Bios 660: Probability and Statistical Inference 1

Homework 2

Due: Tue. September 4, 2018 at the Beginning of Class

Special Note: when turning in homework, please **staple** the answers into **3 groups**: (a) Questions 1-4; (b) Questions 5-8; (c) Questions 9-11.

1. Let $\Omega = \mathbb{R}^2$ and A_n be the interior of the circle with center at $((-1)^n/n, 0)$ and radius 1. Find $\liminf_n A_n$ and $\limsup_n A_n$.
2. Let $\Omega = \mathbb{R}$ and show that $\mathcal{F} = \{A : A \text{ is countable or } A^c \text{ is countable }\}$ is a σ-field.
3. Prove that if $\mathcal{X}_1$ and $\mathcal{X}_2$ are σ-fields, then $\mathcal{X}_1 \cap \mathcal{X}_2$ is also a σ-field.
4. Show that if $\boldsymbol{G}$ is a collection of σ-fields, then $\bigcap_{\mathcal{X} \in \boldsymbol{G}} \mathcal{X}$ is also a σ-field.
5. $\mathcal{X}_1 = \{\phi, A, A^c, \Omega\}$ and $\mathcal{X}_2 = \{\phi, B, B^c, \Omega\}$ are σ-fields, but $\mathcal{X}_1 \cup \mathcal{X}_2$ is not. Why?
6. Let $\{A_n\}$ be a sequence of decreasing sets. Prove that $P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n)$.
7. Show that $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$
8. Casella & Berger, 1.12
9. If $P(E) = 0.9$ and $P(F) = 0.8$, show that $P(E \cap F) \geq 0.7$. Then prove that in general,
$$P(E \cap F) \geq P(E) + P(F) - 1$$
10. Suppose $E_1, E_2, \ldots, E_n$ are events in Ω. Use induction to prove
$$P(E_1 \cap E_2 \cap \cdots \cap E_n) \geq P(E_1) + \cdots + P(E_n) - (n-1)$$
This result is called *Bonferroni's Inequality.*
11. Among the digits 1, 2, 3, 4, and 5 first one digit is chosen, then a second selection is made among the remaining four digits. Assume that all twenty possible results have the same probability. Find the probability that an odd digit will be selected at:
    (a) the first selection
    (b) the second selection
    (c) both times