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Bios 667 HW#5
Due 12/4/17

Step 1: Run proc logistic

model y = time time* group

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parameter	estimate
intercept	-2.509
time	0.249
time* grp	0.266

step 2:
→ proc nlmixed set q=25
w/ parameters from proc logistic and given $G=\begin{bmatrix}4 & 0\\ 0 & 1\end{bmatrix}$

(a)

parameter	Estimate	SE
β_0	4.32	0.478
β_1	-0.0957	0.173
β_2	-0.942	0.356

✓

NOTE sigma estimates (SE)
sigma 11 3.73 (1.59)
sigma 21 -0.0653 (0.318)
sigma 22 0.986 (0.356)
} * used in sims in part c-e ✓

(b) interpret $\beta_0, \beta_1, \beta_2$

β_0 represents time = 0 logit for placebo patients

β_1 the difference in time 0 (baseline) logit due to time

β_2 the difference in logit due to time for treated patients.

(c) ... $\text{logit}(\pi_{ij}) = \beta_0 + b_{i1} + (\beta_1 + b_{i2}) * \text{time} + (\beta_2) * \text{time} \cdot \text{group}$

- Placebo $t=0 \Rightarrow \text{logit} = \beta_0 + b_{i1}$

$E[Y_{0,0}] = 0.949$

- placebo $t=3 \Rightarrow \text{logit} = \beta_0 + b_{i1} + (\beta_1 + b_{i2}) * 3$

$E[Y_{0,3}] = 0.848$

- placebo $t=6 \Rightarrow \text{logit} = \beta_0 + b_{i1} + (\beta_1 + b_{i2}) * 6$

$E[Y_{0,6}] = 0.719$

- Active $t=0 \Rightarrow \text{logit} = \beta_0 + b_{i1}$

$E[Y_{1,0}] = 0.949$

- Active $t=3 \Rightarrow \text{logit} = \beta_0 + b_{i1} + (\beta_1 + b_{i2}) * 3 + \beta_2 * 3$

$E[Y_{1,3}] = 0.621$

- Active $t=6 \Rightarrow \text{logit} = \beta_0 + b_{i1} + (\beta_1 + b_{i2}) * 6 + \beta_2 * 6$

$E[Y_{1,6}] = 0.384$

(d) Find / compute the marginal correlation matrix for placebo at $t = 0, 3, 6$

we know

$$\boxed{\begin{aligned} E[Y_{0,0}] &= 0.949 \\ E[Y_{0,3}] &= 0.848 \\ E[Y_{0,6}] &= 0.719 \end{aligned}} \Rightarrow \boxed{\begin{aligned} E[Y_{0,0} Y_{0,3}] &= 0.816 \\ E[Y_{0,0} Y_{0,6}] &= 0.690 \\ E[Y_{0,3} Y_{0,6}] &= 0.696 \end{aligned}}$$

α

$\text{var}(Y_{00}) = 0.0481$

$\Rightarrow \text{var}(Y_{03}) = 0.129$

$\text{var}(Y_{06}) = 0.202$

$\text{cov}(Y_{00}, Y_{03}) = 0.0104$

$\Rightarrow \text{cov}(Y_{00}, Y_{06}) = 0.00718$

$\text{cov}(Y_{03}, Y_{06}) = 0.0864$

$\rightarrow$

~~2)~~ continued.

⇒ correlation:

$\text{corr}(Y_{00}, Y_{03}) = 0.132$ ✓

$\text{corr}(Y_{00}, Y_{06}) = 0.0728$ ✓ 2

$\text{corr}(Y_{03}, Y_{06}) = 0.536$ ✓

e) Based on model fit, compute the estimated ICC for a placebo pt at $t = 6$.

var_between = 0.150

var - total = 0.202

ICC for placebo, $t = 6$ = 0.741 ✓

(f) Estimate the population proportion of placebo subjects w/ a positive subject specific trend over time:

$P(\beta_2 + b_{i2} > 0) = P(b_{i2} > -\beta_2) = P(-b_{i2} < \beta_2)$ then stdize

$P\left(\frac{-b_{i2}}{\sqrt{g_{22}}} < \frac{\beta_2}{\sqrt{g_{22}}}\right) = \Phi\left(\frac{\beta_2}{\sqrt{g_{22}}}\right) = \theta$

For placebo $\hat{\theta} = \Phi\left(\frac{\hat{\beta}_2}{\sqrt{\hat{g}_{22}}}\right)$

$\hat{g}_{22} = 0.986$

$\hat{\beta}_2 = -0.942$

$\hat{\theta} = \Phi\left(\frac{-0.942}{\sqrt{0.986}}\right)$

To do CI need $\hat{\theta} \pm 1.96\,\widehat{se}$

$\hat{\theta} = \Phi\left(\frac{-0.942}{\sqrt{0.986}}\right)$

$\widehat{se} \Rightarrow$ delta method: $\sqrt{K}\left(\frac{\hat{\beta}_2}{\hat{g}_{22}} - \frac{\beta_2}{g_{22}}\right) \xrightarrow{d} N(0, \Gamma$

$= \frac{\sqrt{\delta^T \Gamma \delta}}{\sqrt{K}}$

... ?

(g) $H_0: g_{22} = 0$ vs. $H_0: g_{22} > 0$ $\alpha = 0.01$

$\Rightarrow$ large-sample dist

$\chi_1^2 = 6.63$ $\Rightarrow$ $\frac{6.63 + 9.210}{2} = \frac{15.8}{2} = 7.92$

at $\alpha = 0.01$

$\chi_2^2 = 9.21$

M_1:

$\nu_{ij} = E[Y_{ij} \mid b_i] = (\beta_1 + b_{i1}) + (\beta_2 + b_{i2}) x_{ij}$

M_2: $\nu_{ij} = E[Y_{ij} \mid b_i] = (\beta_1 + b_{i1}) + \beta_2 x_{ij}$

$-2\log L = 1528.2$

$-2\log L = 1531.5$

$\Rightarrow$ test statistic is $1531.5 - 1528.2 = 3.3$

Note p-value $= \frac{1}{2} P\{\chi_1^2 > \quad\} + P\{\chi_2^2 > \quad\}$

Since the test statistic 3.3 is less than the calculated $\frac{\chi_1^2 + \chi_2^2}{2}$ we reject Ho.