The Negative Binomial

The negative binomial (NB) is given in in C&B as

$$P(Y = y) = \binom{r+y-1}{y} p^r q^y, \quad y = 0, 1, \ldots$$

The parameters are (r, p), and $q := 1 - p$. The mean is $\mu = rq/p$, and the variance is

$$\frac{rq}{p^2} = \frac{\mu}{p} = \mu(1 + \frac{\mu}{r}).$$

In M&N, the variance is

$$\frac{\mu(1+\phi)}{\phi} = \mu(1 + \frac{1}{\phi}).$$

In SAS, the variance is $\mu(1 + k\mu)$.

Relationships:

$$k\mu = \frac{q}{p} = \frac{1}{\phi} = \frac{\mu}{r}, \quad r = \frac{1}{k}, \quad p = \frac{\phi}{1+\phi}.$$

The log-likelihood:

$$l(\mu, k; y) = y \log \frac{k\mu}{1+k\mu} - \frac{1}{k} \log(1 + k\mu) + c(y, k).$$

The canonical link function, $\log\{\mu/(1 + k\mu)\}$, involves k.