Cheynna Crowley
BIOS 667 MT1 Redo

1A FALSE, $\beta$ is not estimated by REML. $\beta$ can be estimated the results from REML and then using a "WLS like" equation.

1B FALSE, $\hat{\beta}$ is usually biased because $W$ (used to calculate $\hat{\beta} = (X^T W X)^{-1} X^T W Y$ where $W = \{\Sigma(\hat{\theta})\}^{-1}$) is a function of $Y$, and is random and does not factor out expectations.

1C FALSE, usually biased except for various circumstances.

2A FIND MEAN AND VARIANCE OF $A_i$

$$\begin{pmatrix} Y_{i1} \\ Y_{i2} \\ Y_{i3} \\ Y_{i4} \end{pmatrix} \sim N\left( \begin{bmatrix} 25 \\ 14 \\ 16 \\ 18 \end{bmatrix}, \begin{array}{c} \begin{array}{cccc} Y_{i1} & Y_{i2} & Y_{i3} & Y_{i4} \end{array} \\ \begin{bmatrix} 50 & 25 & 25 & 20 \\ 25 & 50 & 30 & 25 \\ 25 & 30 & 50 & 25 \\ 20 & 25 & 25 & 50 \end{bmatrix} \end{array} \begin{array}{c} Y_{i1} \\ Y_{i2} \\ Y_{i3} \\ Y_{i4} \end{array} \right)$$

DEFINE $A_i = (Y_{i2} + Y_{i3} + Y_{i4})/3$

$E(A_i) = E((Y_{i2} + Y_{i3} + Y_{i4})/3) = \frac{1}{3} E(Y_{i2} + Y_{i3} + Y_{i4}) = \frac{1}{3}[E(Y_{i2}) + E(Y_{i3}) + E(Y_{i4})] = \frac{1}{3}[14 + 16 + 18] = \frac{1}{3}[48] = 16$

$\Rightarrow E(A_i) = 16$

$VAR(A_i) = VAR(\frac{1}{3}(Y_{i2} + Y_{i3} + Y_{i4})) = \frac{1}{9}(var(Y_{i2} + Y_{i3} + Y_{i4}))$

$= \frac{1}{9}[var(Y_{i2}) + var(Y_{i3} + Y_{i4}) + 2\,cov(Y_{i2}, Y_{i3} + Y_{i4})]$

$= \frac{1}{9}[var(Y_{i2}) + 2\,cov(Y_{i2}, Y_{i3}) + 2\,cov(Y_{i2}, Y_{i4}) + var(Y_{i3}) + var(Y_{i4}) + 2\,cov(Y_{i3}, Y_{i4})] = \frac{1}{9}[50 + 50 + 50 + 2(30) + 2(25) + 2(25)] = \frac{1}{9}[150 + 60 + 50 + 50] = \frac{1}{9}[310] = 34.44$

$\Rightarrow var(A_i) = 310/9 \approx 34.44$

2B $cov(Y_{i1}, A_i) = cov(Y_{i1}, \frac{1}{3}(Y_{i2}+Y_{i3}+Y_{i4})) = \frac{1}{3} cov(Y_{i1}, Y_{i2}+Y_{i3}+Y_{i4})$

$= \frac{1}{3}[cov(Y_{i1}, Y_{i2}) + cov(Y_{i1}, Y_{i3}) + cov(Y_{i1}, Y_{i4})]$

$= \frac{1}{3}[25+25+20] = \frac{1}{3}(70) \approx 23.33$

$\Rightarrow cov(Y_{i1}, A_i) = 70/3$

2C FIND $E(A_i \mid Y_{i1})$ and $var(A_i \mid Y_{i1})$

$E(A_i \mid Y_{i1}) = E(\frac{1}{3}[Y_{i2}+Y_{i3}+Y_{i4}) \mid Y_{i1}) = \frac{1}{3}E[Y_{i2}+Y_{i3}+Y_{i4} \mid Y_{i1}]$

$= \frac{1}{3}[E[Y_{i2} \mid Y_{i1}] + E[Y_{i3} \mid Y_{i1}] + E[Y_{i4} \mid Y_{i1}]]$

we know

$\begin{pmatrix} Y_{i1} \\ Y_{i2} \end{pmatrix} \sim N\left(\begin{pmatrix} 25 \\ 14 \end{pmatrix}, \begin{bmatrix} 50 & 25 \\ 25 & 50 \end{bmatrix}\right)$ $\begin{pmatrix} Y_{i1} \\ Y_{i3} \end{pmatrix} \sim N\left(\begin{pmatrix} 25 \\ 16 \end{pmatrix}, \begin{bmatrix} 50 & 25 \\ 25 & 50 \end{bmatrix}\right)$

$\begin{pmatrix} Y_{i1} \\ Y_{i4} \end{pmatrix} \sim N\left(\begin{pmatrix} 25 \\ 18 \end{pmatrix}, \begin{bmatrix} 50 & 20 \\ 20 & 50 \end{bmatrix}\right)$

since "bivariate normal" $\Rightarrow \begin{cases} E[x_1 \mid x_2] = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2) \\ cov(x_1 \mid x_2) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \\ var(x_1 \mid x_2) = \sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}} \end{cases}$

$E[A_i \mid Y_{i1}] = \frac{1}{3}[E[Y_{i2} \mid Y_{i1}] + E[Y_{i3} \mid Y_{i1}] + E[Y_{i4} \mid Y_{i1}]]$

$\Rightarrow E[Y_{i2} \mid Y_{i1}] = \mu_2 + \Sigma_2 \cdot \Sigma_{22}^{-1}(Y_{i1} - \mu_1) = 14 + 25(50^{-1})(Y_{i1} - 25)$

$E[Y_{i3} \mid Y_{i1}] = 16 + 25(50^{-1})(Y_{i1} - 25)$

$E[Y_{i4} \mid Y_{i1}] = 18 + 20(50^{-1})(Y_{i1} - 25)$

$[A_i \mid Y_{i1}] = \frac{1}{3}\left[16 + \frac{1}{2}(Y_{i1} - 25) + 14 + \frac{1}{2}(Y_{i1} - 25) + 18 + \frac{2}{5}(Y_{i1} - 25)\right]$

$= \frac{1}{3}[48 + \frac{7}{5}(Y_{i1} - 25)] = 16 + \frac{7}{15}(Y_{i1} - 25)$

$ar(A_i \mid Y_{i1}) = var(\frac{1}{3}(Y_{i2} + Y_{i3} + Y_{i4}) \mid Y_{i1})) = \frac{1}{9} var(Y_{i2} + Y_{i3} + Y_{i4} \mid Y_{i1})$

$\frac{1}{9}[var(Y_{i2} \mid Y_{i1}) + var(Y_{i3} + Y_{i4} \mid Y_{i1}) + 2 cov(Y_{i2}, Y_{i3} + Y_{i4} \mid Y_{i1})$

$= [var(Y_{i2} \mid Y_{i1}) + 2 cov(Y_{i2}, Y_{i3} \mid Y_{i1}) + 2 cov(Y_{i2}, Y_{i4} \mid Y_{i1}) + var(Y_{i3} \mid Y_{i1})$

$+ var(Y_{i4} \mid Y_{i1}) + 2 cov(Y_{i3}, Y_{i4} \mid Y_{i1})$

$\boxed{2C}$

$var(A_i | Y_{i1}) = \frac{1}{9}\left[\left(50 - \frac{25^2}{50}\right) + \left(50 - \frac{25^2}{50}\right) + \left(50 - \frac{20^2}{50}\right) + 2\left[ \ldots\right.\right.$

<u>what do we know</u>

$cov(Y_{i2}, Y_{i4} | Y_{i1}) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

$\begin{pmatrix} Y_{i3} | Y_{i1} \\ Y_{i4} \; Y_{i1} \end{pmatrix} \sim N($

$E[Y_{i3} | Y_{i1}] = 16 + 1/2\,(Y_{i1} - 25)$

$E[Y_{i4} | Y_{i1}] = 18 + 2/5\,(Y_{i1} - 25)$

$E[Y_{i2} | Y_{i1}] = 14 + 1/2\,(Y_{i1} - 25)$

$var[Y_{i1}$

$cov(X, Y) = E[XY] - E[X]E[Y]$

$var(Y_{i3} | Y_{i1}) = 50 - \frac{25^2}{50}$ $\quad var(Y_{i4} | Y_{i1}) = 50 - \frac{20^2}{50}$

$var(Y_{i2} | Y_{i1}) = 50 - \frac{25^2}{50}$

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4) $cov(Y_{i1}, A_i) = 70/3 \quad E(A_i) = 16$

$var(A_i) = 310/9$

$\begin{pmatrix} Y_{i1} \\ A_i \end{pmatrix} \sim N\left(\begin{bmatrix} 25 \\ 16 \end{bmatrix}, \begin{bmatrix} 50 & 70/3 \\ 70/3 & 310/9 \end{bmatrix}\right.$

$\Rightarrow \; E[A_i | Y_{i1}] = 16 + 70/3\left(\frac{1}{50}\right)(Y_{i1} - 25)$

$\Rightarrow var(A_i | Y_{i1}) = \frac{310}{9} - \frac{(70/3)^2}{50} = 23.556$

$\boxed{2D}$ $\rightarrow$ $\begin{pmatrix} Y_{i1} \\ Y_{i2} \\ Y_{i3} \\ Y_{i4} \end{pmatrix}$ is a vector of responses for each $i^{th}$ subject. Assuming $j = 1, 2, 3, 4$ corresponds to $t=1, t=2, t=3, t=4$, the problem shows the difference in responses for each timepoint given baseline.

$\rightarrow$ $A_i$ dist. shows the change in the average responses after baseline.

$\boxed{3A}$

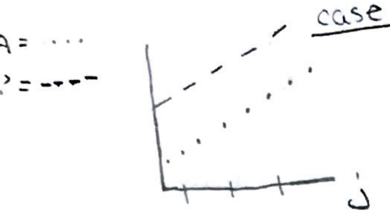
Testing that at each occasion $j$ the true population slopes are equal

⇒ two cases

① equal slopes and equal intercept.

② equal slopes unequal intercept.

case 2




case 1


⇒ testing treatment effect regardless of starting population if the same slope per trt group ⇒ no trt effect.

$\boxed{3B}$ for $j=2$

A = 0.613 (0.202)

P = 0.901 (0.0877)

large sample test to see if Active is different than placebo at $j=2$.

$H_0: \alpha_{12} = \alpha_{02}$

$H_1$: otw

$$\frac{0.613 - 0.901}{\sqrt{0.202^2 + 0.0877^2}} \sim N(0,1)$$

$= -1.31$

⇒ for $\alpha = 0.05$ and $z = -1.31$

pvalue = 0.0885

⇒ we reject Ho and conclude the slope at $j=2$ is different for active and placebo.

$\boxed{3C}$ $H_0: \alpha_0 = \alpha_1$ vs. $H_1: \alpha_0 \neq \alpha_1$ ⇒ $H_0: \begin{pmatrix} \alpha_{02} \\ \alpha_{03} \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \end{pmatrix}$