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Anti-Lock Braking Control using a Sliding Mode like Approach

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Abstract

The paper starts with a brief history of the design of anti-lock brakes (ABS). The advantages of ABS are explained. For analysis and controller design a non-linear longitudinal car model is derived. It is shown that the dynamics can be separated conveniently into two different linear dynamics dependent on the tyre slip. The analysis of the dynamics show the highest possible braking performance. It is further shown that a continuous feedback law will not achieve the maximum braking performance. To achieve the maximum braking performance a sliding mode like controller design approach is suggested. The merits of this controller are shown in an example.

1 Introduction

The origin of anti-lock brake controllers (ABS) lies in the design of the so called anti-skid braking controller. The first anti-skid braking controllers were designed for trains in 1908. After Bosch received a patent in 1936 for an electro-hydraulic anti-lock system such systems were build into aircraft in the 1940's before their introduction to passenger cars in 1969 when Ford built such a system into their motor-cars. The implemented system was marketed under the name "Sure-Track" and due to shortcomings in performance and poor reliability as well as high price it was taken of the market again. After Bosch managed to overcome these shortcomings with a more sophisticated electronic controller design in 1978 the ABS was again put into a car, this time it was a 1979 Mercedes-Benz. After 1984 the ABS was also reintroduced on the American market. Today ABS comes as a standard in nearly every new car.

The advantages of an ABS can be clearly seen when comparing the emergency braking situation of cars with and without ABS. In emergency braking situations the driver wants to reduce the speed of the car as fast as possible, therefore the driver presses the brake pedal as hard as possible. In cars without ABS the wheels will lock and the car will start sliding. This has undesirable effects. Since the car is sliding the friction between tyre and road will have decreased. Hence the distance after which the car will come to a standstill will increase. The tyre wear is not equally distributed over the whole tyre, since the wheel is locked and the tyre is sliding on the very same tyre part. Another undesired effect is that as soon as the wheels lock the car becomes unsteerable. This might be quite dangerous in the case when the driver wants to avoid an obstacle during the braking manoeuvre. In a car with ABS sensors monitor the rotation of the wheels and as soon as the wheels are about to lock the brake pressure is reduced. Therefore the ABS prevents the wheels from locking. Since the wheels are still rolling steerability is maintained, and a higher friction between street and tyre is achieved which leads to a shorter braking distance.

In the following a nonlinear longitudinal car model is presented. It is shown that, partially for analysis, the nonlinear model can be simplified and its linearizations can be used. The analysis will assess maximum braking performance and stability issues. It is shown that a continuous feedback law cannot achieve the maximum braking performance considering the uncertainty with which the friction/slip curves are given. To overcome this problem a sliding mode controller is suggested.

2 Modelling of the longitudinal dynamics

In this section we state the time-varying nonlinear equations of a quarter car model [10]. We simplify and linearize them such that we obtain a suitable representation to carry out linear analysis and controller design.

The time-varying nonlinear equations of a quarter car are as follows:

$$\begin{aligned} \dot{\lambda} &= -\frac{1}{v}\left[\frac{1}{m}(1-\lambda)+\frac{r^2}{J}\right]F_z\mu(\lambda)+\frac{1}{v}\frac{r}{J}T_b \\ \dot{v} &= -\frac{1}{m}F_z\mu(\lambda) \end{aligned}$$

where v: vehicle speed, m: vehicle mass, J: wheel inertia, r: wheel radius, λ: tyre slip, μ: friction function between tyre and road, F_z: vertical force (dynamic load), T_b: brake torque.

The friction coefficient can vary in a very wide range, depending on factors like a) road surface conditions (dry, wet or icy), b) tyre side slip angle, and c) tyre brand (summer tyre, winter tyre).

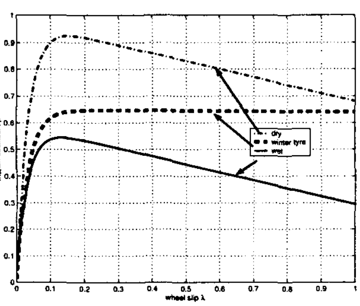

Figure 1: $\mu(\lambda)$ for wet and dry surfaces and winter tyres

The qualitative dependence of λ on surface conditions and tyre brand is shown in figure 1. The task of the ABS controller is to robustly stabilize the system around the maximum friction, such that minimum braking time, i.e. distance, is needed and the car's steerability is maintained. Before we start with the analysis and controller design we cast the nonlinear equations into piecewise linear equations. This is done by approximating the friction/slip curves by piecewise linear functions. After we have found a piecewise linear representation for the friction/slip curves the non-linear model of the braking quarter car is linearized.

In order to cover all possible dynamics we will approximate the $\mu(\lambda)$ with two piecewise linear functions

$$\begin{aligned} \mu &= a\lambda & for \quad \lambda \leq 0.1 \quad (1) \\ \mu &= -\frac{1}{4}\lambda+\frac{3}{4}\pm 0.2 & for \quad \lambda > 0.1 \quad (2) \end{aligned}$$

where $a \in [5.75, 9.75]$ and the notation ± 0.2 means that any arbitrary not necessarily fixed value can be assumed in the interval (-0.2, 0.2). With this approximation we cover most values of μ.

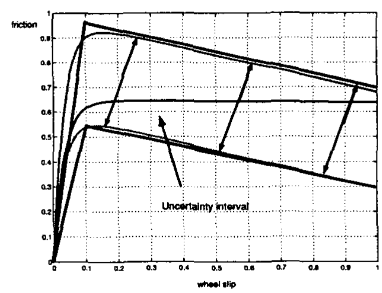

Figure 2: Approximation of $\mu(\lambda)$ with uncertain piecewise linear functions

Since road surfaces can change arbitrarily we need to cope with discontinuous jumps of μ, which may take place at arbitrary times. These unpredictable changes of μ are covered by the affine uncertainty term.

We note that since the mass m of a car is quite large, the term $\mid -\frac{1}{v}\left[\frac{1}{m}(1-\lambda)\right]F_z\mu(\lambda) \mid \ll \mid -\frac{1}{v}\left[\frac{r^2}{J}\right]F_z\mu(\lambda) \mid$, such that we will neglect it. For linearization we approximate the system by the first terms of the Taylor series $f(\lambda, v) \approx f(\lambda_{wp}, v_{wp}) + \frac{df}{d\lambda}\mid_{\lambda_{wp},v_{wp}} (\lambda-\lambda_{wp}) + \frac{df}{dv}\mid_{\lambda_{wp},v_{wp}} (v-v_{wp})$, such that if we do not use a change of coordinates we will get a linear (affine) system description

$$\left[\begin{array}{rcl} \dot{x} & = & A_q x + E_q + Bu^* \\ y & = & C_q x \\ q & = & f(x) \end{array}\right] \qquad (3)$$

where A_q, B and C_q are the system, input and output matrices, respectively, of the linearized system. E_q are the affine terms and f is the function telling which linearization is valid. For each linearization $q \in \{1, 2, \ldots, M\}$, that are linearizations where $\lambda \leq 0.1$

$$A_q = \begin{bmatrix} 0 & -a\frac{F_z}{m} \\ a\frac{F_z r^2 \lambda_{wp}}{v_{wp}^2 J} & -a\frac{F_z r^2}{v_{wp} J} \end{bmatrix} \qquad (4)$$

$$E_q = \begin{bmatrix} 0 \\ -a\frac{F_z r^2 \lambda_{wp}}{v_{wp} J} \end{bmatrix} \qquad (5)$$

and for linearizations $q \in \{M+1, M+2, \ldots, N\}$, that are linearizations where $\lambda > 0.1$

$$A_q = \begin{bmatrix} 0 & \frac{F_z}{4m} \\ \left(-\frac{\lambda_{wp}}{4} + \frac{3}{4}\right)\frac{F_z r^2}{v_{wp}^2 J} \pm 0.2\frac{F_z r^2}{v_{wp}^2 J} & \frac{F_z r^2}{4 v_{wp} J} \end{bmatrix} \qquad (6)$$

$$E_q = \begin{bmatrix} \left(-\frac{3}{4} \pm 0.2\right)\frac{F_z}{m} \\ \left(\frac{\lambda_{wp}}{4} - \frac{3}{2}\right)\frac{F_z r^2}{v_{wp} J} \pm 0.4\frac{F_z r^2}{v_{wp} J} \end{bmatrix} \qquad (7)$$

and $B^T = [0, \frac{r}{J}]$, $u^* = u \cdot v$, $x^T = [v, \lambda]$ for $q \in \{1, 2, \ldots, N\}$. We have now cast the time-varying nonlinear system into piecewise linear systems with uncertainty. In the next section we will analyze the dynamics of the braking car by using its piecewise linear representation.

3 Analysis of the ABS dynamics

3.1 Stability analysis

For the stability analysis we transform the system matrices into controller canonical form $\tilde{A}_q = T A_q T^{-1}$ with T_1 $\forall q \leq M$ and T_2 $\forall q > M$

$$T_1 = \begin{bmatrix} -\frac{m}{aF_z} & 0 \\ 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} \frac{4m}{F_z} & 0 \\ 0 & 1 \end{bmatrix} \qquad (8)$$

such that we get

$$A_q = \begin{bmatrix} 0 & 1 \\ -a^2\frac{F_z^2 r^2 \lambda_{wp}}{m v_{wp}^2 J} & -a\frac{F_z r^2}{v_{wp} J} \end{bmatrix}$$

where $\lambda \leq 0.1$ and

$$A_q = \begin{bmatrix} 0 & 1 \\ \left(-\frac{\lambda_{wp}}{4} + \frac{3}{4}\right)\frac{F_z^2 r^2}{4 m v_{wp}^2 J} \pm 0.2\frac{F_z^2 r^2}{4 m v_{wp}^2 J} & \frac{F_z r^2}{4 v_{wp} J} \end{bmatrix}$$

where $\lambda > 0.1$. Since the system matrices are now given in controller canonical form it is easy to see whether or not the systems are stable. A system is Hurwitz stable if and only if all coefficients in the lowest row of the system matrix in controller canonical form are negative. We now immediately see that the systems $q \in \{1, 2, \ldots, M\}$, i.e. systems where $\lambda \leq 0.1$, are stable since the coefficients in the lower row of the system matrices are negative for all possible parameter variations. However, the systems $q \in \{M+1, M+2, \ldots, N\}$, this are systems where $\lambda > 0.1$, are not globally stable. However, we should not forget that none of the linearizations are valid on the whole state space. So we need to check if the linearizations for $\lambda > 0.1$ converge for $0.1 < \lambda \leq 1$ and $0 < v$. Taking the first equation for $\dot{v}$

$$\dot{v} = \frac{F_z \lambda}{4m} + \left(-\frac{3}{4} \pm 0.2\right)\frac{F_z}{m} < 0 \quad \forall \lambda \qquad (11)$$

we see that the right hand-side of this differential equation remains negative since the maximum value of $\dot{v}$ occurs at $\lambda = 1$, which reduces the equation to

$$\dot{v} = \left(-\frac{1}{2} \pm 0.2\right)\frac{F_z}{m} < 0 \qquad (12)$$

and it is easy to see that it is negative for all possible values. Hence for all values of λ and v the differential equation (DE) converges to values which belong to linearizations (4). The second differential equation for $\dot{\lambda}$

$$\dot{\lambda} = \left(-\frac{3}{4} \pm 0.2\right)\frac{F_z r^2}{vJ} + \frac{F_z r^2 \lambda}{4vJ} < 0 \quad \forall \lambda, v \qquad (13)$$

has a right hand-side which is also negative for all admissible λ and v. This can be easily seen if we substitute for the λ which would make the equation as least negative as possible. This is $\lambda = 1$ which brings the equation into this form:

$$\dot{\lambda} = \left(-\frac{1}{2} \pm 0.2\right)\frac{F_z r^2}{vJ} < 0 \quad \forall v \qquad (14)$$

It is easy to see that this equation is also negative for all admissible velocities v. Hence also this DE converges (tends) to λ which belong to linearizations (4), for all admissible values of the states. We have seen that the linearizations (4), (6) converge individually, for all admissible initial states, to $x \equiv 0$. In general this does not mean that the whole system is stable. However since the states converge for any initial condition of linearizations (6) to states which belong to the linearizations (4) and (4) converges to zero, $x \to 0$ as $t \to \infty$, with out going back to states which belong to (6). Hence the non-linear system is stable. We have seen that the system is stable. It is further desirable to analyse the performance such that we know the maximum deceleration.

3.2 Computation of the maximum deceleration

We would like to compute the maximum deceleration. It is expected that the maximum deceleration is $\dot{v} \approx -g$ if the air resistance is neglected. Remark: in general the air resistance should not be neglected since its contribution especially at higher velocities is considerable particularly when the vehicle is equipped with spoilers.
If we look at the friction/slip curve the highest friction occurs at $\lambda = 0.1$. The models which are valid for $\lambda = 0.1$ are the ones described by (4). We take the first row of (4) $\dot{v} = -a\frac{F_z}{m} \cdot \lambda$ and equate it at $\lambda = 0.1$, hence we obtain $\dot{v} = -\frac{ag}{10}$ since $F_z = g \cdot m$. For the best possible friction at $\lambda = 0.1$ we obtain $\dot{v} = -0.975 \cdot g \approx -g$.
In the next section we proceed with designing a controller, which will achieve this maximum deceleration.

4 Controller design

The objective is to design a controller which decelerates the vehicle as fast as possible and maintains steerability. We have seen that the maximum deceleration is reached at a slip of $\lambda = 0.1$. At such a slip the wheel is far away from being locked, such that we maintain the steerability of the car. We have also seen that it is sensible to approximate the nonlinear car dynamics by (4) and (6). For (4) these are linearizations where $\lambda \leq 0.1$ we would like to increase or maintain λ, i.e. we would like $\dot{\lambda} \geq 0$. For (6) we would like to reduce λ such that we get better steerability and braking performance, i.e. we would like $\dot{\lambda} < 0$. We compute now the control input space in dependence of the state space. Taking

$$\dot{\lambda} = -\frac{1}{v}\left[\frac{1}{m}(1-\lambda) + \frac{r^2}{J}\right] F_z\mu(\lambda) + \frac{1}{v}\frac{r}{J}T_b \quad (15)$$

for $\lambda \leq 0.1$ and $v > 0$ we want to have $\dot{\lambda} \geq 0$. Therefore we take

$$0 \leq -\frac{1}{v}\left[\frac{1}{m}(1-\lambda) + \frac{r^2}{J}\right] aF_z\lambda + \frac{1}{v}\frac{r}{J}T_b \quad (16)$$

hence,

$$T_b \geq \frac{aJ}{r}\left[\frac{1}{m}(1-\lambda) + \frac{r^2}{J}\right] F_z\lambda \quad (17)$$

Using the simplification as before we obtain

$$T_b \geq arF_z\lambda \quad (18)$$

For the maximum value of λ, $\lambda = 0.1$

$$T_b \geq rF_z. \quad (19)$$

For linearizations (6) we desire a negative $\dot{\lambda}$, i.e. $\dot{\lambda} < 0$, hence

$$0 \geq -\frac{1}{v}\left[\frac{1}{m}(1-\lambda) + \frac{r^2}{J}\right] F_z\mu(\lambda) + \frac{1}{v}\frac{r}{J}T_b \quad (20)$$

Simplifying we obtain

$$T_b < \frac{11}{20}rF_z. \quad (21)$$

for values of λ, which are close to 0.1. It is easy to see that there exists no continuous state feedback controller that achieves the desired performance if it is assumed that the friction can vary arbitrarily. This does not mean that there exists no continuous state feedback which stabilizes the system. There are of course continuous controllers which stabilize the system. It can be shown that admissible control inputs are $0 \leq T_b < 0.3rF_z$, which stabilize the system for any initial condition. $T_b \geq 0$ is a technical requirement since the wheels during braking cannot be accelerated. $T_b < 0.3rF_z$ is necessary to ensure that the wheels will not lock.
One possibility is to design a sliding mode controller [9] [7], where the sliding surface is $s = \left(\frac{d}{dt} + K\right)\int_0^t e d\tau$ with $e = \lambda - \lambda_d$, i.e. $\dot{s} = \dot{e} + Ke$. Thus,

$$\dot{s} = -\frac{r^2 F_z\mu(\lambda)}{vJ} + \frac{1}{v}\frac{r}{J}T_b + Ke \quad (22)$$

To stay on the surface $\dot{s} = 0$ is required. Solving for T_b and adding the term which forces the trajectory to stay on the surface we get the control input

$$\hat{T}_b = rF_z\mu(\lambda) - \frac{vJ}{r}Ke \qquad (23)$$

The control input is a function of the friction which is unknown. To overcome this an observer can be designed. However it is known that friction observers have poor performance therefore we would like to pursue a modified strategy.
We suggest a sliding mode controller in the following form: the control input is chosen to be

$$\hat{T}_b = 10rF_z\lambda - \frac{vJ}{r}Ke \qquad (24)$$

for $\lambda \leq 0.08$ until $\lambda \leq 0.1$. Then to avoid chattering

$$\hat{T}_b = -\frac{sK_P + K_i}{s}e \qquad (25)$$

which is valid until $\lambda < 0.08$ or $\lambda > 0.12$ is reached. For $\lambda \leq 0.12$ to $\lambda \leq 0.1$

$$\hat{T}_b = \left(-\frac{1}{4}\lambda + \frac{3}{4} - 0.2\right) rF_z - \frac{vJ}{r}Ke \qquad (26)$$

is valid. In this way we get vector fields that point towards $\lambda = 0.1$ and in directions of smaller velocities v. To avoid chattering the PI controller stabilizes the dynamics around the desired slip $\lambda_d = 0.1$. It can further handel smaller variations in the friction coefficient. For larger variations controller (24) and (26) pushes the trajectory back to $\lambda_d = 0.1$ where the PI controller takes over again.
It needs to be mentioned that such a discontinuous control law, besides the advantages of being robust to variations and uncertainty and achieving high braking performance, has the drawback that it might excite unmodelled dynamics like suspension dynamics etc.. This is undesirable since it reduces passenger comfort.
To illustrate the controller's performance a simulation example is presented.

5 Example

In the example two discontinuous changes to the friction coefficient are made. At $t = 0.5s$ the friction is lowered by 0.3 and at $t = 1.2s$ the friction is increased by 0.28. Figure 3 shows the velocity of the car and of its wheel.

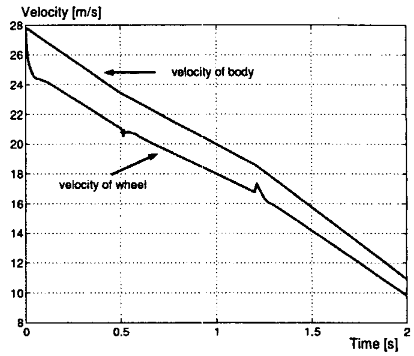

Figure 3: Velocity of the car body and the wheel $\omega * r$

Due to the slip the velocity of the wheel is lower than the velocity of the car body. It can also be seen that the variations in slip result in variations of the wheel velocity. Figure 4 shows the slip λ, and Figure 5 shows the brake torque $\hat{T}_b$

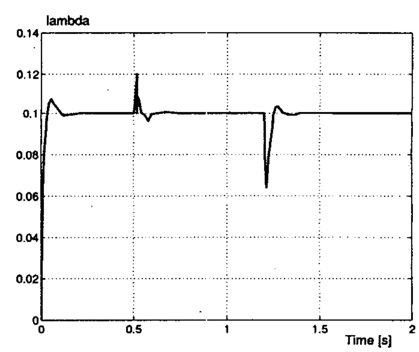

Figure 4: Wheel slip λ

In the first instance controller (24) is active and brings the slip towards $\lambda = 0.1$ by applying maximum torque. Then the PI controller takes over and stabilizes the slip. At $t = 0.5$ the friction is decreased and the slip increases. Controller (26) takes over immediately as $\lambda = 0.12$ and pushes it back where the PI controller takes over again. At time $t = 1.2$ the friction is increased and controller (26) brings the slip back.

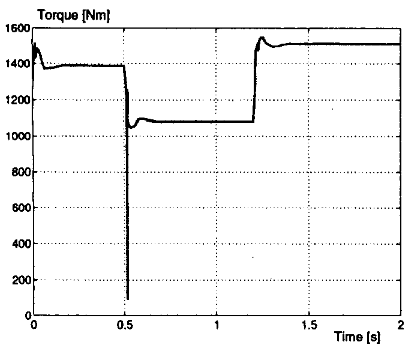

Figure 5: Brake torque $\hat{T}_b$

For tuning the controller needs to be implemented in a real car where its performance can be evaluated, since the final traide off between performance and comfort can only be achieved in the real environment. For tuning we suggest varying the coefficient K. With larger K we will get faster dynamics, such that $|\dot{\lambda}|$ will be larger. For smoother control action K needs to be reduced, such that suspension dynamics are excited less.

6 Conclusions

After a brief introduction to the history of ABS a nonlinear car model was introduced which captured the longitudinal braking dynamics. It was shown that the nonlinear dynamics can be assessed by using two uncertain linear systems. The uncertainties captured the unpredictable changes in road friction due to changes in surface conditions (wet, dry). It was shown that the nonlinear dynamics are stable and that the maximum braking performance occurs at $\lambda = 0.1$. The control input space was computed and it was shown that for $\lambda \leq 0.1$ the slip has to be increased in order to increase the friction, i.e. $\dot{\lambda} \geq 0$. For slips $\lambda > 0.1$, the slip has to be reduced to increase the friction and maintain steerability. It was shown that a continuous feedback could not achieve the maximum braking performance given the range of uncertainty. Therefore it was suggested to design a sliding mode controller. In order to avoid the excitation of unmodelled suspension dynamics a nonlinear discontinuous (sliding mode like) control feedback was chosen, such that a relatively smooth transition at the sliding surface is possible. The controller can now be tuned towards better performance, i.e. increase the bang bang control, or greater passenger comfort.

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