Lecture 7: Deformable mirror modeling and control

Rufus Fraanje
p.r.fraanje@tudelft.nl

TU Delft — Delft Center for Systems and Control

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Delft Center for Systems and Control

Delft University of Technology

Outline

1. Adjustable optics in nature and engineering;
2. Deformable mirrors — modeling;
3. Control of AO systems;
4. Exams

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Adjustable optics

1. Adjustable pupils (size/shape);
2. Adjustable lenses (focus);
3. Adjustable pointing direction (tip/tilt);
4. Deformable mirrors.

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Adjustable optics — Nature

Adjustable pupils (from: Animal Eyes, Land&Nilsson, 2002)

Lens eyes on land 89

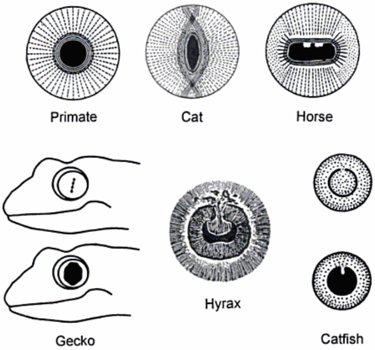

Fig. 5.11 Pupil shapes in vertebrates. *Top row*: round and slit-shaped pupils in mammals, showing how the cat's slit pupil can close further than the circular primate pupil. Iris closer muscles are continuous lines and opener muscles dashed lines. *Bottom row*: gecko pupil contracts to four 'pin-holes' in the light. The hyrax or coney (*Procavia*, a small desert mammal) has a pupil partly closed by a central operculum, which acts as a sunshade. A similar mobile operculum is present in some fish, such as the catfish *Plecostomus*. Combined from Walls (1941).

Adjustable optics — Engineering

Adjustable pupils: (Source: Wikipedia)

“Minolta” lens, $f/1.5 - f/16$

Numerical aperture (NA):

$$\text{NA} = \frac{f}{D} \qquad f/\text{NA} = D$$

NA of E-ELT: 17.7, NA of human eye?

Adjustable optics — Engineering

Adjustable pupils: (Source: Wikipedia)

“Minolta” lens, $f/1.5 - f/16$

$f/32$ (top-left) and $f/5$ (bottom-right)

Numerical aperture (NA):

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Numerical aperture (NA):

$$\mathrm{NA} = \frac{f}{D} \qquad f/\mathrm{NA} = D$$

NA of E-ELT: 17.7, NA of human eye? Answer: $\approx \frac{1.7}{0.7} = 2.4$

Adjustable optics — Nature

Adjustable lenses (from: Animal Eyes, Land&Nilsson, 2002)

Lens eyes on land **83**

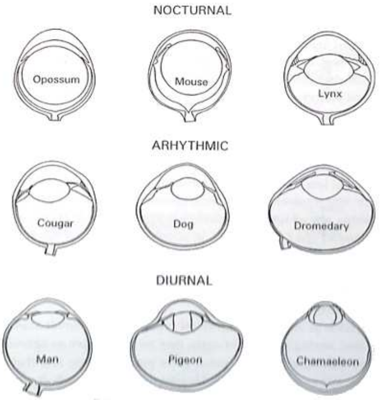


Fig. 5.7 Variations in the structure of eyes from animals with different terrestrial lifestyles (not on the same scale). Nocturnal animals have the biggest lenses and diurnal animals the smallest; animals active day and night (arhythmic) have intermediate eyes. Adapted from Walls (1941).

Adjustable optics — Engineering

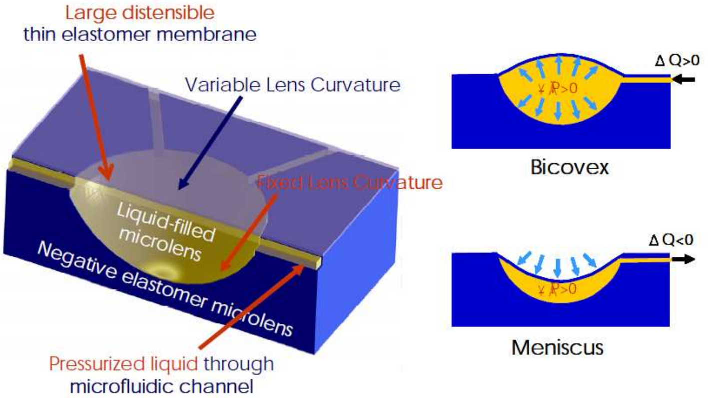
Adjustable lenses (Source: `http://www.nedinsco.nl`):

Day Sight TV Camera VZC-RZ230 MSP

Application	Searching
Characteristics	
Detector	1/2"CCD monochrome
Optics	Continuous Zoom max. 10×
Focus Range	≤ 10m - ∞
Dynamic Range	1 to 150.000 Lux
Wavelength	400 to 1000 nm
Field of View Horizontal	≤ 2,5° to ≤ 24°
Mechanical	
Dimensions	245 × 139 × 111 mm
Weight	2,3 kg
Environmental	
Complies with	MIL-STD 810 & 461
MTBF	≥ 10.000 h

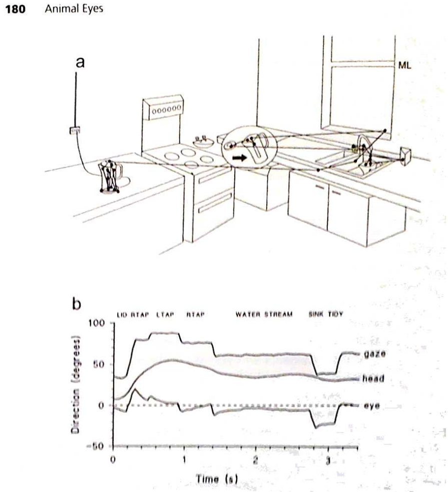
Adjustable optics — Engineering

Adjustable lenses (Source: Jeong et al. (2005), c.f. `http://biopoems.berkeley.edu`)



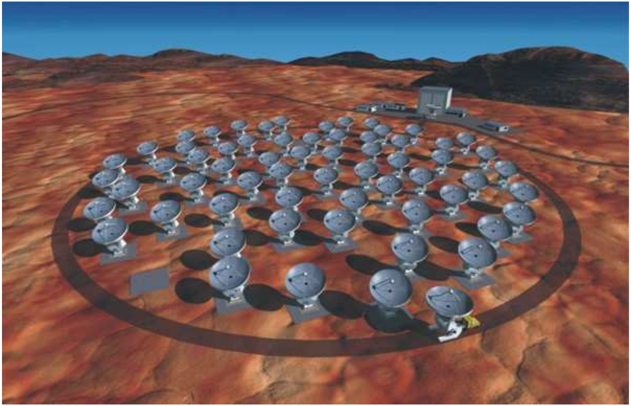
Adjustable optics — Nature

Eye/head movement (from: Animal Eyes, Land&Nilsson, 2002)



Adjustable optics — Engineering

Telescope pointing (Source: `http://www.dailygalaxy.com/photos/uncategorized/2007/08/12/alma_2.jpg`):



ALMA: Atacama Large Millimeter Array (66 telescopes when completed)

Adjustable optics — Nature

Mirrors in scallops (from: Animal Eyes, Land&Nilsson, 2002)

Mirrors in animals **107**

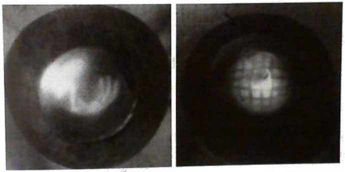


Fig. 6.3 Images in scallops' eyes. *Left*: self-portrait of the author, whose hand is holding the microscope objective used to photograph the eye. *Right*: a grid of 3 mm squares, 15 mm from the eye.

(Source: Wikipedia – Sint-Jakobsschelp)

Adjustable optics — Engineering

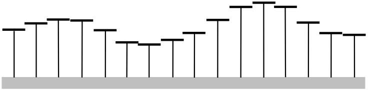
Adjustable mirrors (Keck II, Source: Wikipedia):

Adjustable optics — Engineering

Adjustable mirrors (Electrostatic deformable mirror, Source: Flexible Optical, BV, Rijswijk):

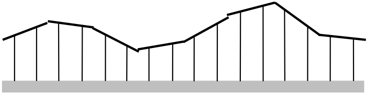
Deformable mirrors

- **Segmented facesheet (piston):**



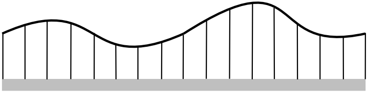
Liquid Crystals, DLP's
very high actuator density

- **Segmented facesheet (piston/tip/tilt):**



very large (M1's) or MEMS devices
high actuator density

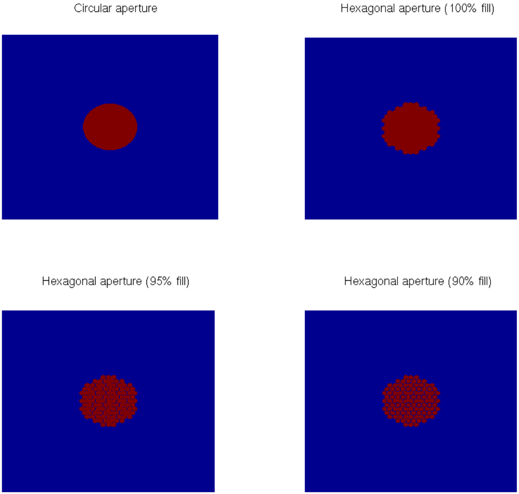
- **Continuous facesheet:**



various types (electrostatic, reluctance, piezo, ...)
medium sized diameters (cm range)
'no' photon loss, low spatial aliasing

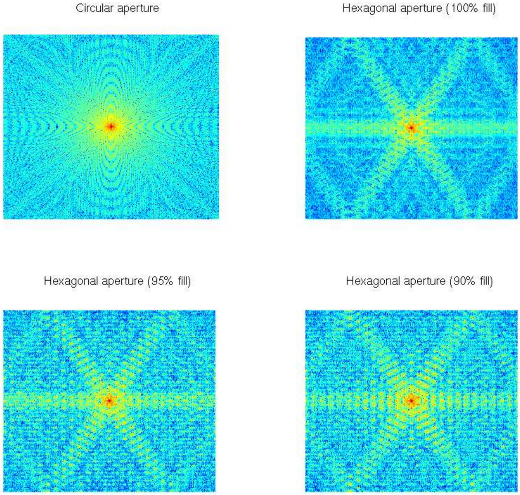
Deformable mirrors

Diffraction effects of hexagonal segmented mirrors: Apertures:



Deformable mirrors

Diffraction effects of hexagonal segmented mirrors: Point spread functions:



Deformable mirrors

Diffraction effects of hexagonal segmented mirrors: Obtained images:

Circular aperture

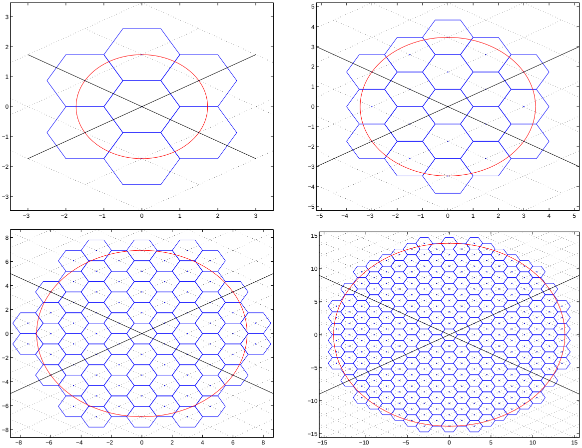
Hexagonal aperture (100% fill)

Hexagonal aperture (90% fill)

Hexagonal aperture (70% fill)

Deformable mirrors

Hexagonal grids



6 corner points:

$$\begin{bmatrix} \pm 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \pm\frac{1}{2} \\ \pm\frac{\sqrt{3}}{2} \end{bmatrix}$$

hex- to rect.-coord.:

$$\begin{bmatrix} x_r \\ y_r \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{3}{2} \\ -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_h \\ y_h \end{bmatrix}$$

rotationally-symmetric over 60^o angles.

Deformable mirrors — modeling

Let $w(x,y,t)$ be the out-of-plane deformation and $p(x,y,t)$ the distributed pressure from, e.g., actuators.

- **Membrane mirror:**

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) w(x,y,t) \; = \; p(x,y,t)$$

+boundary/initial conditions, where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$

- **Thin plate mirror:**

$$\left(\rho h \frac{\partial^2}{\partial t^2} + EI\nabla^4\right) w(x,y,t) \; = \; p(x,y,t)$$

+boundary/initial conditions, where $\nabla^4 = \partial^4/\partial x^4 + 2\partial^4/\partial x^2 \partial y^2 + \partial^4/\partial y^4$.

c.f. Timoshenko et al., *Theory of Plates and Shells*, McGraw-Hill, 1959.

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Deformable mirrors — modeling

Solutions obained by (dynamic or static case):

- finite difference approximation:

$$\left.\frac{\partial w}{\partial x}\right|_{x,y,t} \approx \frac{w(x+\delta x, y, t) - w(x-\delta x, y, t)}{2\delta x}$$

- finite element approximation:

$$w(x,y,t) = \sum_{k=1}^{n} u_k(t) v_k(x,y), \qquad v_k(x,y) \text{ defined over 'finite element'}$$

- modal approximation:

$$w(x,y,t) = \sum_{k=1}^{n} u_k(t) \phi_k(x,y), \qquad \phi_k(x,y) \text{ a solution of homogeneous PDE}$$

- analytical (only for specific forcing functions $p(x,y,t)$ and boundary conditions), e.g., by separation of variables.

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Deformable mirrors — modeling

Static approximations: influence functions

$$w(x,y) \;=\; \sum_{k}^{n} h(x,y,s_k,t_k)u_k$$

- Cubic influence function:

$$h(x,y,s,t) \;=\; A\left(1-3(x-s)^2+2(x-s)^3\right)\left(1-3(y-t)^2+2(y-t)^3\right)$$

- Gaussian influence function:

$$h(x,y,s,t) \;=\; Ae^{r^2/\sigma^2}, \qquad r=\sqrt{(x-s)^2+(y-t)^2}$$

- Circular clamped edge faceplate[1] (polar coordinates):

$$\begin{aligned} h(r,\phi,\rho,\psi) \;&=\; A(1-r^2)(1-\rho^2)(r^2+\rho^2-2r\rho\cos(\phi-\psi)) \\ &\quad x \ln \frac{r^2+\rho^2-2r\phi\cos(\phi-\psi)}{1+r^2\rho^2-2r\rho\cos(\phi-\psi)} \end{aligned}$$

[1] Loktev et al., Comparison study of the performance of piston, thin plate and membrane mirrors for correction of turbulence-induced phase distortions, Optics Communications 192, 2001

Deformable mirrors — modeling

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Deformable mirrors — modeling

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Deformable mirrors — modeling

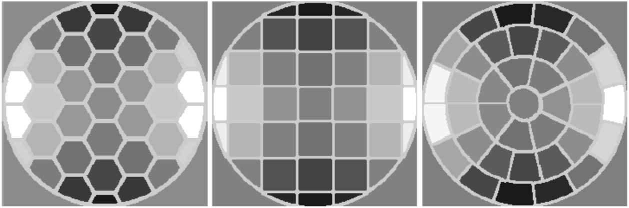
(Continued)

- Supported edge faceplate (c.f. Loktev et al., 2001)
- Free edge faceplate (idem)

Deformable mirrors — modeling

Loktev et al. 2001, various actuator configurations and plate models are compared:

Hexagonal- Square- Ring-layout



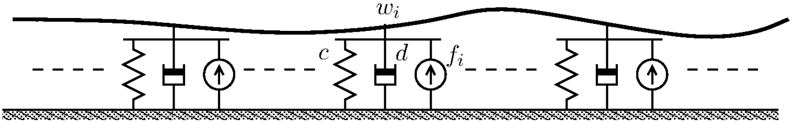
Deformable mirrors — modeling

Residual wavefront aberations for various type of mirrors each with 37 actuators (Credit: Loktev et al, 2001, Table 1 (adjusted))

Type of corrector	Error (%)	B (rad^2)	N_{KL}
(a) Hexagonal structure			
Piston	22.70	0.05282	5
Membrane	9.682	0.00961	29
Continuous faceplate, clamped edge	9.333	0.00892	31
Continuous faceplate, supported edge	9.306	0.00888	32
Continuous faceplate, free edge	9.022	0.00834	33
(b) Ring segments			
Piston	22.55	0.05214	6
Membrane	9.468	0.00918	31
Continuous faceplate, clamped edge	9.267	0.00880	32
Continuous faceplate, supported edge	9.238	0.00874	32
Continuous faceplate, free edge	9.107	0.00850	33
(c) Squares			
Piston	24.26	0.06031	4
Membrane	9.696	0.00963	29
Continuous faceplate, clamped edge	9.351	0.00895	31
Continuous faceplate, supported edge	9.321	0.00890	31
Continuous faceplate, free edge	9.119	0.00852	33

Deformable mirrors — modeling

Example: finite difference discretization over hexagonal grid (Fraanje, 2010)



Thin plate model:

$$\left(EI\nabla^4(1+\eta\frac{\partial}{\partial t})+\rho h\frac{\partial^2}{\partial t^2}\right)w(x,y,t) \;=\; p(x,y,t)$$

Actuator model at location (i,j):

$$p(x_i,y_i,t) \;=\; \frac{1}{A}\left(f(i,j,t)-\underbrace{c(1+\zeta\frac{\partial}{\partial t})w(x_i,y_i,t)}_{\text{spring/damper parallel to force}}\right)$$

Surface area of one hexagon $A=\frac{3\sqrt{3}}{8}\Delta^2$, where Δ distance between two neighboring actuators.

Deformable mirrors — modeling

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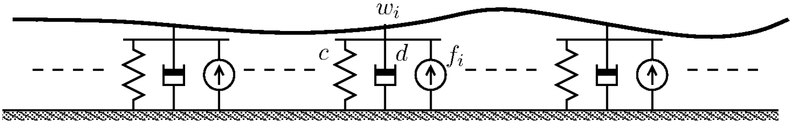

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Surface area of one hexagon $A = \frac{3\sqrt{3}}{8}\Delta^2$, where Δ distance between two neighboring actuators.

Deformable mirrors — modeling

Discretization of biharmonic operator ∇^4 over hexagonal grid:

$$\begin{aligned}\mathcal{G}(S_1, S_2) &= \frac{4}{9}\Big(42 - 10(S_1 + S_1^{-1} + S_1 S_2^{-1} + S_1^{-1} S_2 + S_2 + S_2^{-1}) + \\ &\quad 2(S_1^2 S_2^{-1} + S_1^{-2} S_2 + S_1 S_2 + S_1^{-1} S_2^{-1} + S_1 S_2^{-2} + S_1^{-1} S_2^2) + \\ &\quad (S_1^2 + S_1^{-2} + S_1^2 S_2^{-2} + S_1^{-2} S_2^2 + S_2^2 + S_2^{-2})\Big)\end{aligned}$$

where S_1 and S_2 unit-shifts along principal axes of hexagonal grid.

Then PDE will reduce to interconnected ODE's:

$$\left(\frac{AEI}{\Delta^4}\mathcal{G}(S_1, S_2)\big(1 + \eta\frac{\partial}{\partial t}\big) + c\big(1 + \zeta\frac{\partial}{\partial t}\big) + \rho A h\frac{\partial^2}{\partial t^2}\right) w(i, j, t) \quad = \quad f(i, j, t)$$

(Note: $I = h^3/12$ for thin plates.)

Deformable mirrors — modeling

Discretization of biharmonic operator ∇^4 over hexagonal grid:

$$
\begin{aligned}
\mathcal{G}(S_1, S_2) \quad = \quad & \frac{4}{9}\Big(42 - 10(S_1 + S_1^{-1} + S_1 S_2^{-1} + S_1^{-1} S_2 + S_2 + S_2^{-1}) + \\
& 2(S_1^2 S_2^{-1} + S_1^{-2} S_2 + S_1 S_2 + S_1^{-1} S_2^{-1} + S_1 S_2^{-2} + S_1^{-1} S_2^2) + \\
& (S_1^2 + S_1^{-2} + S_1^2 S_2^{-2} + S_1^{-2} S_2^2 + S_2^2 + S_2^{-2})\Big)
\end{aligned}
$$

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$$
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$$

(Note: $I = h^3/12$ for thin plates.)

Deformable mirrors — modeling

Let $\dot{w} = \partial w/\partial t$ and $\ddot{w} = \partial^2 w/\partial t^2$, then

$$\begin{bmatrix} \dot{w}(i,j,t) \\ \ddot{w}(i,j,t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\left(\frac{Eh^2\mathcal{G}(S_1,S_2)}{12\rho\Delta^4} + \frac{8c}{3\sqrt{3}\rho\Delta^2 h}\right) & -\left(\frac{\eta Eh^2\mathcal{G}(S_1,S_2)}{12\rho\Delta^4} + \frac{8\zeta c}{3\sqrt{3}\rho\Delta^2 h}\right) \end{bmatrix} \begin{bmatrix} w(i,j,t) \\ \dot{w}(i,j,t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{8}{3\sqrt{3}\rho\Delta^2 h} \end{bmatrix} f(i,j,t)$$

$$w(i,j,t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} w(i,j,t) \\ \dot{w}(i,j,t) \end{bmatrix}$$

Stacking over all N nodes (i,j) yields:

$$\begin{aligned} \dot{x}(t) &= (I_N \otimes A_{a,c} + \mathcal{P}_N \otimes A_{b,c})x(t) + (I_N \otimes B_{a,c})f(t) \\ w(t) &= (I_N \otimes C_{a,c})x(t) \end{aligned}$$

where $\mathcal{P}_N$ a sparse *pattern* matrix and ...

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Deformable mirrors — modeling

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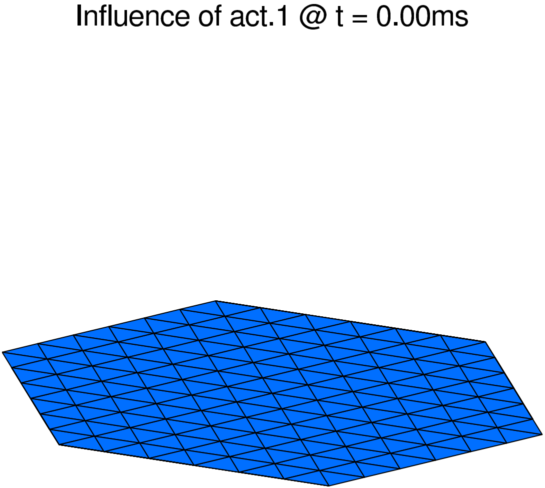
$$\begin{aligned} A_{a,c} &= \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{\zeta c}{m} \end{bmatrix} & A_{b,c} &= \begin{bmatrix} 0 & 0 \\ -\frac{Eh^2}{12\rho\Delta^4} & -\frac{\eta Eh^2}{12\rho\Delta^4} \end{bmatrix} \\ B_{a,c} &= \begin{bmatrix} 0 & \frac{1}{m} \end{bmatrix}^T & C_{a,c} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$

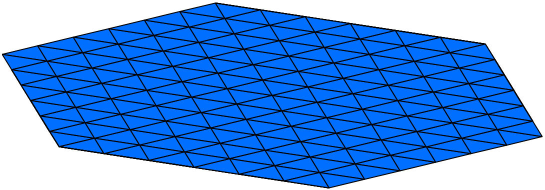
where $m = \rho h \frac{3\sqrt{3}}{8} \Delta^2$ the mass of one hexagon.

Deformable mirrors — modeling



Deformable mirrors — modeling

Influence of act.5 @ t = 0.00ms



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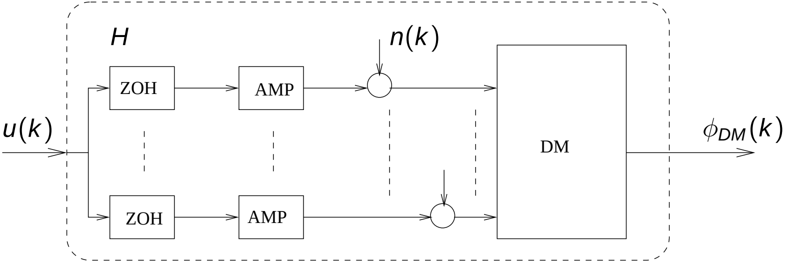
TU Delft
Delft University of Technology

Control of AO systems

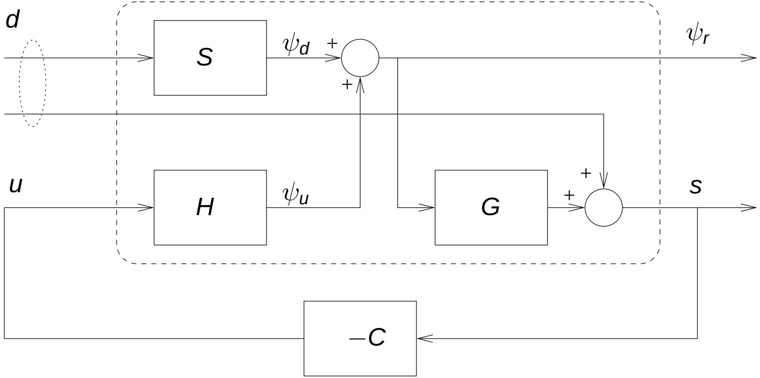


$$\phi_{DM}(k) \; = \; H_{DM}\left(L_{AMP}u(k) + n(k)\right)$$

- Bandwidth of amplifiers;
- Noise-level of amplifiers;
- Dynamics of deformable mirror.

Control of AO systems

Blockscheme calculus:



$$\begin{bmatrix} \psi_r \\ s \end{bmatrix} = \begin{bmatrix} S & H \\ GS & GH \end{bmatrix} \begin{bmatrix} d \\ u \end{bmatrix}$$

$$u = -Cs$$

Solve for ψ_r

Control of AO systems

$$
\begin{bmatrix} \psi_r \\ s \end{bmatrix} = \begin{bmatrix} S & H \\ GS & GH \end{bmatrix} \begin{bmatrix} d \\ u \end{bmatrix}
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$$
u = -Cs
$$

Solving for ψ_r yields:

$$
\begin{aligned}
u &= -CG\psi_r \\
&\Longleftrightarrow \\
\psi_r &= Sd - HCG\psi_r \\
&\Longleftrightarrow \\
(I + HCG)\psi_r &= Sd \\
&\Longleftrightarrow \\
\psi_r &= (I + HCG)^{-1}Sd
\end{aligned}
$$

Control of AO systems

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Control of AO systems

Closed-loop residual wavefront phase:

$$\psi_r = (I + HCG)^{-1} S d$$

Objective: minimize $\| \psi_r \|_2^2$

How to design C?

$$|C| \to \infty \Rightarrow \| \psi_r \|_2^2 \to 0$$

High-gain feedback.

But, ... $(I + HCG)^{-1}$ need to be stable!

Control of AO systems

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Control of AO systems

Example: $C = c$, $H = az^{-1}$, $G = S = 1$, such that

$$\begin{aligned} \psi_r(k) &= d(k) + au(k-1) \\ u(k) &= -c\psi_r(k) \end{aligned}$$

Closed-loop system:

$$\psi_r(k) = d(k) - ac\psi_r(k-1)$$

which is stable if and only if $|ac| < 1, \rightarrow |c| < \frac{1}{|a|}$.

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Control of AO systems

1. PID for MIMO systems;
2. H_2 optimal control;
3. Distributed control.

Control: PID for MIMO systems

Control system:

$$
\begin{aligned}
s(k) &= G\psi_d(k) + \underbrace{GH}_{=P} u(k) \\
u(k+1) &= -C(z)s(k)
\end{aligned}
$$

Design strategy:

- Diagonalize system using SVD of $P = U\Sigma V^T$;
- Design SISO controller.

Decoupled system:

$$
\begin{aligned}
s'(k) &= U^T G\psi_d(k) + \Sigma u'(k) \\
u'(k+1) &= -C'(z)s'(k)
\end{aligned}
$$

where $s'(k) = U^T s(k)$, $u'(k) = V^T u(k)$ and $C(z) = VC'(z)U^T$.
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Control: PID for MIMO systems

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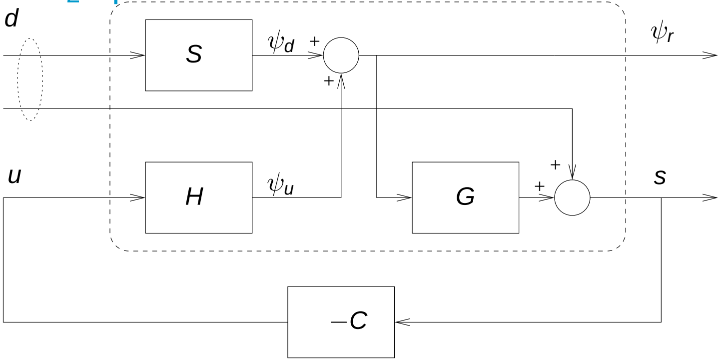
Control: H_2 optimal control



One "big" state-space model:

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) + B_d d(k) \\
\psi_r(k) &= C_\psi x(k) + D_{\psi,u} u(k) + D_{\psi,d} d(k) \\
s(k) &= C_s x(k) + \qquad\qquad D_{s,d} d(k)
\end{aligned}
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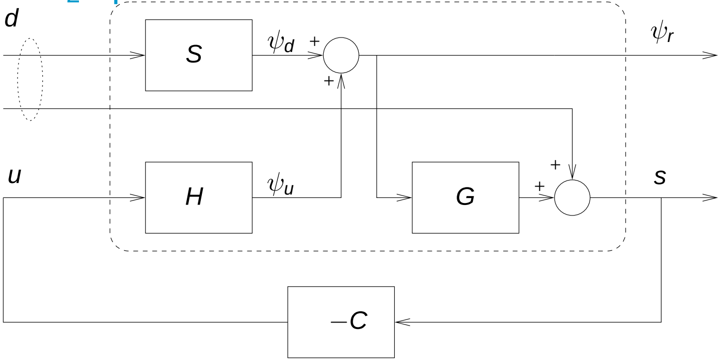
Minimize transfer from $d(k)$ to $\psi_r(k)$: $\mathrm{MSE}(\psi_r)/\mathrm{MSE}(d)$

Control: H_2 optimal control



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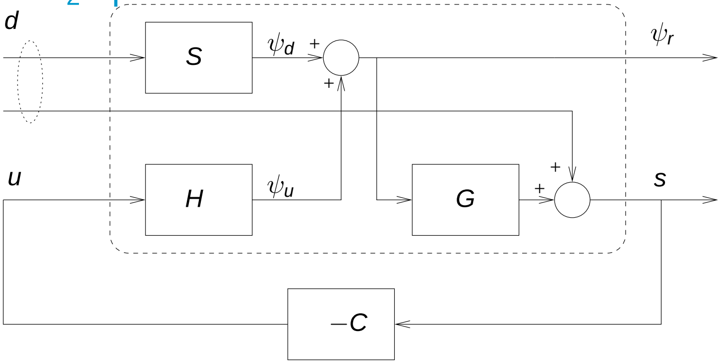

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Control: H_2 optimal control

Solution:

- Kalman filter:

$$\begin{aligned}\hat{s}(k) &= C_s\hat{x}(k)\\ \hat{x}(k+1) &= A\hat{x}(k)+Bu(k)+K(s(k)-\hat{s}(k))\end{aligned}$$

  where K the Kalman gain.

- State-feedback:

$$u(k+1) = -F\hat{x}(k+1)$$

  where F the state-feedback gain.

For static deformable mirrors $\psi_{DM}(k) = Hu(k)$[2]:

- Controller based on predicted wavefront:

$$u(k+1) = -H^{\dagger}\hat{\psi}(k+1)$$

  where H the Moore-Penrose pseudo-inverse and $\hat{\psi}(k+1) = C_{\psi}\hat{x}(k+1)$

[2] c.f., Hinnen, Data-Driven Optimal Control for Adaptive Optics, 2007, page 110

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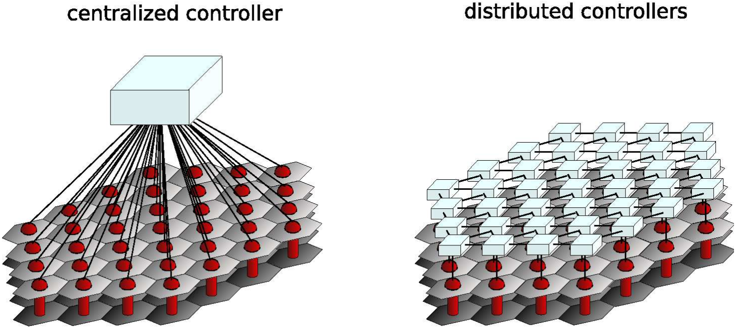
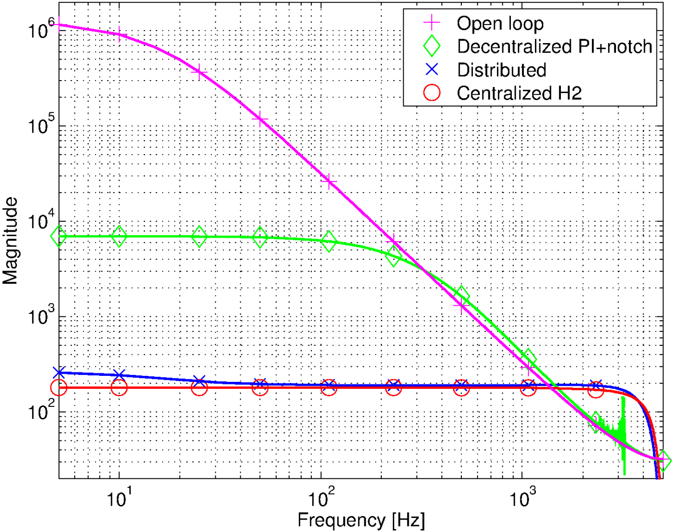
Control: Distributed control

(Fraanje, Massioni, Verhaegen 2011)



Control: Distributed control

(Fraanje, Massioni, Verhaegen 2011)



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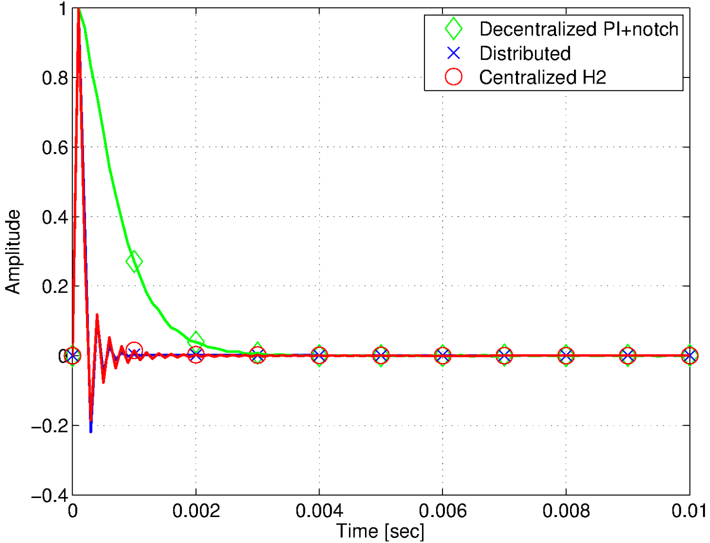

Exams

1. Sparse matrix techniques for wavefront reconstruction;
2. Extended object and wavefront estimation by phase-diversity;
3. Pyramide wavefront sensor;
4. Adaptive optics for the eye;
5. Quantum noise effects in imaging;
6. Data-driven H_2 control;
7. Modeling and validation of deformable mirrors.

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