Computing the Variance of $\widehat{X}_{25}$, $\widehat{X}_{50}$, and $\widehat{X}_{75}$:

In general, let $\theta = \frac{X}{Y}$.

We know from Taylor series expansion that

$$\mathrm{Var}(\theta) \approx \left(\frac{X}{Y}\right)^2 \left\{ \frac{V(X)}{X^2} + \frac{V(Y)}{Y^2} - \frac{2\mathrm{Cov}(X,Y)}{XY} \right\}$$

Now, for $\widehat{X}_{50}$, we have: $\widehat{X}_{50} = \frac{-\widehat{\alpha}}{\widehat{\beta}}$ (see Section 11.2, page 327)

This is analogous to $X = -\widehat{\alpha}$ and $Y = \widehat{\beta}$ above.

$$\text{So } \mathrm{Var}\left(\widehat{X}_{50}\right) \approx \left(\frac{-\widehat{\alpha}}{\widehat{\beta}}\right)^2 \left\{ \frac{V(-\widehat{\alpha})}{(-\widehat{\alpha})^2} + \frac{V\left(\widehat{\beta}\right)}{\left(\widehat{\beta}\right)^2} - \frac{2\mathrm{Cov}\left(-\widehat{\alpha},\widehat{\beta}\right)}{(-\widehat{\alpha})\left(\widehat{\beta}\right)} \right\}$$

$$= \left(\frac{-\widehat{\alpha}}{\widehat{\beta}}\right)^2 \left\{ \frac{V(\widehat{\alpha})}{(\widehat{\alpha})^2} + \frac{V\left(\widehat{\beta}\right)}{\left(\widehat{\beta}\right)^2} - \frac{2\mathrm{Cov}\left(\widehat{\alpha},\widehat{\beta}\right)}{(\widehat{\alpha})\left(\widehat{\beta}\right)} \right\}$$

For $\widehat{X}_{25}$, we have something more complicated:

$$\log\left\{\frac{p_{25}}{1-p_{25}}\right\} = \log\left\{\frac{.25}{.75}\right\} \approx -1.1 = \widehat{\alpha} + \widehat{\beta}\,\widehat{X}_{25}$$

$$\Rightarrow \widehat{X}_{25} = \frac{-1.1-\widehat{\alpha}}{\widehat{\beta}}$$

This is analogous to $X = (-1.1 - \widehat{\alpha})$ and $Y = \widehat{\beta}$ above.

$$\text{So } \mathrm{Var}\left(\widehat{X}_{25}\right) \approx \left(\frac{-1.1-\widehat{\alpha}}{\widehat{\beta}}\right)^2 \left\{ \frac{V(-1.1-\widehat{\alpha})}{(-1.1-\widehat{\alpha})^2} + \frac{V\left(\widehat{\beta}\right)}{\left(\widehat{\beta}\right)^2} - \frac{2\mathrm{Cov}\left(-1.1-\widehat{\alpha},\widehat{\beta}\right)}{(-1.1-\widehat{\alpha})\left(\widehat{\beta}\right)} \right\}$$

$$= \left(\frac{-1.1-\widehat{\alpha}}{\widehat{\beta}}\right)^2 \left\{ \frac{V(\widehat{\alpha})}{(-1.1-\widehat{\alpha})^2} + \frac{V\left(\widehat{\beta}\right)}{\left(\widehat{\beta}\right)^2} + \frac{2\mathrm{Cov}\left(\widehat{\alpha},\widehat{\beta}\right)}{(-1.1-\widehat{\alpha})\left(\widehat{\beta}\right)} \right\}$$

$\mathrm{Var}\left(\widehat{X}_{75}\right)$ is found via the same method with $X = (1.1 - \widehat{\alpha})$ and $Y = \widehat{\beta}$.