

## Homework 9

### Gamma Distribution

Gamma Function:  $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$

$\Gamma(\alpha + 1) = \alpha \Gamma(\alpha)$   $\alpha > 0$

$\Gamma(n) = (n-1)!$   $n \in \mathbb{Z}$

$\Gamma(1/2) = \sqrt{\pi}$

$$f(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$$

$\alpha$  is the shape parameter, influences the peakedness of the distribution

$\beta$  is the scale parameter, influences the spread of the distribution

$EX = \alpha\beta$

$$\int_0^\infty e^{x^2/2} dz = \frac{\sqrt{2\pi}}{2} = \sqrt{\frac{\pi}{2}}$$

### Beta Distribution

$$f(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

$B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx$  Beta Function

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}$$

$$EX^n = \frac{B(\alpha + n, \beta)}{B(\alpha, \beta)} = \frac{\Gamma(\alpha + n)\Gamma(\alpha + \beta)}{\Gamma(\alpha + \beta + n)\Gamma(\alpha)}$$