10/14/2019

Exam 1 Review

2c) $\beta_3 = c_1\mu_1 + c_2\mu_2 + c_3\mu_3$

$= c_1(\beta_1 + \beta_2 + \beta_3) + c_2(\beta_1 + 2\beta_2 + 8\beta_3) + c_3(\beta_1 + 3\beta_2 + 27\beta_3)$

β_3: $1 = c_1 + 8c_2 + 27c_3$

β_2: $0 = c_1 + 2c_2 + 3c_3 \rightarrow c_2 + 2c_3 = 0$

$c_2 = -2c_3$

β_1: $0 = c_1 + c_2 + c_3$

$c_1 = -c_2 - c_3$

$c_2 = -1/6$

$7c_2 + 26c_3 = 1$

$c_1 = \frac{2}{12} - \frac{1}{12}$

$12c_3 = 1$

$= \frac{1}{12}$

$c_3 = 1/12$

$\beta_3 = -\frac{1}{12}\mu_1 - \frac{1}{6}\mu_2 + \frac{1}{12}\mu_3 \rightarrow$ a contrast

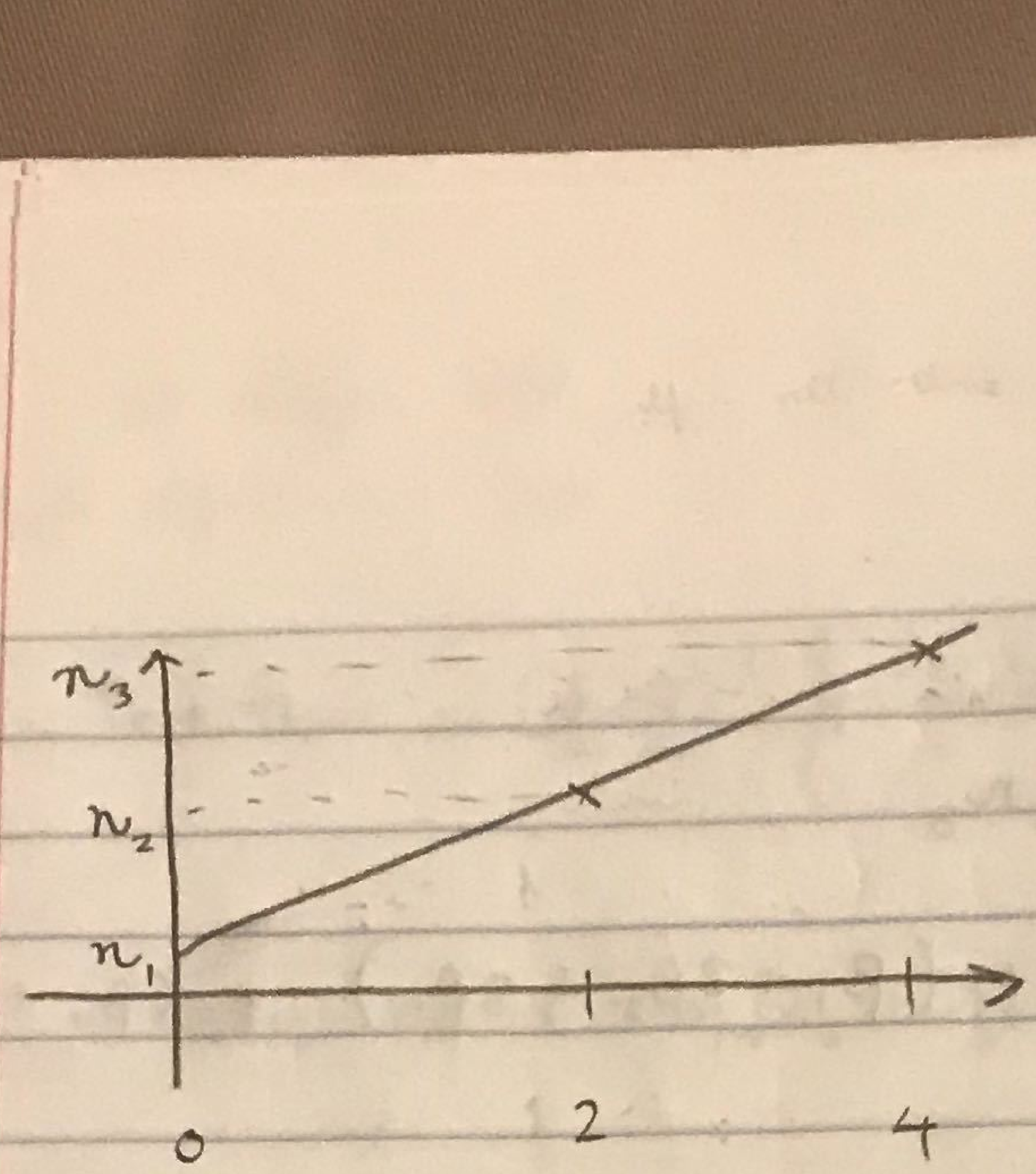
as β_3 is a linear combination of the μ_1, μ_2, μ_3, β_3 is estimable and thus interpretable.

$\beta_3 = \frac{1}{12}(\mu_1 - 2\mu_2 + \mu_3)$

$= \frac{1}{12}(\mu_1 + \mu_3 - 2\mu_2)$

$= \frac{1}{6}\left(\frac{\mu_1 + \mu_3}{2} - \mu_2\right)$

If $\beta_3 = 0$, then μ_2 is the midpoint between μ_1 & μ_2



β_3 is a measure of non-linearity of the time trend of the expected value of the responses

If $\beta_3 = 0$, then this time trend is linear and the increment between μ_1 & μ_2 = the increment between μ_2 & μ_3

2b) Since A_i is a multiple of β_3, the $H_0: E[A_i] = 0$ is saying this time trend is linear (the expected value of the responses is linear w/ respect to time)

2d) $\text{corr}(Z_{i1}^2, Z_{i2}^2)$

Z_{i1} & Z_{i2} is joint bivariate normal (b/c linear combinations of Y_{i1} & Y_{i2})

$$\begin{pmatrix} Z_{i1} \\ Z_{i2} \end{pmatrix} \sim N_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \underset{\text{known } =0.6}{\rho} \\ \rho & 1 \end{bmatrix}\right)$$

$$\text{corr}(Z_{i1}, Z_{i2}) = \text{cov}(Z_{i1}, Z_{i2})$$

b/c $\text{var}(Z_{i1}) = \text{var}(Z_{i2}) = 1$

$$\text{cov}(Z_{i1}, Z_{i2}) = \text{cov}(Y_{i1}, Y_{i2}) = 0.6$$

It is clear that $E[Z_{i1}^2] = 1$

$$\text{Var}(Z_{i1}) = E[Z_{i1}^2] - [E(Z_{i1})]^2$$

$E[Z_{i1}^2 Z_{i2}^2]$ ~~E~~

$$\text{cov}(Z_{i1}^2, Z_{i2}^2) = E\Big[\overbrace{\text{cov}(Z_{i1}^2, Z_{i2}^2 \mid Z_{i1})}^{0 \to Z_{i1} \text{ constant, so } Z_{i1}^2 \text{ is constant}}\Big] + \text{cov}\left(E[Z_{i1}^2 \mid Z_{i1}], E[Z_{i2}^2 \mid Z_{i1}]\right)$$

$$\text{cov}\left(E[Z_{i1}^2 \mid Z_{i1}], E[Z_{i2}^2 \mid Z_{i1}]\right)$$

$$= \text{cov}(Z_{i1}^2, \rho^2 Z_{i1}^2 + 1 - \rho^2)$$

bivariate normal $\to$ give conditional

$$E[Z_{i2}^2 \mid Z_{i1}] = \left(E[Z_{i2} \mid Z_{i1}]\right)^2 + \underbrace{\text{Var}(Z_{i2} \mid Z_{i1})}_{\text{same}}$$

$$= 0 + \rho(Z_{i1} - 0) + (1(1-\rho^2))$$

$$= (\rho Z_{i1})^2 + 1 - \rho^2$$

$$\text{cov}(Z_{i1}^2, \rho^2 Z_{i1}^2 + (1-\rho^2))$$

$$= \rho^2 \text{cov}(Z_{i1}^2, Z_{i1}^2) + \overbrace{\text{cov}(Z_{i1}^2, \underbrace{(1-\rho^2)}_{\text{constant}})}^{0}$$

$$= \rho^2 \cdot \underbrace{\text{var}(Z_{i1}^2)}_{\chi^2_1} = \rho^2 \cdot 2$$

$$\text{cov}(Z_{i1}^2, Z_{i2}^2) = 2\rho^2$$

$$\text{corr}(Z_{i1}^2, Z_{i2}^2) = \frac{2\rho^2}{\sqrt{2 \cdot 2}}$$

$$= \rho^2$$

Even if observations are independent, the residuals will not be

$$\left.\begin{aligned} R_1 &= Y_1 - \hat{\mu}_1 \\ R_2 &= Y_2 - \hat{\mu}_2 \end{aligned}\right\}$$ each is a linear function of all the data, so R_1 & R_2 are correlated

3b) Z_1, Z_2 independent (mean 0, var 1)

$$\text{Correlated}\left[\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}\right] = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$$

subscripts should have given you a hint as to the point of the question

Pick parameters $\underbrace{\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}}$

Compute $\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix}$ ↗ Lower-triangular matrix

☆ Used to simulate data $\underset{\sim}{Y}$ using given $\underset{\sim}{Z}$ (ex: R could give you this)

and your chosen parameters ☆

$\underset{\sim}{Y} = \underset{\sim}{m} + L\underset{\sim}{Z}$ $\quad E(\underset{\sim}{Y}) = \underset{\sim}{m}$

L ↓ our lower-triangular matrix of A's

$\underline{\text{cov}(\underset{\sim}{Y}) = LL^T = \Sigma}$

Choleski Decomposition

☆ When interpreting, look for linearity in stuff ☆

10/14/2019   BIOS 667 ← Lecture

Adjusting for Baseline

	T_1	T_2	T_3	T_4
P				
A				

Look at first two timepoints at first (basic example)

← Baseline

	1	2
P	μ_{11}	μ_{12}
A	μ_{21}	μ_{22}

Treatment effect → In BIOS 663

P	μ_{11}
A	μ_{21}

μ_{ij}

i is group,

j is timepoint of observation

↓

after "treatment" given

↓

Treatment effect could be: $\mu_{21} - \mu_{11}$

(contrast between two groups)

↓

if you really wanted to parallel longitudinal design, j should perhaps $= 2$