Successive 1-Dimensional Maximization

To maximize $L(\alpha, \beta|x)$ over both α and β:

1. Fix α maximize $L(\alpha, \beta|x)$ over β
2. Let $\hat{\beta}(\alpha)$ be the value of β that maxs $L(\alpha, \beta|x)$ for fixed α
3. Profiled likelihood function for α, $H(\alpha|x) = L(\alpha, \hat{\beta}(\alpha)|x)$ depends on α
4. The MLE of β is $\hat{\beta}(\hat{\alpha}_H)$ where $\hat{\alpha}_H$ maxs $H(\alpha|x)$

Invariance Property of MLE

Thm: If $\hat{\theta}$ is the MLE of θ, then for any function $\tau(\theta)$, the MLE of $\tau(\theta)$ is $\tau(\hat{\theta})$

If mapping $\theta \to \tau(\theta)$ is one-to-one, letting $\eta = \tau(\theta)$, the MLE of η is the same since:

$$L^*(\eta|x) = \prod_{i=1}^{n} f(x_i|\tau^{-1}(\eta)) = L(\tau^{-1}(\eta|x)) \text{ and}$$
$$\sup_{\eta} L^*(\eta|x) = \sup_{\eta} L(\tau^{-1}(\eta)|x) = \sup_{\theta} L(\theta|x)$$