MAE 5803 Nonlinear Control Systems **Assigned: Feb 28, 2017**
Homework #4 **Due: Mar 21, 2017**

Submit your answers to all questions below. Show your working steps in sufficient details.

1. (75 pts.)

A single-degree-of-freedom underwater vehicle motion under viscous drag can be modeled as:

$$m\ddot{x} + (\alpha_1 + \alpha_2 \cos^2 x)\,|\dot{x}|\,\dot{x} = u + d$$

where m is the vehicle mass, α_1 and α_2 are time varying coefficients, u is the control input, and d is a time-varying disturbance.

Assume the mass is known, m = 1. The coefficients α_1 and α_2 are not known exactly, but they are bounded by $4 \le \alpha_1 \le 6$ and $1 \le \alpha_2 \le 2$. The magnitude of the disturbance d is bounded by 1. Assume also there is an un-modeled dynamics in the system at the frequency of 4.2 rad/s.

a. Design a *switching controller* for the vehicle to track a desired trajectory $x_d(t)$. Select a set of parameter values that works for your controller. If necessary, briefly explain or justify your selection.

b. Simulate the responses of the system using the switching controller designed in a. for two different values of η (η = 1 and 10). Pick a value of λ that will not excite the high-frequency un-modeled dynamics. For each simulation, display time responses of x and x_d (in one plot), s, tracking error $\tilde{x}$, and control input u for 6 seconds. Do you observe chattering in the control responses? How does η affect the responses?

Note: In the simulation, use $x_d(t) = 2\sin t$. Vary the unknown parameters according to:

$$\alpha_1(t) = 5 + \cos t$$
$$\alpha_2(t) = 1 + |\sin 2t|$$
$$d(t) = \cos 1.3t$$

c. Design a *sliding controller* to eliminate chattering for this vehicle. Indicate the a set of parameter values that can work for your controller. Again, briefly explain or justify your selection when necessary.

d. Simulate the responses of the system using the sliding controller designed in c. for two λ values (choose any two that are “widely” separated). For each λ value selected, simulate the system responses for η = 1 and 10. For each simulation, display time responses of x and x_d (in one plot), s and ϕ (in one plot), tracking error $\tilde{x}$, and control u for 6 seconds. How do λ and η affect the responses of the system? Verify for each simulation whether the magnitude of the tracking error is within the bound predicted by theory.

e. Consider the case where the mass m varies with time. Assume that the mass variation in the operation range of the vehicle is given by $1 \le m \le 2$. Mo dify your sliding controller to make it robust to this mass variation. Indicate the justification for the parameter values that you choose for your controller.

f. Show that your controller work by simulating the responses as in part d. Use the following mass variation in your simulation:

$$m(t) = 2 - |\cos 1.5t|$$

2. (25 pts.)

For the following system:

$$\ddot{x} + \alpha_1(t)|x|\dot{x}^2 + \alpha_2(t)x^3\cos 2x = 5\dot{u} + u$$

where $\alpha_1(t)$ and $\alpha_2(t)$ are unknown time-varying functions with the known bounds

$$\forall t \geq 0, \quad |\alpha_1(t)| \leq 1 \qquad -1 \leq \alpha_2(t) \leq 5$$

a. Design a *switching controller* to track a desired trajectory $x_d(t)$. *Hint:* let $v = 5\dot{u} + u$ and perform the design based on v. Select a set of parameter values including your justification for the selection (when necessary).

b. Perform some simulations to demonstrate that your controller work. Pick your own time-varying functions to represent the desired trajectory to track $x_d(t)$ and the unknown parameters α_1 and α_2 in the simulations. Include plots of both v and u time histories.

c. Do you see chattering in v? Is the chattering observed in u? Is there chattering in the tracking responses of the system? Explain what you observe based on the relation between v and u.