Linearized Ray-Trace Analysis

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Abstract

A new, coordinate-free version of the exact ray-trace equations for optical systems consisting of conic reflecting, refracting and reference surfaces is presented. These equations are differentiated to obtain closed-form optical sensitivity dyadics. For computation, the sensitivities are evaluated in a single global coordinate frame and combined in linearized ray-trace matrix difference equations that propagate the rays and the sensitivities from element to element. One purpose of this analysis is to create optical models that can be directly integrated with models of the instrument structure and control systems for dynamic simulation.

1. Introduction

Large optical instruments such as ground-based or spaceborne astronomical telescopes, or any optical instruments incorporating articulating optical elements, are subject to dynamic motions that affect the beam train geometry. Whether due to deformations of the instrument structure or to controller actions, these dynamic disturbances can deform the beam train alignment and distort element figure, altering the instrument line-of-sight, distorting images, lowering signal-to-noise ratios, reducing the visibility of interference fringes, or decreasing wavefront quality. For large spaceborne optical instruments in particular, optical performance is a function not only of optical parameters, but also of structural configuration and stiffness parameters, of structural thermal qualities, and of control system sensor, actuator and control loop parameters. Our experience in designing and analyzing this type of instrument leads us to conclude that the designer must take dynamic factors explicitly into account. To acheive the best design, it is necessary to trade off optical, structural and control system design variables *as a function of time* to achieve the best system performance while minimizing instrument mass and power requirements. We are in the process of developing new computational tools to support this integrated design task.

To efficiently calculate optical performance of such an instrument in the time domain, and to perform other necessary analyses of controlled optical instruments, we have developed a new formalism for ray tracing general optical beam trains. Our approach is based on the modular coordinate-free version of the exact ray-trace equations summarized in this paper for conic optical elements. For computational purposes, these equations are realized in a single global coordinate frame, eliminating the need to perform coordinate transformations at each new optical element (as is required in more traditional ray-trace techniques[1]). The result is ray-trace computer routines that are fast enough to run in-line with large dynamic simulations.

In addition to the exact ray-trace equations, we have derived coordinate-free, closed-form *differential* ray-trace equations for the calculation of the optical sensitivities of rays to (conic) optical element motions and design parameters. These equations, summarized below, can be used to create linearized ray-trace optical models for optical tolerancing and integrated system analysis. The sensitivities are also required for other purposes, such as for determining the transformations between optical sensor signals and the corresponding control system actuator actions.

Our approach has been realized in the Controlled Optics Modeling Package (COMP), an optical model generating computer tool which computes linearized and exact optical models in the form of fixed matrices and nonlinear subroutines. These optical models are combined with standard structure and control models to obtain fully integrated, end-to-end *system* models that can be exercised in static and dynamic, time- and frequency-domain analyses and simulations to determine total system performance as a function of optical, structural and control system parameters. This enables the cross-disciplinary design trade studies required for integrated design.

Due to space constraints, the discussion in this paper is limited to a partial summary of the exact ray-trace equations and the linearized differential ray-trace equations. The full set of equations, their derivations and examples of their use in the context of large space optical instrument design and analysis are given in Refs. 2-4.

2. A Note on Notation

Most of this work is conducted in coordinate-free vector notation. Vectors and dyadics are signified by bold face type, with over-arrows signifying general vectors and hats signifying unit-vectors. Vector super-cross ($\vec{\mathbf{v}}^{\times}$) signifies the "cross-dyadic" $\tilde{\mathbf{v}}$[5]. Where convenient, we also use matrix notation, signified by plain type, with column matrices distinguished by arrows and unit-column matrices by hats. Matrix super-cross signifies the cross product matrix operator:

$$\vec{v}^{\times} = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

I and I are the unit dyadics and identity matrices, respectively. $\mathbf{P}_u = \mathbf{I} - \widehat{\mathbf{u}}\widehat{\mathbf{u}}$ is the "projection dyadic," for which the product $\mathbf{P}_u \cdot \vec{\mathbf{v}}$ is the component of $\vec{\mathbf{v}}$ perpendicular to $\hat{\mathbf{u}}$.

3. Exact Ray-Trace Equations

This section considers the tracing of a ray past a single reflective or refractive element with a conic-of-revolution surface, one of many, perhaps, in a beam train. We assume (without loss of generality) an element numbering scheme, subscripting each surface in the order that the ray strikes it. Doing this, the ray segment incident on the i^{th} element is the ray segment output from the $i\text{-}1^{th}$ element and is denoted $\hat{\mathbf{r}}_{i-1}$. If the surface is reflective, the next ray segment ($\hat{\mathbf{r}}_i$) can be determined from the mirror properties at the point of incidence, by applying the Law of Reflection. Similarly, if the surface effect is refractive, $\hat{\mathbf{r}}_i$ is determined by applying Snell's Law. In what follows we present coordinate-free versions of these laws.

A conic-of-revolution surface can be represented in vector three-space in terms of a "surface dyadic" **M** and a vector $\vec{\mathbf{N}}_0$. Denoting the surface principal axis direction as $\hat{\psi}$, the surface eccentricity as e, and its focal length as f (Fig. 1), **M** is

$$\mathbf{M} \equiv \left(\mathbf{I} - e^2\widehat{\psi}\widehat{\psi}\right). \tag{1}$$

The principal axis $\hat{\psi}$ is defined as pointing from the surface vertex towards its focus. For flat surfaces we say that $e = 0$, $f = \infty$, and $\hat{\mathbf{N}} = \vec{\mathbf{N}} = \hat{\psi}$. Writing $\vec{\rho}$ for a vector from the mirror vertex to an arbitrary point on the surface, the surface is defined by

$$\vec{\rho} \cdot \mathbf{M} \cdot \vec{\rho} + 2\vec{\mathbf{N}}_0 \cdot \vec{\rho} = 0 \tag{2}$$

$\vec{\mathbf{N}}_0$ is a vector parallel to the surface normal at the vertex of the surface with magnitude equal to the radius of curvature evaluated at the mirror vertex:

$$\vec{\mathbf{N}}_0 = -f(1+e)\hat{\psi}. \tag{3}$$

The vector $\vec{\mathbf{N}}$ at an arbitrary surface point $\vec{\rho}$ with respect to the vertex is:

$$\vec{\mathbf{N}} = \vec{\mathbf{N}}_0 + \mathbf{M}\vec{\rho}. \tag{4}$$

$\vec{\mathbf{N}}$ is parallel to the surface normal at $\vec{\rho}$; its magnitude is the local radius of curvature.

Now consider $\vec{\rho}$ to designate not an arbitrary point, but rather the point of incidence on the surface of a ray originating at a point $\vec{\mathbf{p}}$ (with respect to the surface vertex) with direction $\hat{\mathbf{r}}_{i-1}$, as sketched in Fig. 1. To avoid ambiguity in the discussion of refractive elements, we adopt a convention that the unit normal $\hat{\mathbf{N}}$ is pointed away from the incident ray direction $\hat{\mathbf{r}}_{i-1}$:

$$\hat{\mathbf{N}} = -\mathrm{sign}(\hat{\mathbf{i}} \cdot \vec{\mathbf{N}})\frac{\vec{\mathbf{N}}}{\mathbf{N}} \tag{5}$$

where the magnitude N (the radius of curvature at $\vec{\rho}$) is

$$\mathrm{N} = |\vec{\mathbf{N}}|. \tag{6}$$

The point of incidence is determined as:

$$\vec{\rho} = \vec{\mathbf{p}} + \mathrm{L}\hat{\mathbf{i}} \tag{7}$$

where L is the geometrical length of the incident ray. Substituting for $\vec{\rho}$ in Eq. 2, L is a solution of

$$\left(\hat{\mathbf{r}}_{i-1} \cdot \mathbf{M} \cdot \hat{\mathbf{r}}_{i-1}\right)\mathrm{L}_g^2 + 2\hat{\mathbf{r}}_{i-1} \cdot \left(\mathbf{M} \cdot \hat{\mathbf{p}} + \vec{\mathbf{N}}_0\right)\mathrm{L}_g + \hat{\mathbf{p}} \cdot \left(\mathbf{M} \cdot \hat{\mathbf{p}} + 2\vec{\mathbf{N}}_0\right) = 0. \tag{8}$$

The optical pathlength of the ray is L_g scaled by the index of refraction n of the medium that contains it:

$$\mathrm{L} = n\mathrm{L}_g \tag{9}$$

The Law of Reflection states that the effect of a reflective surface on a ray $\hat{\mathbf{r}}_{i-1}$ incident upon it is to reverse the normal component of $\hat{\mathbf{r}}_{i-1}$, while leaving the parallel components of $\hat{\mathbf{r}}_{i-1}$ unchanged (Fig. 1). In coordinate-free notation this is written in terms of the "reflection dyadic" **R** as

$$\hat{\mathbf{r}}_i = \mathbf{R} \cdot \hat{\mathbf{r}}_{i-1} \tag{10}$$

where **R** is defined as

$$\mathbf{R} = \mathbf{I} - 2\hat{\mathbf{N}}\hat{\mathbf{N}} \tag{11}$$

This dyadic form of the law of reflection is derived in Ref. 2; it apparently first appeared in Ref. 6.

The refraction of a ray by a surface is governed by Snell's Law (Fig. 1):

$$n_a \sin \phi_a = n_b \sin \phi_b \tag{12}$$

Here n_a is the index of refraction of the medium containing the incident ray and n_b is the index of refraction of the medium containing the refracted ray. ϕ_a and ϕ_b are the angles of the incident ray $\hat{\mathbf{r}}_{i-1}$ and refracted ray $\hat{\mathbf{r}}_i$ with respect to the normal $\hat{\mathbf{N}}$; $\hat{\mathbf{r}}_i$ is in the plane defined by $\hat{\mathbf{N}}$ and $\hat{\mathbf{r}}_{i-1}$. A coordinate-free version of Snell's Law (derived in Ref. 2) is:

$$\hat{\mathbf{r}}_i = \mu \hat{\mathbf{r}}_{i-1} - \frac{1 - \mu^2}{\sqrt{1 - \mu^2 + \mu^2 \left(\hat{\mathbf{N}} \cdot \hat{\mathbf{r}}_{i-1}\right)^2} - \mu \hat{\mathbf{N}} \cdot \hat{\mathbf{r}}_{i-1}} \hat{\mathbf{N}} \tag{13}$$

where

$$\mu = n_a / n_b \tag{14}$$

Our method for tracing rays through a beam train is summarized as follows. We start at the aperture of the system, defining a ray starting point $\vec{\mathbf{p}}_0$ and direction $\hat{\mathbf{i}}_0$. Equations 7 and 8 are applied to find the point of incidence on the first surface $\vec{\rho}_1$. The reflected or refracted ray $\hat{\mathbf{r}}_1$ is determined using Eqs. 11 or 13. For each subsequent surface, the incident ray direction is the reflected or refracted ray direction of the previous surface and the incident ray starting position is the point of incidence on the previous surface ($\vec{\mathbf{p}}_i = \vec{\rho}_{i-1}$). The pathlength is accumulated as:

$$L_i = \sum_{j=1}^{i} L_j. \tag{15}$$

4. Optical Sensitivities for Single Surfaces

In this section we present equations for the sensitivities of rays to element and incident-ray perturbations. We present sensitivities for reflected rays only; refraceted ray sensitivities are presented in Ref. 3. The sensitivities can be combined to form linearized ray-trace equations valid for perturbed rays that are "close" to exact nominal rays. By "close" we mean that the ray directions are close enough to the nominal that the small angle approximation holds. The perturbed ray direction is then:

$$\hat{\mathbf{r}}_{pert} = \hat{\mathbf{r}}_{nom} + d\hat{\mathbf{r}}. \tag{16}$$

We assume that the transverse aberration (beamwalk) $\vec{\gamma}$ of the perturbed ray away from the nominal ray is small compared to the curvature of each element surface, so that the movement of the reflection (or refraction) point is confined to a plane tangent to the surface at the nominal reflection point. We assume the motions $\vec{\theta}$ and $\vec{\delta}$ of the elements are small. Finally, we neglect second and higher order perturbations. The validity of these approximations is a function of the particular element type and the particular system geometry and should be examined on a case-by-case basis.

Given these approximations, perturbed rays can be computed in terms of deviations from exactly-traced nominal rays (Fig. 2). The perturbation of reflected or refracted ray direction is $d\hat{\mathbf{r}}_i$. Perturbations of the position of the ray are decomposed into the transverse aberration $\vec{\gamma}_i$, which is perpendicular to nominal ray, and the OPD - denoted dL_i - which is parallel to the nominal ray. These ray perturbations are functions of the incident ray direction perturbation $d\hat{\mathbf{r}}_{i-1}$, the incident ray beamwalk $\vec{\gamma}_{i-1}$, and tilt and translation perturbations of the i^{th} element $\vec{\theta}$ and $\vec{\delta}$. To first order:

$$d\hat{\mathbf{r}}_i = \frac{\partial \hat{\mathbf{r}}_i}{\partial \hat{\mathbf{r}}_{i-1}} \cdot d\hat{\mathbf{r}}_{i-1} + \frac{\partial \hat{\mathbf{r}}_i}{\partial \vec{\gamma}_{i-1}} \cdot \vec{\gamma}_{i-1} + \frac{\partial \hat{\mathbf{r}}_i}{\partial \vec{\theta}_i} \cdot \vec{\theta}_i + \frac{\partial \hat{\mathbf{r}}_i}{\partial \vec{\delta}_i} \cdot \vec{\delta}_i \tag{17}$$

$$\vec{\gamma}_i = \frac{\partial \vec{\gamma}_i}{\partial \hat{\mathbf{r}}_{i-1}} \cdot d\hat{\mathbf{r}}_{i-1} + \frac{\partial \vec{\gamma}_i}{\partial \vec{\gamma}_{i-1}} \cdot \vec{\gamma}_{i-1} + \frac{\partial \vec{\gamma}_i}{\partial \vec{\theta}_i} \cdot \vec{\theta}_i + \frac{\partial \vec{\gamma}_i}{\partial \vec{\delta}_i} \cdot \vec{\delta}_i \tag{18}$$

$$dL = \frac{\partial L}{\partial \hat{\mathbf{r}}_{i-1}} \cdot d\hat{\mathbf{r}}_{i-1} + \frac{\partial L}{\partial \vec{\gamma}_{i-1}} \cdot \vec{\gamma}_{i-1} + \frac{\partial L}{\partial \vec{\theta}_i} \cdot \vec{\theta}_i + \frac{\partial L}{\partial \vec{\delta}_i} \cdot \vec{\delta}_i \tag{19}$$

In the rest of this section we give equations for sensitivity dyadics for reflected rays. Space available does not permit us to give refracted-ray sensitivities or any derivations. The interested reader is referred to Ref. 2. We start by listing terms that are common to all conic surfaces. The curvature of the mirror surface at $\vec{\rho}$ can be expressed as the spatial gradient of the normal:

$$\frac{d\hat{N}}{d\vec{\rho}} = -\mathrm{sign}(\hat{i}\cdot\vec{N})\left[\frac{1}{N}\mathbf{I} - \frac{\hat{N}}{N^2}\left(\vec{\rho}\cdot\mathbf{M} + \vec{N}_0\right)\right]\cdot\mathbf{M} = -\mathrm{sign}(\hat{i}\cdot\vec{N})\,\frac{1}{N}\mathbf{P}_N\cdot\mathbf{M} \tag{20}$$

Changes in the point of incidence with incident-ray beamwalk and direction change are:

$$\frac{\partial\vec{\rho}_i}{\partial\vec{\gamma}_{i-1}} = \left(\mathbf{I} - \frac{\hat{r}_{i-1}\hat{N}_i}{\hat{r}_{i-1}\cdot\hat{N}_i}\right). \tag{21}$$

$$\frac{\partial\vec{\rho}_i}{\partial\hat{r}_{i-1}} = L_i\left(\mathbf{I} - \frac{\hat{r}_{i-1}\hat{N}_i}{\hat{r}_{i-1}\cdot\hat{N}_i}\right) \tag{22}$$

Changes in the point of incidence with surface translation perturbations are:

$$\frac{\partial\vec{\rho}_i}{\partial\vec{\delta}_i} = -\left(\mathbf{I} - \frac{\hat{r}_{i-1}\hat{N}_i}{\hat{r}_{i-1}\cdot\hat{N}_i}\right) \tag{23}$$

If rotational perturbations of the surface take place about points other than the point of incidence, the rotations couple into translational effects. Denoting the point of rotation with respect to the surface vertex as $\vec{q}$, the coupling is:

$$\frac{\partial\vec{\delta}_i}{\partial\vec{\theta}_i} = -\left(\vec{p} - \vec{q}\right)^{\times} \tag{24}$$

The following are for reflecting surfaces only. The change in the reflected ray direction due to changes in the normal are:

$$\frac{\partial\hat{r}_i}{\partial\hat{N}_i} = -2\left(\left(\hat{N}_i\cdot\hat{r}_{i-1}\right)\mathbf{I} + \hat{N}_i\hat{r}_{i-1}\right). \tag{25}$$

The sensitivities of the reflected ray direction to incident ray direction and beamwalk:

$$\frac{\partial\hat{r}_i}{\partial\hat{r}_{i-1}} = \mathbf{R}_i + \frac{\partial\hat{r}_i}{\partial\hat{N}_i}\cdot\frac{\partial\hat{N}_i}{\partial\vec{\rho}_i}\cdot\frac{\partial\vec{\rho}_i}{\partial\hat{r}_{i-1}}. \tag{26}$$

$$\frac{\partial\hat{r}_i}{\partial\vec{\gamma}_{i-1}} = \frac{\partial\hat{r}_i}{\partial\hat{N}_i}\cdot\frac{\partial\hat{N}_i}{\partial\vec{\rho}_i}\cdot\frac{\partial\vec{\rho}_i}{\partial\vec{\gamma}_{i-1}}. \tag{27}$$

The sensitivities of the reflected ray beamwalk to incident ray direction and beamwalk:

$$\frac{\partial\vec{\gamma}_i}{\partial\hat{r}_{i-1}} = \mathbf{L}\mathbf{R}_i. \tag{28}$$

$$\frac{\partial\vec{\gamma}_i}{\partial\vec{\gamma}_{i-1}} = \mathbf{R}_i. \tag{29}$$

To first order, incident ray direction changes and beamwalk have no effect on the total pathlength. The effects of mirror translation and rotation on the reflected ray direction are given by:

$$\frac{\partial\hat{r}_i}{\partial\vec{\delta}_i} = \frac{\partial\hat{r}_i}{\partial\hat{N}_i}\cdot\frac{\partial\hat{N}_i}{\partial\vec{\rho}_i}\cdot\frac{\partial\vec{\rho}_i}{\partial\vec{\delta}_i} \tag{30}$$

$$\frac{\partial\hat{r}_i}{\partial\vec{\theta}_i} = \frac{\partial\hat{r}_i}{\partial\hat{N}_i}\cdot\frac{\partial\hat{N}_i}{\partial\vec{\theta}_i} + \frac{\partial\hat{r}_i}{\partial\vec{\delta}_i}\cdot\frac{\partial\vec{\delta}_i}{\partial\vec{\theta}_i} \tag{31}$$

A simpler alternative to Eq. 32 valid only for flat mirrors is:

$$\frac{\partial\hat{r}_i}{\partial\vec{\theta}_i} = -2\hat{r}_i^{\times}\cdot\mathbf{P}_N \tag{32}$$

The effect of mirror translation and rotation on beamwalk and pathlength:

$$\frac{\partial\vec{\gamma}_i}{\partial\vec{\delta}_i} = \mathbf{P}_r\cdot\frac{\hat{r}_{i-1}\hat{N}_i}{\hat{r}_{i-1}\cdot\hat{N}_i}. \tag{33}$$

$$\frac{\partial \vec{\gamma}_i}{\partial \vec{\theta}_i} = \frac{\partial \vec{\gamma}_i}{\partial \vec{\delta}_i} \cdot \frac{\partial \vec{\delta}_i}{\partial \vec{\theta}_i} \tag{34}$$

$$\frac{\partial L_i}{\partial \vec{\delta}_i} = -\left(\frac{1-\hat{\mathbf{r}}_i \cdot \hat{\mathbf{r}}_{i-1}}{\hat{\mathbf{r}}_{i-1} \cdot \hat{\mathbf{N}}_i}\right)\hat{\mathbf{N}}_i . \tag{35}$$

$$\frac{\partial L_i}{\partial \vec{\theta}_i} = \frac{\partial L_i}{\partial \vec{\delta}_i} \cdot \frac{\partial \vec{\delta}_i}{\partial \vec{\theta}_i} \tag{36}$$

The refracted ray sensitivities are similar in form to those for reflected rays[2].

5. Linearized Models of Beam Trains

In creating optical models for integration with structural and control system models it is convenient to pick a coordinate system and recast the ray-trace and sensitivity equations into matrix notation. Choosing the instrument *structural* coordinate frame for the optical calculations allows us to parameterize the optical models by structural elements, which enables direct concatenation of the optical and structural models to produce an integrated computer model. Control system elements, such as mirror gimbals and optical sensors, can also be directly incorporated into the formulation of the optical model, as discussed in Ref. 3. This section summarizes the linearized ray-trace equations in matrix notation.

The perturbation state vector at the i^{th} element can be written as a 7×1 column matrix $\vec{x}_i$, where:

$$\vec{x}_i = \begin{bmatrix} d\hat{r}_i \\ \vec{\gamma}_i \\ dL_i \end{bmatrix} \tag{37}$$

Similarly, $\vec{u}_i$ is a 6×1 column matrix consisting of the small translation and rotation of the i^{th} element:

$$\vec{u}_i = \begin{bmatrix} \vec{\theta}_i \\ \vec{\delta}_i \end{bmatrix} \tag{38}$$

Equations 17-19 can now be written as a state difference equation:

$$\vec{x}_i = \frac{\partial \vec{x}_i}{\partial \vec{x}_{i-1}}\vec{x}_{i-1} + \frac{\partial \vec{x}_i}{\partial \vec{u}_i}\vec{u}_i \tag{39}$$

where $\partial\vec{x}_i/\partial\vec{x}_{i-1}$ is the transition matrix from the $i\text{-}1^{th}$ to the i^{th} element,

$$\frac{\partial \vec{x}_i}{\partial \vec{x}_{i-1}} = \begin{bmatrix} \frac{\partial \hat{r}_i}{\partial \hat{r}_{i-1}} & \frac{\partial \hat{r}_i}{\partial \vec{\gamma}_{i-1}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \frac{\partial \vec{\gamma}_i}{\partial \hat{r}_{i-1}} & \frac{\partial \vec{\gamma}_i}{\partial \vec{\gamma}_{i-1}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \frac{\partial L_i}{\partial \hat{r}_{i-1}} & \frac{\partial L_i}{\partial \vec{\gamma}_{i-1}} & 1 \end{bmatrix} \tag{40}$$

and $\partial\vec{x}_i/\partial\vec{u}_i$ is the influence matrix at the i^{th} element,

$$\frac{\partial \vec{x}_i}{\partial \vec{u}_i} = \begin{bmatrix} \frac{\partial \hat{r}_i}{\partial \vec{\theta}_i} & \frac{\partial \hat{r}_i}{\partial \vec{\delta}_i} \\ \frac{\partial \vec{\gamma}_i}{\partial \vec{\theta}_i} & \frac{\partial \vec{\gamma}_i}{\partial \vec{\delta}_i} \\ \frac{\partial L_i}{\partial \vec{\theta}_i} & \frac{\partial L_i}{\partial \vec{\delta}_i} \end{bmatrix} . \tag{41}$$

Sensitivities of the ray perturbation state at one element to ray or element perturbations at another element are easily calculated as products of these matrices. The sensitivity of the ray at the n^{th} element to the ray at the i^{th} element is:

$$\frac{\partial \vec{x}_n}{\partial \vec{x}_i} = \frac{\partial \vec{x}_n}{\partial \vec{x}_{n-1}} \cdots \frac{\partial \vec{x}_{i+1}}{\partial \vec{x}_i}. \tag{42}$$

The sensitivity of the ray at the n^{th} element to perturbations of the i^{th} element is:

$$\frac{\partial \vec{x}_j}{\partial \vec{u}_i} = \frac{\partial \vec{x}_j}{\partial \vec{x}_{j-1}} \cdots \frac{\partial \vec{x}_{i+1}}{\partial \vec{x}_i} \frac{\partial \vec{x}_i}{\partial \vec{u}_i}. \tag{43}$$

A complete linear model of a beam train combines the effects of perturbations of the rays and elements as seen at a reference surface. Tracing m rays through a system of n elements, the combined linear model is the matrix C:

$$\begin{bmatrix} \vec{x}_{n_{ray\,1}} \\ \vdots \\ \vec{x}_{n_{ray\,m}} \end{bmatrix} = C \begin{bmatrix} \vec{x}_0 \\ \vec{u}_1 \\ \vdots \\ \vec{u}_n \end{bmatrix} = \begin{bmatrix} \left[\frac{\partial \vec{x}_n}{\partial \vec{x}_0} \; \frac{\partial \vec{x}_n}{\partial \vec{u}_1} \cdots \frac{\partial \vec{x}_n}{\partial \vec{u}_n} \right]_{ray\,1} \\ \vdots \\ \left[\frac{\partial \vec{x}_n}{\partial \vec{x}_0} \; \frac{\partial \vec{x}_n}{\partial \vec{u}_1} \cdots \frac{\partial \vec{x}_n}{\partial \vec{u}_n} \right]_{ray\,m} \end{bmatrix} \begin{bmatrix} \vec{x}_0 \\ \vec{u}_1 \\ \vdots \\ \vec{u}_n \end{bmatrix} \tag{44}$$

Computing the C matrix in the structural model coordinate frame, the $\vec{u}_i$'s generally correspond directly to particular structural model outputs, and the integration of linear structural and optics models requires only the matrix multiplication of Eq. 44.

6. Conclusion and Acknowledgement

We have presented coordinate-free equations for ray-tracing and differential ray-tracing. These formulas have proven useful in creating linear and nonlinear optical models for integration with flexible-structure and control system models for end-to-end performance analysis of large controlled optical instuments. This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

8. References

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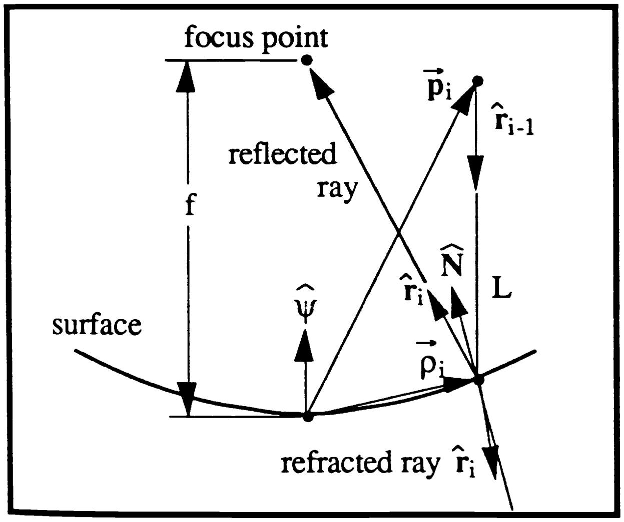

Figure 1. Exact Ray-Trace Geometry

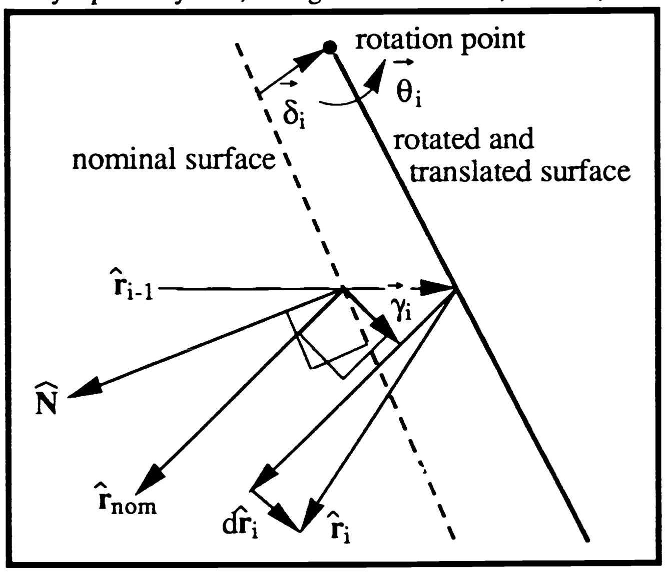

Figure 2. Perturbation Geometry