MAE 5803 - Homework #1 Problem #2

Tim Coon: 19, January 2017

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```
clear; close all; clc;
```

Consider the following second-order system

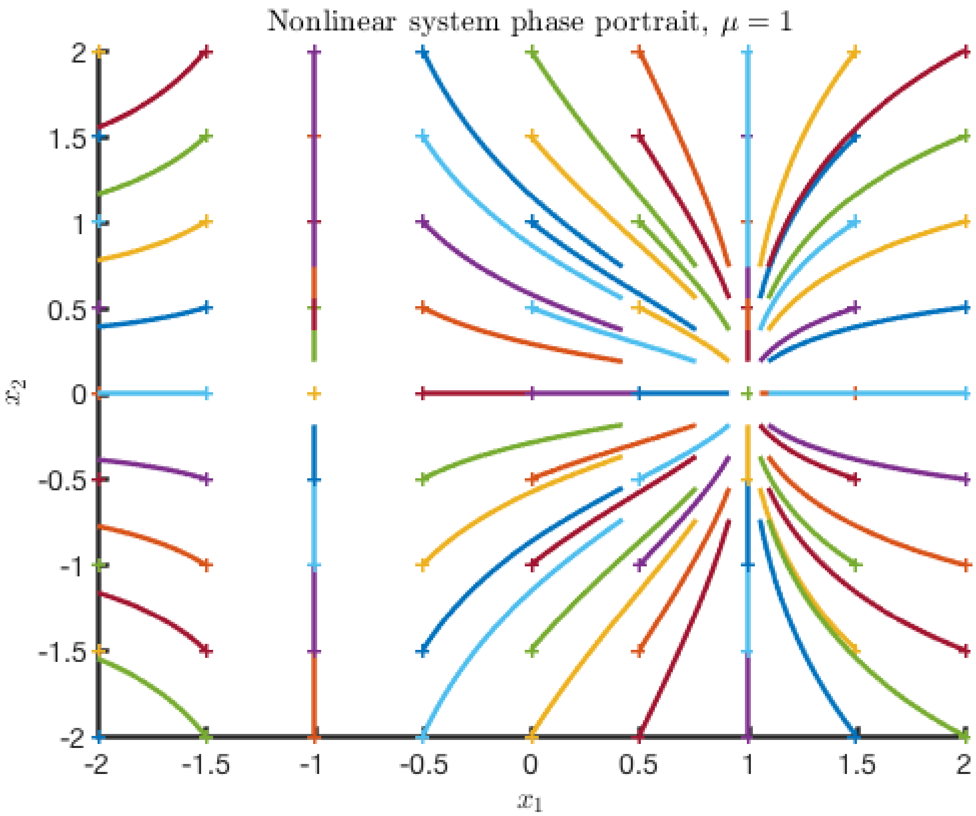
$$\dot{x}_1 = \mu - x_1^2$$

$$\dot{x}_2 = -x_2$$

a) Identify Singular points

For $\mu = 1$, find the singular points of the system, then determine the stability of the singular points by analyzing the linearized equation about each singular point. Generate the phase portrait of the system using MATLAB® to confirm your analysis. Frame your plot so that the horizontal and vertical axes range from -2 to 2.

```
mu = 1;
tspan = [0 1];
figure();
hold on
for x1 = -2:.5:2
    for x2 = -2:.5:2
        X0 = [x1; x2];
        [t,X] = ode45(@P2stateEqn,tspan,X0,[],mu);
        h = plot(X(:,1),X(:,2));
        c = get(h,'color');
        plot(X0(1),X0(2),'+','color',c);
    end
end
axis([-2 2 -2 2])
xlabel('$x_1$')
ylabel('$x_2$')
title('Nonlinear system phase portrait, $\mu = 1$')
hold off
```



First Singular Point

Singular point at (1,0) is a stable node as confirmed by negative eigenvalues of the linearized system. Let $\xi_1 = x_1 - 1$ so $\bar{\xi} = (0, 0)$.

$$\dot{\xi}_1 = \mu - \xi_1^2 - 2\xi_1 - 1$$

$$\dot{\xi}_2 = -\xi_2$$

```
eValue1 = eig([-2 0; 0 -1])
```

```
eValue1 =

    -2
    -1
```

Second Singular Point

Singular point at (-1,0) is a saddle point As confirmed by the opposite signs of the two eigenvalues on the real axis in the linearized system. Let $\xi_1 = x_1 + 1$ so $\bar{\xi} = (0, 0)$.

$$\dot{\xi}_1 = \mu - \xi_1^2 + 2\xi_1 - 1$$

$$\dot{\xi}_2 = -\xi_2$$

```
eValue2 = eig([2 0; 0 -1])
```

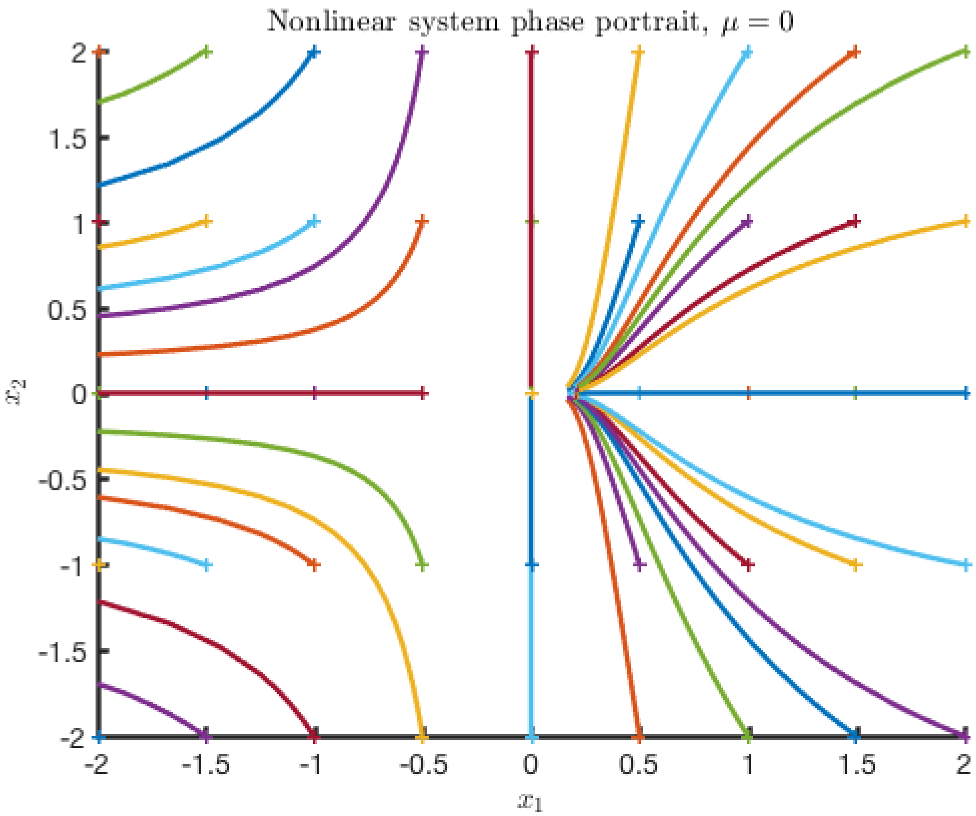
```
eValue2 =

    -1
     2
```

b) Let $\mu = 0$.

Repeat part (a) for $\mu = 0$.

```
mu = 0;
tspan = [0 4];
figure();
hold on
for x1 = -2:.5:2
    for x2 = -2:1:2
        X0 = [x1; x2];
        [t,X] = ode45(@P2stateEqn,tspan,X0,[],mu);
        h = plot(X(:,1),X(:,2));
        c = get(h,'color');
        plot(X0(1),X0(2),'+','color',c);
    end
end
axis([-2 2 -2 2])
xlabel('$x_1$')
ylabel('$x_2$')
title('Nonlinear system phase portrait, $\mu = 0$')
hold off
```



Singular Point

Singular point at (0,0) is an unstable node. As confirmed by the linearized system having an eigenvalue at the origin.

```
eValue1 = eig([0 0; 0 -1])
```

```
eValue1 =

    -1
     0
```

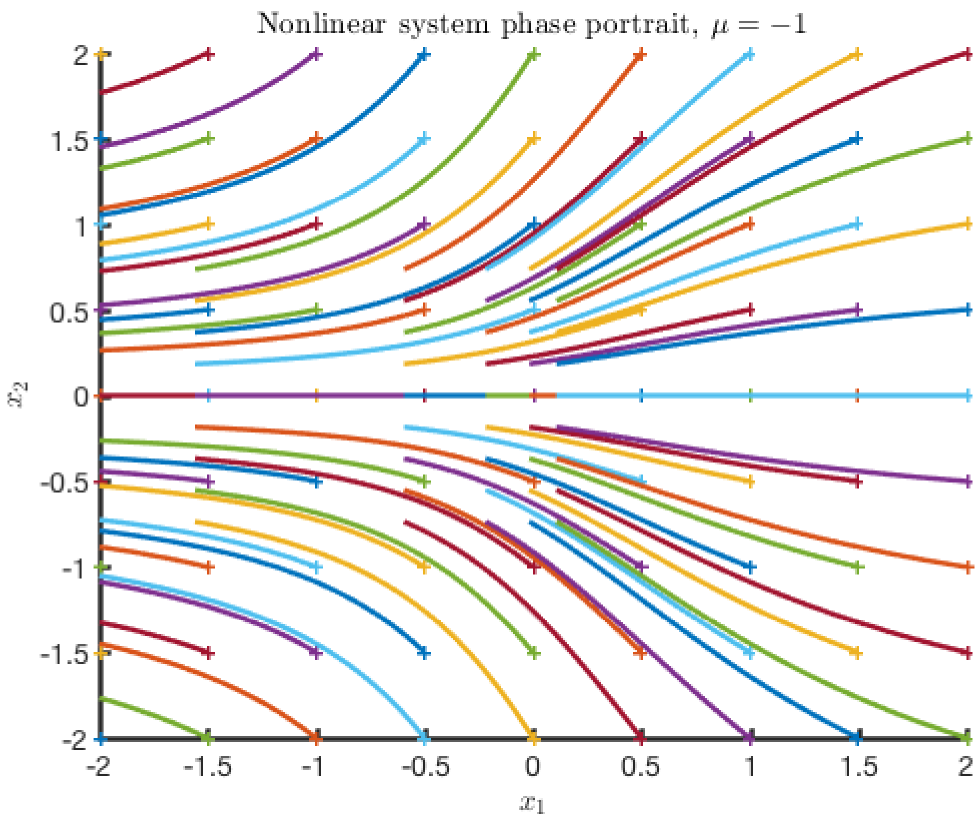
c) Let $\mu = -1$

Repeat again part (a) for $\mu = -1$.

The linearized systems look the same because μ only affects the forcing function

```
mu = -1;
tspan = [0 1];
figure();
hold on
for x1 = -2:.5:2
    for x2 = -2:.5:2
        X0 = [x1; x2];
        [t,X] = ode45(@P2stateEqn,tspan,X0,[],mu);
        h = plot(X(:,1),X(:,2));
        c = get(h,'color');
        plot(X0(1),X0(2),'+','color',c);
```

```
    end
end
axis([-2 2 -2 2])
xlabel('$x_1$')
ylabel('$x_2$')
title('Nonlinear system phase portrait, $\mu = -1$')
hold off
```



No Singular Points

There are no singular points within the range $-2 \leq x_1, x_2 \leq 2$.

d) Comments

What phenomenon do you observe as the parameter, μ, varies as in the above? Explain the reason for your answer.

Because $\dot{x}_2 = -x_2$ always, it is clear the slope towards the horizontal axis decays exponentially in all scenarios. Effectively, μ changes the initial rate of change of the solution in the negative x_1-direction which becomes more negative as the function moves away from the vertical axis. Thus, decreasing μ serves to increase the initial rate of change in the negative x_1-direction and the singular points first merge at the origin, then move quickly in the negative x_1-direction together.

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