MAE 5803 - Homework #1 Problem #3

Table of Contents

Tim Coon: 25, January 2017

```
clear; close all; clc;
```

Consider the following second-order system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 + (\mu - x_1^2)x_2$$

a) Plot Eigenvalues

Find the eigenvalues of the linearized system about the equilibrium point, (0,0). Express your answer in terms of μ. Sketch in the complex plane the variation of the locations of these eigenvalues as μ varies from -0.5 to 0.5.

$$A = \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{\bar{x}=\bar{0}}$$

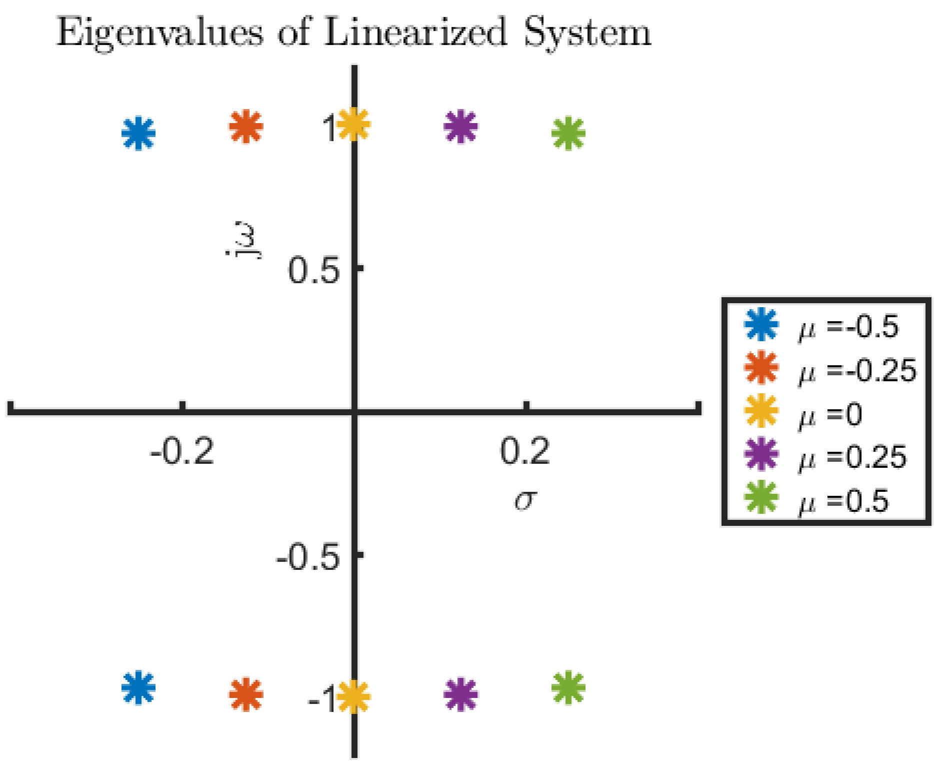
$$\frac{\partial f_1}{\partial x_1} = 0 \quad \frac{\partial f_1}{\partial x_2} = 1 \quad \frac{\partial f_2}{\partial x_1} = -1 - 2x_1x_2 \quad \frac{\partial f_2}{\partial x_2} = \mu - x_1^2$$

$$A = \begin{pmatrix} 0 & 1 \\ -1 & \mu \end{pmatrix}$$

Calculate the eigenvalues for samples of the range of μ specified

```
mu = [-0.5:0.25:0.5];
eValue = zeros(2,length(mu));
figure(1)
hold on
for i = 1:length(mu)
    A =  [0 1; -1 mu(i)];
    eValue(:,i) = eig(A);
    plot(real(eValue(:,i)),imag(eValue(:,i)),'*','MarkerSize',12)
end
```

```
ax = gca;
ax.XAxisLocation = 'origin';
ax.YAxisLocation = 'origin';
ylim([-1.2 1.2]);
title('Eigenvalues of Linearized System')
xlabel('$\sigma$'); ylabel('j$\omega$');
legend(strcat('\mu =
 ',strread(num2str(mu),'%s')),'Location','EastOutside')
hold off
```

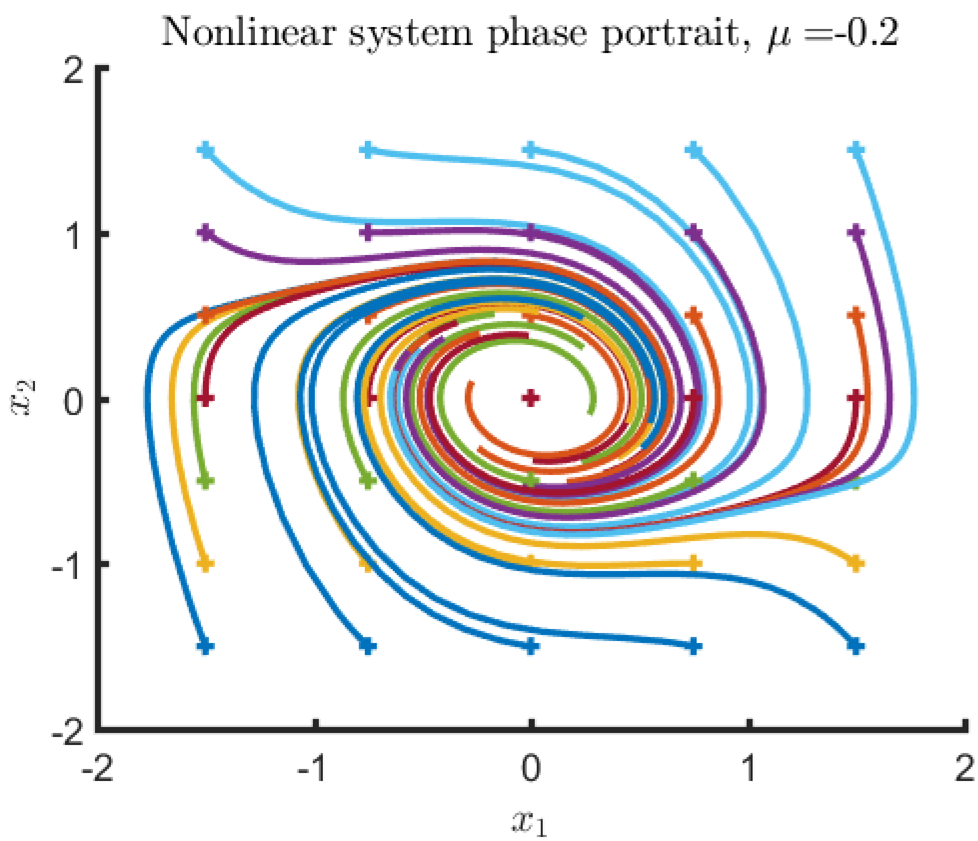


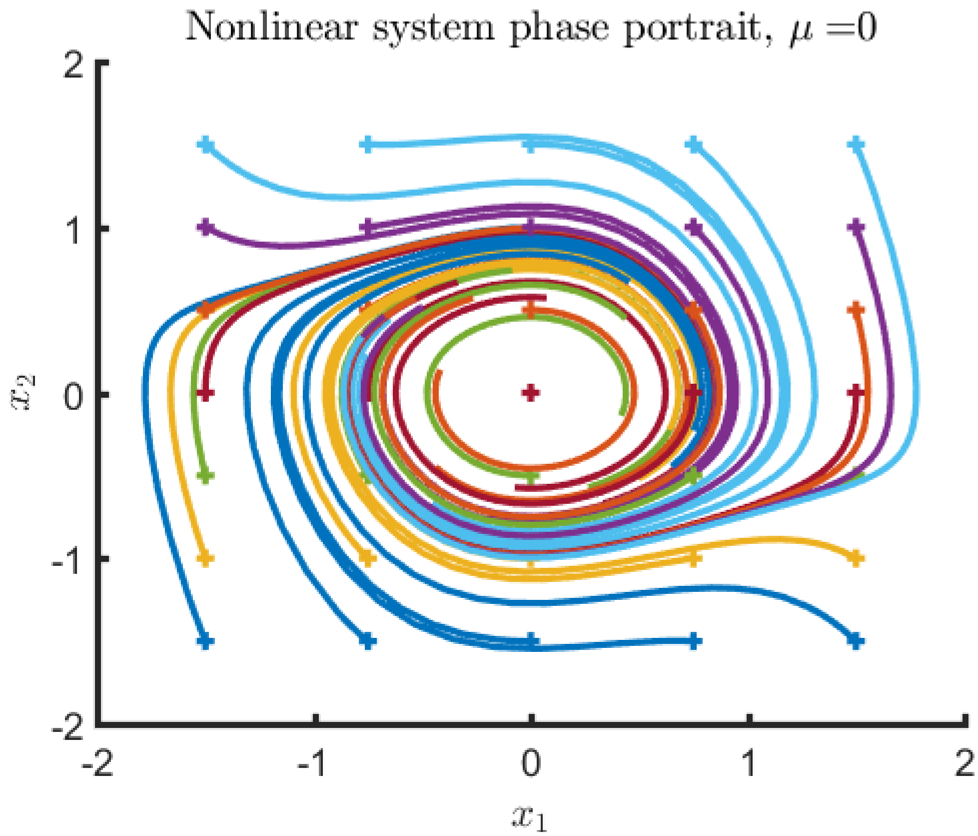
b) Nonlinear Phase Portraits

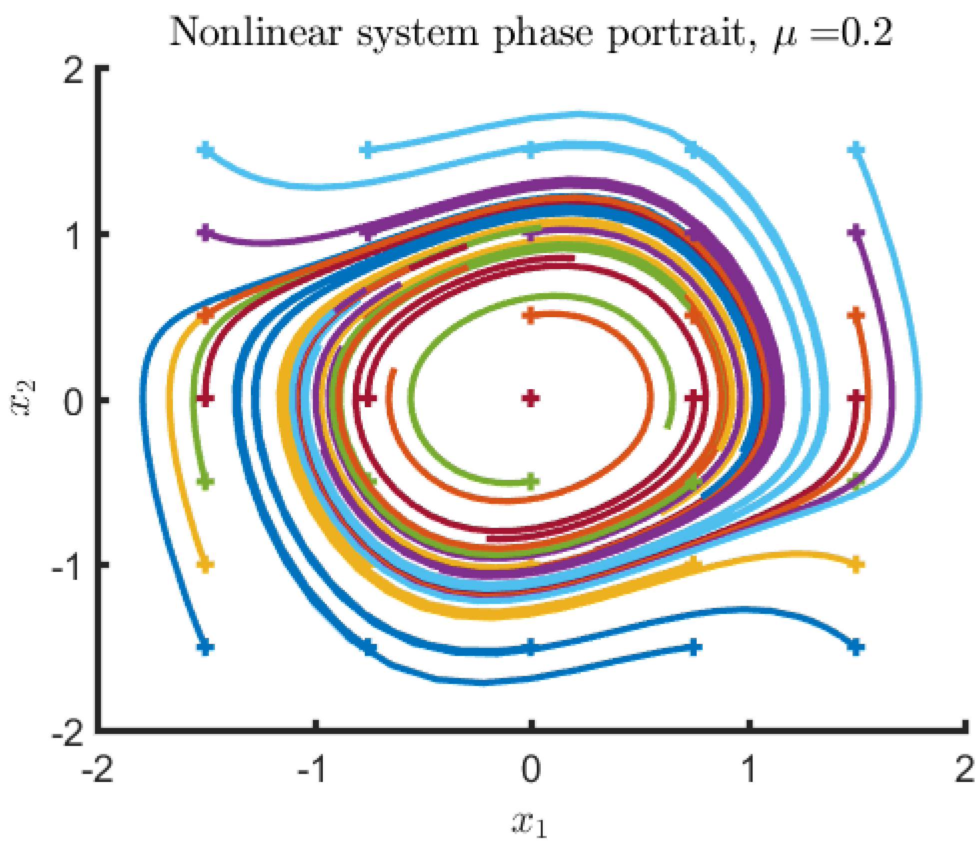
Draw the phase portraits of the system using MATLAB for $\mu = -0.2$, $\mu = 0$, and $\mu = 0.2$. Use -2 to 2 range of values for the horizontal and vertical axes.

```
mu = [-0.2 0 0.2];
for i = 1:length(mu)
    figure()
    hold on
    for x1 = -1.5:.75:1.5
        for x2 = -1.5:.5:1.5
            tspan = [0 5];
            x0 = [x1; x2];
            [t,x] = ode45(@P3stateEqn,tspan,x0,[],mu(i));
            h = plot(x(:,1),x(:,2));
```

```
            c = get(h,'color');
            plot(x0(1),x0(2),'+','color',c);
        end
    end
    axis([-2 2 -2 2])
    xlabel('$x_1$')
    ylabel('$x_2$')
    title(strcat('Nonlinear system phase portrait, $\mu =$ ',
 num2str(mu(i))))
    hold off
end
```




Nonlinear system phase portrait, μ =0
2
1
0
-1
-2
-2
-1
0
1
2
x_2
x_1


Nonlinear system phase portrait, μ =0.2
2
1
0
-1
-2
-2
-1
0
1
2
x_2
x_1

d) Observations

What phenomenon do you observe as the parameter, μ, varies from negative to positive? Justify your answer using Poincare-Bendixson Theorem.

1. If the real parts of all eigenvalues are negative, then $\mathbf{x} = \mathbf{0}$ is locally asymptotically stable
2. If the real part of at least one eigenvalue is positive, then $\mathbf{x} = \mathbf{0}$ is locally unstable
3. If the real part of at least one eigenvalue is equal to zero, then the local stability of $\mathbf{x} = \mathbf{0}$ cannot be concluded

From the plot of eigenvalues, it can be seen, as μ goes from negative to positive, the real part of the eigenvalues moves from positive to negative even as the equilibrium point remains on the origin. Thus, the stability of origin transitions from locally asymptotically stable to unstable. At $\mu = 0$, the eigenvalues have real parts equal to zero and stability is not concluded by eigenvalue analysis, but the phase portrait reveals the origin is stable in this system.

Poincare-Bendixson Theorem: If a trajectory of a second-order autonomous system remains in a finite region (Ω), then one of the following is true:

1. The trajectory goes to an equilibrium point
2. The trajectory tends to a stable limit cycle
3. The trajectory itself is a limit cycle

The system having $\mu > 0$ is unstable at the origin, by Slotine-Li Definition 3.3. However, there is a limit cycle all trajectories tend toward regardless of origin.

Notice this system exhibits bifurcation as μ transitions from negative to positive The system switches from having a stable node at the origin when the real parts of the eigenvalues are negative and transitions to having a stable limit cycle when the real parts of eigenvalues become positive and the point at the origin is no longer stable.

Published with MATLAB® R2016a