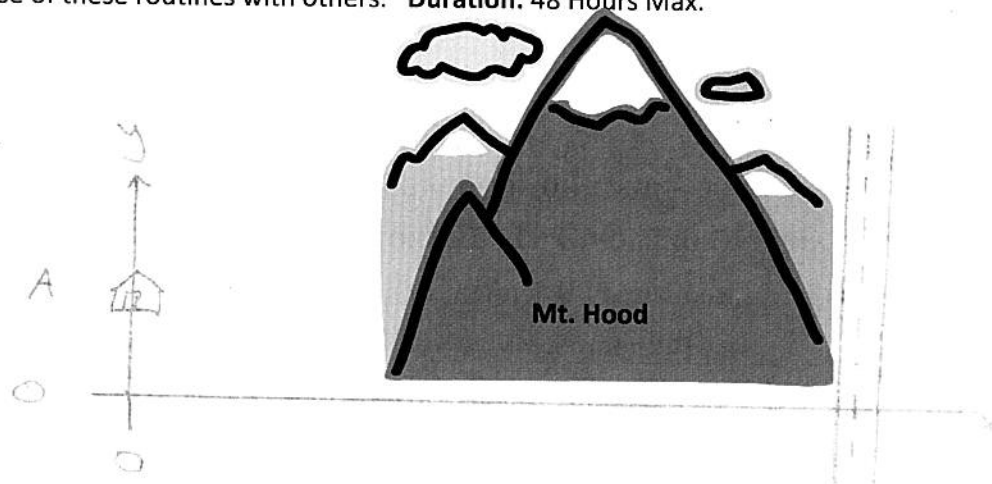
AFIT GRADUATE SCHOOL OF ENGINEERING AND MANAGEMENT

DOCTORAL EXAM COVERING OPTIMIZATION

for Mr. Tim Coon

December 2014

Take Home. (open notes, text and MATLAB) Do not discuss questions or solutions with anyone. You may use any MATLAB routines developed by you as well as by others, but you may no longer discuss the use of these routines with others. **Duration:** 48 Hours Max.



Question: You are on a trek from your house at point A to the road (which is parallel to the y axis) on the far right. You want to make the trip while expending the minimum amount of energy. Obstacles along the way cause different amounts of energy consumption. Your job is to find the path, y(x) for (x≥0), that takes you from your house at A (x=0, y=1) to the road (x=3) while consuming minimum energy. The energy consumed along the way is described by:

$$E = \int_0^3 \left(1 + \left(\frac{dy}{dx}\right)^2\right)\left(1 + e^{-\left[(x-2)^2 + (y-2)^2\right]}\right) dx$$

Solve this problem using a direct pseudospectral collocation method. You may, use MATLAB to assist you in solving the minimization problem. (GPOPS, Fmincon, etc) Show all work. Compare the results obtained as the number of collocation points increases.

Description and Comparison

The min energy problem is solved using the x-position as the independent variable, rather than time. Because the integrand of the cost function is strictly positive and regular, it is unlikely for a minimum energy path to double back in the x-direction, so using x-position as the independent variable is acceptable. Evaluating the cost is more simple because the integrand of the cost functional is taken with respect to the x-direction. The MATLAB© code utilizes GPOPS-2©, Ver. 2.0 to solve the optimal control problem as follows.

Optimal Control Problem

Find the control, **u**, in the set of admisible controls, **U**, that minimizes the following cost functional

$$\min_{\mathbf{u}\in\mathbf{U}} \quad E = \int_0^3 \left[1 + \left(\frac{dy}{dx}\right)^2\right] \left[1 + \exp\left(-\left[(x-2)^2 + (y-2)^2\right]\right)\right] \tag{1}$$

subject to the dynamic constraint

$$y\prime = u \tag{2}$$

where the control is

$$u = \frac{dy}{dx} \tag{3}$$

with the boundary conditions,

$$x0 = 0 \qquad y(x0) = 1 \tag{4a}$$

$$xf = 3 \qquad y_{min} \leq y(xf) \leq y_{max} \tag{4b}$$

Solution

The MATLAB© code and plots are attached. In the figure, solutions determined using a different number of collocation points and mesh intervals are plotted along with the "energy terrain" of the area. Neither the path nor the cost change significantly with the number of collocation points or the number of mesh intervals. The optimal path is reasonable as it moves toward the lowest energy area at the base of the mountain without making significant trajectory changes ($\frac{dy}{dx}$). The choice of x as the independent variable is further verified because the optimal control vector (the slope) does not approach the user-defined limits on either end ($u_{min} = -10$, $u_{max} = 10$).

Furthermore

I removed the control contribution to the cost functional, Equation (5) and the results vary more significantly with the number of collocation points.

$$\min_{\mathbf{u}\in\mathbf{U}} \quad E = \int_0^3 \left[1 + \exp\left(-\left[(x-2)^2 + (y-2)^2\right]\right)\right] \tag{5}$$