

Problem 1

(a)

$i = 1$ Placebo $i = 2$ Active Treatment

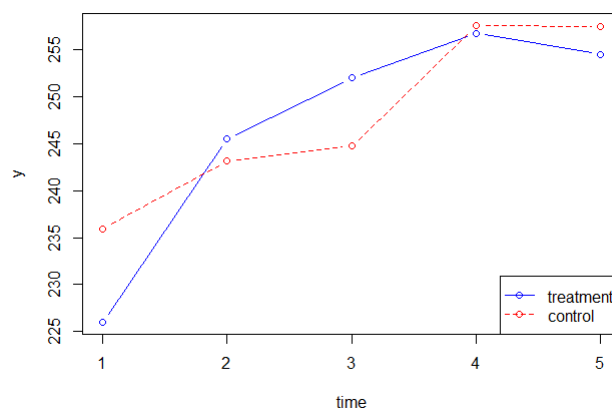
$$\mu_1 = \begin{bmatrix} 235.93 & 243.17 & 244.76 & 257.60 & 257.48 \end{bmatrix}^T$$

$$\mu_2 = \begin{bmatrix} 226.02 & 245.53 & 252.02 & 256.80 & 254.55 \end{bmatrix}^T$$

$$\Sigma_1 = \begin{bmatrix} 3080.44 & 2342.72 & 2158.73 & 2404.83 & 2086.78 \\ 2342.72 & 2755.49 & 2261.28 & 2392.10 & 2123.54 \\ 2158.73 & 2261.28 & 2267.72 & 2184.92 & 1828.96 \\ 2404.83 & 2392.10 & 2184.92 & 2666.96 & 2012.98 \\ 2086.78 & 2123.54 & 1828.96 & 2012.98 & 2439.19 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 1962.46 & 1302.20 & 1150.85 & 952.35 & 1009.28 \\ 1302.20 & 1715.22 & 1109.22 & 1023.43 & 1199.37 \\ 1150.85 & 1109.22 & 1553.90 & 696.86 & 1265.57 \\ 952.35 & 1023.43 & 696.86 & 1147.61 & 866.61 \\ 1009.28 & 1199.37 & 1265.57 & 866.61 & 2545.69 \end{bmatrix}$$

Means over time



(b)

Imposing the estimability restrictions and fitting the general two-way anova model

For $j = (2, 3, 4, 5)$ we have:

$$E[Y_{ij}] = \mu + \alpha_2 I(\text{group} = 2) + \beta_i I(\text{time} = j) + \gamma_{2j} I(\text{group} = 2) I(\text{time} = j)$$

Month	0	6	12	20	24
P	μ	$\mu + \beta_2$	$\mu + \beta_3$	$\mu + \beta_4$	$\mu + \beta_5$
A	$\mu + \alpha_2$	$\mu + \alpha_2 + \beta_2 + \gamma_{22}$	$\mu + \alpha_2 + \beta_3 + \gamma_{23}$	$\mu + \alpha_2 + \beta_4 + \gamma_{24}$	$\mu + \alpha_2 + \beta_5 + \gamma_{25}$

$$\text{Treatment effect} = \delta = (\gamma_{22}, \gamma_{23}, \gamma_{24}, \gamma_{25})^T$$

δ represents the treatment effect on the changes in mean cholesterol level over time

$\hat{\delta}$	\hat{se}
$\gamma_{22} = 12.272$	6.194
$\gamma_{23} = 16.418$	6.752
$\gamma_{24} = 4.977$	7.011
$\gamma_{25} = 6.903$	9.887

(c) Wald Chi-square test

$$H_0 : \delta = 0$$

$$H_1 : \delta \neq 0$$

$$\text{Wald } \chi^2 = 7.86 \text{ df}=4$$

$$\text{p-value} = .0968 > .05 \text{ Thus fail to reject } H_0$$

Not enough evidence to suggest δ is not equal to 0. The mean response profiles are statistically the same for the two groups.

(d)

Month	0	6	12	20	24
P	μ	$\mu + \beta_2$	$\mu + \beta_3$	$\mu + \beta_4$	$\mu + \beta_5$
A	μ	$\mu + \beta_2 + \gamma_{22}$	$\mu + \beta_3 + \gamma_{23}$	$\mu + \beta_4 + \gamma_{24}$	$\mu + \beta_5 + \gamma_{25}$

$$\text{Treatment effect} = \delta = (\gamma_{22}, \gamma_{23}, \gamma_{24}, \gamma_{25})^T$$

δ represents the treatment effect on the changes in mean cholesterol level over time

$\hat{\delta}$	\hat{se}
$\gamma_{22} = 9.478$	5.596
$\gamma_{23} = 12.886$	5.85
$\gamma_{24} = 1.614$	6.208
$\gamma_{25} = 3.003$	9.064

(e)

$$t = \text{time} \quad x = I(\text{group2})$$

$$\text{group 1: } E(Y_{ij}) = \beta_1 + \beta_2 * t_{ij} + \beta_3 * t_{ij}^2$$

$$\text{group 2: } E(Y_{kj}) = \alpha_1 + \alpha_2 * t_{kj} + \alpha_3 * t_{kj}^2$$

$$\text{Restrictions: } E(Y_{i1}) = E(Y_{k1}), \quad \beta_1 = \alpha_1$$

Month	0	6	12	20	24
P	μ	$\mu + 6t + 36t^2$	$\mu + 12t + 144t^2$	$\mu + 20t + 400t^2$	$\mu + 24t + 576t^2$
A	μ	$\mu + 6t + 36t^2$ $+ 6xt + 36xt^2$	$\mu + 12t + 144t^2 +$ $+ 12xt + 144xt^2$	$\mu + 20t + 400t^2 +$ $20xt + 400xt^2$	$\mu + 24t + 576t^2 +$ $24xt + 576xt^2$

$$\text{Treatment effect} = \boldsymbol{\delta} = (xt, xt^2)^T$$

$\boldsymbol{\delta}$ represents the treatment effect on the changes in mean cholesterol level over time

$$\left[\begin{array}{c|c} \hat{\boldsymbol{\delta}} & \hat{se} \\ \hline xt = 1.937 & .823 \\ xt^2 = -.086 & .037 \end{array} \right]$$

Wald Chi-square test

$$H_0 : \boldsymbol{\delta} = 0$$

$$H_1 : \boldsymbol{\delta} \neq 0$$

$$\text{Wald } \chi^2 = 5.89 \text{ df}=2$$

p-value= .0527 > .05 Thus fail to reject H_0

Not enough evidence to suggest $\boldsymbol{\delta}$ is not equal to 0. The mean response profiles are statistically the same for the two groups.

(f)

Treatment Effect on the changes in mean cholesterol level over time $= \boldsymbol{\lambda} = (\alpha_2, \gamma_{23}, \gamma_{24}, \gamma_{25})$

$$\left[\begin{array}{c|c} \hat{\boldsymbol{\lambda}} & \hat{se} \\ \hline \alpha_2 = 12.272 & 6.194 \\ \gamma_{23} = 4.028 & 5.979 \\ \gamma_{24} = -8.07 & 6.097 \\ \gamma_{25} = -5.226 & 8.901 \end{array} \right]$$

(g) Wald Chi-square test

$$H_0 : \boldsymbol{\lambda} = 0$$

$$H_1 : \boldsymbol{\lambda} \neq 0$$

$$\text{Wald } \chi^2 = 7.99 \text{ df}=4$$

p-value= .0919 > .05 Thus fail to reject H_0

Not enough evidence to suggest $\boldsymbol{\lambda}$ is not equal to 0. The mean response profiles are statistically the same for the two groups.

(h)

Treatment Effect on the changes in mean cholesterol level over time $= \boldsymbol{\lambda}^* = (\alpha_2, \gamma_{23}, \gamma_{24}, \gamma_{25})$

$\hat{\lambda}^*$	\hat{se}
$\alpha_2 = 9.023$	5.653
$\gamma_{23} = 4.144$	5.937
$\gamma_{24} = -7.394$	6.03
$\gamma_{25} = -5.218$	8.842

- (i) Wald Chi-square test
 $H_0 : \lambda^* = 0$
 $H_1 : \lambda^* \neq 0$
Wald $\chi^2 = 6.54$ df=4
p-value= .1623 > .05 Thus fail to reject H_0
Not enough evidence to suggest λ^* is not equal to 0. The mean response profiles are statistically the same for the two groups.
- (j) Full LRT test
 $H_0 : \lambda^* = 0$
 $H_1 : \lambda^* \neq 0$
 $-2\text{Log}(L)_{Full} = 3218.6$ df=8
 $-2\text{Log}(L)_{Reduced} = 3225.1$ df=4
 $\chi^2 = 3225.1 - 3218.6 = 6.5$ df=4
p-value= .165 > .05 Thus fail to reject H_0
Not enough evidence to suggest λ^* is not equal to 0. The mean response profiles are statistically the same for the two groups.
- (k) You need the following assumptions to guarantee $\lambda = \lambda^*$:
1) No interaction
2) At baseline the two groups have the same mean