Bios 661: 1 – 5; Bios 673: 2 – 6.

1. C&B 8.20
2. C&B 8.22
3. C&B 8.28
4. C&B 8.31
5. Let $X_1, \ldots, X_n$ be random sample of size n having a pdf of the form $f(x|\theta) = 1/\theta$, $0 < x < \theta$, and zero elsewhere. Let $X_{(n)}$ be the maximum order statistics. One would reject $H_0 : \theta = 1$ and accept $H_1 : \theta \neq 1$ if either $X_{(n)} \leq 1/2$ and $X_{(n)} > 1$. Find the power function $\beta(\theta)$ of the test for $\theta > 0$,.
6. Let $X_1, \ldots, X_n$ be a random sample from a population with probability density function
$$f(x|\theta) = \frac{1}{\sqrt{2\pi}\theta x} \exp\left\{-\frac{1}{2}\left(\frac{\log x}{\theta}\right)^2\right\}, \quad x > 0, \quad \theta > 0.$$
This is a pdf of what we call "log-normal distribution" and could be a possible distribution other than exponential (or Gamma family) for a variable having only positive values (with only positive domain). You may check C&B to see its relationship with the normal distribution.
   (a) Let $T = \sum_{i=1}^n (\log X_i)^2$. Show that $P(T > t|\theta = \theta_2) > P(T > t|\theta = \theta_1)$, for all $\theta_2 > \theta_1$ and any constant value $t > 0$.
   (b) Show that there is an uniformly most powerful test of null hypothesis $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$, and find the rejection region of such test.
7. [Bios 763 class discussion] Let $X_1, \ldots, X_n$ be a random variable having probability density $f(x|\theta) = \exp\{w(\theta)t(x) - \xi(\theta)\}h(x)$, where $w(\theta)$ is an increasing and differentiable function of $\theta \in \Theta \subset \mathcal{R}$.
   (a) Show that $\ell(\hat{\theta}) - \ell(\theta_0)$ is increasing (or decreasing) in t when $\hat{\theta} > \theta_0$ (or $\hat{\theta} < \theta_0$), where $\ell(\theta)$ is the log-likelihood function, $\hat{\theta}$ is the MLE of θ, and $\theta_0 \in \Theta$.
   (b) For testing $H_0 : \theta_1 \leq \theta \leq \theta_2$ versus $H_1 : \theta < \theta_1$ or $\theta > \theta_2$, show that there is a likelihood ratio test whose rejection region is equivalent to $T(X) < c_1$ or $T(X) > c_2$ for some constant c_1 and c_2.