ACTIVE VIBRATION CONTROL OF OPTICAL SPACE SYSTEMS

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ABSTRACT
A spatio-temporal filter (STF) based active vibration control method is presented as an approach to meet the motion stability requirements for next generation, segmented, optical space systems. STF theory is presented and experiment results obtained on the Air Force Research Laboratory's (AFRL) Precision Deployable Optical Structure (PDOS) and the Mid-Deck Active Control Experiment II (MACE II) hardware are shown. Simulation results are also shown for AFRL's Deployable Optical Telescope (DOT).

NOMENCLATURE.

$x(t)$	Time domain data points
ϕ	Modal or eigenvector
$\eta(t)$	Modal coordinate response
λ	System pole
ω_d	Damped natural frequency
ω_n	Undamped natural frequency
σ	Real part of system pole
l	Modal participation vector
ψ	Spatio-Temporal Filter vector

1. INTRODUCTION

Many next generation optical space systems such as space based lasers, space observing space telescopes and earth observing space telescopes will utilize very large and/or segmented mirror elements in order to achieve the resolution desired. These systems will require that the elastic deformation of and/or relative motion between elements be reduced to unprecedented levels. The Deployable Optical Telescope (DOT) experiment being developed by the Air Force Research Laboratory (AFRL) requires relative piston motion between mirror segments to be below 10^{-9} meters (10 nanometers). This is roughly the deflection of a street manhole cover under the weight of a dime.

In the context of these position stability requirements the space environment is not "quiet". Disturbances from reaction wheels, slewing maneuvers and, particularly for space based chemical laser applications, pumps and fluid forces, can cause "large" vibrations.

Active vibration control is a candidate solution for achieving the element stability required, however, classic modern control techniques have disadvantages. Perhaps the most significant of these disadvantages is the need for a complete and accurate model of the plant dynamics. The dynamic response of real-world structural systems is typically very complex. It is difficult to create robust, control oriented models of these systems, particularly if they are time varying which is often the case. To insure control stability and performance these models must include the effects of all dynamics in the frequency range of interest, meaning, typically, many structural vibration modes must be accurately modeled. In many real world applications this modeling effort can represent the majority of the total effort required to develop an active control strategy. Alternative approaches that do not require such an extensive and accurate system model are preferable.

Sheet Dynamics, Ltd., (SDL) is working with AFRL to develop active control concepts that require little knowledge of plant dynamics in order to implement effective, multi-input, multi-output vibration control systems. In addition, the systems will accommodate sensor and actuator failures and can be easily updated to account for changing plant dynamics. These spatio-temporal filtering (STF) algorithms will be demonstrated on the ground based DOT experiment and on the MACE II experiment scheduled to be conducted on the International Space Station in 2000.

2. SPATIO-TEMPORAL FILTER BASED CONTROL

The concept of Spatio-Temporal Filtering is simple; the dynamic response characteristics of a complex, multi-input, multi-output system with many structural modes can be decomposed into its fundamental components; simple first order systems described by a single pole value and input and output scaling coefficients. These simple, canonical components can easily be controlled or monitored independent of the complexities of the rest of the system.

STF is a sensor/actuator array based approach. It utilizes the sensing and actuation capability of multiple sensors and actuators in an integrated manner rather than individually. It is made feasible by recent developments in lower cost sensors and actuators and associated digital signal processing electronics. It is now feasible to use larger numbers of sensors and actuators for structural vibration control and monitoring.

This approach is unlike classical system identification/model based control. Rather than attempting to identify a complete and accurate model of a complex, possibly time varying system, STF based control attempts to make all of the system response except that associated with the modes of interest completely unobservable. This enables dealing with only the modes contributing to the phenomenon of interest – the rest of the system characteristics are of no concern and are not considered or unnecessarily accounted for; they are intentionally made invisible. This approach allows complex systems that are difficult to accommodate with conventional methods to be controlled and monitored.

Figure 1 illustrates using STF based control for vibration suppression of two critical low frequency modes of a complex system. The figure shows frequency response functions (FRFs) of a physical response, uncontrolled modal coordinates extracted with STF, controlled modal coordinate responses and the resulting closed loop physical system response.

The merit of this basic approach has been recognized by others in the past, however, a practical implementation has not previously been achieved. Meirovitch [1] proposed using strictly spatial or modal filters to implement Independent Modal Space Control (IMSC). The method required a distributed parameter model of the system in order to calculate modal filter coefficients. This is not possible for most systems of practical interest. The STF approach differs in two ways; 1) it filters in both space and time is to achieve higher accuracy with fewer sensors and; 2) a practical, reference model based approach is used to adaptively calculate spatio-temporal weighting coefficients. This approach requires minimal knowledge of the system, and given sufficient redundancy, corrects for sensor and actuator failures.

2.1 STF Background

STF is an extension of modal, or spatial, filtering which has been investigated by the authors and other researchers for some time[1-6]. Modal filtering utilizes the characteristic that the dynamic response of any real structure is composed of a sum of individual modal responses, each behaving as a single-degree-of-freedom (SDOF) system and each having a particular response shape or eigenvector. The modal filter approach applies sets of scalar, spatial weighting coefficients to responses measured by each element of a sensor array to extract these individual canonical modal responses from the global response of the structure. Note the modal filter is **NOT** related to a conventional bandpass filter. Each channel of the modal filter has output across the entire frequency band, however, it is the output associated with just a single mode.

The basis for modal filtering is the standard modal coordinate transformation that is utilized to simplify the solution, understanding, and analysis of systems of linear differential equations. For simplicity consider the undamped, structural case where the common discrete system model consists of a second order linear differential equation with N by N matrix coefficients of mass and stiffness terms.

$$M\ddot{x} + Kx = f \qquad (1)$$

Equation **Error! Reference source not found.** may be solved for N linearly independent eigenvectors ϕ_r. Since the eigenvectors are linearly independent the system response, x, may be represented as a linear combination of the eigenvectors weighted by the

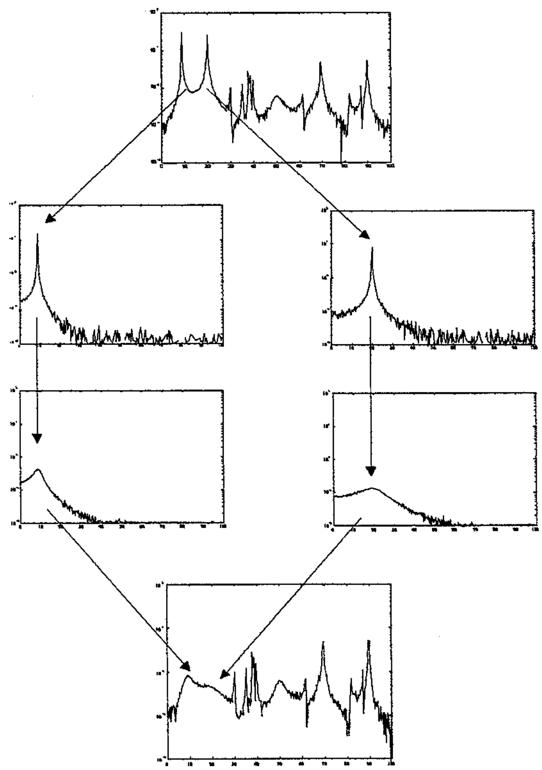

Figure 1. STF Based Structural Vibration Control: Uncontrolled Physical Response; STF Modal Coordinate Estimates – Uncontrolled; STF Modal Coordinate Estimates – Controlled; Resulting Controlled Physical Response.

canonical degrees of freedom or modal coordinates, $\eta_r(t)$.

$$\begin{aligned} x(t) &= \sum_{r=1}^{N} [\phi_r \eta_r(t)] \\ &= \Phi\eta(t) \end{aligned} \qquad (2)$$

For the physical case, where control and monitoring is being conducted on a structure, the response vector, $x(t)$, is measured with sensors located at corresponding physical locations and measurement directions. A modal filter is applied to the measured response data, $x(t)$, to extract the modal coordinate response(s), $\eta_i(t)$, of interest. To extract the modal coordinate response for the $i'th$ mode, a vector of spatial weighting coefficients, ψ_i, is sought which has the following characteristics;

$$\begin{aligned}\psi_i^T \phi_r &= 0 \quad i \neq r \\ &= 1 \quad i = r\end{aligned} \tag{3}$$

The inner product between the modal filter vector, ψ_i, and response vector, $x(t)$, is formed which is equivalent to forming a weighted average of the response signals measured at different locations on the structure.

$$\begin{aligned}\psi_i^T x(t) &= \psi_i^T \sum_{r=1}^{N} [\phi_r \eta_r(t)] \\ &= \psi_i^T \phi_i \eta_i(t) \\ &= \eta_i(t)\end{aligned} \tag{4}$$

The resulting scalar quantity is the modal coordinate response, $\eta_i(t)$, for the $i'th$ mode and the vector, ψ_i, is the associated modal filter vector. The above discussion holds for the damped case as well, both proportional and non-proportional [3].

2.2 Spatio-Temporal Filtering

The spatio-temporal filter is a generalization of the spatial or modal filter which extends the capabilities by utilizing temporal information; two dimensional filtering in the space and time dimensions. This reduces the number of sensors required, allows dissimilar sensors to be integrated, and accommodates sensor dynamics.

In order to extract the modal coordinate response of interest with modal filters, the modal vectors, as sampled at the sensor locations, must be linearly independent [3]. This dictates that at least as many sensors as there are independent modes contributing to the measured response are required. Even with modern low cost sensors and DSP electronics this will be considered a disadvantage in some applications.

The modal filter estimates the modal coordinate response at time k by forming a weighted summation of sensor signals measured at different spatial locations at time k;

$$\hat{\eta}_k = \psi^T x_k \tag{5}$$

An N_i'th order spatio-temporal filter also utilizes N_i past samples of the response information;

$$\hat{\eta}_k = \psi^T \begin{Bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-Nt} \end{Bmatrix} = \psi^T X_k \tag{6}$$

This introduces a different N_i'th order finite impulse response (FIR) or all-zero filter on each sensor channel. The FIR filters perform different functions depending on the specific implementation; pole-zero cancellation if spatial resolution alone is insufficient to separate modes, accommodating the relative phase between sensors caused by complex (non-proportional damping) modes, selective differentiation if a nonhomogeneous sensor array is used, and correction for some sensor dynamics. Most likely a combination of the above characteristics will be manifested in the temporal filter component of the STF.

2.3 Reference Model Solution

This section introduces a reference model approach for adaptively calculating and updating the STF filter coefficients with very little a priori information. This enables the method to be applied to any arbitrarily complex real-world structure and accommodate sensor and actuator failures in a manner which it transparent to control and monitoring algorithms.

The structure of the STF filter estimation problem is similar to other estimation problems in that an error is defined which is a function of the parameters (in this case STF filter vector coefficients) to be estimated. The parameters are estimated by minimizing the error. As with other estimation problems, different solution methods may be employed to minimize the error to arrive at a solution that is optimal in some sense.

The subscript denoting mode number on the variables is dropped. The development is applicable to any single mode. Discrete time is assumed with the subscript now indicating sample number.

For clarity, first consider a single input STF (no temporal information) estimation problem. An error is defined which is the difference between the true modal coordinate, η_k, and the modal coordinate estimated by the STF, $\hat{\eta}_k$, at time k.

$$\begin{aligned}e_k &= \eta_k - \hat{\eta}_k \\ &= \eta_k - \psi_k^T X_k\end{aligned} \tag{7}$$

The true modal coordinate, however, is not known. Indeed, estimating η_k is the purpose of the STF. A reference modal coordinate, $\eta_k^{(r)}$, which is highly correlated with the true modal coordinate may be generated by driving a SDOF reference system constructed from only the pole of the mode of interest. The first order, discrete time reference system is;

$$\eta_{k+1}^{(r)} = z_\lambda \eta_k^{(r)} + f_k \tag{8}$$

z_λ is the Z domain pole $z_\lambda = e^{\lambda \Delta t}$. In this case the driving force, f_k, is the measured control force. The reference modal coordinate is then used in Equation (7) in place of the true modal coordinate to calculate the error. The solution problem is to minimize this error over a number of time steps to estimate ψ. The structure of this problem is illustrated in Figure 2.

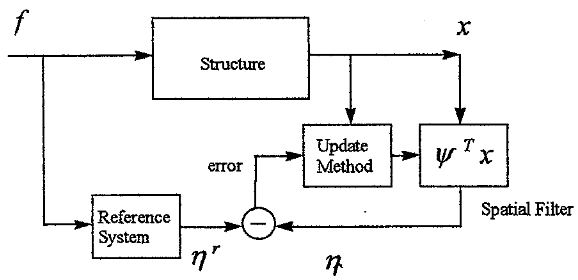

Figure 2: Structure of the STF Estimation Problem.

For the multi-input case, the total modal coordinate response of each mode is due to multiple input forces. The effect of each input is described by an unknown vector of, possibly, complex force appropriation coefficients, l (also called modal participation vectors) [3]. The reference model becomes;

$$\eta_{k+1}^{(r)} = z_\lambda \eta_k^{(r)} + l^T f_k \tag{9}$$

where f_k is now a vector of applied forces. In this case the reference system is driven with a modal force consisting of the sum of the input forces weighted by the force appropriation vector coefficients. An equivalent reference modal coordinate may be generated by driving N_i reference models;

$$\begin{aligned} \eta_{k+1}^{(r1)} &= z_\lambda \eta_k^{(r1)} + f_k^{(1)} \\ &\vdots \\ \eta_{k+1}^{(rN_i)} &= z_\lambda \eta_k^{(rN_i)} + f_k^{(N_i)} \end{aligned} \tag{10}$$

by the unweighted N_i forces and using the force appropriation vector to form a weighted average of the Ni reference modal coordinates.

$$\begin{aligned} \eta_k^{(r)} &= l^T \begin{Bmatrix} \eta_k^{(r1)} \\ \vdots \\ \eta_k^{(rN_i)} \end{Bmatrix} \\ &= l^T \eta_k^r \end{aligned} \tag{11}$$

Note the distinction between $\eta_k^{(r)}$ which is the scalar modal coordinate and η_k^r which is a vector of partial modal coordinate responses associated with the individual input forces. The general spatio-temporal filter error with both input and output temporal filtering is then given in Equation (12).

A trivial solution which must be avoided is the zero error solution where both the response STF filter vector, ψ, and force appropriation vector, l, are zero. This may be accomplished by artificially normalizing one of the coefficients to, for instance, unity. This has the drawback that, if the coefficient is physically close to zero amplitude, the problem is ill conditioned and an inaccurate solution results. The preferable solution is to impose a norm constraint on the solution vector where, for instance, the norm of the solution vector is constrained to unity. Additional norm constraint considerations arise when the updating the STF inside a control loop. In this case the norm constraint must be applied only to the response weighting filter coefficients.

$$\begin{aligned} e_k &= \eta_k^{(r)} - \hat{\eta}_k \\ &= l^T \begin{Bmatrix} \eta_k^r \\ \eta_{k-1}^r \\ \vdots \\ \eta_{k-Nti}^r \end{Bmatrix} - \psi^T \begin{Bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-Nto} \end{Bmatrix} \\ &= \begin{Bmatrix} \psi \\ l \end{Bmatrix}^T \begin{Bmatrix} -x_k \\ \vdots \\ -x_{k-Nto} \\ \eta_k^r \\ \vdots \\ \eta_{k-Nti}^r \end{Bmatrix} \end{aligned} \tag{12}$$

Different solution methods have been utilized for the STF filter estimation method. Of most interest are adaptive, on-line methods since they recover from sensor and actuator failure. In the event of a sensor failure the response related STF filter coefficients update to continue to optimally estimate the modal coordinate response with the remaining sensing capacity. Provided sufficient sensing capability remains to estimate the modal coordinates, a controller utilizing these outputs for feedback control would be unaffected by the sensor failure. Multiple input modal controllers apply a control force vector which is either l, the force appropriation vector component of the STF or a direct function of it. The "force" driving the reference models is generally the command to the control actuators. An actuator failure is reflected in the associated force appropriation coefficient estimated by the STF and is inherently accommodated.

To date, two different real-time adaptive STF update methods have been utilized; Least Mean Squares (LMS) and Recursive Least Squares (RLS) [7].

The benefits of the RLS approach are superior convergence speed and accuracy. It has a higher computational demand, however, multiple modes can be estimated with little additional computational burden. The additional computational burden to update a STF with Ns sensors and Nt time taps is approximately 2*Ns*Nt floating point multiplies and adds.

2.4 STF Based Control

The STF is not, in itself, a vibration controller. However, design of very effective STF based, multiple-input, multiple-output, active vibration suppression controllers can entail selection of merely a single scalar control gain parameter for each mode to be controlled.

The reference model utilized to adaptively update the STF coefficients can take the form of a position, velocity or acceleration output model. The adaptive STF will attempt to form a modal coordinate output that matches the reference model. For resonant vibration suppression a modal coordinate velocity output is often desired to utilize directly for rate feedback control.

In this case the control command for each mode consists of;

$$f_c^{(i)} = \hat{\dot{\eta}}^{(i)}\alpha^{(i)}\nu^{(i)} \tag{13}$$

where $f_c^{(i)}$ is the control command (generally a force command) vector output to control the i'th mode, $\hat{\dot{\eta}}^{(i)}$ is the estimate of the modal coordinate velocity of the i'th mode generated by the STF, $\alpha^{(i)}$ is the control gain and $\nu^{(i)}$ is the forcing vector. The theoretical control gain required to achieve a certain level of damping can be calculated, however, in practice it is often more effective to manually adjust control gain.

A number of considerations may effect the choice of forcing vector. In general, the forcing vector should be chosen to project strongly on the force appropriation vector (FAV). This is desired since the resulting modal control force is the inner product of the FAV and the control force vector, $\ell^{(i)^T} f_c^{(i)}$. A good choice of control force vector to maximize this inner product is the FAV vector itself. Since the STF automatically generates the FAV vector it is the logical choice. For each modal controller the control force is;

$$f_c^{(i)} = \hat{\dot{\eta}}^{(i)}\alpha^{(i)}\ell^{(i)} \tag{14}$$

Control design, then, consists of choosing the control gain, $\alpha^{(i)}$, for each controlled mode. Multiple modal controllers are run in parallel to control multiple modes. In this case the physical control force command is the sum of the individual modal control forces.

3. STF CONTROL OF OPTICAL SPACE SYSTEMS

The AFRL DOT is a functional, ground based model of a 5 meter diameter sparse array, deployable optical space telescope (Figure 3). It has three deployable 60 cm primary mirrors (1.7 meter array diameter) and a deployable secondary mirror tower. The objective of the DOT program is to demonstrate autonomous deployment, phase capture and maintenance of the primary mirrors in the presence of realistic operating vibration excitation, including reaction wheel forces, slewing maneuvers and chemical laser disturbances.

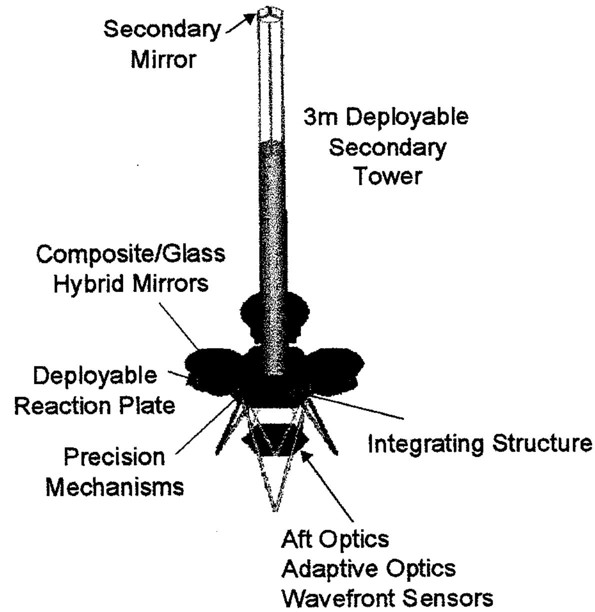

Figure 3: Deployable Optical Telescope Experiment.

An evolutionary step in the DOT program was the Precision Deployable Optical Structure (PDOS), a single, full size deployable "petal" of the 5 meter telescope concept. PDOS consists of a 1.5 meter diameter mirror simulator on a deployable carbon composite boom hinged from an isolated granite slab (Figure 4). Mirror piston, tip and tilt motion is sensed and controlled with three laser interferometer/piezoelectric stack actuator pairs.

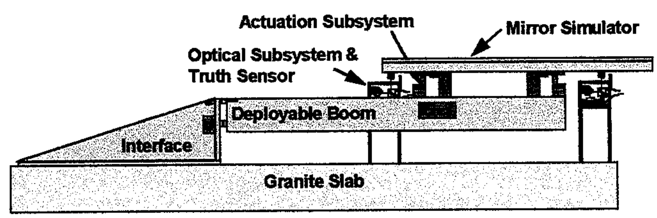

Figure 4: PDOS Experiment Hardware.

The primary resonant response on the mirror is due to the cantilever mode of the deployed boom. An STF controller was implemented using the three interferometers and piezoelectric actuators to control this mode. Figure 5 and Figure 6 show the open and closed loop response of Interferometer 1 due to a vertical impact on the boom. In Figure 6 the STF controller is turned on at the 6 second mark, immediately damping out the boom mode. The remaining oscillation is a 1.8 Hertz bounce mode of the granite slab on it's isolators which is uncontrollable.

At the time of writing the DOT hardware was not operational so control algorithm development is being conducted in simulation. Initial experimental work will be conducted without the secondary tower since it will not be installed until mid 2000. In this configuration the DOT structure is quite stiff, with the first elastic mode at approximately 80 Hz. Figure 7 shows open and closed loop frequency response

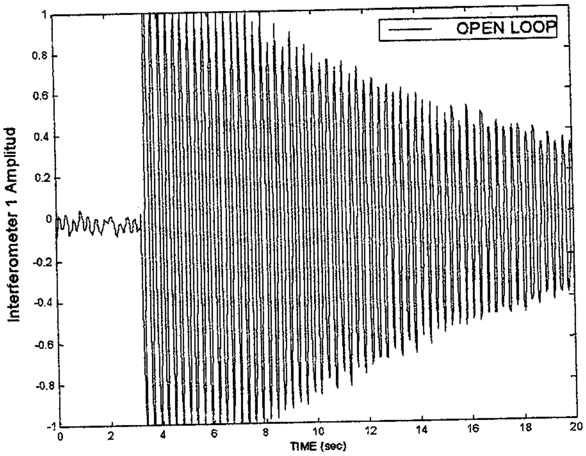

Figure 5: PDOS Vertical Open Loop Free Decay.

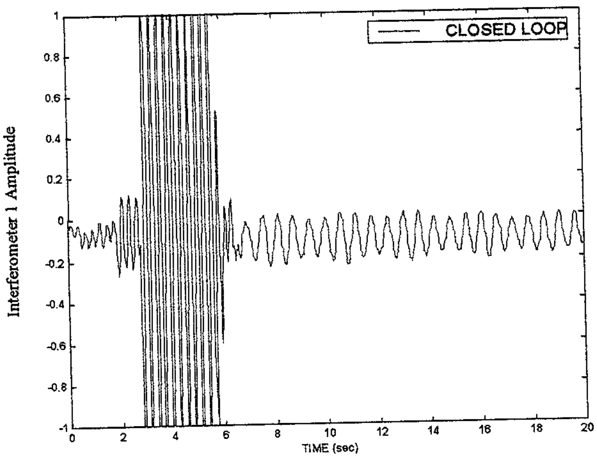

Figure 6: PDOS Vertical Closed Loop Free Decay.

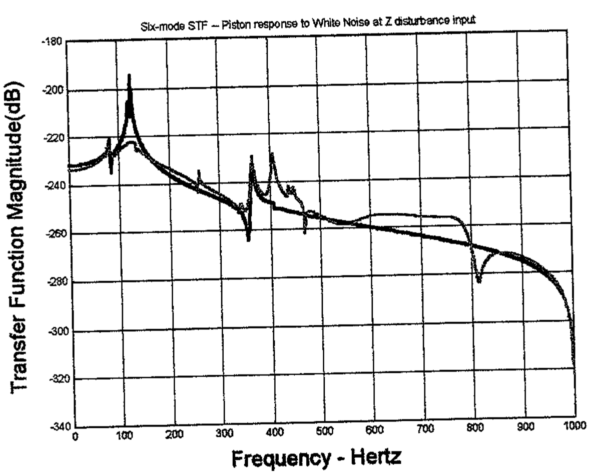

Figure 7: DOT Control Simulation Results.

functions (FRFs) from a vertical disturbance force on the integrating structure to a mirror piston motion, with and without STF control. This is a 40 mode, finite element derived simulation model characterized by clusters of three very close modes due to the three identical deployable segments. The control system utilizes three piezo stack actuators and a measure of tip, tilt and piston motion on each of the three primary mirrors.

For the closed loop data in Figure 7 STF controllers are implemented on the modes at 80.84, 81,0 82.42, 121.0, 121.7 and 125.8 Hertz. Reductions in resonant mode amplitude of 30 db are achieved, although, some higher frequency modes have been adversely affected.

The original MACE I experiment was conducted in the space shuttle mid-deck. The goal was to test how well precision pointing controllers could be designed and tested in a 1 g earth environment for operation in a 0 g environment. The MACE II experiment is being conducted on the International Space Station in mid-2000 to test the effectiveness of adaptive or self-learning pointing control approaches in a 0 g environment. The MACE hardware, shown during the MACE I experiment in the shuttle mid-deck in Figure 8, consists of a 1.5 meter reconfigurable, flexible structure. Rigid body motion is controlled by three reaction wheels at the center of the bus. Precision pointing is achieved using two-axis gimbals located at the ends of the bus. An active piezoelectric strut completes the actuator suite. Sensors include strain gages, digital encoders and rate gyros.

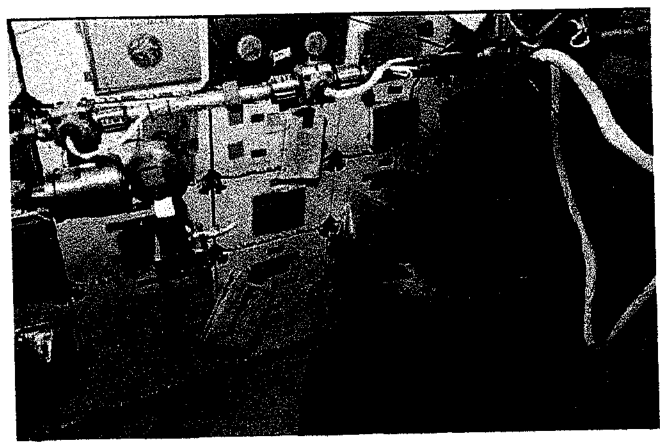

Figure 8: MACE I Flight Experiment

Preliminary ground experiments have been conducted on the MACE II hardware at AFRL. STF filters and force appropriation vectors for the first four structural modes were identified. The STF filters were configured to estimate the velocity corresponding to the targeted modes. These estimated modal velocities were then fed back to increase the damping of the targeted modes. The open/closed loop FRFs between the secondary Z gimbal disturbance and strain gage No. 1 are shown in Figure 9. Greater than 20 db reduction in the amplitude of major resonant peaks is obtained.

Resonant mode control results obtained on PDOS, the MACE II hardware and on the DOT model simulation were very good. In all three cases, however, it was clear that control of resonant vibration was not sufficient to meet the control objectives. Much of the RMS disturbance response is due to forced, non-resonant response of the systems.

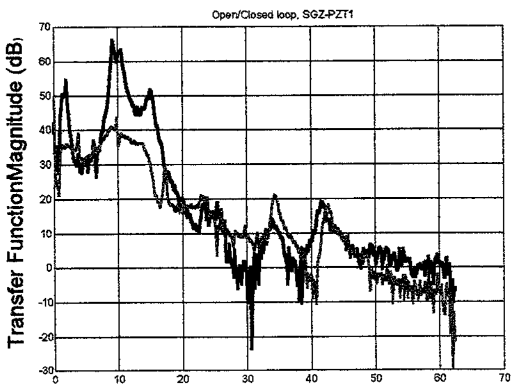

Figure 9. FRF from Secondary Z Gimbal Disturbance to Strain Gauge Number One.

To address this need a modified STF control approach is being developed which controls off-resonant, residual flexibility or residual mass response. Figure 10 shows open and closed loop FRFs between an X-Axis DOT base disturbance input and a mirror piston response. Figure 11 shows the associated open and closed loop time responses for the proposed test facility ambient floor disturbance. This disturbance is primarily in the 10 to 30 Hertz frequency band (recall the first DOT mode is at approximately 80 Hertz). Figure 11 shows the RMS time response amplitude is significantly reduced. This approach is being refined and it is expected that it will complement the STF resonant response control approach described in detail in this paper.

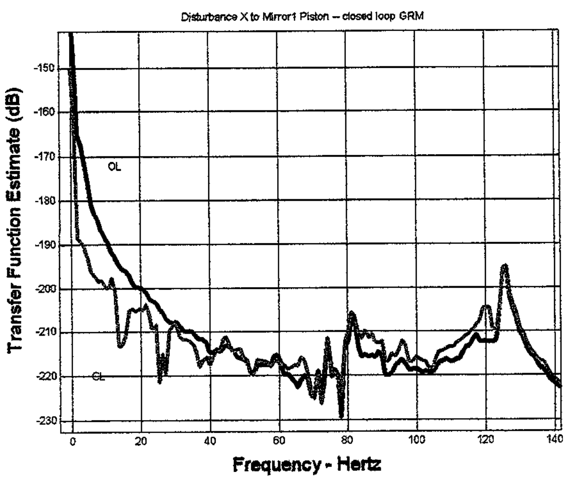

Figure 10: Low Frequency, Off-Resonance Control Performance of Modified STF Controller.

4 CONCLUSIONS

STF based control has been demonstrated to be an effective control approach for control of resonant vibration in optical space systems. It is currently being extended to accommodate off-resonant forced vibration. Demonstration on DOT hardware will be conducted early in 2000.

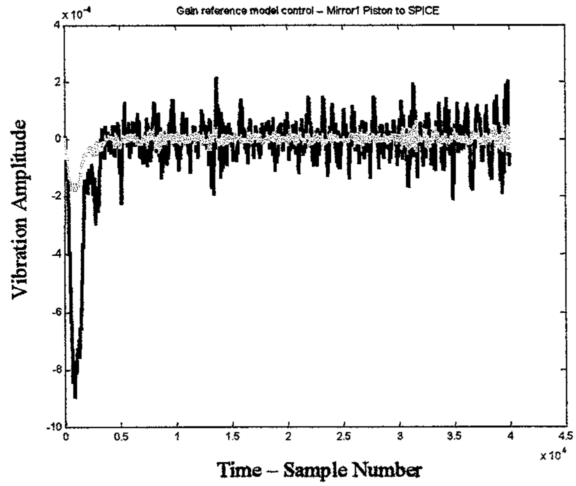

Figure 11: Open and Closed Loop Piston Response to SPICE Disturbance.

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