Stochastic Ordering
X is stochastically greater than y if:
$F_X(t) \leq F_Y(t) \; \forall \; t$
$F_X(t) < F_Y(t)$ for some t
equivalently:
$P(X > t) \geq P(Y > t) \; \forall \; t$
$P(X > t) > P(Y > t)$ for some t

Median m
$P(X \leq m) \geq 1/2 \quad P(X \geq m) \geq 1/2$
$\int_{-\infty}^{m} f(x)\mathrm{d}x = \int_{m}^{\infty} f(x)\mathrm{d}x = 1/2$

Symmetric at point a
$\forall \; \epsilon > 0 \; f(a+\epsilon) = f(a-\epsilon)$

Mode a
$f(x)$ is unimodal with mode a if $a \geq x \geq y$ then
$f(a) \geq f(x) \geq f(y)$ and if $a \leq x \leq y$ then $f(a) \geq f(x) \geq f(y)$.

Geometric Series
$\sum_{n=1}^{\infty} ar^{n-1} \; s = \dfrac{a_1}{1-r}$

Finite $s = \dfrac{a_1(1-r^n)}{1-r}$

$\lim\limits_{n\to\infty} \left(1 + \dfrac{a_n}{n}\right)^n = e^a$

$M_X(t) = E(e^{tx}) = \sum_{n=0}^{\infty} \dfrac{t^n}{n!} E(X^n)$

$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\mathrm{d}x$

$Y = g(x)$ monotone function

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \begin{cases} f_X(g^{-1}(y)) \dfrac{d}{dy} g^{-1}(y) & \text{if g increasing} \\ -f_X(g^{-1}(y)) \dfrac{d}{dy} g^{-1}(y) & \text{if g decreasing} \end{cases}$$

$\min\limits_{b} E(X-b)^2 = E(X-EX)^2$

Bernoulli $p^x(1-p)^{1-x} \quad x = 0, 1$

Binomial $\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

Poisson $\dfrac{e^{-\lambda}\lambda^y}{y!}$

Hypergeometric $f_x(X) = \dfrac{\binom{M}{x}\binom{N-M}{k-x}}{\binom{N}{k}}$

Hypergeometric $\rightarrow$	Binomial $\rightarrow$	Poisson
N$\rightarrow \infty$,	$n \rightarrow \infty$,	$\lambda = np$
$M \rightarrow \infty$,	$p \rightarrow \infty$,	
$M/N \rightarrow p$	$np \rightarrow \lambda$	

Geometric $f(x) = p(1-p)^{x-1}$ $x = 1, 2, \dots$
$F(x) = 1 - (1-p)^x$

Memoryless Property Suppose $k > i$ then:
$P(X > k | X > i) = P(X > k - i)$

Negative Binomial number of failures before s^{th} success
$f(x) = \binom{s+x-1}{x} p^s q^x \quad x = 0, 1, 2, \dots$
no closed form for cdf.
$\lim\limits_{s \to \infty}$ we have poisson.

Uniform $U(a, b)$
$f(y) = \dfrac{1}{b-a} \quad a \leq y \leq b$

$$F(y) = \int_a^y \frac{1}{b-a} \mathrm{d}x = \begin{cases} 0 & y < a \\ \dfrac{y-a}{b-a} & a \leq y \leq b \\ 1 & y > b \end{cases}$$

Exponential $X \sim exp(\lambda)$
$f(y) = \lambda e^{-\lambda y} \quad y \geq 0$
$F(y) = \int_0^y \lambda e^{-\lambda x} dx = 1 - e^{-\lambda y} \quad y \geq 0$

Normal Distribution $Y \sim N(\mu, \sigma^2)$
$f(y) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(y-\mu)^2/2\sigma^2} \quad -\infty < y < \infty$
cdf no closed form
$\Phi(x) = F(X) = P(Y \leq x)$ for standard normal

Standardization
$Y \sim N(u, \sigma^2) \iff Z = \dfrac{Y - \mu}{\sigma} \sim N(0, 1)$
Shifting and scaling
$Z \sim N(0, 1) \iff Y = \sigma Z + \mu \sim N(\mu, \sigma^2)$

Gamma function
$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} \mathrm{d}x$
if a is an integer, $\Gamma(a) = (a-1)!$

Weibull
$f(y) = \dfrac{\beta}{\alpha} \left(\dfrac{y-v}{a} \right)^{\beta - 1} \exp \left[- \left(\dfrac{y-v}{\alpha} \right)^\beta \right] \quad y \geq v$
$F(y) = 1 - \exp \left[- \left(\dfrac{y-v}{\alpha} \right)^\beta \right] \quad y \geq v$
Usual case $v = 0$
If $\beta = 1$ we get exponential with parameter $\lambda = 1/\alpha$

Cauchy Distribution

$f(y) = \dfrac{1}{\pi} \dfrac{1}{1 + (y-\mu)^2/\sigma^2} \quad -\infty < y < \infty$

if $\mu = 0,\ \sigma = 1$ we have t-distribution with 1 degree of freedom.

Moments of Cauchy are not defined, its quantiles are.

Beta

$f(y) = \dfrac{y^{a-1}(1-y)^{b-1}}{B(a,b)} \quad 0 \leq y \leq 1$

Where $B(a,b)$ is the complete Beta function:

$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\mathrm{d}x = \dfrac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$

$\Gamma(a)$ is the complete gamma function

If a and b are integers, $B(a,b)$ can be calculated in closed form.

Location and Scale families

Let $f(x)$ be any pdf. Then the family of pdfs:

$f_{\mu,\sigma}(x) = \dfrac{1}{\sigma} f\left(\dfrac{x-\mu}{\sigma}\right) \quad \mu \in \mathbb{R},\ \sigma > 0$

is called a location-scale family

If $\sigma = 1$ we get a location family.

Multinomial Probabilities

$P(F) = p_1 \quad P(PS) = p_2 \quad P(S) = p_3 \quad (p_1 + p_2 + p_3) = 1$

Suppose in sample size n: s_1=number of failures, s_2=number of partial successes, s_3= number of successes

$P(s_1, s_2, s_3) = \dfrac{n!}{s_1!s_2!s_3!} p_1^{s_1} p_2^{s_2} p_3^{s_3}$

Generalizing to k classes gives us:

Multinomial Distribution:

$P(s_1, s_2, \ldots, s_k) = \dfrac{n!}{s_1!s_2!\ldots s_k!} p_1^{s_1} p_2^{s_2} \cdots p_k^{s_k}$

where $\sum_{i=1}^k s_i = n$ and $\sum_{i=1}^k p_i = 1$