Bios 660/Bios 672 (3 Credits)
Probability and Statistical Inference I

Homework 8

Due: Tue. October 23, 2018 at the Beginning of Class

Special Note: when turning in homework, please **staple** the answers into **3 groups**: (a) Questions 1-5; (b) Questions 6-10; (c) Questions 11-15.

1. (optional) Exercise 4.8 of Gut (1st ed, Page 78): Recall the following result:

Let X be a random variable and a and b be real numbers. Then $$\phi_{aX+b}(t) = e^{ibt}\phi_X(at)$$

   Let $X \sim N(\mu, \sigma^2)$. Use the expression for the characteristic function of the standard normal distribution and Theorem 4.8 (above result) to show that $\phi_X(t) = e^{it\mu - \sigma^2 t^2/2}$.

2. Let X be a poisson RV with parameter λ. Show that the PMF $p_X(k)$ increases monotonically with k up to the point where k reaches the largest integer not exceding λ and after tha point decreases monotinically with k. What is the definition of the point where k reaches the largest integer not exceding λ?

3. Casella and Berger 3.3

4. Casella and Berger 3.5

5. (optional) Casella and Berger 3.6

6. (optional) Casella and Berger 3.9

7. Casella and Berger 3.10

8. (optional) Casella and Berger 3.13

9. Casella and Berger 3.15

10. Casella and Berger, 3.16

11. One thousand independent rolls of a fair die will be made. Compute an approximation to the probability that 6 will appear between 150 and 200 times.

12. (optional) The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio, what is the probability that it will be working after an additional 8 years?

13. (optional) If X is an exponential random variable with mean $1/\lambda$. Show that:

$$E[X^k] = \frac{k!}{\lambda^k}, \text{ for } k = 1, 2, \ldots$$

14. If X is an exponential random variable with parameter λ, and $c > 0$, show that cX is exponential with parameter λ/c.

15. A particular parameterization of the Weibull pdf is given as:

$$f_X(s) = \frac{\beta}{\alpha}\left(\frac{s-\nu}{\alpha}\right)^{\beta-1} \exp\{-(\frac{s-\nu}{\alpha})^\beta\} \text{ for } s > \nu.$$

Show that a plot of $\log(\log(1 - F(x))^{-1})$ agains $\log x$ will be a straight line with slope β when $F(\cdot)$ is a Weibull distribution function with parameters γ and β. Show also that approximately 63.2 percent of all observations from such a distribution will be less than γ.