$(X, Y) \sim$ bivariate normal

$X \sim N(\mu_1, \sigma_{11})$, $Y \sim N(\mu_2, \sigma_{22})$ $cov(X,Y) = \sigma_{12} = \sigma_{21}$

Then: $\rho = \dfrac{\sigma_{12}}{\sqrt{\sigma_{11}\sigma_{22}}}$

$X|Y \sim N(\mu_1 + \sigma_{12}\sigma_{22}^{-1}(Y-\mu_2),\ \sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{21})$

$var(X|Y) = \sigma_{11}(1-\rho^2)$

$P(B) = E[P(B|X)]$

$var(X) = E[X^2] - [EX]^2$

Mixed 1

- residual, w/in subject variance

  $var(\varepsilon_i) = cov(Y_i|b_i) = R_i$ (usually $\sigma_w^2 I_{n_i \times n_i}$)

  variance from the subject-specific mean

- between-subject variance:

  $cov(b_i) = G$

$\mu_{ij} = E[Y_{ij}] = x_{ij}\beta$

$var(Y_{ij}) = z_{ij}^T G z_{ij} + R_{jj}^i$

$cov(Y_{ij}, Y_{ik}) = z_{ij}^T G z_{ik} + R_{jk}^i$

- Total Variance of one cluster:

$$\Sigma_i = cov(Y_i) = \underbrace{Z_i^T G Z_i}_{\text{between}} + \underbrace{R_i}_{\text{w/in}}$$

ICC: ① correlation of observations w/in a cluster

② Proportion of total variation due to var between clusters

Wald Test:

$$W = \frac{(\hat{\theta} - \theta_0)^2}{var(\hat{\theta})}$$

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + \underbrace{Bias(\hat{\theta}, \theta)^2}_{\downarrow \; (E[\hat{\theta} - \theta])^2}$$

Prediction mean-squared error:

$$MSE: \boxed{var(\hat{b}_i - b_i)} = G - Q_i^T \{ \Sigma_i - cov(\hat{\mu}_i) \} Q_i$$

$$Q_i = \Sigma_i^{-1} Z_i G \, .$$