MAE 5803 - Homework #1 Problem #3

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```
clear; close all; clc;
```

Consider the following second-order system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - (\mu - x_1^2)x_2$$

a)

Find the eigenvalues of the linearized system about the equilibrium point, (0,0). Express your answer in terms of μ. Sketch in the complex plane the variation of the locations of these eigenvalues as μ varies from -0.5 to 0.5.

About (0,0), the squared term can be canceled because $x_1 < 0$, so $x_1^2 << 0$. The linearized state equation is then:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - \mu x_2$$

The A matrix is

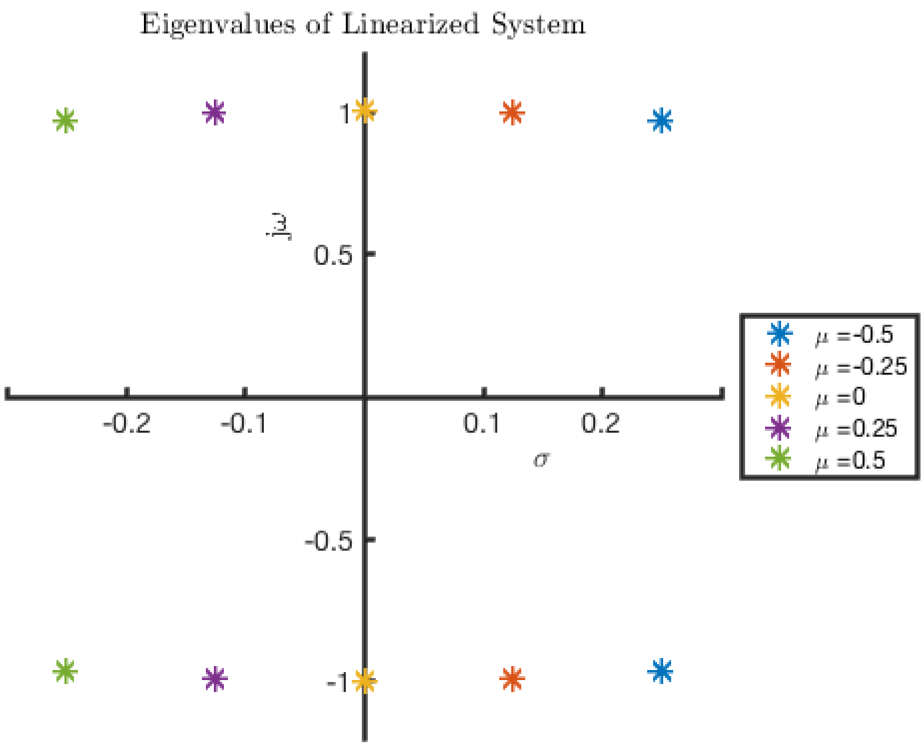
$$A = \begin{pmatrix} 0 & 1 \\ -1 & -\mu \end{pmatrix}$$

Calculate the eigenvalues for samples of the range of μ specified

```
mu = [-0.5:0.25:0.5];
eValue = zeros(2,length(mu));
figure(1)
hold on
for i = 1:length(mu)
    A =  [0 1; -1 -mu(i)];
    eValue(:,i) = eig(A);
    plot(real(eValue(:,i)),imag(eValue(:,i)),'*','MarkerSize',12)
end
ax = gca;
ax.XAxisLocation = 'origin';
ax.YAxisLocation = 'origin';
ylim([-1.2 1.2]);
title('Eigenvalues of Linearized System')
```

```
xlabel('$\sigma$'); ylabel('j$\omega$');
legend(strcat('\mu = ',strread(num2str(mu),'%s')),'Location','EastOutside')
hold off

%
```



Linearize using jacobian

This is a more formal method of obtaining the A matrix. It yields the same result as above.

$$A = \left.\frac{\partial \bar{f}}{\partial \bar{x}}\right|_{\bar{x}=\bar{0}}$$

$$\frac{\partial f_1}{\partial x_1} = 0$$

$$\frac{\partial f_1}{\partial x_2} = 1$$

$$\frac{\partial f_2}{\partial x_1} = -1 + 2x_1 x_2$$

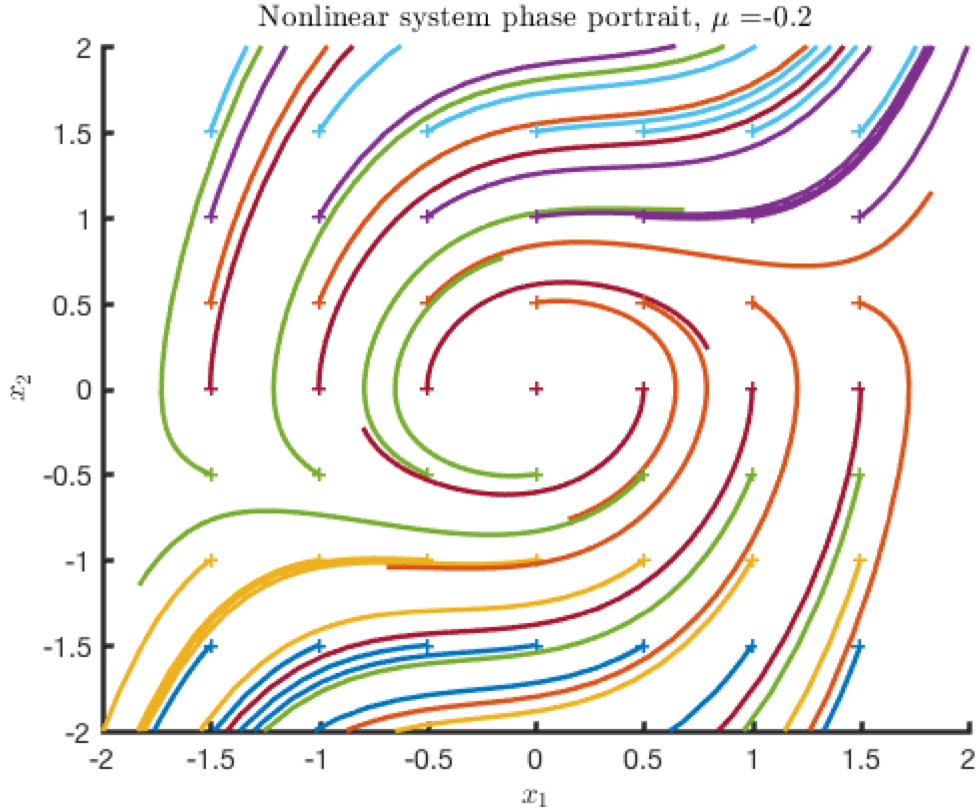
$$\frac{\partial f_2}{\partial x_2} = -\mu + x_1^2$$

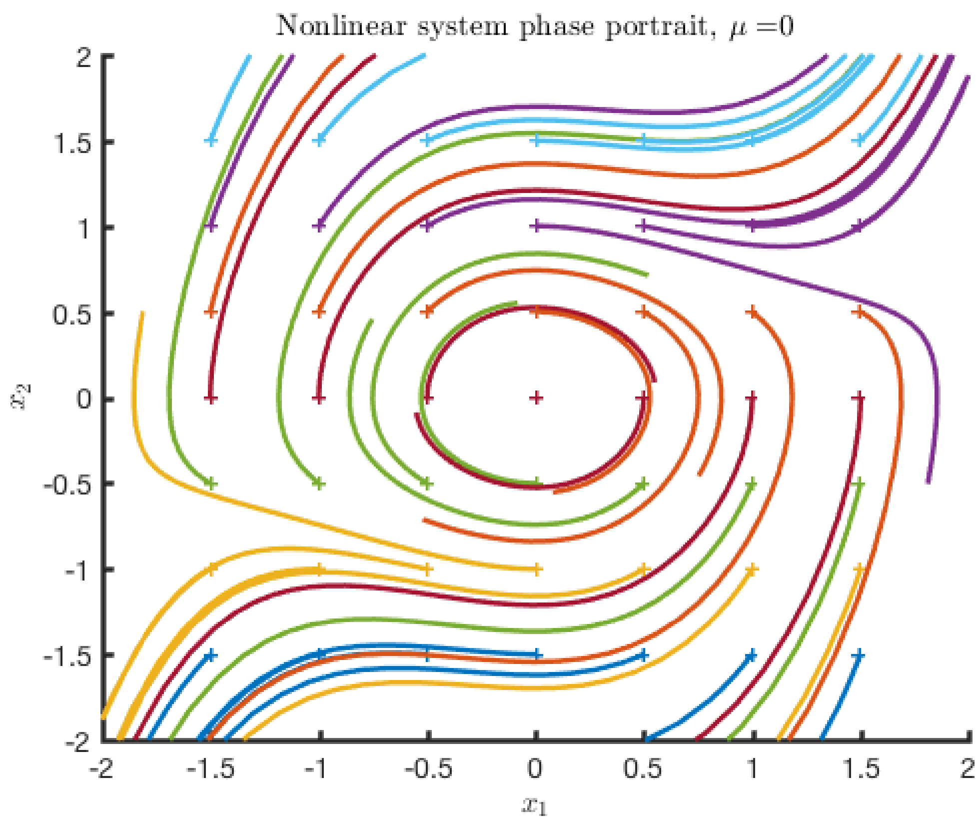
$$A = \begin{pmatrix} 0 & 1 \\ -1 & -\mu \end{pmatrix}$$

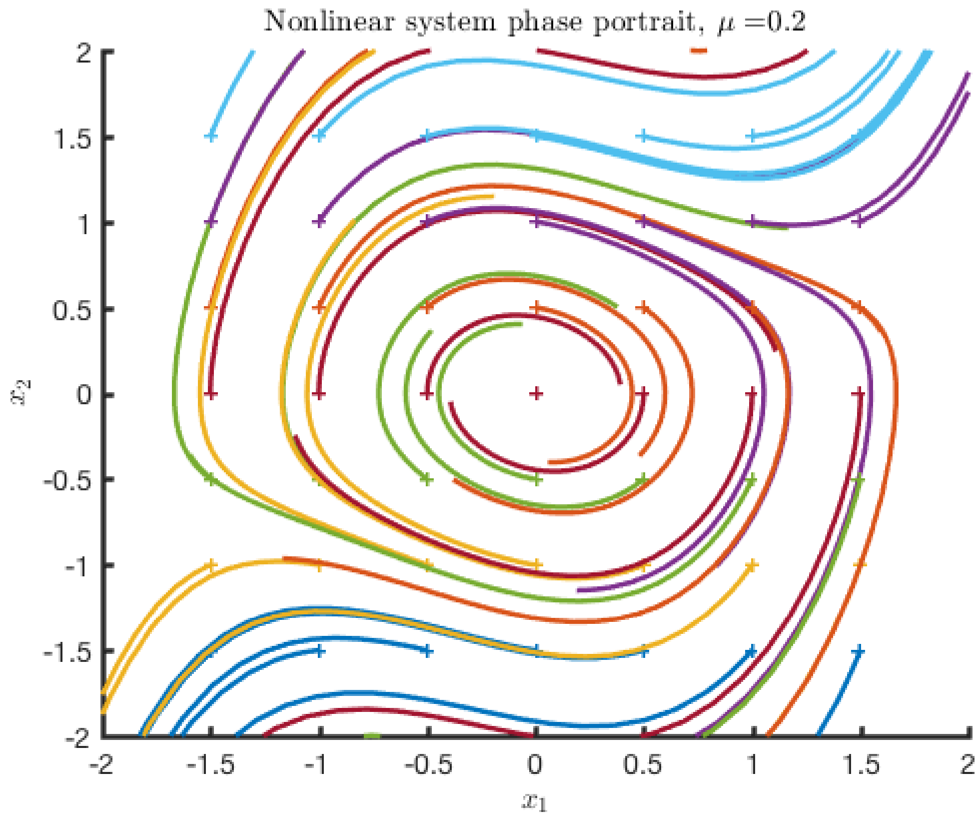
b)

Draw the phase portraits of the system using MATLAB for $\mu = -0.2$, $\mu = 0$, and $\mu = 0.2$. Use -2 to 2 range of values for the horizontal and vertical axes.

```
mu = [-0.2 0 0.2];
for i = 1:length(mu)
    figure()
    hold on
    for x1 = -1.5:.5:1.5
        for x2 = -1.5:.5:1.5
            tspan = [0 3];
            x0 = [x1; x2];
            [t,x] = ode45(@P3stateEqn,tspan,x0,[],mu(i));
            h = plot(x(:,1),x(:,2));
            c = get(h,'color');
            plot(x0(1),x0(2),'+','color',c);
        end
    end
    axis([-2 2 -2 2])
    xlabel('$x_1$')
    ylabel('$x_2$')
    title(strcat('Nonlinear system phase portrait, $\mu =$ ', num2str(mu(i))))
    hold off
end
```


Nonlinear system phase portrait, μ =-0.2
x_2
x_1
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Nonlinear system phase portrait, μ =0
x_2
x_1
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2



d)

What phenomenon do you observe as the parameter, μ, varies from negative to positive? Justify your answer using Poincare-Bendixson Theorem.

1. If the real parts of all eigenvalues are negative, then $\mathbf{x} = \mathbf{0}$ is locally asymptotically stable
2. If the real part of at least one eigenvalue is positive, then $\mathbf{x} = \mathbf{0}$ is locally unstable
3. If the real part of at least one eigenvalue is equal to zero, then the local stability of $\mathbf{x} = \mathbf{0}$ cannot be concluded

From the plot of eigenvalues, it can be seen, as μ goes from negtive to positive, the real part of the eigenvalues moves from positive to negative. Thus, the system transitions from locally asymptotically stable to locally unstable. At $\mu = 0$, the eigenvalues have real parts equal to zero, but the phase portrait reveals the system is unstable at this point.

Poincare-Bendixson Theorem: If a trajectory of a second-order autonomous system remains in a finite region (Ω), then one of the following is true:

1. The trajectory goes to an equilibrium point
2. The trajectory tends to a stable limit cycle
3. The trajectory itself is a limit cycle

Consider the region defined by the ball centered on the origin with a radius greater than the distance from the origin to the initial point. The system with negative real parts of all eigenvalues goes towards the equilibrium point at the origin. The system with zero real parts to the eigen values are found to increase indefinitely, if only slowly when near the origin. Thus, there is no region in which these trajectories are contained and PB Theorem does not apply. The same goes for the eigenvalues having positive real parts and the rate of increase increases with μ.

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