BIOS 667, Spring 2018 Midterm

Show your work. All numerical expressions must be simplified as much as possible (Example: $\sqrt{9}/(1+\sqrt{4})$ should be simplifed to 1).

1. In the TLC study, in the Placebo group, suppose that, using i to index subjects, the vector consisting of the first three outcomes $(Y_{i1}, Y_{i2}, Y_{i3})^\top$ is distributed as multivariate normal with mean $(25, 15, 12)^\top$ in μg/dL, and covariance matrix

$$50 \begin{bmatrix} 1 & 0.5 & 0.3 \\ & 1 & 0.4 \\ & & 1 \end{bmatrix}.$$

   Define $A_i = (Y_{i2} + Y_{i3})/2 - Y_{i1}$.

   (a) Find the mean and variance of A_i.

   (b) Find $\text{cov}(A_i, Y_{i1})$.

   (c) Find expressions for the conditional mean and conditional variance of A_i given Y_{i1}.

2. Suppose we have three observations with covariate values $x_1 = -2, x_2 = 0, x_3 = 1$, and mean vector, η, given by:

$$\eta_i = \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 \qquad i = 1, 2, 3.$$

   (a) Is $\beta_2 - \beta_3$ estimable? Justify.

   (b) Is β_1 estimable? Justify.