# Laboratory exercise no. 4: Application of box models to Upper Danube catchment simulation.

Aim: The aim of the laboratory is to calculate a mean residence time of water in the river catchment.

Laboratory programme:

1. Get acquainted with the methodology and modelling object
2. Writing of the programme code implementing the above methodology
3. Selection of the best transit function
4. Calculation of water mean residence time in the system using trial-error method
5. Implementing of the inverse modelling methodology
6. Discussion of the results and conclusions

In box models a modeling object is treated as so called "black-box". The information about the object characteristics is often obtained from the tracer experiments allowing to calculate the response of the system for pulse input function. Based on that response it is possible to calculate system response for any other extortion assuming linearity of the object.

$$C(t) = \int_{-\infty}^{t} C_{in}(t')g(t-t')\exp[-\lambda(t-t')]dt'$$

where:
$C(t)$ – output function
$C_{in}(t)$ – input function
$g(t)$ – transit time distribution function
$\lambda$ - radioactive tracer decay constant
$t$ – time variable
$t'$ – integration parameter

For the objects like ground water reservoirs, river catchments and other hydrological systems the most popular transit functions are:

1. Piston-flow model

$$g(t-t') = \delta((t-t')-t_t)$$

2. Exponential model

$$g(t-t') = t_t^{-1}\exp(-(t-t')/t_t)$$

3. Dispersion model

$$g(t-t') = (4\pi Pe(t-t')/t_t)^{-1/2} \cdot 1/(t-t') \cdot \exp\left[-\frac{(1-(t-t')/t_t)^2}{4Pe(t-t')/t_t}\right]$$

where: Pe – Peclet number equal to vx/D
$t_t$ – mean residence time

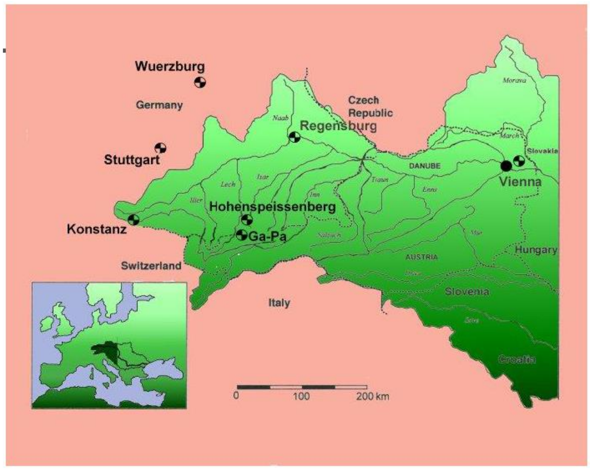
Pe and $t_t$ are the model parameters.

It is also allowed to use a combination of above functions forming parallel, serial or combined configurations.

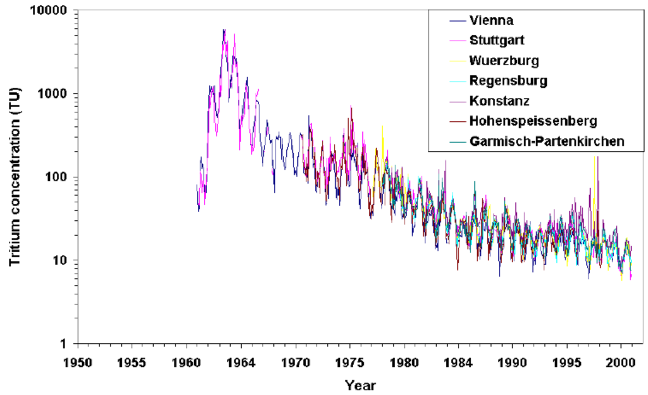
The modelling object is upper part of the Danube river catchment. The tracer used in modelling is tritium ($^3H$).

- Longest available $^3H$ record
- Catchment area: 817000 km$^2$
- River length: 2857 km

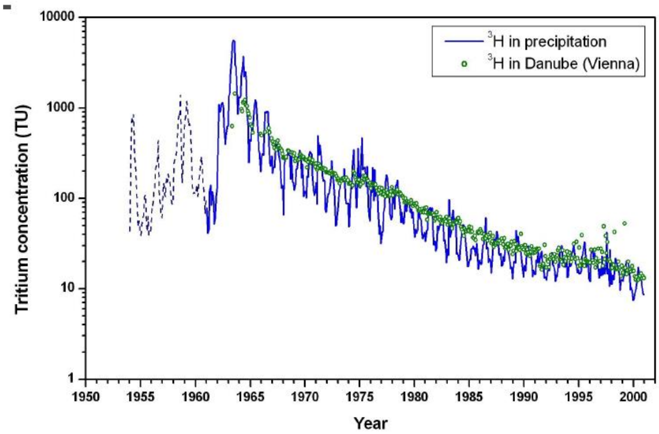
Catchment area is presented on the figure below:




There are several atmospheric precipitation monitoring stations located in the study area allowing for the preparation of input function. The tritium in precipitation record for those stations is presented on the next figure:




The input function used in the modelling exercise is calculated as a mean value of tritium concentration in the precipitation, weighted by the precipitation amount for each station. Moreover the input function is extender back to the year 1953 by the extrapolation based on the data originating from Ontario, Canada. Obtained input function (weighted average tritium content in the precipitation on the area of catchment) along with the output (tritium concentration in the river water in Vienna) is presented on the next figure:




Both datasets are provided by the tutor.

Laboratory outline:

1. Writing of the programme code calculating the convolution integral (eq. 1) for selected transit time distribution function (piston-flow, exponential and dispersion model).
2. Analysis of the optimal model.
3. Calculation of the mean residence time of the water in upper Danube catchment using trial-error method.
4. Assuming the objective function
5. Extending the code by implementing the automatic optimization of the objective function.
6. Comparison of the results obtained by inverse modelling and trial-error method.
7. Computer programme can be written in any programming language or software environment. Recommended environment is MATLAB.
8. Programme code supplemented with appropriate comments should be included as a part of a report prepared in pdf format.
9. The report must include the conclusion.