Bios 660/Bios 672 (3 Credits)
Probability and Statistical Inference I

Homework 6

Due: Tue. October 9, 2018 at the Beginning of Class

Special Note: when turning in homework, please **staple** the answers into **3 groups**: (a) Questions 1-5; (b) Questions 6-10; (c) Questions 11-14.

1. Let K be a random variable that takes with equal probability $1/(2n+1)$, the integer values in the interval $[-n, n]$. Find the PMF of the random variable $Y = \log(X)$ where $X = a^{|K|}$ and $a > 0$.

2. Let X be a random variable with PMF

$$p_X(x) = \begin{cases} x^2/a & \text{if } x = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find a and $E[X]$

   (b) What is the PMF of the Random variable $Z = (X - E[X])^2$?

   (c) Using part (b), compute the variance of X.

   (d) Compute the variance of X using the formula $\text{var}(X) = \sum_x (x - E[X])^2 p_X(x)$.

3. Let a and b be positive integers with $a \leq b$ and let X be a random variable that takes as values, with equal probability, the powers of 2 in the interval $[2^a, 2^b]$. Find the expected value and the variance of X.

4. As an advertising campaign, a chocolate factory places golden tickets in some of its candy bars with the promise that a golden ticket is worth a trip through the chocolate factory, and all the chocolate you can eat for life. If the probability of finding a golden ticket is p, find the mean and variance of the number of candy bars you need to eat to find a ticket.

5. You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

6. Casella and Berger, Exercise 2.2

7. Casella and Berger, Exercise 2.6
8. Casella and Berger, Exercise 2.9
9. Casella and Berger, Exercise 2.10
10. Casella and Berger, Exercise 2.14
11. Casella and Berger, Exercise 2.17
12. Casella and Berger, Exercise 2.18
13. Casella and Berger, Exercise 2.26
14. Casella and Berger, Exercise 2.27