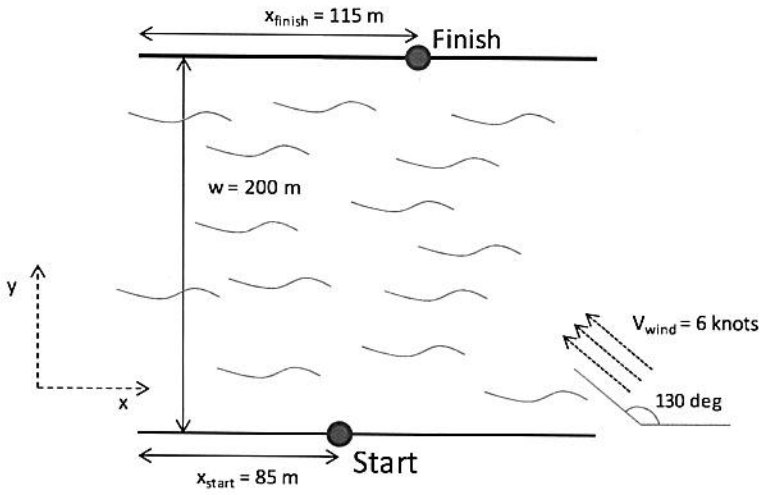
Optimal Control Qual Exam for Tim Coon
Take-Home (48 hours)

Numerical Optimization of a River Crossing:

Your task is to cross a river in minimum time using a sailboat (you may assume a point mass model). A diagram of the river crossing is shown in the below figure.



The river is flowing *from left to right* with $V_{river} = 4*V_{max}*y*(w-y)/w^2$, where V_{max} is 32 knots. Your solution to this problem must include a diagram of the crossing profile (e.g. path as a function of x and y) and a plot of the sail angle vs. time for the optimal path. You may not use GPOPS for this problem, but you may use any method within Matlab (e.g. *fmincon*, *fsolve*, etc.) that you deem appropriate. In addition to the above mentioned plots, please include:

a) A formal statement of the discretized static optimization problem that you used to solve for your optimal path. Include your objective function and ALL constraints.
b) The augmented Lagrangian function. Write out each of the terms expanded as much as possible, but leave in terms of *i*. That is, expand the state terms but not time other than the first and last time steps as is traditional.
c) The similarly expanded Hamiltonian, if you did not expand as part of b).
d) Perform an analysis (experimenting is sufficient) to determine the value of V_{max} that results in an optimal path with the minimum crossing time (an optimum of optimums, so to speak). Why is the river velocity that results in this not zero?
e) What is the minimum crossing time if you are not required to finish at a specific location on the far bank (use V_{max} = 32 knots)? Which far bank location results in this minimum crossing time?
f) *Optional:* If you solve this problem by supplying the gradients to the optimizer in Matlab, you will automatically get full credit. You must also state why your code does not require the explicit calculation of the lagrange multiplier, v.

$\therefore$ b/c $(x-x_f)+(y-y_f)=0 \equiv (x-x_f)=0 \;\&\; (y-y_f)=0$

b/c $(x-x_f)\le 0 \;\&\; (y-y_f)\le 0$

INCLUDE ANY CODE YOU WRITE (comments are helpful)

Only Dr. Cobb or Maj. Dillsaver may answer questions. Open book, (your) open notes. You may use any code from Mech 622 that you consider helpful. Do not use the internet.