Bios 660/Bios 672 (3 Credits)
Probability and Statistical Inference I

Homework 12

Due: Thursday. November 29, 2018 at the Beginning of Class

Special Note: No grouping needed for this assignment.

1. Casella and Berger, 4.62
2. Suppose (X, Y) follows a bivariate normal distribution with the pdf function:

$$\begin{aligned} f_{XY}(x,y) &= \left(2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}\right)^{-1} \\ &\times \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2\right.\right. \\ &- \left.\left.2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right)+\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right]\right\} \end{aligned}$$

   for $-\infty < x < \infty$ and $-\infty < y < \infty$. Show that

   (a) The marginal distribution of X is $N(\mu_X, \sigma_X^2)$.

   (b) The conditional distributions is:

$$Y|X \sim N\left[\mu_Y + \rho(\sigma_Y/\sigma_X)(x-\mu_X),\ \sigma_Y^2(1-\rho^2)\right]$$

   (c) For any constants a and b, the distribution of $aX + bY$ is

$$N(a\mu_X + b\mu_Y, a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho\sigma_X\sigma_Y)$$

3. Suppose the momment generating function of $[X, Y]'$ is

$$\psi_{X,Y}(t,u) = exp\{2t + 3u + t^2 + atu + 2u^2\}.$$

   (a) Determine a so that $X + 2Y$ and $2X - Y$ become independent.

   (b) Compute $P(X + 2Y < 2X - Y)$ with a as in part (a).

4. Let X_1 and X_2 be independent $N(0,1)$-distributed random variables. Set $Y_1 = X_1 - 3X_2 + 2$ and $Y_2 = 2X_1 - X_2 - 1$. Determine the distribution of

   (a) $\boldsymbol{Y} = (Y_1, Y_2)$, and

   (b) $Y_1 | Y_2 = y$.

5. Casella and Berger, 5.24

6. The geometric distribution is a discrete analog of the exponential distribution in the sense of lack of memory. More precisely, show that if X_1 and X_2 are independent $geom(p)$ distributed random variables, then $X_{(1)}$ and $X_{(2)} - X_{(1)}$ are independent.