MAE 5803 Nonlinear Control Systems **Assigned: Jan 31, 2017**
Homework #2 **Due: Feb 9, 2017**

Submit your answers to all questions below. Show your working steps in sufficient details.

1. (15 pts.)

   The norm used in the definitions of stability need not be the usual Euclidian norm. If the state space is of finite dimension *n* (i.e. the state vector has *n* components), stability and its type are independent of the choice of norm (all norms are "equivalent"), although a particular choice of norm may make analysis easier. For *n* = 2, draw the regions corresponding to the following norms:

   a. $\|\mathbf{x}\|^2 = x_1^2 + x_2^2 \le 1$ (Euclidian norm)

   b. $\|\mathbf{x}\|^2 = x_1^2 + 5x_2^2 \le 1$

   c. $\|\mathbf{x}\| = |x_1| + |x_2| \le 1$

   d. $\|\mathbf{x}\| = \sup(|x_1|, |x_2|) \le 1$

2. (25 pts.)

   For the following systems, find the *equilibrium points* and determine *their stability*. Indicate whether the stability is *asymptotic*, and whether it is *global*.

   a. $\dot{x} = -x^3 + \sin^4 x$

   b. $\dot{x} = (5 - x)^5$

   c. $\ddot{x} + \dot{x}^5 + x^7 = x^2 \sin^8 x \cos^2 3x$

   d. $\ddot{x} + (x-1)^4 \dot{x}^7 + x^5 = x^3 \sin^3 x$

   e. $\ddot{x} + (x-1)^2 \dot{x}^7 + x = \sin \dfrac{\pi x}{2}$

3. (15 pts.)

   Analyze the *stability* of the dynamics describing a mass sinking in a viscous fluid below:

   $$\dot{v} + 2a|v|v + bv = 0 \quad ; \quad a > 0,\ b > 0$$

   where *v* is the normalized sinking speed.

4. (20 pts.)

   Consider the following second order electrical system with nonlinear inductance, resistance, and capacitance:

   $$L(\dot{q})\,\ddot{q} + R(\dot{q}) + C(q) = 0$$

   where q is the charge in the capacitor, $L(\dot{q})$ is strictly positive, and both $R(\dot{q})$ and $C(q)$ are continuous and of the same sign as their arguments.

   Determine the *equilibrium point(s)* of the system and determine *stability*. Also indicate whether the stability is asymptotic, and under what conditions it is global.

5. (25 pts.)

Consider the system:

$$\dot{x} = 4x^2 y - f_1(x)[x^2 + 2y^2 - 4]$$
$$\dot{y} = -2x^3 - f_2(y)[x^2 + 2y^2 - 4]$$

where the continuous functions f_1 and f_2 have *the same sign as their argument*.

a. Show that $x^2 + 2y^2 = 4$ is the *limit cycle* of the system. Determine whether this is a *clockwise* or *counter-clockwise* limit cycle in the phase plane.

b. Show that the system tends toward the limit cycle *independent* of the explicit expressions of f_1 and f_2.