Problem 13.1

Part 0

Simple Statistics

Clinic 1, Active Treatment

Visit	*N*	*Mean*	*SD*
0	27	0.33	0.48
1	27	0.52	0.51
2	27	0.59	0.50
3	27	0.63	0.49
4	27	0.44	0.51

Clinic 1, Placebo

Visit	*N*	*Mean*	*SD*
0	29	0.31	0.47
1	29	0.41	0.50
2	29	0.34	0.48
3	29	0.41	0.50
4	29	0.31	0.47

Clinic 2, Active Treatment

Visit	*N*	*Mean*	*SD*
0	27	0.56	0.51
1	27	0.85	0.36
2	27	0.81	0.40
3	27	0.81	0.40
4	27	0.78	0.42

Clinic 2, Placebo

Visit	*N*	*Mean*	*SD*
0	28	0.61	0.50
1	28	0.57	0.50
2	28	0.43	0.50
3	28	0.50	0.51
4	28	0.57	0.50

Covariance Matrices

Clinic 1, Active Treatment

$$\begin{bmatrix} 0.23 & 0.17 & 0.10 & 0.13 & 0.08 \\ 0.17 & 0.26 & 0.10 & 0.12 & 0.07 \\ 0.10 & 0.10 & 0.25 & 0.11 & 0.15 \\ 0.13 & 0.12 & 0.11 & 0.24 & 0.09 \\ 0.08 & 0.07 & 0.15 & 0.09 & 0.26 \end{bmatrix}$$

Clinic 1, Placebo

$$\begin{bmatrix} 0.23 & 0.17 & 0.10 & 0.13 & 0.08 \\ 0.17 & 0.26 & 0.10 & 0.12 & 0.07 \\ 0.10 & 0.10 & 0.25 & 0.11 & 0.15 \\ 0.13 & 0.12 & 0.11 & 0.24 & 0.09 \\ 0.08 & 0.07 & 0.15 & 0.09 & 0.26 \end{bmatrix}$$

Clinic 2, Active Treatment

$$\begin{bmatrix} 0.26 & 0.09 & 0.03 & 0.07 & 0.05 \\ 0.09 & 0.13 & 0.05 & 0.05 & 0.08 \\ 0.03 & 0.05 & 0.16 & 0.12 & 0.07 \\ 0.07 & 0.05 & 0.12 & 0.16 & 0.07 \\ 0.05 & 0.08 & 0.07 & 0.07 & 0.18 \end{bmatrix}$$

Clinic 2, Placebo

$$\begin{bmatrix} 0.26 & 0.09 & 0.03 & 0.07 & 0.05 \\ 0.09 & 0.13 & 0.05 & 0.05 & 0.08 \\ 0.03 & 0.05 & 0.16 & 0.12 & 0.07 \\ 0.07 & 0.05 & 0.12 & 0.16 & 0.07 \\ 0.05 & 0.08 & 0.07 & 0.07 & 0.18 \end{bmatrix}$$

Part 1

Model for the log odds that respiratory status is classified as good
Assuming separate pairwise log odds ratios among the five binary responses
where $i = 0$ is the placebo (reference) and $i = 1$ is the active treatment
$j = 0, 1, 2, 3, 4$ for the j_{th} occasion time 0 (baseline) is the reference
Under the constraint that the treatment groups have the same mean at time 0
we have $\beta_0 = \gamma_{0j} = \gamma_{i0} = 0$
$\eta_{ij} = \mu + \beta_j I(\text{time}_j) + \gamma_{ij} I(\text{time}_j) * I(\text{treatment}_i)$

Parameter	*estimate*	$\hat{se}$
$\hat{\mu}$	$-.199$	.191
$\hat{\beta}_1$	.151	.262
$\hat{\beta}_2$	$-.274$	.282
$\hat{\beta}_3$	.012	.263
$\hat{\beta}_4$	$-.058$	.257
$\hat{\gamma_{11}}$	.838	.337
$\hat{\gamma_{12}}$	1.347	.380
$\hat{\gamma_{13}}$	1.154	.361
$\hat{\gamma_{14}}$	.718	.355

$$\text{Treatment effect} = \boldsymbol{\delta} = (\gamma_{11}, \gamma_{12}, \gamma_{13}, \gamma_{14})^T$$

$\boldsymbol{\delta}$ represents the treament effect on changes in the log odds that respiratory status is classified as good

Testing the null hypothesis of no effect of treatment on changes in the log odds that respiratory status is classified as good based on the empirical standard errors

Wald χ^2 Test

$$H_0 : \boldsymbol{\delta} = 0$$

$$H_1 : \boldsymbol{\delta} \neq 0$$

$$\chi^2 = 16.91 \text{ with } df = 4$$

p-value = .002 < .05Thus reject H_0

Part 2

Since the null hypothesis was rejected that there is evidence to suggest a statistically significant effect of treatment on changes in the log odds that respiratory status is classified as good

Part 3a

Model for the log odds that respiratory status is classified as good
$\eta_{ijk} = \mu + \beta_j + \alpha_k + \gamma_{ij} + \tau_{jk} + \lambda_{ijk}$
where $i = 0$ is the placebo (reference) and $i = 1$ is the active treatment
$j = 0, 1, 2, 3, 4$ for the j_{th} occasion time 0 (baseline) is the reference
$k = 1, 2$ is the k_{th} center with center 1 as the reference

$$\text{Treatment effect} = \boldsymbol{\delta} = (\lambda_{112}, \lambda_{122}, \lambda_{132}, \lambda_{142})^T$$

$\boldsymbol{\delta}$ represents the treament effect on changes in the log odds that respiratory status is classified as good

Parameter	*estimate*	$\hat{se}$
$\hat{\lambda_{112}}$	1.203	.759
$\hat{\lambda_{122}}$	.862	.803
$\hat{\lambda_{132}}$	.749	.761
$\hat{\lambda_{142}}$	.507	.773

Testing if the effect of treatment is the same in the two clincs

Wald χ^2 Test

$H_0 : \boldsymbol{\delta} = 0$ (The treament effect is the samin in the two clinics)

$$H_1 : \boldsymbol{\delta} \neq 0$$

$$\chi^2 = 3.10 \text{ with } df = 4$$

p-value = .541 > .05 Thus fail to reject H_0

Since we failed to reject the null hypothesis there is not enough evidence to suggest a statistically significant difference in effect of treatment between the two clinics on changes in the log odds that respiratory status is classified as good

Part 3b

We drop the 3 variable interactions since the results from part a showed it was non-significant.
Model for the log odds that respiratory status is classified as good
$\eta_{ijk} = \mu + \beta_j + \alpha_k + \gamma_{ij} + \tau_{jk}$
where $i = 0$ is the placebo (reference) and $i = 1$ is the active treatment
$j = 0, 1, 2, 3, 4$ for the j_{th} occasion time 0 (baseline) is the reference
$k = 1, 2$ is the k_{th} center with center 1 as the reference

$Parameter$	$estimate$	$\hat{se}$
$\hat{\mu}$	$-.771$	.289
$\hat{\beta}_1$	.202	.295
$\hat{\beta}_2$	$-.046$	.330
$\hat{\beta}_3$	.289	.297
$\hat{\beta}_4$	$-.127$	.356
$\hat{\alpha_2}$	1.103	.400
$\hat{\gamma_{11}}$	.892	.348
$\hat{\gamma_{12}}$	1.389	.389
$\hat{\gamma_{13}}$	1.195	.368
$\hat{\gamma_{14}}$	.774	.380
$\hat{\tau_{12}}$	$-.071$	.417
$\hat{\tau_{22}}$	$-.437$	.470
$\hat{\tau_{32}}$	$-.531$	.428
$\hat{\tau_{42}}$	.153	.461

$\hat{\mu}$ is the log odds ratio of good respiratory status at baseline for the placebo group at center 1.
$\hat{\beta}_j$ the difference in the log odds ratio of good respiratory status at occasion j relative to baseline for patients receiving placebo at center 1.
$\hat{\alpha_2}$ is the difference in the log odds ratio of good respiratory status for patients at center 2 compared to center 1
$\hat{\gamma_{1j}}$ is the interaction effect of treatment and time on the log odds ratio of good respiratory status.
$\hat{\tau_{j2}}$ is the interaction effect of time and center 2 on the log odds ratio of good respiratory status.

Part 4

Table of estimated probabilities that respiratory status is classified as good as a function of both time and treatment group clinic

Time	Clinic 1		Clinic 2	
	Placebo	*Active*	*Placebo*	*Active*
0	.316	.316	.582	.582
1	.361	.580	.614	.795
2	.306	.639	.614	.775
3	.382	.671	.522	.783
4	.289	.589	.469	.756

Based on the table it appears that the treatment has an effect on the probability good respiratory status within in each clinic, but it does not appear that clinic has a significant effect on the probability good respiratory status