Identifiability

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Definition: We say that two pdf's or pmf's with the same domain "are equal" or "are the same" if they are equal at each point in their domain (essentially, they are copies of each other).

First, the context: A family (or collection or set) of densities (pdf's or pmf's) for a random scalar or vector Y. The family is indexed by a scalar or vector parameter θ

We say that θ is identifiable if no two different values of θ define the same density. i.e. there is a one-to-one correspondence between the space for θ and the family of densities.

Example 1: $Y \sim N(\theta_1, \theta_2)$. θ is identifiable because if we change either θ_1 or θ_2, or both, we get a different pdf. If two normal densities are not equal, they must have two different values of θ (the vector), i.e. they must have two different means or two different variances (or both).

Example 2: $Y \sim N(\theta_1 + \theta_2, \theta_3)$. θ is not identifiable because, for example, $\theta = (1, 1, 1)$ and $(0, 2, 1), (-10, 12, 1)$ and $(2, 0, 1)$ all define the same density. Also, $\theta = (k, 2 - k, 1)$ defines the same pdf, whatever k is.

Example 3: $Y \sim N(\theta_1, \theta_2\theta_3)$. θ is not identifiable because, for example, $\theta = (1, 1, 4)$ and $(1, 4, 1), (1, 2, 2)$ and $(1, 16, 1/4)$ all define the same density. Also, $\theta = (1, k, 4/k)$ defines the same pdf for all $k > 0$.

If the above definition applies to a certain subset of θ (or generally, a certain function of θ) [with the rest of θ held constant], we say that that subset (or that function) of θ is identifiable. Similarly, if the definition fails to apply for a subset or a function of θ, we say that that subset or that function of θ is non-identifiable.

Example 4: In Example 2, θ_3 is identifiable, but (θ_1, θ_2) is not.

Exercise:
Suppose that Y is multivariate normal with mean vector 0 and covariance matrix

$$\Sigma = \begin{bmatrix} g_{11} + \sigma^2 & g_{11} + \rho\sigma^2 & g_{11} + \rho\sigma^2 \\ g_{11} + \rho\sigma^2 & g_{11} + \sigma^2 & g_{11} + \rho\sigma^2 \\ g_{11} + \rho\sigma^2 & g_{11} + \rho\sigma^2 & g_{11} + \sigma^2 \end{bmatrix}.$$

Note: $\sigma^2 > 0, g_{11} > 0, \rho \in (-0.5, 1)$. Is the vector (g_{11}, σ^2, ρ) identifiable?