Problem 1

$$P(max(X_1, X_2) > m) = 1 - P(X_1 \leq m, X_2 \leq m)$$
Since the Xs are iid we have:
$$= 1 - P(X_1 \leq m)P(X_2 \leq m)$$
$$= 1 - P(X_1 \leq m)^2$$
$$= 1 - (1/2)^2 = 3/4$$
Generalizing this result:
$$P(max(X_1, \dots, X_n) \leq m) = 1 - P(X_i \leq m,\ i = 1, \dots, n)$$
$$= 1 - P(X_1)P(X_2 \leq m) \cdots P(X_n \leq m)$$
$$= 1 - [P(X_1 \leq m)]^n = 1 - (1/2)^n$$

Problem 2

$$f_X(x) = \frac{1}{\theta} \quad 0 < x < \theta \quad F_X(x) = \frac{1}{\theta}x$$
Let $U = X_{(1)} \quad V = X_{(n)}$
$$f_{U,V}(u, v) = \frac{n!}{(1-1)!(n-1-1)!(n-n)!}\frac{1}{\theta^2}[\frac{1}{\theta}u]^{1-1}[\frac{1}{\theta}(u-v)]^{n-1-1}[1-\frac{1}{\theta}v]^{n-n}$$
$$f_{U,V}(u, v) = \frac{n(n-1)}{\theta^n}(v-u)^{n-2} \quad 0 < u < v < \theta$$
Let $Z = U/V \quad W = V$

Then $U = ZW \quad V = W$

$0 < z < 1(\text{since } u < v) \quad 0 < w < \theta$
$$J = \begin{bmatrix} w & 0 \\ z & 1 \end{bmatrix} = |w|$$
$$f_{Z,W}(z, w) = \frac{n(n-1)}{\theta^n}(w - zw)^{n-2}|w|$$
$$= \frac{n(n-1)}{\theta^n}w^{n-2}(1-z)^{n-2}w$$
$$f_{Z,W}(z, w) = \frac{n(n-1)}{\theta^n}w^{n-1}(1-z)^{n-2} \quad 0 < z < 1,\ 0 < w < \theta$$
Since f_{ZW} can be factored into $(f_Z)(f_W)$ they are independent

Thus $\frac{X_{(1)}}{X_{(n)}}$ and $X_{(n)}$ are independent random variables

Problem 3

(a)

(b)

Problem 4

(a)

(b)

Problem 5

(a)

(b)

(c)

(d)