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Integral Nested Sliding Mode Control for Antilock Brake System ⋆

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Abstract: An integral nested Sliding Mode (SM) Control is proposed for an Antilock Brake System (ABS) control problem by employing integral SM and nested SM concepts. This controller has robustness against matched and unmatched perturbations, and the capability to reduce the sliding functions gains. Application to an ABS is presented as a simulation example.

Keywords: Brake Control, Antilock Braking Systems (ABS), Sliding Mode Control, Automotive Control, Integral Controller

1. INTRODUCTION

The ABS control is a very important problem and its objective is to obtain desired vehicle motion providing adequate vehicle stability. The main difficulty arising in the ABS design is due to the high nonlinearity and uncertainty in the problem. This problem has been widely studied and various controllers have been proposed using SM technique (see Tan and Chin [1991], Drakunov et al. [1995], Unsal and Pushkin [1999], Hadri et al. [2002], Ming-Chin and Ming-Chang [2003]). In this work we design a new controller on the basis of integral Sliding Mode (SM) (Utkin et al. [2009]) in combination with nested SM (see González Jiménez and Loukianov [2008] and Huerta et al. [2008]) in order to achieve robustness to matched, and unmatched perturbations, and ensure output tracking. Theorically, this integral nested SM control can guarantee the robustness of the system throughout the entire response starting from the initial time instance and reduce the sliding functions gains in comparison with standard SM. In spite of the mentioned above works we consider the real situation: the control input can take only two values "0" or "1" depending on the corresponding valve being open or closed.

The work is organized as follows. The mathematical model for the longitudinal movement of a vehicle, including the brake system is presented in Section 2. In Section 3 a SM controller for ABS is designed. The simulation results are presented in Section 4 to verify the robustness and performance of the proposed control strategy. Finally, some conclusions are presented in Section 5.

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2. MATHEMATICAL MODEL

In this section, the model of a pneumatic brake system is under consideration. The specific configuration of this system includes the next: brake disks, which hold the wheels, as a result of the increment of the air pressure in the brake cylinder (Fig. 1).

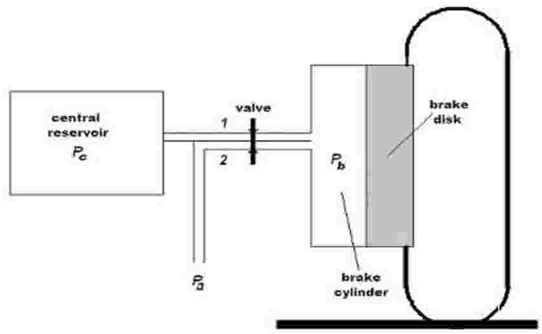

Fig. 1. Pneumatic brake model

The entrance of the air trough the pipes from the central reservoir and the expulsion from the brake cylinder to the atmosphere is regulated by a common valve. This valve allows only one pipe to be open, when 1 is open 2 is closed and vice versa. The time response of the valve is considered small, compared with the time constant of the pneumatic systems.

We study the task of control of the wheels rotation, such that the longitudinal force, due to the contact of the wheel with the road, is near from the maximum value in the period of time valid for the model. This effect is reached as a result of the ABS valve throttling.

2.1 Wheel motion equations

To describe the wheels motion we will use a partial mathematical model of the dynamic system (Novozhilov et al. [2000], Kruchinin et al. [2001] and Magomedov et al. [2001]), the dynamics of the angular momentum change relative to the rotation axis are given by

$$I_y \frac{d\Omega_y}{dt} = FR - L \tag{1}$$

where Ω_y is the wheel angular velocity, I_y is the wheel inertia moment, R is the wheel radius, F is the contact force and L is the brake torque.

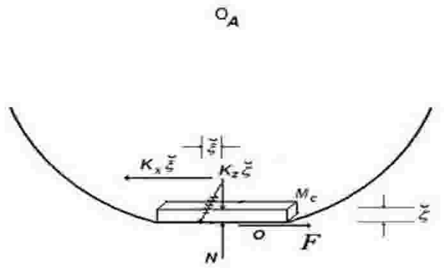

Fig. 2. Model for the contact element of the tire.

The expression for longitudinal component of the contact force in the motion plane according to experimental results (Pacejka [1981]) is equal

$$F = \nu N \phi(s) \tag{2}$$

where ν is the friction coefficient between the wheel and the road, N is the normal reaction and s is the slip rate

$$s = \frac{V_x - \Omega_y R - \frac{d\hat{\xi}}{dt}}{V_x} \tag{3}$$

V_x is the longitudinal velocity of the wheel mass center and $\hat{\xi}$ is the longitudinal deformation of the tire contact area element.
In (2), the function $\phi(s)$ represents a friction/slip characteristic relation between the tyre and road surface. Here, we use the Pacejka model (Bakker et al. [1989]), defined as follows

$$\phi(s) = D \sin \left(C \arctan \left(Bs - E \left(Bs - \arctan (Bs)\right)\right)\right). \tag{4}$$

In general, this model produces a good approximation of the tyre/road friction interface. With the following parameters $B = 10$, $C = 1.9$, $D = 1$ and $E = 0.97$ that function represents the friction relation under a dry surface condition. A plot of this function is shown in Fig. 3.

The motion equation of the contact element with mass M_c is described by the tire longitudinal deformation. The interaction between this element and the rigid part of the wheel can be described by the following viscoelastic forces model

$$M_c \frac{d}{dt}\left(V_x - \Omega_y R + \frac{d}{dt}\hat{\xi}\right) = F - c_x \frac{d}{dt}\hat{\xi} - k_x \hat{\xi} \tag{5}$$

where c_x and k_x are the longitudinal constants of viscous and elastic behavior of tire model, respectively. The model (5) to be used is similar to the description of the first waveform model in (Jansen et al. [1998]).

The equations (1)-(5) characterize the wheel motion.

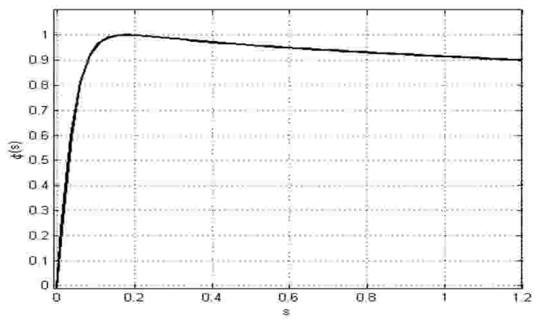

Fig. 3. Characteristic function $\phi(s)$

2.2 Pneumatic brake system equations

We suppose that the brake torque L is proportional to the pressure P_m in the brake cylinder:

$$L = k_L P_m \tag{6}$$

with $k_L > 0$.

For the brake system we use an approximated model of pressure changes in the brake cylinder due to the opening of the valve with a first order relation (Clover and Bernard [1998]):

$$T_e \frac{dP_m}{dt} + P_m = P_* \tag{7}$$

where P_* is the valve input signal.

We suppose that opening and closing of the valves are momentary and the parameters of the equation (7) are given by the following rules:

- If $P_* = P_c = const$ then $T_e = T_{in}$
- If $P_* = P_a = 0$ then $T_e = T_{out}$

where P_c is the pressure inside the central reservoir, P_a is the atmospheric pressure, that we consider equal to zero; T_{in} and T_{out} are the time constants of internal and external pipelines, respectively.

2.3 The vehicle motion equation

The vehicle longitudinal motion dynamics are represented by

$$M \frac{dV_x}{dt} = P_x - F_{ax} + M V_y \Omega_z.$$

Assuming that there is no the lateral motion, $V_y = 0$, the last equation evolves in

$$M \frac{dV_x}{dt} = P_x - F_{ax} \tag{8}$$

where M is the vehicle mass; F_{ax} is the aerodynamic drag force, which is proportional to the vehicle velocity and is defined as

$$F_{ax} = \frac{1}{2}\rho C_d A_f \left(V_x + V_{wind}\right)^2$$

where ρ is the air density, C_d is the aerodynamic coefficient, A_f is the frontal area of vehicle, V_{wind} is the wind velocity; the contact force P_x is modeled of the form

$$P_x = -\nu N \varphi(s)$$

The dynamic equations of the whole system (1)-(8) can be rewritten in a more useful form

$$\begin{aligned} & I_y \frac{d\Omega_y}{dt} = -\nu N R \phi(s) - L \\ & M_c \frac{d^2\hat{\xi}}{dt^2} + c_x \frac{d\hat{\xi}}{dT} + k_x \hat{\xi} = \\ & \quad -\frac{M_c R}{I_y} L - \left(\frac{M_c R^2}{I_y} + 1\right) \nu N \phi(s) \\ & T_e \frac{dL}{dt} - k_L P_* + L = 0 \end{aligned} \tag{9}$$

with

$$s = 1 - \Omega_y \frac{R}{V_x} - \frac{1}{V_x}\frac{d\hat{\xi}}{dt}. \tag{10}$$

3. INTEGRAL NESTED SM CONTROL FOR ABS

To design an integral nested SM control for (9) and (10), the system is represented using the state variables

$$x = \left[x_1, x_2, x_3, x_4, x_5\right]^T = \left[\Omega_y, P_m, \hat{\xi}, \frac{d\hat{\xi}}{dt}, V_x\right]^T$$

with initial conditions $x_0 = x(0)$ results the following form:

$$\begin{aligned} \dot{x}_1 &= c_1\left(RF - k_L x_2\right) \\ \dot{x}_2 &= -c_2 x_2 + bu \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -a_{41}x_3 - a_{42}x_4 - a_{43}x_2 - f_4(x_1, x_4) \\ \dot{x}_5 &= -c_3\left(F + f_5(x_5)\right) \end{aligned} \tag{11}$$

with output

$$y = s = h(x) = 1 - R\frac{x_1}{x_5} - \frac{x_4}{x_5}$$

where $b = c_2 k_L P_c$, $c_1 = 1/I_y$, $c_2 = 1/T_e$, $c_3 = 1/M$, $a_{41} = k_x/M_c$, $a_{42} = c_x/M_c$, $a_{43} = (Rk_L)/I_y$ $f_4(x_1) = k\nu N\phi(x_1)$ and $f_5(x_5) = d\left(x_5^2 + V_{wind}\right)^2$, with $k = \left(R^2/I_y + 1/M_c\right)$ and $d = (\rho C_d A_f)/(2M)$.

To calculate the control law, we obtain the output dynamics

$$\dot{y} = a_1(x_1, x_5) + a_2(x_5)x_2 + f_1(x)$$

where $a_1(x_1, x_5) \triangleq a_1 = \frac{R}{I_y x_5}\left(RF + \frac{F I_y x_1}{M x_5}\right)$, $a_2(x_5) \triangleq a_2 = (Rk_L)/(I_y x_5)$ and $f_1(x_4, x_5) = -\frac{d(x_4/x_5)}{dt} + \Delta(\nu)$. The term $\Delta(\nu)$ contains variations of the friction parameter ν.

Throughout the development of the controller, we will use the following assumptions:

A1) All the state variables are available for measurement.
A2) The term $f_1(x)$ includes the friction force variation, it is considered as unmatched bounded perturbation

$$\|f_1(x)\| < \beta_1 < \infty \tag{12}$$

A3) The sign function can be approximated by the sigmoid function as is shown by the following limit:

$$\lim_{\varepsilon \to \infty} sigm(\varepsilon S) = sign(S). \tag{13}$$

The Fig. 3 shows the approximation for various values of the sigmoid function slope.

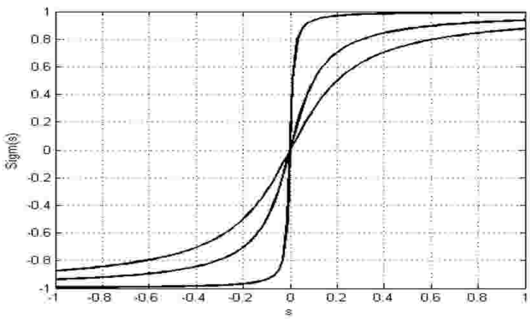

Fig. 4. Sigmoid function for various values of the slope ε

3.1 Control formulation

The control objective is to design an Integral Nested Sliding Mode controller to obtain output trajectory tracking in despite of the system perturbations. Define $y_{ref}(t)$ as the desired trajectory of the relative slip.

Let $y_{ref}(t)$ be a twice differentiable function, but with unknown derivatives, now define the output tracking error as $e_1 = y(t) - y_{ref}(t) \triangleq \alpha_1(x, t)$ then its derivative is

$$\dot{e}_1 = a_1 + a_2 x_2 + g_1(x, t) \tag{14}$$

where $g_1(x, t)$ is the unmatched perturbation term defined by

$$g_1(x, t) = f_1(x) - \dot{y}_{ref}. \tag{15}$$

Considering x_2 as virtual control in (14), we propose

$$x_{2ref} = x_{2,0} + x_{2,1} \tag{16}$$

where $x_{2,0}$ is the nominal part of the virtual control and $x_{2,1}$ is the part which will be designed to reject the perturbation in (14) (see Utkin et al. [2009]). Now, we define a new error variable e_2 as

$$e_2 = x_2 - x_{2ref} \triangleq \alpha_2(x, t) \tag{17}$$

and two auxiliar variables σ_1 and σ_2 of the form

$$\sigma_1 = e_1 + w_1 \tag{18}$$

$$\sigma_2 = e_2 + w_2 \tag{19}$$

where x_{2ref} is the desired value of x_2 to obtain the control aim, w_1 and w_2 are integral variables used to reduce the control gain, σ_1 and σ_2 are pseudo-sliding functions proposed to attenuate the perturbation terms. All the variables will be designed later. Using equations (16)-(19) we obtain

$$x_2 = \sigma_2 + x_{2,0} + x_{2,1} - w_2. \tag{20}$$

Taking the derivative of σ_1 results in

$$\dot{\sigma}_1 = a_1 + a_2 x_2 + \dot{w}_1 + g_1(t). \tag{21}$$

Substituting (20) in (21) yields

$$\dot{\sigma}_1 = a_1 + a_2\left(\sigma_2 + x_{2,0} + x_{2,1} - w_2\right) + \dot{w}_1 + g_1(t)$$

or

$$\dot{\sigma}_1 = a_1 + a_2 \left(e_2 + x_{2,0} + x_{2,1}\right) + \dot{w}_1 + g_1(t). \tag{22}$$

Choosing the dynamics of the integral variable w_1 as

$$\dot{w}_1 = -a_1 - a_2 \left(e_2 + x_{2,0}\right) \tag{23}$$

with $w_1(0) = -e_1(0)$, the nominal part $x_{2,0}$ is designed to eliminate the old known dynamics in (14) and assign the desired dynamics $-k_{10}e_1$, $k_{10} > 0$

$$x_{2,0} = -\frac{1}{a_2}\left(a_1 + k_{10}e_1\right). \tag{24}$$

Substituting (23) and (24) in (22) yields

$$\dot{\sigma}_1 = a_2 x_{2,1} + g_1(t). \tag{25}$$

To attenuate the perturbation term $g_1(x,t)$ in (25) using the integral SM technique and to enforce a sliding motion on $\sigma_1 = 0$ the virtual control $x_{2,1}$ is chosen as

$$x_{2,1} = -k_{11} sigm\left(\varepsilon_1\sigma_1\right)$$

with $k_{11} > 0$.
Using (11) and (17), straightforward calculations reveal

$$\dot{e}_2 = -c_2 e_2 + bu + f_{2e}(x) \tag{26}$$

where

$$f_{2e}(x) = -c_2 x_{2ref} + \frac{\partial\alpha_2(x,t)}{\partial x_1}\dot{x}_1 + \frac{\partial\alpha_2(x,t)}{\partial x_3}\dot{x}_3 + \ldots + \frac{\partial\alpha_2(x,t)}{\partial \sigma_1}\dot{\sigma}_1 + \frac{\partial\alpha_2(x,t)}{\partial x_5}\dot{x}_5.$$

To induce sliding mode in (26) we choose the control signal as

$$u = 0.5\mathrm{sign}\left(-e_2\right) + 0.5. \tag{27}$$

3.2 Stability analysis

Using the new variables e_1, e_2 and σ_1 the extended closed loop system (11), (22) and (27) is presented as

$$\dot{e}_1 = -k_{10}e_1 - a_2 k_{11} sigm(\varepsilon_1\sigma_1) + g_1(x,t) \tag{28}$$

$$\dot{\sigma}_1 = -k_{11} sigm(\varepsilon_1\sigma_1) + g_1(x,t) \tag{29}$$

$$\dot{e}_2 = -c_2 e_2 + f_{2e}(x) + 0.5b\mathrm{sign}\left(-e_2\right) + 0.5b \tag{30}$$

$$\begin{aligned} \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -a_{41}x_3 - a_{42}x_4 - a_{43}x_2 - f_4(x_1,x_4) \\ \dot{x}_5 &= -c_3\left(F + f_5(x_5)\right). \end{aligned} \tag{31}$$

The stability of (28) - (31) can be is studied step by step:
A) SM stability of the projection motion (30);
B) SM stability of the projection motion (29);
C) SM dynamics (28) stability in the neighborhood of SM manifold $e_2 = 0$ and $\sigma_1 = 0$.

We use the following assumptions:

$$|f_{2e}(x)| \leqslant \alpha_2 |e_2| + \beta_2 \tag{32}$$

$$|g_1(x,t)| \leqslant \alpha_1 |\sigma_1| + \beta_1 \tag{33}$$

$$\begin{aligned} |\dot{g}_1(x,t)| &\leqslant \alpha_0 |\sigma_2| + \beta_0 \\ \varsigma &= \dot{\sigma}_1 \end{aligned} \tag{34}$$

with $\alpha_0 > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, $\beta_0 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $c_2 > \alpha_2$ and $b > |f_{2e}(x)|$.

A) The system (30) can be presented as follows:

Case 1, $e_2 > 0$, then

$$\dot{e}_2 = -c_2 e_2 + f_{2e}(x). \tag{35}$$

In this case, under condition (32) the solution of (30) is ultimately bounded by (Khalil [2001])

$$|e_2(t)| \leqslant \delta_0, \quad \delta_0 = \frac{\beta_2}{c_2 - \alpha_2}. \tag{36}$$

Case 2, $e_2 < 0$, then

$$\dot{e}_2 = -c_2 e_2 + f_{2e}(x) + b. \tag{37}$$

In this case $\dot{e}_2 > 0$, therefore, there is a time t_1 such that

$$e_2(t_1) = 0.$$

B) To analyze stability of (29) we use $V_1 = (1/2)\sigma_1^2$. Using (29) we have

$$\dot{V}_1 = \sigma_1\left(-k_{11} sigm(\varepsilon_1\sigma_1) + g_1(x,t)\right). \tag{38}$$

Now we establishe the following equality:

$$sigm(\varepsilon_1\sigma_1) = sign\left(\sigma_1\right) - \Delta\left(\varepsilon_1,\sigma_1\right) \tag{39}$$

where $\Delta(\varepsilon_1,\sigma_1)$ is the difference between the sign and sigmoid functions. It is evidently that $\Delta(\varepsilon_1,\sigma_1)$ is bounded. That is, there is a constant $0 < \gamma_1 < 1$ such that $|\Delta(\varepsilon_1,\sigma_1)| < \gamma_1$. Then using (39) the derivative (38) becomes

$$\begin{aligned} \dot{V}_1 &\leqslant -|\sigma_1|\left(k_{11}\left(1 - |\Delta\left(\varepsilon_1 \quad ,\sigma_1\right)|\right) - |g_1(x,t)|\right) \\ &\leqslant -|\sigma_1|\left(k_{11}\left(1-\gamma_1\right) - \beta_1\right) - \alpha_1|\sigma_1|). \end{aligned} \tag{40}$$

Under the condition $k_1\left(1-\gamma_1\right) > \beta_1$ we have $\dot{V}_1 < 0$, and hence $\sigma_1(t)$ converges to the region given by

$$\|\sigma_1(t)\| \leqslant \delta_1, \quad \delta_1 = \frac{k_{11}\left(1-\gamma_1\right) - \beta_1)}{\alpha_1}. \tag{41}$$

Now, consider the derivative $\varsigma = \dot{\sigma}_1$ (29) described by

$$\dot{\varsigma} = -k_2\varsigma + \dot{g}_1(x,t), \;\; k_2 = k_{11}\varepsilon_1(1 - \tanh^2(\varepsilon_1\sigma_1(1)) \tag{42}$$

and $V_2 = (1/2)\varsigma^2$. Then using (42) and (34) the straightforward calculations gives

$$\dot{V}_2 = \varsigma\left(-k_2\varsigma + \dot{g}_1(x,t)\right) \leqslant -|\varsigma|\left(\left(k_2 - \alpha_0\right)|\varsigma| - \beta_1\right) \tag{43}$$

Under the condition $k_2 > \alpha_0$ the derivative $\varsigma = \dot{\sigma}_1$ is ultimately bounded by

$$|\varsigma(t)| \leqslant \delta_2, \quad \delta_2 = \frac{\beta_1}{k_2 - \alpha_0}. \tag{44}$$

C) Stability of the equation (28) in the neighborhood of the sliding manifold $e_2 = 0$ and $\sigma_1 = 0$ is studied by $V_0 = (1/2)e_1^2$. Using (36) and (43) we have

$$\begin{aligned} \dot{V}_1 &= e_1[-k_{10}e_1 - a_2 k_{11} sigm(\varepsilon_1\sigma_1) + g_1(x,t)] \\ &\leqslant -|e_1|\left(k_{10}|e_1| - \delta_0 - \delta_2\right) \end{aligned} \tag{45}$$

If $k_{10} > 0$, then the control error $e_1(t)$ converges to an arbitrary small neighborhood of the equilibrium point, defined by $|e_1(t)| < \delta$, $\delta = (\delta_0 + \delta_2)/k_{10}$. Moreover it is possible to show that an equilibrium point of the residual dynamics (31) is exponentially stable, therefore the control objectives are fulfilled, and the desired performance of the closed-loop system is obtained.

4. SIMULATION RESULTS

To show the effectiveness of the proposed control law, simulations have been carried out on one wheel model design example, the system parameters used are listed in Table 1. In order to maximize the friction force, in the simulations we suppose that slip tracks a constant signal.

$$y_{ref} = 0.205$$

which produces a value close to the maximum of the function $\phi(s)$. The parameters used in the control law are $k_{10} = 8700$, $k_{11} = 1000$ and $\varepsilon_1 = 100$.

TABLE 1
Values of Parameters

Parameter	Value	Parameter	Value
C_x	10	V_{wind}	-5
K_x	9000	v	0.8
M	2396	B	10
I_y	18.9	C	1.9
R	0.535	D	1
K_L	1000	E	0.97
ρ	1.225	N	23504.76
C_d	0.65	P_a	0
A_f	6.6	P_c	8

On other hand, to show robustness property of the control algorithm in presence of parametric variations we introduce a change of the friction coeficient ν which produces a diferent contact force, namely $\hat{F}$. Then, $\nu = 0.5$ for $t < 1$ s, $\nu = 0.52$ for $t \in [1, 2)$ s, and $\nu = 0.5$ for $t > 2$ s. It is worth mentioning that just the nominal values were considered in the control design.

In the figures 5 and 6 are shown, respectively, the slip and the friction function ϕ in the braking process

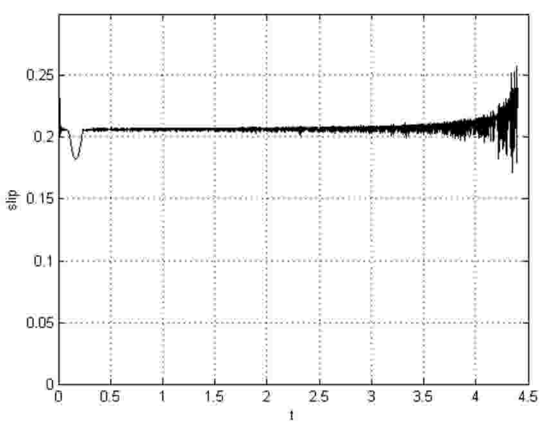

Fig. 5. Slip performance in the braking process

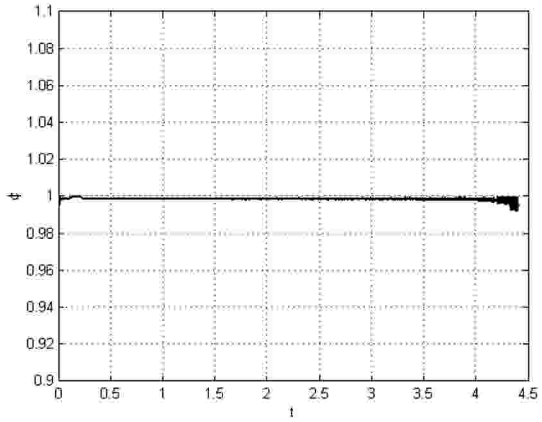

Fig. 6. Performance of ϕ in the braking process

while Figs. 7 and 8 summarize the behavior of the error variables e_1 and e_2 respectively.

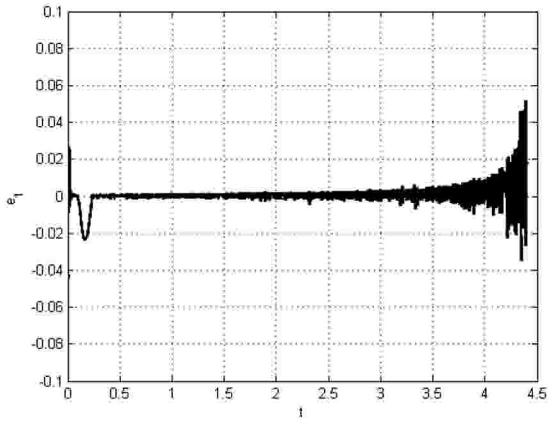

Fig. 7. Tracking error, $e_1 = y - y_{ref}$

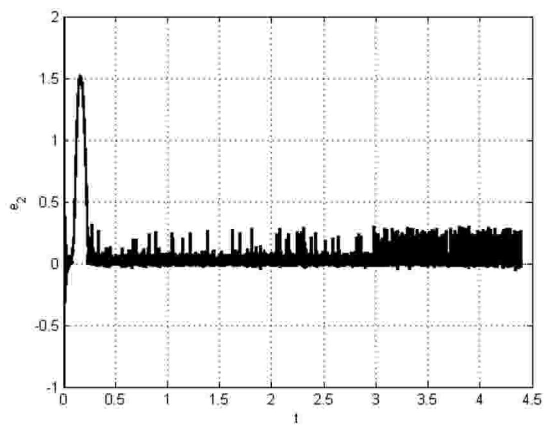

Fig. 8. Tracking error, $e_2 = x_2 - x_{2ref}$

In Fig. 9 the longitudinal speed V_x and the linear wheel speed $\Omega_y R$ are showed; it is worth noting that the slip controller should be turn off when the longitudinal speed V_x is close to zero. In Fig. 10 the control action is shown.

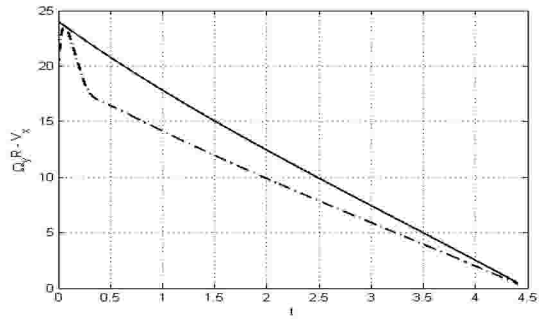

Fig. 9. Longitudinal speed V_x (solid) and linear wheel speed $\Omega_y R$ (dashed)

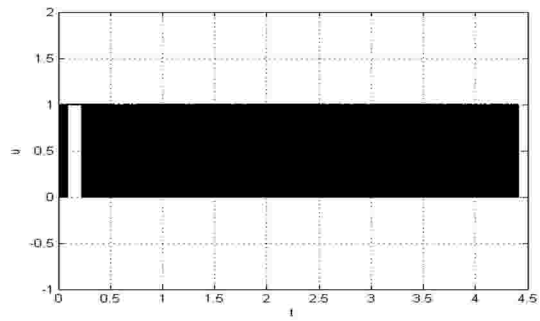

Fig. 10. Control input

Finally, in Fig. 11 the nominal F, and the $\hat{F}$ contact force are shown.

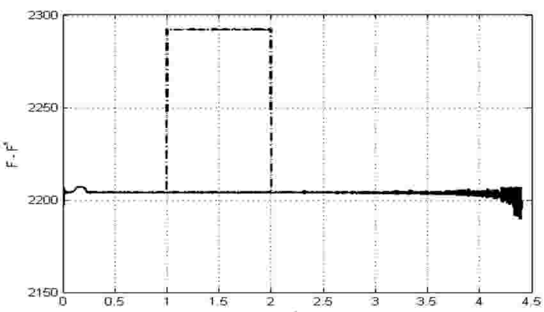

Fig. 11. Nominal F (solid) and $\hat{F}$ (dashed) contact forces

5. CONCLUSION

In this work an integral nested sliding mode control for ABS has been proposed. The simulation results show good performance and robustness of the closed-loop system in presence of both the matched and unmatched perturbations, namely, parametric variations and neglected dynamics.

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