

Homework 9

Gamma Distribution

Gamma Function: $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$

$\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$ $\alpha > 0$

$\Gamma(n) = (n-1)!$ $n \in \mathbb{Z}$

$\Gamma(1/2) = \sqrt{\pi}$

$f(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$

α is the shape parameter, influences the peakedness of the distribution

β is the scale parameter, influences the spread of the distribution

$EX^v = \frac{\beta^v \Gamma(v + \alpha)}{\Gamma(\alpha)}$

$\Gamma(\alpha + v) = \int_0^\infty x^{v+\alpha-1} e^{-x} dx$

$EX = \alpha\beta$

$\int_0^\infty e^{-x^2/2} dz = \frac{\sqrt{2\pi}}{2} = \sqrt{\frac{\pi}{2}}$

$\int_0^\infty x^2 e^{-x^2} dx$ is the same

Beta Distribution

$f(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}$

Beta Function: $B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx$

$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}$

$EX^n = \frac{B(\alpha + n, \beta)}{B(\alpha, \beta)} = \frac{\Gamma(\alpha + n)\Gamma(\alpha + \beta)}{\Gamma(\alpha + \beta + n)\Gamma(\alpha)}$

Mgfs

Negative Binomial Mgf $\left(\frac{p}{1 - (1-p)e^t} \right)^r$

Exponential Families

A family of pdfs or pmfs is called an exponential family if it can be expressed as

$$f(x|\theta) = h(x)c(\theta) \exp \left(\sum_{i=1}^k w_i(\theta) t_i(x) \right)$$

Homework 11

Conditional Probability

$$f(y|x) = \frac{f(x,y)}{f(x)}$$