

Simulation of Brownian Motion – Laboratory nr 1

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Date: 11.03.2024

1. Introduction

Brownian motion is the random motion of the particles suspended in the fluid, which was first observed by the scottish botanist Robert Brown in 1827 and first explained by Albert Einstein in 1905. This explanation of Brownian motion served as convincing evidence that atoms and molecules exist, and was further verified experimentally by Jean Perrin in 1908. Perrin was awarded the Nobel Prize in Physics in 1926 "for his work on the discontinuous structure of matter". The direction of the force of atomic bombardment is constantly changing, and at different times the particle is hit more on one side than another, leading to the seemingly random nature of the motion.

2. Execution of the exercise

2.1 Creating simulation

At first we need to create a program to simulate brownian motion. Programming language used in the simulation is Python 3.10. Class „Particle” was created to simulate single particle movement:

```
import numpy as np

class Particle:
    def __init__(self, x=0, y=0):

        self.x = x
        self.y = y
        self.lista_x = [x]
        self.lista_y = [y]

    def make_step(self):

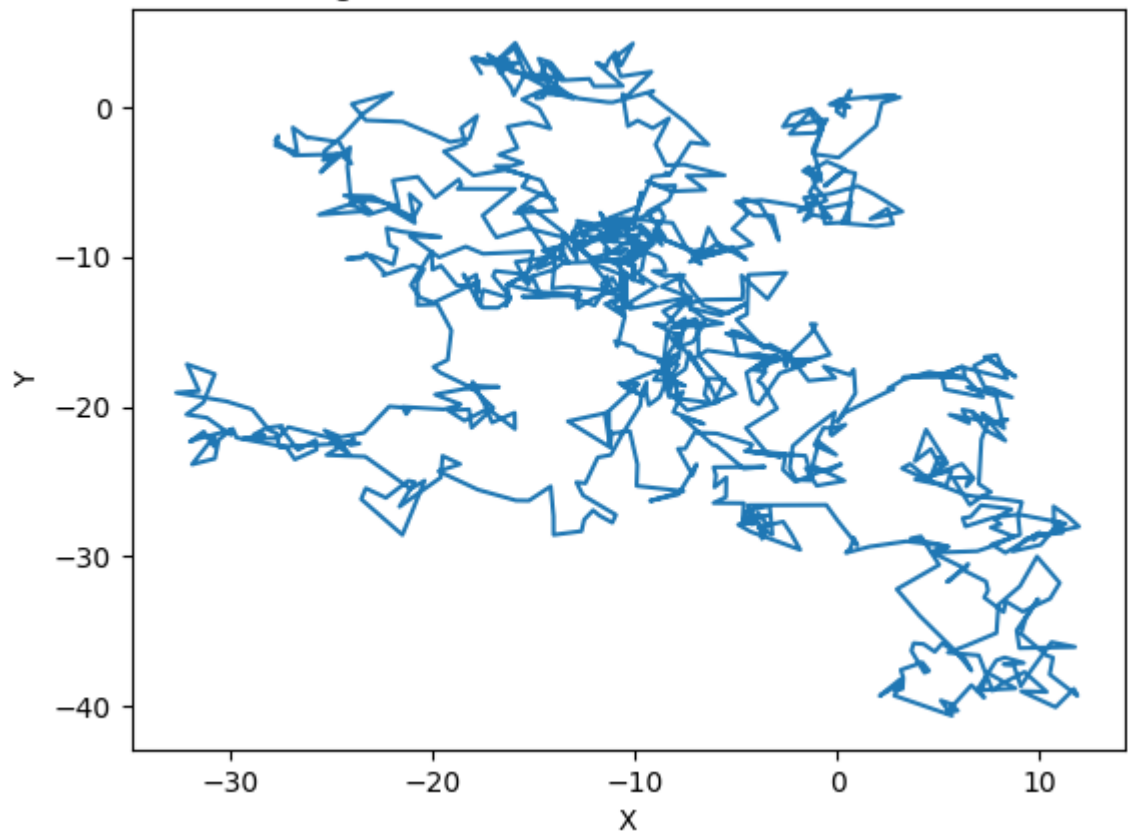
        self.x += np.random.normal(0, 1, 1)[0]
        self.y += np.random.normal(0, 1, 1)[0]

        self.lista_x.append(self.x)
        self.lista_y.append(self.y)
```

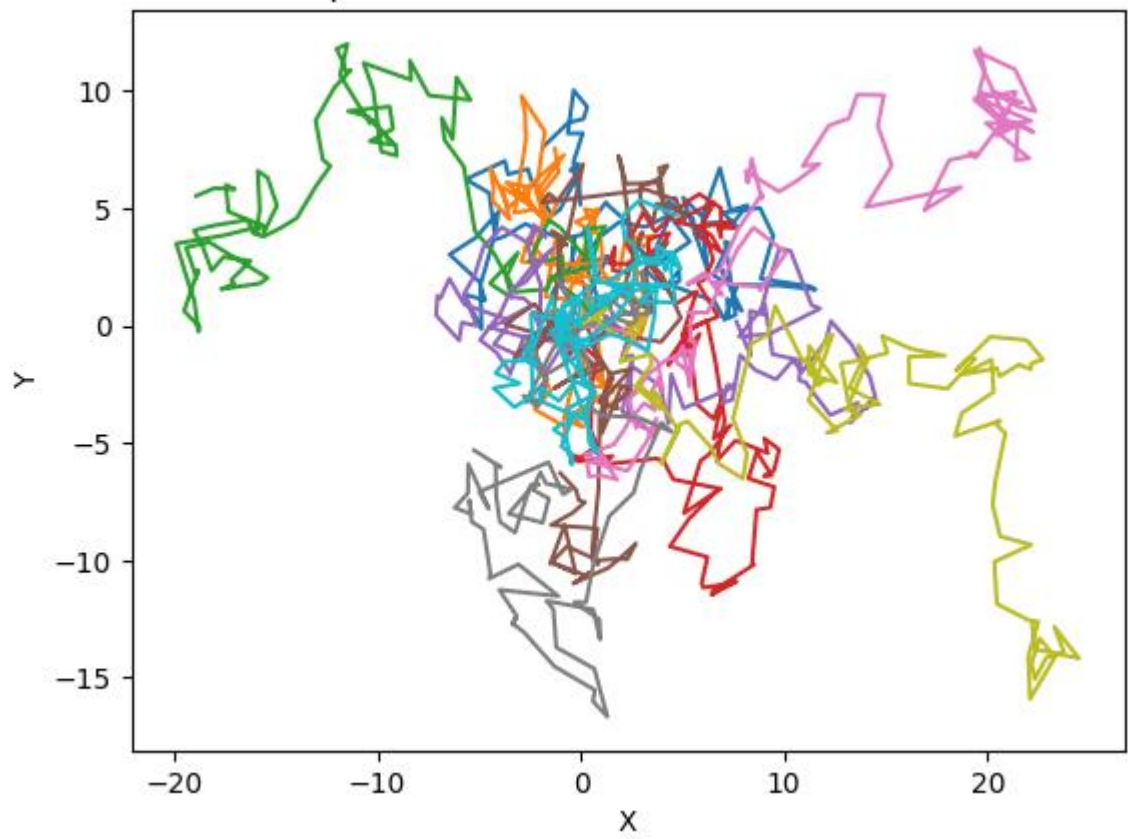
2.2 Executing simulation

Next, the movement of single, ten, hundred and thousand particles was presented. Each of these plots presents particles after 100 iterations.

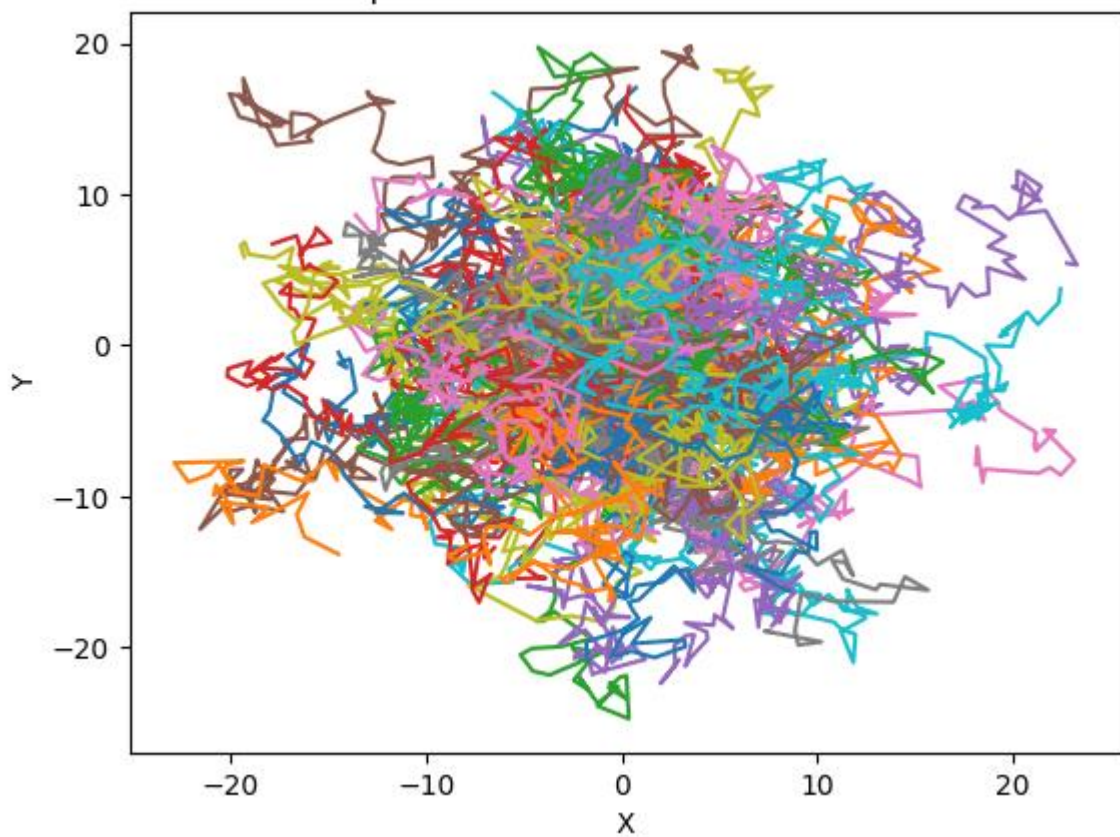
Single cell movement after 100 iterations



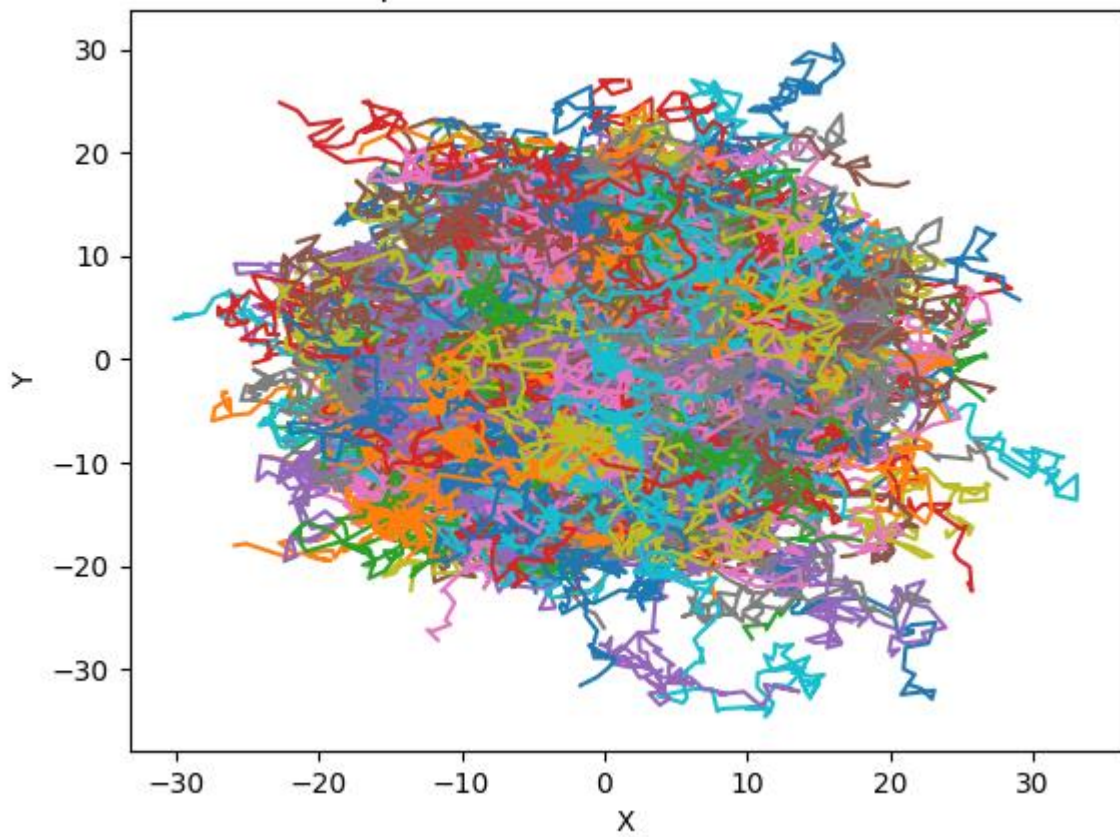
Ten particles movement after 100 iterations



Hundred particles movement after 100 iterations

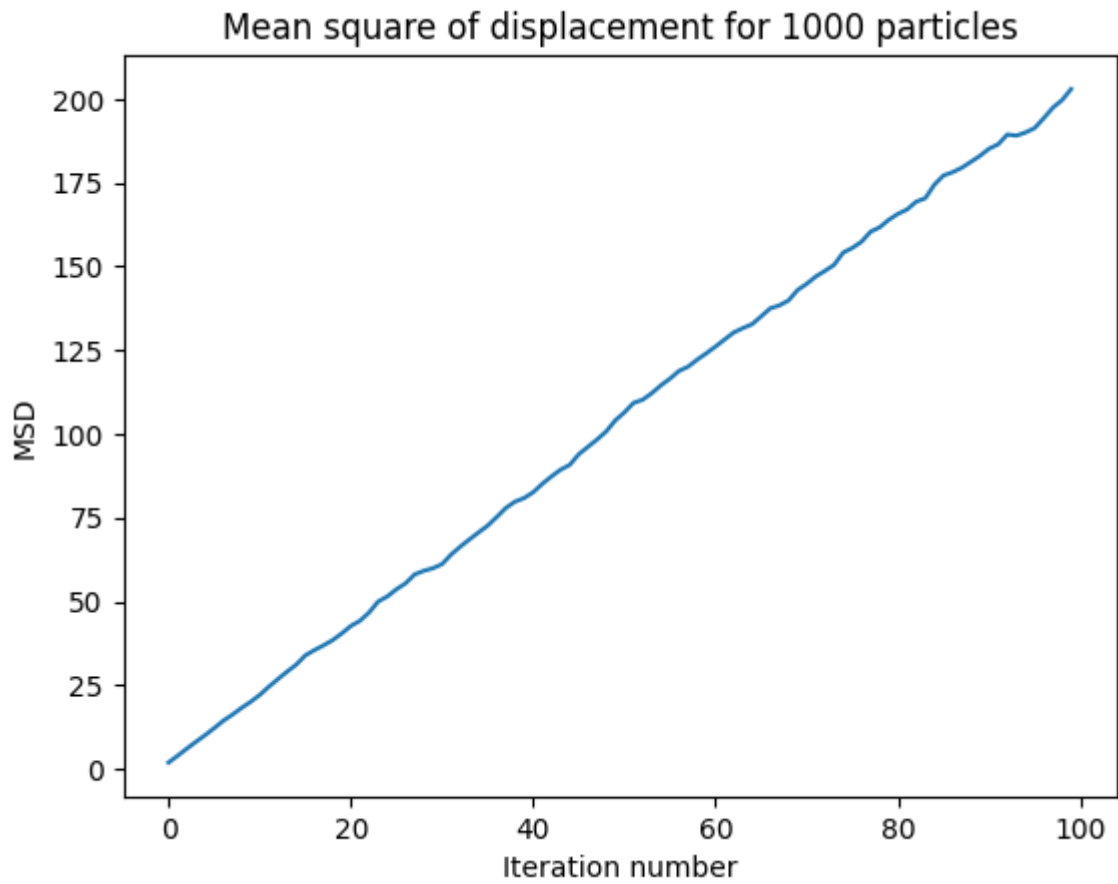


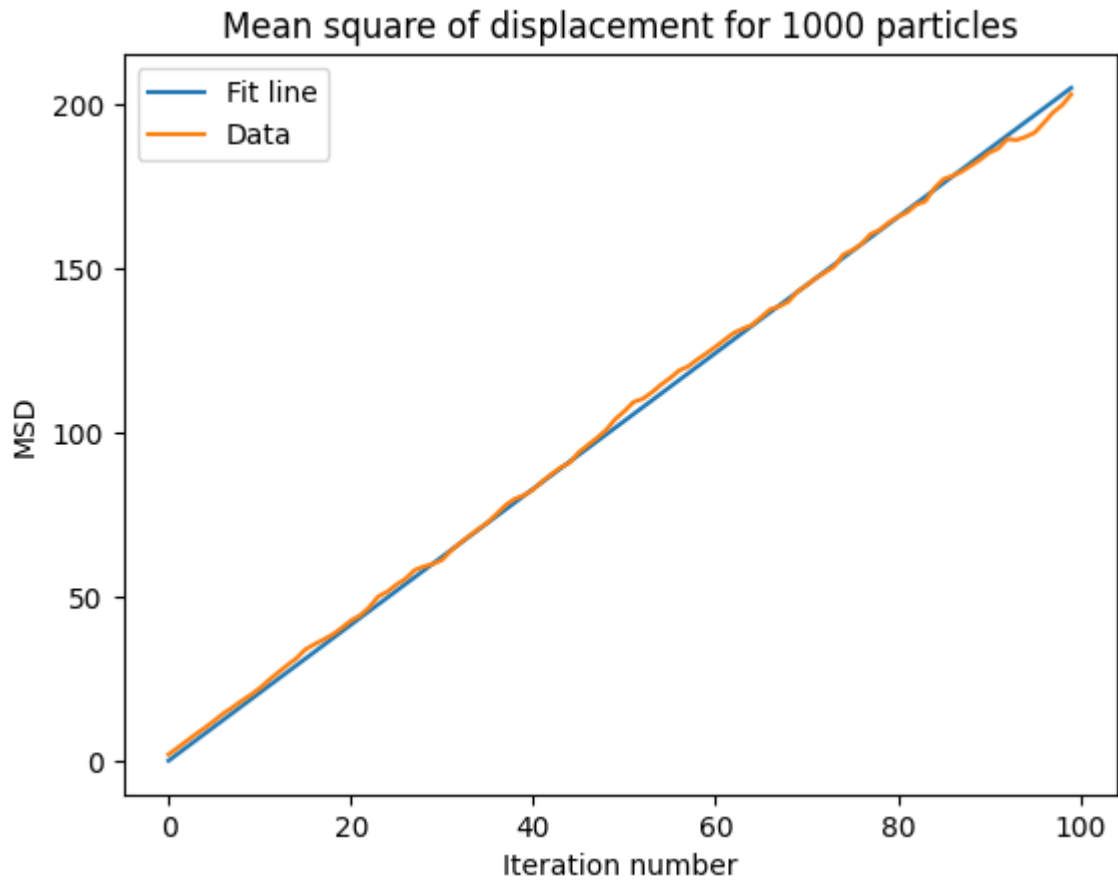
Thousand particles movement after 100 iterations



2.3 Mean square displacement

Next, on the basis of trajectory of thousand particles, mean square displacement was calculated for each iteration. Results are presented below:



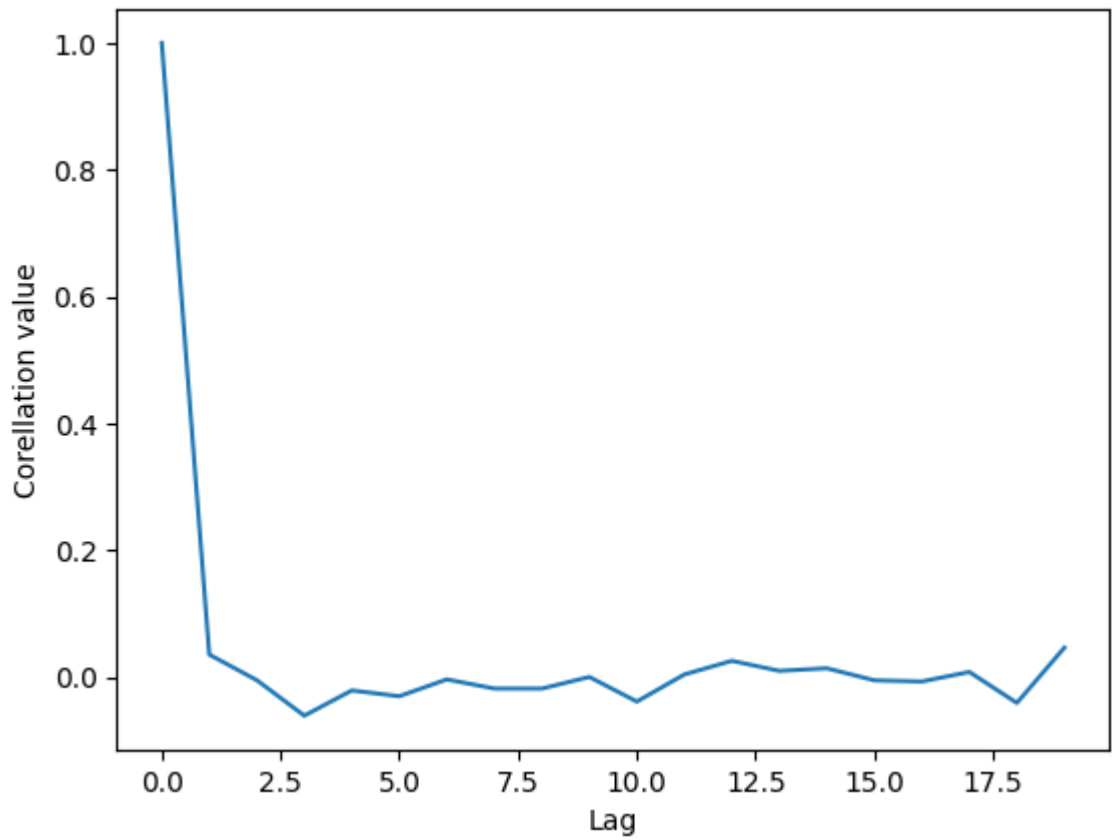


Coefficient of fitted line was calculated as of: 2.070

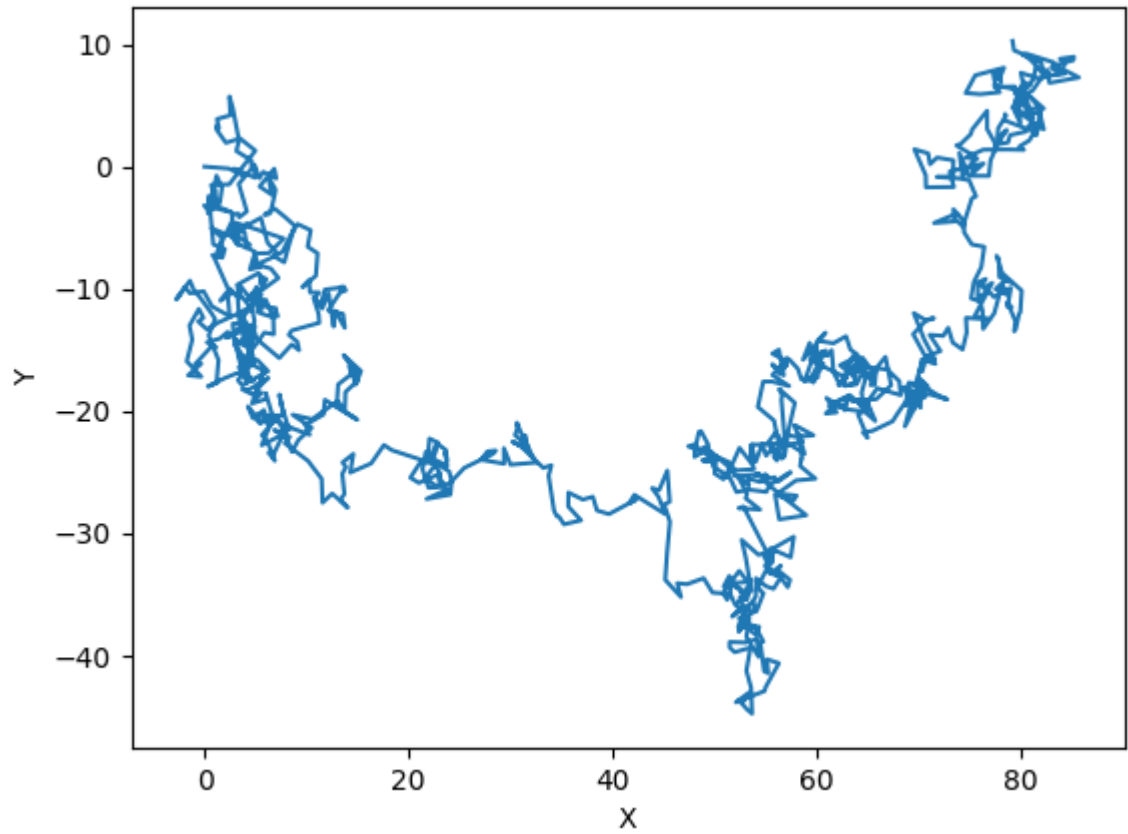
2.4 Autocorellation

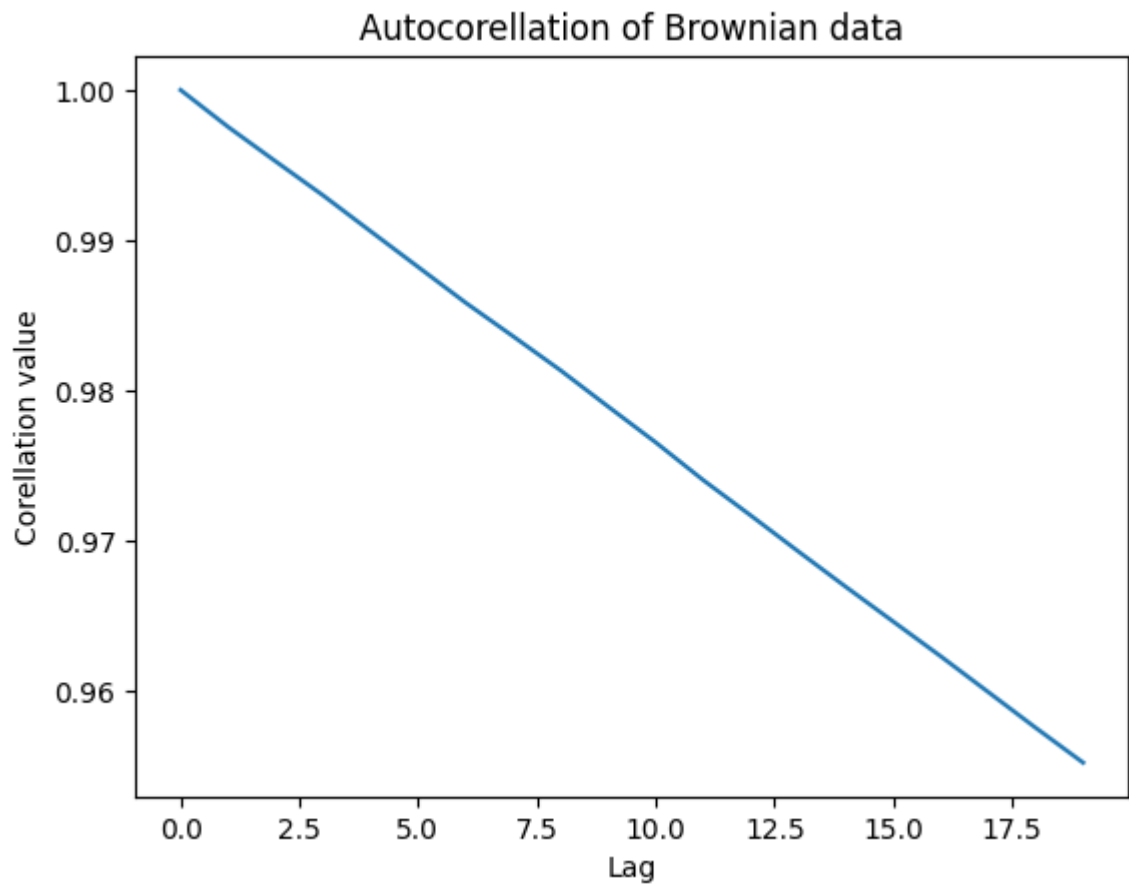
Next thing that was done, was calculating autocorellation for the single trajectory of a particle. Randomly generated data don't posses this trait, but brownian particle trajectory is autocorellated.

Autocorellation of Random data



Single cell movement after 1000 iterations

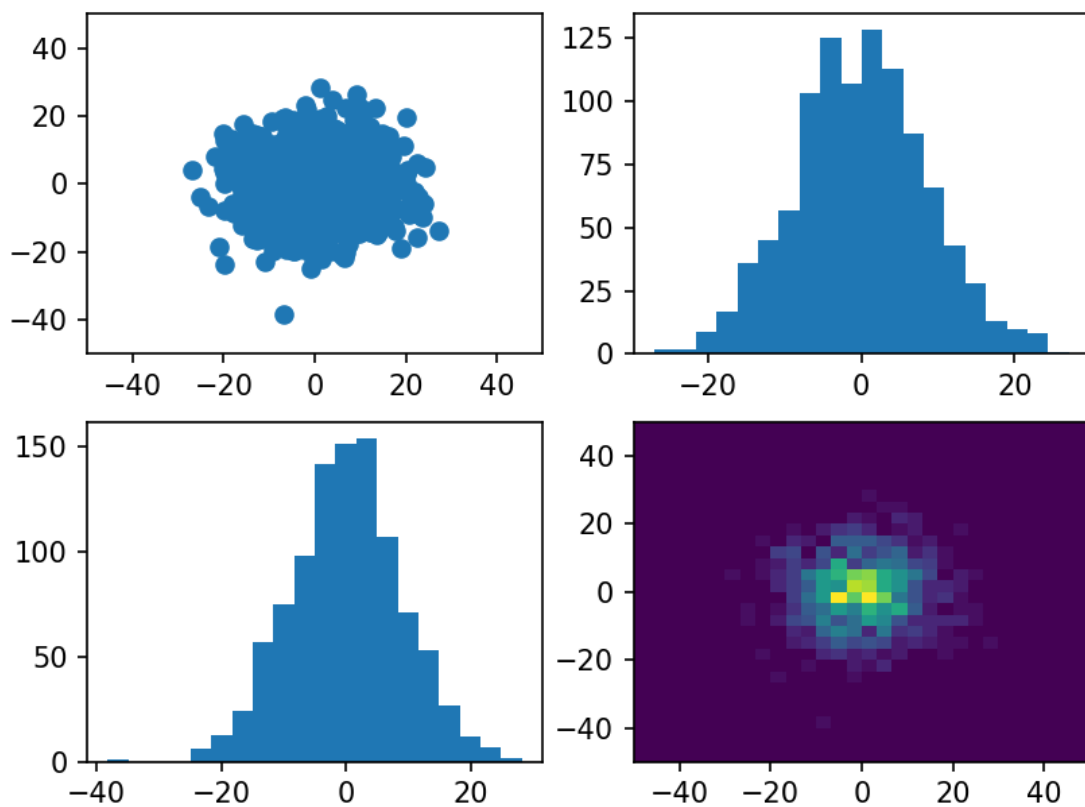
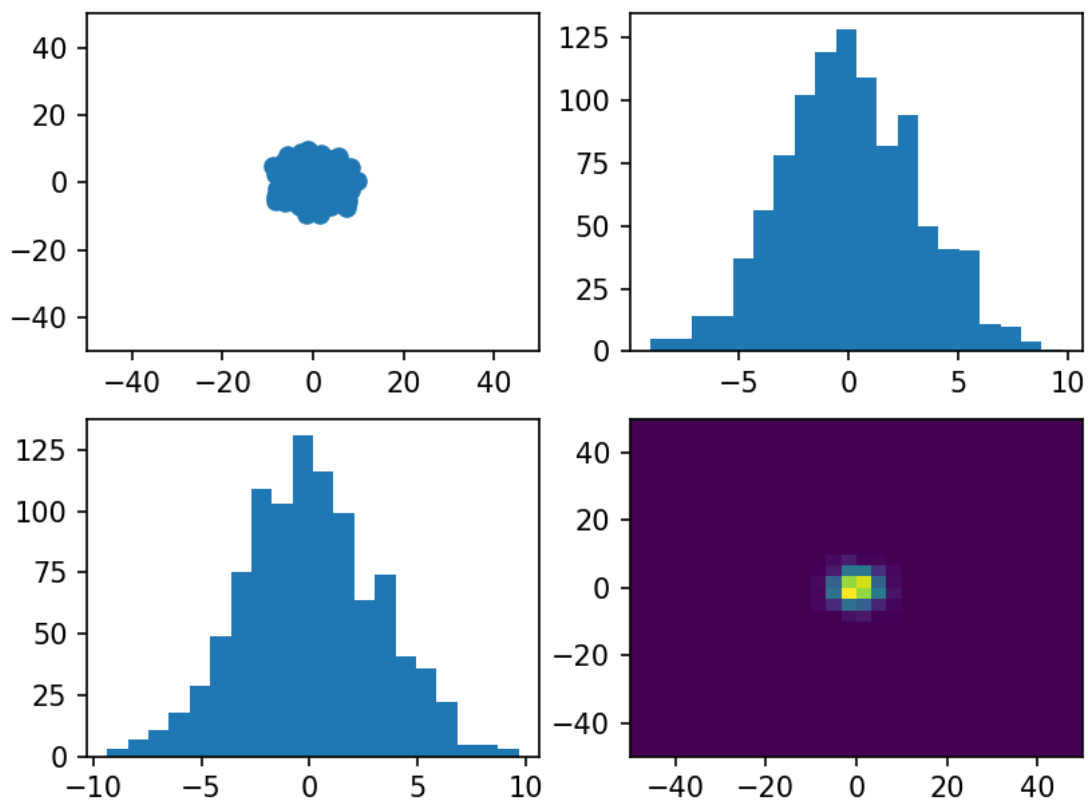


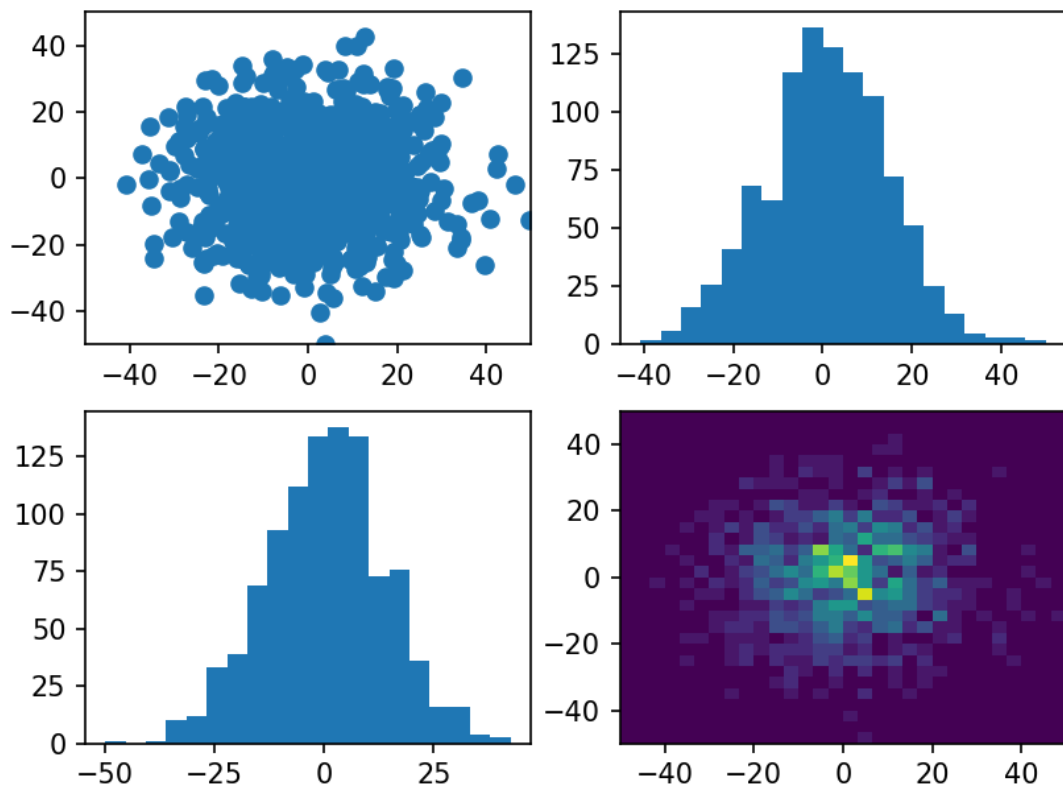


We see the expected results on the above plots.

2.5 Density plot

Lastly, the heatmap of density points was created and evaluated over time.





On the histograms we see that X&Y coordinates and density of the particles is being shifted towards the rim, but still the most dense point on the Surface is starting point (0, 0).

3. Conclusion

Brownian motion is not solely based on random distribution (as implemented in the model), but motion on the basis of bell curve distribution is excellent for the sake of simulation. With 'random walk' we can describe and predict different subjects in the fields of physics, chemistry, economics, psychology and other related sciences.

From the conducted laboratory we can conclude a few things: 'random walk' is an acceptable approximation of Brownian motion, Brownian motion is random on the scale of one particle, but on the larger scale submits to the rules (such as mean square distance dependant on time, iterations), Brownian motion is in fact autocorrelated in contrast to random generated data.