Q Can all points be equally likely

2. Let S2 be a sample space with countably infinite disjoint events corresponding to countably infinite points.

Assume the events E, Ez, have the same probability.

By the property of probability measure,

P(E, UE, U --) = S. P(E, S.

Case 1: P(Ei) = 0 for all it N. Then I P(Ei) = 0.

This wortedicts the property of probability measure

that I P(Ei) (for all Ei & S2) = 1

Case 2: P(Ei) > o for all it N. Then \$\frac{\mathcal{D}}{\text{i}}, P(\mathcal{E}i) = \pi .

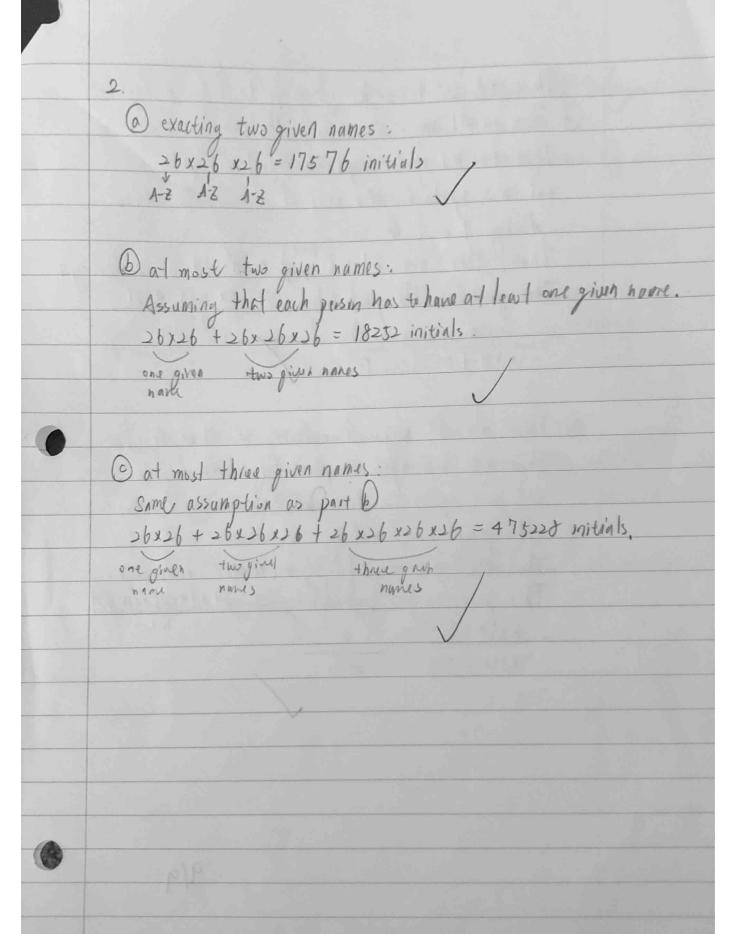
This contradicts the property of probability menone that \$\frac{\mathcal{D}}{\text{i}}P(\mathcal{E}i) = 1.

Thus, P(Ei) for it I cannot all'equal and all prints cannot be equally likely.

Des. A example would be Let So be a sample space countably infinite grants

events E, E, ... correspondy to countably infinite grants

Let (on) = (\frac{1}{20}) be an infinite sequence. Each Ex iEN corresponds to the ith term in an. Thus, P(Ex)= (it) >0. So each point has positive probability. Since \$\frac{1}{2}n = 1, it also satisfy the probability means of \$\frac{1}{2}p(\mathbb{E}i) = 1 = P(N). Thus, all points can have positive probability onlesses the properties of probability measure,



3. There no total number of ways to order no digits.

(a) There are 2! ways of ordering the two digits.

There are not spots to place the two digits

and no spots for the rest of the digits of the placing I and 2.

Thus, there are 2! x (no!) x (no.2)! or derings.

This gives us the probability of 2x(no.1) (no.2)!

= 2x(no.1) = 2.

There are 3! ways of ordering the three lights

There are no spots to place the three digits

arl no 3 spots to place the rest of the digits

after placy 1, 2, and 3.

Thus there are 3! x (n-2) x (n-3)! orderings.

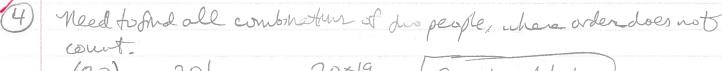
This gives us the probability of 3! x (n-2) x (n-3)!

- 6x (n-2)

(n) (n+1) (n-2) = n-n.



0



$$\binom{20}{2} = \frac{20!}{2!(20-18)!} = \frac{20\times19}{2} = \boxed{190 \text{ handshakes}}$$

(5) need to find combinations of 7 from (0 people, and order does matter.

$$(10P7) = \frac{10!}{(10-7)!} = [604,800]$$
 district results

(6) (a) For the unns and balls problem, we know that the amount of possiblities (assuming some of the unna can be left empty, is:

$$\frac{(n+r-1)!}{n!(r-1)!} = \frac{(n+r-1)}{r-1} = \frac{11!}{3!(8)!} = \frac{11!}{3!(8)!} = \frac{165}{165}$$
 ways

(b) For urns & bells problem, we know that the amt of poss, belters, assuming each uspe has at least one ball, is: $\binom{n-1}{r-1} = \binom{7}{3} = \boxed{35 \text{ ways}}$

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7. (a) If an investment must be made in each opportunity.
  "The minimal investments are 2,2,3, and 4 thousand dollars for
  the 4 possible opportunities and we have 201 thousand dollars in
  total, the problem can be simplified as investing
          20-(2+2+3+4) = 9 thousand dollars on the 4 opportunities
   as X1, X2, X3, X4 respectively.
            X1+ X2+ X3+X4 = 9 and X27,0, 2=1,2,3,4
     220 different investment strategies are available.
  (b) investments must be made in at least 3 of the 4 opportun
   be the same as in (a) that there exist 220 different strategie
 2) If we don't make investment on opportunity 1, the problem is
  the same as investing 20-(2+3+4) = 11 thousand dollars on the 3 opportunities \chi_2, \chi_3, \chi_4 respectively:
   / X+X3+X4=11 and X270 ==2,3,4
     3) If we don't make investment on opportunity 2, because the minima
  investment of opportunity 2 is the same as opportunity I, so as is shown in (2), \binom{11+3-1}{3-1} = \binom{13}{2} = 78 strategies are available
     1 If we don't make investment on opportunity 3, the problem is
     the some as investing 20-(2+2+4)=12 thousand dollars on the
     3 opportunities X1, X2, X4 respectively
   V X1+X2+X4 = 12 and X27,0, i=1,2,4
     : \binom{12+3-1}{3-1} = \binom{14}{2} = \frac{14!}{2!12!} = 91 strategies are available
    If we don't make investment on opportunity 4, the problem is the same as investing 20 - (2+2+3) = 13 thousand dollars on the
    3 opportunities X_1, X_2, X_3 respectively.

X_1+X_2+X_3=13 and X_1>0, i=1,2,3
    (13+3-1) = (15) = 105 different strategies are available.
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7. (b) continued

.. In summary, if investments must be made in at least 3 of the opportunities,

$$\binom{9+4+1}{4-1}$$
 + $2 \times \binom{11+3-1}{3-1}$ + $\binom{12+3+1}{3-1}$ + $\binom{13+3-1}{3+1}$

$$= {\binom{3}{3}} + 2 \times {\binom{13}{2}} + {\binom{14}{2}} + {\binom{15}{2}}$$

$$= 220 + 2 \times 78 + 91 + 105$$

5/2 different investment strategies are avoidable.

8. (1) Let N=No=

$$\sum_{r=0}^{n_0} \binom{n_0}{r} \cdot \alpha^r b^{n_0-r} = \binom{1}{0} \alpha^0 b^{1-0} + \binom{1}{1} \cdot \alpha^1 \cdot b^0 = \alpha + b$$

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^r b^{n-r} holds for n=n_0=1 and n=n_0=0$$

Assume
$$(a+b)^{n-1} = \sum_{r=0}^{n-1} {n-1 \choose r} a^r b^{(n-1)-r}$$
 holds

$$(a+b)^{N} = (a+b) \cdot (a+b)^{N-1} = (a+b) \cdot \begin{bmatrix} \sum_{j=0}^{N-1} {n+j \choose j} a^{\gamma} b^{(N+j)-\gamma} \end{bmatrix}$$

$$= \sum_{j=0}^{N-1} {n+j \choose j} \cdot a^{\gamma+1} \cdot b^{(N+j)-\gamma} + \sum_{j=0}^{N-1} {n+j \choose j} \cdot a^{\gamma} \cdot b^{(N+j)-\gamma+1}$$

$$= \sum_{r=1}^{n} \frac{\binom{n-1}{r-1} \cdot \alpha^r \cdot b^{n-r} + \sum_{r=0}^{n-1} \binom{n-1}{r} \cdot \alpha^r \cdot b^{n-r}}{r-1}$$

$$= \left[\binom{n-1}{n-1} \cdot \alpha^{n} \cdot b^{0} + \sum_{\gamma=1}^{n-1} \binom{n-1}{\gamma-1} \alpha^{\gamma} b^{n-\gamma} \right] + \left[\binom{n-1}{0} \cdot \alpha^{0} b^{n} + \sum_{\gamma=1}^{n-1} \binom{n-1}{\gamma} \cdot \alpha^{\gamma} b^{n-\gamma} \right]$$

$$= a^{n}b^{0} + a^{0}b^{n} + \sum_{r=1}^{n-1} \{ [(x-1) + (x-1)] \cdot a^{r}b^{n-r} \}$$

$$= a^{n}b^{o} + a^{o}b^{n} + \frac{n-1}{2}(\frac{n}{2})a^{\gamma}b^{n-\gamma}$$

$$= \binom{n}{0} a^{0} b^{n-0} + \sum_{r=1}^{n-1} \binom{n}{r} a^{r} b^{n-r} + \binom{n}{n} a^{n} b^{n-n}$$

$$= \sum_{\gamma=0}^{n} \alpha^{\gamma} b^{\gamma-\gamma}$$

:
$$(a+b)^n = \sum_{r=0}^n a^r b^{n-r}$$
 holds for $n=0,1,2,...$