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ST 732, TEST 1, SPRING 2007, SOLUTIONS

Please sign the following pledge certifying that the work on this test is your own:

"I have neither given nor received unauthorized aid on this test."

Signature: ______________________________

Printed Name: ______________________________

There are SIX questions, some with multiple parts. For each part of each question, please write your answers in the space provided. If you need more space, continue on the back of the page and indicate clearly where on the back you have continued your answer. Scratch paper is available from the instructor.

You are allowed ONE (1) SHEET of HANDWRITTEN NOTES (FRONT ONLY). Calculators are NOT allowed (you will not need one). NOTHING should be on your desk but this test paper, your one page of notes, and scratch paper given to you by the instructor.

Points for each part of each problem are given in the left margin. TOTAL POINTS = 100.

In all problems, all symbols and notation are defined exactly as they are in the class notes.

Note: My answers are much more detailed than I expected your answers to be!

[5 points] 1. A study was conducted in which $m = 50$ units were randomly sampled from a single population. Each unit was observed at 0, 1.5, 3.0, 4.5, 6.0 hours, and the sample covariance matrix $\widehat{\boldsymbol{\Sigma}}$ and the corresponding sample correlation matrix $\widehat{\boldsymbol{\Gamma}}$ based on the data from the m units were calculated as:

$$\widehat{\boldsymbol{\Sigma}} = \begin{pmatrix} 10.28 & 5.18 & -0.78 & -0.30 & 0.44 \\ 5.18 & 11.17 & 7.16 & 1.39 & 1.49 \\ -0.78 & 7.16 & 17.92 & 11.05 & 0.12 \\ -0.30 & 1.39 & 11.05 & 16.90 & 6.70 \\ 0.44 & 1.49 & 0.12 & 6.70 & 18.80 \end{pmatrix}, \quad \widehat{\boldsymbol{\Gamma}} = \begin{pmatrix} 1.00 & 0.48 & -0.058 & -0.02 & 0.03 \\ 0.48 & 1.00 & 0.51 & 0.10 & 0.10 \\ -0.06 & 0.51 & 1.00 & 0.63 & 0.01 \\ -0.02 & 0.10 & 0.63 & 1.00 & 0.38 \\ 0.03 & 0.10 & 0.10 & 0.38 & 1.00 \end{pmatrix}.$$

For each of (i) and (ii) below, you must give an explanation to receive credit!

(i) Which covariance model would you choose to describe the true pattern of variance and correlation in the population? Explain your reasons for your choice.

The diagonal elements of $\widehat{\boldsymbol{\Sigma}}$, which estimate overall variance in the population at each time point, appear to increase over time, suggesting that the true population variances may not be the same at each time point. This might suggest a heterogeneous model is appropriate, although some of you felt that the increase was not profound enough to abandon a homogeneous assumption.

The sample correlation matrix $\widehat{\boldsymbol{\Gamma}}$ suggests that observations one time interval (1.5 hours – these times are equally-spaced) are positively correlated and with correlation of similar magnitude (roughly 0.5), but observations 2 or more time intervals apart show negligible correlation, with all the estimates ≤ 0.10 in absolute value. The correlations one time interval apart are not exactly the same, as this is an estimate, but they are in a similar "ballpark" suggesting that maybe the true population correlations could be the same. Likewise, the off-diagonal elements are very close to zero with the possible exception of 0.10; again, as this is a sample estimate, if the true correlation was zero, such an estimate might still be obtained. These correlations are*much* smaller than the 1-time-interval correlations. These observations are consistent with a *heterogeneous one-dependent* covariance structure.

Some of you thought an AR(1) (or Markov, which is equivalent to AR(1) here as the times are equally-spaced) is also a possibility. The "drop-off" in correlation after lag 1 does seem a bit more precipitous than would be predicted by these models (i.e., $0.5^2 = 0.25$, $0.5^3 = 0.125$, etc). However, again, this is sample information, so it might be possible.

(ii) Does a particular source of variation/correlation appear to be "dominant?" If so, identify the source and say why you think this is the case. If not, explain why you do not think so.

Here, correlation drops off substantially when observations are more than one time interval (1.5 hours) apart. This is characteristic of the tendency for deviations due to "within-unit fluctuations" to become less alike the farther apart in time they are. This suggests that, in terms of contribution to the overall pattern of correlation, the *within-unit* source of correlation is dominant.

2. In another study, m subjects were randomly sampled from a single population. The intention of the study was for each subject to be observed at times 0 (baseline), 2, 4, 6, 8, 10, and 12 months.

   Let $\boldsymbol{Y}_i$ be the vector of *observed* responses for a subject who missed his scheduled visits at months 2, 4, and 10.

[5 points] (a) Write down the covariance matrix for $\boldsymbol{Y}_i$ if it is assumed that the covariance structure for the vector of *intended* responses follows a homogeneous AR(1) model.

The observations are equally-spaced, with a time interval of 2 months, so this model is reasonable. The *actual* times for this subject are (0,6,8,12), which correspond to intended times indexed by (1,4,5,7). So, for example, 0 and 6 are 3 time intervals apart, 8 and 12 are 2 intervals apart, and so on. With the homogeneity, assuming common variance σ^2 at all time points, we thus have:

$$\sigma^2 \begin{pmatrix} 1 & \rho^3 & \rho^4 & \rho^6 \\ & 1 & \rho^1 & \rho^3 \\ & & 1 & \rho^2 \\ & & & 1 \end{pmatrix},$$

where ρ is the correlation parameter for the AR(1) structure.

[5 points] (b) Write down the covariance matrix for $\boldsymbol{Y}_i$ if it is assumed that the covariance structure for the vector of *intended* responses follows a heterogeneous compound symmetry model.

Here, variance changes with time. There are 7 *intended* time points, so let the (unequal) variances at times 0, 2, 4, 6, 8, 10, 12 be $\sigma_1^2, \sigma_2^2, \ldots, \sigma_7^2$. As above, we have only seen the observations at times indexed by (1,4,5,7). Thus, using this "intended" indexing, the matrix is

$$\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_4 & \rho\sigma_1\sigma_5 & \rho\sigma_1\sigma_7 \\ & \sigma_4^2 & \rho\sigma_4\sigma_5 & \rho\sigma_4\sigma_7 \\ & & \sigma_5^2 & \rho\sigma_5\sigma_7 \\ & & & \sigma_7^2 \end{pmatrix},$$

where ρ is the assumed constant correlation for the compound symmetry correlation structure.

3. In the notation in the notes, the statistical model underlying the univariate repeated measures analysis of variance is

$$Y_{h\ell j} = \mu_{\ell j} + b_{h\ell} + e_{h\ell j} = \mu + \tau_\ell + \gamma_j + (\tau\gamma)_{\ell j} + b_{h\ell} + e_{h\ell j}, \tag{1}$$

where $Y_{h\ell j}$ is the response for the hth unit in group ℓ at time j, $\ell = 1, \ldots, q$, $j = 1, \ldots, n$, and $h = 1, \ldots, r_\ell$ in group ℓ; μ, the τ_ℓ, the γ_j, and the $(\tau\gamma)_{\ell j}$ are fixed parameters;

$$\mu_{\ell j} = \mu + \tau_\ell + \gamma_j + (\tau\gamma)_{\ell j}$$

is the mean response for group ℓ at time j; and the $b_{h\ell}$ and $e_{h\ell j}$ are random components.

[5 points] (a) Recall that an alternative way to write model (1) is by using a *single index* i, where i is determined by the unique values of h and ℓ. In matrix notation, the model is written as

$$\boldsymbol{Y}_i' = \boldsymbol{a}_i'\boldsymbol{M} + \mathbf{1}b_i + \boldsymbol{e}_i',$$

where $\boldsymbol{M}$ depends on the $\mu_{\ell j}$. Suppose that $q = 4$ and $n = 5$. Write down $\boldsymbol{M}$ in this case, and write down $\boldsymbol{a}_i'$ for a unit in group 3.

$$\boldsymbol{M} = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} & \mu_{14} & \mu_{15} \\ \mu_{21} & \mu_{22} & \mu_{23} & \mu_{24} & \mu_{25} \\ \mu_{31} & \mu_{32} & \mu_{33} & \mu_{34} & \mu_{35} \\ \mu_{41} & \mu_{42} & \mu_{43} & \mu_{44} & \mu_{45} \end{pmatrix}.$$

$$\boldsymbol{a}_i' = (0, 0, 1, 0)'$$

[5 points] (b) Suppose we are interested in the null hypothesis that the pattern of change in mean response over time is similar for all groups. For the scenario in (a), with $q = 4$ and $n = 5$, express this hypothesis in the form $H_0 : \boldsymbol{CMU} = \mathbf{0}$ by giving the form of $\boldsymbol{C}$ and $\boldsymbol{U}$.

The null hypothesis is that of parallelism – the alternative would be that the pattern of change (over time) is different among the groups somehow. Thus, we need to take differences over time and then ask whether there are difference across groups, which gives the matrices

$$\boldsymbol{C} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}, \quad \boldsymbol{U} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

A few of you wrote down this $\boldsymbol{C}$ matrix but the $\boldsymbol{U}$ matrix that has one column with all values equal to 1/5; that is, the $\boldsymbol{U}$ matrix for "averaging" across time. Thus, your choices correspond to the test of main effect of group, which asks whether there are differences across groups, averaged across time (so "averaging away" the pattern of change over time).

Parts (c) and (d) of this problem are on the next page

[5 points] (c) State the standard assumptions made on the random components in model (1), defining any symbols you use. In terms of these symbols, write down the overall correlation between observations taken at times 1 and 2 on a unit in group 3 in a situation where $q = 4$ and $n = 5$.

The usual assumption is that $b_{h\ell}$, which represents among-unit variation, are independent of $e_{h\ell j}$ and independent of each other. Furthermore, $b_{h\ell} \sim \mathcal{N}(0, \sigma_b^2)$ and $e_{h\ell j} \sim \mathcal{N}(0, \sigma_e^2)$, where the among- and within-subject variances σ_b^2 and σ_e^2 are assumed the same for all individuals and time points. Under these assumptions, we know that the overall covariance matrix of a data vector is compound symmetric and the same for all groups.. Thus, the overall correlation between any pair of measurements in any group is the same. We showed in the notes that this correlation is

$$\frac{\sigma_b^2}{\sigma_b^2 + \sigma_e^2}.$$

[5 points] (d) A friend in another department is going to conduct a longitudinal study as part of his dissertation research. He tells you that he intends to use model (1) along with the standard assumptions that go with it as the basis for his analysis. What advice do you give him?

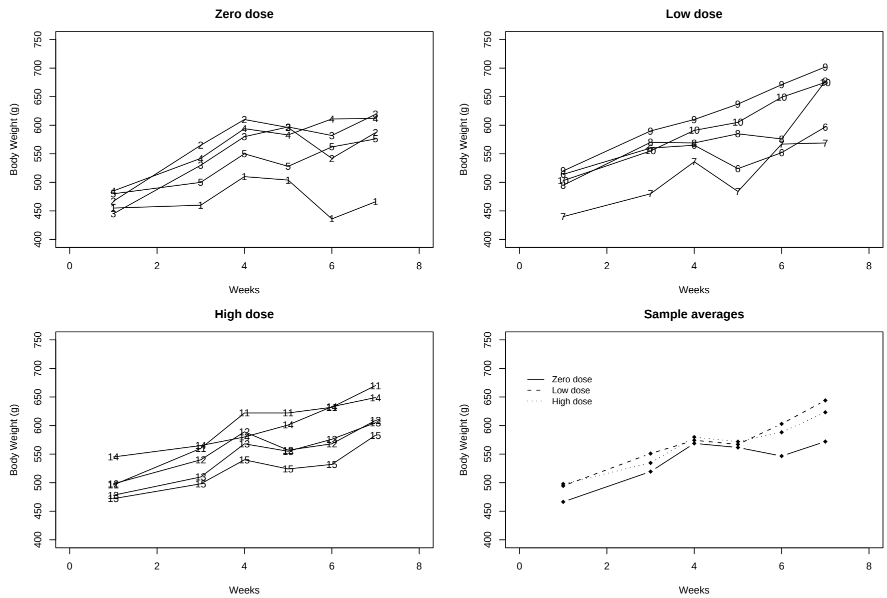
You might tell him that there are some serious disadvantage to using this model, and that there are more modern methods that allow the analyst much more latitude in describing what is going on. The model imposes some restrictions that may end up not being true for his data.

The disadvantages include:

- Need for balance - even if your experiment is designed to be balanced, missing observations may occur, complicating the analysis with this model, which assumes balance.
- Assumed form of covariance matrix (compound symmetric or, at best, type H) that is *the same* for each group – this may not be true!
- The model does not explicitly acknowledge time – it treats time as a series of categories. So, even if you believe that the pattern of change in mean response is smooth over time, you can't take advantage of this. Moreover, if under this belief you want to estimate means at time points between those in your study or estimate quantities having to do, for example, with the rate of change, you can't do this with this model. The analysis focuses mainly on hypothesis testing.

There are further reasons. If you mentioned some of these, that was good.

4. Recall the guinea pig diet study, which was introduced in Chapter 1 of the class notes. $m = 15$ guinea pigs were randomly assigned to 3 groups, 5 pigs per group. At the beginning of the study (baseline, week 0) all pigs were given the same growth-inhibiting substance and were then treated identically until the end of week 4 (week 4). At this point, they were started on one of three vitamin E supplement doses, depending on the group to which they had been randomized (zero, low, or high dose). For each pig, body weight (g) was recorded at weeks 1, 3, 4, 5, 6, and 7. The plots of the data presented in Chapter 1 are reproduced below:



Letting Y_{ij} be the body weight measured for the ith pig at the jth week, denoted by t_{ij}, the investigators adopted the following statistical model:

$$
\begin{aligned}
Y_{ij} &= \beta_{01} + \beta_{11} t_{ij} + \beta_{21}(t_{ij} - 4)_+ + \epsilon_{ij}, \quad i \text{ from zero dose group} \\
&= \beta_{02} + \beta_{12} t_{ij} + \beta_{22}(t_{ij} - 4)_+ + \epsilon_{ij}, \quad i \text{ from low dose group} \qquad (2) \\
&= \beta_{03} + \beta_{13} t_{ij} + \beta_{23}(t_{ij} - 4)_+ + \epsilon_{ij}, \quad i \text{ from high dose group,}
\end{aligned}
$$

$$
\begin{aligned}
x_+ &= x \quad \text{if } x \geq 0 \\
&= 0 \quad \text{if } x < 0.
\end{aligned}
$$

[5 points] (a) Assuming this model, write down an expression for the rate of change of mean body weight *after* initiation of vitamin E for the low dose group.

This problem is a generalization to 3 groups of Problem 4 in Homework 3.

For group 2, the low dose group, the model says that mean body weight *before* week 4 is $\beta_{02} + \beta_{12} t_{ij}$. The model also says that mean body weight *after* week 4 is

$$\beta_{02} + \beta_{12} t_{ij} + \beta_{22}(t_{ij} - 4) = \beta_{02} + \beta_{12}(4) + (\beta_{12} + \beta_{22})(t_{ij} - 4).$$

(This is similar for the other groups.) So if we take week 4 as the "origin," for the phase *after* week 4, the rate of change for group 2 is

$$\beta_{12} + \beta_{22}.$$

Parts (b) and (c) are on the next page.

[5 points] (b) Because the pigs in all three groups were treated *identically* until the end of week 4, the investigators wondered whether or not a simpler model that reflects this fact could be adopted. Collect all the parameters that describe the mean body weight trajectories in model (2) on the previous page in a parameter vector $\boldsymbol{\beta}$, and give the form of $\boldsymbol{\beta}$. Then write down the matrix $\boldsymbol{L}$ corresponding to the null hypothesis of the form $H_0 : \boldsymbol{L\beta} = \mathbf{0}$ addressing this issue.

Let

$$\boldsymbol{\beta} = (\beta_{01}, \beta_{02}, \beta_{03}, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{21}, \beta_{22}, \beta_{23})'.$$

As we discussed in part (a), the model in group $k = 1, 2, 3$ prior to week 4, while pigs were all treated identically, is

$$\beta_{0k} + \beta_{1k} t_{ij}.$$

If the pigs were treated identically until the end of week 4, we would expect the means across the three groups to be the same at any $t_{ij} \leq 4$. The only way this could be the case is if all the β_{0k} are the same and all the β_{1k} are the same. The $\boldsymbol{L}$ matrix is

$$\boldsymbol{L} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}.$$

In fact, because this is a *randomized study*, we might well expect the β_{0k} = mean body weight for group k at time 0 to be the same, regardless of whether they were treated identically afterwards. So, in summary, unless there was a serious problem with the randomization leading to different means at time 0, if the pigs were treated identically, we'd expect identical results throughout time up to week 4.

A few of you characterized "being treated identically" as leading to identical patterns of change until 4 weeks, but with possibly different intercepts. That is, you tested whether or not the β_{1k} are the same only. Credit was given for this, but, under the circumstances, including randomization, the interpretation above is probably more appropriate. It could be possible that, if the randomization was compromised, so that the pig groups represented different subpopulations with different characteristics, that they could be treated identically and still show different patterns of mean change. Under correct randomization, the groups would had the same characteristics on average, so that identical treatment would be reasonably thought to lead to the same rates of mean change.

[5 points] (c) The investigators' key question was whether or not the pattern of change in mean body weight *after* the groups were put on their assigned vitamin E doses was the same in all groups. *Assuming that your null hypothesis in (b) is true*, write down a new model that incorporates this. Collect all the parameters that describe the mean body weight trajectories in your new model into a parameter vector $\boldsymbol{\beta}$, and give the form of $\boldsymbol{\beta}$. Then write down the matrix $\boldsymbol{L}$ corresponding to the null hypothesis of the form $H_0 : \boldsymbol{L}\boldsymbol{\beta} = \mathbf{0}$ addressing this issue.

Under the hypothesis in (b), the model becomes

$$\begin{aligned} Y_{ij} &= \beta_0 + \beta_1 t_{ij} + \beta_{21}(t_{ij} - 4)_+ + \epsilon_{ij}, \quad i \text{ from zero dose group} \\ &= \beta_0 + \beta_1 t_{ij} + \beta_{22}(t_{ij} - 4)_+ + \epsilon_{ij}, \quad i \text{ from low dose group} \\ &= \beta_0 + \beta_1 t_{ij} + \beta_{23}(t_{ij} - 4)_+ + \epsilon_{ij}, \quad i \text{ from high dose group,} \end{aligned}$$

so that the mean trajectories before week 4 are identical. We have

$$\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_{21}, \beta_{22}, \beta_{23})'.$$

The slopes after week 4 are thus $\beta_1 + \beta_{2k}$ for each group $k = 1, 2, 3$, and the question is whether these slopes are identical. This will be the case if $\beta_{21} = \beta_{22} = \beta_{23}$, which gives

$$\boldsymbol{L} = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}.$$

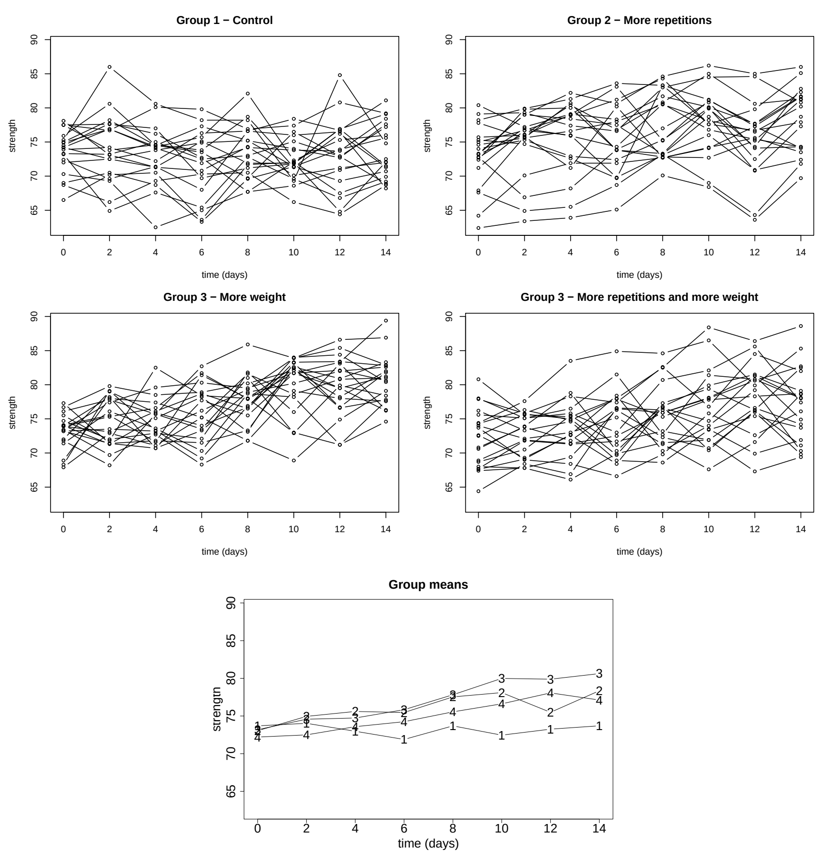
In fact, if this null hypothesis is true, the profiles are all identical (with possibly different slopes before and after 4 weeks).

5. A team of exercise physiologists conducted a study to investigate the effects of modifying weight-training programs on the strength of men who already follow a weight-training program regularly. The investigators recruited $m = 80$ such men to participate in the study, who were randomly assigned to four groups of 20 men each:

   - Group 1: Continue with current weight-training program (control group)
   - Group 2: Increase the number of repetitions used in the current program but continue to use the same amount of weight
   - Group 3: Increase the amount of weight used in the current program but continue to use the same number of repetitions
   - Group 4: Increase both the number of repetitions and the amount of weight used over those in the current program

   A strength measurement was taken on each man at baseline (day 0), before he started his assigned modified program. Each man then followed his assigned modified program for the next two weeks, and measures of strength were recorded for each on days 2, 4, 6, 8, 10, 12, and 14. The higher the strength measure, the stronger the man.

   Because all of these men were accustomed to working out regularly, it is not surprising strength measurements were recorded for all 80 men at each intended time. The data are shown below:



   The investigators decided to adopt the statistical model for univariate repeated measures analysis of variance in Equation (1) in Problem 3, along with its standard assumptions, as the framework in which they would address their scientific questions. On the 5 pages following the next one, you will find some SAS code and excerpts of its output; use this information to answer the following questions.

[5 points] (a) Based on the code and output, do you feel comfortable about the validity of inferences based on this statistical model? Explain your answer, citing specific information in the output that leads you to feel the way you do.

The inferences based on this model and its standard assumptions will only be valid if the overall covariance matrix for each group is *the same* and of Type H. The test for Sphericity based on orthogonal components yields a p-value of 0.03, suggesting there is evidence that this is violated. This could be the case either because Type H is violated, the covariance matrices are not the same in each group, or both. From the `proc corr` results, which give the standard deviations at each time point for each group and the sample correlation matrix, there seems to be some suggestion that variances (standard deviations) are a little larger in some groups than others (although possibly constant over time in each group) and the pattern of correlation may be different across groups. For some groups, it looks like it could be compound symmetric; for others, it is not clear, and the magnitudes of the estimated correlations don't seem necessarily the same. Of course, this is only sample evidence, but taken all together with the test, it does raise concerns. I would be wary about trusting inferences in this case.

[5 points] (b) Regardless of your answer to (a), assuming that the statistical model (1) is correct, write down an estimate of a quantity that gives an idea of the extent of variation due to within-subject sources.

The question is asking for an estimate of σ_e^2, the within-subject variance. This is estimated by the mean square for "within-subject error" at the bottom of the ANOVA table, which is given by the `Error(day)` row in the `proc glm` output. The value is 9.3329.

[5 points] (c) Regardless of your answer to (a), assuming that the statistical model (1) is correct, is there evidence in these data to suggest that the pattern of change in mean strength may be different depending on the type of weight training the man followed over the two weeks? Cite specific information in the output in support of your answer.

This is a question about parallelism, which here is represented by the `day*group` interaction. From the output, the F statistic is 4.57, which has an associated p-value that is < 0.0001, so less than any reasonable level of significance. There appears to be very strong evidence that the pattern of change in mean strength may not be the same for all groups.

[5 points] (d) The investigators expected that men who *both* increased the number of repetitions *and* the amount of weight used would show a constant rate of increase in mean strength that eventually "levels off" toward the end of the study period. On the other hand, they expected men who increased repetitions but not weight or increased weight but not repetitions to show a constant rate of increase in mean strength that does not "level off" over the entire period, and men who did neither to show no change. Is there evidence in these data that is consistent with this claim? Cite specific information in the output in support of your answer (assuming as in (b) that the model is correct).

There are a number of ways to think about this question – here's one. The expectation is that the mean profile for the men in group 4 might show *curvature* – going up at a relatively constant rate but then "bending over" and "leveling off" at the later times. In contrast, groups 2 and 3 should show mean profiles that do not "bend over" but rather look like straight lines, and that for group 1 would be "flat" (a straight line with 0 slope). Thus, we expect one profile that might resemble a quadratic function during the time frame of the study, with the rest straight lines. The test of whether there is a difference across groups in quadratic effects over time (the second polynomial contrast in the $\boldsymbol{U}$ matrix for polynomial contrasts) would address this issue, as the expectation is that the groups differ in this effect in that one group shows curvature while the rest do not. This may be addressed by the test for `group` in the `day_2` polynomial contrast table in the output. The relevant F statistic is 2.08 with a p-value of 0.11. Based on this, there is not enough evidence to suggest that there is such a difference. If we look at group differences in the linear effects, addressed by the test for `group` in the `day_1` polynomial contrast table, the evidence of differences is very strong. The impression from these results is that, although the overall linear trends differ, there is not strong enough evidence to suggest that at least one of the groups "levels" off.

Another way to think about this would be to examine the Helmert contrasts, which compare the mean at the current time to subsequent times. If one of the groups "levels off," two others keep going with a constant rate of change, and the fourth is "flat," we'd expect differences to in how the each mean compares to the average of its predecessors potentially to show difference across groups throughout time because of the "flat" one, for which the mean compared to the average of its predecessors will tend to be close to 0, while these comparisons in the other groups will show differences for the first few contrasts where the mean profiles are showing a rise. If there really is a "leveling off," we might expect that the group differences in the last few contrasts might become diluted a bit, because then there would now be two groups for which the current mean is not really different from its predecessors. The p-values for the `group` effects in each Helmert contrast, although not tracking exactly with this interpretation, do show stronger early differences and less profound later differences for the most part, so aren't inconsistent with it.

Overall, the evidence is a bit inconclusive.

```
/* Note: the variable ``age'' is not used in this problem */

data strength1;
  infile "strength.dat";
  input subject day strength age group;
  dayplus=day/2+1;
run;

proc sort data=strength1; by group subject age; run;
data strength2(keep=day0 day2 day4 day6 day8 day10 day12 day14 group subject age);
  array ee{8} day0 day2 day4 day6 day8 day10 day12 day14;
  do dayplus=1 to 8;
     set strength1;
     by group subject age;
     ee{dayplus}=strength;
     if last.subject then return;
  end;
run;

proc sort data=strength2; by group; run;
proc corr cov; by group; var day0 day2 day4 day6 day8 day10 day12 day14; run;

title "UNIVARIATE REPEATED MEASURES ANOVA 1";
proc glm data=strength2;
  class group;
  model day0 day2 day4 day6 day8 day10 day12 day14 = group /nouni;
  repeated day 8 (0 2 4 6 8 10 12 14) polynomial / printe printm summary nom;
run;

title "UNIVARIATE REPEATED MEASURES ANOVA 2";
proc glm data=strength2;
  class group;
  model day0 day2 day4 day6 day8 day10 day12 day14 = group /nouni;
  repeated day 8 (0 2 4 6 8 10 12 14) helmert / printe printm summary nom;
run;
```

-- group=1 ---

The CORR Procedure

Simple Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
day0	20	73.69500	3.16119	1474	66.50000	78.10000
day2	20	74.03000	4.99348	1481	64.90000	86.00000
day4	20	72.97500	4.20412	1460	62.50000	80.60000
day6	20	71.89000	4.85580	1438	63.30000	79.80000
day8	20	73.69000	3.82387	1474	67.70000	82.10000
day10	20	72.46000	3.09676	1449	66.20000	78.40000
day12	20	73.25500	5.14684	1465	64.40000	84.80000
day14	20	73.70000	4.06072	1474	68.20000	81.10000

Pearson Correlation Coefficients, N = 20
Prob > |r| under H0: Rho=0

	day0	day2	day4	day6	day8	day10	day12	day14
day0	1.00000	0.59186	0.62127	0.64103	0.41076	0.25428	0.63185	0.13358
		0.0060	0.0035	0.0023	0.0720	0.2793	0.0028	0.5745

day2	0.59186	1.00000	0.80107	0.67464	0.29699	0.23534	0.66674	0.14650
	0.0060		<.0001	0.0011	0.2035	0.3179	0.0013	0.5377
day4	0.62127	0.80107	1.00000	0.74807	0.46046	0.38458	0.64287	0.29322
	0.0035	<.0001		0.0001	0.0410	0.0941	0.0022	0.2096
day6	0.64103	0.67464	0.74807	1.00000	0.56914	0.30031	0.68538	0.32949
	0.0023	0.0011	0.0001		0.0088	0.1983	0.0009	0.1560
day8	0.41076	0.29699	0.46046	0.56914	1.00000	0.26558	0.61094	0.41857
	0.0720	0.2035	0.0410	0.0088		0.2578	0.0042	0.0662
day10	0.25428	0.23534	0.38458	0.30031	0.26558	1.00000	0.39370	0.44893
	0.2793	0.3179	0.0941	0.1983	0.2578		0.0859	0.0471
day12	0.63185	0.66674	0.64287	0.68538	0.61094	0.39370	1.00000	0.50280
	0.0028	0.0013	0.0022	0.0009	0.0042	0.0859		0.0238
day14	0.13358	0.14650	0.29322	0.32949	0.41857	0.44893	0.50280	1.00000
	0.5745	0.5377	0.2096	0.1560	0.0662	0.0471	0.0238	

-- group=2 ---

Simple Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
day0	20	73.05000	4.65103	1461	62.40000	80.40000
day2	20	74.95000	4.82346	1499	63.40000	79.90000
day4	20	75.60500	5.30387	1512	63.90000	82.20000
day6	20	75.46500	5.01474	1509	65.10000	83.60000
day8	20	77.53500	4.85899	1551	70.10000	84.60000
day10	20	78.10500	4.78578	1562	68.40000	86.20000
day12	20	75.49000	5.42188	1510	63.60000	85.00000
day14	20	78.35000	4.68182	1567	69.70000	86.00000

Pearson Correlation Coefficients, N = 20
Prob > |r| under H0: Rho=0

	day0	day2	day4	day6	day8	day10	day12	day14
day0	1.00000	0.68371	0.71232	0.56352	0.64148	0.62796	0.58187	0.73398
		0.0009	0.0004	0.0097	0.0023	0.0030	0.0071	0.0002
day2	0.68371	1.00000	0.89223	0.65796	0.47961	0.66281	0.64388	0.67045
	0.0009		<.0001	0.0016	0.0324	0.0014	0.0022	0.0012
day4	0.71232	0.89223	1.00000	0.69738	0.49787	0.67596	0.66362	0.75020
	0.0004	<.0001		0.0006	0.0255	0.0011	0.0014	0.0001
day6	0.56352	0.65796	0.69738	1.00000	0.51167	0.61537	0.60225	0.51104
	0.0097	0.0016	0.0006		0.0211	0.0039	0.0050	0.0213
day8	0.64148	0.47961	0.49787	0.51167	1.00000	0.66249	0.73175	0.62482
	0.0023	0.0324	0.0255	0.0211		0.0015	0.0002	0.0032
day10	0.62796	0.66281	0.67596	0.61537	0.66249	1.00000	0.79252	0.70931
	0.0030	0.0014	0.0011	0.0039	0.0015		<.0001	0.0005
day12	0.58187	0.64388	0.66362	0.60225	0.73175	0.79252	1.00000	0.74895
	0.0071	0.0022	0.0014	0.0050	0.0002	<.0001		0.0001
day14	0.73398	0.67045	0.75020	0.51104	0.62482	0.70931	0.74895	1.00000
	0.0002	0.0012	0.0001	0.0213	0.0032	0.0005	0.0001	

-- group=3 ---

Simple Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
day0	20	73.18500	2.56890	1464	67.90000	77.30000
day2	20	74.57500	3.47985	1492	68.20000	79.80000
day4	20	74.73500	3.17362	1495	70.70000	82.50000
day6	20	75.84000	4.26768	1517	68.30000	82.70000
day8	20	77.82000	3.57353	1556	71.80000	85.90000
day10	20	79.99000	4.25242	1600	68.90000	84.00000
day12	20	79.88500	4.26013	1598	71.20000	86.60000
day14	20	80.62500	3.62214	1613	74.60000	89.40000

Pearson Correlation Coefficients, N = 20
Prob > |r| under H0: Rho=0

	day0	day2	day4	day6	day8	day10	day12	day14
day0	1.00000	-0.13305	0.19529	0.37240	0.17542	0.09871	0.03331	0.34344
		0.5760	0.4093	0.1059	0.4595	0.6789	0.8891	0.1382
day2	-0.13305	1.00000	0.19071	0.18741	0.41685	0.53986	0.27292	0.25840

	day0	day2	day4	day6	day8	day10	day12	day14
	0.5760		0.4206	0.4288	0.0675	0.0140	0.2443	0.2713
day4	0.19529	0.19071	1.00000	0.41639	0.35909	-0.11132	-0.19242	0.14955
	0.4093	0.4206		0.0678	0.1200	0.6403	0.4164	0.5292
day6	0.37240	0.18741	0.41639	1.00000	0.33857	0.46782	0.37243	0.75971
	0.1059	0.4288	0.0678		0.1442	0.0375	0.1059	0.0001
day8	0.17542	0.41685	0.35909	0.33857	1.00000	0.10607	0.42993	0.28069
	0.4595	0.0675	0.1200	0.1442		0.6563	0.0585	0.2306
day10	0.09871	0.53986	-0.11132	0.46782	0.10607	1.00000	0.54040	0.46579
	0.6789	0.0140	0.6403	0.0375	0.6563		0.0139	0.0385
day12	0.03331	0.27292	-0.19242	0.37243	0.42993	0.54040	1.00000	0.61779
	0.8891	0.2443	0.4164	0.1059	0.0585	0.0139		0.0037
day14	0.34344	0.25840	0.14955	0.75971	0.28069	0.46579	0.61779	1.00000
	0.1382	0.2713	0.5292	0.0001	0.2306	0.0385	0.0037	

--- group=4 ---

Simple Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
day0	20	72.20500	4.31173	1444	64.40000	80.80000
day2	20	72.49500	3.15436	1450	67.80000	77.60000
day4	20	73.58500	4.18422	1472	66.10000	83.50000
day6	20	74.24500	4.77510	1485	66.60000	84.90000
day8	20	75.55000	4.48500	1511	68.60000	84.60000
day10	20	76.61500	5.35206	1532	67.60000	88.40000
day12	20	78.06500	5.09554	1561	67.30000	86.40000
day14	20	77.12000	5.34293	1542	69.40000	88.60000

Pearson Correlation Coefficients, N = 20
Prob > |r| under H0: Rho

	day0	day2	day4	day6	day8	day10	day12	day14
day0	1.00000	0.71006	0.57958	0.17837	0.41025	0.53072	0.54294	0.61127
		0.0005	0.0074	0.4518	0.0724	0.0161	0.0134	0.0042
day2	0.71006	1.00000	0.81986	0.43844	0.60940	0.68032	0.51743	0.71243
	0.0005		<.0001	0.0531	0.0043	0.0010	0.0195	0.0004
day4	0.57958	0.81986	1.00000	0.43853	0.61582	0.58172	0.55900	0.54792
	0.0074	<.0001		0.0531	0.0038	0.0071	0.0104	0.0124
day6	0.17837	0.43844	0.43853	1.00000	0.41197	0.45609	0.31560	0.34567
	0.4518	0.0531	0.0531		0.0711	0.0433	0.1753	0.1355
day8	0.41025	0.60940	0.61582	0.41197	1.00000	0.68700	0.59515	0.65908
	0.0724	0.0043	0.0038	0.0711		0.0008	0.0056	0.0016
day10	0.53072	0.68032	0.58172	0.45609	0.68700	1.00000	0.68701	0.77865
	0.0161	0.0010	0.0071	0.0433	0.0008		0.0008	<.0001
day12	0.54294	0.51743	0.55900	0.31560	0.59515	0.68701	1.00000	0.59572
	0.0134	0.0195	0.0104	0.1753	0.0056	0.0008		0.0056
day14	0.61127	0.71243	0.54792	0.34567	0.65908	0.77865	0.59572	1.00000
	0.0042	0.0004	0.0124	0.1355	0.0016	<.0001	0.0056	

UNIVARIATE REPEATED MEASURES ANOVA 1

The GLM Procedure
Class Level Information

Class	Levels	Values
group	4	1 2 3 4

Number of Observations Read	80
Number of Observations Used	80

day_N represents the nth degree polynomial contrast for day

M Matrix Describing Transformed Variables

	day0	day2	day4	day6
day_1	-.5400617249	-.3857583749	-.2314550249	-.0771516750
day_2	0.5400617249	0.0771516750	-.2314550249	-.3857583749
day_3	-.4308202184	0.3077287274	0.4308202184	0.1846372365
day_4	0.2820380374	-.5237849266	-.1208734446	0.3626203338
day_5	-.1497861724	0.4921545664	-.3637664186	-.3209703694
day_6	0.0615457455	-.3077287274	0.5539117094	-.3077287274
day_7	-.0170697185	0.1194880298	-.3584640895	0.5974401492

day8	day10	day12	day14

day_1	0.0771516750	0.2314550249	0.3857583749	0.5400617249
day_2	-.3857583749	-.2314550249	0.0771516750	0.5400617249
day_3	-.1846372365	-.4308202184	-.3077287274	0.4308202184
day_4	0.3626203338	-.1208734446	-.5237849266	0.2820380374
day_5	0.3209703694	0.3637664186	-.4921545664	0.1497861724
day_6	-.3077287274	0.5539117094	-.3077287274	0.0615457455
day_7	-.5974401492	0.3584640895	-.1194880298	0.0170697185

Sphericity Tests

Variables	DF	Mauchly's Criterion	Chi-Square	Pr > ChiSq
Transformed Variates	27	0.5640592	42.05855	0.0325
Orthogonal Components	27	0.5640592	42.05855	0.0325

Tests of Hypotheses for Between Subjects Effects

Source	DF	Type III SS	Mean Square	F Value	Pr > F
group	3	1315.217125	438.405708	4.88	0.0037
Error	76	6826.570125	89.823291		

Univariate Tests of Hypotheses for Within Subject Effects

Source	DF	Type III SS	Mean Square	F Value	Pr > F	Adj Pr > F G - G	Adj Pr > F H - F
day	7	1461.931250	208.847321	22.38	<.0001	<.0001	<.0001
day*group	21	896.264375	42.679256	4.57	<.0001	<.0001	<.0001
Error(day)	532	4965.126875	9.332945				

Analysis of Variance of Contrast Variables

day_N represents the nth degree polynomial contrast for day

Contrast Variable: day_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	1373.313574	1373.313574	99.46	<.0001
group	3	652.449789	217.483263	15.75	<.0001
Error	76	1049.355327	13.807307		

Contrast Variable: day_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	1.1183601	1.1183601	0.12	0.7275
group	3	57.0557768	19.0185923	2.08	0.1098
Error	76	694.8464583	9.1427166		

Contrast Variable: day_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	9.9841212	9.9841212	1.12	0.2942
group	3	47.5357462	15.8452487	1.77	0.1599
Error	76	680.1291477	8.9490677		

Contrast Variable: day_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	2.0961437	2.0961437	0.22	0.6402
group	3	25.3594619	8.4531540	0.89	0.4511
Error	76	723.1927386	9.5156939		

Contrast Variable: day_5

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	59.1220009	59.1220009	8.32	0.0051
group	3	67.8367770	22.6122590	3.18	0.0287
Error	76	540.2486854	7.1085353		

Contrast Variable: day_6

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	4.0703712	4.0703712	0.57	0.4513
group	3	26.7600114	8.9200038	1.26	0.2955
Error	76	539.6831780	7.1010944		

Contrast Variable: day_7

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	12.2266785	12.2266785	1.26	0.2652
group	3	19.2668131	6.4222710	0.66	0.5782
Error	76	737.6713395	9.7062018		

UNIVARIATE REPEATED MEASURES ANOVA 2

(Note: all of the output will be the same as above except for the following:)

day_N represents the contrast between the nth level of day and the mean of subsequent levels

M Matrix Describing Transformed Variables

	day0	day2	day4	day6
day_1	1.000000000	-0.142857143	-0.142857143	-0.142857143
day_2	0.000000000	1.000000000	-0.166666667	-0.166666667
day_3	0.000000000	0.000000000	1.000000000	-0.200000000
day_4	0.000000000	0.000000000	0.000000000	1.000000000
day_5	0.000000000	0.000000000	0.000000000	0.000000000
day_6	0.000000000	0.000000000	0.000000000	0.000000000
day_7	0.000000000	0.000000000	0.000000000	0.000000000

	day8	day10	day12	day14
day_1	-0.142857143	-0.142857143	-0.142857143	-0.142857143
day_2	-0.166666667	-0.166666667	-0.166666667	-0.166666667
day_3	-0.200000000	-0.200000000	-0.200000000	-0.200000000
day_4	-0.250000000	-0.250000000	-0.250000000	-0.250000000
day_5	1.000000000	-0.333333333	-0.333333333	-0.333333333
day_6	0.000000000	1.000000000	-0.500000000	-0.500000000
day_7	0.000000000	0.000000000	1.000000000	-1.000000000

Analysis of Variance of Contrast Variables

day_N represents the contrast between the nth level of day and the mean of subsequent levels

Contrast Variable: day_1

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	554.2540816	554.2540816	57.88	<.0001
group	3	288.4638571	96.1546190	10.04	<.0001
Error	76	727.7894082	9.5761764		

Contrast Variable: day_2

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	297.6704201	297.6704201	30.44	<.0001
group	3	271.5622049	90.5207350	9.26	<.0001
Error	76	743.1315417	9.7780466		

Contrast Variable: day_3

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	339.4056050	339.4056050	28.65	<.0001
group	3	184.1183350	61.3727783	5.18	0.0026
Error	76	900.2628600	11.8455639		

Contrast Variable: day_4

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	463.082820	463.082820	28.54	<.0001
group	3	62.104586	20.701529	1.28	0.2887
Error	76	1232.948219	16.223003		

Contrast Variable: day_5

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	54.1753472	54.1753472	4.30	0.0414
group	3	121.9551528	40.6517176	3.23	0.0270
Error	76	956.6172778	12.5870694		

Contrast Variable: day_6

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	5.7781250	5.7781250	0.51	0.4784
group	3	63.5271250	21.1757083	1.86	0.1435
Error	76	865.1147500	11.3830888		

Contrast Variable: day_7

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Mean	1	48.050000	48.050000	2.76	0.1005
group	3	148.315000	49.438333	2.84	0.0432
Error	76	1321.115000	17.383092		

6. Consider the weight-training data in Problem 5. One of the investigators felt that whether or not a man is middle-aged (defined as age 45 years or greater) may play a role in the strength men exhibit from regular weight training on average and how mean strength might increase if men modify their weight training programs. He suggested that the team consider a different statistical model. Letting Y_{ij} = strength measurement for man i at time t_{ij} and defining

$$a_i = 0 \text{ if man } i < 45 \text{ and } a_i = 1 \text{ if man } i \geq 45$$

(thus, $a_i = 0$ corresponds to "younger," and $a_i = 1$ corresponds to "middle-aged"), he proposed the model

$$\begin{aligned} Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{11} + \beta_{11a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 1} \\ Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{12} + \beta_{12a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 2} \\ Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{13} + \beta_{13a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 3} \\ Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{14} + \beta_{14a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 4} \end{aligned} \tag{3}$$

Define

$$\boldsymbol{\beta} = (\beta_0, \beta_{01a}, \beta_{02a}, \beta_{03a}, \beta_{04a}, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{11a}, \beta_{12a}, \beta_{13a}, \beta_{14a})'$$

[5 points] (a) The investigators first fit this model several times, each time assuming something different about the ϵ_{ij}. The basic code they used looked like

```
proc mixed data=strength1;
  class group subject;
  model strength = group group*age group*day group*age*day/ noint solution;
  repeated / STUFF;
run;
```

Here, `STUFF` looked different depending on the assumption on the ϵ_{ij}. In the following table, *AIC* and *BIC* values resulting from taking `STUFF` to be several different things are given:

STUFF	*AIC*	*BIC*
`type=un subject=subject r rcorr;`	3418.0	3503.7
`type=un subject=subject group=group r rcorr;`	3490.4	3833.4
`type=cs subject=subject r rcorr;`	3406.6	3411.4
`type=cs subject=subject group=group r rcorr;`	3392.5	3411.6
`type=csh subject=subject r rcorr;`	3405.6	3427.1
`type=csh subject=subject group=group r rcorr;`	3429.9	3515.6
`type=ar(1) subject=subject r rcorr;`	3430.9	3435.7
`type=ar(1) subject=subject group=group r rcorr;`	3433.2	3452.3

Based on these results, state the particular covariance model among those considered by the investigators that you would assume for this analysis and explain why you chose it, citing specific information here and in Problem 5 that you may have used to decide. (Give the *name* of the model, not just the SAS `type` designation.)

As is often the case in real life, the two information criteria do not agree: AIC prefers a homogeneous compound symmetric model that is different in each group, while BIC prefers (just barely) a common homogeneous compound symmetric model. So this evidence is suggesting a compound symmetric structure, but whether it is common to groups is in question. The test for Sphericity using the mean model in problem 5 rejected common Type H, and we noted that there were some dissimilarities across groups in the sample variances/correlations. I would probably go with the assumption of homogeneous compound symmetry different in each group, but it's really a toss-up.

Parts (b)–(e) of this problem on the next pages.

For convenience, here is the statistical model again:

$$
\begin{aligned}
Y_{ij} &= \beta_0 + \beta_{0a}a_i + (\beta_{11} + \beta_{11a}a_i)t_{ij} + \epsilon_{ij}, \quad i \text{ in group } 1 \\
Y_{ij} &= \beta_0 + \beta_{0a}a_i + (\beta_{12} + \beta_{12a}a_i)t_{ij} + \epsilon_{ij}, \quad i \text{ in group } 2 \\
Y_{ij} &= \beta_0 + \beta_{0a}a_i + (\beta_{13} + \beta_{13a}a_i)t_{ij} + \epsilon_{ij}, \quad i \text{ in group } 3 \\
Y_{ij} &= \beta_0 + \beta_{0a}a_i + (\beta_{14} + \beta_{14a}a_i)t_{ij} + \epsilon_{ij}, \quad i \text{ in group } 4
\end{aligned}
$$

$$\boldsymbol{\beta} = (\beta_0, \beta_{01a}, \beta_{02a}, \beta_{03a}, \beta_{04a}, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{11a}, \beta_{12a}, \beta_{13a}, \beta_{14a})'$$

The investigators assumed one of the covariance models. On page 18 you will find the code they ran and excerpts of its output; use this information to answer the following:

[5 points] (b) Based on this model, is there evidence to suggest that the mean strength of men who follow a weight-training program regularly differs between middle-aged men and "younger" men, prior to the men adopting any modification to their programs? From the output, give the value of a test statistic and associated p-value appropriate for testing the relevant null hypothesis H_0, and state your conclusion regarding the strength of the evidence against H_0. (You need not state H_0.)

The question is whether $\beta_{0a} = 0$. This is addressed by contrast B, which yields a very very small p-value. There is very strong evidence that the mean strength differs between middle-aged and younger men.

In fact, the estimated mean strength for younger men is 74.99 from the `Solution for Fixed Effects`. That for middle-aged men is $74.99+(-5.06) = 69.96$, suggesting that this is because middle-aged men aren't as strong as their younger counterparts (even if they work out with weights!)

[5 points] (c) Based on this model, is there evidence to suggest that the pattern of change of mean strength over the time period of the study differs between middle-aged men and "younger" men in at least one of the groups? From the output, give the value of a test statistic and associated p-value appropriate for testing the relevant null hypothesis H_0, and state your conclusion regarding the strength of the evidence against H_0. (You need not state H_0.)

The pattern of mean change (rate of change) in group $k = 1, 2, 3, 4$ is $\beta_{1k} + \beta_{1ka}a_i$), and $\beta_{1ka} = 0$ means that in group k there is no difference in rate of change between the two types of men for that group. The null hypothesis is that there is no such difference in any of the groups vs. the alternative there is such a difference in one of the groups. The null is thus $\beta_{11a} = \beta_{12a} = \beta_{13a} = \beta_{14a} = 0$. This is addressed by contrast D, for which the p-value is 0.188. There is not enough evidence to suggest that the rate of change differs between the two types of men for at least one of the groups.

Parts (d)–(f) of this problem on the next page.

For convenience, here is the statistical model again:

$$\begin{aligned}
Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{11} + \beta_{11a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 1} \\
Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{12} + \beta_{12a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 2} \\
Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{13} + \beta_{13a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 3} \\
Y_{ij} &= \beta_0 + \beta_{0a} a_i + (\beta_{14} + \beta_{14a} a_i) t_{ij} + \epsilon_{ij}, \quad i \text{ in group 4}
\end{aligned}$$

$$\boldsymbol{\beta} = (\beta_0, \beta_{01a}, \beta_{02a}, \beta_{03a}, \beta_{04a}, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{11a}, \beta_{12a}, \beta_{13a}, \beta_{14a})'$$

[5 points] (d) Regardless of your answer to (c), is there evidence that the difference between the rate of change of mean strength for middle-aged men and the rate of change of mean strength for "younger" men is not the same in all groups? From the output, give the value of a test statistic and associated p-value appropriate for testing the relevant null hypothesis H_0, and state your conclusion regarding the strength of the evidence against H_0. (You need not state H_0.)

The rate of change for younger men in group k is β_{1k} and that for middle-aged men is $\beta_{1k} + \beta_{1ka}$). If the difference in rate of change between the two types of men were the same for all groups, then the β_{1ka}, $k = 1, 2, 3, 4$, which represent these differences for each group, would have to be the same, i.e., $\beta_{11a} = \beta_{12a} = \beta_{13a} = \beta_{14a}$. This is addressed by contrast E. The corresponding p-value is 0.124. There is not enough evidence to suggest that this difference in rate of change is different for at least one of these groups.

[5 points] (e) Based on the fit, give an estimate of the rate of change of mean strength for "younger" men who increased repetitions but did not increase the amount of weight used (group 2).

We are interested in the estimate of β_{12}. We can read this right off of the `Solution for Fixed Effects` table - it is given by the `day*group` effect for `group 2`. The estimate is 0.3298 strength units/day.

[5 points] (f) One of the investigators fit the model under both the assumption that all strength measurements from all men are mutually independent (so using ordinary least squares) and using `proc mixed` as on the next page with `INVESTIGATORS' STUFF` equal to

```
type=cs subject=subject r rcorr;
```

He noticed that the estimates of the elements of $\boldsymbol{\beta}$ from both fits were *identical*. Thus, he suggested to his colleagues that they just base all of their inferences on the ordinary least squares results.

Is this a good idea? Why or why not?

NO! Even though the estimates from OLS are identical to those from the fancier longitudinal data model, the estimated standard errors for them under the OLS analysis will not take into account the correlation in the data, because OLS assumes all observations are mutually independent. As a result, these estimated standard errors will misrepresent the true sampling variation! Test and confidence intervals will be misleading if based on these standard errors and the investigators could thus be led to erroneous conclusions. The longitudinal analysis, on the other hand, takes account of the correlation, so that standard errors based on it give a more realistic picture of the sampling variation.

A number of you noted that this is not a good idea because OLS does not take into account the correlation expected in longitudinal data, but you did not explain why that is a bad thing. Given that the estimates from OLS and the longitudinal analysis are the same, it would be important to explain to a (non-statistician) investigator why this is undesirable; otherwise, he is unlikely to care!

```
data strength1;
  infile "strength.dat";
  input subject day strength age group;
run;
proc mixed data=strength1;
  class group subject;
  model strength = age group*day group*age*day/ solution;
  repeated / INVESTIGATORS' STUFF;
  contrast 'A' intercept 1 age 1 / chisq;
  contrast 'B' intercept 0 age 1 / chisq;
  contrast 'C' group*day 1  0  0  0  group*age*day 0  0  0  0,
               group*day 0  1  0  0  group*age*day 0  0  0  0,
               group*day 0  0  1  0  group*age*day 0  0  0  0,
               group*day 0  0  0  1  group*age*day 0  0  0  0 / chisq;
  contrast 'D' group*day 0  0  0  0  group*age*day 1  0  0  0,
               group*day 0  0  0  0  group*age*day 0  1  0  0,
               group*day 0  0  0  0  group*age*day 0  0  1  0,
               group*day 0  0  0  0  group*age*day 0  0  0  1 / chisq;
  contrast 'E' group*day 0  0  0  0  group*age*day 1 -1  0  0,
               group*day 0  0  0  0  group*age*day 1  0 -1  0,
               group*day 0  0  0  0  group*age*day 1  0  0 -1 / chisq;
  contrast 'F' group*day 1 -1  0  0  group*age*day 0  0  0  0,
               group*day 1  0 -1  0  group*age*day 0  0  0  0,
               group*day 1  0  0 -1  group*age*day 0  0  0  0 / chisq;
  contrast 'G' group*day 1 -1  0  0  group*age*day 1 -1  0  0,
               group*day 1  0 -1  0  group*age*day 1  0 -1  0,
               group*day 1  0  0 -1  group*age*day 1  0  0 -1 / chisq;
run;
```

Solution for Fixed Effects

Effect	group	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept		74.9985	0.4160	78	180.29	<.0001
age		-5.0646	0.6793	78	-7.46	<.0001
day*group	1	-0.09148	0.05757	552	-1.59	0.1126
day*group	2	0.3298	0.05571	552	5.92	<.0001
day*group	3	0.5617	0.05405	552	10.39	<.0001
day*group	4	0.4255	0.07184	552	5.92	<.0001
age*day*group	1	0.2181	0.09697	552	2.25	0.0249
age*day*group	2	-0.03868	0.1005	552	-0.38	0.7006
age*day*group	3	-0.07384	0.1058	552	-0.70	0.4857
age*day*group	4	0.01147	0.09467	552	0.12	0.9036

Contrasts

Label	Num DF	Den DF	Chi-Square	F Value	Pr > ChiSq	Pr > F
A	1	78	16957.3	16957.3	<.0001	<.0001
B	1	78	55.58	55.58	<.0001	<.0001
C	4	552	164.68	41.17	<.0001	<.0001
D	4	552	6.18	1.54	0.1862	0.1878
E	3	552	5.79	1.93	0.1222	0.1236
F	3	552	82.20	27.40	<.0001	<.0001
G	3	552	14.45	4.82	0.0024	0.0026