

Bios 660/Bios 672 (3 Credits)

Probability and Statistical Inference 1

Homework 5

Due: Tu. October 2, 2018 at the Beginning of Class

Special Note: when turning in homework, please **staple** the answers into **3 groups**: (a) Questions 1-5; (b) Questions 6-10; (c) Questions 11-16.

1. We roll two fair 6-sided dice. Each outcome is assumed to be equally likely. Find:
 - (a) The probability that doubles are rolled
 - (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled.
 - (c) Find the probability that at least one die roll is a 6.
 - (d) Given that the two dice land on different numbers, find the conditional probability that at least one die rolled is a 6.
2. Each of k jars contains m white and n black balls. A ball is randomly chosen from jar 1 and transferred to jar 2, then a ball is randomly chosen from jar 2 and transferred to jar 3, etc. Finally, a ball is chosen randomly from jar k . Show that the probability that the last ball is white is the same as the probability that the first ball is white, i.e. it is $\frac{m}{m+n}$.
3. Show that
$$P(A|B) = P(C|B)P(A|B \cap C) + P(C^c|B)P(A|B \cap C^c)$$
assuming all conditioning events have positive probability.
4. Let A and B be independent events. Prove that A and B^c are independent and that A^c and B^c are also independent.
5. The UNC mens soccer team has 2 games scheduled for the weekend. It has a 0.4 probability of not losing the first game and a 0.7 probability of not losing the second game, independently of the first. If it does not lose a particular game, the team is equally likely to win or tie, independently of what happens in the other game. The team will receive 2 points for a win, 1 for a tie, and 0 for a loss. Find the PMF of the number of points that the team earns over the weekend.

6. Harry and Ron play a chess match in which the first player to win a game wins the match. After 10 successive draws, the match is declared drawn. Each game is won by Ron with probability 0.4, won by Harry with probability 0.3, and is a draw with probability 0.3, independently of the previous games.
- (a) What is the probability that Harry wins the match?
 - (b) What is the PMF of the duration of the match?
7. Casella & Berger, 1.33
8. Casella & Berger, 1.36
9. Casella & Berger, 1.38
10. Casella & Berger, 1.39
11. Casella & Berger, 1.44
12. Casella & Berger, 1.45
13. Casella & Berger, 1.47
14. Casella & Berger, 1.53
15. Casella & Berger, 1.54
16. Casella & Berger, 1.55