

## BIOSTATISTICS 667

### Homework 1

1. In “tlc004.sas” there are two 2-sample tests; one is based on lead levels and the other is based on differences from baseline. Comment on each of these as a test of a “treatment effect” (i.e. is it a legitimate/valid test of treatment effect, and why). The “why” is important; a simple yes/no answer will be considered a no answer (zero credit).
2. For the TLC data discussed in class, let  $\mu$  be the  $4 \times 1$  mean vector for weeks 0, 1, 4 and 6. Test the null hypothesis that there is a linear trend in the four elements of  $\mu$  against an unrestricted alternative. Do this separately for each group (“Active” and “Placebo”). Describe your methods. Present and interpret your findings. It is important to note that the four time points are not equally-spaced (Hint: write down  $H_0$  in some detail). Do both an exact test, assuming normality, and a “large-sample” test.

Note: For background, three papers from the TLC study have been posted in the “papers” folder.

[As always in this class, do NOT submit any computer code, not even in an appendix, unless the problem asks you *explicitly* to submit your code. Do NOT refer to any computer statements or options you might have used in SAS, R, etc. Use the language of statistics, but do not assume that your reader knows SAS or R or whatever software you used. Do not copy computer output verbatim into your homework.]

3. Let  $X$  be an  $n \times m$  matrix with orthogonal columns and 1’s in the first column. Clearly  $X^\top X$  is diagonal. But what can we say about  $X^\top W X$  if  $W$  is a positive definite matrix? But what can we say about  $X^\top W X$  if  $W$  has the form  $W = aI + bJ$  where  $a$  and  $b$  are real numbers,  $I$  is the identity matrix and  $J$  is a matrix of 1’s?
4. In the one-sample problem (normal distribution), derive the MLE of the variance. Derive the MLE based on the REML likelihood. Are they identical?
5. Using the Bock(1975) data, use Hotelling’s  $T^2$  to test the null hypothesis that the mean is constant over time against an unrestricted alternative. Which of the tests reported in the textbook (H&G) is it (cite where it appears in the book).