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TCSS 343

Homework #3

Due 11/22/2020

2.1 UNDERSTAND

1.

$$T(n) = \begin{cases} c, & if\ n < 125 \\ 25T\left(\frac{n}{125}\right) + \sqrt[3]{n^2}, & if\ n \geq 125 \end{cases}$$

Master's Theorem:

$$a = 25, \qquad b = 125, \qquad f(n) = \sqrt[3]{n^2}$$

$$n^{\log_b a} = n^{\log_{125} 25} = n^{\frac{2}{3}}$$

$$\sqrt[3]{n^2} \in \ \Theta\left(n^{\frac{2}{3}}\right)$$

$$T(n) \in \ \Theta\left(n^{\frac{2}{3}} \log n\right)$$

2.

$$T(n) = \begin{cases} c, & if\ n < 16 \\ 8T\left(\dfrac{n}{16}\right) + \sqrt[3]{n^2}\log n, & if\ n \geq 16 \end{cases}$$

Master's Theorem:

$$a = 8, \qquad b = 16, \qquad f(n) = \sqrt[3]{n^2}\log n$$

$$n^{\log_b a} = n^{\log_{16} 8} = n^{\frac{3}{4}}$$

$$\lim_{n\to\infty} \frac{n^{\frac{2}{3}}\log n}{n^{\frac{3}{4}}} = \lim_{n\to\infty} \frac{\log n}{n^{\frac{1}{12}}} = \lim_{n\to\infty} \frac{1/n}{\frac{1}{12}n^{-\frac{11}{12}}} = \lim_{n\to\infty} \frac{12n^{\frac{11}{12}}}{n} = \lim_{n\to\infty} \frac{12}{n^{\frac{1}{12}}} = 0$$

$$\sqrt[3]{n^2}\log n \in \ O\left(n^{\frac{3}{4}-\epsilon}\right)\ for\ 0 < \epsilon < \frac{1}{12}$$

$$T(n) \in \ \Theta(n^{\frac{3}{4}})$$

3.

$$T(n) = \begin{cases} c, & if\ n < 49 \\ 343T\left(\frac{n}{49}\right) + n^2 \log\log n\,, & if\ n \geq 49 \end{cases}$$

Master's Theorem:

$$a = 343, \qquad b = 49, \qquad f(n) = n^2 \log\log n$$

$$n^{\log_b a} = n^{\log_{49} 343} = n^{1.5}$$

$$\lim_{n\to\infty} \frac{n^2 \log\log n}{n^{1.5+\epsilon}} = \lim_{n\to\infty} n^{0.5-\epsilon} \log\log n = \infty$$

$$n^2 \log\log n \in \ \Omega(n^{1.5+\epsilon})\ for\ 0 < \epsilon \leq 0.5$$

$$af\left(\frac{n}{b}\right) = 343(\frac{n}{49})^2 \log\log(\frac{n}{49})\,, \qquad kf(n) = kn^2 \log\log n$$

$$343(\frac{n}{49})^2 \log\log(\frac{n}{49}) \leq kn^2 \log\log n\ \ for\ 0.9 < k < 1\ and\ n > 49$$

$$T(n) \in \ \Theta(n^2 \log\log n)$$

4.

$$T(n) = \begin{cases} c, & if\ n < 7 \\ 49T\left(\frac{n}{7}\right) + n^2 \log\log n\,, & if\ n \geq 7 \end{cases}$$

Master's Theorem:

$$a = 49, \qquad b = 7, \qquad f(n) = n^2 \log\log n$$

$$n^{\log_b a} = n^{\log_7 49} = n^2$$

$$\lim_{n\to\infty} \frac{n^2 \log\log n}{n^{2+\epsilon}} = \lim_{n\to\infty} \frac{\log\log n}{n^{\epsilon}} = \lim_{n\to\infty} \frac{1/n \log n}{\epsilon n^{\epsilon-1}} = \lim_{n\to\infty} \frac{1}{\epsilon n^{\epsilon} \log n} = 0\ for\ any\ \epsilon > 0$$

$$therefore\ f(n) \notin \Omega(n^{2+\epsilon})$$

$$\lim_{n\to\infty} \frac{n^2 \log\log n}{n^{2-\epsilon}} = \lim_{n\to\infty} n^{\epsilon} \log\log n = \infty\ for\ any\ \epsilon > 0$$

$$therefore\ f(n) \notin O(n^{2-\epsilon})$$

$$\lim_{n\to\infty} \frac{n^2 \log\log n}{n^2} = \lim_{n\to\infty} \log\log n = \infty$$

$$therefore\ f(n) \notin \Theta(n^2)$$

The Master Theorem cannot be applied here.

5.

$$T(n) = \begin{cases} c, & if\ n < 7 \\ 49T\left(\frac{n}{7}\right) + n^2 \log n\,, & if\ n \geq 7 \end{cases}$$

Master's Theorem:

$$a = 49, \qquad b = 7, \qquad f(n) = n^2 \log n$$

$$n^{\log_b a} = n^{\log_7 49} = n^2$$

$$n^2 \log n \in \Theta(n^2\,(\log n)^d)\ for\ d = 1$$

$$T(n) \in \Theta(n^2\,(\log n)^{d+1})$$

$$T(n) \in \Theta(n^2\,(\log n)^2)$$

2.2 EXPLORE

1.

$$T(n)\begin{cases} c, & if\ n \le 5 \\ 3T\left(\frac{3n}{5}\right) + 2n^2 + 2n(\log n)^2, & if\ n > 5 \end{cases}$$

2.

Master's Theorem:

$$a = 3, \qquad b = \frac{5}{3}, \qquad f(n) = 2n^2 > \ 2n(\log n)^2$$

$$n^{\log_b a} = \ n^{\log_{\frac{5}{3}} 3} \approx n^{2.15}$$

$$\lim_{n\to\infty} \frac{2n^2}{n^{2.15-\epsilon}} = \lim_{n\to\infty} \frac{2}{n^{.15-\epsilon}} = 0\ for\ 0 < \epsilon < \ .15$$

$$2n^2 \in O(n^{2.15})$$

$$T(n) \in \Theta(n^{2.15})$$

3.

$$T(n)\begin{cases} c, & if\ n \le 1 \\ 2T\left(\frac{n}{\sqrt{2}}\right) + n^2 + n^2(\log(\frac{2n}{3}))^2), & if\ n > 1 \end{cases}$$

4.

Master's Theorem:

$$a = 2, \qquad b = \sqrt{2}, \qquad f(n) = n^2(\log n)^2 > \ n^2$$

$$n^{\log_b a} = \ n^{\log_{\sqrt{2}} 2} \approx n^2$$

$$n^2(\log n)^2 \in \Theta(n^2(\log n)^d) \ for\ d = 2$$

$$T(n) \in \Theta(n^2(\log n)^{d+1})$$

$$T(n) \in \Theta(n^2(\log n)^3)$$

2.3 EXPAND

1.

Median of Two Sorted Lists (MOTSL)

Input conditions: sorted array A[a…m], sorted array B[b…n], and empty array C[c…m + n].

Output conditions: median of the merged array C[c…m + n].

2.

(a)

MOTSL (A[a…m], B[b…n], C[c…m + n])

if $(c \geq m + n)$

return C[$\left\lfloor \frac{m+n}{2} \right\rfloor + 1$]

else if $(a \leq b\ or\ b \geq n)$

c = a

return MOTSL(A[a + 1…m], B[b…n], C[c + 1… m + n])

else if $(b \leq a\ or\ a \geq m)$

c = b

return MOTSL(A[a…m], B[b + 1…n], C[c + 1… m + n])

end MOTSL

(b)

The total elements, m + n, is referred to as n here.

$$T(n) = \begin{cases} c, & if\ n \leq 1 \\ T(n-1) + d, & if\ n > 1 \end{cases}$$

$$T(n) \in \Theta(n)$$

3.

(a)

We can use the medians of the two lists to find that the median of the collection is one of medians of the two lists or is an element whose value is between them. This set of elements that are between the medians of the two lists, including those medians, can be used as pivots in a QuickSearch algorithm to find the median of the collection. This reduces the QuickSearch instance of this problem to a sub-instance by using a set of pivots instead of using random/arbitrary pivots.

(b)

I am at a loss for the rest of 2.3.