Games of chance are probably as old as the human desire to get something for nothing.

Lightner, 1991

BIOS 660/672, Midterm 1 Fall, 2017

1. An elevator takes six passengers and stops at ten floors. Consider the passengers to be distinct
a) What is the number of ways for passengers to get off the elevator? (9 pts)
b) What is the number of ways for passengers to get off the elevator if no two passengers get off at the same floor? (9 pts)
c) Now consider the passengers indistinguishable. What is the number of ways for passengers to get off the elevator (no constraints as in a)? (9 pts)

2. Let X and Y be random variables resulting in flipping two fair coins, each taking the values -1 if a coin lands on a tail and 1 otherwise. That is, $P(X = 1 \text{ and } Y = 1) = P(X = 1)P(Y = 1) = 0.25$, and similarly for $P(X = 1 \text{ and } Y = -1)$ etc. Let $Z = XY$. To show that two random variables are independent, one needs to show, for all pairs of outcomes of the two variables, that the probability of intersection of the two events is equal to the product of probabilities. Note that {1,1}, {1,-1}, {-1,1},{-1,-1} represent all pairs of outcomes for X and Z.
   a) Are X and Z independent? (9 pts)
   b) Are X, Y and Z mutually independent? (9 pts)

3. At the beginning of a study of a group of subjects, 15% were classified as heavy smokers, 30% as light smokers, and 55% as nonsmokers. In the study, it was determined that the annual death rates of the heavy and light smokers were five and three times that of the nonsmokers, respectively. A randomly selected participant died over a one-year period; calculate the probability that the participant was a nonsmoker. (10 pts)

4. Two brothers, Nicholas and Alexander, are given a set of 10 cars each. The cars in each set are numbered from 1 to 10. The sets are identical. Each brother is instructed to select 4 different cars from the set. For example, a possible selection is that Nicholas choses cars {1,3,4,10} and Alexander {2,3,4,6}. Brothers' selections are independents of each other. All choices are equally likely.
   a) Let X_1 be a random variable that is equal to 1 if Nicholas selects car number 1, and 0 otherwise. What is the distribution of this random variable? (9 pts)
   b) Let Y_1 be a random variable that is equal to 1 if both brothers select car number 1, and 0 otherwise. What is the distribution of this random variable? (9 pts)
   c) Let M be a random variable that is equal to the number of cars that were selected by both Nicholas and Alexander, $M = Y_1 + \ldots + Y_{10}$, where Y_i be a random variable that is equal to 1 if both brothers select car number i, and 0 otherwise. What is the range of M? What is the distribution of M? (9 pts)

5. N observations are sampled with replacement from a set, A, of N observations $A=\{Z_1, \ldots, Z_N\}$ to create a bootstrap set B.

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a) What is the probability that an observation Z_1 was not selected to be in B? Give an approximate value of this probability when N is large. (9 pts)

b) If $N = 100$, what is the expected number of observations that are in A but not in B? (9 pts)