Homework 6, BIOS 767
Spring, 2017

1. Problem **14.1**.
   14.1.11 Using the estimates from 14.1.10 with 50 quadrature points, compute a decomposition of the total variance into "within" and "between" components, and hence compute an intra-class correlation. Do this calculation at week 0 and at week 48 in each treatment group. Also, compute the estimated correlation between the outcomes at weeks 0 and 48 in each treatment group. Convert the correlations to odds ratios and compare them (informally) to the estimates obtained in 14.1.1.

   The problem states "The variable Month denotes the exact timing of measurements in months." What is a "month" in this problem?

2. High-order moments and correlations: Suppose that the $d \times 1$ random vector Z is distributed such that $\mathrm{E}[Z_i] = 0, \mathrm{var}(Z_i) = 1, 1 \leq i \leq d$, and $\mathrm{cov}(Z) = R$, a positive definite matrix with elements $\rho_{ij} = \mathrm{E}[Z_i Z_j]$. The indices (i, j, k, l) in this problem are distinct and range between 1 and d.

   The moments $\mathrm{E}[Z_i^3]$, $\mathrm{E}[Z_i^2 Z_j]$ and $\mathrm{E}[Z_i Z_j Z_k]$ are all called third moments. The moment $\mathrm{E}[Z_i Z_j Z_k]$ is known as a third-order correlation.

   Now suppose that Z is distributed as multivariate normal. Derive explicit expressions for the three third moments given above. Simplify them as much as possible.

   How many third moments are there for d variables, $d \geq 1$?

   Derive an explicit expression for the fourth-order correlation $\mathrm{E}[Z_i Z_j Z_k Z_l]$ and simplify it as much as possible.

   Note: In the multivariate normal, all high-order moments are determined by the first two moments. But that is not necessarily the case in other distributions.