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Target Slip Tracking Using Gain-Scheduling for Antilock Braking Systems

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Abstract

In this paper, a gain-scheduling scheme is proposed for optimal target slip tracking of an antilock braking system. The study is based on the fact that for certain road surface condition, the tire force characterization, i.e. the force-slip curve, is indeed varying with respect to different vehicle forward speed. The optimal slip at which the braking force achieves its maximum is not, in contrast to what most slip-regulator design is based upon, a constant. This paper proposes a new ABS controller which regulates the slip to different levels depending on the instant vehicle forward speed along the braking process using gain-scheduling. The control algorithm, coupled with a feedback linearization compensation outer loop, is applied to a quarter-car ABS model. Simulations are performed using MATLAB/SIMULINK. It is shown that the gain-scheduling slip-regulator improves the vehicle braking performance as compared to constant slip-regulators.

1. Introduction

Antilock braking systems can significantly improve the vehicle handling and safety performance. A complete ABS system is a highly nonlinear system which includes various mechanical, hydraulic and electric components. The task of control design for an ABS system is to adjust the caliper pressure adequately so that the wheels are prevented from locking and the directional stability is maintained while at the same time a short braking distance is achieved. Various intuitive tuning methods and simple control algorithms have been used in the industry to design ABS controllers [5, 4, 6].

Generally, the slip-friction force characterization of tires serves as a guideline for ABS controller design in such a way that an ABS system is kept working within or close to the optimal (maximal) friction force range. The slip-friction characterization is a curve obtained mostly from test data to best match the actual friction force between the road surface and the tires of a motor vehicle. The study of the tire force versus the vehicle slip coefficient relation can be traced back to more than two decades ago [2] and most early works on tire force modeling are based on collections of test data. In recent years, there have been some analytic way to generate the slip-friction curve [1]. In general, the shape of the slip-friction curve is affected by several factors. In particular, vehicle forward speed and road surface conditions are two most important factors that must be taken into consideration in tire force modeling.

Commonly, an ABS controller is designed to maintain the vehicle slip at certain level to achieve maximal friction force without losing vehicle stability. Various control algorithms have been used to design such slip-regulators. [5] proposes to use feedback linearization of nonlinearity together with a high-gain linear model following feedback loop in ABS controller designs. An ABS controller based on constant target slip regulation has its limitation because the actual tire force model is vehicle speed-dependent. As a motor vehicle decelerates during braking, the slip-friction characteristics change in its peak values (sometime sharply) as well as in the optimal slips where the peak values occur. This in turn makes the constant target slip regulating inadequate for maximizing the braking effort consistently. A figure shown in the appendix is the resemblance of a set of slip-friction curves taken from [2] for different vehicle speeds at 20, 40, and 60 mph. Obviously, if the optimal slip for 40 mph curve is chosen as the desired target slip during the whole braking process from 60 mph to complete stop, the vehicle is unstable at high speed range and underbraking at low speed range.

In this paper, a gain-scheduling scheme is used to design an ABS controller which, at ideal operating condition, regulates the target slip to the respective optimal values at different vehicle speed. The controller is divided into two loops. The inner loop is a local feedback which is basically a standard slip regulator with a given target slip. The outer loop is a high-level gain-scheduling scheme which adjust both target slip and control signals adaptively according to measured vehicle speed. In the next section, a simplified quarter car model is provided and the simulation setup is described in details. The control algorithms and the simulation results are provided and compared in Section 3. Evaluations and some concluding remarks are given in Section 4.

2. Quarter-Car Model

The model used in this paper is extremely simple, yet very useful in preliminary evaluation of ABS performance. It serves as basis in providing some guidelines for the more complete ABS control design. The vehicle dynamics included in the models are very simple with emphasis on different tire models. Several assumptions are made in the model development. First, the model only includes the longitudinal dynamics of the vehicle. Second, since this is a quarter-car model, no interaction between the four wheels are taken into consideration.

2.1. Nomenclature

M	mass of vehicle
I_ω	wheel moment of inertia about axes
R_e	effective rolling radius
v	longitudinal vehicle velocity
ω	angular wheel velocity
κ	wheel slip coefficient
μ	tire friction coefficient
F_z	normal tire force
F_x	longitudinal tire force
F_{ax}	longitudinal aerodynamic force
T_d	driving torque

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T_b	braking torque
A	effective front vehicle area
ρ_0	air density
C_x	aerodynamic force coefficient
A_{ax}	longitudinal drag coefficient

2.2. Simplified Quarter-Car Model

The simplified model of a quarter-car model consists of vehicle dynamics and wheel dynamics. The longitudinal vehicle dynamics is simply described by

$$M\dot{v} = 4F_x - F_{ax}. \tag{1}$$

The wheel relational dynamics the axle is described by

$$I_\omega \dot{\omega} = T_d - \operatorname{sgn}(\omega) T_b - F_x R_e. \tag{2}$$

. In those two equations, M and I_ω are the mass of the vehicle and the moment of inertia of the wheel respectively, v and ω are the vehicle forward speed and the wheel angular velocity respectively, F_x and F_{ax} are the tire force and aerodynamic drag force, T_b and T_d are the braking and driving torque, and finally R_e is the effective rolling radius. The drag force is calculated from a simplified aerodynamics law as

$$F_{ax} = A_{ax} v^2$$

where $A_{ax} = \frac{1}{2}\rho A C_x$ with ρ being the air density, A the longitudinal effective area, and C_x the longitudinal aerodynamic force coefficient.

2.3. Tire Model

The braking effect comes from the friction force F_x between the tires and the road surface, therefore, tire model is one of the most important aspects of the ABS model. In general, F_x is affected by various factors. In [1], a full tire model including longitudinal and lateral forces as well as self-aligning moment is described. In particular, the longitudinal tire force is given as

$$F_x = f(\kappa, F_z, \alpha, \gamma, \mu)$$

where F_z is the normal force on the tire, α is the slip angle (zero as there is no turning), γ is the camper angle (zero in our model), and μ is the road friction coefficient. The longitudinal slip κ is defined as

$$\kappa = \frac{R_e \omega - v}{v}$$

and is a measure of the relative motion between the tire and road surface. In a simplified description of vehicle braking process on a homogeneous road surface, the Pacejka tire model assumes that the tire friction coefficient μ is solely determined by wheel slip κ and the friction force is usually characterized by the so-called slip-friction curve.

The slip-friction relation is mainly obtained from road test [2]. [1] also provides an analytic method to calculate the slip-friction relation based on tire dynamics analysis and validation with test data. Although the Pacejka tire model is widely used in the automotive industry, its lack of speed-dependency may have limited its accuracy in predicting the ABS performance using computer simulation.

The overall ABS system (with Pacejka tire model) is modeled using both MATRIX-X on SPARC II and MATLAB/SIMULINK on 486PC. The illustrative block-diagram is omitted in here.

3. Controller Design

Several factors need to be taken into consideration when the slip-friction curve is used in the ABS controller design. First, it should be understood that these curves only represent the static characterization of a tire model since most tire force test data are obtained at constant vehicle speed. The transient slip-friction relation which characterize the tire force during a real braking process is discussed in [7]. Second, since there is an inevitable gap between the tire model used in simulation and reality, the controller design based on the simplified Pacejka tire model is in a way not reflective to what an improved controller can really achieve. In this paper, the focus of the controller design is on exploiting the possibility of improving braking performance by employing a gain scheduling nonlinear controller.

The first step to accomplish the controller design is to construct standard slip regulator. For this purpose, it is convenient to work with the slip dynamics derived from the vehicle and wheel dynamic equations. From the definition of the slip, the slip dynamics can be described as

$$\dot{\kappa} = R_e \frac{\dot{\omega}}{v} - (1+\kappa)\frac{\dot{v}}{v} \tag{3}$$

Substituting the wheel dynamics (2) into Eq. (3), one obtains

$$\dot{\kappa} = R_e \frac{-F_x R_e - \operatorname{sgn}(\omega) T_b}{I_\omega v} - (1+\kappa)\frac{\dot{v}}{v}$$

which, together with

$$\dot{v} = -\frac{A_{ax}}{M} v^2 + \frac{4F_x}{M}$$

characterizes the vehicle and wheel dynamics in terms of state variable κ and v. Notice that $F_x = F_x(\kappa, v)$. The state-space equations of the model becomes

$$\begin{aligned} \dot{\kappa} &= R_e \frac{-F_x R_e - \operatorname{sgn}(\omega) T_b}{I_\omega v} - (1+\kappa)\frac{4F_x - A_{ax}}{M v} && (4) \\ \dot{v} &= -\frac{A_{ax}}{M} v^2 + \frac{4F_x}{M} && (5) \end{aligned}$$

with $\omega = (\kappa+1)v/R_e$, T_b being the control variable, and $F_x = F_x(\kappa, v)$ given by the slip-friction curves. Eqs. (4) and (5) can be written more compactly as

$$\begin{aligned} \dot{\kappa} &= f_1(\kappa, v) + g_1(\kappa, v) T_b \\ \dot{v} &= f_2(\kappa, v) + g_2(\kappa, v) T_b \end{aligned}$$

where

$$\begin{aligned} f_1 &= -\frac{R_e^2 F_x}{I_\omega v} + \frac{(1+\kappa)(-4F_x + A_{ax} v^2)}{M v} \\ g_1 &= -\frac{R_e \operatorname{sgn}(\omega)}{I_\omega v} \\ f_2 &= \frac{4F_x - A_{ax} v^2}{M} \\ g_2 &= 0. \end{aligned}$$

Using a global feedback linearization control law $T_b = (-f_1 + T_b')/g_1$, i.e.

$$T_b = \frac{\operatorname{sgn}(\omega)}{M R_e}((1+\kappa)(A_{ax} v^2 - 4F_x) I_\omega - F_x M R_e^2 - M I_\omega v T_b')$$

where T_b' is the new control variable to be designed, the closed-loop system dynamics becomes

$$\begin{aligned} \dot{\kappa} &= T_b' \\ \dot{v} &= \frac{1}{M}(4F_x - A_{ax} v^2). \end{aligned}$$

Since $A_{ax} \geq 0$, $F_x \leq 0$ and $v \geq 0$ based on the practical situation in the braking process, a simple Lyapunov argument shows that the system can be stabilized at the equilibrium point $(\kappa_T, 0)$ with

$$T_b' = -\gamma(\kappa - \kappa_T).$$

Note that theoretically, the slip-friction curve does not effect the slip dynamics when such a nonlinear control law is employed.

A similar procedure leads to a different representation of the quarter-car dynamics using κ and ω as state variables. With the definition of κ and the wheel dynamics equation, the new representation is

$$\begin{aligned} \dot{\kappa} &= f_1(\kappa, \omega) + g_1(\kappa, \omega) T_b \\ \dot{\omega} &= f_2(\kappa, \omega) + g_2(\kappa, \omega) T_b \end{aligned}$$

The feedback linearization control law is now chosen as

$$\begin{aligned} T_b = & -4F_x I_\omega (1+\kappa)^2 - F_x M R_e^2 (1+\kappa) \\ & + A_{ax} I_\omega R_e^2 \omega^2 - I_\omega M R_e \omega T_b' (1+\kappa) M R_e \operatorname{sgn}(\omega) \end{aligned}$$

where T_b' is again a new control variable. The closed-loop system dynamics becomes

$$\begin{aligned} \dot{\kappa} &= T_b' \\ \dot{\omega} &= -\frac{F_x R_e}{I_\omega} - \frac{\operatorname{sgn}(\omega)}{I_\omega} T_b. \end{aligned}$$

When T_b is substituted into the second equation, the closed-loop system dynamics becomes rather complicated. Because of this complication, the state-space model in terms of κ and v is used for controller design purpose throughout this paper.

3.1. Control with Pacejka Tire Model

Consider the closed-loop system given by Eqs. (6) and (6). For a Pacejka tire model, the friction force is described as velocity-independent. In order to achieve the maximal braking effort, a slip-regulator is designed to keep the slip at its optimal point so that the friction is maintained maximal constantly. Given the tire friction coefficient μ, the longitudinal tire force can be calculated as

$$F_x = \mu \times F_z.$$

The optimal slips which yield maximal tire force on different road surfaces are shown in Table 3.1.. As can be easily

κ	Road 1	Road 2	Road 3	Road 4
−0.1400	−0.2287	−0.4575	−0.6862	−0.9149
−0.1500	−0.2288	−0.4575	−0.6863	−0.9150
−0.1600	−0.2284	−0.4568	−0.6852	−0.9136

Table 1: Optimal slips ranges for different road surfaces

seen, the optimal slip is around −0.15. Now if this is used as the target slip in the slip-regulator design and the inner-loop controller is set to be a high-gain linear feedback

$$T_b' = -\gamma(\kappa - \kappa_T)$$

the closed-loop system becomes

$$\begin{aligned} \dot{\kappa} &= -\gamma(\kappa - \kappa_T) \\ \dot{v} &= \frac{1}{M}(4F_x - A_{ax} v^2) \end{aligned}$$

Simulation is performed using the MATLAB/SIMULINK ABS model with Pacejka tire model. A set of the physical parameters of a passenger car is used and the initial speed is $v = 30$ m/sec. Figures given in the appendix shows the comparison results when different target slip κ_T's (constant) are used. The study on the slip, vehicle speed, wheel speed, and stopping distance profiles shows very close results. This implies that choosing an optimal target slip with great accuracy may not be very critical with a speed-independent tire model.

3.2. Control with Speed-Dependent Tire Model

Although braking performance is not sensitive to the variations in target slips with the Pacejka tire model, this is not the case with a *speed-dependent* tire model. This is because in the speed-dependent tire model the magnitude of the tire force may vary sharply even with slight variation in slip values. In this section, a new ABS model with speed-dependent slip-friction characterization in the tire model is constructed first. Since there is no such model available to the authors, an artificially constructed slip-friction profile which resemble, to some extent, the test results reported in [2] is obtained by modifying the calculation algorithm provided in the Pacejka tire modeling procedure [1]. Figure provided in the appendix shows three slip-friction curves corresponding to the vehicle speed at 20, 40 and 60 mph. It is obvious that the lower the velocity, the larger the peak friction force. It is also clear that the variations between the optimal slip values at different velocities are relatively small.

The ABS model is set up as follows. The tire force F_x in the block-diagram is evaluated on-line depending on both the vehicle speed and and tire slip. Different from the Pacejka tire model based ABS control (slip-regulating), the target slip in this case is changing on-line and the control algorithm becomes a switching feedback. To be more specific, the control law is chosen as a switching feedback law

$$T_b = \frac{1}{g_1(\kappa, v)}(-f_1(\kappa, v) - \gamma(\kappa - \kappa_T(t))$$

where $\kappa_T(t)$ is now *time-varying*, and the closed-loop system becomes

$$\begin{aligned} \dot{\kappa} &= \gamma(\kappa - \kappa_T(t)) \\ \dot{v} &= \frac{1}{M}(4F_x(t) - A_{ax} v^2) \end{aligned}$$

The time history of the system dynamics can be divided into three stages with different speed ranges. For the given curves at 60, 40 and 30 mph, the switchings are set to take place at 50 and 30 mph respectively. This gives a threshold of ±10 mph for each slip-friction curve. Table 3.2. shows the optimal slips at different vehicle speed obtained from the speed-dependent slip-friction curves. The target slips at which the tire forces achieve their maximum for different vehicle speed are −0.0367, −0.0700 and −0.1067 respectively. In real simulation, the slip-friction data is stored in a table. Two search algorithms are used for obtaining the optimal target slip for the current vehicle speed range and the instantaneous friction coefficient μ (and consequently the tire force) at given vehicle speed and actual wheel slip. The switching control strategy is described in Table 3.2.. It is noted that the closed-loop system dynamics at each vehicle speed range is stable in the sense of Lyapunov (with $v > 0$). Based on the results in [3] that the overall closed-loop system with the proposed slip-tracking switching (gain-scheduling) control is also stable in the sense of Lyapunov (with $v > 0$). For comparison purpose, simulation results with different controller setup are plotted together. The simulation for braking distance, slip coefficient, vehicle speed and wheel speed are shown in the appendix. In each plot, the first curve is the result with the proposed switching (gain-scheduling) control. The rest three curves are from the cases

κ	$v = 60$ mph	$v = 40$ mph	$v = 20$ mph
...	...	...	...
-0.0333	-0.2556	-0.4586	-0.5752
-0.0367	-0.2557	-0.4719	-0.6050
-0.0400	-0.2551	-0.4823	-0.6307
...	...	...	...
-0.0667	-0.2431	-0.5113	-0.7390
-0.0700	-0.2413	-0.5115	-0.7450
-0.0733	-0.2396	-0.5114	-0.7500
...	...	...	...
-0.1033	-0.2251	-0.5015	-0.7672
-0.1067	-0.2237	-0.4999	-0.7673
-0.1100	-0.2223	-0.4982	-0.7671
...	...	...	...

Table 2: Optimal slips ranges at different vehicle speeds

v (mph)	$v_0 \geq v > 50$	$50 \geq v > 30$	$30 \geq v > 0$
κ_T	-0.0367	-0.0700	-0.1067
μ_{max}	-0.2557	-0.5115	-0.7673

Table 3: Feedback law and corresponding tire force switching rules

where the controllers are chosen to be non-switching with fixed target slips which are locally optimal on slip-friction curves at different vehicle speed. The tire forces are calculated on-line (switching).

It should be understood that two types of switching exist in the proposed gain-scheduling ABS control setup. The feedback law (target slip) switching and tire force switching. The plots shown are all based on the speed-dependent tire model. Therefore the tire force switching occurs implicitly. For the effect of the gain-scheduling on ABS control performance, however, it can be seen that the results with gain-scheduling are improved over those without it by about 8%. Comparisons with results based on speed-independent tire model and non-switching feedback are, although expected to be significant, not provided here. The gap should reflect the effect of modeling error and consequently the difference between simulation and reality.

It should also be understood that the speed-dependent tire model is artificially constructed with close resemblance to some of the available data. The variation in this model will definitely affect the results obtained in this paper.

4. Conclusion

This paper considers a gain-scheduling control algorithm for a quarter-car model antilock brake system. The motivation for using the switching controller instead of constant slip regulator comes from the fact that the actual slip-friction characterization of a tire model is vehicle-speed dependent. At different vehicle speed range along the braking process, the optimal slip at which the maximal tire force (braking) occurs, as well as the maximal tire force is also different. Therefore using the Pacejka tire model (speed-independent slip-friction characterization) in a real ABS controller design not only sacrifices the accuracy of the model, but also limits the possible improvement of ABS controller design. The simulation results shown in the paper provides us some evidence that a speed-dependent tire model and a corresponding switching controller may have the potential to further improve the performance of the over-all ABS.

Two remarks need to be made on the results of this paper. First, the speed-dependent tire model used in the study is artificially created, rather than taken from any literature. To this end, the study for such a tire model is much needed. Second, the ABS hardware model is not included in this preliminary study. To apply similar control algorithms to the ABS model with the ABS hardware is currently underway.

References

[1] E. Bakker, H. B. Pacejka, and L. Lidner. A new tire model with an application in vehicle dynamics studies. In *SAE Technical Paper*, No. 890087 1989.

[2] J. L. Harned, L. E. Johnston, and G. Scharpf. Measurement of tire brake force characteristics as related to wheel slip (antilock control system design. In *SAE Technical Paper*, No. 690214 1969.

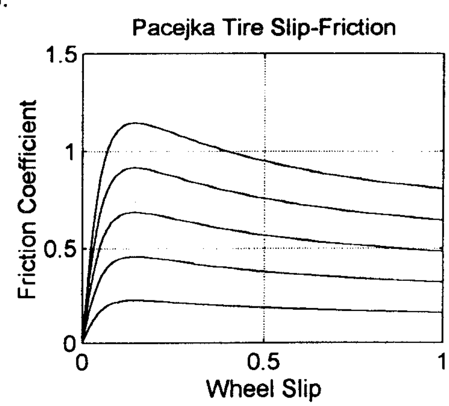
[3] Y. Liu and W. B. Gao. Switching stabilization control for a set of time-varying nonlinear systems. In *IFAC Nonlinear Control System Design Symposium*, Capri, Italy, 1989.

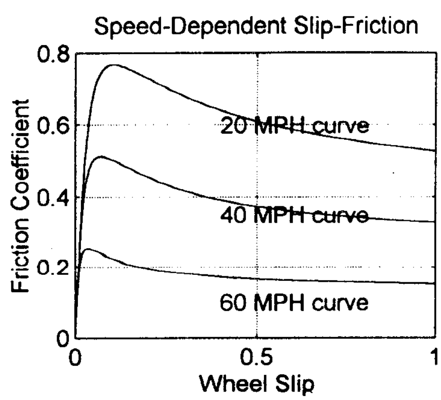
[4] M. Ohba, H. Kuraoka, N. Matsumoto, N. Ohka, and T. Tabe. Automotive antiskid system based on modern control theory. In *Conference on Applied Motion Control*, pages 253–257, 1986.

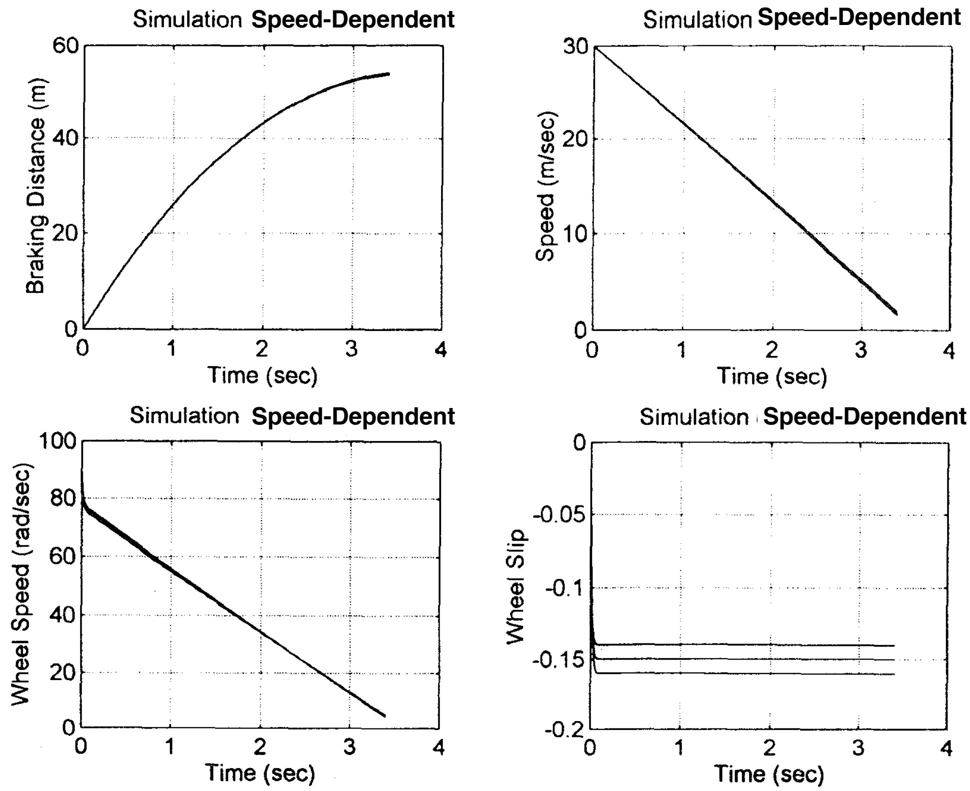
[5] J. Sun. Controller and speed estimator design for an antilock braking system. In *Technical Report, Wayne State University*, 1993.

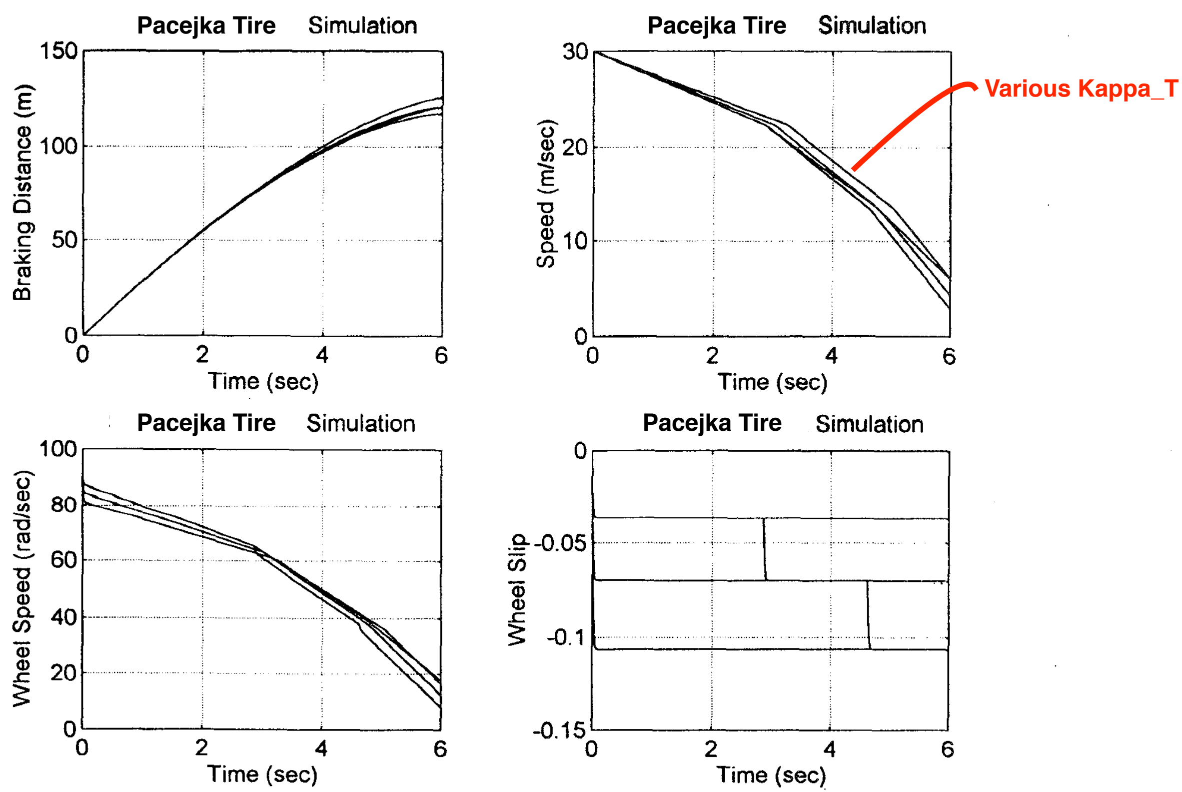
[6] H. S. Tan and M. Tomizuka. Discrete-time controller design for robust vehicle traction. In *American Control Conference*, Pittsburg, PA, 1989.

[7] A. van Zanten, R. Erhardt, and A. Lutz. Measurement and simulation of transients in longitudinal and lateral tire forces. In *SAE Technical Paper Series 900210*, Detroit, MI, 1990.






Simulation Speed-Dependent
Braking Distance (m)
Time (sec)
Simulation Speed-Dependent
Speed (m/sec)
Time (sec)
Simulation Speed-Dependent
Wheel Speed (rad/sec)
Time (sec)
Simulation Speed-Dependent
Wheel Slip
Time (sec)


Pacejka Tire Simulation
Braking Distance (m)
Time (sec)
Pacejka Tire Simulation
Speed (m/sec)
Time (sec)
Various Kappa_T
Pacejka Tire Simulation
Wheel Speed (rad/sec)
Time (sec)
Pacejka Tire Simulation
Wheel Slip
Time (sec)