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TECHNICAL PAPER

Optimization of a stochastic dynamical system

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Abstract This paper analyzes a simple stochastic dynamical system and a target performance to be achieved by this system. Such target performance is the height of the FRF at given frequency bands. The practical interest on this performance is in percussion drilling, where the idea is to improve the drilling process by self-excitation of the imposed vibro-impact action. However, some parameters of the system are modeled as random variables, thus, requiring the use of stochastic optimization. To model the uncertainties of the system, the parametric approach is used and the probability density functions are derived using the Maximum Entropy Principle. To take into account the uncertainties of the dynamical system in the optimization process, a multi-objective optimization of some statistical characteristics of a distance between the response of the system and the target performance is proposed. The global and bounded Nelder–Mead optimization algorithm is employed to optimize the stochastic function. The results showed that when the uncertainties are considered, the optimum design is different from the deterministic optimization results and that the robust optimization is a very useful tool to deal with uncertainties in dynamical systems.

Keywords Stochastic dynamics · Robust optimization · Stochastic optimization · Target performance

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List of symbols

$\boldsymbol{M}$	Mass matrix
$\boldsymbol{C}$	Dumping matrix
$\boldsymbol{K}$	Stiffness matrix
$\boldsymbol{u}$	Displacement vector
$\dot{\mathbf{u}}$	Velocity vector
$\ddot{\boldsymbol{u}}$	Acceleration vector
$\boldsymbol{f}$	Force vector
t	Time
h	Frequency response function
$\hat{\boldsymbol{u}}$	Fourier transform of $\boldsymbol{u}$
$\hat{f}$	Fourier transform of $\boldsymbol{f}$
K_i	Random stiffness of the ith mass of the system
$\underline{k}_i$	Mean value of the ith mass of the system
$E\{\ldots\}$	Mean operator
$\mathrm{var}\{\ldots\}$	Variance operator
K_{rand}	Random stiffness matrix
$\hat{U}_i$	Random displacement of the ith d.o.f.
H	Random frequency response function
d	Distance between the system response and the target performance

B_i	Frequency bands i
t_i	Target peak in the frequency band B_i
p_i	Peak in the frequency band B_i
$\boldsymbol{s}$	Design vector
D	Random variable representing the distance between the system response and the target performance of the stochastic problem
P_i	Random peak in the frequency band B_i of the stochastic problem
H_{Bi}	Ramdom response of the system in the frequency band B_i
J^{opt}	Objective function of the RDO
C_{adm}	Feasible set
E^* and var^*	Utopia points

Greek symbols

ω	Frequency
δ_i	The coefficient of variation of the random variable i
Γ	Gamma function
α	Weighting factor
β	Positive parameter that controls the length of the Gaussians

Subscripts

rand	Random
1	Relative to the first d.o.f. or the frequency band B_1
2	Relative to the second d.o.f. or the frequency band B_2
K	Relative to the random variable K
B_i	Relative to the frequency band B_i
adm	Admissible/feasible set
min	Relative to the lower bound of the design variable
max	Relative to the upper bound of the design variable

1 Introduction

Optimization has become a very important tool in several fields, especially in engineering design. Although deterministic optimization methods have been widely applied, it is difficult to find examples of systems to be optimized, in any field, that do not include some level of uncertainty, for example, on its parameters, geometry, boundary conditions, or even in the very model being used.

Two optimizations under uncertainty techniques that have been widely applied to engineering problem are the reliability-based design optimization [1–3] and the robust design optimization (RDO) [4, 5]. The former is mainly concerned with the inclusion of probability constraints in the optimization problem, while the latter aims at obtaining optimal designs that are not very sensitive to the variability of the system parameters. Several forms of objective function have been proposed in the RDO, for instance: minimization of the mean and/or variance of the response of the system under consideration. Stochastic programming [6], Taguchi methodology [7], and optimization methods [8] have been applied to solve RDO. Several papers have dealt with robust optimization in engineering in the form of a multi-objective optimization problem, since a widely applied objective function is the simultaneous minimization of the mean and variance of the system response. To deal with multi-objective optimization, among others, the weighted sum, compromise approach, and the preference aggregation methods have been employed [4].

The application of the RDO to dynamical systems is recent [5, 9–13]. In this paper, we are concerned with the RDO of a simple dynamical system with only two degrees of freedom in order to focus on the robust optimization. A target performance optimization specifying the height for the FRF at given frequency bands of the system is proposed,—in other words, we seek the set of system parameters that leads to a system response (peaks) as close as possible to a performance defined a priori. This approach differs, for instance, from a classical analysis which would be to maximize the distance between the load frequency and the resonance frequency, or to minimize the peak of the response. What motivates this unusual analysis is the percussion drilling [14–16]. The idea is to improve the drilling process by self-excitation of the imposed vibro-impact action. In this case, we want to work near resonance (to efficiently use the energy), but we also want to avoid too much displacement, so that the equipment is preserved.

The uncertainties on the stiffness of such system are taken into account by modeling them as random variables. To accomplish it, the parametric approach is used and the probability density functions are derived using the Maximum Entropy Principle [17–19]. Then, the objective function of the RDO problem is constructed as the minimization of the mean and the variance of the difference between the system performance and the target one. The Global and Bounded Nelder–Mead (GBNM) algorithm is employed as optimizer due to its ability to handle non-convex and multimodal functions [20, 21].

This paper is organized as follows: The deterministic dynamical problem is presented in Sect. 2 and the probabilistic model is presented in Sect. 3. The robust optimization problem is defined in Sect. 4 and the optimization algorithm is explained in Sect. 5. Finally, the numerical results are presented in Sect. 6 and the concluding remarks are given in Sect. 7.

2 Deterministic model

A simple dynamical system is considered in order to focus on the robust optimization strategy applied. The optimization procedure proposed may be easily extended to more complex dynamical problems, yet it is interesting to note that, even for a simple system, the RDO presents results that are not trivial. Figure 1 shows the two-degrees-of-freedom dynamical system that is analyzed [22].

This linear system has two natural frequencies and two normal modes. The dynamics of the system is given by:

$$\boldsymbol{M}\boldsymbol{u}(t) + \boldsymbol{C}\underline{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{f}(t) \tag{1}$$

with initial conditions $u(0) = u_0$ and $\dot{u}(0) = v_0$, in which

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}; \; C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \tag{2}$$

$$\boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \tag{3}$$

and

$$\boldsymbol{f}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}; u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{4}$$

The mass, damping, and stiffness matrices, denoted by $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$, are real symmetric positive-definite. The external force is represented by the vector $\boldsymbol{f} = (f_1, f_2)^{\mathrm{T}}$, where f_1 and f_2 are the forces applied on the masses m_1 and m_2. The displacements of the masses are denoted by u_1 and u_2, which are the components of the vector $\boldsymbol{u}$. Let $\boldsymbol{f}(t) = (f_1(t), 0)^{\mathrm{T}}$ be the input force applied on the system, and let $\boldsymbol{u}(t) = (u_1(t), u_2(t))^{\mathrm{T}}$ be the corresponding output. Let $\hat{f}_1$ be the Fourier transform of f_1, $\hat{u}_1$ be the Fourier transform of u_1, and $\hat{u}_2$ be the Fourier transform of u_2. In the frequency domain Eq. (1) is written as:

$$\hat{\boldsymbol{u}} = \left(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K}\right)^{-1}\hat{\boldsymbol{f}}. \tag{5}$$

where $i = \sqrt{-1}$ and ω is the frequency. In this paper, the deterministic frequency response function of interest is denoted by h and is defined as

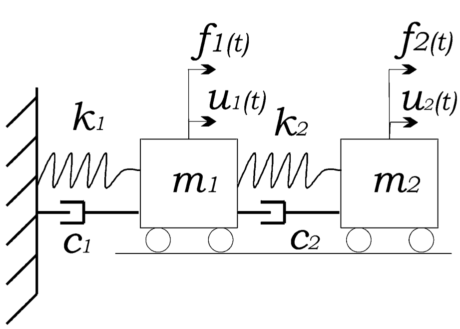

Fig. 1 Two d.o.f. system used

$$h(\omega) = \left|\frac{\hat{u}_2(\omega)}{\hat{f}_1(\omega)}\right| \tag{6}$$

3 Probabilistic model

We consider in the design problem that the stiffnesses of the two springs are uncertain,—e.g. they were taken from a batch. As the manufacturing process is not perfect, there are uncertainties in the values of the stiffness and they may differ from the nominal value. A parametric probabilistic approach has been employed to model the uncertainties on the stiffness which have mean values $\underline{k}_1$ and $\underline{k}_2$, their nominal values, and associated random variables K_1 and K_2 (the capital letter is used for the random variables).

The choice of the probability distribution function is crucial since all the stochastic simulations depend on it. The Maximum Entropy Principle [17–19] has been employed to construct the probability density function of the random variables in a way that only the known information is used. This information is: (1) the stiffness is always positive, $K_1 > 0$ and $K_2 > 0$, (2) the mean values are known ($E\{K_1\} = \underline{k}_1$ and $E\{K_2\} = \underline{k}_2$), and (3) $E\{K_2^{-2}\} = c_1 < +\infty$ and $E\{K_2^{-2}\} = c_2 < +\infty$, so that the response of the system is a second-order random variable (this means that from the measured displacement and the stiffness the force could be computed, i.e. the inverse problem is well posed).

It follows that K_1 and K_2 are Gamma random variables: $K_1 \sim$ Gamma $(\underline{k}_1, \delta_{K1})$ and $K_2 \sim$ Gamma $(\underline{k}_2, \delta_{K2})$, where $\delta_K = \sigma_K/\underline{k}$ is the coefficient of variation and σ_K is the standard deviation. In terms of probability density functions:

$$p_{K_1}(k_1) = \mathbf{1}_{]0,+\infty[}(k_1)\frac{1}{\underline{k}_1}\left(\frac{1}{\delta_{K_1}^2}\right)^{\frac{1}{\delta_{K_1}^2}}\frac{1}{\Gamma\left(\frac{1}{\delta_{K_1}^2}\right)}\left(\frac{k_1}{\underline{k}_1}\right)^{\frac{1}{\delta_{K_1}^2}-1}\mathrm{e}^{\left(-\frac{k_1}{\delta_{K_1}^2\underline{k}_1}\right)} \tag{7}$$

and

$$p_{K_2}(k_2) = \mathbf{1}_{]0,+\infty[}(k_2)\frac{1}{\underline{k}_2}\left(\frac{1}{\delta_{K_2}^2}\right)^{\frac{1}{\delta_{K2}^2}}\frac{1}{\Gamma\left(\frac{1}{\delta_{K_2}^2}\right)}\left(\frac{k_2}{\underline{k}_2}\right)^{\frac{1}{\delta_{K_2}^2}-1}\mathrm{e}^{\left(-\frac{k_2}{\delta_{K_2}^2\underline{k}_2}\right)} \tag{8}$$

which are Gamma probability density functions. $\Gamma(z) = \int_0^{+\infty} t^{z-1}\mathrm{e}^{-1}\mathrm{d}t$ is the Gamma function defined for $z > 0$. The random stiffness matrix has the following form:

$$\mathbf{K}_{\text{rand}} = \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \tag{9}$$

where the boldface is used to represent a random matrix. The Monte Carlo method [24] is used to generate the random variables K_1 and K_2. The stochastic dynamical equation is written as:

$$\hat{\boldsymbol{U}} = \left(-\omega^2 \boldsymbol{M} + i\omega \boldsymbol{C} + \boldsymbol{K}_{rand}\right)^{-1} \hat{\boldsymbol{f}} \tag{10}$$

and the random frequency response function H is the randomization of h:

$$H(\omega) = \left| \frac{\hat{U}_2(\omega)}{\hat{f}_1(\omega)} \right| \tag{11}$$

4 Robust design optimization

Here, we wish to obtain a structural design for which the response is as close as possible to a pre-defined target performance. Thus, we aim at minimizing the distance between the system response and the target performance. Such distance d in a deterministic problem would be defined as:

$$d(\mathbf{s}) = \left[\frac{p_1(\mathbf{s}) - t_1}{t_1}\right]^2 + \left[\frac{p_2(s) - t_2}{t_2}\right]^2, \tag{12}$$

where s is the variable design vector $\boldsymbol{s} = (k_1, k_2)$, t_1 and t_2 are the target peaks in the frequency bands B_1 and B_2, respectively. For $i = 1$ and 2, p_i is the peak given by $p_i = \max\ (h_{\mathrm{B}i})$ where $h_{\mathrm{B}i}$ is the response of the system in the frequency band B_i. Thus, d measures how close the performance of the system (peaks p_1 and p_2) is to the target performance (peaks t_1 and t_2). Figure 2 shows a frequency response function with two targets t_1 in B_1 and t_2 in B_2.

In the associated stochastic problem, the peaks will be random variables P_1 and P_2 and, consequently, the distance is also a random variable D. So,

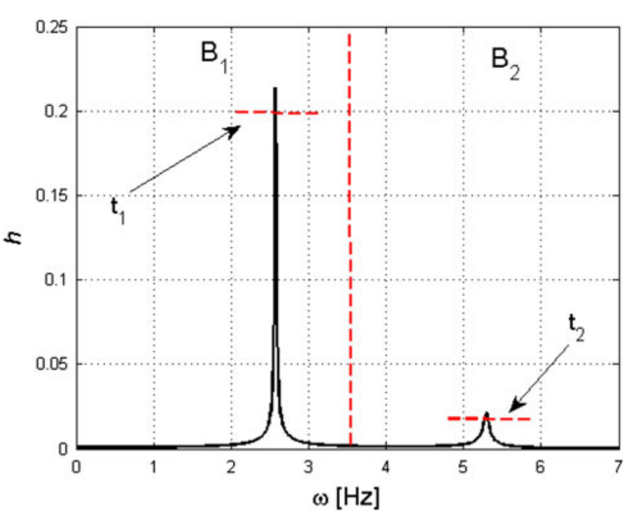

Fig. 2 Frequency response function with targets t_1 and t_2

$$D(\underline{\mathbf{s}}) = \left[\frac{P_1(\underline{\mathbf{s}}) - t_1}{t_1}\right]^2 + \left[\frac{P_2(\underline{\mathbf{s}}) - t_2}{t_2}\right]^2 \tag{13}$$

where $\underline{\boldsymbol{s}}$ is the mean value of the design vector $\underline{\boldsymbol{s}} = (\underline{k}_1, \underline{k}_2)$. For $i = 1$ and 2, P_i is the peak given by $P_i = \max\ (H_{\mathrm{B}i})$ where $H_{\mathrm{B}i}$ is the random response of the system in the frequency band B_i. Thus, D has mean $E\{D\}$ and variance $\mathrm{var}\{D\}$.

As already commented, the aim of this paper is to pursue the RDO of a dynamical system, in other words, the system is optimized taking into account uncertainties. To accomplish this, we propose to minimize some statistical characteristics of D [Eq. (15)]. First, the stochastic optimization problem is posed as:

$$J^{\mathrm{opt}} = \arg \min_{\underline{\mathbf{s}} \in C_{\mathrm{adm}}} J(\underline{\mathbf{s}}) \tag{14}$$

where J is comprised by some statistical characteristics of the distance D and the feasible set is defined by $C_{\mathrm{adm}} = \{\underline{\boldsymbol{s}} = (\underline{k}_1, \underline{k}_2);\ \underline{k}_{1\min} \le \underline{k}_1 \le \underline{k}_{1\max};\ \underline{k}_{2\min} \le \underline{k}_2 \le \underline{k}_{2\max}\}$. To take into account the uncertainties of the parameters, we propose to minimize simultaneously the mean and the variance of D, what leads to a multi-objective optimization problem similar to what was done in references [5, 10]. The first step to solve the multi-objective optimization problem is to find the Utopia points by minimizing individually the mean and the variance as single objective functions. The Utopia points are denoted by E^* and var^*. Since there are only two objective functions, they may be combined into a single objective function using the weighted sum method. Therefore, the objective function becomes:

$$J(\underline{\mathbf{s}}) = \alpha \frac{E\{D(\underline{\mathbf{s}})\}}{E^*} + (1 - \alpha) \frac{\mathrm{var}\{D(\underline{\mathbf{s}})\}}{\mathrm{var}^*} \tag{15}$$

where $\alpha\ \varepsilon\ [0, 1]$ is the weighting factor. Then, J is minimized for different values of α between 0 and 1 in order to construct the Pareto frontier obtaining tradeoffs between the two objectives of the problem. The function to be minimized is non-convex and multimodal as will be shown in the numerical analysis (Sect. 8). Thus, in order to get good results, the use of a global optimizer becomes mandatory. The globalized and bounded Nelder–Mead algorithm is employed here and it is described in the sequel.

5 Optimization algorithm

As the function under analysis is non-convex and multimodal, the utilization of a global optimization algorithm is required. In this framework, stochastic or hybrid stochastic/deterministic methods are often used. The simplest approach is a random search, where a new point is randomly generated and examined. It is kept if its performance is better than the previous iteration, if not it is rejected and the old point is kept. Of course, this procedure leads to a very high computational cost. Thus, several classes of global optimization algorithms have been developed to perform the search in a more efficient way. One of them is the coupling of global and local optimization algorithms. For instance, any local optimizer can be turned into a global one by restarting the search randomly. The global and bounded Nelder–Mead (GBNM) optimization algorithm does it in an interesting way. The restart procedure uses an adaptive probability density constructed using the memory of past local searches. The algorithm is fully described in [20]. Here, the main parts of the algorithm are detailed, especially the probabilistic restart.

The local search of the GBNM is performed by the Nelder–Mead algorithm, which is a classic zero order method that is based on the comparison of function values at the $n + 1$ vertices of a simplex. Some modifications have been implemented in the GBNM such as the handling of constraints by penalization, bounds through projection and the degeneracy of the simplex. The stopping criteria of the local searches are when the simplex is flat, small or degenerated. When one of such criteria is achieved, the search is restarted, which is described in the sequel.

The probability of having sampled a point $\underline{s}$ in the GBNM is described by a Gaussian-Parzen-window approach [23]:

$$f(\underline{s}) = \frac{1}{N}\sum_{i=1}^{N} f_i(\underline{s}) \tag{16}$$

where N is the number of points $\underline{s}_{(i)}$ already sampled. Such points come from the memory kept from the previous local searches, being, in the present version of the algorithm, all the starting points and local optima already found. $f_i(\underline{s})$ is the normal multidimensional probability density function given by:

$$f_i(\underline{s}) = \frac{1}{(2\Pi)^{\frac{n}{2}}\det(\Sigma)^{\frac{1}{2}}} \times \mathrm{e}^{\left(-\frac{1}{2}\left(\underline{s}-\underline{s}_{(i)}\right)^T \Sigma^{-1}\left(\underline{s}-\underline{s}_{(i)}\right)\right)} \tag{17}$$

where n is the problem dimension and Σ is the covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & & \\ & \cdots & \\ & & \sigma_n^2 \end{bmatrix} \tag{18}$$

and variances are estimated by the relation:

$$\sigma_j^2 = \beta\left(s_j^{\max} - s_j^{\min}\right)^2 \tag{19}$$

where β is a positive parameter that controls the length of the Gaussians, and $s_j^{\max}$ and $s_j^{\min}$ are the bounds of the jth variable ($j = 1$, 2 in our case). To keep the method simple, the variances are kept constant during the optimization. After each local minimum is found, N points are randomly sampled ($\boldsymbol{s}_1$, $\boldsymbol{s}_2$, …, $\boldsymbol{s}_N$) to restart the next search and the one that minimizes Eq. (16) is selected. The stopping criterion of the global optimization is the maximum number of function evaluations $n_{\max}$ defined a priori by the user.

6 Numerical analysis

In this section some results are discussed. Section 6.1 shows the surface generated by d (Eq. 12) considering the deterministic problem. The convergence of the Monte Carlo simulations to evaluate the mean and variance of D (Eq. 13) are in Sect. 6.2. In Sect. 6.3, the robust optimization problem is solved and the results are compared to the ones of the deterministic optimization.

The data used in the simulations are: $m_1 = 1.5$ kg, $m_2 = 0.75$ kg, $c_1 = 0.5$ Ns/m, $c_2 = 0.05$ Ns/m, $900 \leq k_1 \leq 1100$ N/m, $130 \leq k_2 \leq 170$ N/m, $t_1 = 0.2$ m/N, $t_2 = 0.02$ m/N, $\mathrm{B}_1 = [0, 3{:}5]$ Hz, $\mathrm{B}_2 = [3{:}5, 7]$ Hz, $\delta K_1 = 0.025$, $\delta K_2 = 0.025$. The parameters used in the GBNM are shown in Table 1.

6.1 Surface generated by d

Figure 3 shows an approximation of the surface generated employing Eq. (12) in function of $s = (k_1, k_2)$. It is noticed that, for the target chosen, we get a very complicated surface with many local minima. This point must be emphasized: note that the system considered for the analysis is very simple, nevertheless it turns out that the optimization problem is non-convex and multimodal.

6.2 Convergence of the stochastic solution

Monte Carlo simulations are employed to compute the mean and variance at each point of the robust optimization. The typical convergence curves for the estimator of the mean and variance of D are shown in Figs. 4 and 5.

In all the numerical experiments of the next section, the sample size used is 2000.

Table 1 Parameters used for the optimization algorithm

Parameter	Value
N	10
$n_{\max}$	5,000
Simplex size	5
β	0.01

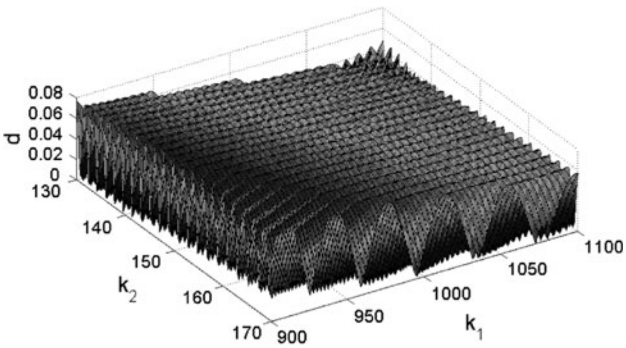

Fig. 3 Surface generated by J using D_1

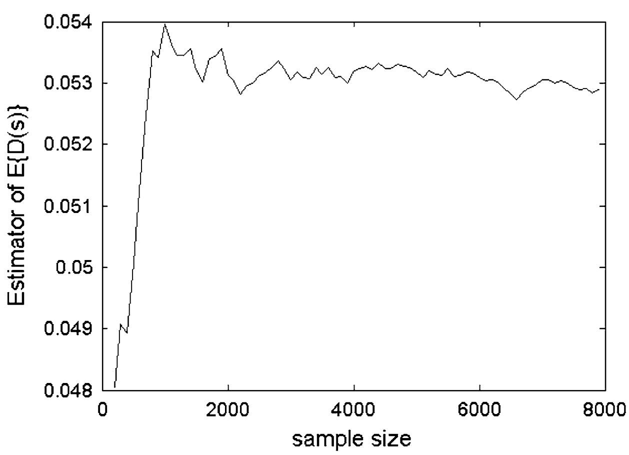

Fig. 4 Convergence of the mean

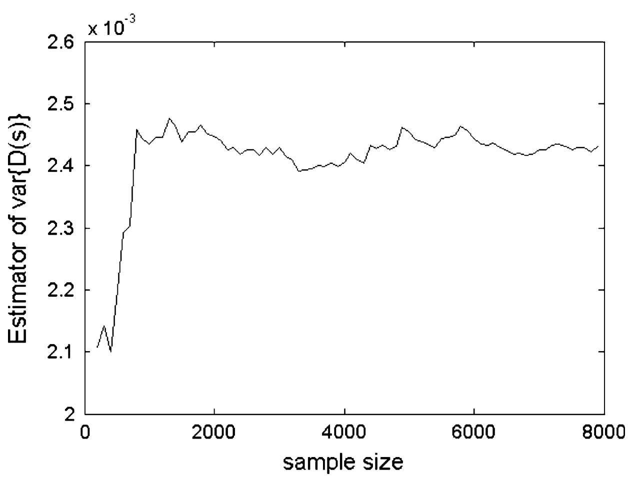

Fig. 5 Convergence of the variance

7 Robust design optimization

Before presenting the results of the RDO, a comment has to be made regarding the computational cost of the optimization. As shown in Sect. 6.2, each point evaluated by the GBNM in the RDO requires a sample of 2,000 Monte Carlo

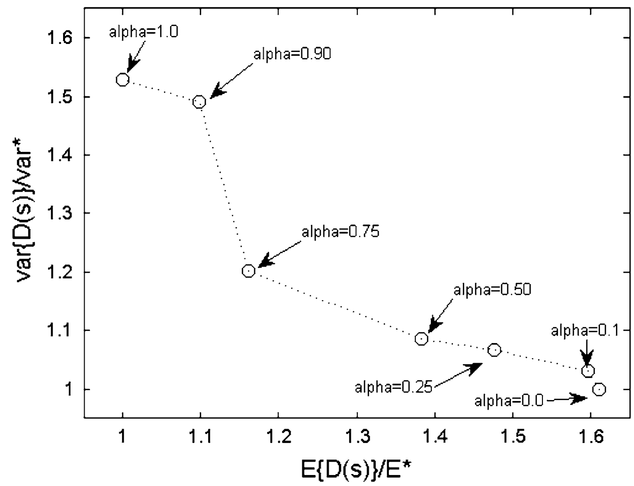

Fig. 6 Pareto front

Table 2 Results of the multi-objective optimization

α	$E\{K_1\}$	$E\{K_2\}$	$E\{D(\underline{s})\}$	$\mathrm{var}\{D(\underline{s})\}$
0.00	1096.2	139.7	0.0481	0.2528×10^{-3}
0.10	1097.1	140.3	0.0477	0.2690×10^{-3}
0.25	1090.0	141.2	0.0441	0.2890×10^{-3}
0.50	1094.9	144.8	0.0413	0.2993×10^{-3}
0.75	1099.9	151.5	0.0347	0.3648×10^{-3}
0.90	1097.0	158.6	0.0328	0.5617×10^{-3}
1.00	1100.0	161.6	0.0299	0.5905×10^{-3}

simulations, which costs 85.70 s in a Intel Pentium M 1.6 GHz processor. Thus, each point in the Pareto frontier (Fig. 6) took approximately 5 days to be computed.

Figure 6 shows the results in the objective space, formed by the normalized values of $E\{D(s)\}$ versus $\mathrm{var}\{D(s)\}$ for each α considered. The trade-off between the mean and the variance can clearly be observed.

The results of the robust optimization using different values of α are shown in Table 2. It can be seen that when the value of α changes, different optimal results are obtained. The higher the α is, the better the mean value of the response is and the worse the variance of response is. Then, it is up to the designer to choose his preferred trade-off between the mean and the variance of the response for the structure under analysis.

A good feature of the GBNM algorithm is that it provides several local optima of the optimization problem. Such feature is explored in Fig. 7, where the five best local optima of four different situations are shown: deterministic problem and robust problems for $\alpha = [0.0, 0.5, 1.0]$. Note that the bigger symbol of each situation gives the best design found, except for the deterministic case, where, of course, the response of all the five designs coincides. It can

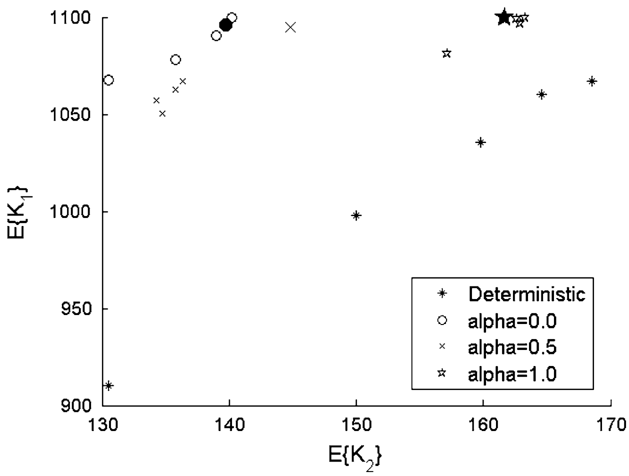

Fig. 7 Five best local minima found for the deterministic case and for the RDO for different α

Table 3 Mean and variance of the deterministic optima

$E\{K_1\}$	$E\{K_2\}$	$E\{D(\underline{s})\}$	$\mathrm{var}\{D(\underline{s})\}$
998.08	150.00	0.0458	0.0017
1060.18	164.55	0.0402	0.0012
1066.78	168.56	0.0405	0.0014
910.07	130.52	0.0585	0.0027
1035.36	159.79	0.0443	0.0016

be seen that when the uncertainties of the system are considered, the optimum design changes, even in the case where only the mean of the target function is minimized ($\alpha = 1.0$). To see how different the deterministic and the robust optimization results are, Table 3 shows the mean and variance values of the five best local minima of the deterministic optimization. Table 3 shows clearly that the mean and variances of the deterministic optima are much higher than the optimum results of the robust optimization (Table 2), especially when the variance is considered. One sees then, when the uncertainties are not considered, the deterministic optimization results are poor. Moreover, the results show that the robust optimization is a very useful tool to deal with uncertainties in dynamical systems.

8 Conclusion

This paper dealt with the target performance optimization of a simple stochastic dynamical system subject to uncertainties in two parameters. To consider such uncertainties of the system, the parametric approach was used and the probability density functions were derived using the Maximum Entropy Principle. To take into account the uncertainties of the dynamical system in the optimization, a multi-objective optimization of some statistical characteristics of a distance between the response of the system and the target performance was proposed and solved using the weighted sum approach. The GBNM algorithm was employed in the optimization due to its ability to handle non-convex and multimodal functions. The results showed that: (1) even for a very simple system, the optimization problem can be complicated, (2) the GBNM has successfully dealt with the optimization of the stochastic function; (3) when the uncertainties are considered, the robust optimum design is different from the deterministic optimum, and (4) the robust optimization is a very useful tool to deal with uncertainties in dynamical systems.

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