2017 2e

$$S(t) = P(Y_1 > T) = \exp(-t/\beta) \text{ (from part b)}$$
$$\text{Let } \tau\beta = \exp(-t/\beta) = S(t)$$
$$\text{WTS: } E(V_1|U) \text{ is an unbiased estimator of } S(t)$$
$$E(E(V_1|U)) = E(V_1)$$
$$E(V_1) = S(t) \text{ from part c}$$
$$\text{Thus } E(E(V_1|U)) = S(t)$$
$$\text{Therefore } E(V_1|U) \text{ is an unbiased estimator of } S(t)$$
$$\text{WTS } U = \sum Y_i \text{ is a css for } \beta$$
$$f_y(y|\beta) = \beta^{-n} \exp\left(-\sum Y_i/\beta\right), y > 0 \beta > 0$$
$$h(x) = I(y > 0) \quad c(\beta) = \beta^{-n}$$
$$w(\beta) = -1/\beta \quad t(x) = \sum Y_i$$
$$\text{Thus } f(y|\beta) = h(x)c(\beta)\exp(w(\beta)t(x)), 0 < \beta < \infty$$
$$T(x) = \sum Y_i = U \text{ is complete because:}$$
$$\{w(\beta) : \beta \in (0, \infty)\} \text{ contains an open set in } R^1$$
$$w(\beta) = -1/\beta, \beta > 0 = (-\infty, 0) \text{ in } R^{-1} \text{ (the range contains an open set in } R^1)$$
$$\text{We have: } V_1 \text{ an unbiased estimator of } \tau(\beta) = S(t), U \text{ a CSS for } \beta$$
$$\text{Thus by lehmann-scheffe thm } E(V_1|U) \text{ is the UMVUE for } \tau(\beta) = S(t)$$