Problem 1

In order for the sum to be greater than or equal to k, we need at least k ones. The number of ways to get k ones is:

$$\left(\sum_{i=1}^{n} x_i = k\right) \rightarrow \binom{n}{k}$$

Then we have to take in account all of the ways to get $k+1, k+2, ..., n$ ones. Summing all of these possibilities gives us:

$$\sum_{i=k}^{n} \binom{n}{i}$$

Problem 2

We want to show $\displaystyle\sum_{i=0}^{n}(-1)^i\binom{n}{i} = 0$ for $n > 0$

The binomial theorem states:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \quad \forall\ n \in \mathbb{N} \tag{1}$$

Let $x = -1, y = 1, k = i$

Then (1) becomes $\displaystyle\sum_{i=0}^{n} \binom{n}{i}(-1)^i(1)^{n-i} = (-1+1)^n$

Then we have $\displaystyle\sum_{i=0}^{n} \binom{n}{i}(-1)^i(1) = 0^n$

$$= \sum_{i=0}^{n} \binom{n}{i}(-1)^i = 0$$

Therefore $\displaystyle\sum_{i=0}^{n} \binom{n}{i}(-1)^i = 0$ for $n > 0$

Problem 3

(a) $\frac{4\binom{13}{5}}{\binom{52}{5}} \approx .001980$

(b) $\frac{13\binom{4}{2}\binom{12}{3}4^3}{\binom{52}{5}} \approx .422569$

(c) $\frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}11 * 4}{\binom{52}{5}} \approx .047539$

(d) $\frac{13\binom{4}{3}\binom{12}{2} * 4^2}{\binom{52}{5}} \approx .021128$

(e) $\frac{13 * 12 * 4}{\binom{52}{5}} \approx .000240$

Problem 4

$$\frac{16 * 4}{52 * 51} + \frac{4 * 16}{52 * 51} = \frac{16 * 4 * 2}{52 * 51} \approx .048265$$

Problem 5