Homework 9

Gamma Distribution

Gamma Function: $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}\ dt$
$\Gamma(\alpha+1) = \alpha\Gamma(\alpha)\ \alpha > 0$
$\Gamma(n) = (n-1)! \quad n \in \mathbb{Z}$
$\Gamma(1/2) = \sqrt{\pi}$
$f(x|\alpha,\beta) = \dfrac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$
α is the shape parameter, influences the peakedness of the distribution
β is the scale parameter, influences the spread of the distribution
$EX^v = \dfrac{\beta^v\Gamma(v+\alpha)}{\Gamma(\alpha)}$
$\Gamma(\alpha+v) = \int_0^\infty x^{v+\alpha-1}e^{-x}\ \mathrm{d}x$
$EX = \alpha\beta$
$\int_0^\infty e^{-x^2/2}\ dz = \dfrac{\sqrt{2\pi}}{2} = \sqrt{\dfrac{\pi}{2}}$
$\int_0^\infty x^2e^{-x^2}$ is the same

Beta Distribution

$f(x|\alpha,\beta) = \dfrac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$
Beta Function: $B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\ \mathrm{d}x$
$B(\alpha,\beta) = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$
$EX^n = \dfrac{B(\alpha+n,\beta)}{B(\alpha,\beta)} = \dfrac{\Gamma(\alpha+n)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+n)\Gamma(\alpha)}$

Mgfs

Negative Binomial Mgf $\left(\dfrac{p}{1-(1-p)e^t}\right)^r$

Exponential Families

A family of pdfs or pmfs is called an exponential family if it can be expressed as

$$f(x|\theta) = h(x)c(\theta)\exp\left(\sum_{i=1}^k w_i(\theta)t_i(x)\right)$$