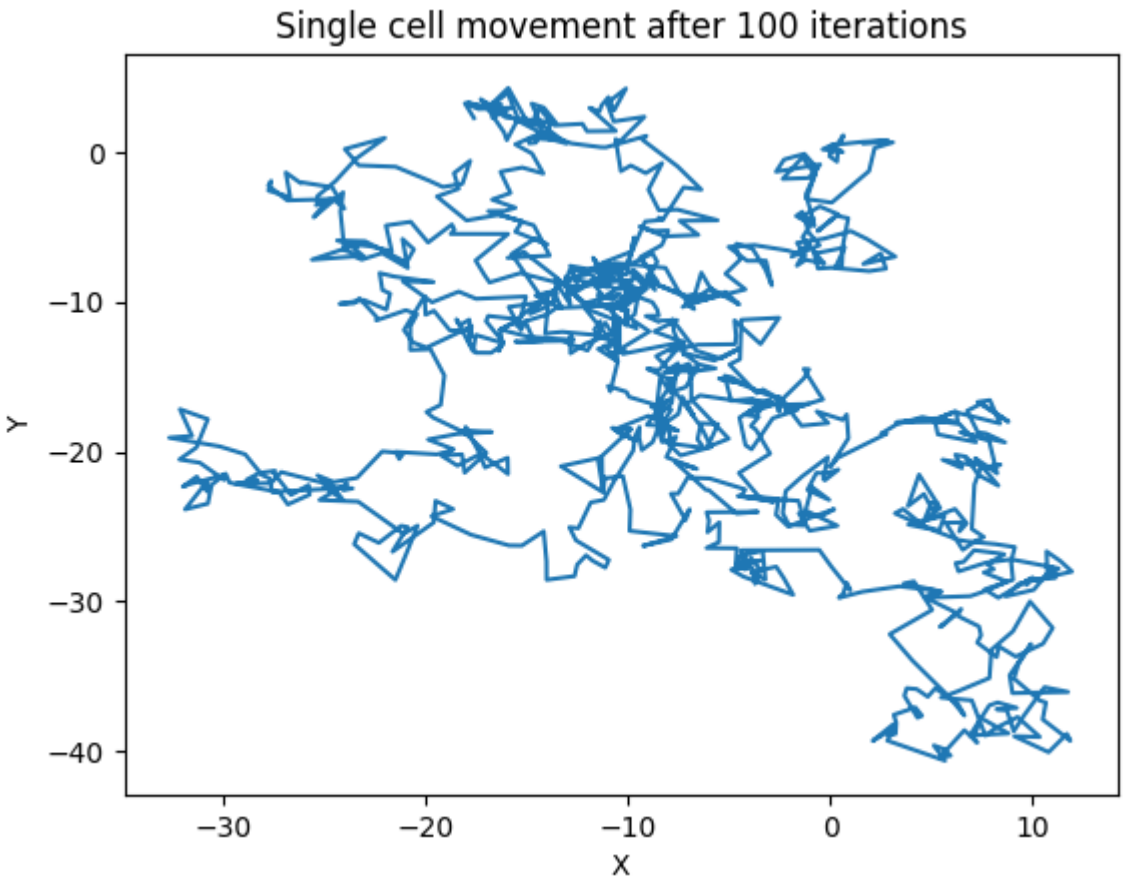
2.1 Creating simulation

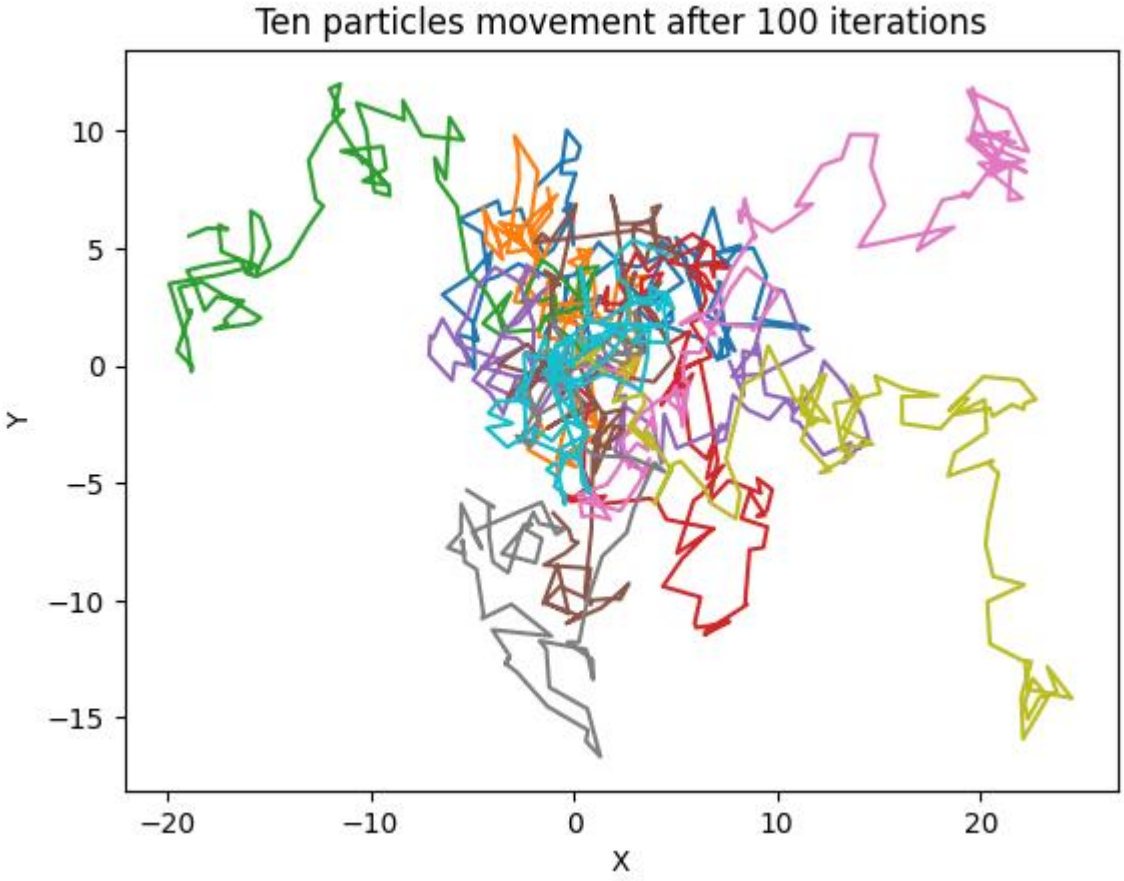
At first we need to create a program to simulate brownian motion. Programming language used in the simulation is Python 3.10. Class „Particle" was created to simulate single particle movement. We are going to simulate brownian motion based on a 'random walk' simulation:

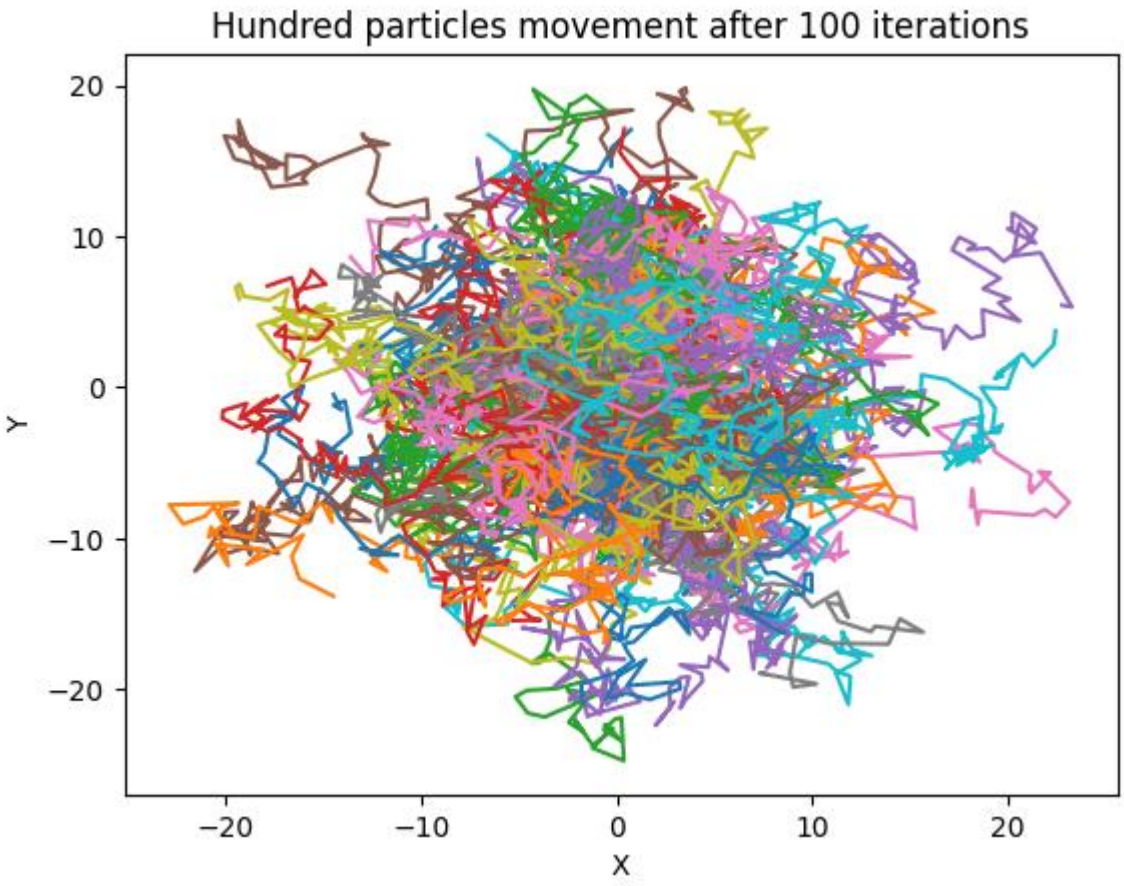
```python
import numpy as np

class Particle:
    def __init__(self, x=0, y=0):

        self.x = x
        self.y = y
        self.lista_x = [x]
        self.lista_y = [y]

    def make_step(self):

        self.x += np.random.normal(0, 1, 1)[0]
        self.y += np.random.normal(0, 1, 1)[0]

        self.lista_x.append(self.x)
        self.lista_y.append(self.y)
```

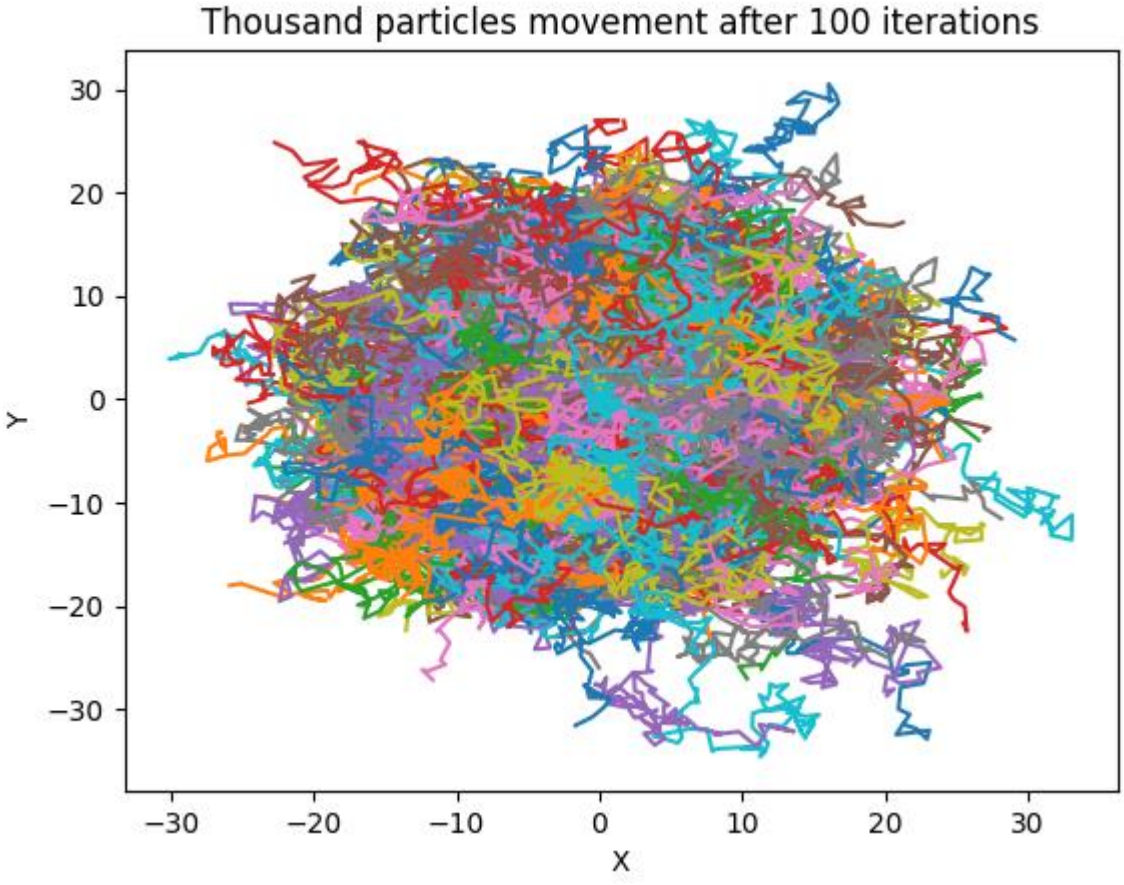
2.2 Executing simulation

Next, the movement of single, ten, hundred and thousand particles was presented. Each of these plots presents particles after 100 iterations.


Single cell movement after 100 iterations
0
−10
−20
−30
−40
Y
−30
−20
−10
0
10
X

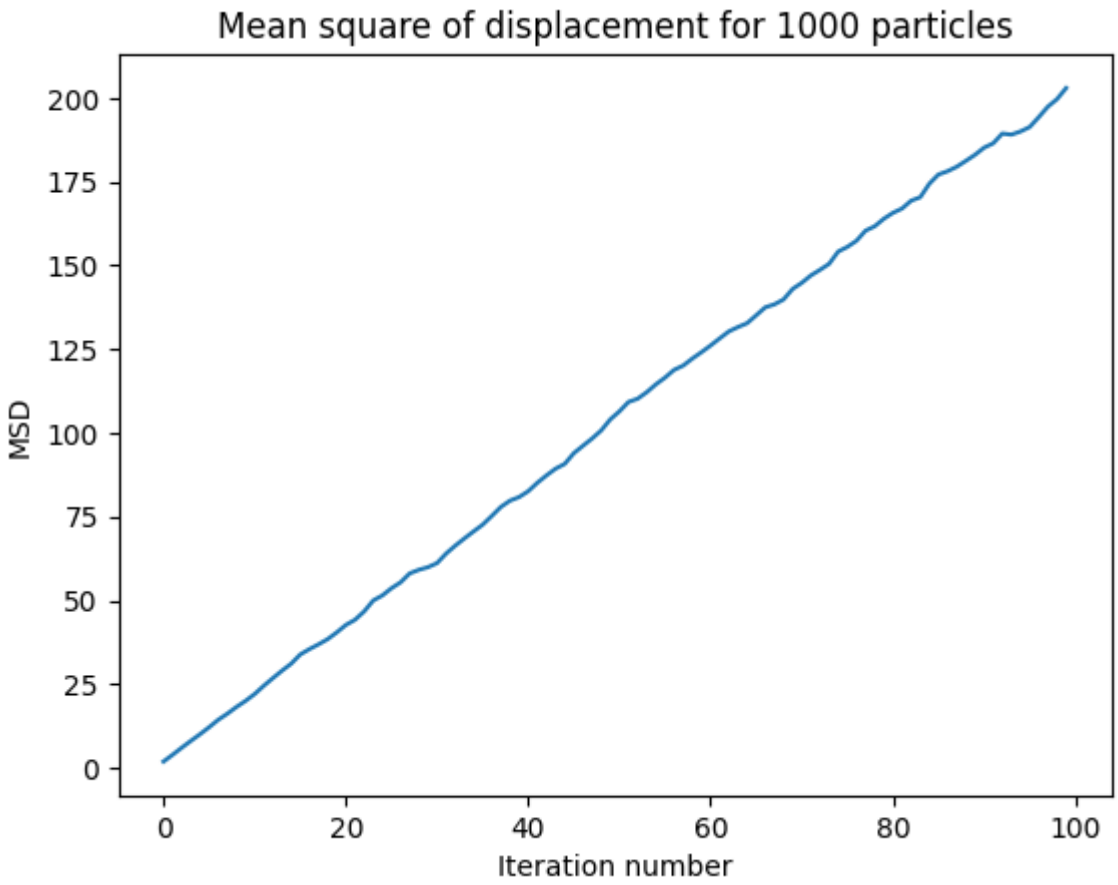

Ten particles movement after 100 iterations
10
5
0
−5
−10
−15
Y
−20
−10
0
10
20
X

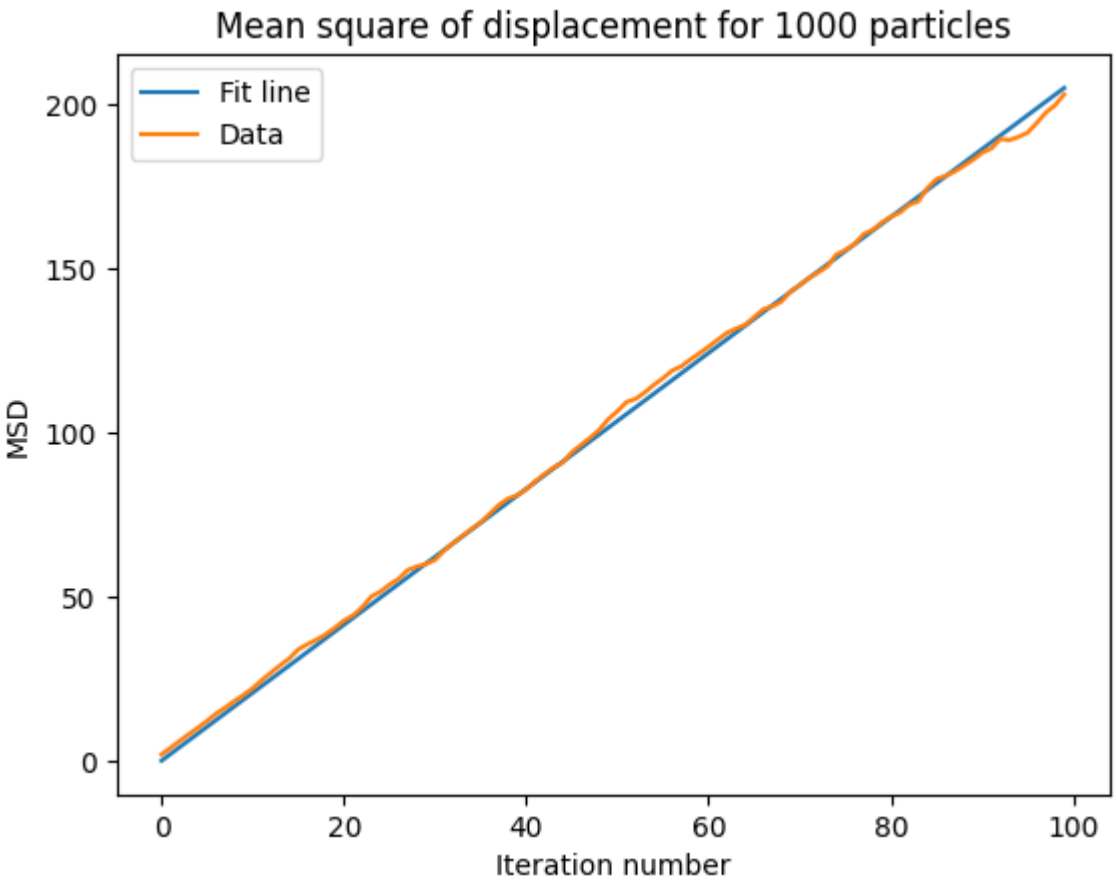

Hundred particles movement after 100 iterations
20
10
0
−10
−20
Y
−20
−10
0
10
20
X


Thousand particles movement after 100 iterations
30
20
10
0
−10
−20
−30
Y
−30
−20
−10
0
10
20
30
X

2.3 Mean square displacement

Next, on the basis of trajectory of thousand particles, mean square displacement was calculated for each iteration. Results are presented below:



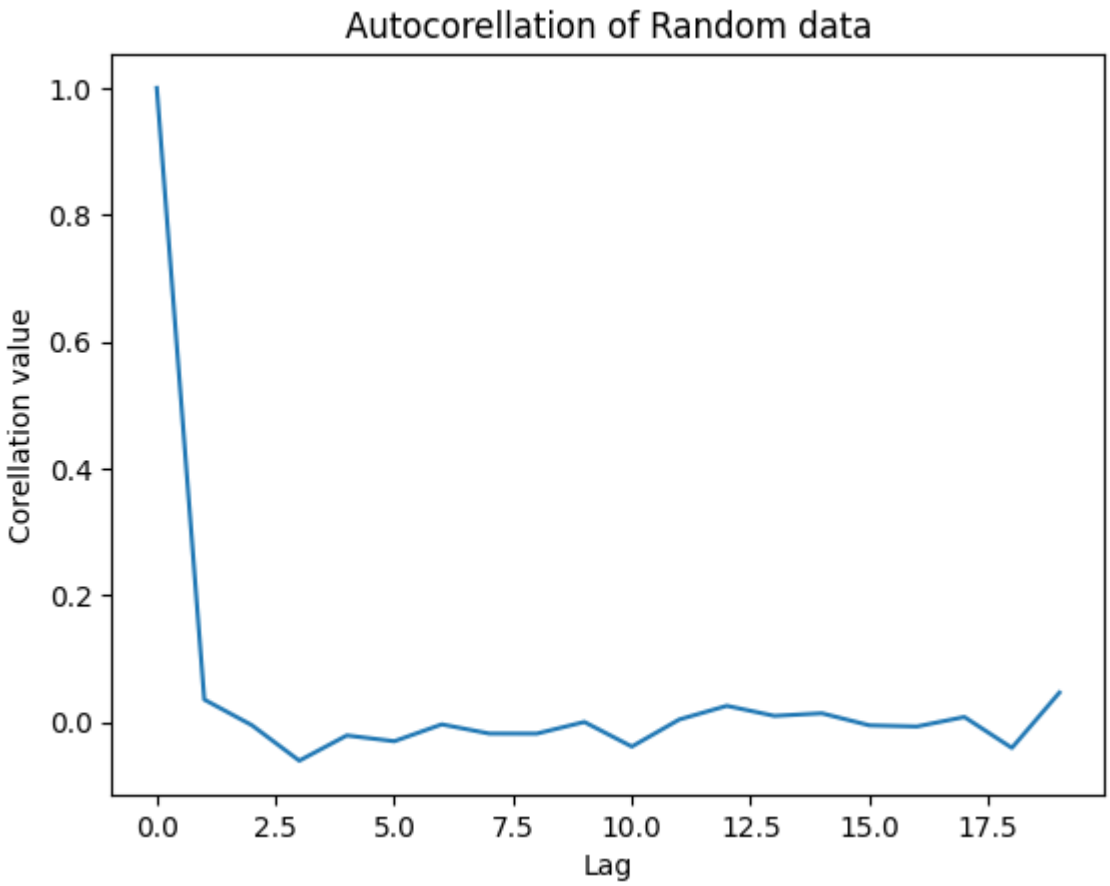


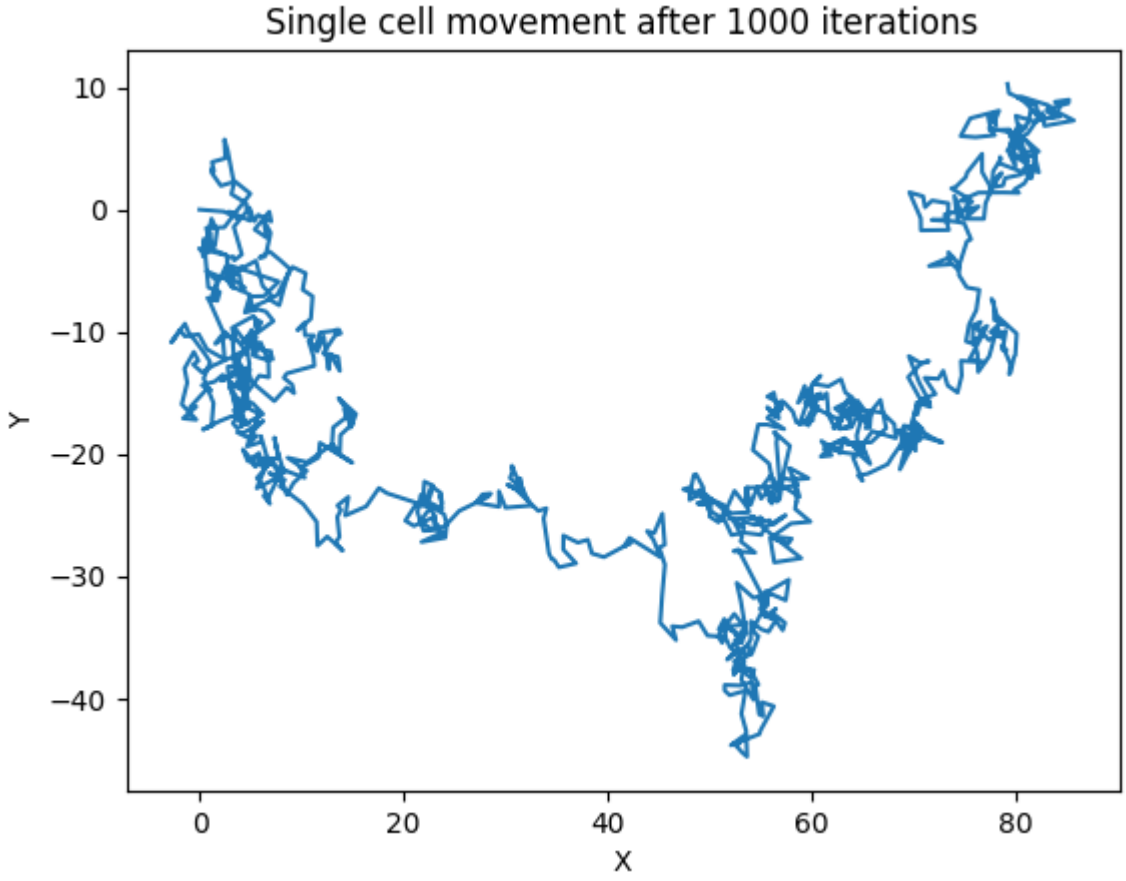
Coefficient of fitted line was calculated as of: 2.070

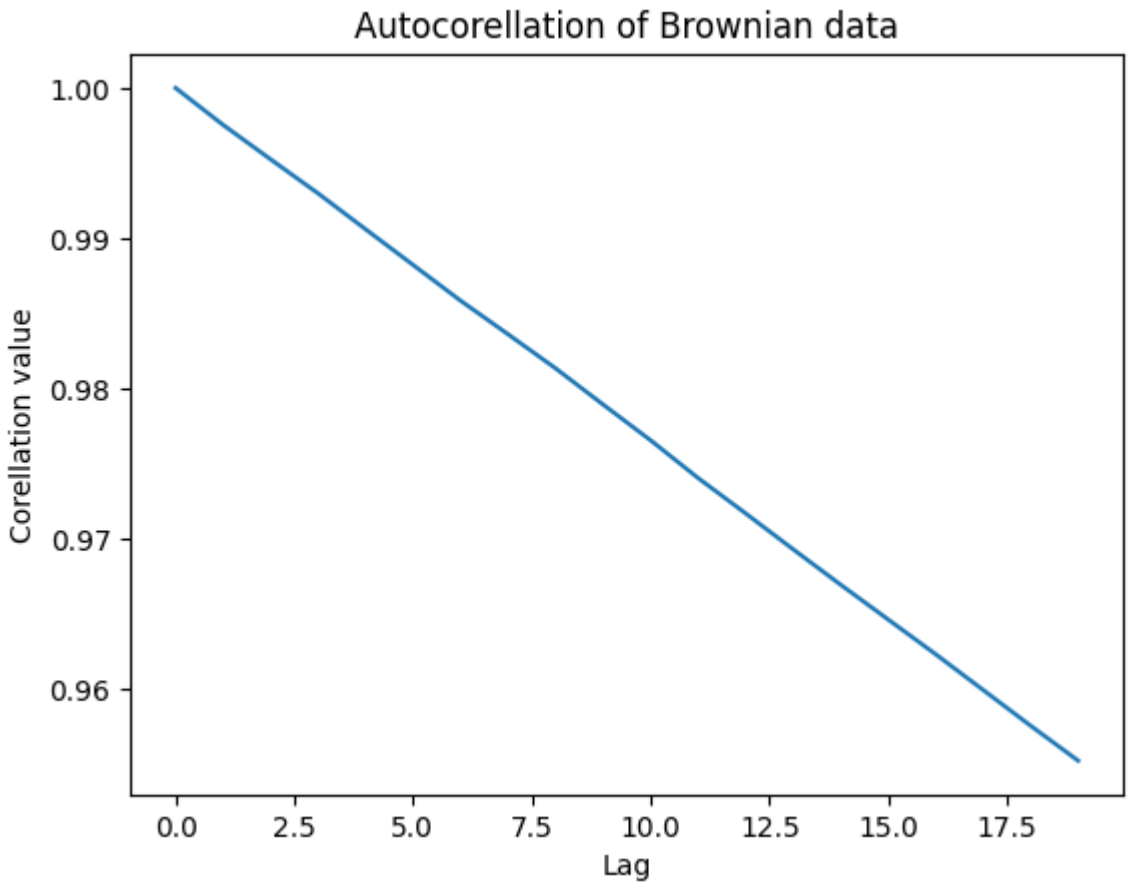
As we can observe, MSD is correlated to the number of iteration or the time passed. It means that single particle walk is random, but pack of particles have some properties which are constant.

2.4 Autocorellation

Next thing that was done, was calculating autocorellation for the single trajectory of a particle. Randomly generated data don't posses this trait, but brownian particle trajectory is autocorellated.


Autocorellation of Random data
Corellation value
1.0
0.8
0.6
0.4
0.2
0.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
Lag

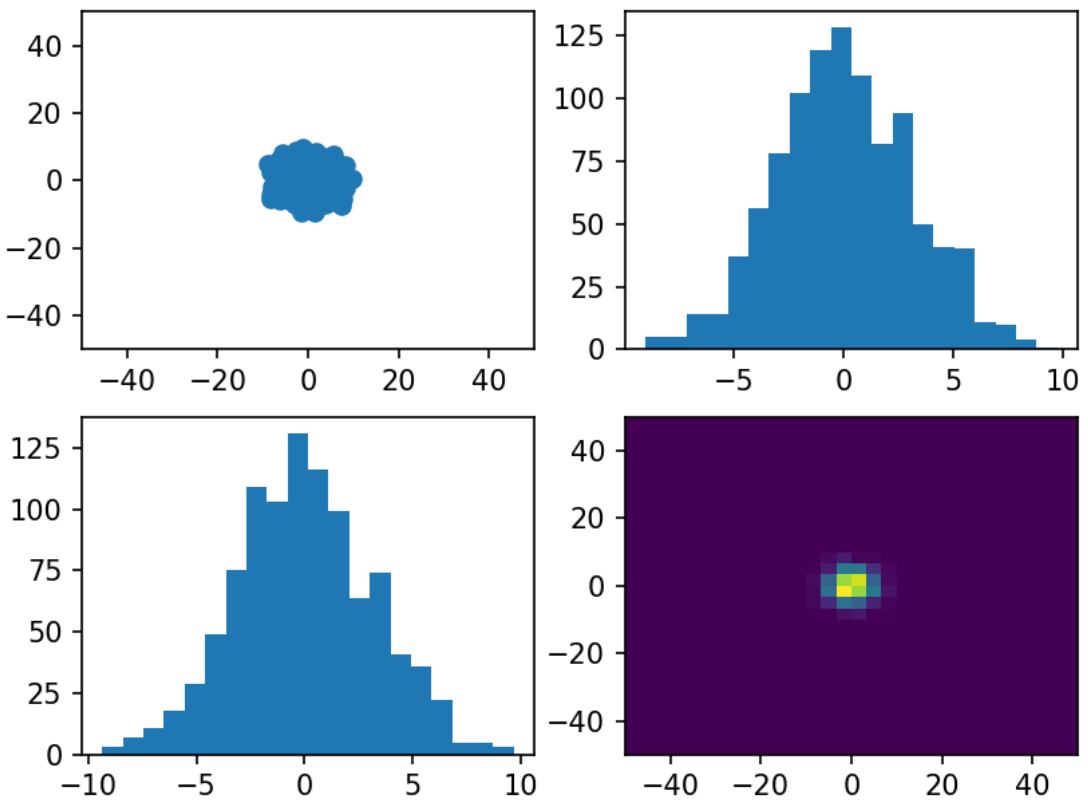

Single cell movement after 1000 iterations
Y
10
0
−10
−20
−30
−40
0
20
40
60
80
X

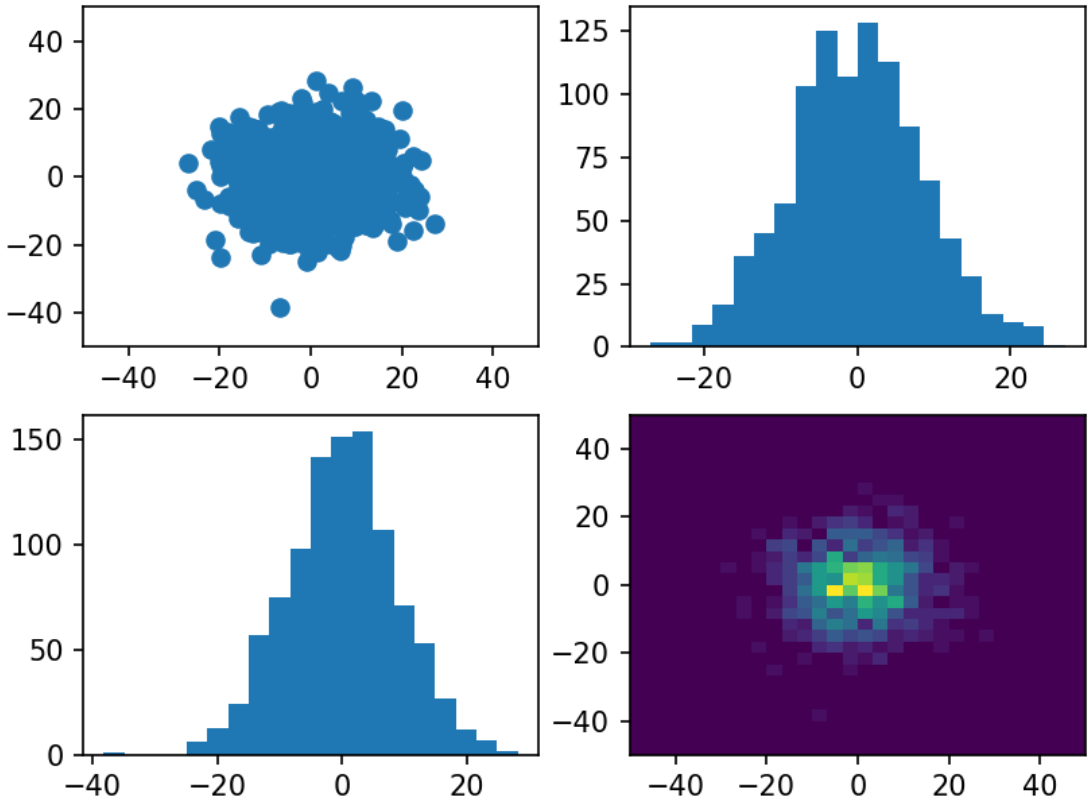


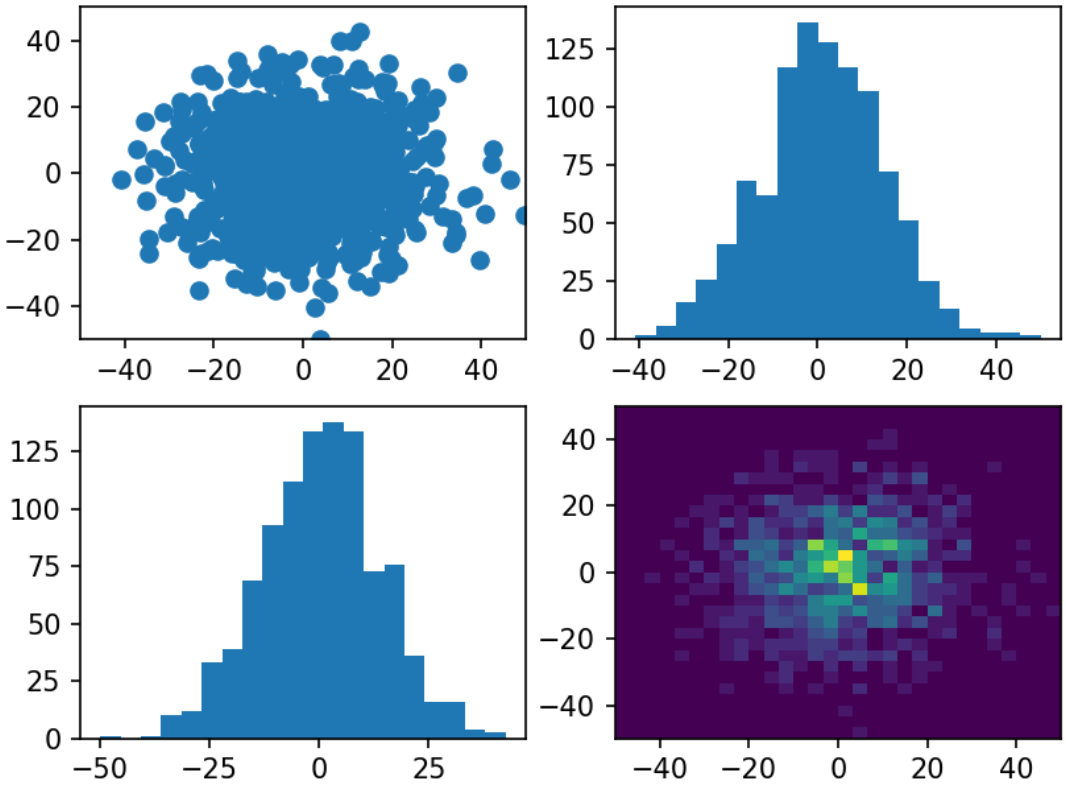
We see the expected rusults on the above plots.

2.5 Density plot

Lastly, the heatmap of density points was created and evaluated over time.


40
20
0
−20
−40
−40 −20 0 20 40
125
100
75
50
25
0
−5 0 5 10
125
100
75
50
25
0
−10 −5 0 5 10
40
20
0
−20
−40
−40 −20 0 20 40


40
20
0
−20
−40
−40 −20 0 20 40
125
100
75
50
25
0
−20 0 20
150
100
50
0
−40 −20 0 20
40
20
0
−20
−40
−40 −20 0 20 40



On the histograms we see that X&Y coordiantes and density of the particles is being shifted towards the rim, but still the most dense point on the Surface is starting point (0, 0).

3. Conclusion

Brownian motion is not solely based on random distribution (as implemented in the model), but motion on the basis of bell curve distribution is excellent for the sake of simulation. With ‘random walk’ we can describe and predict diffrent subject in the fields of physics, chemistry, economics, psychology and other related sciences.

From the conducted laboratory we can conclude few things: ‘random walk’ is an acceptable aproximation of brownian motion, brownian motion is random on the scale of one particle, but on the larger scale submits to the rules (such as mean square distance dependant on time, iterations), brownian motion is in fact autocorellated in contrast to random generated data.

4. Repository

Github: https://github.com/tycjantyc/Modeling_of_Physical_Systems