1 Problem 5

Suppose $P(A \cup B) = P(A \cap B)$
We can write $P(A) = P(A \cap B) + P(A \cap B^c)$
and $P(B) = P(A \cap B) + P(A^c \cap B)$
Then

$$P(A) + P(B) = 2P(A \cap B) + P(A \cap B^c) + P(A^c \cap B) \tag{1}$$

By definition $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Since $P(A \cup B) = P(A \cap B)$ we have:

$$P(A) + P(B) = 2P(A \cap B) \tag{2}$$

Combining (1) and (2) we have:
$2P(A \cap B) = 2P(A \cap B) + P(A \cap B^c) + P(A^c \cap B)$
Thus $P(A \cap B^c) + P(A^c \cap B) = 0$
Since probability is nonnegative
$P(A \cap B^c) = 0$ and
$P(A^c \cap B) = 0$
Thus $P(A) = P(B)$