Dynamics of Longitudinal Vehicle Traction

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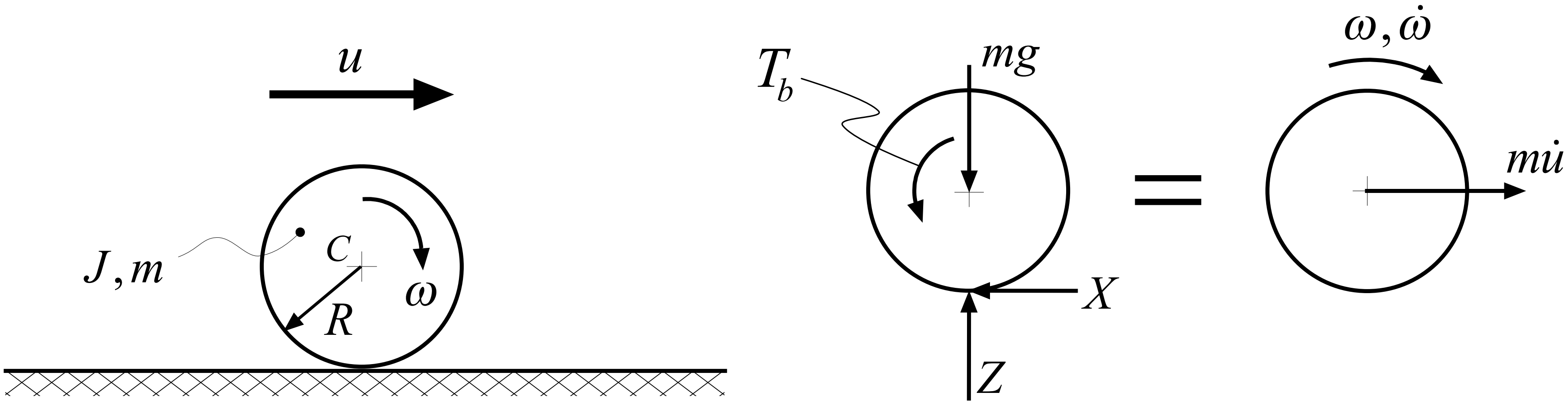
Outline

- Introduction
- Single-Wheel Models
  - Braking (SWBM)
  - Acceleration (SWAM)
- Two-Wheel Model
  - Braking (2WBM)
- Traction Control
- Conclusions and Directions for Future Work

Single-Wheel Braking Model



- Governing Equations

$$
\begin{aligned}
Z &= mg \\
m\dot{u} &= -X \\
J\dot{\omega} &= RX - T_b
\end{aligned}
$$

Assumptions and Restrictions

- Longitudinal motion only, i.e., no cornering.
- Driveline drag, aerodynamic drag, and rolling resistance not included.
- Constant or slowling varying brake torque.
- Vehicle travels on a homogeneous surface.

The Tire/Road Interface

- Friction Law (Creep Force Equation)

$$X = \mu(s) Z$$

μ longitudinal force coefficient

s longitudinal wheel slip

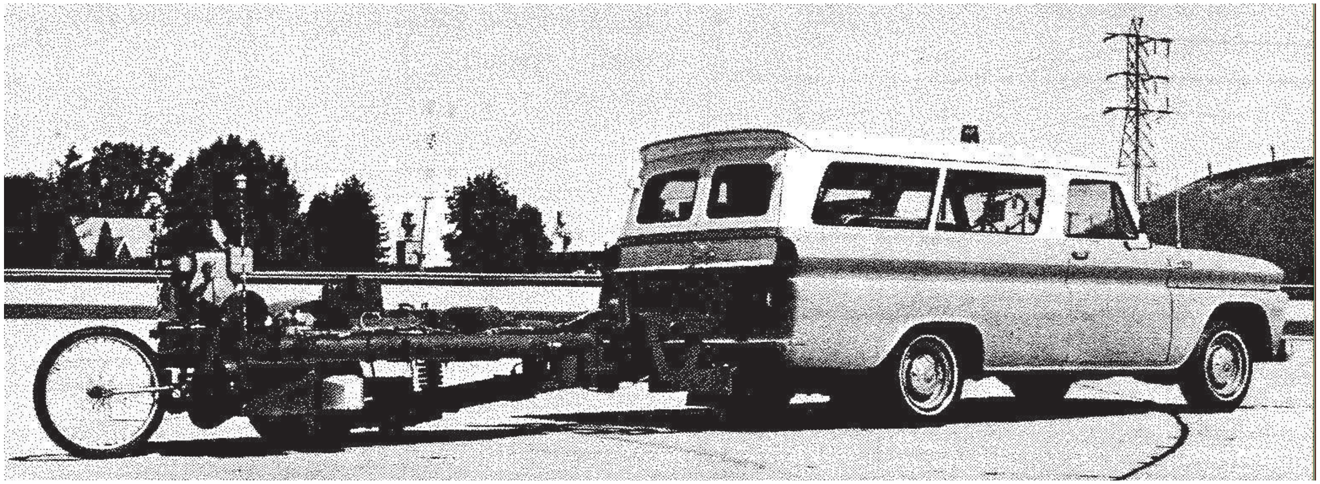
Longitudinal Wheel Slip

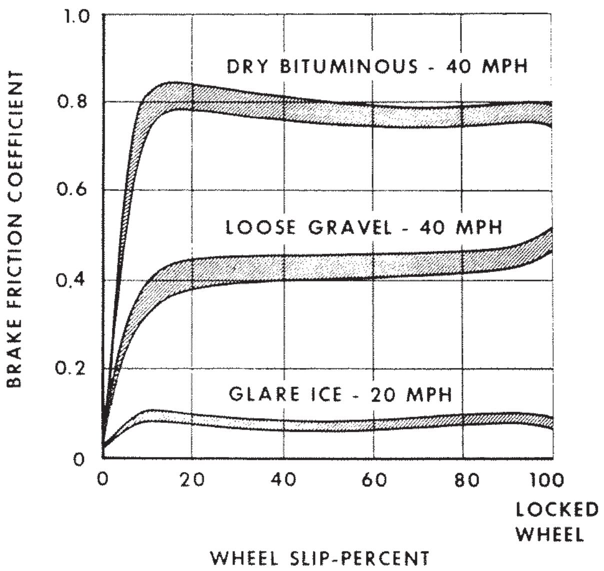
- Dimensionless measure of the difference between u and ωR

$$s \equiv \frac{u - \omega R}{\text{max}(u, \omega R)}, \quad \begin{cases} \omega R \leq u & (\text{Braking}) \\ \omega R \geq u & (\text{Acceleration}) \end{cases}$$

$$\begin{array}{llcl} s = -1 & (u = 0) & \Leftrightarrow & \text{pure slip} \\ s = 0 & (u = \omega R) & \Leftrightarrow & \text{no slip, or free rolling} \\ s = 1 & (\omega R = 0) & \Leftrightarrow & \text{lockup} \end{array}$$

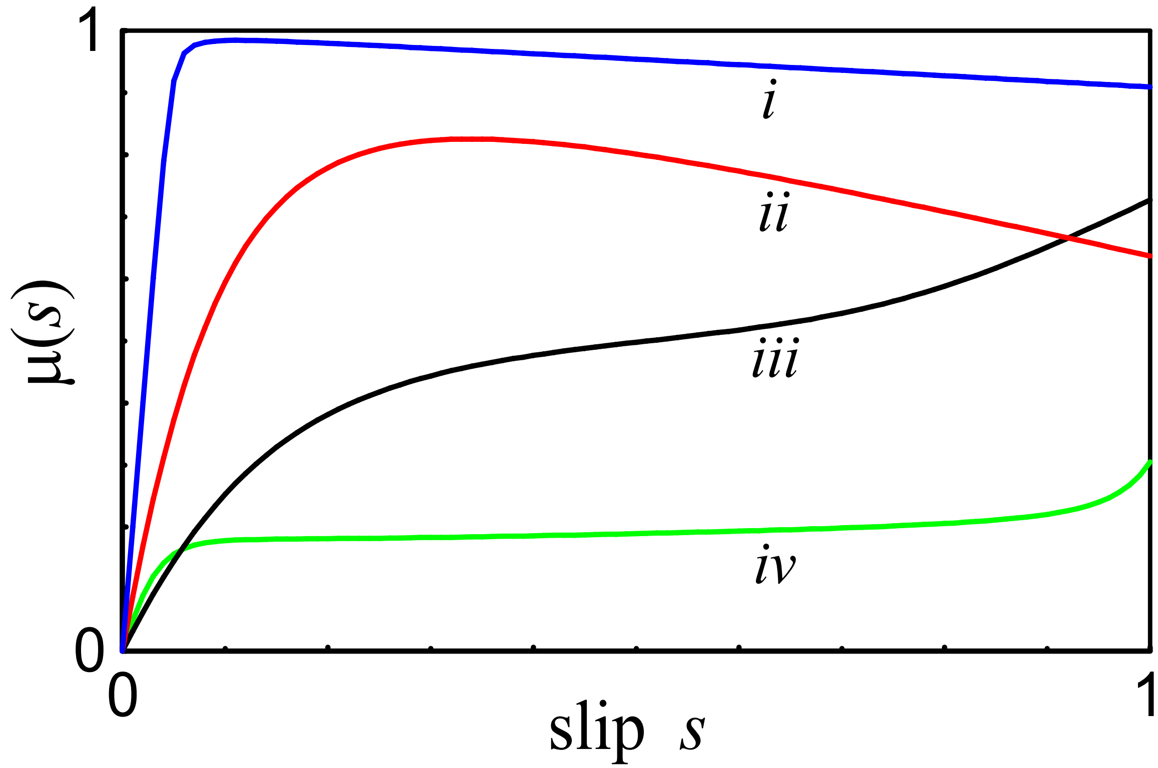
Friction Model





Taken from Goodenow, *et al.*, 1968, SAE paper no. 690214.

Typical Friction Characteristics



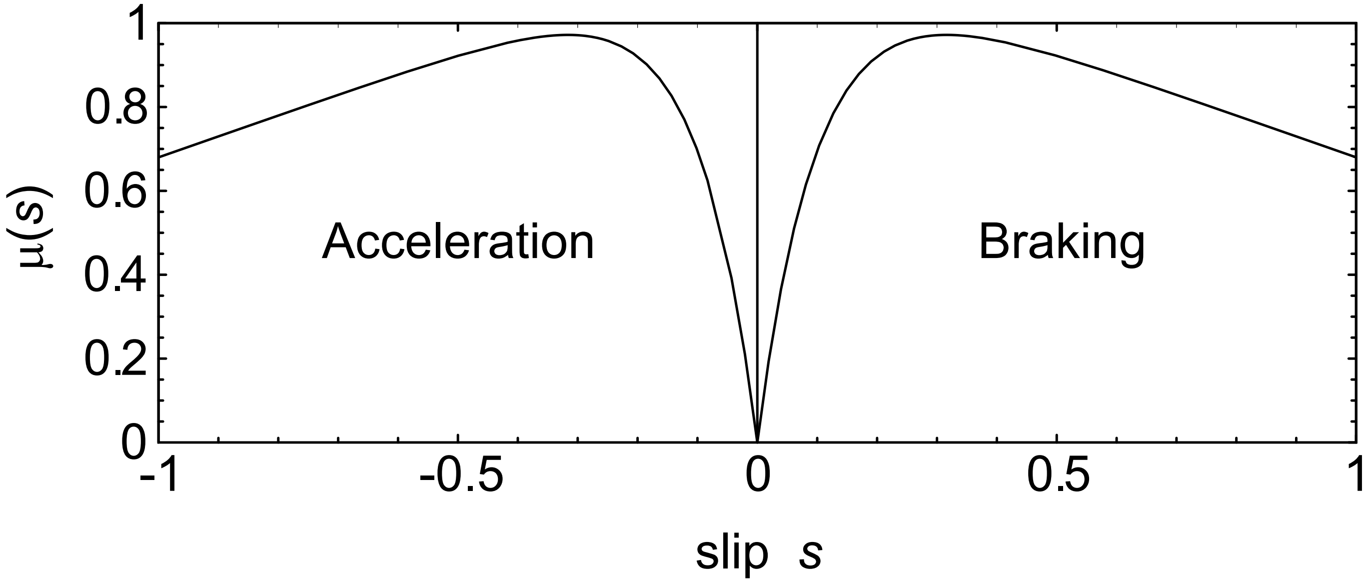
$i)$ Dry Asphalt

$ii)$ Wet Asphalt

$iii)$ Gravel

$iv)$ Packed Snow

Friction Characteristics Employed



$s_p = 0.316$

$\mu(s_p) = 0.972$

$$\mu(s) = 1.18\left(1 - e^{-10s}\right) - s/2 \qquad \text{(braking)}$$

$$s \longrightarrow -s \qquad \text{(acceleration)}$$

Equations of Motion—(u, ω)

- u and ω are coupled in a complicated way via the slip.

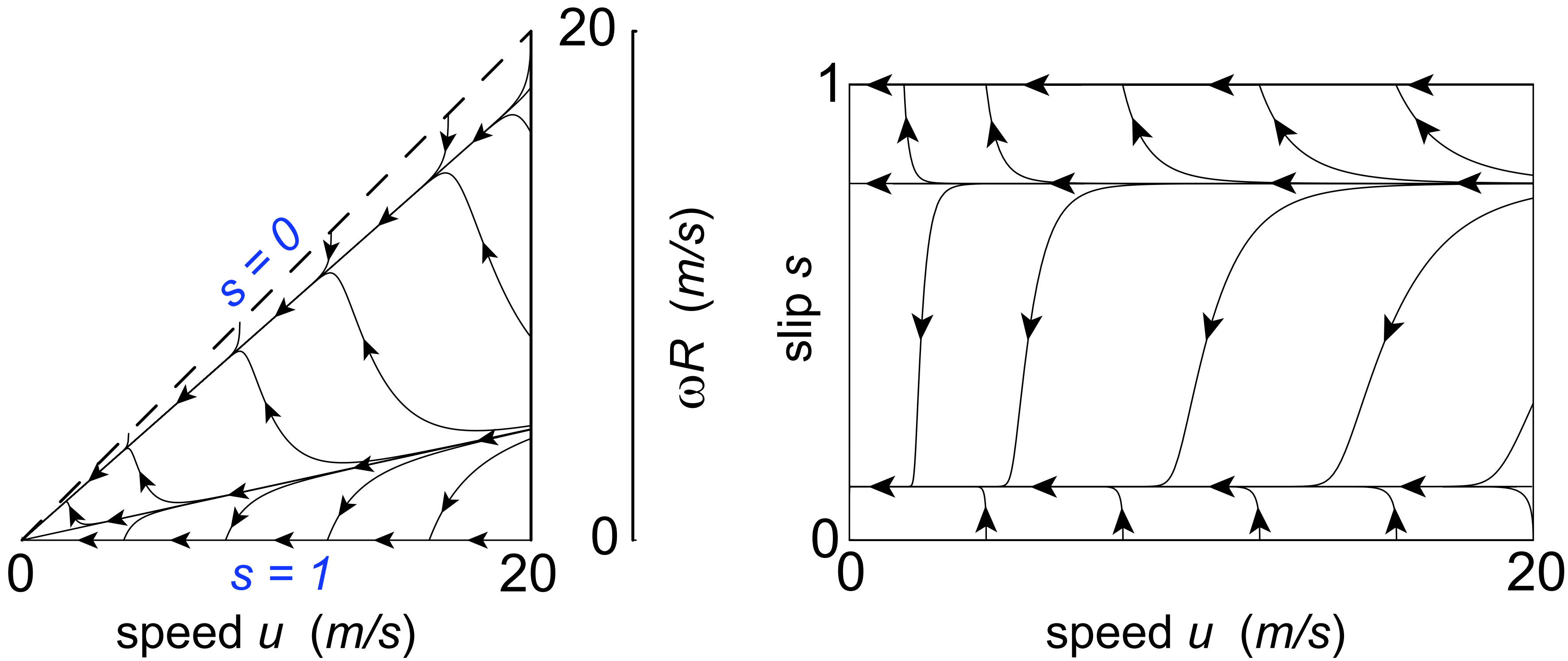
$$
\begin{aligned}
\dot{u} &= -\mu(s)g \\
\dot{\omega} &= \frac{\mu(s)mgR - T_b}{J}
\end{aligned}
$$

where

$$s = \frac{u - \omega R}{u}$$

Choice of Dynamic States

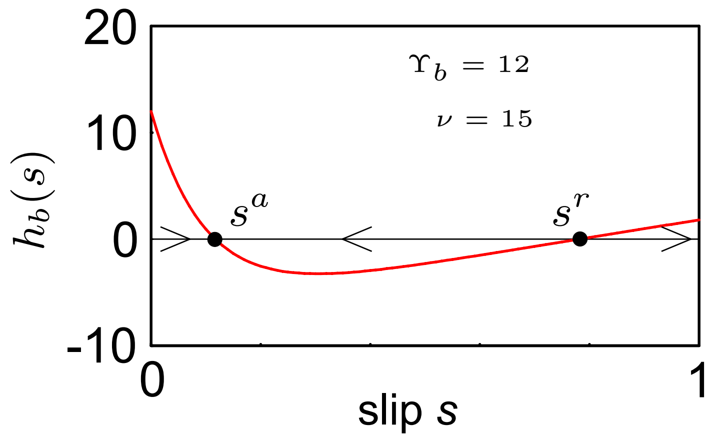
$(u, \omega R)$ vs. (u, s) State Space



Equations of Motion—(u, s)

- Governing equations in terms of u and s

$$\left.\begin{array}{rcl} \dot{u} &=& -\mu(s)g \\ \dot{s} &=& \frac{g}{u}h_b(s) \end{array}\right\} \quad s \in I,\ u \in \mathbb{R}_+$$

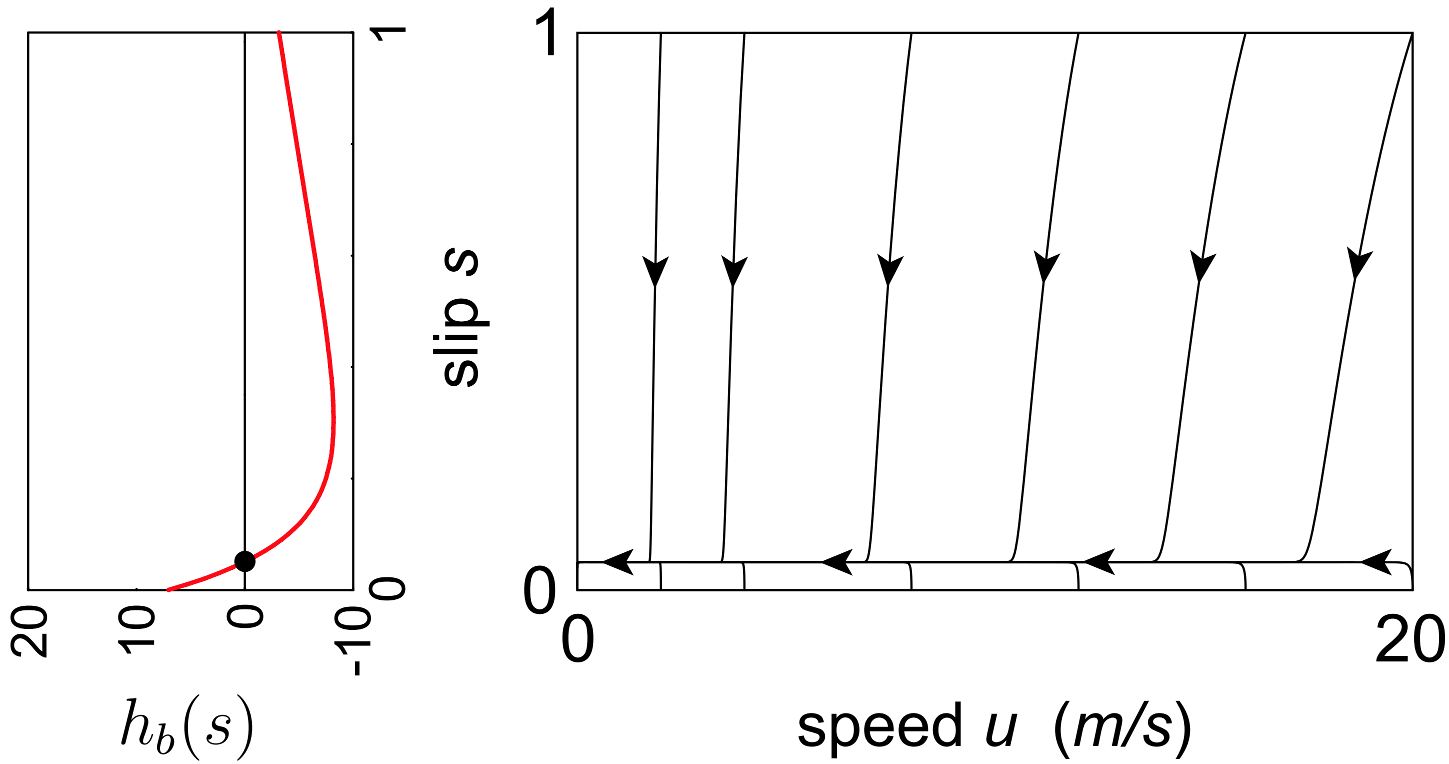


$$\Upsilon_b = \frac{R}{Jg}T_b$$

$$\nu = \frac{mR^2}{J}$$

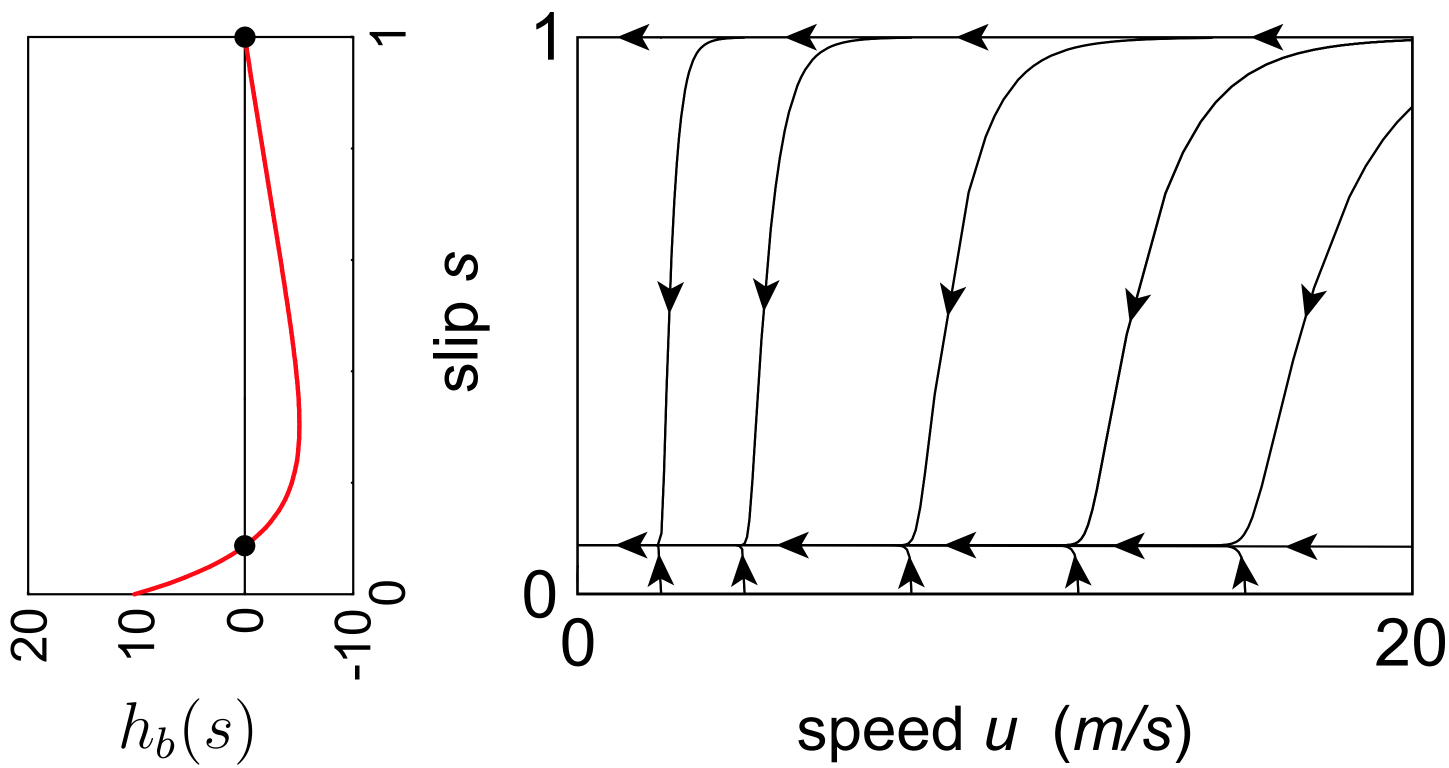
$$h_b(s) = \mu(s)(s - 1 - \nu) + \Upsilon_b$$

State Space Description



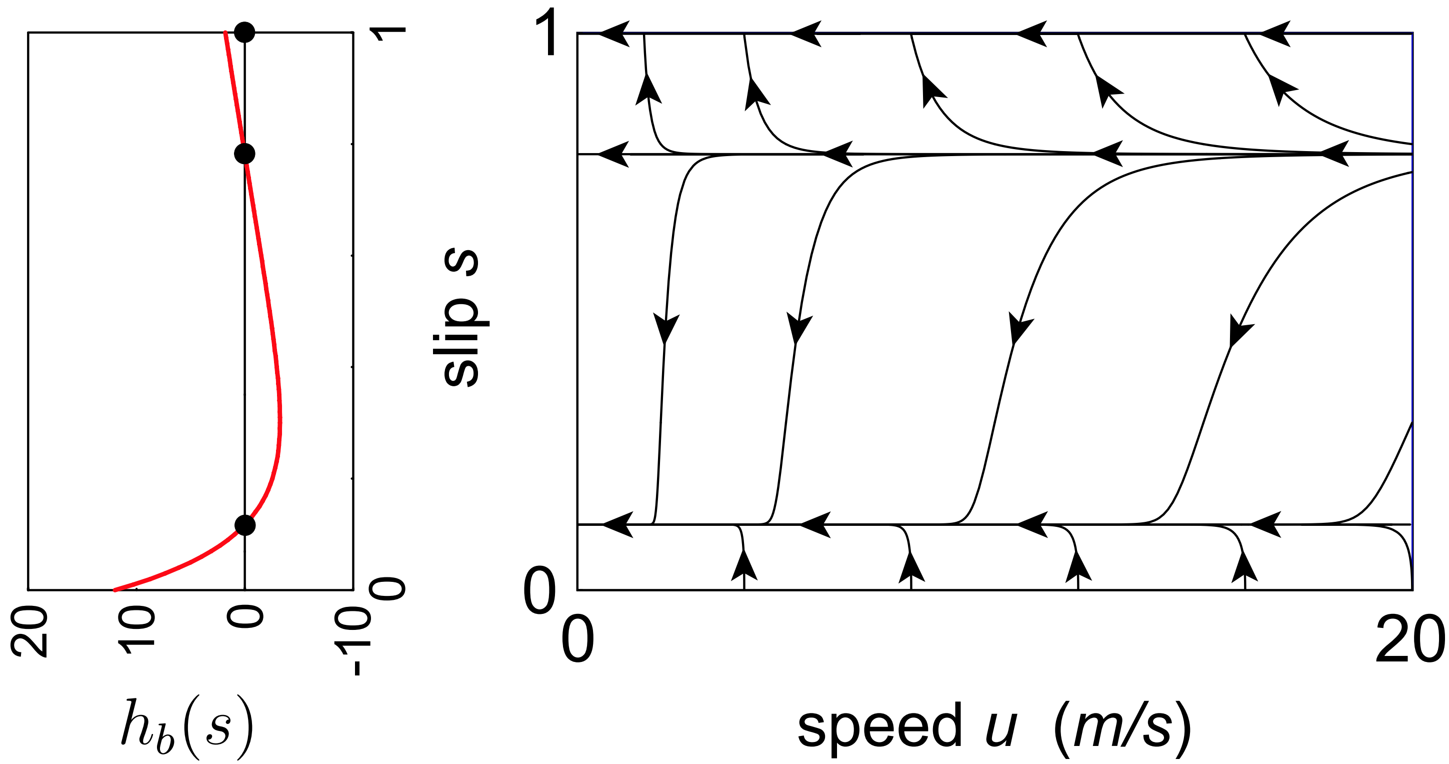
$\Upsilon_b = 7$ Stable Braking

State Space Description



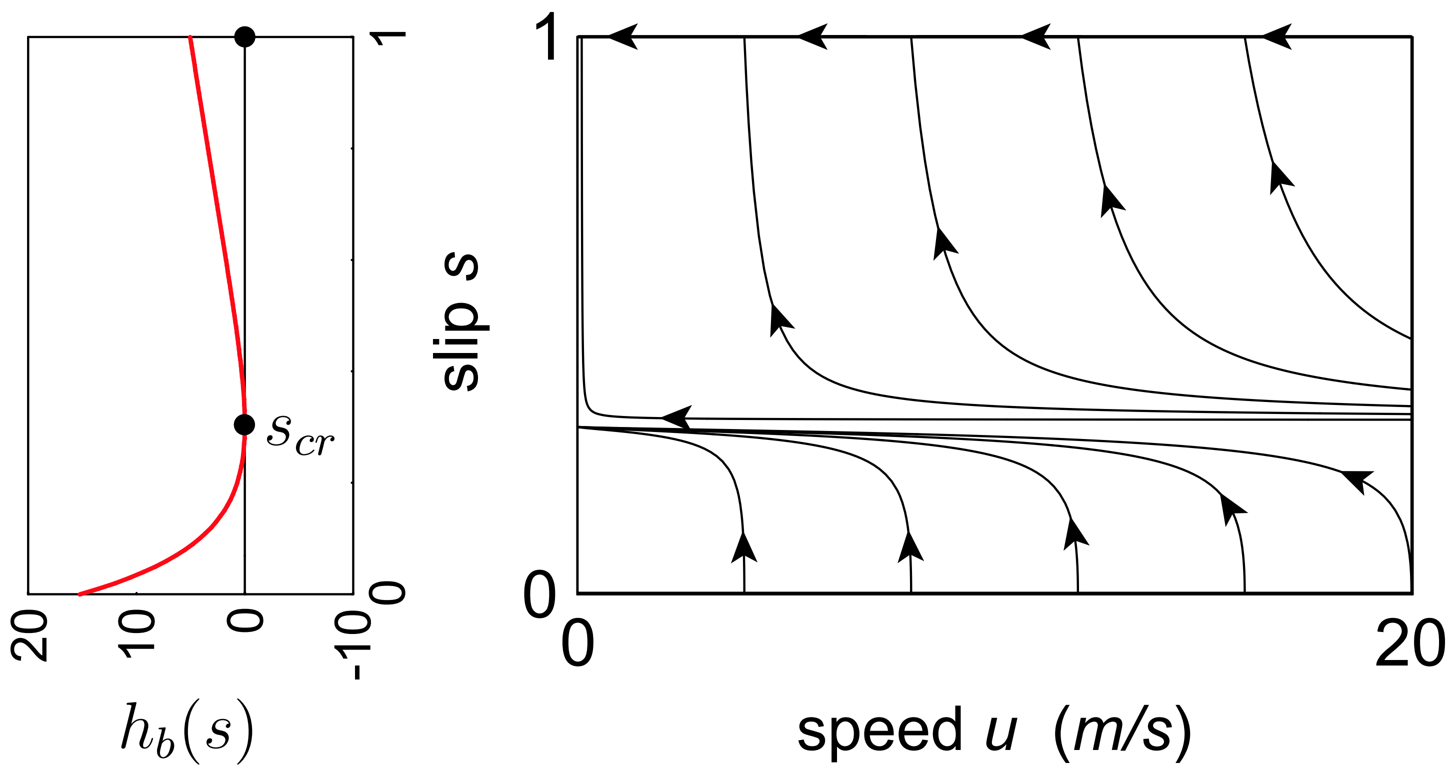
$\Upsilon_b^L = 10.199$ Impending Possible Lockup

State Space Description



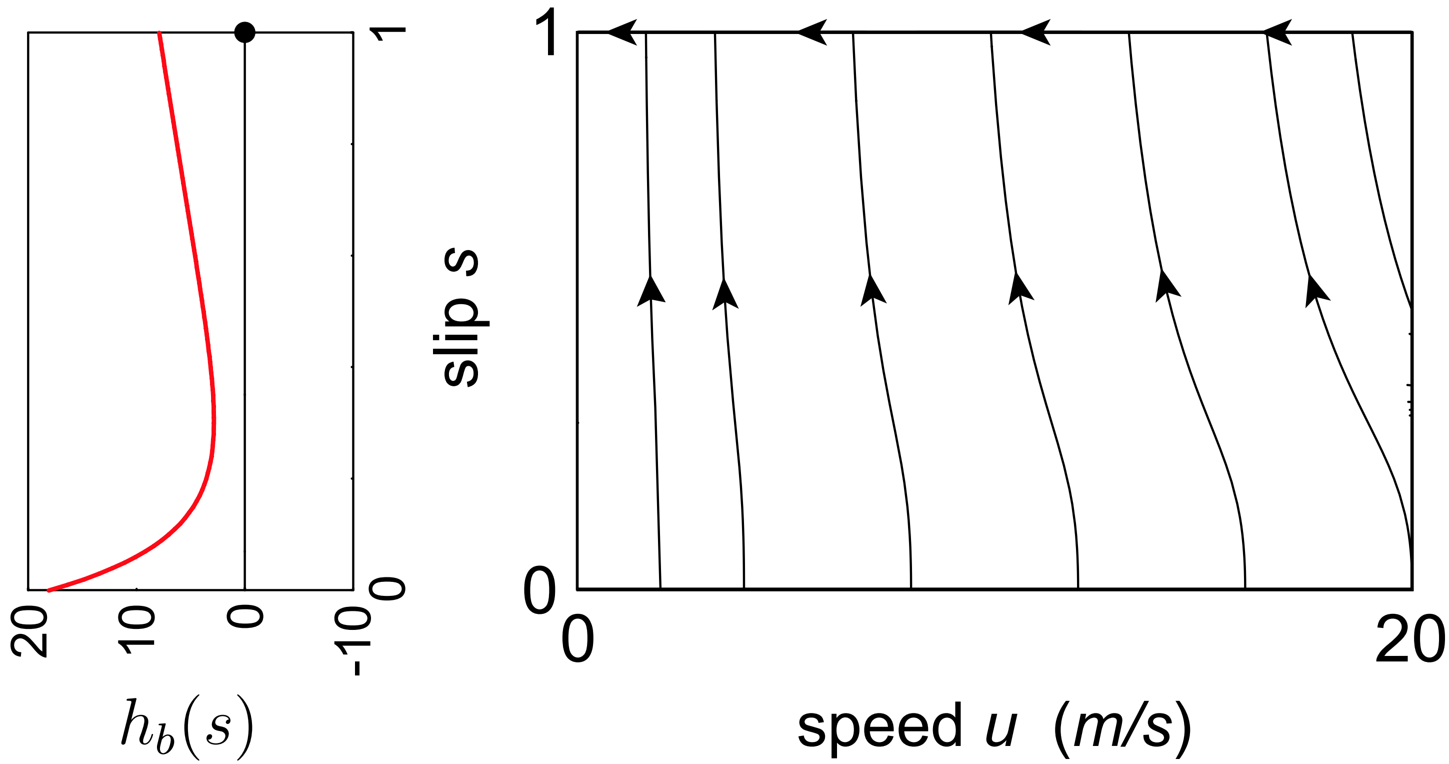
$\Upsilon_b = 12$ Possible Lockup

State Space Description



$\Upsilon_b^{cr} = 15.250$ Impending Guaranteed Lockup

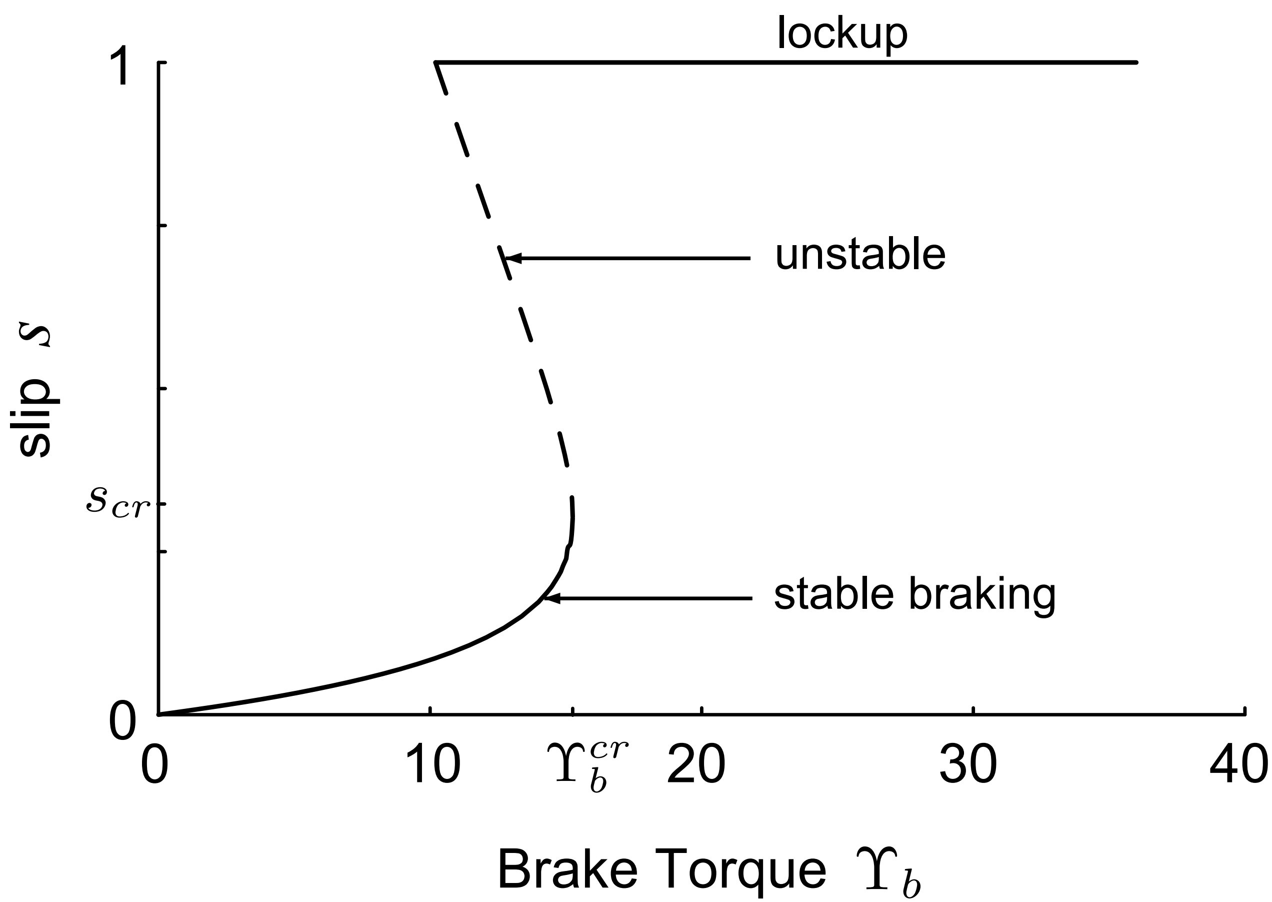
State Space Description



$\Upsilon_b = 18$

Guaranteed Lockup

Bifurcation of Slip Dynamics



$$\Upsilon_b^{cr} = 15.250$$
$$s_{cr} = 0.304$$
$$s_p = 0.316$$

Lockup Instability

- Maximum brake torque

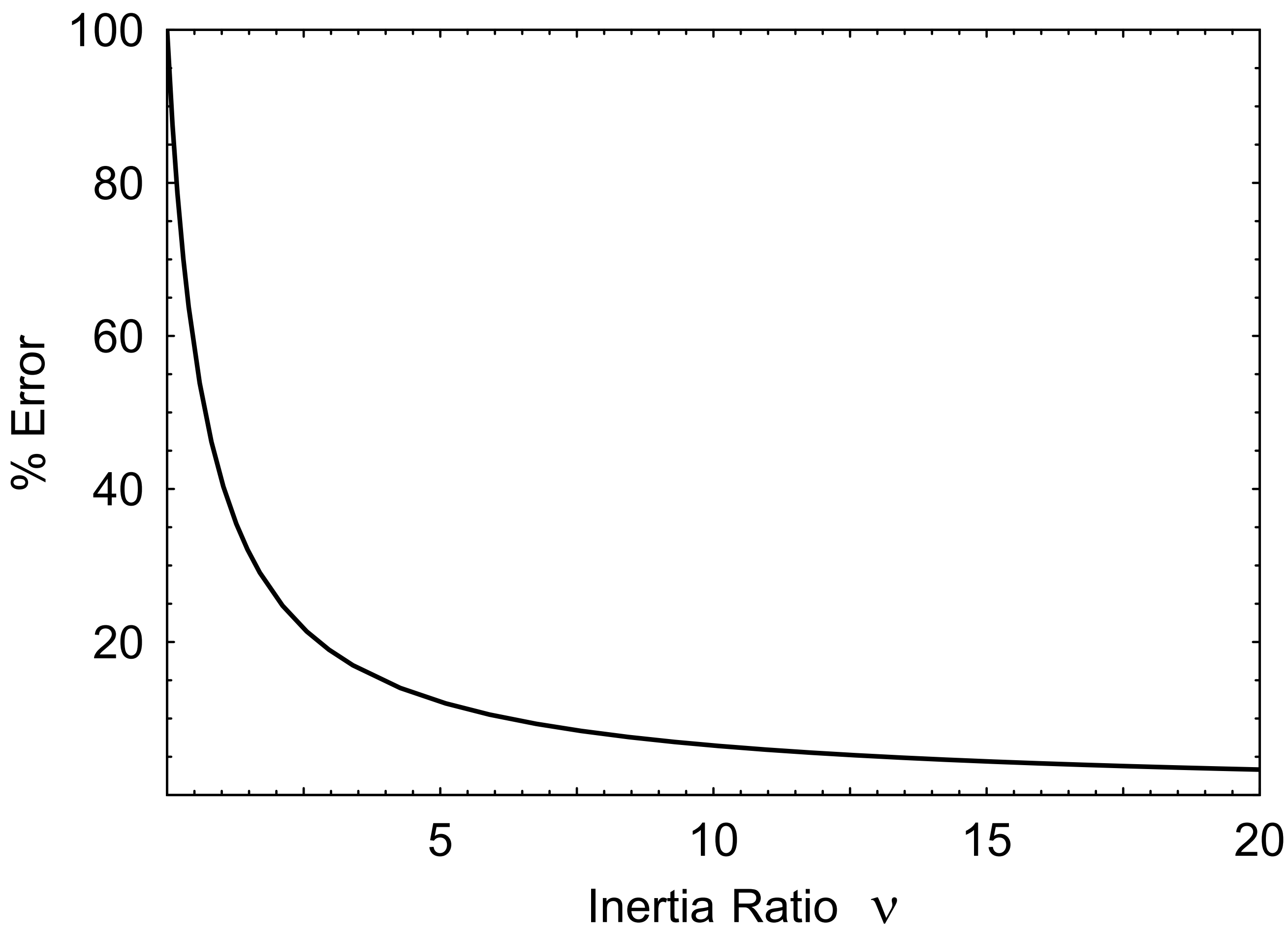
$$\Upsilon_b^{cr} = \begin{cases} \nu\mu(s_p) & \text{(TextBook}^\dagger\text{)} \\ \nu\mu(s_{cr})\left[1 + \frac{1}{\nu}(1 - s_{cr})\right] & \text{(Actual)} \end{cases}$$

$h'(s_{cr}) = 0$

† Fundamental Assumptions:

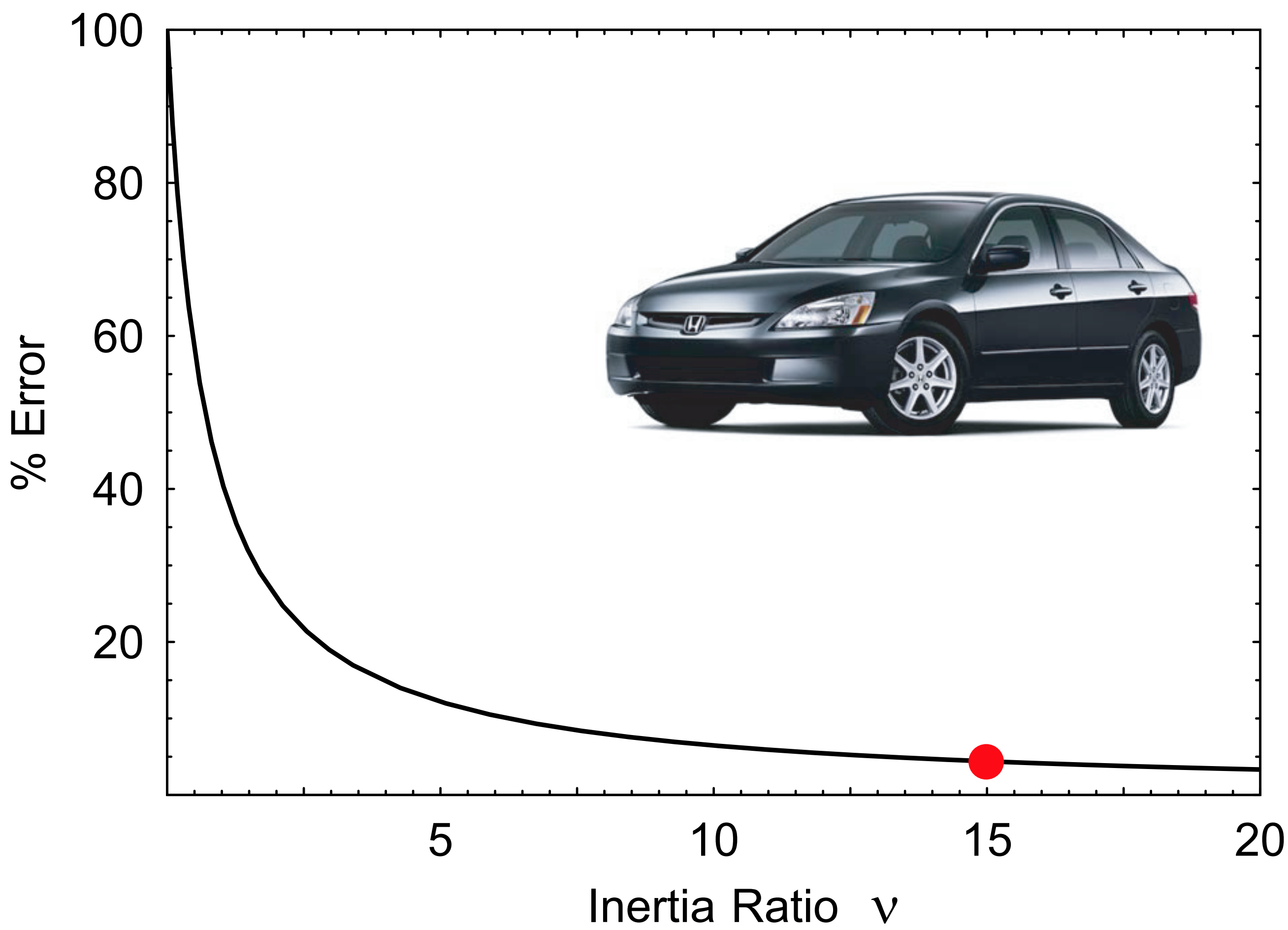
1. $T_b = mgR\mu(s_p)$
2. $\nu \gg 1.$
3. s_p can be reached.

Percent Error in Υ_b^{cr}



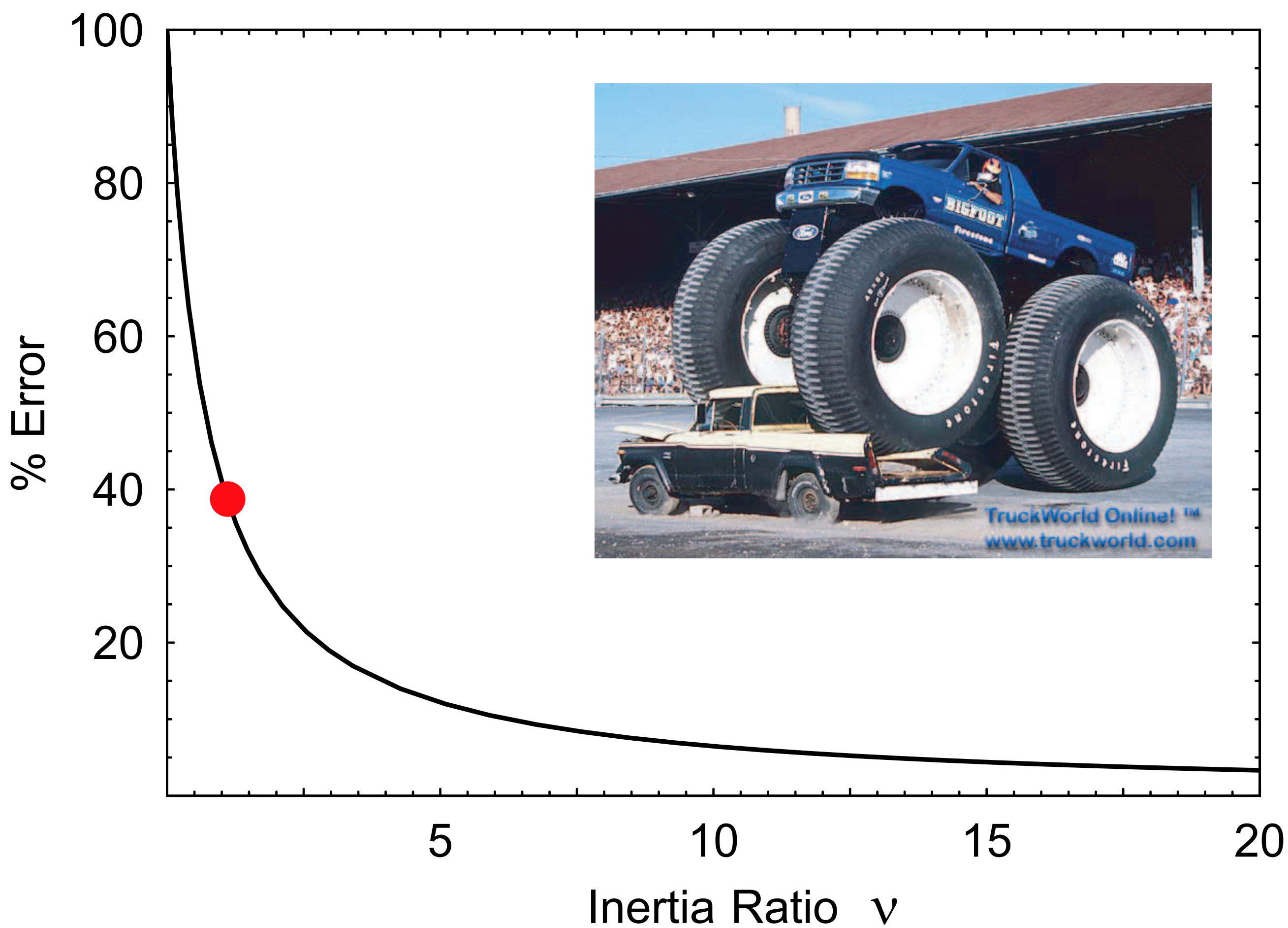
$$\%\,\text{Error} = \left| \frac{\Upsilon_b^{cr}|_{\text{actual}} - \Upsilon_b^{cr}|_{\text{textbook}}}{\Upsilon_b^{cr}|_{\text{actual}}} \right| 100\%$$

Percent Error in Υ_b^{cr}



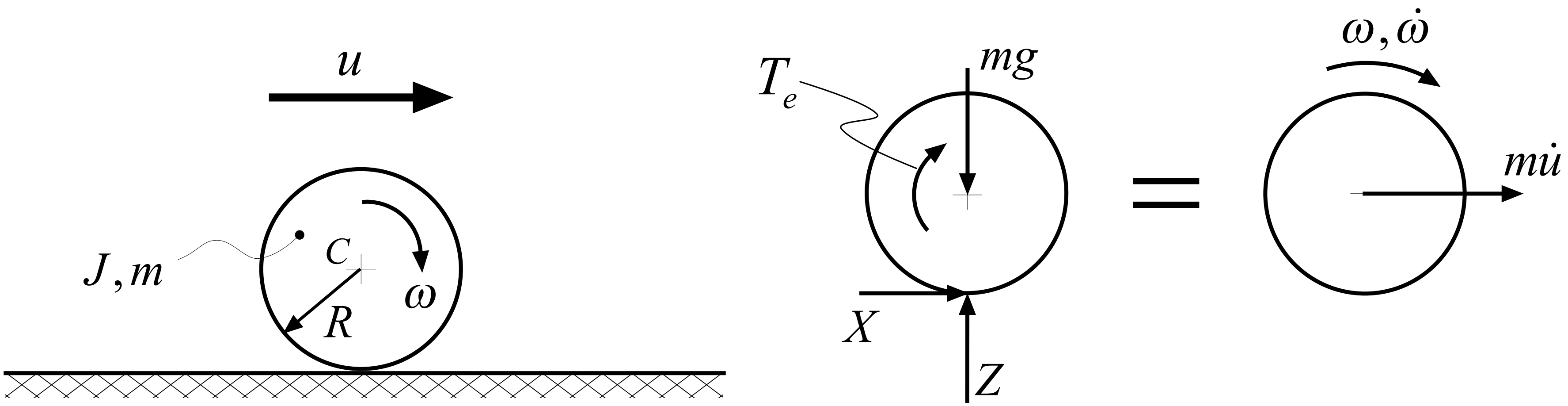
$$\%\,\text{Error} = \left| \frac{\Upsilon_b^{cr}|_{\text{actual}} - \Upsilon_b^{cr}|_{\text{textbook}}}{\Upsilon_b^{cr}|_{\text{actual}}} \right| 100\%$$

Percent Error in Υ_b^{cr}



$$\% \, \text{Error} = \left| \frac{\Upsilon_b^{cr}|_{\text{actual}} - \Upsilon_b^{cr}|_{\text{textbook}}}{\Upsilon_b^{cr}|_{\text{actual}}} \right| 100\%$$

Single-Wheel Acceleration Model



- Governing Equations

$$
\begin{aligned}
Z &= mg \\
m\dot{u} &= X \\
J\dot{\omega} &= -RX + T_e
\end{aligned}
$$

Equations of Motion—(u, s)

- Governing equations in terms of u and s

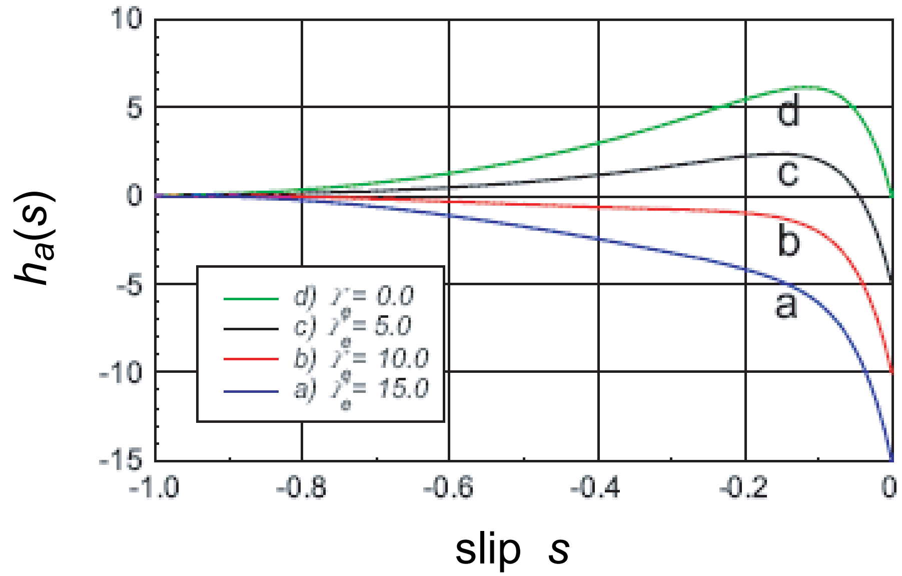
$$\left.\begin{array}{rcl} \dot{u} & = & \mu(s)g \\ \dot{s} & = & \frac{g}{u}h_a(s) \end{array}\right\} \qquad s \in [-1, 0], \;\; u \in \mathbb{R}_+$$

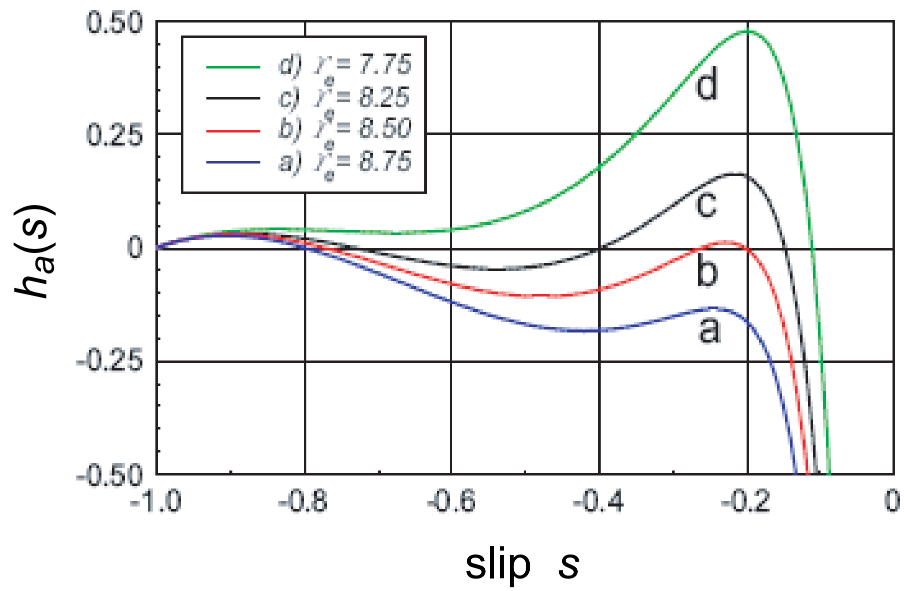
where

$$h_a(s) = (s+1)^2 \left[(s+1)^{-1}\mu(s) + \nu\mu(s) - \Upsilon_e\right]$$

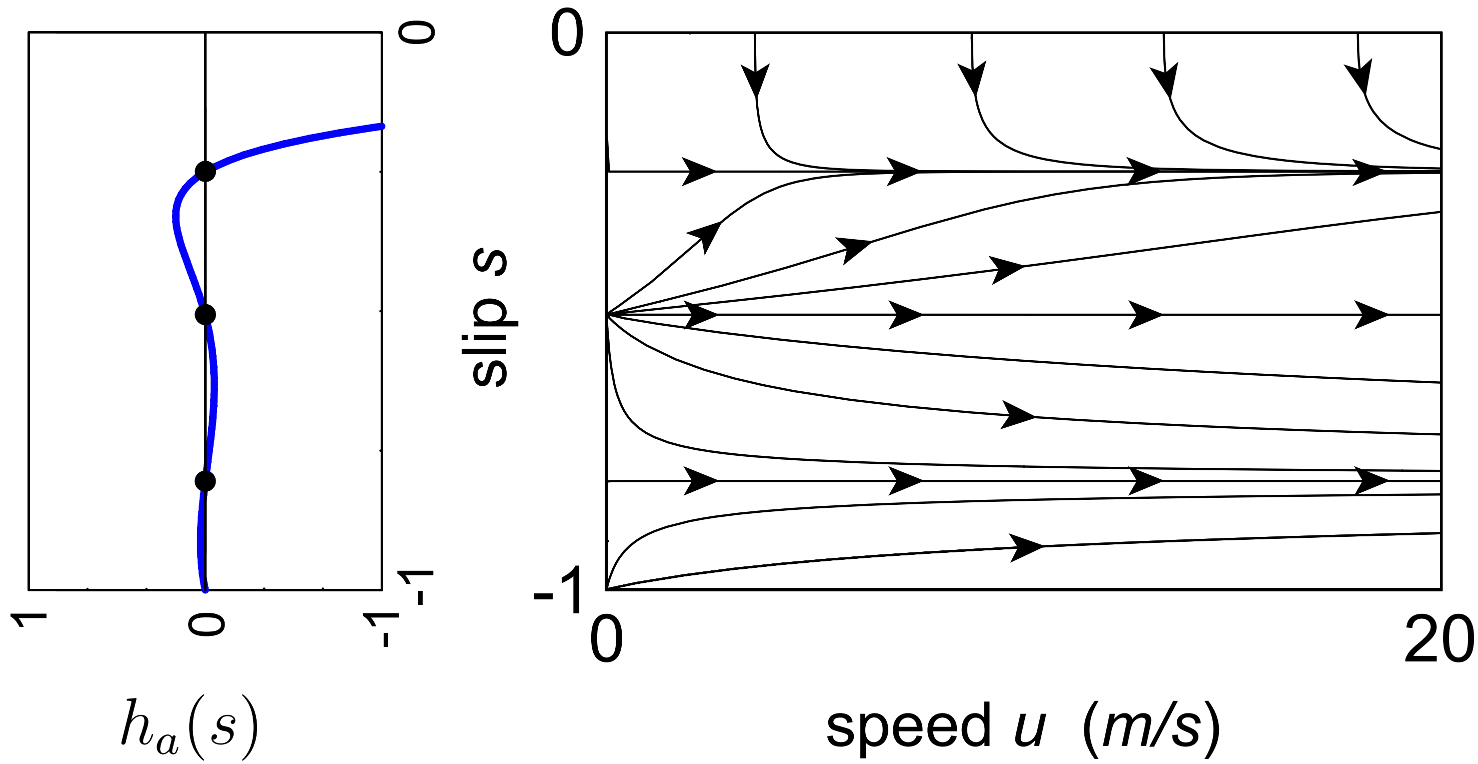
$$s = \frac{u - \omega R}{\omega R}, \qquad \nu = \frac{mR^2}{J}, \qquad \Upsilon_e = \frac{R}{Jg}T_e$$

Equations of Motion—(u, s)



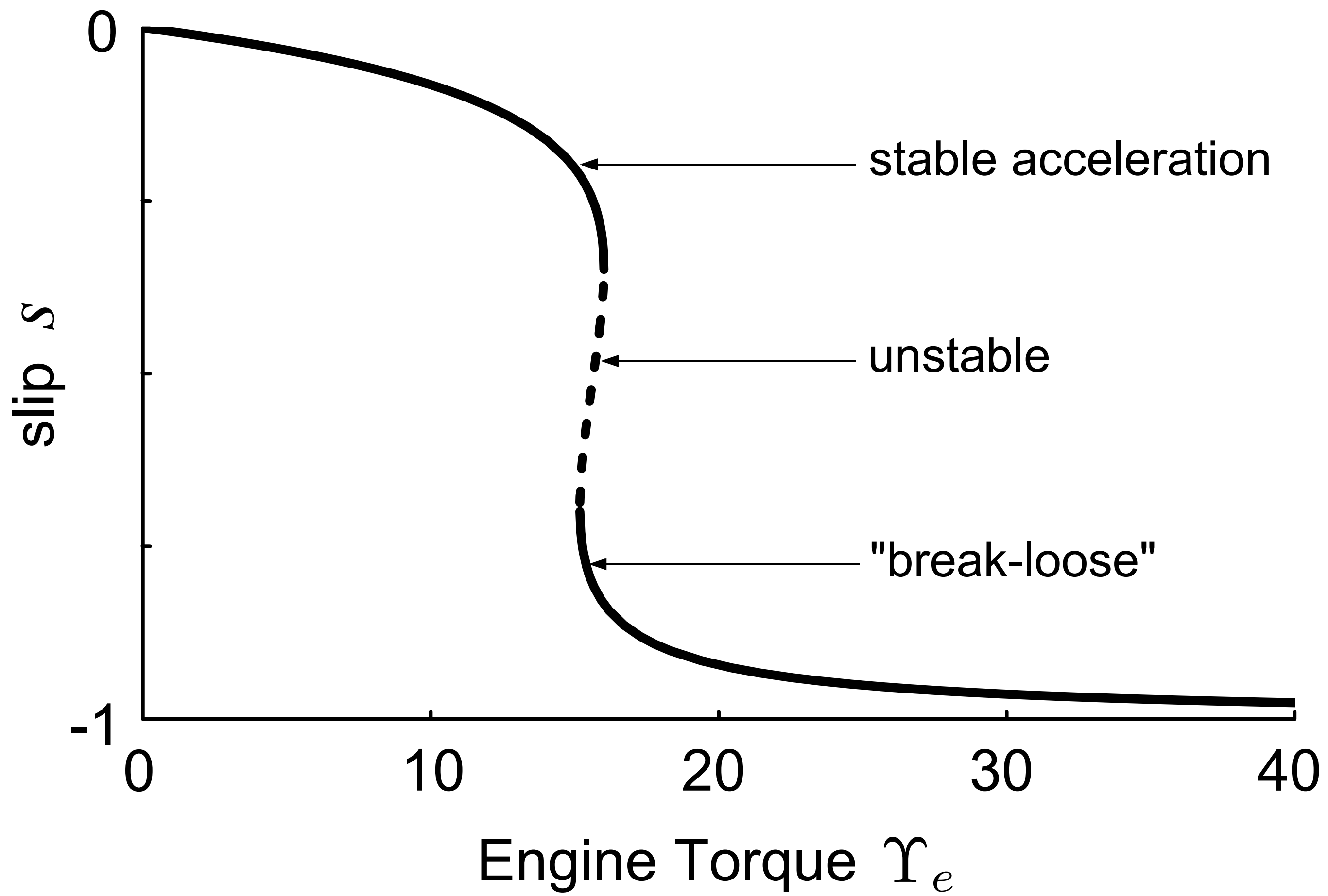


Example State Space Description

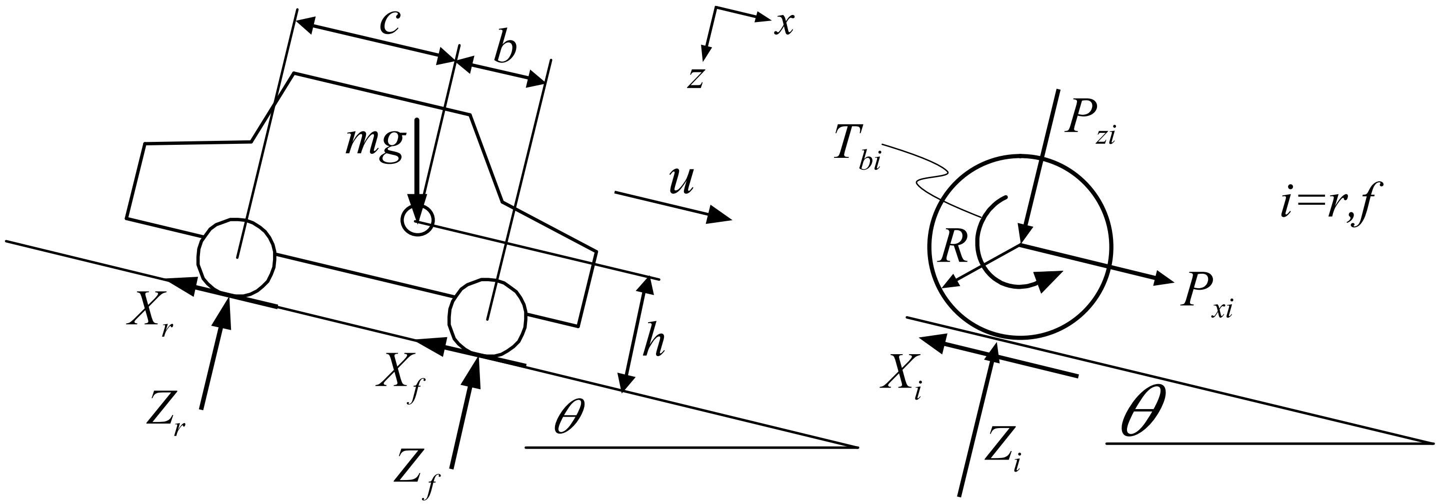


$$\Upsilon_e = 15.65$$

Bifurcation of Slip Dynamics



Two-Wheel Braking Model



- Governing Equations

$$\left.\begin{aligned} mg\sin\theta - X_r - X_f &= m\dot{u} \\ mg\cos\theta - Z_r - Z_f &= 0 \\ h\left(X_r + X_f\right) + cZ_r - bZ_f &= 0 \end{aligned}\right\} \qquad \text{(Vehicle)}$$

$$RX_i - T_{bi} = J\dot{\omega} \qquad (i = r, f) \qquad \text{(Wheels)}$$

Two-Wheel Braking Model (cont.)

- Friction Laws:

$$X_i = \mu(s_i) Z_i \quad (i = r, f)$$

- Dynamic Load Transfer:

$$\begin{aligned} Z_r &= mg\left(\frac{b}{l}\cos\theta - \frac{h}{l}\sin\theta\right) + m\dot{u}\frac{h}{l} \\ Z_f &= mg\left(\frac{c}{l}\cos\theta + \frac{h}{l}\sin\theta\right) - m\dot{u}\frac{h}{l} \end{aligned}$$

Equations of Motion—(u, s)

- Governing equations in terms of u and s

$$\left.\begin{aligned} \dot{u} &= -g\left(\Lambda_b(\mathbf{s})\cos\theta - \sin\theta\right) \\ \dot{s}_r &= \tfrac{g}{u} h_{br}(\mathbf{s}) \\ \dot{s}_f &= \tfrac{g}{u} h_{bf}(\mathbf{s}) \end{aligned}\right\} \quad s \in I,\ \ u \in \mathbb{R}_+$$

where

$$\mathbf{s} = (s_r, s_f)$$

$$h_{bi}(\mathbf{s}) = (1 - s_i)\left(\Lambda_b(\mathbf{s})\cos\theta - \sin\theta\right) - \mu(s_i)\nu\lambda_i(\mathbf{s}) + \Upsilon_{bi}$$

Equations of Motion (cont.)

$$
\begin{aligned}
\Lambda_b(\mathbf{s}) &= \frac{\mu(s_r)\frac{b}{l} + \mu(s_f)\frac{c}{l}}{1 + \frac{h}{l}\left(\mu(s_r) - \mu(s_f)\right)} \\
\lambda_r(\mathbf{s}) &= \left(\frac{b}{l} - \Lambda_b(\mathbf{s})\frac{h}{l}\right)\cos\theta \\
\lambda_f(\mathbf{s}) &= \left(\frac{c}{l} + \Lambda_b(\mathbf{s})\frac{h}{l}\right)\cos\theta
\end{aligned}
$$

$$
s_i = \frac{u - \omega_i R}{u}, \qquad \nu = \tfrac{mR^2}{J}, \qquad \Upsilon_{bi} = \frac{R}{Jg}T_{bi}
$$

Slip Dynamics

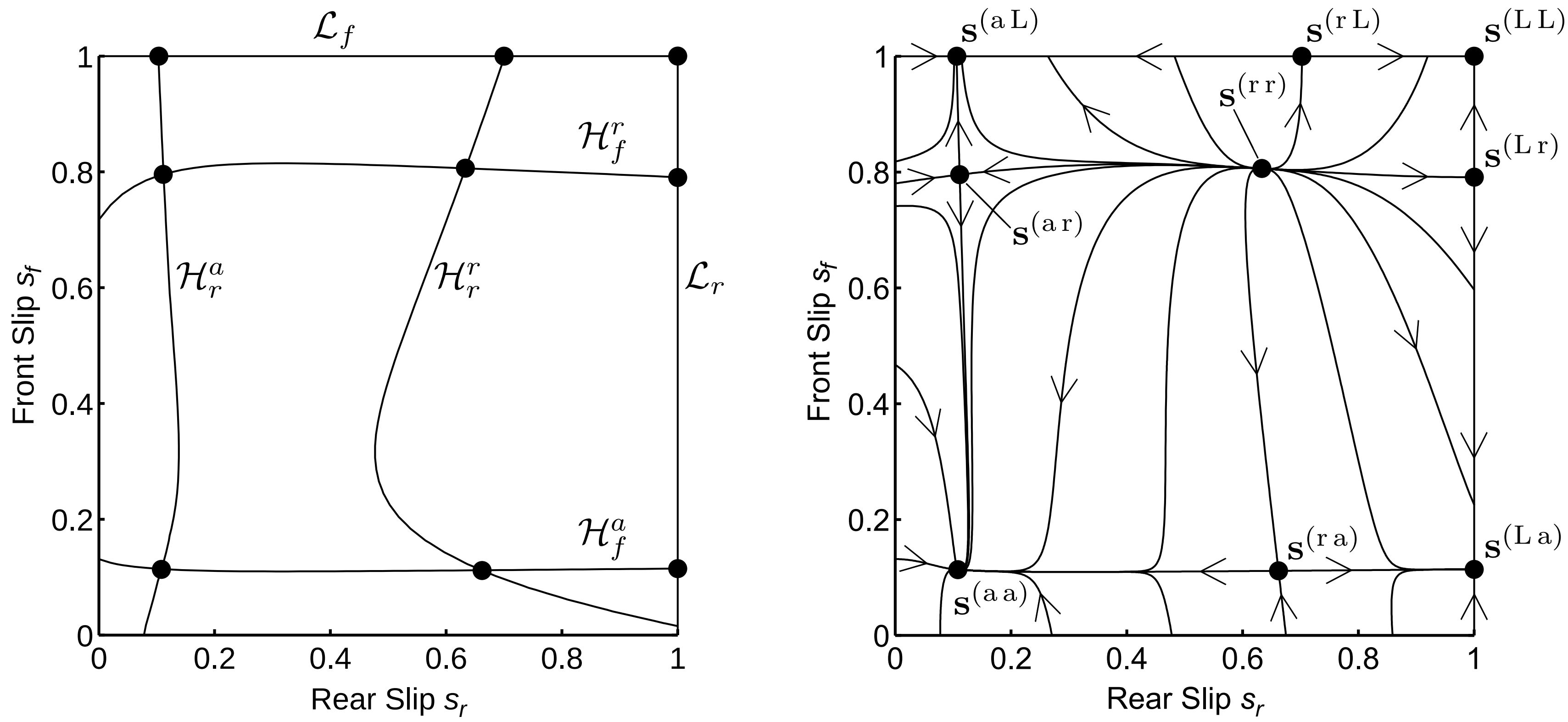
- The qualitative nature of the slip dynamics are decoupled from speed dynamics.

$$\frac{ds_r}{ds_f} = \frac{\dot{s}_r}{\dot{s}_f} = \frac{\frac{g}{u} h_{br}(\mathbf{s})}{\frac{g}{u} h_{bf}(\mathbf{s})} = \frac{h_{br}(\mathbf{s})}{h_{bf}(\mathbf{s})}$$

- Investigate planar system with fixed u (non-uniform time scaling)

$$\begin{aligned} \dot{s}_r &= \frac{g}{u} h_{br}(\mathbf{s}) \\ \dot{s}_f &= \frac{g}{u} h_{bf}(\mathbf{s}) \end{aligned}$$

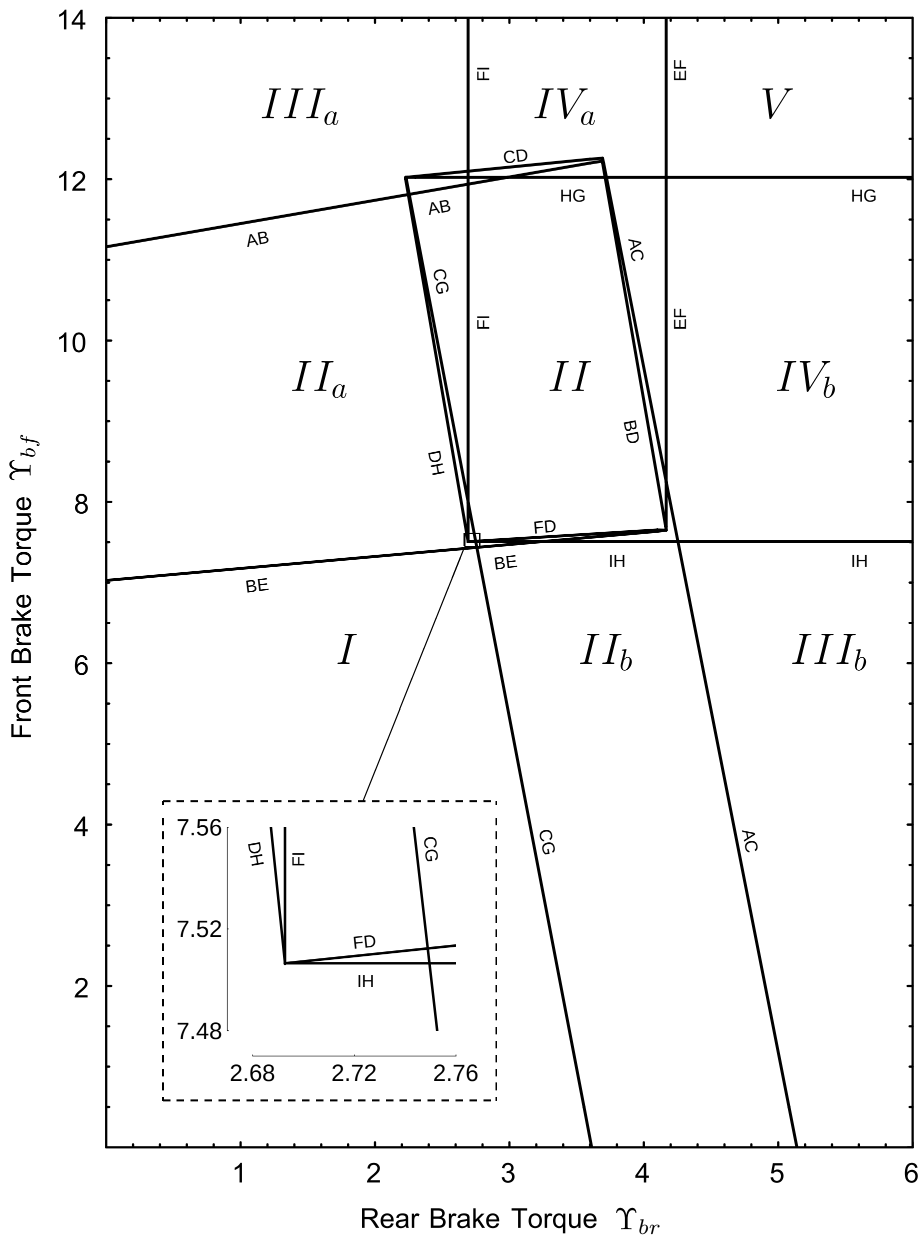
Example State Space Description

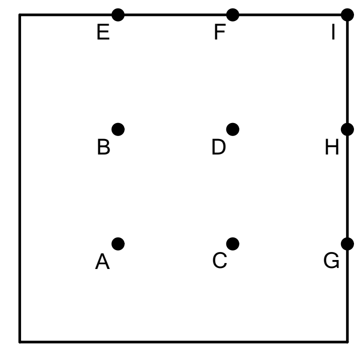


$$\frac{h}{l} = 0.125, \quad \frac{c}{l} = 0.6, \quad , \quad \theta = 0, \quad \nu = 15, \quad \Upsilon_{br} = 3.4, \quad \Upsilon_{bf} = 9$$

Bifurcation of Slip Dynamics

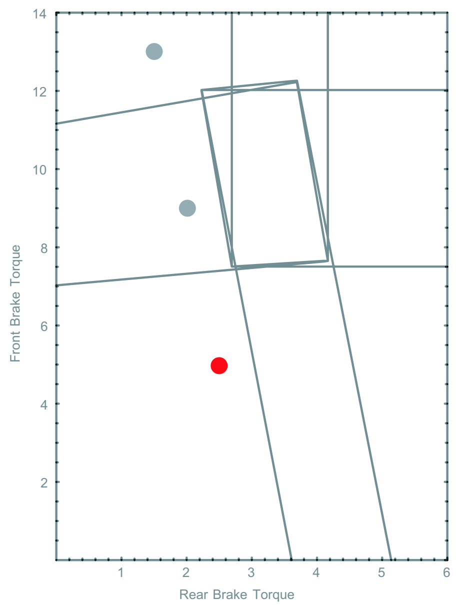
$$h/l = 0.125$$
$$c/l = 0.6$$
$$b/l = 0.4$$
$$\theta = 0$$
$$\nu = 15$$

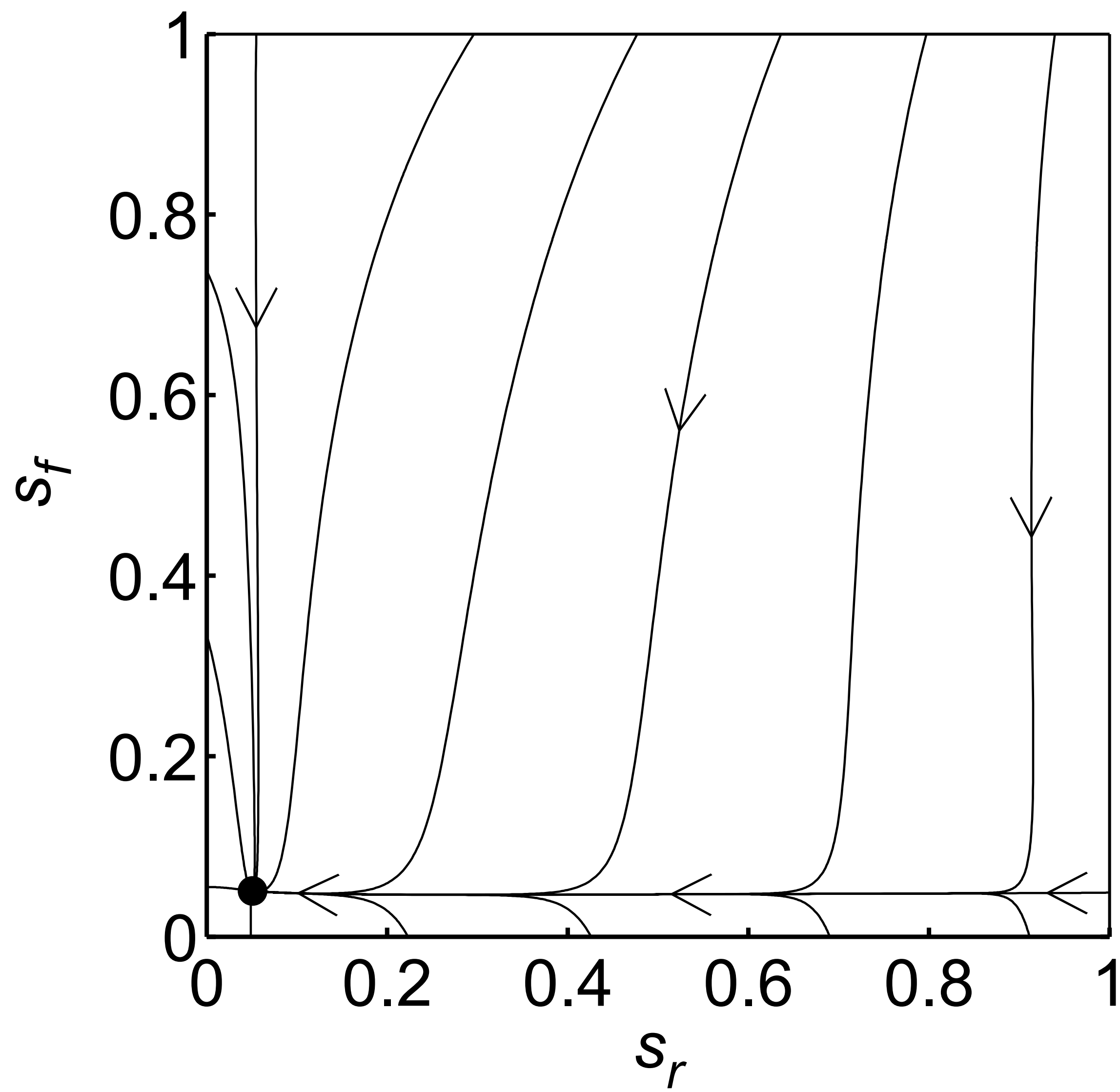




Letter	$\mathbf{s}^*$
A	$\mathbf{s}^{(\mathrm{a\,a})}$
B	$\mathbf{s}^{(\mathrm{a\,r})}$
C	$\mathbf{s}^{(\mathrm{r\,a})}$
D	$\mathbf{s}^{(\mathrm{r\,r})}$
E	$\mathbf{s}^{(\mathrm{a\,L})}$
F	$\mathbf{s}^{(\mathrm{r\,L})}$
G	$\mathbf{s}^{(\mathrm{L\,a})}$
H	$\mathbf{s}^{(\mathrm{L\,r})}$
I	$\mathbf{s}^{(\mathrm{L\,L})}$

Example State Space Descriptions





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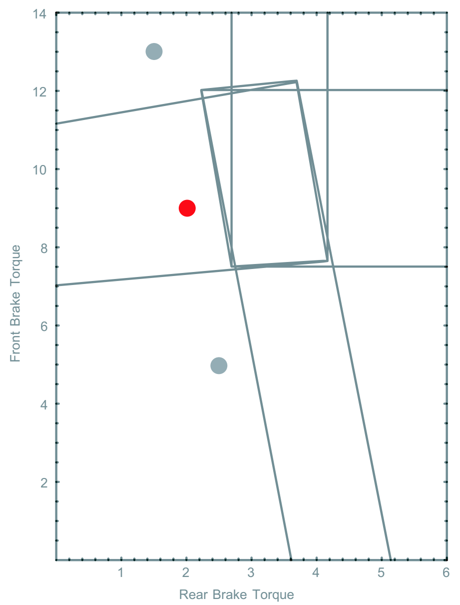

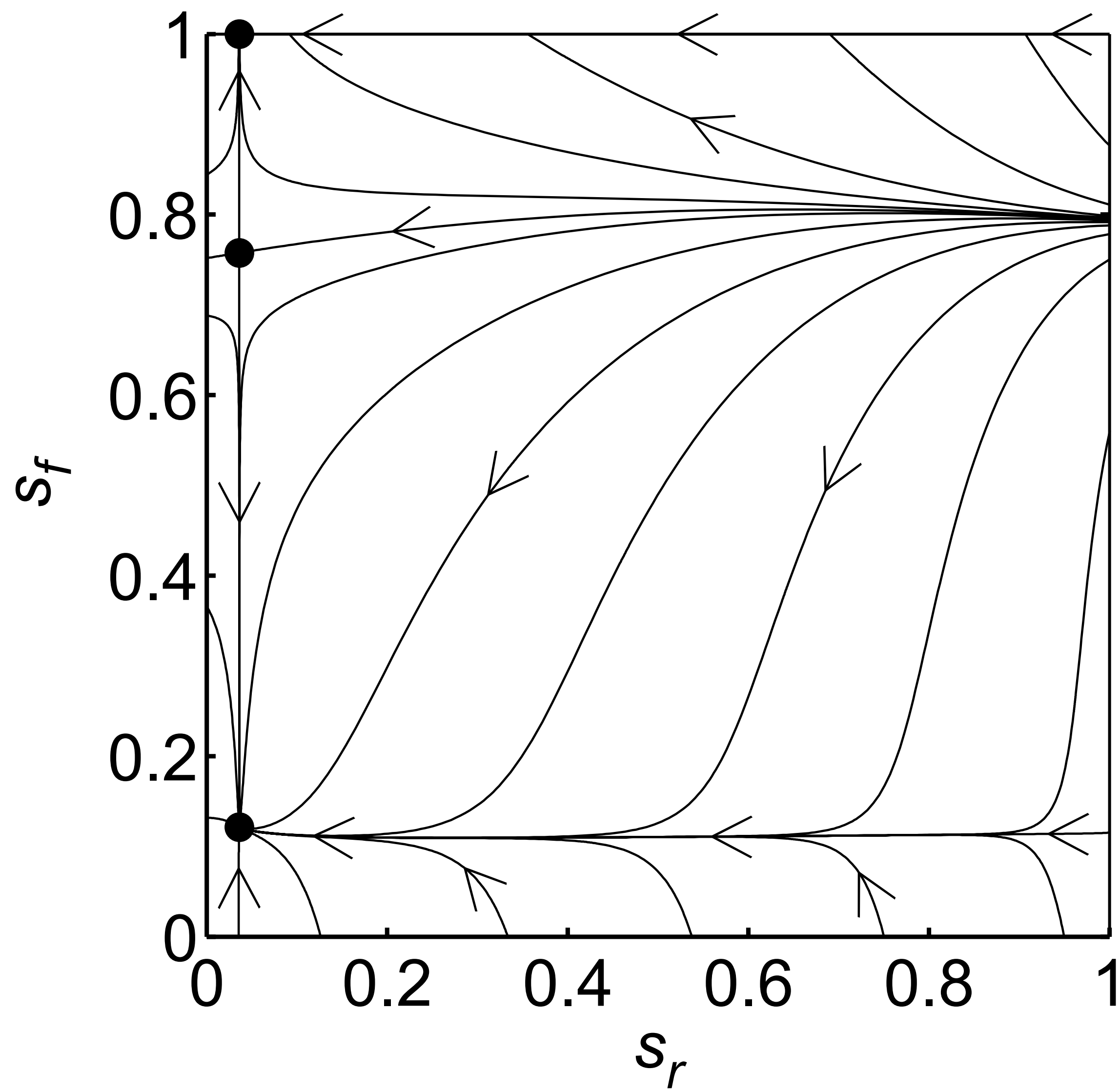

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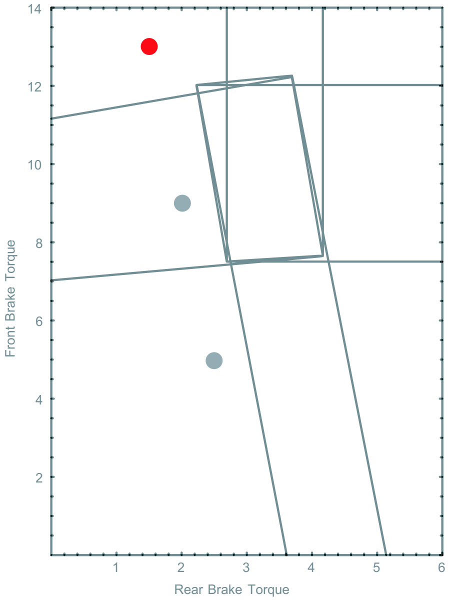

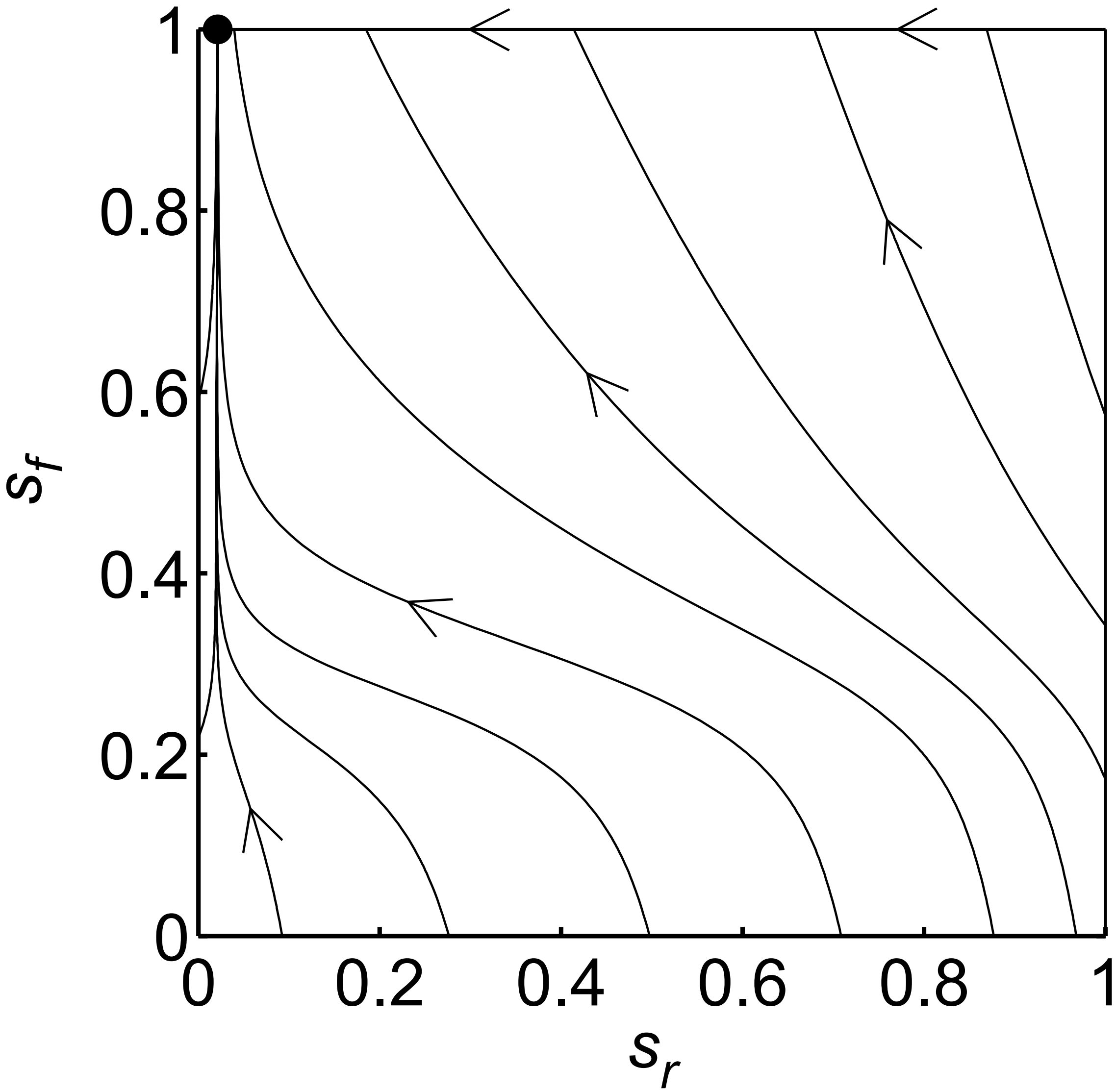

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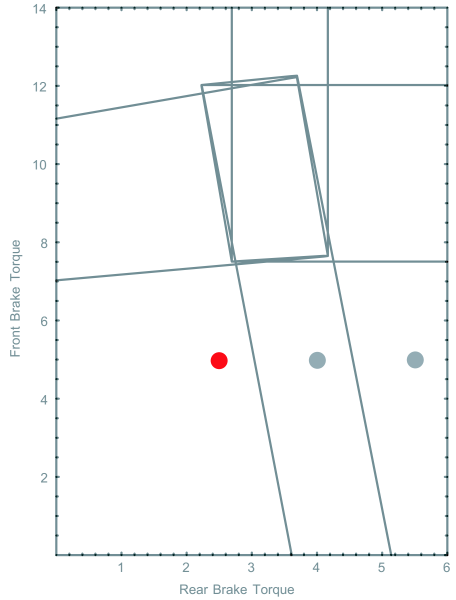

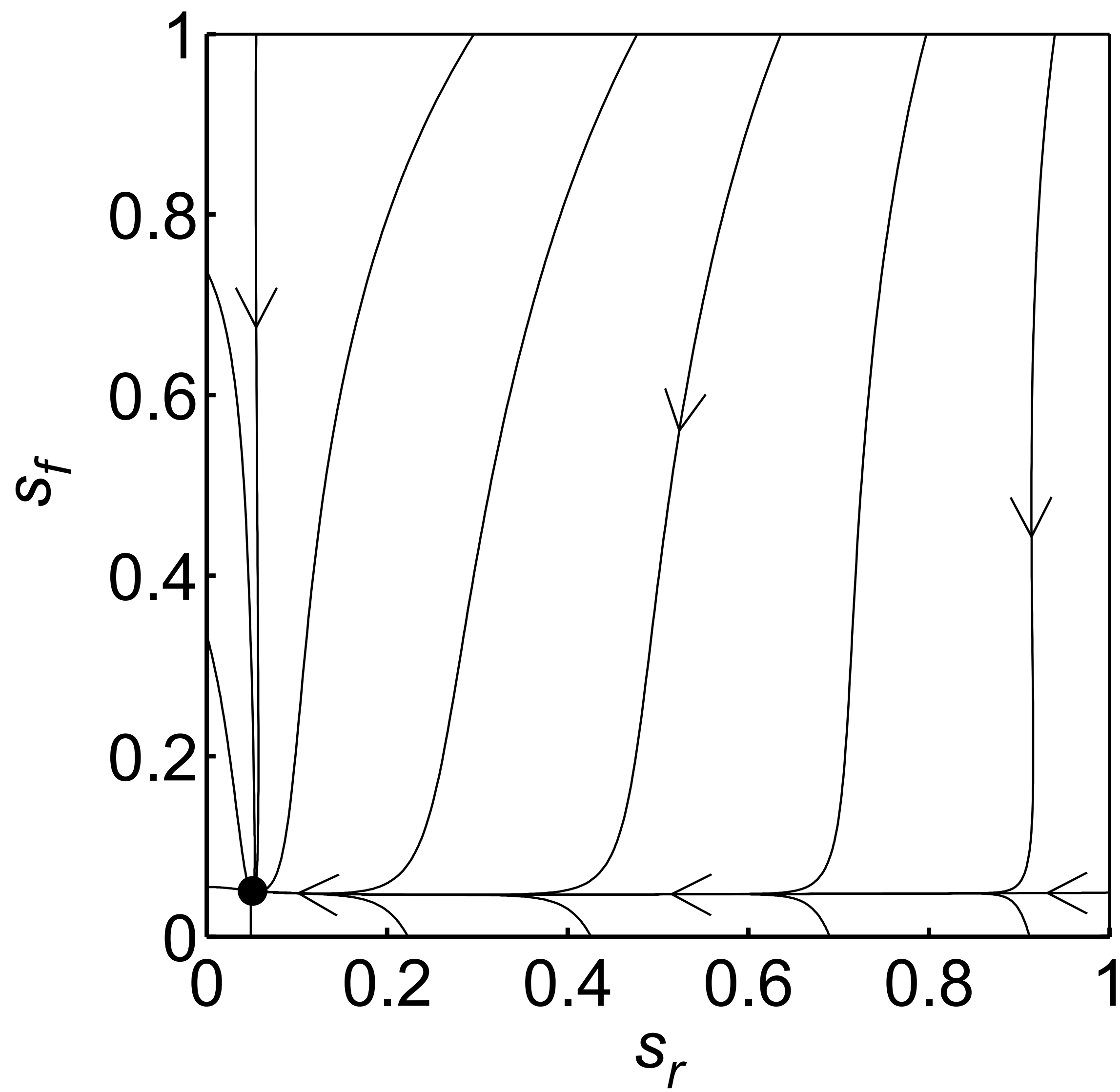

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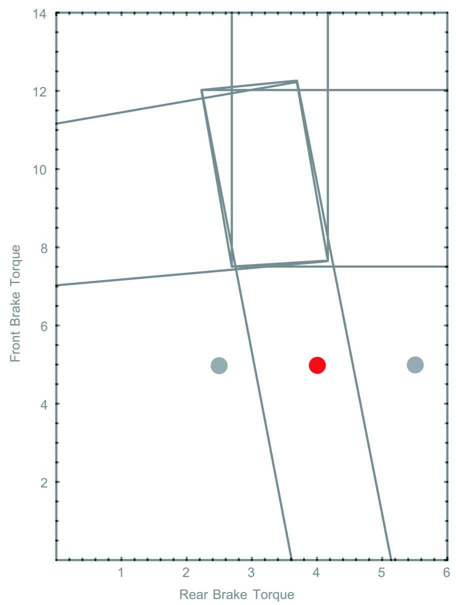

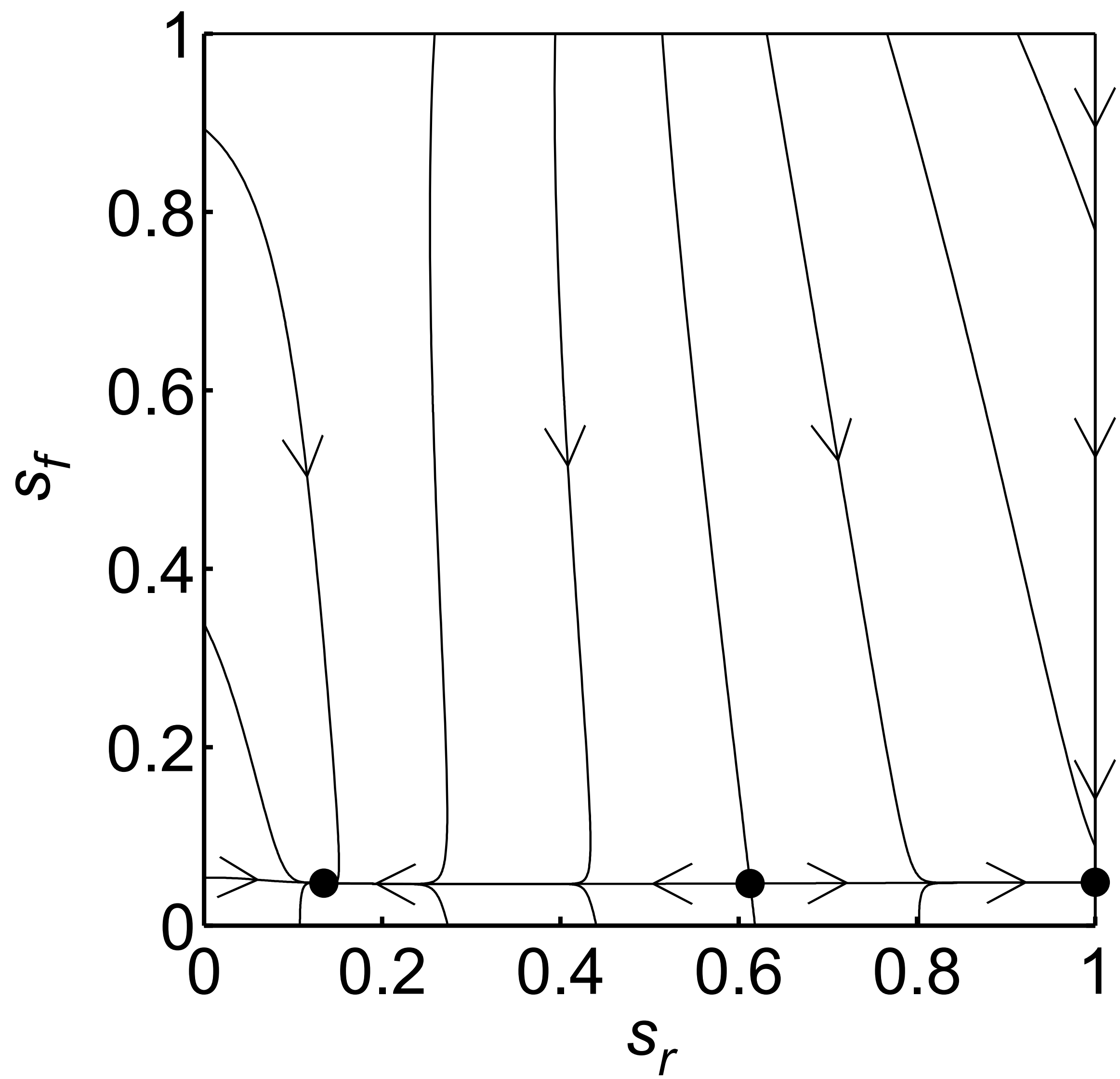

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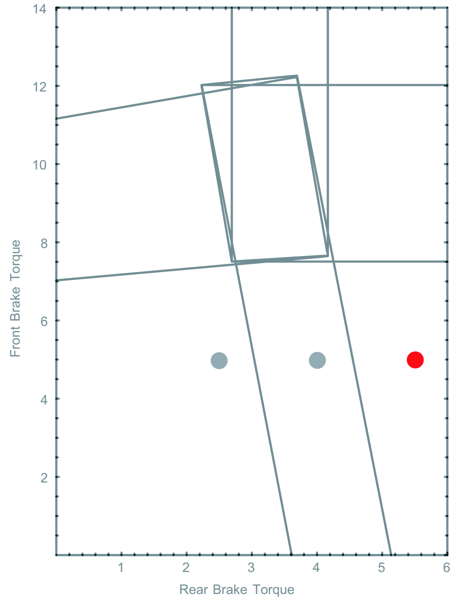

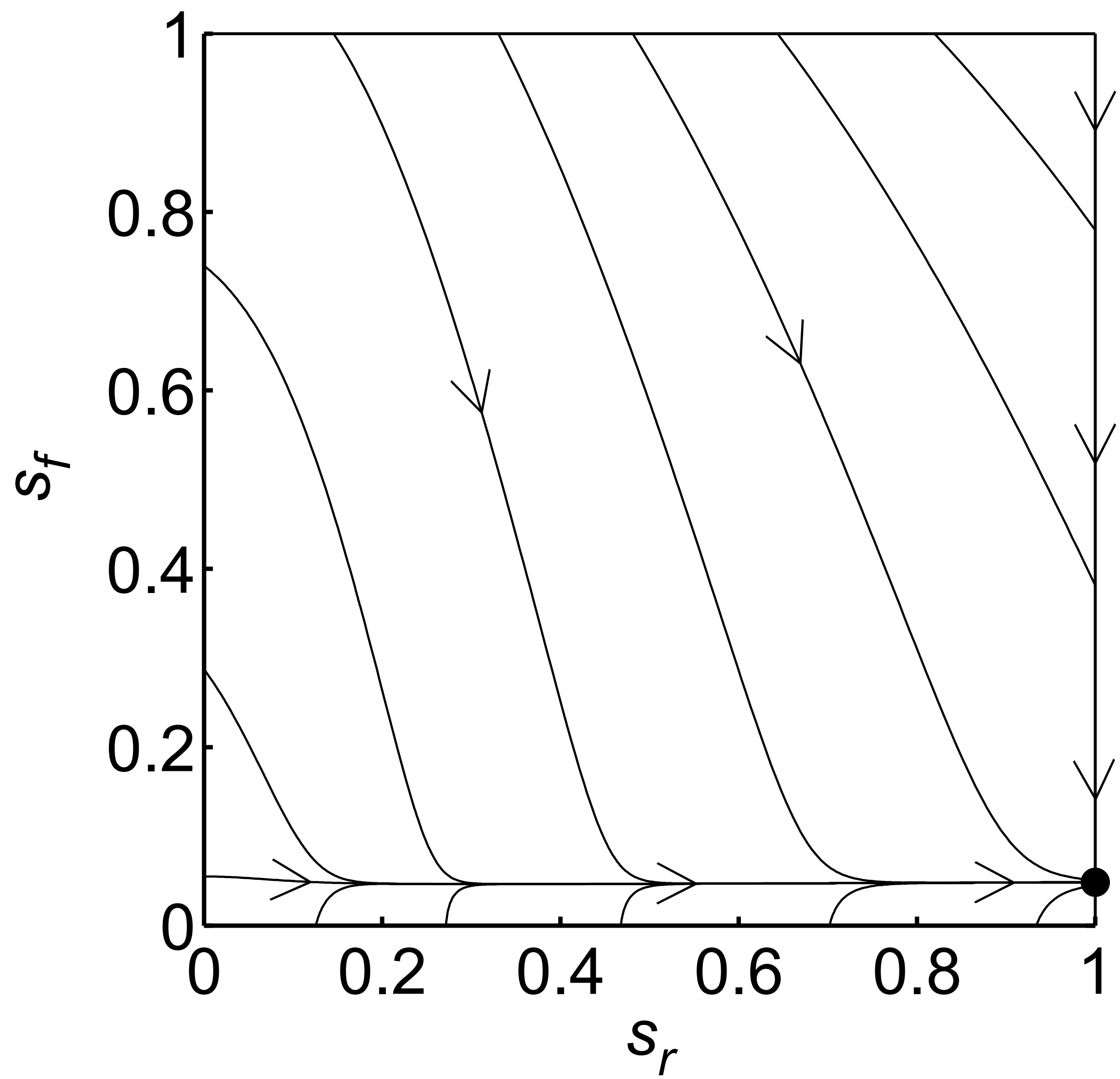

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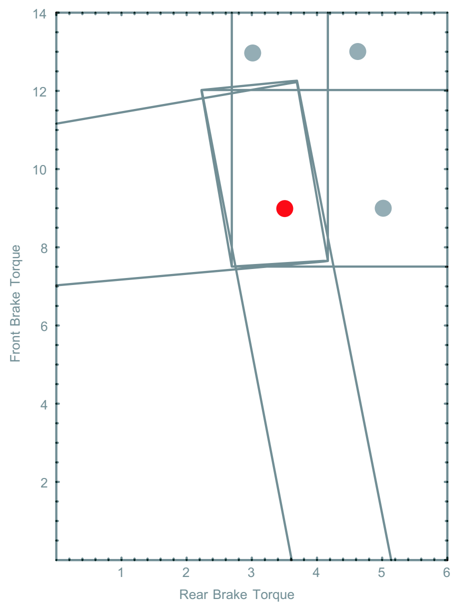

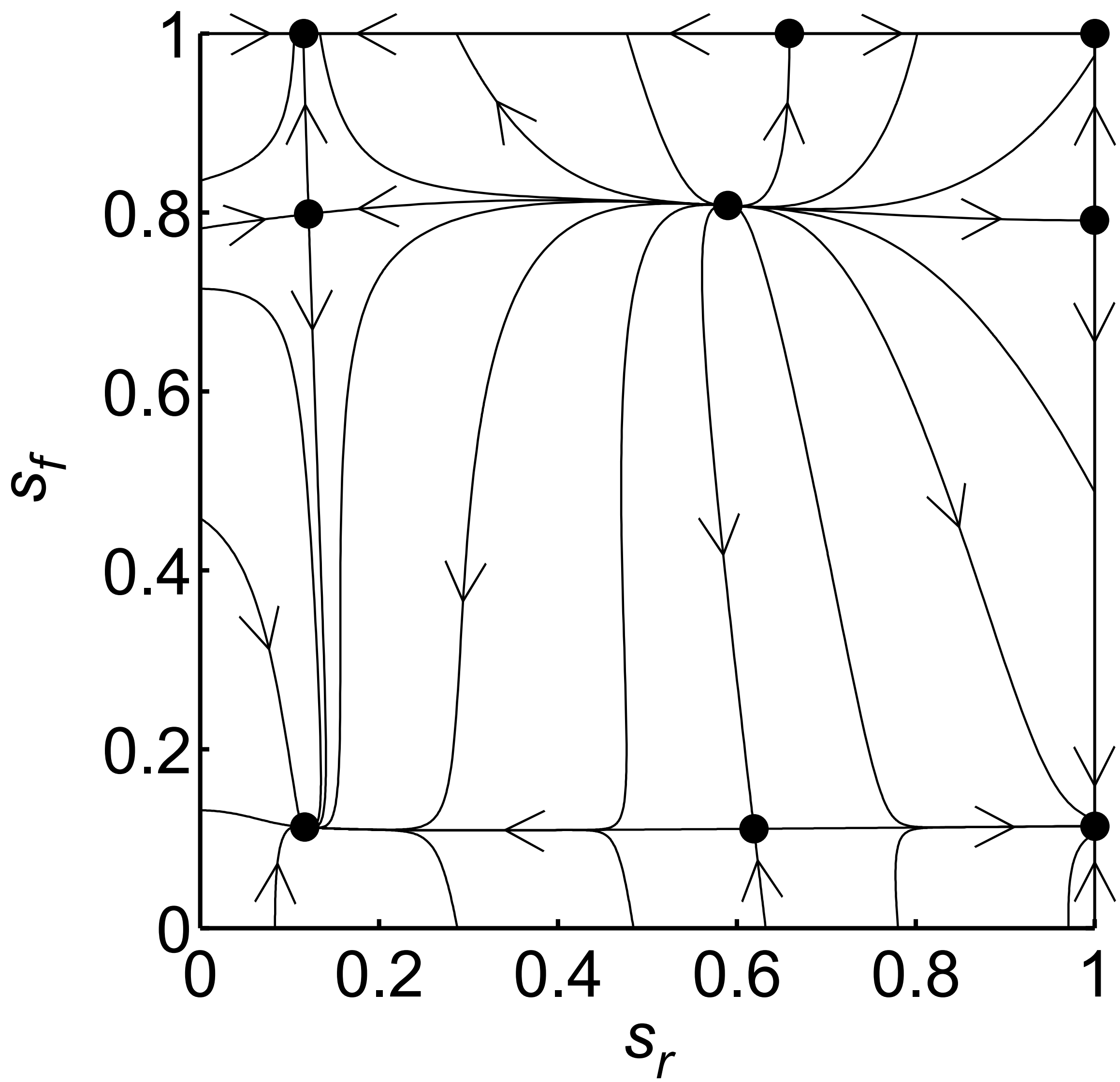

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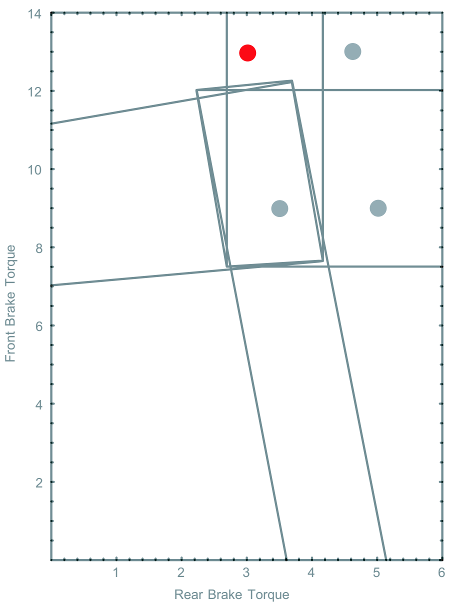

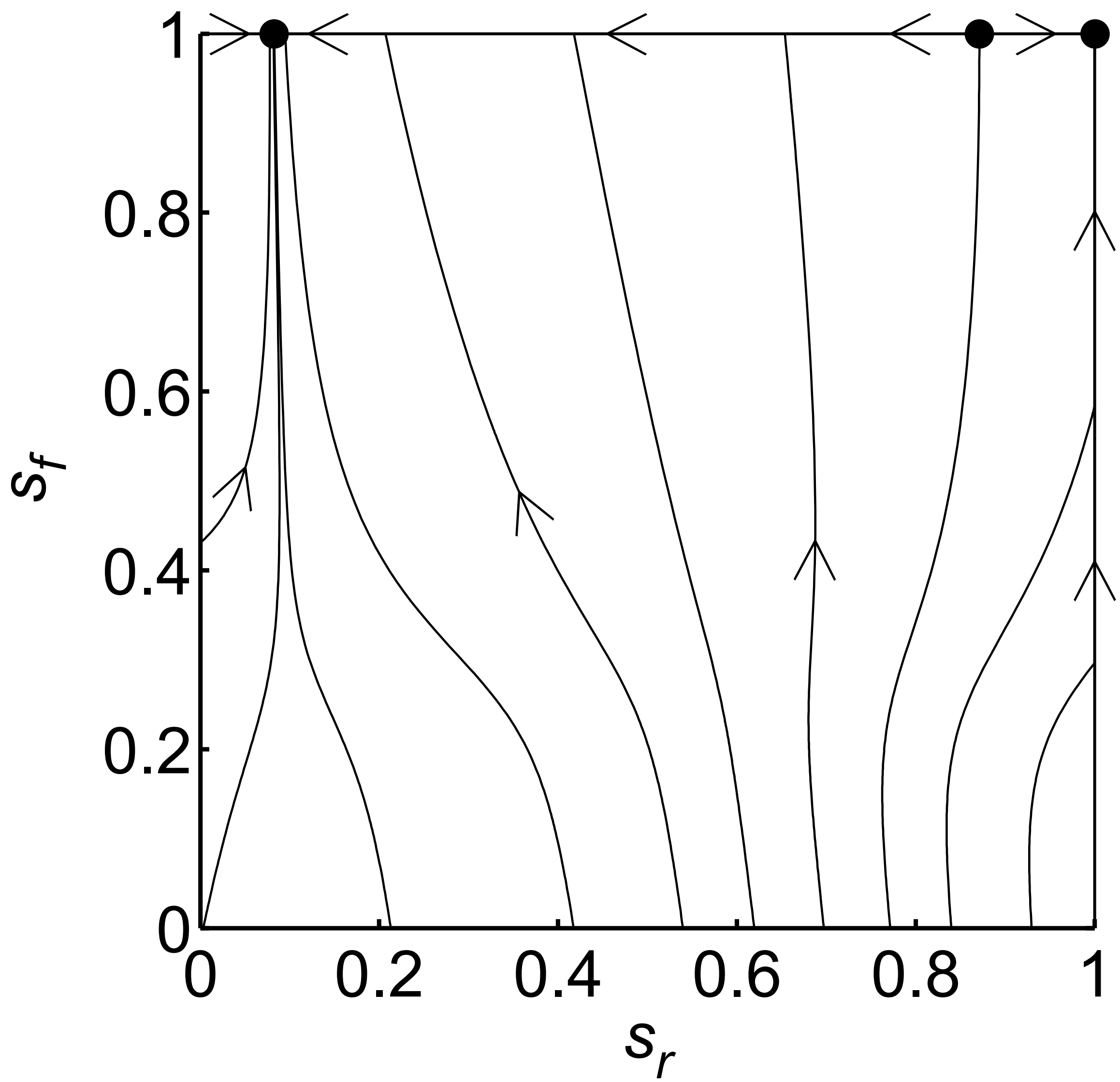

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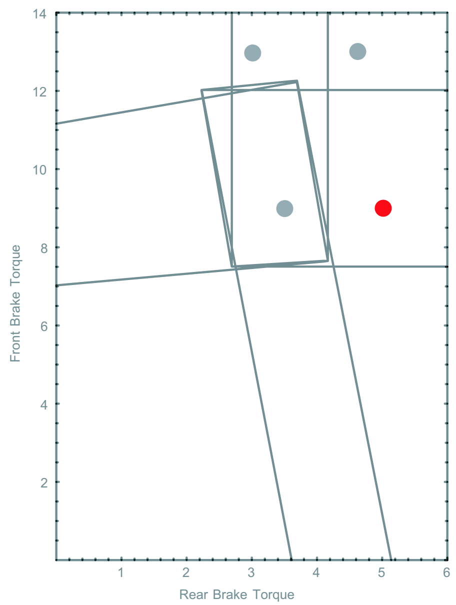

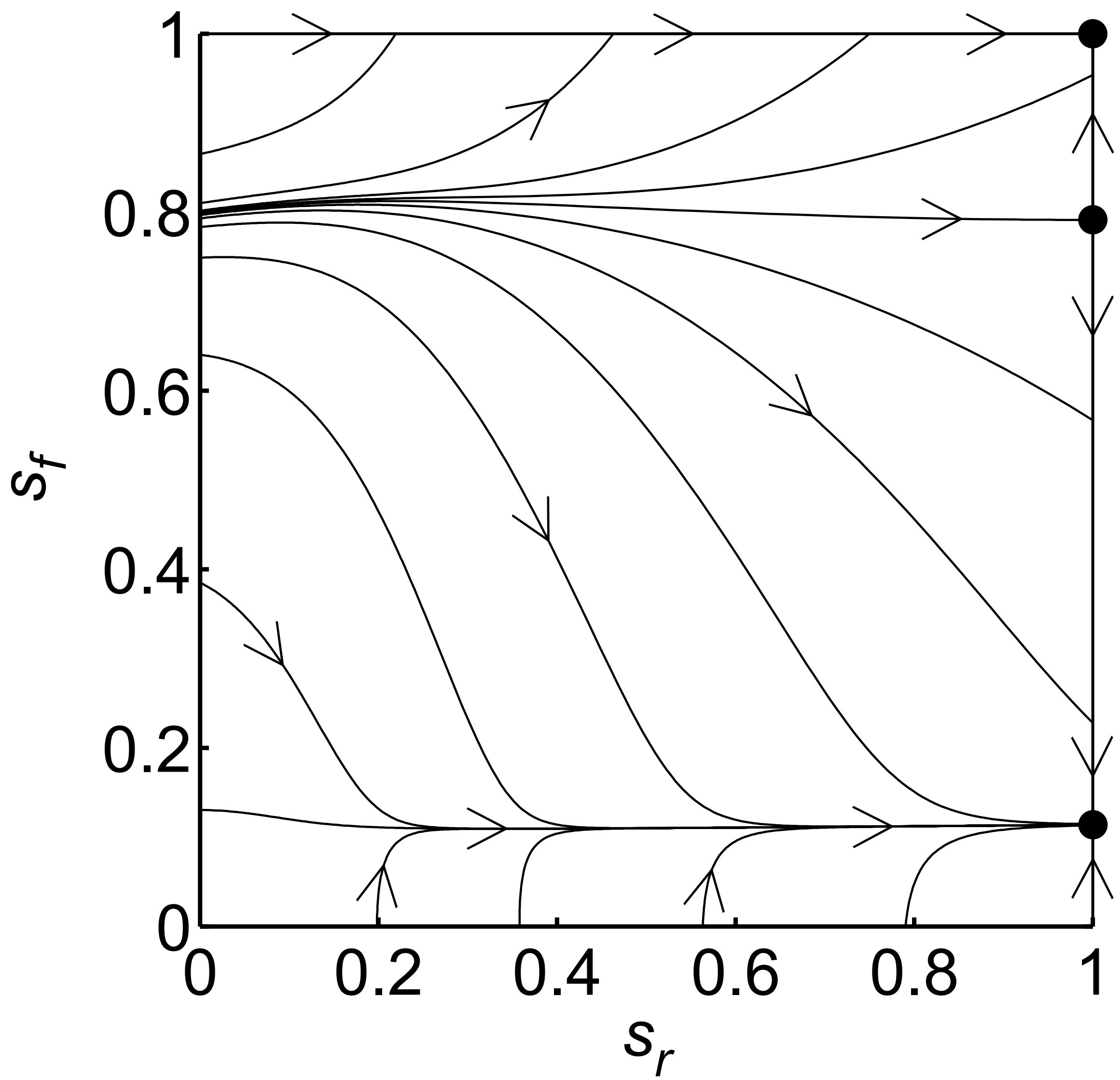

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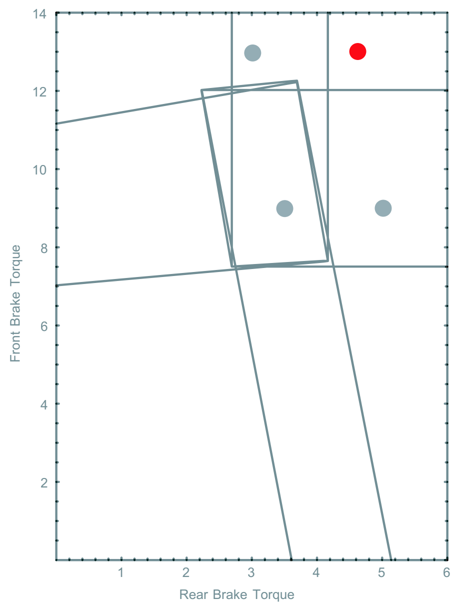
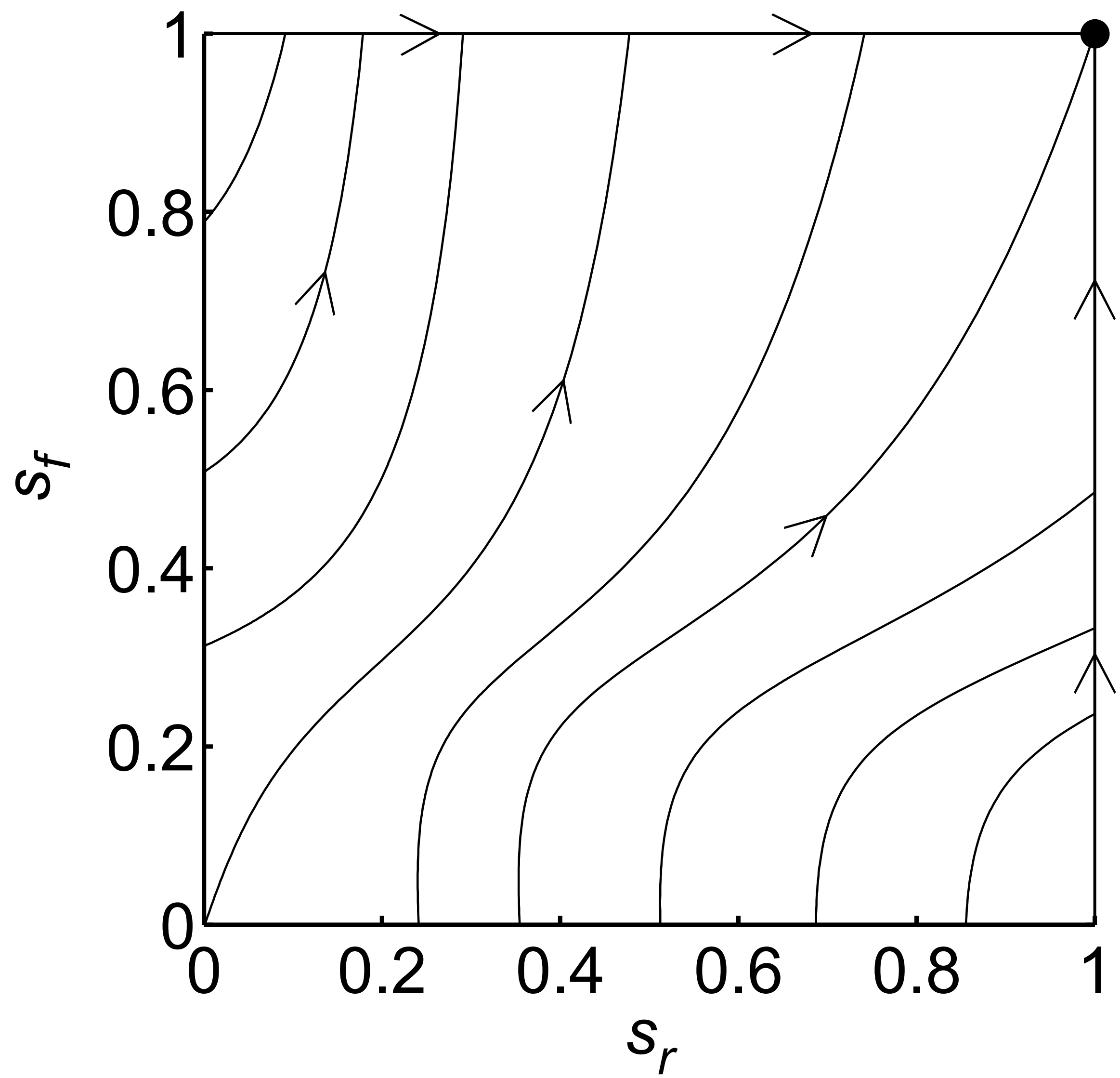
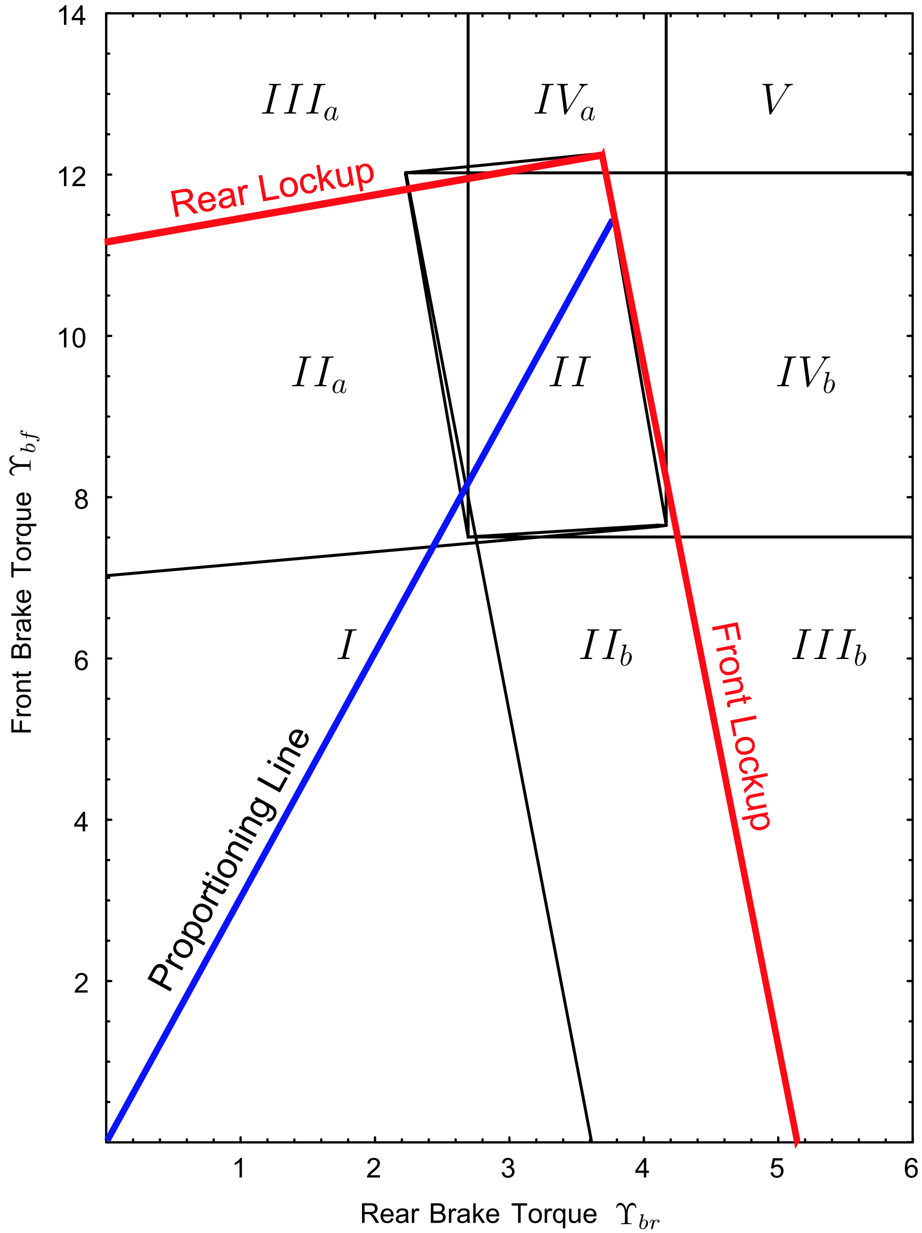




Brake Proportioning

$$h/l = 0.125$$
$$c/l = 0.6$$
$$b/l = 0.4$$
$$\theta = 0$$
$$\nu = 15$$



FRONT LOCKUP

Loss of Steerability

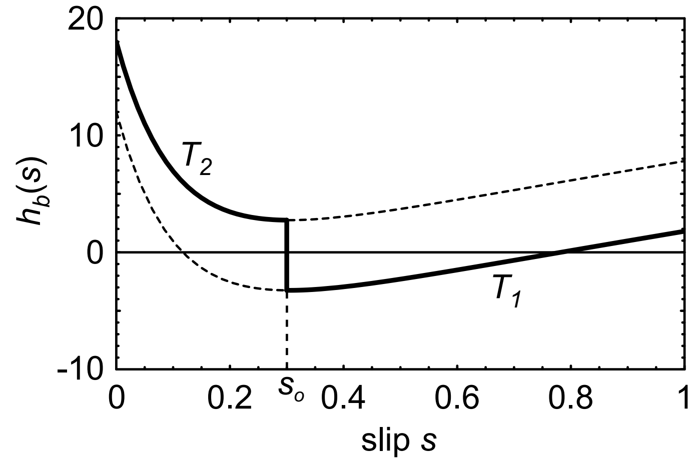
REAR LOCKUP

Yaw Instability

Bang-Bang Control

- Control Torque:

$$T_{bC}(s) = \begin{cases} T_2 & \text{if} \quad s < s_o \\ T_1 & \text{if} \quad s > s_o \end{cases}$$



where s_o is chosen near s_p.

- A more advanced strategy: Vary s_o s.t. $|\dot{u}|$ is maximized.

Brake Torque Dynamics

- Brake torque takes finite time to switch.

$$T_{bC} = \gamma P$$

where

$$\begin{aligned} \gamma &= \text{gain} \\ P &= \text{brake line pressure} \end{aligned}$$

- Pressure dynamics:

$$\dot{P} + \sigma P = w$$

where

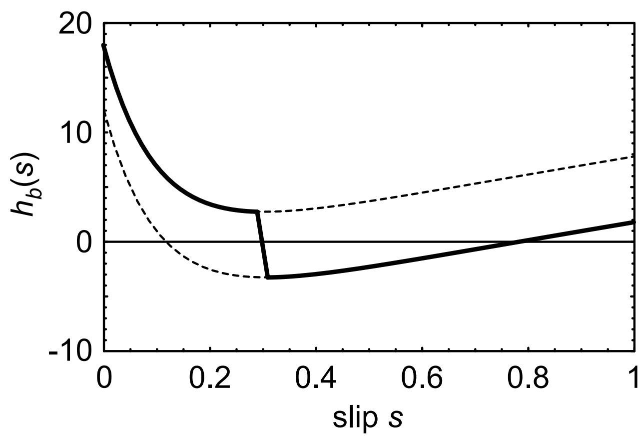
$$w = \text{command signal}$$

Backstepping

- Error between actual and desired pressure:

$$
\begin{aligned}
z &= P - \frac{T_{bC}(s)}{\gamma} \\
\dot{z} &= \dot{P} - \frac{1}{\gamma}\frac{\partial T_{bC}}{\partial s}\dot{s} \equiv f(z) \qquad (1)
\end{aligned}
$$

- T_{bC} must be smoothed.



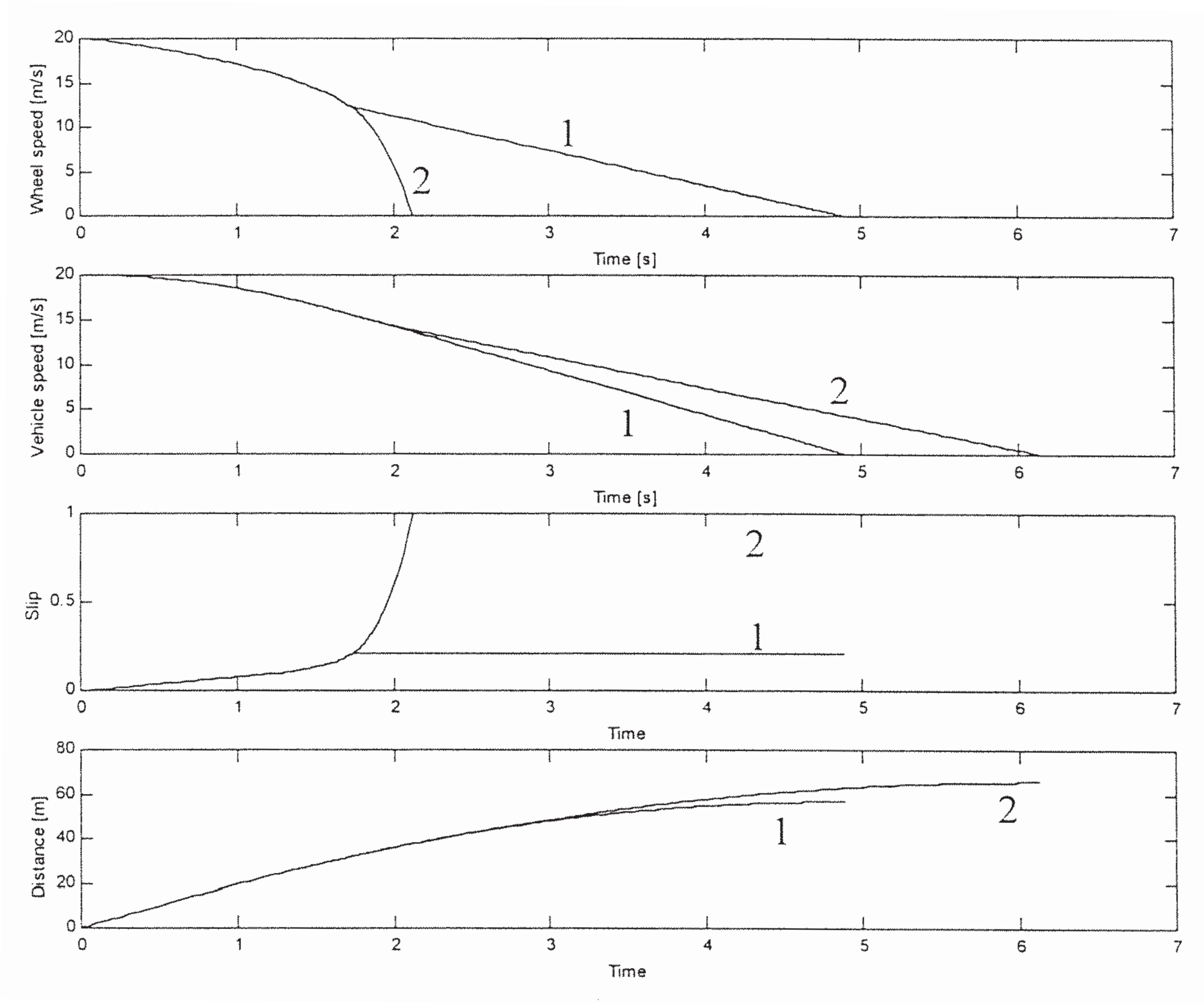
Backstepping (cont.)

- Select $\dot{z} = f(z)$ s.t. $z = 0$ is GAS, e.g., $\dot{z} = -\lambda z$, or $\dot{z} = -\beta\,\text{sgn}(z)$.
- Command Signal

$$w = \sigma P + \frac{g}{\gamma u}\frac{\partial T_{bC}}{\partial s} h_C(s) + f\left(P - \frac{T_{bC}(s)}{\gamma}\right)$$

- $s = s_o$ is GAS.
- s, P, and u must be measured.

Simulation Results



Conclusions

- Use of wheel slip s provides new insight into vehicle traction.
- Entire dynamics are captured by $h_b(s)$ (SWBM), $h_a(s)$ (SWAM), and $h_{bi}(\mathbf{s})$ (2WBM).
- New lockup threshold for SWBM.
- Complete description (stability and bifurcation) of 2WBM dynamics in terms of brake torques.
- Useful example for teaching nonlinear control methods.

Directions for Future Work

- Effects of system parameters and inclines;
- Extension of the approach to vehicle acceleration;
- Selection of brake proportioning schedules;
- Effects of cornering;
- Effects of rolling and air resistance on the dynamic model;
- Incorporation of these models into ABS/TCS development, where slip plays a central role.

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