Bios 660/Bios 672 (3 Credits)
Probability and Statistical Inference 1

Homework 3

Due: Tu. September 11, 2018 at the Beginning of Class

Special Note: when turning in homework, please **staple** the answers into **3 groups**: (a) Questions 1-3; (b) Questions 4-6; (c) Questions 7-9.

1. Consider an experiment whose sample space consists of a countably infinite number of points. Show that not all points can be equally likely. Can all points have positive probability?

2. How many different sets of initials can be formed if every person has one surname and

   (a) exactly 2 given names

   (b) at most 2 given names

   (c) at most 3 given names

3. The numbers $1, 2, \ldots, n$ are arranged in random order. Find the probability that the digits (a) 1 and 2, (b) 1,2, and 3 appear as neighbors in the order.

4. Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

5. 7 gifts are to be distributed among 10 children. How many distinct results are possible if no child is to receive more than 1 gift?

6. If 8 identical blackboards are to be divided among 4 schools how many divisions are possible? How many divisions are possible if each school must receive at least 1 board?

7. We have 20 thousand dollars that must be invested among 4 possible opportunities. Each investment must be integral in units of 1 thousand dollars, and there are minimal investments that need to be made if one is to invest in these opportunities. The minimal investments are 2, 2, 3, and 4 thousand dollars, respectively, for the 4 opportunities. How many different investment strategies are available if (a) an investment must be made in each opportunity (b) investments must be made in at least 3 of the 4 opportunities?

8. Use mathematical induction to prove the Binomial Theorem:

$$(a+b)^n = \sum_{r=0}^{n} \left(\begin{array}{c} n \\ r \end{array} \right) a^r b^{n-r}$$

9. Prove the following result:

$$\left(\begin{array}{c} n+1 \\ r \end{array} \right) = \left(\begin{array}{c} n \\ r-1 \end{array} \right) + \left(\begin{array}{c} n \\ r \end{array} \right)$$