Problem 1

(a)

$$\text{Given } Y|X \sim N(X,X) \quad X \sim U(0,1)$$
$$E(Y) = E(E(Y|X)) = E(X) = 1/2$$
$$Var(Y) = Var(E(Y|X)) + E(Var(Y|X)) = Var(X) + E(X) = 1/12 + 1/2 = 7/12$$
$$Cov(Y,X) = E(XY) - E(X)E(Y) = E(E(XY|X)) - 1/4$$
$$E(E(XY|X)) = E(XE(Y|X)) = E(X^2) = Var(X) + E(X)^2 = 1/12 + 1/4$$
$$Cov(Y,X) = 1/12 + 1/4 - 1/4 = 1/12$$

(b)

$$Cov(Y-X,X) = E((Y-X)X) - E(Y-X)E(X) = E(YX-X^2) - [E(Y)-E(X)](1/2)$$
$$= E(YX) - E(X^2) - (1/2 - 1/2)(1/2) = E(YX) - (1/2 + 1/4) - 0$$
$$E(YX) = E(E(XY|X)) = 1/12 + 1/4 \text{(from part a)}$$
$$Cov(Y-X,X) = (1/12 + 1/4) - (1/12 + 1/4) = 0$$
$$\text{Cov=0 does not imply independence unless jointly normal}$$
$$\text{Let } T = Y - X$$
$$P(T|X) = P(T) \text{ for all } (t,x) \text{ if independent}$$
$$P(T|X = 0 = y) = 1$$
$$P(T = y) \neq 1$$
$$\text{Thus T depends on X for some } (t,x)$$
$$\text{Therefore T and X are not independent}$$
$$\text{Thus} Y - X \text{ and } X \text{are not independent}$$

Problem 2

(a)

(b)

(c)

Problem 3

(a)

(b)

(c)

Problem 4

(a)

(b)

(c)

(d)

Problem 5

(a)

(b)

(c)

(d)