MAE 5803 Nonlinear Control Systems **Assigned: Feb 14, 2017**
Homework #3 **Due: Feb 23, 2017**

Submit your answers to all questions below. Show your working steps in sufficient details.

1. (20 pts.)
 Consider the system:
 $$\mathbf{A}_1\ddot{\mathbf{y}} + \mathbf{A}_2\dot{\mathbf{y}} + \mathbf{A}_3\mathbf{y} = \mathbf{0}$$
 where the $2n \times 1$ vector $\mathbf{x} = [\mathbf{y}^T \quad \dot{\mathbf{y}}^T]^T$ is the state vector, and the $n \times n$ matrices $\mathbf{A}_j$ are all *symmetric positive definite*. Show that:
 a. $\mathbf{0}$ is a *unique equilibrium point* of the system.
 b. the system is *globally asymptotically stable*.

2. (15 pts.)
 Find a *Lyapunov function* to describe the system:
 $$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}\mathbf{x}$$

3. (20 pts.)
 Determine the *stability* of the following systems. If it is stable, indicate whether the stability is asymptotic and whether it is global.
 a. $\dot{\mathbf{x}} = \begin{bmatrix} -1 & 2\sin t \\ 0 & -(t+1) \end{bmatrix}\mathbf{x}$
 b. $\dot{\mathbf{x}} = \begin{bmatrix} -1 & e^{2t} \\ 0 & -2 \end{bmatrix}\mathbf{x}$

4. (15 pts.)
 Consider a scalar, lower bounded, and twice differentiable function $V(t)$ for which
 $$\forall t \geq 0 \quad ; \quad \dot{V}(t) \leq 0$$
 Show that $\forall t \geq 0 :\ \dot{V}(t) = 0$ implies $\ddot{V}(t) = 0$

5. (30 pts.)
 The mathematical model of a nonlinear pendulum is given by:
 $$\ddot{x} - \omega_0^2 x \sin x = u$$
 where ω_0^2 is a positive constant smaller than 2 (not known exactly), and u is the control input.
 a. Using Lyapunov's direct method, design a controller that will make the equilibrium point at the origin *globally asymptotically stable*. Justify your design properly.

b. Simulate the time responses of the pendulum, $x(t)$, for a few cases (at least 3) with different ω_0^2 values within the specified bound. For each case, plot the responses with and without control.