In terms of sets, we have the following interpretations:

- $x \in \bigcup_{i \in I} A_i$ means that x is in at least one of the A_i sets.
- $x \in \bigcap_{i \in I} A_i$ means that x is in all of the A_i sets.

So this means that

1. $\bigcap_{N \geq 1} \bigcup_{n \geq N} A_n$ are all elements somewhere in $A_N, A_{N+1}, A_{N+2}, \ldots$, *no matter how large* N *is*. Being a member of this set is logically equivalent to being "in infinitely many of the A_i sets".
2. $\bigcup_{N \geq 1} \bigcap_{n \geq N} A_n$ are all elements in every single one of $A_N, A_{N+1}, A_{N+2}, \ldots$ for some N. Being a member of this set is logically equivalent to being "in all *but* finitely many the A_i sets".

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A member of

$$\bigcup_{N=1}^{\infty} \bigcap_{n \geq N} A_n$$

is a member of at least one of the sets

$$\bigcap_{n \geq N} A_n,$$

meaning it's a member of either $A_1 \cap A_2 \cap A_3 \cap \cdots$ or $A_2 \cap A_3 \cap A_4 \cap \cdots$ or $A_3 \cap A_4 \cap A_5 \cap \cdots$ or $A_4 \cap A_5 \cap A_6 \cap \cdots$ or $\ldots$ etc. That means it's a member of all except finitely many of the A.

A member of

$$\bigcap_{N=1}^{\infty} \bigcup_{n \geq N} A_n$$

is a member of **all** of the sets

$$\bigcup_{n \geq N} A_n,$$

so it's a member of $A_1 \cup A_2 \cup A_3 \cup \cdots$ and of $A_2 \cup A_3 \cup A_4 \cup \cdots$ and of $A_3 \cup A_4 \cup A_5 \cup \cdots$ and of $A_4 \cup A_5 \cup A_6 \cup \cdots$ and of $\ldots$ etc. That means no matter how far down the sequence you go, it's a member of at least one of the sets that come later. That means it's a member of infinitely many of them, but there might also be infinitely many that it does not belong to.