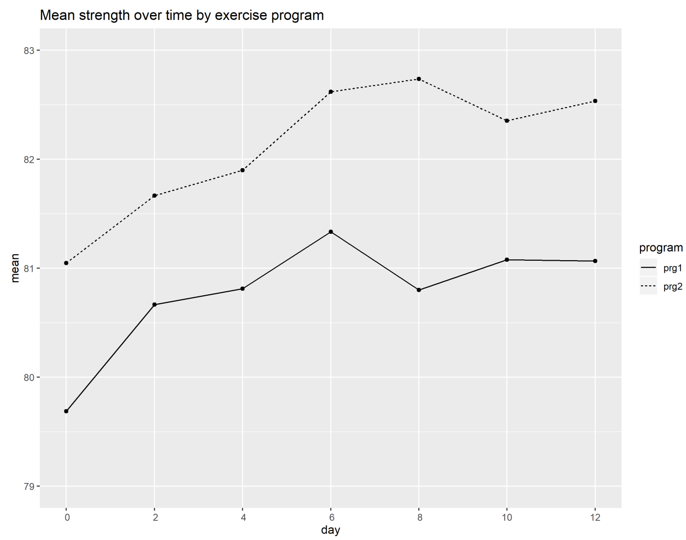
Problem 8.1

Part 1



Looking at the graph we see that the mean strength tends to increase over time for both treatment groups. Treatment 1 decreases between day 6 and day 8, and for treatment 2, the mean strength decreases between day 8 and day 10. Treatment 2 has a higher mean strength than treatment 2 at baseline. The mean differences in strength over time between treatment 2 and treatment 1 are shown in the following table.

day	μ_{diff}
0	1.36
2	1.00
4	1.09
6	1.29
8	1.94
10	1.28
12	1.47

Part 3

$E(Y_{ij}|b_i) = \beta_1 + b_{1i} + \beta_2 time + \beta_3 treatment1 + \beta_4 time * treatment1 + b_{2i} time$
Where $time = (0, 2, 4, 6, 8, 10, 12)$ is the time in days when the measurement occurred
treatment1 is the indicator of the subject being in program 1

Estimated Regression Coefficients (fixed effects) and standard errors

Parameter	*estimate*	*se*
β_1	81.264	.7
β_2	$-.169$	.045
β_3	-1.131	1.064
β_4	$-.052$	.067

Estimated Covariance of the random effects and standard errors

Parameter	*estimate*	*se*
$Var(b_{1i}) = g_{11}$	9.953	2.457
$Var(b_{2i}) = g_{12}$	$-.017$	.112
$Cov(b_{1i}, b_{2i}) = g_{22}$	.034	.01
$Var(\epsilon_i) = \sigma_w^2$	.665	.073

(a) Estimated variance of the random intercepts $= 9.953$

(b) Estimated variance of the random slopes $= .034$

(c) Estimated correlation between the random intercepts and slopes $= -.029$

(d) 95% CI baseline strength for program1
$(\hat{\beta}_1 + \hat{\beta}_3) \pm \sqrt{g_{11}} = (73.949, 86.315)$
95% CI baseline strength for program2
$\hat{\beta}_1 \pm \sqrt{g_{11}} = (75.081, 87.447)$
There is a large variability in the baseline strength in each program.
Approximately 95% of the subjects in program 1 have a baseline strength between 73.949 and 86.315
Approximately 95% of the subjects in program 2 have a baseline strength between 75.081 and 87.447

(e) 95% CI change in baseline strength for program1
$(\hat{\beta}_2 + \hat{\beta}_4) \pm \sqrt{g_{22}} = (-.244, .478)$
Approximately 95% of the subjects in program 1 have a change in strength between -.244 and .478
95% CI change in baseline strength for program2
$\hat{\beta}_2 \pm \sqrt{g_{22}} = (-.192, .53)$
Approximately 95% of the subjects in program 1 have a change in strength between -.192 and .53
There is a large variability in the change in baseline strength in each program.

Part 4

LRT test comparing model with two random effects and model with one random effect using $\alpha = .05$
$H_0 : g_{22} = 0$
The critical value is a 50:50 mixture of chi-square distributions with 1 and 2 degrees of freedom
critical value = 5.14
For the model with two random effects, $-2logL = 818.5$
For the model with one random effect, $-2logL = 881.2$
$\chi^2_{obs} = 881.2 - 818.5 = 62.7 > 5.14$ Reject H_0
Conclude there is significant evidence that the model with random slopes and random intercepts is a better fit than the model with random intercepts only.

Part 5

Mean intercept and mean slope in each program

Program	*Intercept*	*Slope*
1	80.132	.117
2	81.263	.169

Part 6

From the table in the previous part, we see that the slope is .117 for program 1 and .169 for program 2. The estimated slope for program 2 is .052 units higher than for program 1.
This suggests that program 2 increases mean strength at higher rate than program 1.
Running a type 3 wald test on the interaction term
$H_0 : \beta_4 = 0$
Wald $\chi^2_1 = .59$ p-value= $.44 > .05$ Fail to reject H_0, the interaction term is non significant
Thus conclude the changes in strength are not statistically significant between the two programs.

Part 7

$\sigma^2_w = .665 \quad \sigma^2_b = 9.953$
$Var(Y_{i1}|b_i) = \sigma^2_w = .665$
$Var(Y_{i1}) = \sigma^2_w + \sigma^2_b = .665 + 9.953 = 10.618$
The difference between the variance estimates is whether the estimate accounts for the between subject variance. In this case, the between subject variance is a lot higher than the within subject variance.

Part 8

Empirical BLUP b_i		
id	intercept	slope
1	-1.052	-0.077
2	2.909	0.228
3	1.642	-0.034
4	0.856	-0.021
5	-0.001	0.264
6	-3.997	-0.157
7	1.963	0.150
8	-2.608	0.228
9	5.125	0.065
10	-4.707	0.177
11	-3.390	-0.140
12	4.029	0.064
13	-1.199	0.110
14	-2.386	-0.320
15	-1.583	-0.404
16	4.399	-0.133
17	2.690	-0.213
18	-6.403	-0.040
19	1.407	-0.189
20	5.167	-0.130
21	1.234	0.103
22	-2.110	-0.120
23	-2.610	0.172
24	6.704	0.216
25	-0.547	0.022
26	0.713	0.180
27	-2.171	0.051
28	-1.783	0.216
29	2.253	-0.146
30	-0.208	0.084
31	-3.124	-0.092
32	1.063	-0.035
33	-1.787	-0.097
34	-0.122	-0.145
35	4.292	-0.102
36	-3.326	0.228
37	-1.332	0.035

In order to obtain the Predicted empirical BLUP intercept and slope estimates for each subject we must add the Mean intercept and mean slope for the subjects respective program

Predicted empirical BLUP intercept and slope		
id	intercept	slope
1	79.080	0.040
2	83.041	0.345
3	81.774	0.083
4	80.989	0.096
5	80.131	0.381
6	76.135	-0.040
7	82.095	0.267
8	77.524	0.345
9	85.257	0.182
10	75.425	0.294
11	76.742	-0.023
12	84.161	0.181
13	78.933	0.227
14	77.746	-0.203
15	78.549	-0.287
16	84.531	-0.016
17	83.953	-0.044
18	74.860	0.129
19	82.670	-0.020
20	86.430	0.039
21	82.497	0.272
22	79.153	0.049
23	78.654	0.342
24	87.967	0.385
25	80.716	0.191
26	81.976	0.349
27	79.092	0.220
28	79.480	0.385
29	83.516	0.024
30	81.055	0.253
31	78.139	0.077
32	82.326	0.134
33	79.476	0.072
34	81.141	0.024
35	85.555	0.067
36	77.937	0.397
37	79.931	0.204

Part 9

Subject 24 OLS Estimates		
Parameter	Estimate	se
Intercept	87.8	.787
Slope	.45	.161

Part 10

For subject 24, the empirical blup slope is .385 and the intercept is 87.967.
The empirical blup intercept is slightly higher but the OLS estimated slope is higher.
The empirical blup are predicted and the OLS estimates are estimated. A prediction usually has larger uncertainty than a related estimate, due to the added uncertainty in the outcome of that random variable. This is the main reason why the OLS estimates and the empirical BLUP differ.
Also for subject 24, two of the seven observations are missing which could effect the subjects predicted response.