

If X less than full rank ($r < p$) then collinearity exists among the columns of X

$$X_{n \times p} = X_{*,(n \times r)} V'_{+, (r \times p)}$$

Suppose we have less than full rank model:

$$y_{n \times 1} = X_{n \times p} \beta_{p \times 1} + \epsilon_{n \times 1}$$

Define $X_{*,(n \times r)} V_{+, (p \times r)}$ with $\beta_{*, (r \times 1)}$

Then equivalent full rank model:

$$y_{n \times 1} = X_{*,(n \times r)} \beta_{*, (r \times 1)} + \epsilon_{n \times 1} \text{ with } \hat{\beta}_* = (X_*' X_*)^{-1} X_*' y$$

Many possible choices for V_+ such as the set of eigenvectors of $X'X$ corresponding to non-zero eigenvalues.