MAE 5803 - Homework #1 Problem #1

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Contents


```
clear; close all; clc;
warning('off','MATLAB:ode45:IntegrationTolNotMet') % suppress ode45 warnings
set(0,'defaulttextinterpreter','latex')
```

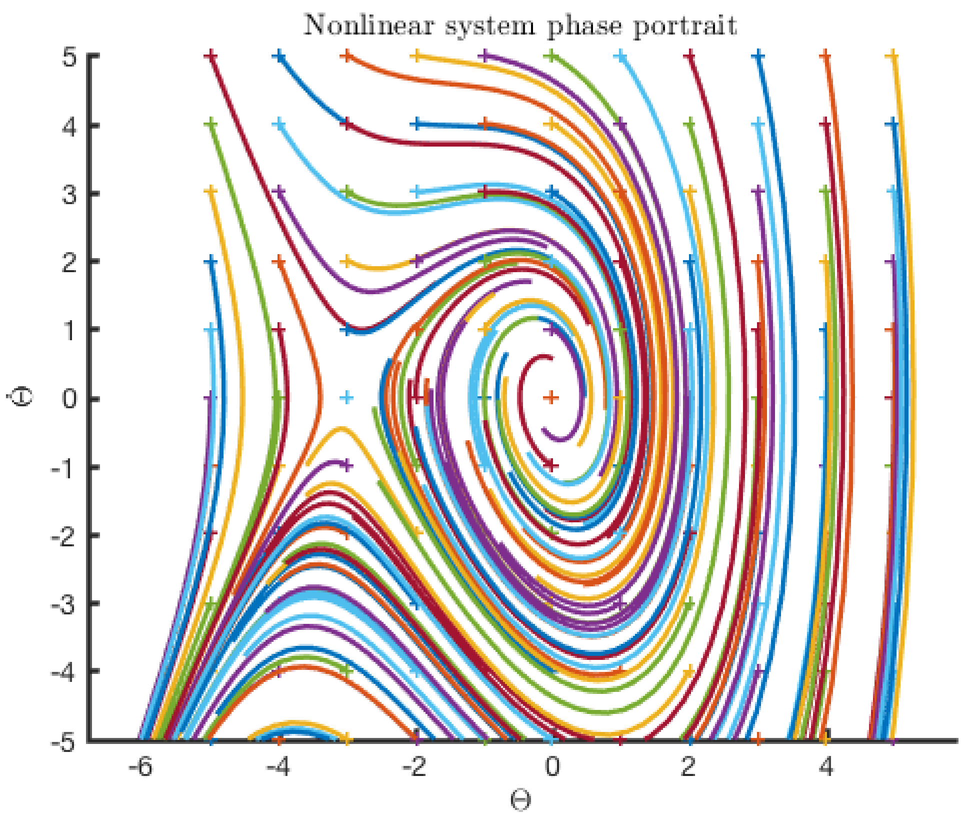
Second-Order Nonlinear State Equation:

$$\ddot{\Theta}_{(t)} + 0.6\dot{\Theta}_{(t)} + 3\Theta_{(t)} + \Theta^2_{(t)} = 0$$

Draw the phase portrait

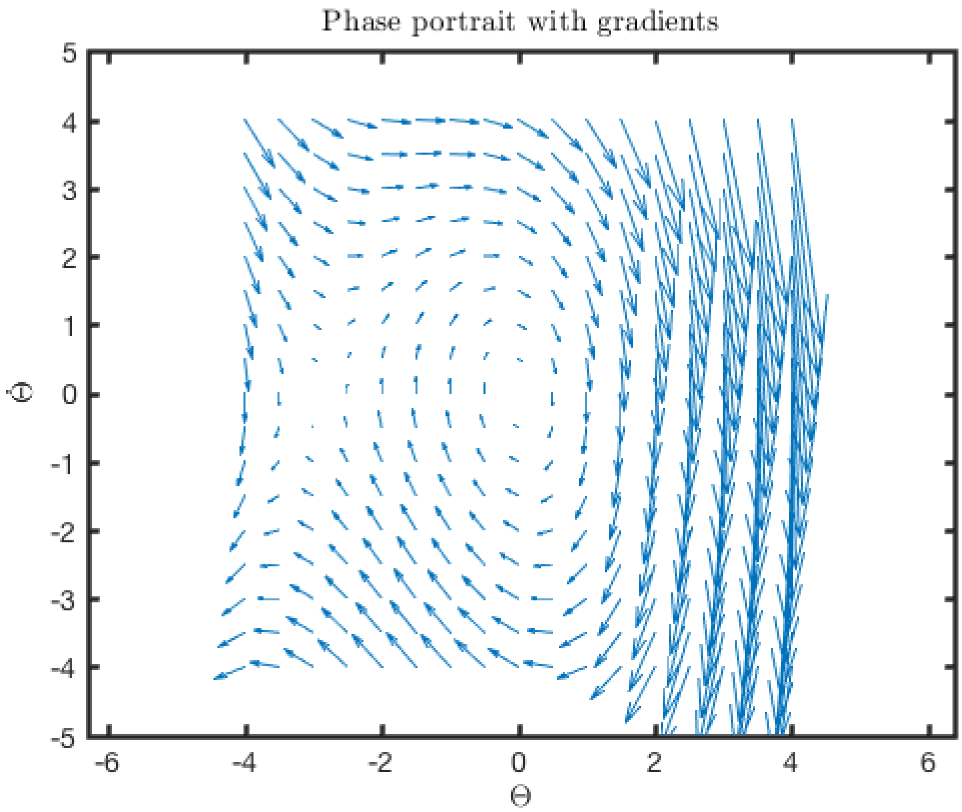
Numerically integrate the state equation using ode45 starting at various points in the plane. The Plus (+) marks indicate starting points for each simulated trajectory.

```
tspan = [0 2];
figure();
hold on
for i = -5:1:5
    for j = -5:1:5
        T0 = [i; j];
        options = odeset('RelTol',1e-4,'AbsTol',1e-7);
        [t,T] = ode45(@P1stateEqn,tspan,T0,[],1);
        h = plot(T(:,1),T(:,2));
        c = get(h,'color');
        plot(T0(1),T0(2),'+','color',c);
    end
end
axis([-5 5 -5 5])
axis equal
xlabel('$\Theta$')
ylabel('$\dot{\Theta}$')
title('Nonlinear system phase portrait')
hold off
```



Plot the field of the phase portrait

```
[x1, x2] = meshgrid(-4:0.5:4, -4:0.5:4);
x1dot = x2;
x2dot = -0.6*x2 - 3.*x1 - x1.^2;
figure()
quiver(x1,x2,x1dot,x2dot,'AutoScaleFactor',5)
axis([-5 5 -5 5])
axis equal
xlabel('$\Theta$')
ylabel('$\dot{\Theta}$')
title('Phase portrait with gradients')
```



a) From the phase portrait, identify the singular points of the system and determine their types (stable node, unstable focus, etc.).

Reference Slotine, Section 2.5 There are two singular points.

First Singular Point

The first is a singular point at the origin. It is a stable focus as evidenced by the negative real parts of the eigenvalues with nonzero imaginary parts.

The first-order system is:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -0.6x_2 - 3x_1 - x_1^2$$

```
eValue1 = polyeig(1,0.6,3)
```

```
eValue1 =

  -0.1000 + 0.5686i
  -0.1000 - 0.5686i
```

Second Singular Point

The second sigular point is a saddle point at (-3,0). As suggested in the text, shift the solution so this point is at the origin and specify new state variables, $\bar{X}$ accordingly. Rewrite the system of equations with the new state variables and find the eigenvalues of this system. One eigenvalue is positive and the other negative with no imaginary parts. Thus, the singular point is a saddle point.

$\bar{x}_{equil} = (-3, 0)$

$\xi_1 = x_1 + 3$

$\bar{\xi}_{equil} = (0, 0)$

$\dot{\xi}_1 = \xi_2$

$\dot{\xi}_2 = 0.6\xi_2 + 3\xi_1 - \xi_1^2 - 9$

```
eValue2 = polyeig(1,0.6,-3)
```

```
eValue2 =

   -0.4859
    0.6859
```

b) Obtain the linearized equations about the singular points of the system. Then, determine the eigenvalues of each linearized equation to determine the stability of the corresponding singular point.

The singular point is at the origin and, near the origin, Θ and Θ_{dot} are very small. If $\Theta < 1$, then $\Theta^2 << 1$ and we have justification for neglecting the last term of the original second-order state equation.

$\ddot{\Theta}_{(t)} + 0.6\dot{\Theta}_{(t)} + 3\Theta_{(t)} = 0$

The linearized first-order system about $x = (0, 0)$ is:

$\dot{x}_1 = x_2$

$\dot{x}_2 = -0.6x_2 - 3x_1$

The linearized first-order system about $x = (-3, 0)$ is:

$\dot{\xi}_1 = \xi_2$

$\dot{\xi}_2 = 0.6\xi_2 + 3\xi_1 - 9$

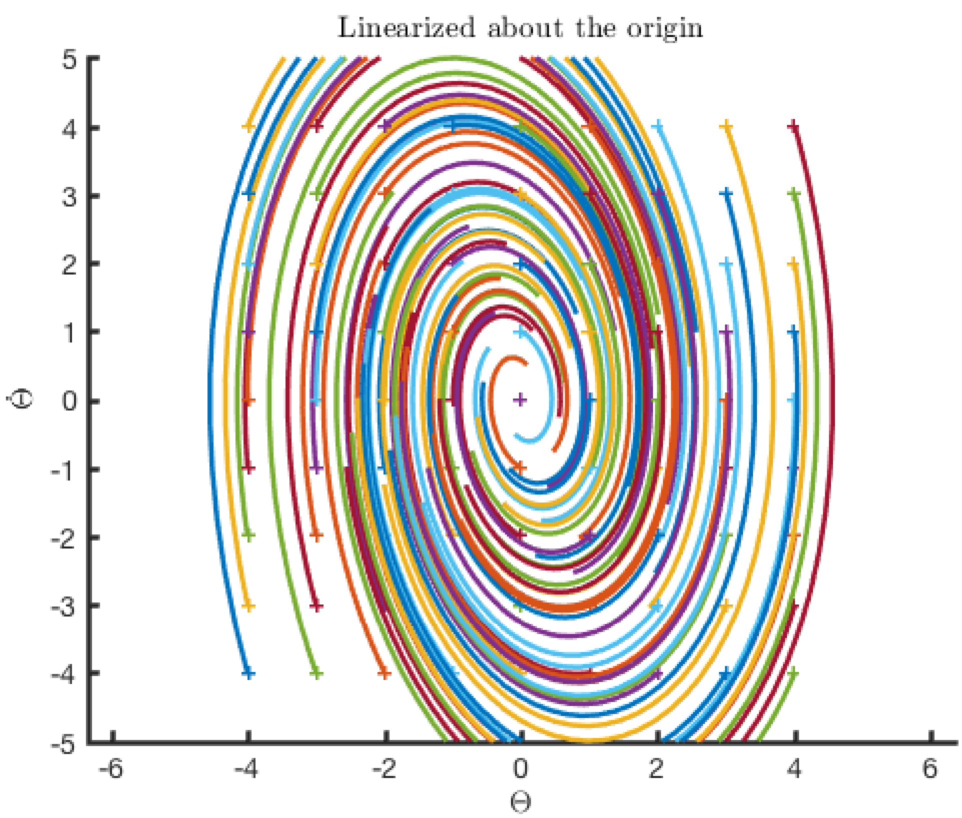
c) Draw also the phase portraits of the linearized equations. Does the phase portrait of the nonlinear system in the neighbothood of the singular points compare well with the phase portraits of the linearized equations?

Plot the same as nonlinear, but use linear equations in function file.

Linearized about the singular point at the origin

The phase portrait of the linearized system looks very similar to that of the nonlinear system near the Θ-axis where the effects of the squared term are inconsequential. The linearized system is always stable, with trajectories tending toward the origin.

```
tspan = [0 2];
figure();
hold on
for i = -4:1:4
    for j = -4:1:4
        T0 = [i; j];
        [t,T] = ode45(@P1stateEqn,tspan,T0,[],2);
        h = plot(T(:,1),T(:,2));
        c = get(h,'color');
        plot(T0(1),T0(2),'+','color',c);
    end
end
axis([-5 5 -5 5])
axis equal
xlabel('$\Theta$')
ylabel('$\dot{\Theta}$')
title('Linearized about the origin')
hold off
```

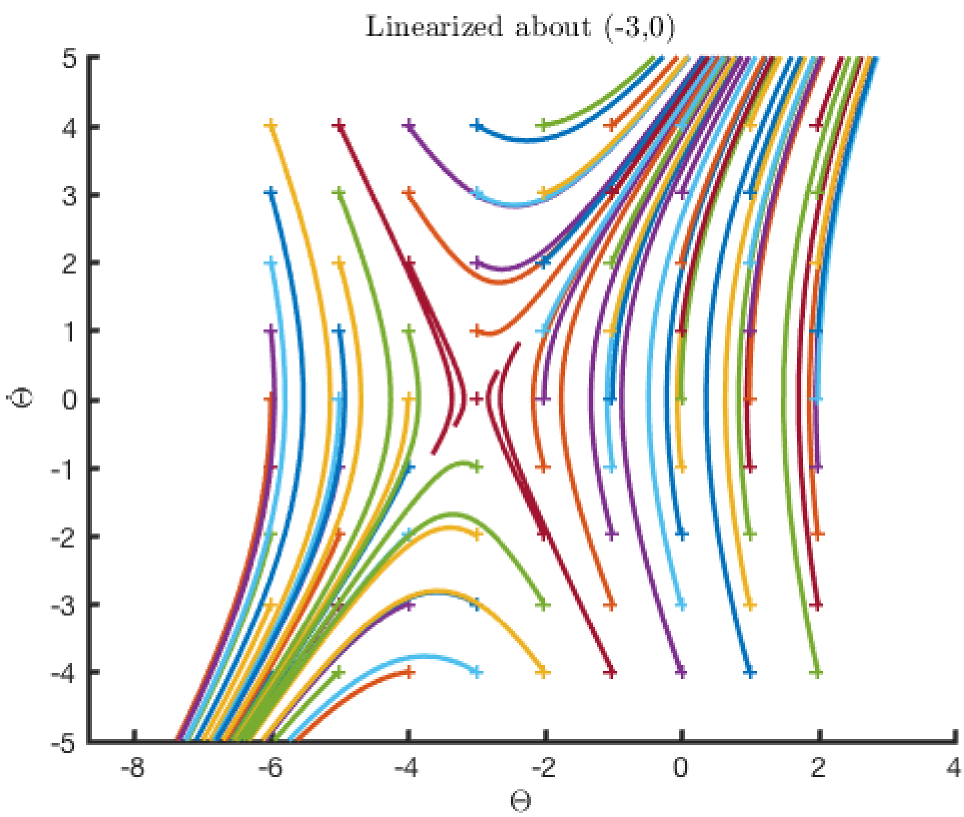


Linearized about the singular point at (-3, 0)

The phase portrait of the linzearized system looks very similar to that of the nonlinear system near $\bar{x} = (-3, 0)$ where the squared term is canceled by the x_1 term preceding it in the nonlinear first-order system.

```
tspan = [0 2];
figure();
hold on
for i = -6:1:2
    for j = -4:1:4
        T0 = [i; j];
        [t,T] = ode45(@P1stateEqn,tspan,T0,[],3);
```

```
        h = plot(T(:,1),T(:,2));
        c = get(h,'color');
        plot(T0(1),T0(2),'+','color',c);
    end
end
axis([-5 1 -5 5])
axis equal
xlabel('$\Theta$')
ylabel('$\dot{\Theta}$')
title('Linearized about (-3,0)')
hold off
```



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