MAE 5803 - Homework #1 Problem #2

Table of Contents

Tim Coon: 25, January 2017

```
clear; close all; clc;
```

Consider the following second-order system

$$\dot{x}_1 = \mu - x_1^2$$

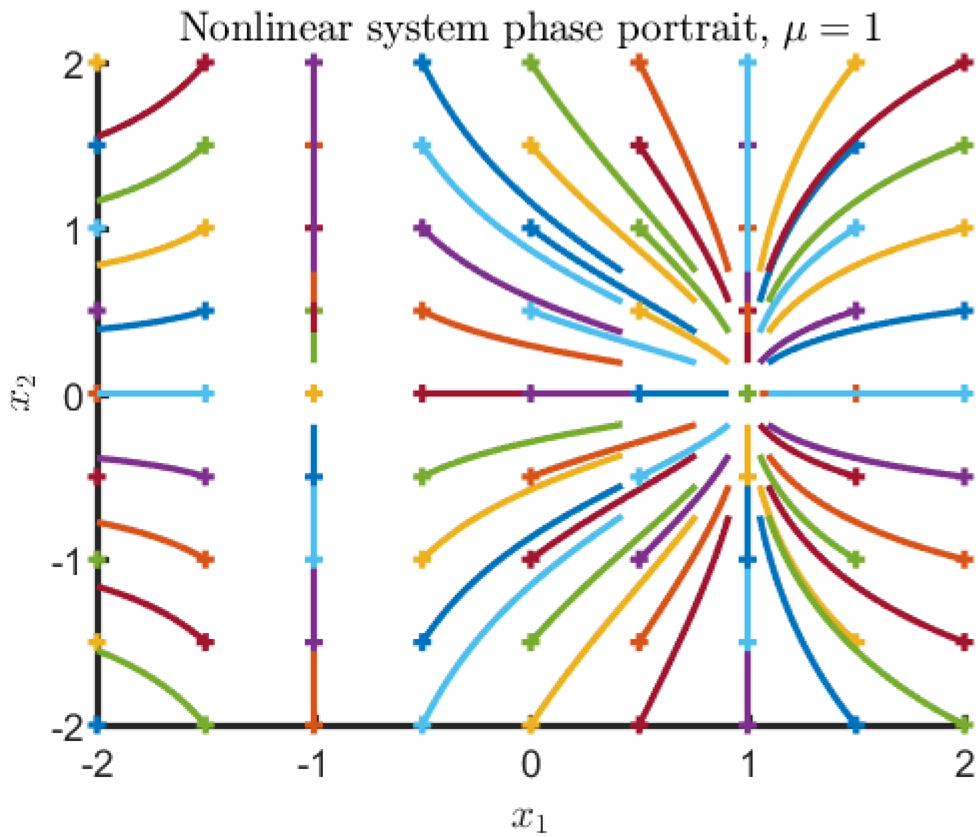
$$\dot{x}_2 = -x_2$$

a) Identify Singular points

For $\mu = 1$, find the singular points of the system, then determine the stability of the singular points by analyzing the linearized equation about each singular point. Generate the phase portrait of the system using MATLAB® to confirm your analysis. Frame your plot so that the horizontal and vertical axes range from -2 to 2.

```
mu = 1;
tspan = [0 1];
figure();
hold on
for x1 = -2:.5:2
    for x2 = -2:.5:2
        X0 = [x1; x2];
        [t,X] = ode45(@P2stateEqn,tspan,X0,[],mu);
        h = plot(X(:,1),X(:,2));
        c = get(h,'color');
        plot(X0(1),X0(2),'+','color',c);
```

```
    end
end
axis([-2 2 -2 2])
xlabel('$x_1$')
ylabel('$x_2$')
title('Nonlinear system phase portrait, $\mu = 1$')
hold off
```



First Singular Point

The first singular point is a stable node at (1,0). Use the Jacobian to linearize about this point. Both eigenvalues have negative real parts, supporting the ID as a stable focus.

$$A_1 = \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{\bar{x}=(1,0)}$$

$$\frac{\partial f_1}{\partial x_1} = -2x_1 \quad \frac{\partial f_1}{\partial x_2} = 0 \quad \frac{\partial f_2}{\partial x_1} = 0 \quad \frac{\partial f_2}{\partial x_2} = -1$$

$$A_1 = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$$

```
eValue1 = eig([-2 0; 0 -1])
```

```
eValue1 =

    -2
    -1
```

Second Singular Point

The second singular point is a saddle point at (-1,0). Use the Jacobian to linearize about this point. Both eigenvalues have negative real parts, supporting the ID as a stable focus.

$$A_2 = \frac{\partial \bar{f}}{\partial \bar{x}}\bigg|_{\bar{x}=(-1,0)}$$

$$A_2 = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

```
eValue2 = eig([2 0; 0 -1])
```

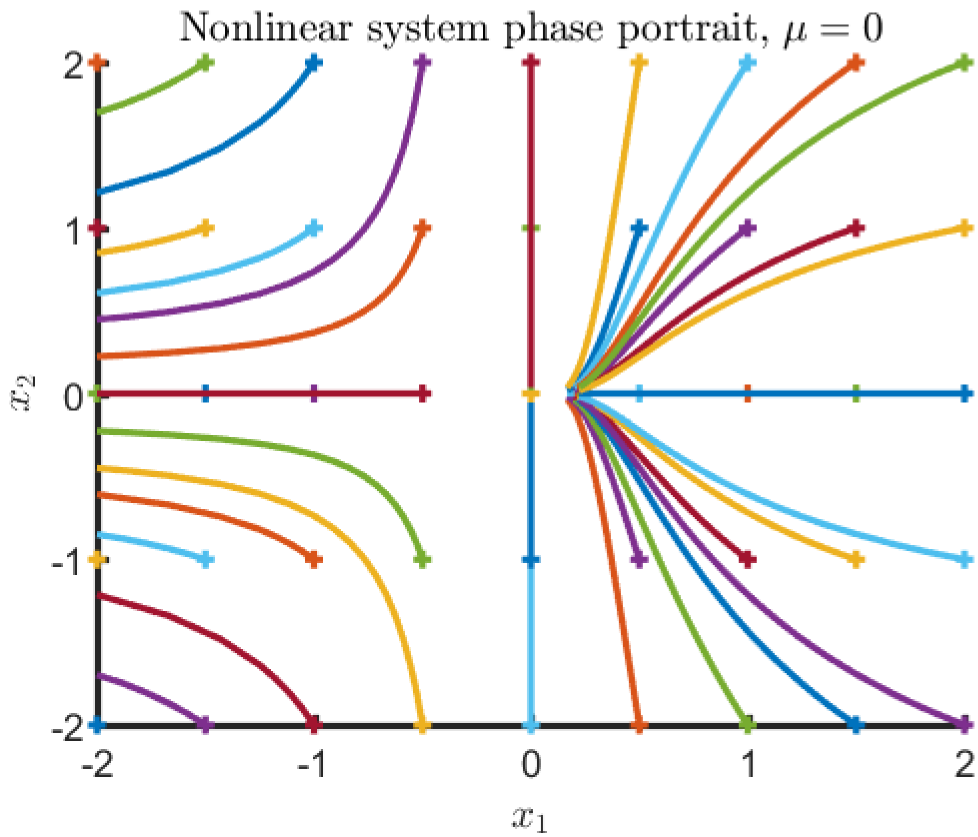
```
eValue2 =

    -1
     2
```

b) Let $\mu = 0$.

Repeat part (a) for $\mu = 0$.

```
mu = 0;
tspan = [0 4];
figure();
hold on
for x1 = -2:.5:2
    for x2 = -2:1:2
        X0 = [x1; x2];
        [t,X] = ode45(@P2stateEqn,tspan,X0,[],mu);
        h = plot(X(:,1),X(:,2));
        c = get(h,'color');
        plot(X0(1),X0(2),'+','color',c);
    end
end
axis([-2 2 -2 2])
xlabel('$x_1$')
ylabel('$x_2$')
title('Nonlinear system phase portrait, $\mu = 0$')
```

```
hold off
```



Singular Point, $\mu = 0$

The singular point at the origin is an unstable node. Use the Jacobian to linearize about this point. One eigenvalue at the origin of the complex plane with no negative eigenvalues means the stability of the system cannot be determined by the eigenvalues alone. From the phase portrait, it is clear any state in the right-half plane tends toward the origin. This would indicate stability were it mirrored by the left-half plane. However, any state in the left-hand plane escapes along the negative x_2 axis, so the node is unstable.

$$A_2 = \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{\bar{x}=(0,0)}$$

$$A_2 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

```
eValue1 = eig([0 0; 0 -1])
```

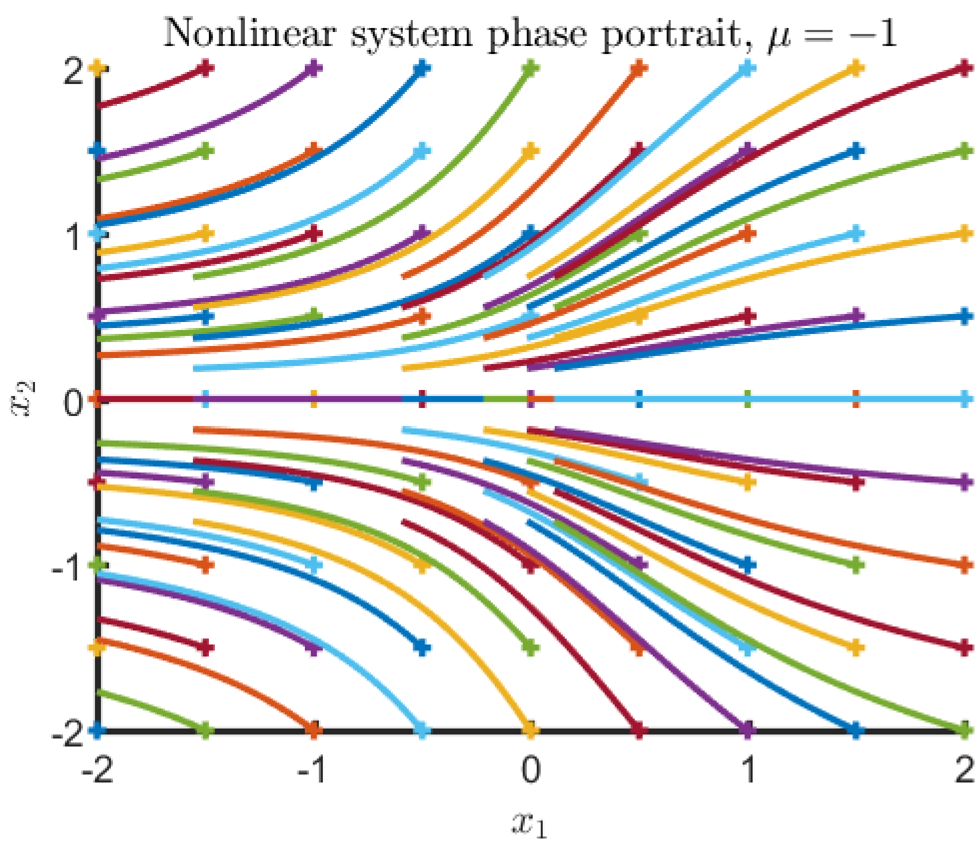
```
eValue1 =

    -1
     0
```

c) Let $\mu = -1$

Repeat again part (a) for $\mu = -1$.

The linearized systems look the same because μ only affects the forcing function

```
mu = -1;
tspan = [0 1];
figure();
hold on
for x1 = -2:.5:2
    for x2 = -2:.5:2
        X0 = [x1; x2];
        [t,X] = ode45(@P2stateEqn,tspan,X0,[],mu);
        h = plot(X(:,1),X(:,2));
        c = get(h,'color');
        plot(X0(1),X0(2),'+','color',c);
    end
end
axis([-2 2 -2 2])
xlabel('$x_1$')
ylabel('$x_2$')
title('Nonlinear system phase portrait, $\mu = -1$')
hold off
```



No Singular Points for $\mu = -1$

There are no singular points within the range $-2 \leq x_1, x_2 \leq 2$.

d) Comments

What phenomenon do you observe as the parameter, μ, varies as in the above? Explain the reason for your answer.

Because $\dot{x}_2 = -x_2$ always, it is clear the slope towards the horizontal axis decays exponentially in all scenarios. Effectively, μ changes the initial rate of change of the solution in the negative x_1-direction which becomes more negative as the function moves away from the vertical axis. Thus, decreasing μ serves to increase the initial rate of change in the negative x_1-direction and the singular points first merge at the origin, then move quickly in the negative x_1-direction together.

Published with MATLAB® R2016a