BIOS 667, Spring 2017 Midterm

1. Consider the model $\mathrm{E}[Y] = X\beta, \mathrm{cov}(Y) = \Sigma$, where Y is an $n \times 1$ random vector distribted as multivariate normal with mean $X\beta$ and covariance matrix Σ; β is $p \times 1$ and Σ depends on a $q \times 1$ vector θ that is functionally unrelated to β.

   (a) True or false: The REML estimator of the regression coefficients β are obtained by maximizing the REML likelihood with respect to β. Explain briefly (1–3 sentences).

   (b) True or false: The REML estimator of β is unbiased. Explain briefly.

   (c) True or false: The maximum-likelihood (full likelihood, not REML likelihood) estimator of β is unbiased. Explain briefly.

2. In the TLC study, in the "Active" group, suppose that, using i to index subjects, the otucome vector $(Y_{i1}, Y_{i2}, Y_{i3}, Y_{i4})^\top$ is distributed as multivariate normal with mean $(25, 14, 16, 18)^\top$ in μg/dL, and covariance matrix

$$50 \begin{bmatrix} 1 & 0.5 & 0.5 & 0.4 \\ & 1 & 0.6 & 0.5 \\ & & 1 & 0.5 \\ & & & 1 \end{bmatrix}.$$

   Define $A_i = (Y_{i2} + Y_{i3} + Y_{i4})/3$.

   All numerical values must be simplified and reduced as much as possible.

   (a) Find the mean and variance of A_i.

   (b) Find $\mathrm{cov}(Y_{i1}, A_i)$.

   (c) Find the conditional mean and variance of A_i given Y_{i1}.

   (d) What is the point of this question?

3. In the TLC study, one of the study statisticians asked for fitting 6 separate linear regression models: regression of Y_{ij} on Y_{i1} (with intercept), $j = 2, 3, 4$, for i in the Active group, and separately for i in the Placebo group. The estimated slopes (standard errors) were:

	A	P
j=2	0.613(0.202)	0.901(0.0877)
j=3	0.600(0.208)	0.961(0.0898)
j=4	0.912(0.231)	0.848(0.106)

   (a) The said statistician wanted to test the hypothesis that, at each occasion j, the corresponding true population slopes, say α_{1j} (A) and α_{0j} (P) are equal in the two groups. Why?

   (b) Perform the test for $j = 2$ only (the test can be either exact or approximate).

   (c) The tests above for $j = 2, 3, 4$ are correlated. How would you test all three hypotheses as a single hypothesis, $H_0 : \alpha_0 = \alpha_1$ (vectors) versus $H_0 : \alpha_0 \neq \alpha_1$? You can use the full original data set.