Bios 660/Bios 672 (3 Credits)
Probability and Statistical Inference 1

Homework 4

Due: Tu. September 18, 2018 at the Beginning of Class

Special Note: when turning in homework, please **staple** the answers into **3 groups**: (a) Questions 1-3; (b) Questions 4-6; (c) Questions 7-9.

1. Determine the number of vectors $(x_1, x_2, \ldots, x_n)$ such that x_i is either 0 or 1 and

$$\sum_{i=1}^{n} x_i \geq k$$

2. Show that for $n > 0$

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$$

3. If it is assumed that all $\binom{52}{5}$ poker hands are equally likely, what is the probabilty of being dealt:

   (a) a flush? (all 5 cards have same suit)

   (b) one pair? (the cards have denomination a,a,b,c,d where a,b,c,d are distinct)

   (c) two pairs? (the cards have denomination a,a,b,b,c where a,b,c are distinct)

   (d) three of a kind? (the cards have denomination a,a,a,b,c where a,b,c are distinct)

   (e) four of a kind? (the cards have denomination a,a,a,a,b where a,b are distinct)

4. Two cards are selected randomly from an ordinary playing deck. What is the probability that they form a blackjack, i.e. they add up to 21?

5. If 4 married couples are arranged in a row, find the probability that no husband sits next to his wife.

6. Suppose that n balls are randomly distributed in N compartments. Find the probability that m balls will fall in the first compartment. Assume that all N^n arrangements are equally likely.

7. A group of 6 women and 6 men are randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of men?

8. A car is parked among N cars in a row (not at either end). On his return, the owner finds that exactly r of the N paces are still occupied. What is the probability that both neighboring places are empty?

9. We are given three coins: one has heads on both faces, the second has tails on both faces, and the third has a head in one face and a tail in the other. We choose a coin at random, toss it, and it comes up heads. What is the probability that the opposite face is tails?