1 PMI

If T is a subset of $\mathbb{N}$ such that:

1. $1 \in T$
2. $\forall\ k \in \mathbb{N}$, if $k \in T$, then $(k+1) \in T$,
   Then $T = \mathbb{N}$

Use induction to prove statements in the form of $(\forall\ n \in \mathbb{N})(P(n))$

Goal is to prove $T = \mathbb{N}$

- Basis Step: Prove $P(1)$
- Inductive Step: Prove $\forall\ k \in \mathbb{N}$ if $P(k)$ is true, then $P(k+1)$ is true.
- Conclude $P(n)$ is true $\forall\ n \in \mathbb{N}$

Start inductive step by assuming $P(k)$ is true.
This is called the **inductive hypothesis**

The key is to discover how $P(k+1)$ is related to $P(k)$ for an arbitrary $k \in \mathbb{N}$

2 Induction Proof Example

Proof by induction:

$\forall\ n \in \mathbb{N}$ let $P(n)$ be:

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

Basis Step: $1 = \frac{1(1+1)}{2}$

$= 1$

Thus $P(1)$ is true

Inductive Step: Let $k \in \mathbb{N}$ and assume $P(k)$is true:

$$\sum_{j=1}^{k} j = \frac{k(k+1)}{2} \tag{1}$$

We will prove $P(k+1)$ is true:

$$\sum_{j=1}^{k+1} j = \frac{(k+1)((k+1)+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} \tag{2}$$

add $(k+1)$ to both sides of (1)

$$\sum_{j=1}^{k} j + (k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{(k+1)(k+2)}{2}$$

Which is the same as (2)

Hence the inductive step has been established

and by PMI we have proven that:

$$\forall\ n \in \mathbb{N}\ \sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$