MAE 5803 Nonlinear Control Systems **Assigned: Mar 30, 2017**
Homework #5 **Due: Apr 11, 2017**

Submit your answers to all questions below. Show your working steps in sufficient details.

1. (30 pts.)

   Consider a third order system:

   $$\dddot{x} + a_1 \dot{x}^2 \cos x + a_2 \sin 2x = u$$

   where a_1 and a_2 are unknown constants and u is the control input.

   a. Design an adaptive controller for the system to track a desired trajectory $x_d(t)$.

   b. Simulate the responses of the system using the adaptive controller designed in a. to track the trajectory given by $x_d = \sin 0.8t$. You are free to choose the values for the control parameters (**P**, λ, and k) to achieve the tracking goal within the following ranges:

   $\mathbf{P} \le 5\mathbf{I}$ (**I** is identity matrix) $\quad \lambda \le 4 \quad k \le 10$

   Assume the actual values of a_1 and a_2 are 2 and 5, respectively. For the simulation, display time responses of x and x_d (in one plot), estimates of a_1 and a_2 (in one plot), and control input u for 20 seconds. Submit only one set of plots that show good tracking behavior.

   c. Simulate the responses of the system in the following cases (but you don't need to submit all your plots, only the ones to support your answers):

   (i) For fixed λ and k, vary **P**

   (ii) For fixed **P** and k, vary λ

   (iii) For fixed λ and **P**, vary k

   Based on these simulations, describe the effects of **P**, λ, and k on the tracking performance, parameter convergence, and the magnitude of control inputs.

2. (30 pts.)

   Suppose there is an unknown but bounded disturbance $d(t)$ in the system:

   $$\dddot{x} + a_1 \dot{x}^2 \cos x + a_2 \sin 2x = u + d(t)$$

   where $|d(t)| \le D$.

   a. Design an adaptive tracking controller for this system that is robust to the specified disturbance.

   b. Simulate the responses of the system to track $x_d = \sin 0.8t$ using the controller designed in 2a. assuming D = 0.5. Assume the actual values of a_1 and a_2 are 2 and 5, respectively. Choose a set of control parameters within the ranges specified in 1b. and submit only a set of simulated responses that will yield sufficiently good tracking. Vary D in your simulations and describe its effect on the resulting control system behavior (submit relevant plots to support your answer).

3. (40 pts.)

Consider the dynamics of a two-link manipulator used in Example 9.1 to 9.3 in the textbook:

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

where

$$H_{11} = a_1 + 2a_3 \cos q_2 + 2a_4 \sin q_2$$

$$H_{12} = H_{21} = a_2 + a_3 \cos q_2 + a_4 \sin q_2$$

$$H_{22} = a_2$$

$$h = a_3 \sin q_2 - a_4 \cos q_2$$

with

$$a_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$$

$$a_2 = I_e + m_e l_{ce}^2$$

$$a_3 = m_e l_1 l_{ce} \cos \delta_e$$

$$a_4 = m_e l_1 l_{ce} \sin \delta_e$$

a. Simulate the response of the manipulator ($q_1(t)$ and $q_2(t)$) using a PD controller with $\mathbf{K}_D = 100\,\mathbf{I}, \mathbf{K}_P = 20\mathbf{K}_D$ starting from zero initial conditions to the desired final positions of q_{d_1} =1 and $q_{d_2} = 2$. Use the following parameter values in the simulation:

   $m_1 = 1 \quad l_1 = 1 \quad m_e = 2 \quad \delta_e = 30^o \quad I_1 = .12 \quad l_{c1} = 0.5 \quad I_e = 0.25 \quad l_{ce} = 0.6$

b. Design an adaptive controller of the form $\boldsymbol{\tau} = \mathbf{Y}\hat{\mathbf{a}} - \mathbf{K}_D\mathbf{s}$ for the manipulator assuming no initial knowledge of the parameters $a_1, a_2, a_3,$ and a_4. In the Example 9.3, the expressions for the components of the matrix $\mathbf{Y}$ are given as

   $$Y_{11} = \ddot{q}_{r1} \qquad Y_{12} = \ddot{q}_{r2} \qquad Y_{21} = 0 \qquad Y_{22} = \ddot{q}_{r1} + \ddot{q}_{r2}$$

   $$Y_{13} = (2\ddot{q}_{r1} + \ddot{q}_{r2}) \cos q_2 - (\dot{q}_2 \dot{q}_{r1} + \dot{q}_1 \dot{q}_{r2} + \dot{q}_2 \dot{q}_{r2}) \sin q_2$$

   $$Y_{14} = (2\ddot{q}_{r1} + \ddot{q}_{r2}) \sin q_2 + (\dot{q}_2 \dot{q}_{r1} + \dot{q}_1 \dot{q}_{r2} + \dot{q}_2 \dot{q}_{r2}) \cos q_2$$

   $$Y_{23} = \ddot{q}_{r1} \cos q_2 + \dot{q}_1 \dot{q}_{r1} \sin q_2$$

   $$Y_{24} = \ddot{q}_{r1} \sin q_2 - \dot{q}_1 \dot{q}_{r1} \cos q_2$$

   Simulate the responses of the manipulator using this adaptive controller with two sets of desired trajectories:

$$q_{d_1} = 1 - e^{-t} \qquad q_{d_2} = 2(1 - e^{-t})$$

and

$$q_{d_1} = 1 - \cos 2\pi t \qquad q_{d_2} = 2(1 - \cos 2\pi t)$$

Use $\mathbf{\Lambda} = 20\,\mathbf{I}$ and $\mathbf{K}_D = 100\,\mathbf{I}$ in the simulation with the following $\mathbf{P}$ values:

$$\mathbf{P}_1 = \mathrm{diag}(0.6, 0.1, 0.1, 0.06)$$

$$\mathbf{P}_2 = 200\,\mathbf{P}_1$$

$$\mathbf{P}_3 = 0.1\,\mathbf{P}_1$$

Discuss the tracking performance and parameter convergence for the different cases tried.