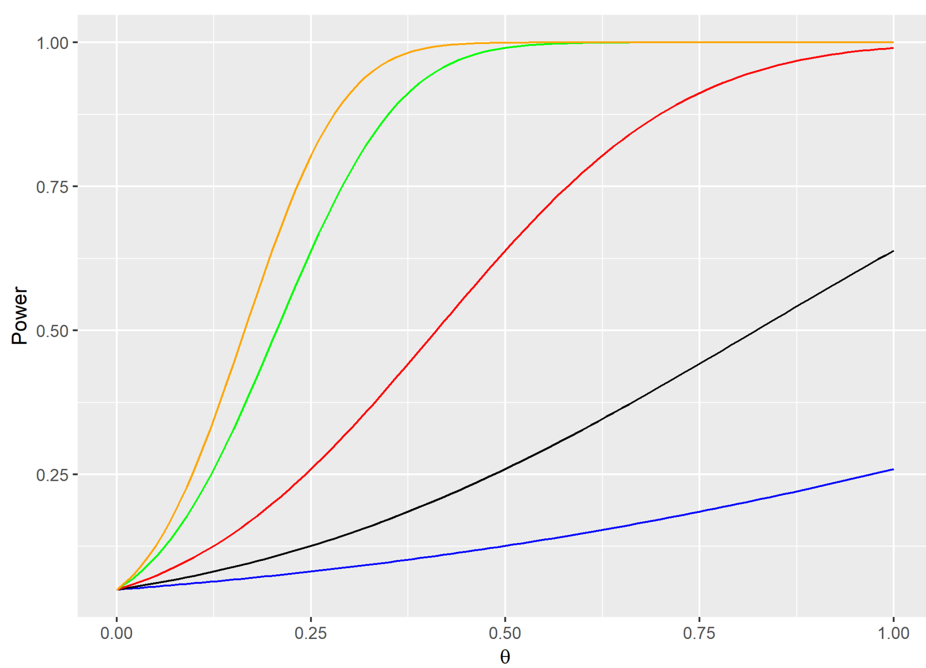
Problem 1

(a)

$$n = 1, 4, 16, 64, 100 \quad \alpha = .05$$
$$X \sim N(\mu, \sigma^2) \quad (\sigma^2 \text{ known})$$
$$H_0 : \mu \leq 0 \text{ vs } H_1 : \mu > 0$$
$$L(\theta|x) = \prod_{i=1}^{n} (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$
$$= (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x-\mu)^2}{2\sigma^2}\right)$$
$$\lambda(x) = \frac{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x-\mu_0)^2}{2\sigma^2}\right)}{(2\pi\sigma^2)^{-n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x-\bar{x})^2}{2\sigma^2}\right)}$$
$$= \exp\left(\left[-\sum_{i=1}^{n}(x_i-\mu_0)^2 + \sum_{i=1}^{n}(x_i-\bar{x})^2\right]/2\sigma^2\right)$$
$$R = \{x : \exp\left(-n(\bar{x}-\mu_0)^2/(2\sigma^2)\right) \leq c\}$$
$$R = \{x : (\bar{x}-\mu_0)^2 \geq \frac{-2\sigma^2\log(c)}{n}\}$$
$$R = \{x : |\bar{x}-\mu_0| \geq \sqrt{-\frac{2\sigma^2\log(c)}{n}}\}$$
$$R = \{x : |\bar{x}-\mu_0| \geq \frac{\sigma}{\sqrt{n}}\sqrt{-2\log(c)}\}$$
$$\Leftrightarrow R = \{x : \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \geq \sqrt{-2\log(c)} \text{ or } \leq -\sqrt{-2\log(c)}\}$$
$$c_1^* = \sqrt{-2\log(c)} \quad c_2^* = -\sqrt{-2\log(c)}$$
$$R = \{x : \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \geq c_1^* \text{ or } \leq c_2^*\}$$
$$R = \{x : \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \geq c_1^*\} \text{ (follows direction of } H_1)$$
$$\alpha = .05 = P\left(\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \geq c_1^*\right)$$
$$.05 = P(Z > c_1^*) \quad Z \sim N(0,1)$$
$$.05 = 1 - P(Z \leq c_1^*)$$
$$P(Z \leq c_1^*) = .95$$
$$qnorm(.95) = c_1^* = 1.645$$

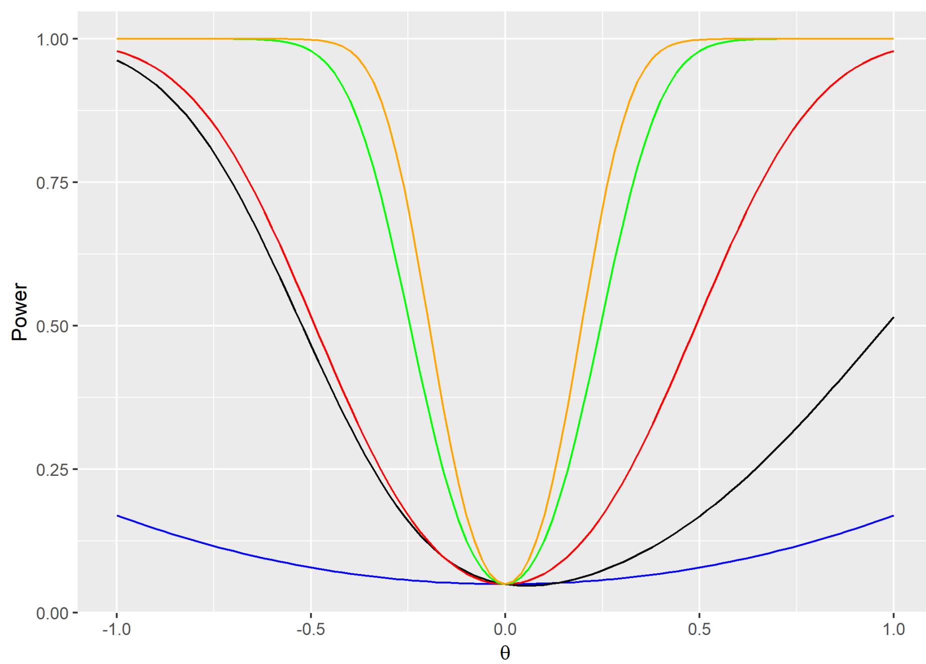
$$\beta(\mu) = P\left(\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} > c_1^*\right)$$
$$= P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > c_1^* + \frac{\mu_0-\mu}{\sigma/\sqrt{n}}\right)$$
$$= P(Z > 1.645 + \frac{\mu_0-\mu}{\sigma/\sqrt{n}})$$
$$\text{Since } \mu_0 = 0 \text{ we have:}$$
$$\beta(\mu) = P(Z > 1.645 - \frac{\sqrt{n}\mu}{\sigma})$$
$$= 1 - P(Z \leq 1.645 - \frac{\sqrt{n}\mu}{\sigma})$$
$$Power = 1 - P(Z \leq 0) = .5 \text{ if } 1.645 = \frac{\sqrt{n}\mu}{\sigma} \Rightarrow \mu = \frac{1.645\sigma}{\sqrt{n}}$$



(b)

$$H_0 : \mu = 0 \text{ vs } H_1 : \mu \neq 0$$
$$R = \{\lambda(x) < c\}$$
$$\Leftrightarrow R^* = \{x : \bar{x} \geq c_1^* \text{ or } \leq c_2^*\}$$

$$\alpha/2 = .025 = P\left(\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \geq c^*\right)$$
$$.025 = 1 - P(Z < c^*) \Rightarrow .975 = P(Z < c^*)$$
$$c^* = qnorm(.975) = 1.96$$
$$-c^* = qnorm(1-.975) = -1.96$$
$$\beta(\mu) = P\left(-c^* \leq \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \leq c^*\right)$$
$$= P\left(-1.96 \leq \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \leq 1.96\right)$$
$$\beta(\mu) = P(-1.96 - \sqrt{n}\mu/\sigma \leq Z \leq 1.96 + \sqrt{n}\mu/\sigma)$$
$$Power = .5 \text{ if } \mu = \pm 1.96\sigma/\sqrt{n}$$



Problem 2

$$X_1, \ldots, X_n \sim f(x|\theta,\lambda) = \frac{1}{\lambda}e^{-(x-\theta)/\lambda}I_{[\theta,\infty)}(x)$$
$$H_0 : \theta \leq 0 \text{ vs } H_1 : \theta > 0$$

$$L(\theta,\lambda|x)=\prod_{i=1}^n \frac{1}{\lambda}e^{-(x_i-\theta)/\lambda}I_{[\theta,\infty)}(x_i)$$
$$L(\theta,\lambda|x)=\lambda^{-n}\exp\left(-(\sum_{i=1}^n x_i-n\theta)/\lambda\right)I_{[\theta,\infty)}(x_{(1)})$$
$L(\theta,\lambda|x)$ is an increasing function of θ if $x_{(1)}\geq\theta$ for any λ
$$\hat{\theta}_{MLE}=x_{(1)}$$
$$\ell=-n\log(\lambda)-\frac{\sum_{i=1}^n x_i-n\hat{\theta}}{\lambda}$$
$$\frac{\partial\ell}{\partial\lambda}=\frac{-n}{\lambda}+\frac{\sum_{i=1}^n x_i-n\hat{\theta}}{\lambda^2}=0$$
$$\lambda=\frac{\sum_{i=1}^n x_i-n\hat{\theta}}{n}=$$
$$\hat{\lambda}=\bar{x}-\hat{\theta}=\bar{x}-x_{(1)}$$
$$\frac{\partial\ell^2}{\partial\lambda^2}=\frac{n}{\lambda^2}-2\frac{\sum_{i=1}^n x_i-n\hat{\theta}}{\lambda^3}$$
$$=\frac{n}{(\bar{x}-x_{(1)})^2}-2n\left(\frac{\bar{x}-x_{(1)}}{(\bar{x}-x_{(1)})^3}\right)$$
$$\frac{\partial\ell^2}{\partial\lambda^2}=-\left(\frac{n}{(\bar{x}-x_{(1)})^2}\right)<0$$
Thus $\hat{\theta}=x_{(1)},\ \hat{\lambda}=\bar{x}-x_{(1)}$ are the unrestricted MLEs
Under $H_0:\theta\leq 0$
$$\hat{\theta}_0=\begin{cases}0 & \text{if } x_{(1)}>0\\ x_{(1)} & \text{if } x_{(1)}\leq 0\end{cases}$$
For $x_{(1)}>0,\ \hat{\theta}_0=0$ we have:
$$\ell=-n\log(\lambda)-\frac{n\bar{x}}{\lambda}$$
$$\frac{\partial\ell}{\partial\lambda}=-\frac{n}{\lambda}+\frac{n\bar{x}}{\lambda^2}=0$$
$$\hat{\lambda}_0=\bar{x}$$
$$\lambda(x)=\frac{\sup_{\Theta_0}L(\theta,\lambda|x)}{\sup_{\Theta}L(\theta,\lambda|x)}=\begin{cases}1 & \text{if } x_{(1)}\leq 0\\ \dfrac{L(\theta_0,\lambda_0|x)}{L(\hat{\theta},\hat{\lambda}|x)} & \text{if } x_{(1)}>0\end{cases}$$
$$\frac{L(\theta_0,\lambda_0|x)}{L(\hat{\theta},\hat{\lambda}|x)}=\frac{\bar{x}^{-n}\exp\left(-(\sum_{i=1}^n x_i-n*0))/\bar{x}\right)}{(\bar{x}-x_{(1)})^{-n}\exp\left(-(\sum_{i=1}^n x_i-nx_{(1)})/(\bar{x}-x_{(1)})\right)}$$
$$=\left(\frac{\bar{x}-x_{(1)}}{\bar{x}}\right)^n\exp\left(-n-\frac{-n(\bar{x}-x_{(1)})}{\bar{x}-x_{(1)}}\right)$$

$$\frac{L(\theta_0,\lambda_0|x)}{L(\hat{\theta},\hat{\lambda}|x)} = (1-x_{(1)}/\bar{x})^n$$
$$\lambda(x) = \begin{cases} 1 & \text{if } x_{(1)} \leq 0 \\ (1-x_{(1)}/\bar{x})^n & \text{if } x_{(1)} > 0 \end{cases}$$
$$T(X) = x_{(1)}/\bar{x}$$

for $x_{(1)} > 0,\ \lambda(x)$ is a montone increasing function of $T(X)$ for every $\bar{x} > x_{(1)}$

Thus the MLR property holds

$$R = \{x : (1-x_{(1)}/\bar{x})^n \leq c\}$$
$$R = \{x : n\log(1-x_{(1)}/\bar{x}) \leq \log(c)\}$$
$$R = \{x : \exp(\log(1-x_{(1)}/\bar{x})) \leq \exp(\log(c)/n)\}$$
$$R = \{x : -x_{(1)}/\bar{x} \leq \exp(\log(c)/n) - 1\}$$
$$R = \{x : x_{(1)}/\bar{x} \geq 1 - c^{1/n}\}$$

Thus the rejection region is $R^* = \{x : x_{(1)}/\bar{x} \geq c^*\} \quad c^* = 1 - c^{1/n}$

Problem 3

$$X_1,\dots,X_n \sim N(0,\sigma^2)$$
$$H_0 : \sigma = \sigma_0 \text{ vs } H_1 : \sigma = \sigma_1$$
$$\sigma_0 < \sigma_1$$
$$L(x|\sigma^2) = (2\pi)^{-n/2}(\sigma^2)^{-n/2}\exp\left(\frac{-\sum_{i=1}^n x_i^2}{2\sigma^2}\right)$$
$$\ell(x|\sigma^2) \propto (-n/2)\log(\sigma^2) - \frac{\sum_{i=1}^n x_i^2}{2\sigma^2}$$
$$\frac{\partial\ell}{\partial\sigma^2} = (1/2)\left(-\frac{n}{\sigma^2} + \frac{\sum_{i=1}^n x_i^2}{(\sigma^2)^2}\right) = 0$$
$$\hat{\sigma^2}_{MLE} = \frac{\sum_{i=1}^n x_i^2}{n}$$
$$\frac{\partial\ell}{\partial(\sigma^2)^2}^2(\hat{\sigma^2}) = \frac{n}{2(\hat{\sigma^2})^2} - \frac{\sum_{i=1}^n x_i^2}{(\hat{\sigma^2})^3}$$
$$= \frac{n/2}{(\sum_{i=1}^n x_i^2/n)^2} - \frac{\sum_{i=1}^n x_i^2}{(\frac{\sum_{i=1}^n x_i^2}{n})^3}$$
$$= \frac{n/2-n}{(\sum_{i=1}^n x_i^2/n)^2} = \frac{-n/2}{(\sum_{i=1}^n x_i^2/n)^2} < 0$$

$$\text{Thus } \hat{\sigma^2} = \frac{\sum_{i=1}^n x_i^2}{n} \text{ is the unrestricted MLE}$$

Since we have simple vs composite

We can use (N-P) Lemma to construct a UMP level α test

$$R = \{x : \frac{f(x|\sigma_1)}{f(x|\sigma_0)} > c\}$$

$$\frac{f(x|\sigma_1)}{f(x|\sigma_0)} = \left(\frac{2\pi\sigma_1^2}{2\pi\sigma_0^2}\right)^{-n/2} \frac{\exp\left(-\sum_{i=1}^n x_i^2/2\sigma_1^2\right)}{\exp\left(-\sum_{i=1}^n x_i^2/2\sigma_0^2\right)}$$

$$= \left(\frac{\sigma_0}{\sigma_1}\right)^n \exp\left(\frac{1}{2}\sum_{i=1}^n x_i^2(1/\sigma_0^2 - 1/\sigma_1^2)\right)$$

$$R = \{x : \sum_{i=1}^n x_i^2 > \frac{2\log(c(\sigma_0/\sigma_1)^n)}{(1/\sigma_0^2 - 1/\sigma_1^2)}\}$$

$$c^* = \frac{2\log(c(\sigma_0/\sigma_1)^n)}{(1/\sigma_0^2 - 1/\sigma_1^2)}$$

$$R = \{x : \sum_{i=1}^n x_i^2 > c^*\}$$

$$\alpha = P(\sum_{i=1}^n x_i^2 > c^*)$$

$$\sum_{i=1}^n X_i^2/\sigma_0^2 \sim \chi_n^2$$

$$\alpha = P(\sum_{i=1}^n X_i^2/\sigma_0^2 > c^*/\sigma_0^2) = P(\chi_n^2 > c^*/\sigma_0^2)$$

$$c^*/\sigma_0^2 = \chi_{n,\alpha}^2$$

$$c^* = \sigma_0^2\chi_{n,\alpha}^2$$

Problem 4

(a)

$$f(x|\theta) = (1-\theta) + \frac{\theta}{2\sqrt{x}} \quad 0 < x < 1,\ 0 \le \theta \le 1$$

$$f(x|\theta) = \begin{cases} 1 & \text{if } \theta = 0 \\ \dfrac{1}{2\sqrt{x}} & \text{if } \theta = 1 \end{cases}$$

$$R = \{x : \frac{f(x|\theta=1)}{f(x|\theta=0)} > c\}$$
$$\frac{f(x|\theta=1)}{f(x|\theta=0)} = \frac{\prod_{i=1}^n (2\sqrt{x_i})^{-1}}{\prod_{i=1}^n 1}$$
$$= \frac{1}{2^n \prod_{i=1}^n x_i^{1/2}}$$
$$R = \{x : \frac{1}{2^n \prod_{i=1}^n x_i^{1/2}} > c\}$$
$$= \{x : -n\log(2) - \frac{1}{2}\sum_{i=1}^n \log(x_i) > \log(c)\}$$
$$= \{x : -\sum_{i=1}^n \log(x_i) > 2(\log(c) + n\log(2))\}$$
$$R = \{x : -\sum_{i=1}^n \log(x_i) > c^*\}$$
$$c^* = 2(\log(c) + n\log(2))$$
$$\text{Let } Y_i = -\log(x_i)$$
$$e^{-y_i} = x_i$$
$$J = |-e^{-y_i}|$$
$$f_{y_i}(y) = f_{x_i}(e^{-y_i})|e^{-y_i}|$$
$$\text{Since } \theta = 0 \quad f_{x_i}(e^{-y_i}) = 1$$
$$f_{y_i}(y) = 1e^{-y_i} = e^{-y_i} \quad 0 < y_i < \infty$$
$$Y_i \sim Exp(1)$$
$$\sum_{i=1}^n y_i \sim gamma(n, 1)$$
$$\alpha = P(-\sum_{i=1}^n log(x_i) > c^*|\theta = 0)$$
$$c^* = gamma(n, 1, 1-\alpha)$$
$$R = \{x : -\sum_{i=1}^n \log(x_i) > gamma(n, 1, 1-\alpha)\}$$

(b)

$$n = 5 \quad \alpha = .01$$

$$R = \{x : -\sum_{i=1}^{n} \log(x_i) > gamma(5, 1, .99)\}$$
$$qgamma(p = .99, shape = 5, scale = 1) = 11.6$$
$$R = \{x : -\sum_{i=1}^{n} \log(x_i) > 11.6\}$$
$$Power = P(-\sum_{i=1}^{n} \log(x_i) > 11.6|\theta = 1)$$
$$Y_i = -\log(x_i)$$
$$e^{-y_i} = x_i$$
$$J = | - e^{-y_i}|$$
$$f_{y_i}(y) = f_{x_i}(e^{-y_i})|e^{-y_i}|$$
$$\text{Since } \theta = 1 \quad f_{x_i}(e^{-y_i}) = \frac{1}{2\sqrt{e^{-y_i}}}$$
$$f_{y_i}(y) = \frac{1}{2\sqrt{e^{-y_i}}} e^{-y_i} = \frac{1}{2} e^{-y_i/2}$$
$$Y_i \sim Exp(2)$$
$$\sum_{i=1}^{n} Y_i \sim gamma(n, 2)$$
$$\text{Since } n = 5 \quad \sum_{i=1}^{n} Y_i \sim gamma(5, 2)$$
$$Power = P(-\sum_{i=1}^{5} \log(x_i) > 11.6|\theta = 1)$$
$$Power = pgamma(q = 11.6, shape = 5, scale = 2) = .6872817 \approx .69$$

(c)

$$P(TypeIError) = .01 \quad P(TypeIIError) = .01$$
$$Power = 1 - .01 = .99$$
$$.99 = P(-\sum_{i=1}^{n} \log(x_i) > gamma(n, 1, .99)|\theta = 1)$$
$$-\sum_{i=1}^{n} \log(x_i) \sim gamma(n, 2)$$
$$\text{Using R, testing } n = 1 : 55 \text{ into}$$
$$1 - pgamma(shape = n, scale = 2, q = qgamma(shape = n, scale = 1, p = .99))$$

$$n = 45 \; p = 0.9891953 \quad n = 46 \; p = 0.9904418$$

Thus sample size needed is 46

Finding approximate sample size

$$P(-\sum_{i=1}^{n} \log(x_i) > gamma(n, 1, .99)|\theta = 1)$$

$$CLT : \sqrt{n}(\bar{Y} - E(Y_1)) \xrightarrow{d} N(0, Var(Y_1))$$

$$.99 \approx P(\sum_{i=1}^{n} Y_i > Gamma(n, 1, .99)|\theta = 1)$$

$$.99 = P(\frac{\sqrt{n}(\bar{Y} - E(Y_1))}{\sqrt{Var(Y_1)}} > \frac{\sqrt{n}Gamma(n, 1, .99)/n) - 2}{\sqrt{Var(Y_1)}})$$

$$\sqrt{Var(Y_1)} = \sqrt{4} = 2 \quad E(Y_1) = 2$$

$$P(\sqrt{n}(\bar{Y} - 2)/2) > [\sqrt{n}gamma(n, 1, .99)/(n) - 2]/2) = .99$$

Let $Z = \sqrt{n}(\bar{Y} - 2)/2 \sim N(0, 1)$

$$P(Z > (\sqrt{n}gamma(n, 1, .99)/(n) - 2)/2) = .99$$

$$1 - P(Z \leq (\sqrt{n}gamma(n, 1, .99)/(n) - 2)/2) = .99$$

$$P(Z \leq (\sqrt{n}gamma(n, 1, .99)/(n) - 2)/2) = .01$$

$$qnorm(.01) = -2.33$$

$$\sqrt{n}gamma(n, 1, .99)/(n) - 2)/2 = -2.33$$

Using R to Solve:

$$sqrt(n) * (qgamma(shape = n, scale = 1, p = .99)/(n) - 2)/2 = -2.33$$

We get $n \approx 52$

(d)

$$f(x|\theta) = (1 - \theta) + \frac{\theta}{2\sqrt{x}}$$

$$L'(\theta) = -1 + \frac{1}{2\sqrt{x}}$$

if $L'(\theta) > 0$ then $\hat{\theta} = 1$

if $L'(\theta) < 0$ then $\hat{\theta} = 0$

if $L'(\theta) = 0$ then $0 \leq \hat{\theta} \leq 1$

$$-1 + \frac{1}{2\sqrt{x}} = 0$$

$$2\sqrt{x} = 1$$

$$x = 1/4$$

$$\hat{\theta} = \begin{cases} 1 & \text{if } 0 < x \leq 1/4 \\ 0 & \text{if } 1/4 < x < 1 \end{cases}$$

$$E(\hat{\theta}) = 1 * P(X \leq 1/4) + 0 * P(X > 1/4)$$

$$E(\hat{\theta}) = \int_0^{1/4} 1(1 - \theta + \frac{\theta}{2\sqrt{x}})\mathrm{d}x$$

$$= \Big|_0^{1/4} (1-\theta)x + \theta x^{1/2} = (1-\theta)(1/4) + (1/2)\theta = (1/4)(\theta+1)$$

$$E(\hat{\theta}) = (1/4)(\theta+1) \text{ (biased)}$$

$$E(4\hat{\theta} - 1) = \theta \text{ (unbiased)}$$

$$T(X_1) = a + b\hat{\theta} = -1 + 4\hat{\theta}$$

The problem with $T(X_1)$ as an estimator of θ is:

$$\hat{\theta} = 0 \Rightarrow T(X_1) = 4(0) - 1 = -1$$

$$\hat{\theta} = 1 \Rightarrow T(X_1) = 4(1) - 1 = 3$$

Neither of these results make sense because

they are outside of the parameter space $0 \leq \theta \leq 1$

Problem 5

(a)

$$Y_i \sim Pois(\theta x_i)$$

$$f(y|\theta) = \prod_{i=1}^{n} f(y_i|\theta) = \prod_{i=1}^{n} \frac{(\theta x_i)^{y_i} e^{-\theta x_i}}{y_i!}$$

$$= \left(\prod_{i=1}^{n} \frac{x_i^{y_i}}{y_i!}\right) \theta^{\sum_{i=1}^n y_i} e^{-\theta \sum_{i=1}^n x_i}$$

$$H_0 : \theta = 1 \text{ vs } \quad H_1 : \theta > 1$$

Since we have simple vs composite

We can use (N-P) Lemma, to construct a UMP level α test

$$R = \{y : \frac{f(y|\theta)}{f(y|\theta = 1)} > c\}$$

$$\frac{f(y|\theta)}{f(y|\theta=1)} = \frac{\left(\prod_{i=1}^n \frac{x_i^{y_i}}{y_i!}\right) \theta_1^{\sum_{i=1}^n y_i} e^{-\theta_1 \sum_{i=1}^n x_i}}{\left(\prod_{i=1}^n \frac{x_i^{y_i}}{y_i!}\right) 1^{\sum_{i=1}^n y_i} e^{-\sum_{i=1}^n x_i}}$$

$$= \frac{\theta_1^{\sum_{i=1}^n y_i} e^{-\theta_1 \sum_{i=1}^n x_i}}{e^{-\sum_{i=1}^n x_i}}$$
$$R = \{y : \theta_1^{\sum_{i=1}^n y_i} e^{-\sum_{i=1}^n x_i(\theta_1 - 1)} > c\}$$
$$\alpha = \sup_{\theta \in H_0} P(\theta_1^{\sum_{i=1}^n y_i} e^{-\sum_{i=1}^n x_i(\theta_1 - 1)} > c)$$
$$\alpha = P_{\theta=1}(\theta_1^{\sum_{i=1}^n y_i} e^{-\sum_{i=1}^n x_i(\theta_1 - 1)} > c)$$
$$R \Leftrightarrow R^* = \{y : T(y) \geq c_1^* \text{or} \leq c_2^*\} \text{ (MLR property)}$$
$$T(y) = \sum_{i=1}^n y_i$$
$$\{y : \sum_{i=1}^n y_i \geq c_1^* \text{or} \leq c_2^*\}$$
$$R^* = \{y : \sum_{i=1}^n y_i \geq c_1^*\} \text{ (R follows the direction of } H_1)$$
$$\alpha = P(\sum_{i=1}^n y_i \geq c_1^* | \theta = 1)$$
$$\sum_{i=1}^n y_i \sim Pois(\theta \sum_{i=1}^n x_i) = Pois(\sum_{i=1}^n x_i) \text{ (since } \theta = 1)$$

(b)

$$H_0 : \theta \leq 1 \text{ vs } H_1 : \theta > 1$$
Since we have composite vs composite

We can use (K-R) Theorem, to construct a UMP level α test
$$f(y|\theta) = \left(\prod_{i=1}^n \frac{x_i^{y_i}}{y_i!}\right) \theta^{\sum_{i=1}^n y_i} e^{-\theta \sum_{i=1}^n x_i}$$
$$\frac{f(y|\theta_1)}{f(y|\theta_0)} \text{ increasing function of } T(y) \text{ if } \theta_1 > \theta_0 \text{ (MLR)}$$
$$T(y) = \sum_{i=1}^n y_i$$
$$\frac{f(y|\theta_1)}{f(y|\theta_0)} = \frac{\left(\prod_{i=1}^n \frac{x_i^{y_i}}{y_i!}\right) \theta_1^{\sum_{i=1}^n y_i} e^{-\theta_1 \sum_{i=1}^n x_i}}{\left(\prod_{i=1}^n \frac{x_i^{y_i}}{y_i!}\right) \theta_0^{\sum_{i=1}^n y_i} e^{-\theta_0 \sum_{i=1}^n x_i}}$$
$$\frac{f(y|\theta_1)}{f(y|\theta_0)} = \frac{\theta_1}{\theta_0} e^{-(\theta_1 - \theta_0) \sum_{i=1}^n x_i}$$

$$\theta_1 > \theta_0 \Rightarrow \frac{\theta_1}{\theta_0} > 0 \Rightarrow \frac{f(y|\theta_1)}{f(y|\theta_0)} \text{ is an increasing function of } \sum_{i=1}^{n} y_i$$
$$R = \{y : \sum_{i=1}^{n} y_i \geq c_1^*\}$$
$$c_1^* = 3 \text{ (from part a)}$$

(c)

$$\sum_{i=1}^{n} y_i \sim Pois(\sum_{i=1}^{n} x_i) = Pois(.8)$$
$$\alpha = .05 = P(\sum_{i=1}^{n} y_i \geq c_1^* | \theta = 1)$$
$$.05 = 1 - P(\sum_{i=1}^{n} y_i < c_1^*)$$
$$.05 = 1 - P(\sum_{i=1}^{n} y_i \leq c_1^* - 1)$$
$$P(\sum_{i=1}^{n} y_i \leq c_1^* - 1) = .95$$
$$P(X \leq 2) = .952 \quad X \sim Pois(.8)$$
$$P(\sum_{i=1}^{n} y_i \leq c_1^* - 1) \approx P(X \leq 2)$$
$$\text{Thus } c_1^* = 3$$
$$1 - P(\sum_{i=1}^{n} y_i \leq 3 - 1) = 1 - .952 = .048$$

(d)

$$\sum_{i=1}^{n} y_i \sim Pois(\theta \sum_{i=1}^{n} x_i) = Pois(5 * .8) = Pois(4)$$
$$Power = P(\sum_{i=1}^{n} y_i \geq 3 | \theta = 5)$$

$$= 1 - P(\sum_{i=1}^{n} y_i < 3)$$
$$= 1 - P(\sum_{i=1}^{n} y_i \leq 2)$$
$$P(X \leq 2) = .238 \quad X \sim Pois(4)$$
$$= 1 - P(\sum_{i=1}^{n} y_i \leq 2) \approx 1 - P(X \leq 2)$$
$$= 1 - .238 = .762$$
$$Power = .762$$