MAE 5803 Nonlinear Control Systems **Assigned: Jan 19, 2017**
Homework #1 **Due: Jan 26, 2017**

Submit your answers to all questions below. Show your working steps in sufficient details

1. (25 pts.)
   Draw the *phase portrait* of the following system using MATLAB:
   $$\ddot{\theta}(t) + 0.6\,\dot{\theta}(t) + 3\theta(t) + \theta^2(t) = 0$$
   a. From the phase portrait, identify the *singular points* of the system and determine *their types* (stable node, unstable focus, etc.).
   b. Obtain *linearized equations* about the singular points of the system. Then determine eigenvalues of each linearized equation to determine the *stability* of the corresponding singular point.
   c. Draw also the *phase portraits* for the linearized equations. Does the phase portrait of the nonlinear system in the neighborhood of the singular points compare well with the phase portraits of the linearized equations?

2. (25 pts.)
   Consider the following second order system:
   $$\dot{x}_1 = \mu - x_1^2$$
   $$\dot{x}_2 = -x_2$$
   a. For $\mu = 1$, find the *singular points* of the system, then determine the *stability* of the singular points from analyzing the linearized equation about each singular point. Generate the *phase portrait* of the system using MATLAB to confirm your analysis. Frame your plot so that the horizontal and vertical axes range from −2 to 2.
   b. Repeat part a. for $\mu = 0$.
   c. Repeat again part a. for $\mu = -1$.
   d. What *phenomenon* do you observe as the parameter μ varies as in the above? Explain the reason for your answer.

3. (25 pts.)
   Consider a system described by:
   $$\dot{x}_1 = x_2$$
   $$\dot{x}_2 = -x_1 + (\mu - x_1^2)x_2$$
   a. Find the *eigenvalues* of the linearized system about the equilibrium point (0,0). Express your answer in terms of μ. Sketch in the *complex plane* the variation of the locations of these eigenvalues as μ varies from −0.5 to 0.5.
   b. Draw the *phase portraits* of the system using MATLAB for $\mu = -0.2$, $\mu = 0$, and $\mu = 0.2$. Use −2 to 2 range of values for the horizontal and vertical axes.
   c. What *phenomenon* do you observe as the parameter μ varies from negative to positive? Justify your answer using *Poincare-Bendixson theorem*.

4. (25 pts.)
   Consider the following *Van der Pol equation*:

$$\dot{x}_1 = -x_2$$
$$\dot{x}_2 = x_1 - (1 - x_1^2)x_2$$

   a. Draw the *phase portrait* of the system about the equilibrium point $(x_1, x_2) = (0, 0)$. Be sure to include a sufficiently wide area of the state space to capture the possible limit cycle in the system.
   b. Is the limit cycle *stable*? Explain your answer.
   c. Determine the stability of the *equilibrium point* at the origin (be specific on the type of stability). Determine its *region of attraction* if it is asymptotically stable.