Bios 660/Bios 672 (3 Credits)
Probability and Statistical Inference I

Homework 10

Due: Tue. November 13, 2018 at the Beginning of Class

Special Note: when turning in homework, please **staple** the answers into **3 groups**: (a) Questions 1-3; (b) Questions 4-6; (c) Questions 7-9.

1. Gut, Chapter I, problem 10: Show that if $X \sim \chi_m^2$ and $Y \sim \chi_n^2$ are independent random variables, then $\frac{X/m}{Y/n} \sim F_{m,n}$.

2. Gut, Chapter I, problem 21: The random vector $\boldsymbol{X} = [X_1, X_2, X_3]'$ has density function:
$$f_{\boldsymbol{X}}(\boldsymbol{X}) = \begin{cases} \frac{2}{2e-5} x_1^2 x_2 e^{x_1 x_2 x_3}, & \text{for } 0 < x_1, x_2, x_3 < 1 \\ 0 & \text{Otherwise} \end{cases}$$
Determine the distribution of $X_1 \cdot X_2 \cdot X_3$.

3. Gut, Chapter II, problem 2: Let X and Y be independent $\exp(1)$ random variables. Find the conditional distribution of X given $X + Y = c$ (c is a positive constant).

4. A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period, and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying:
$$-2 \le x \le 4, \quad -1 \le y - x \le 1$$

   (a) Find the Marginal PMFs and the means of X and Y.

   (b) Find the mean of the trader's profit.

5. Casella and Berger, 4.1

6. Casella and Berger, 4.4

7. Casella and Berger, 4.5

8. Casella and Berger, 4.7

9. Casella and Berger, 4.9