# BIOSTATISTICS 667 (Fall 2019)
## Homework 4

1. Suppose that the random variable $Y$ is distributed as Bernoulli with mean $\mu$, $0 < \mu < 1$,

$$P(Y = 1) = \mu = 1 - P(Y = 0).$$

Show that this distribution belongs to the exponential family of distributions. Identify the canonical parameter, the canonical statistic, the canonical link function, the cumulant function and the variance function. Use the notation $\theta$ for the canonical parameter and $V(\mu)$ for the variance function. Develop an expression for the deviance.

2. Suppose that $Y = (Y_1, \cdots, Y_n)^\top$ is a response vector with $E[Y] = X\beta, \text{var}(Y_i) = 1, i = 1, \cdots, n$, and $\text{cov}(Y_i, Y_j) = \rho, i \neq j$, $-1/(n-1) < \rho < 1$. Let $\Sigma$ denote the covariance matrix of $Y$. The $n \times p$ design matrix $X$ contains the intercept and $p - 1$ covariates and has full column rank (the rank is $p$, and $p \leq n$). The parameter $\rho$ is known.

Show that the ordinary least-squares and the weighted least-squares estimators of $\beta$ are identical. The OLS estimator is

$$\hat{\beta}_{OLS} = (X^\top X)^{-1} X^\top Y,$$

while the WLS estimator is

$$\hat{\beta}_{WLS} = (X^\top W X)^{-1} X^\top W Y,$$

where $W = \Sigma^{-1}$.

Hints: $\Sigma = aI + bJ$ where $I$ is the identity matrix and $J$ is a matrix of 1's, both $n \times n$. Further, $W = cI + dJ$ where $a, b, c, d$ are certain constants. The problem can be solved in less than half a page and *without* finding explicit expressions for the two estimators.

3. The last two parts of 11.2: TBA