Cauchy-Schwartz Inequality
$Cov(X,Y) \leq \sqrt{Var(X)Var(Y)}$

Equivalently: $Var(X) \geq \dfrac{[Cov(X,Y)]^2}{Var(Y)}$

CRLB
Let $X_1, \ldots, X_n$ be a sample with pdf $f(x|\theta)$ and let $W(\boldsymbol{X}) = W(X_1, \ldots, X_n)$ be any unbiased estimator of $\tau(\theta)$ satisfying:
$\frac{\partial}{\partial\theta}E(W(\boldsymbol{X})) = \int_{\chi} \frac{\partial}{\partial\theta}[W(\boldsymbol{x})f(\boldsymbol{x}|\theta)]\mathrm{d}x$
and $Var(W(\boldsymbol{X})) < \infty$
Then: $Var(W(\boldsymbol{X})) \geq \dfrac{\{\frac{\partial\tau(\theta)}{\partial\theta}\}^2}{E(U(\theta|\boldsymbol{x}))^2}$
Where **score function** $U(\theta|\boldsymbol{x}) = \frac{\partial}{\partial\theta}\log(f(\boldsymbol{x}|\theta))$

Thm: If W is the best unbiased estimator (UMVUE) for $\tau(\theta)$ then it is unique

$$Var(W(\boldsymbol{X})) \geq \frac{\{\frac{\mathrm{d}}{\mathrm{d}\theta}E(W(\boldsymbol{X}))\}^2}{E\left[\{\frac{\partial}{\partial\theta}\log(f(\boldsymbol{X}|\theta))\}^2\right]}$$

If $X_1, \ldots, X_n$ are iid with pdf $f(x|\theta)$ then:

$$Var(W(\boldsymbol{X})) \geq \frac{\{\frac{\mathrm{d}}{\mathrm{d}\theta}E(W(\boldsymbol{X}))\}^2}{nE\left[\{-\frac{\partial^2}{\partial\theta^2}\log(f(X_1|\theta))\}\right]}$$

Numerator is 1 if using θ

Numerator is $\left(\dfrac{d\tau(\theta)}{d\theta}\right)^2$ if using $\tau(\theta)$

Rao-Blackwell Theorem
Let W be any unbiased estimator of $\tau(\theta)$, and let T be an SS for θ.
Define $\phi(T) = E(W|T)$
Then $E(\phi(T)) = \tau(\theta)$ and $Var(\phi(T)) \leq Var(W)$ for all θ

Lehmann-Sheffe Theorem
W unbiased estimator of $\tau(\theta)$, T CSS for θ. Then $\phi(T) = E(W|T)$ is the UMVUE for $\tau(\theta)$ and is unique

LRT Statistic: $\lambda(x) = \dfrac{L(\hat{\theta}_0|x)}{L(\hat{\theta}|x)}$

Rejection Region: $R = \{x : \lambda(x) \leq c\}$
Want to find an equivalent region using unrestricted MLE $\hat{\theta}$:
$R = \{x : \lambda(x) \leq c\} \Leftrightarrow R^* = \{x : \hat{\theta} \geq c^* \text{ or } \hat{\theta} \leq c^*\}$
$\hat{\theta} \geq c^*$ or $\hat{\theta} \leq c^*$ follows direction of H_1