Numerical atmospheric turbulence models and LQG control for adaptive optics system

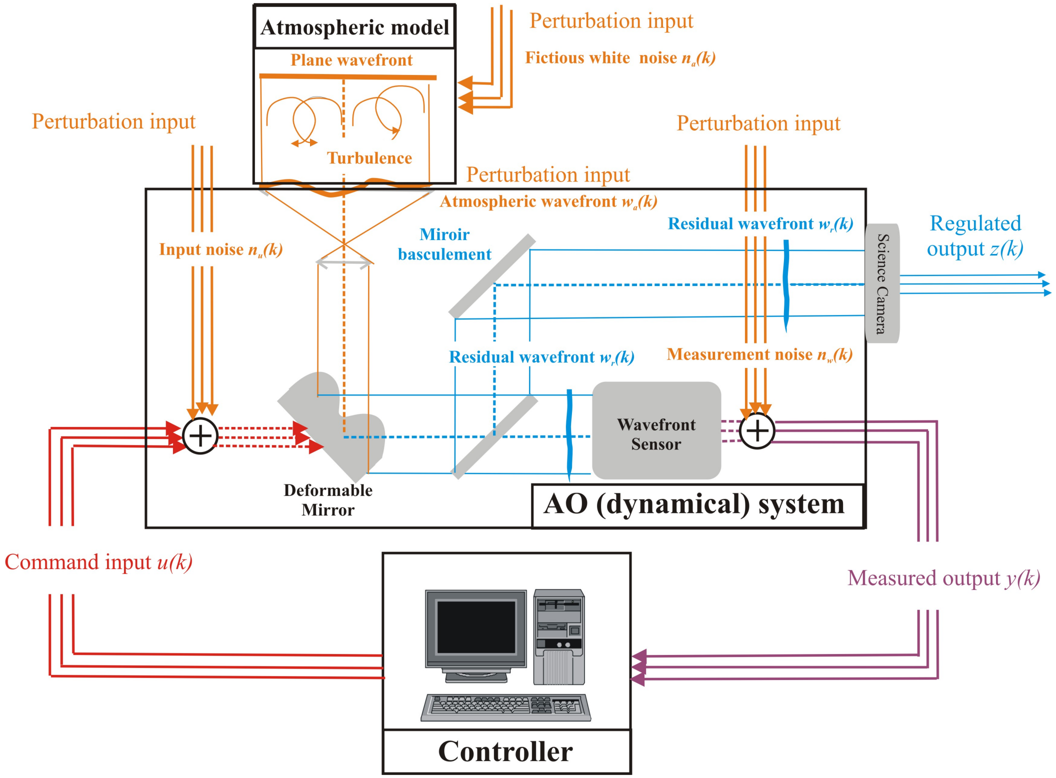
Jean-Pierre FOLCHER, Marcel CARBILLET

UMR6525 H. Fizeau, Université de Nice Sophia-Antipolis/CNRS/Observatoire de la Côte d'Azur, France

AO FOR ETL 2, Victoria , BC, CANADA, September 25-30, 2011

Context

- **AO (dynamical) system**: a relationship between (regulated, measured) output signals and (command, perturbation) input signals.
- **Control objective**: maintain the regulated output signal close to zero despite the perturbation input.
- **Feedback concept** : generate the command input calculated from the measured output using a **controller**.

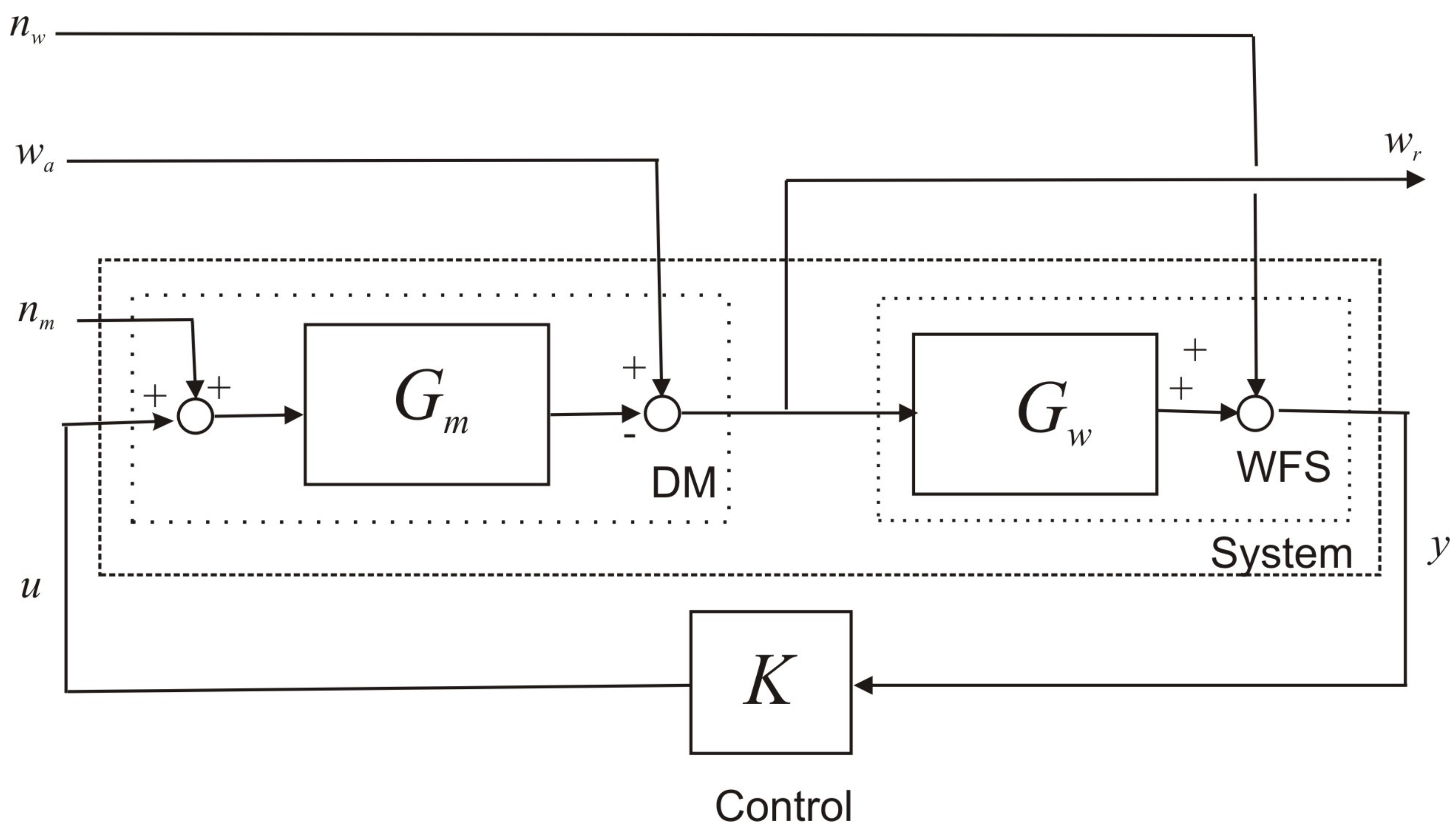


Context: cont'd

- **Temporal error causes** of the residual wavefront
  - DM dynamics,
  - WFS delay (exposure time and read out of the CCD camera),
  - computational delay of the control law.
- **Image quality degradation:** when atmospheric wavefront dynamic is fast relative to cumulative loop delay.
- **For high AO system performance:** ability of the controller
  - to have a reasonable complexity (to limit computational delay)
  - to take into account the temporal evolution of the atmospheric wavefront
- **Convenient approach:** Linear Quadratic Gaussian control (minimum-variance control)
  - optimal state-feedback control of the DM
  - optimal estimation of the atmospheric wavefront
- **In many AO applications :** wind velocities and the strength of atmospheric turbulence change rapidly.
- **A compelling issue:** find a 'robust' controller despite atmospheric turbulence variations.

Adaptive optics loop

Figure: AO discrete-time system block-diagram.



- $u \in \mathbf{R}^{n_u}$ is the DM command input, $y \in \mathbf{R}^{n_y}$ is the WFS discrete-time measurement & $n_w \in \mathbf{R}^{n_y}$ is an additive perturbation.
- $w_a \in \mathbf{R}^{n_b}$ is the atmospheric wavefront, $w_m \in \mathbf{R}^{n_b}$ is the mirror shape correction & $w_r \in \mathbf{R}^{n_b}$ is the residual wavefront.

Multivariable transfer function

Residual wavefront in the z-domain is

$$\mathcal{Z}\{w_r\} = (I + L(z))^{-1}\,\mathcal{Z}\{w_a\} + (I + L(z))^{-1}\,G_m(z)K(z)\mathcal{Z}\{n_w\}\ ,$$

where $L(z) = G_m(z)K(z)G_w(z)$ is the loop transfer function.

Disturbance rejection performance entirely determined by

- the sensitivity transfer function $T_{11}(z) = (I + L(z))^{-1}$
- the disturbance rejection transfer function $T_{12}(z) = (I + L(z))^{-1}\,G_m(z)K(z)$

which have to be 'small' in a given frequency range.

No assumption is made

- for the type of the controller (integral, LQG, ...)
- for the set of the perturbation inputs (deterministic, stochastic)

Mean-square error performance

Residual wavefront variance is the sum of the atmospheric wavefront contribution and the WFS noise contribution.

$$\mathbf{E}\left[\|w_r(k)\|^2\right] = \frac{T}{2\pi}\int_0^{\frac{2\pi}{T}} \mathbf{Tr}\left(T_{11}(e^{j\omega T})S_{w_a}(\omega)T_{11}(e^{-j\omega T})^T\right)d\omega\ldots$$
$$+\frac{T}{2\pi}\int_0^{\frac{2\pi}{T}} \mathbf{Tr}\left(T_{12}(e^{j\omega T})S_{n_w}(\omega)T_{12}(e^{-j\omega T})^T\right)d\omega$$

where

- S_{n_w}: power spectral densities of n_w (taken constant)
- $S_{w_a} = G_a(e^{j\omega T})G_a(e^{-j\omega T})^T$: power spectral density of w_a
- $G_a(z)$: the transfer function of the atmospheric model.

LQG design find the optimal K which minimize $\mathbf{E}\left[\|w_r(k)\|^2\right]$ for a unique atmospheric model $G_a(z)$.

How to obtain a 'robust' LQG controller ?

ensuring performance for a set of atmospheric model...

$\rightarrow$ computation of a **nominal and worst case atmopsheric model**

For a set of temporal evolutions of turbulent wavefronts

- identification (Burg algorithm) of second order diagonal AR model (to take into account the oscillating behavior of time evolution) $G_a(z)$

$$A_0 w_a(k) + A_1 w_a(k-1) + A_2 w_a(k-2) = n_a(k-1) \ ,$$

  where input $n_a \in \mathbf{R}^{n_b}$ is a zero-mean white stochastic process with unitary covariance matrix.
- numeric evaluation of the frequency response of $G_a(z)$
- numeric computation of
  - the nominal AR model (with a mean frequency response)
  - the worst case AR model (with a worst case frequency response)

How to obtain a 'robust' LQG controller ? cont'd

$\rightarrow$ design of a **nominal LQG controller** and a **worst case LQG controller**

- Consider the **augmented system**

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(k) + w(k) \\ y(k) &= \begin{bmatrix} C_1 & C_2 \end{bmatrix} x(k) + v(k) \ , \end{aligned} \quad (1)$$

  - where $w(k)$ and $v(k)$ are respectively the Gaussian state/measurement noises with covariance $\mathbf{E}\left[w(k)w^T(l)\right] = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \begin{bmatrix} 0 & B_2^T \end{bmatrix} \delta(k-l)$ and $\mathbf{E}\left[v(k)v^T(l)\right] = V\delta(k-l)$.
  - with state space matrices A_1, A_2, B_1, B_2, C_1, C_2 (given in the paper).

- and the quadratic cost criterion (to minimize)

$$J = \lim_{K\rightarrow\infty} \frac{1}{K} \mathbf{E} \left[\sum_{k=0}^{K-1} x(k)^T Q x(k) + u(k)^T R u(k) \right] \ ,$$

  - where the weighting matrix $Q = Q^T \geq 0$ is chosen such that $x(k)^T Q x(k) = \|w_r(k)\|^2$
  - where weighting matrice $R = R^T > 0$ is fixed to ensure a reasonable peak input command.

How to obtain a 'robust' LQG controller ? cont'd

- The controller (linear quadratic regulator + linear optimal state estimator) is described by

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ -\mathbf{L}_2 C_2 & A_2 - \mathbf{L}_2 C_2 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & \mathbf{L}_2 \end{bmatrix} \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}$$

$$u(k) = \begin{bmatrix} -\mathbf{K}_1 & -\mathbf{K}_2 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix},$$

  - Optimal state feedback gains

$$\mathbf{K}_1 = (B_1^T P_{11} B_1 + R)^{-1} B_1^T P_{11} A_1, \quad \mathbf{K}_2 = (B_1^T P_{11} B_1 + R)^{-1} B_1^T P_{12} A_2,$$

    where $P_{11} = P_{11}^T \geq 0$ is the solution of an algebraic Riccati equation, P_{12} is the solution of a Sylvester equation,
  - Optimal observer gain $\mathbf{L}_2 = A_2 X_{22} C_2^T \left(C_2 X_{22} C_2^T + V \right)^{-1}$,
    where $X_{22} = X_{22}^T \geq 0$ is the solution of an algebraic Riccati equation.

Results

Main parameters

- `Software Package CAOS` numerical modeling.
- 1000×1 ms wavefronts propagated through an evolving 3-layers turbulent atmosphere ($r_0 = 10\ cm$ at $\lambda = 500\ nm$, $\mathcal{L}_0$=25 m, wind velocities=8–16 m/s).
- 8-m telescope, 0.1 obstruction ratio.
- Wavefronts projected over a Zernike polynomials base of size $n_b = 44$.
- DM with 77 actuators using a influence function description.
- 8×8 ($\Rightarrow$52) subaperture Shack-Hartmann WFS (8×8 0.2" px/subap., λ_0=700 nm).
- DM influence matrix M_w and WFS influence matrix M_w determined numerically (computed within system calibration simulation).

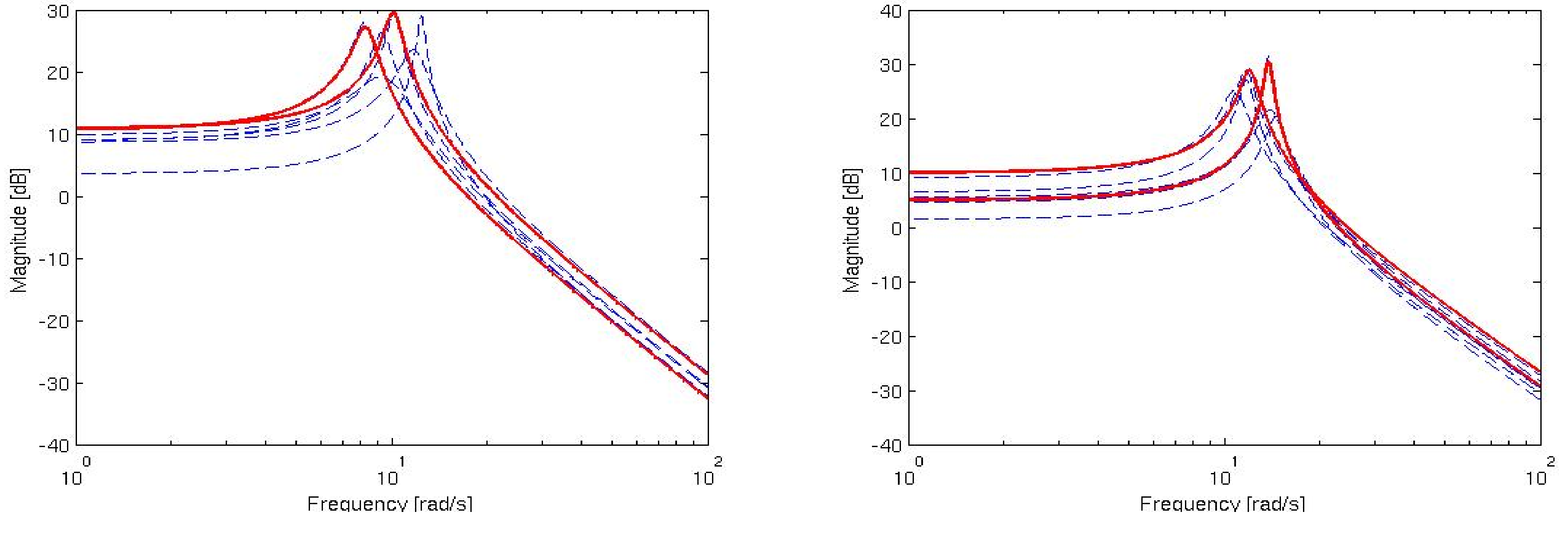
Results: cont'd

Nominal and worst case atmopsheric model

For a set of 6 temporal evolutions of turbulent wavefronts

- identification (Burg algorithm) of the 6 AR diagonal models $G_a(z)$
- evaluation of the 6 frequency responses
- numeric computation of
  - the nominal AR model (with a mean frequency response)
  - the worst case AR model (with a worst case frequency response)

Figure: Bode magnitude plots of transfert function $G_a^{(10)}(z)$, $G_a^{(20)}(z)$ for the 6 identified model (dashed line), for the nominal AR model (plain line), and worst case AR model (plain line).



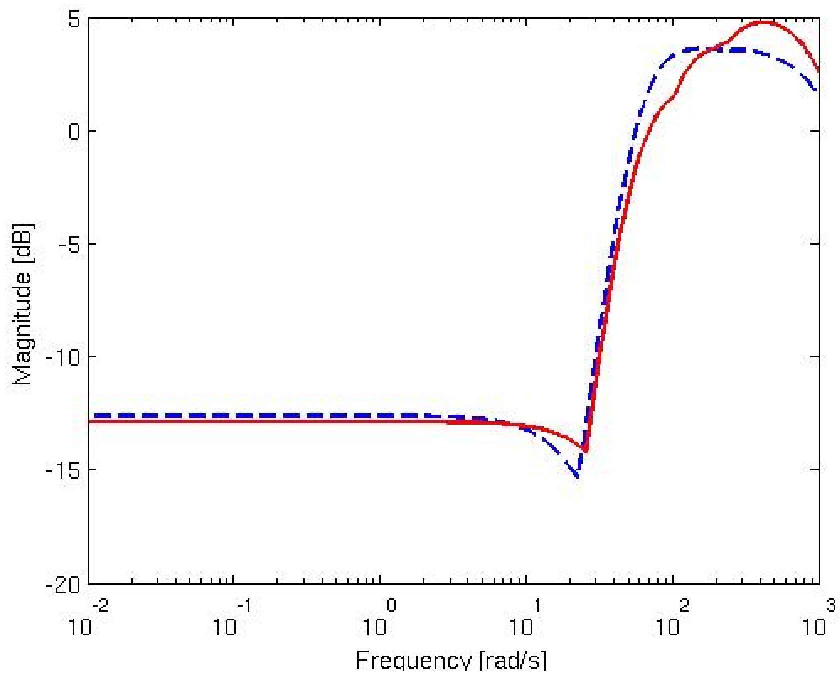
Results: cont'd

LQG controller design

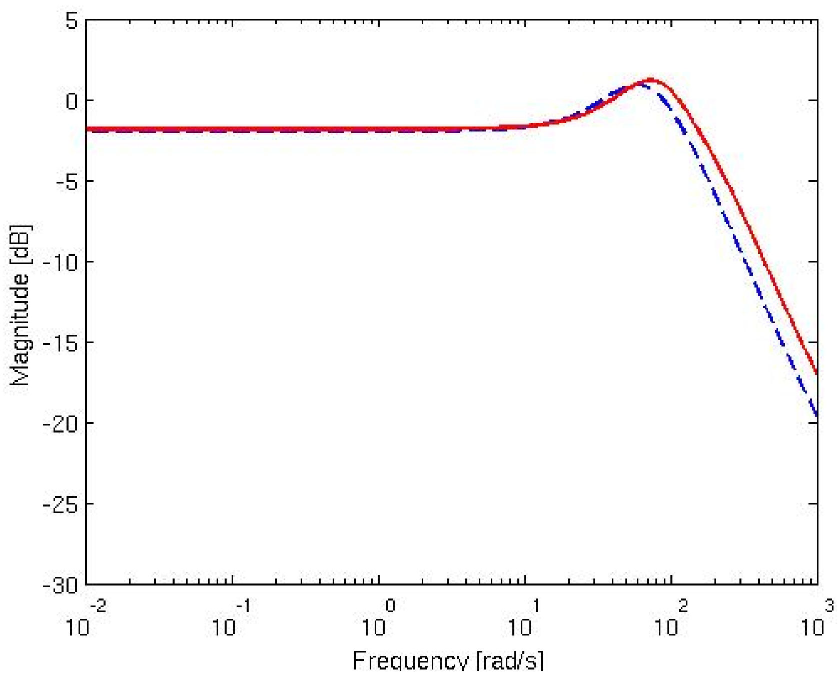
- weighting matrice $R = 10^{-2}I$ & WFS noise level $V = 10^{-2}I$
- Nominal LQG controller: obtain using the nominal AR model in (1)
- Worst case LQG controller: obtain using the the worst case AR model in (1)

Frequency responses

Figure: Maximum sigular values $\bar{\sigma}(T_{11}(e^{j\omega T}))$ and $\bar{\sigma}(T_{12}(e^{j\omega T}))$ for the nominal LQG controller (dashed line) and for the worst case LQG controller (plain line).



(a) $\bar{\sigma}(T_{11}(e^{j\omega T}))$



(b) $\bar{\sigma}(T_{12}(e^{j\omega T}))$

Results: cont'd

Time responses of the residual wavefronts for six simulated atmospheric wavefront sequences

Table: Standard deviation of the atmospheric wavefront sequences.

All modes standard deviation					
Sequence 1	Sequence 2	Sequence 3	Sequence 4	Sequence 5	Sequence 6
$\sim 1481\ nm$	$\sim 1280\ nm$	$\sim 1048\ nm$	$\sim 1034\ nm$	$\sim 1503\ nm$	$\sim 1190\ nm$

Table: Standard deviation of the residual wavefront for the two designed LQG controllers.

Controllers	All modes standard deviation					
	Sequence 1	Sequence 2	Sequence 3	Sequence 4	Sequence 5	Sequence 6
Nominal LQG law	$\sim 367\ nm$	$\sim 310\ nm$	$\sim 294\ nm$	$\sim 294\ nm$	$\sim 367\ nm$	$\sim 325\ nm$
Worst case LQG law	$\sim 365\ nm$	$\sim 307\ nm$	$\sim 291\ nm$	$\sim 291\ nm$	$\sim 362\ nm$	$\sim 322\ nm$

A reference value (Noll residual): 278 nm