Question 1

2015, MS-2

① 50 SIDS - 1 exp.
50 control - 8 exp.

a) While controls were matched on age and that is a good start, addl factors such as SES, educ, etc. could confound the relationship under examination and would need to be controlled for.
(so, relatively appropriate but could be better) DON'T HAVE to be mothers

b)

	exp	un	
case	1	49	50
control	8	42	50

$OR = \frac{ad}{bc} = \frac{42}{8(49)}$

$\ln(OR) \pm 1.96\,(SE\ln(OR))$

$SE\ln(OR) = \sqrt{\frac{1}{1} + \frac{1}{49} - \frac{1}{8} + \frac{1}{42}}$

$H_0: OR = 1$

c) From above, the OR is a measure of strength of association

For b), use X^2 test $\Rightarrow \sum \frac{(O-E)^2}{E}$ compare to c.v. df = 1

d) $\frac{\text{total who use}}{\text{total } n} = \frac{8}{50} = 0.16$ ~~= 0.09~~ CI for proportion = $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$

↑ use controls b/c representative

e) Still pretty good, but could be better (non have to be moms → good)

f) No ⇒ we would how many of each pair were exp/exp etc.

(probably)

i) now cohort study

p illicit drugs = $\frac{\#\text{ illicit}}{\text{total}} = \frac{(7+1008)}{(7+21+1008+3720)}$ $\quad p \pm z_{1-\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}$

+ CI formula

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ii) RR (or OR)

$= \left(\frac{7/(7+1008)}{21/(21+3720)}\right)$

+ CI formula

$SE(\ln(OR)) = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{21}} + \frac{1}{n_{12}} + \frac{1}{n_{22}}}$

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iii) Results could have been affected by confounding (self-report, confounding, bias in women sampled, recall bias)

2015 exam review

① cases = 50 women w/ SIDS babies (1 used)
controls = 50 women in same age range (8 used)

a) choice of controls?

Good they were matched on age, but the lack of babies is troubling as mothers and non-mothers may not have the same behavior patterns. It would also be good to match on other possible confounding vars like geography etc.

b)

	drug ~~SIDS~~	no drug ~~non-SIDS~~	
cases	1	49	50
controls	8	42	50
			100

⇒ measure of assoc = OR w/ CI, or chi-square test

c) OR for strength of assoc.

d) use controls only. calculate 95% CI for p

e) much better! ← matched case-control

f) no - because they were individually matched, we need the full data on the pairs to determine the number concordant etc.

g) new cohort
use all women

h) RR for ~~risk SIDS | drug~~ / risk SIDS | no drug

i)

Question 2

2015, MS-2

Type 1 → pancreas unable to produce insulin to absorb glucose (LESS insulin)
Type 2 → cells don't respond to insulin (MORE insulin)

Infant feeding / breastfeeding can be protective.

children 10-13 with Type 1 and 2 diabetes *** does the data exclusively contain diabetics ***

glycemic control → HbA1C → % ⇒ HIGHER = WORSE
LOWER = BETTER

Questions

1) Longer breastfeeding associated w/ better control (LOWER Hb)
2) Relationship the same for T1 and T2 diabetics

a) $\vec{y} = \vec{X}\vec{\beta} + \vec{\varepsilon}$

$\vec{y}$ = n x 1 vector of HbA1C %'s
$\vec{X}$ = n x 4 matrix
$\vec{\beta}$ = 4 x 1 matrix of coefficients
$\vec{\varepsilon}$ = n x 1 vector of random errors

$X = [$ interept | breast | T2_ind | interatn
β_0 | β_1 | β_2 | β_3

make a model based on Q then use it for next Q's.

---

b)

i. Type of diabetes unrelated to HbA1C

$H_0: \beta_2 = \beta_3 = 0$

$\Rightarrow C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\theta_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

test both ME and interaction

ii. Breastfeeding is unrelated to HbA1C

$H_0: \beta_1 = \beta_3 = 0$

$\Rightarrow C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\theta_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

same

b) continued. $6 * \beta_1 = 0.5$

iii. In children breastfed 6 months HbA1C is 0.5 higher in children w/ T2 than T1 diab $\rightarrow \beta_2 * 1$

H_0: $\underbrace{\beta_0 + \beta_1(6)}_{\text{T1 Hb}} = \underbrace{\beta_0 + \beta_1(6) + \beta_2(1) + \beta_3(6)}_{\text{T2 Hb}}$

T2 avg = 0.5 times ~~(6 months times the T1 average)~~

$\Rightarrow H_0: \beta_2 + 6\beta_3 = 0.5$

$C = (0\ 0\ 1\ 6) \quad \theta_0 = 0.5$

ask for clarification on problems like this in exam.

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iv. Effect of diabetes type does not depend on # ~~months~~ of breastfeeding.

$\Rightarrow$ no interaction

$H_0: \beta_3 = 0$

$C = (0\ 0\ 0\ 1) \quad \theta_0 = 0$

---

v. Children w/ T1 who were not breastfed have Hb that is no diff. from kids w/ T1 who were breastfed for 3 months.

$H_0: \beta_0 + \beta_1(0) = \beta_0 + \beta_1(3)$

$\Rightarrow H_0: \beta_1 * 3 = 0$

$C = (0\ 3\ 0\ 0) \quad \theta_0 = 0$

c) Hb is lower in T II diab.

$$(X'X) = \begin{pmatrix} 67 & 105 \\ 105 & 113 \end{pmatrix} \quad \begin{matrix} n = 67 \\ \Sigma X_i \\ \Sigma X_i^2 \end{matrix} \qquad X = \begin{pmatrix} 1 & \# \\ \vdots & \# \\ 1 & \# \end{pmatrix}$$

$(X'y) = 766.2$

$CSS = 278.385$

$SSE = 277.999$

i. Compute the L.S. Est.

$(x'x)^{-1}(x'y) \Rightarrow$ do in R

ii. Test whether there is a linear assoc.

$H_0: \beta_1 = 0$ (no assoc)

either use $C = (0\ 1)$ $\theta_0 = 0$

or Wald: $\dfrac{\hat{\beta}_1}{\sqrt{\text{Var}\beta_1}} \sim t_{df\,error}$

$\sqrt{\text{Var}\beta_1}$ ↗ $\left\{\sigma^2 (x'x)^{-1}\right\}_{11}$

iii. compute a 95% prediction interval for someone (some T2 diab. child) breasfed = 6

$X_f = (1\ 6)$

then follow formulas.

Question 3 | 2015, MS-2

6456 men/women aged 65-89 years
diagnosed on a continuum:

- cognitively normal = N
- mildly impaired } 'cognitive impairment' = M
- dementia = D

race ⇒ B/W (ind.)
gender ⇒ F/M (ind.)
age ⇒ yrs (cont.)

cogimp = 1 if M or D
dementia = 1 if D
mci = 1 if M
} 0 otherwise

---

a) Write down the algebraic expression for modeling the prevalence (PERCENTAGE) of cognitive impairment as a function of race, gender, age. Assume ME only.

cogimp ~ Bern(p) p = prev. of cognitive impairment

$E(\text{cogimp}) = \beta_0 + \beta_1(\text{RACE}) + \beta_2(\text{GENDER}) + \beta_3(\text{AGE})$

β_0 = probability/prevalence of cognitive impairment for an individual who is white, male, and 0 years old

β_1 = inc/dec (change) in log odds of cogimp for individuals who are black

β_2 = inc/dec (change) in log odds of cogimp for individuals who are female

β_3 = inc/dec in cogimp with every one additional year of age.

b) Modify the model to test the effect that the effect of race differs between men and women.

$E(\text{cogimp}) = \beta_0 + \beta_1 \text{race} + \beta_2 \text{gender} + \beta_3 \text{age} + \beta_4 \text{race} \times \text{gender}$

β_4 = the additional (above + beyond the M.E.'s) change in log odds, for a black female of cogimp.

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c) Now we care about the prevalence of dementia.
hog. Reg. run in SAS.
Write algebraic expression for this model

$E(\text{dementia}) = \beta_0 + \beta_1(\text{race}) + \beta_2(\text{gender}) + \beta_3(\text{age})$

Prob dementia = 1 ⇒ prev. dementia

race: ⇓ =1 if black

gender: ⇓ =1 if female

age: ⇓ years

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d) prob. dementia for black, male, 85 years

$X = (1 \;\; 1 \;\; 0 \;\; 85)$

$$p = \frac{\exp(X\beta)}{1 + \exp(X\beta)} = \frac{\exp(-14.5105 + 1.090 + 0.14776(85))}{1 + \text{same}} = 0.2943$$

The probability of an 85 year old black male having dementia (being diagnosed w/ dementia) is 0.2943 (29.43%).

e) Calculate measures of association and 95% CI's for the effects of race, gender, and 5 year ↑ in prev. Summarize in 2-3 sentences.

i. race: $OR = \exp(1.0890) = 2.97$

$95\%\ CI = \exp(1.0890 \pm 1.96(0.11901)) = (2.3532, 3.7518)$

ii. gender: $OR = \exp(-0.1670) = 0.8462$

$95\% = \exp(-0.1670 \pm 1.96(0.1166)) = (0.6733, 1.0634)$

iii. age: $OR = \exp(0.1476 * 5)$ SE*5 $= 2.0917$

$95\% = \exp(0.1476*5 \pm 1.96(0.0107)) = (2.0483, 2.1361)$

⇒ higher odds of having dementia if black (significant)
slightly lower odds if female (but N.S.)
higher odds w/ a 5 year ↑ in age (significant, quite precise).

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f) Now interested in the prevalence of MCI. Can the same model be used? Describe/justify whatever answer.

Technically it can, although it does not make much sense. The non-MCI group includes normal cognition AND dementia diagnoses. Perhaps better to model prev. of MCI among pts. w/o dementia, i.e. restrict the sample.

Question 4

2015, MS-2
Some answers.
* checked w/ SAS!

Twenty rats @ 4 doses (5 rats/dose)

doses = 0, 1, 2, 3 mg/day.

d_i = dose i indicator

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a) First linear model: $\alpha_0 d_0 + \alpha_1 d_1 + \alpha_2 d_2 + \alpha_3 d_3$ ⇒ CELL MEANS NO INT.

Interpret the estimate of α_3 and its SE. corresponding p-val and relevance.

α_3 = average weight gain for rats on dose 3 mg/day.

SE = sqrt(MSE * $(X'X)^{-1}_{33}$)

↳ variance is the MSE divided by the group sample (how much variance is contributed by that group)

SE is sqrt(variance)

↳ SE $\bar{X} = \sqrt{\frac{\sigma^2}{n}}$ $\quad Var(\bar{X}) = \frac{\sigma^2}{n}$

$$\left(\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}\right)^{-1}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

p-val = < 0.0001

⇒ The estimate (161.40) is significantly different from 0. ← mean.

Not super relevant.

b) Now consider $\mu + \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 d_3$

Interpret γ_1

Estimate γ_1 and its SE

γ_1 = difference in mean wt gain of rats on dose 1 compared to rats on dose = 0 mg/day (μ)

estimate = $\alpha_1 - \alpha_0 = 18$ mg/day

SE = $\sqrt{\sigma^2(\frac{2}{5})} = 4.6411$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1 \end{pmatrix}$$

---

c) Test the null that the 4 groups have the same mean wt gain.

Report stat, distn, p-val, results

$$\begin{pmatrix} 1/n & -1/n & -1/n & -1/n \\ -1/n & 2/n & 1/n & 1/n \\ -1/n & 1/n & 2/n & 1/n \\ -1/n & 1/n & 1/n & 2/n \end{pmatrix}$$

$H_0: \alpha_0 = \alpha_1 = \alpha_2 = \alpha_3$ (this one)

or $H_0: \gamma_1 = \gamma_2 = \gamma_3 = 0$

$$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \quad \theta_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$F = \frac{\{(\hat{\theta} - \theta_0)' M^{-1} (\hat{\theta} - \theta_0)\} / \text{rank}(c)}{MSE}$$

$F \sim 3, 16$ (95% critical value 4.07)

$F = 60.27979$

$\Rightarrow$ very significant.

evidence the mean wt gains are not the same.

$$(X'X)^{-1} = \begin{pmatrix} 1/5 & & & 0 \\ & 1/5 & & \\ & & 1/5 & \\ 0 & & & 1/5 \end{pmatrix}$$

d) $\beta_0 + \beta_1(\text{dose})$ dose = 0, 1, 2, 3 (continuous)

β_1 = expected ↑ in wt. gain for every 1 mg increase in dose.

p-val = $< 0.0001 \Rightarrow$ slope is significantly diff. from 0.
(there is a relationship)

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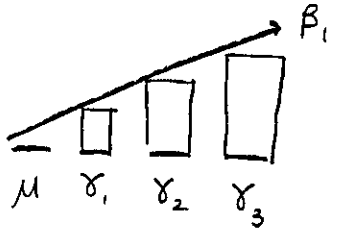
e) Using the replicates (5 rats/dose), test the linearity assumption from d). Report test stat, null distn, p-val, interpretation.

H_0: $3\gamma_1 = \gamma_3$
$2\gamma_1 = \gamma_2$

$\Rightarrow C = \begin{pmatrix} 0 & 3 & 0 & -1 \\ 2 & 2 & -1 & 0 \end{pmatrix}$ $\theta_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$



$\sim F_{2,16}$

$= 0.1508$

p-val = 0.2971

$(X'X)^{-1}$ from reg. coding.

$\Rightarrow$ at the 0.05 level, we fail to reject the null hypothesis.
There is no evidence d) violates the linearity assumption.

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f) brief explanation

- rats at higher doses gain more weight
relationship is linear → i.e. if you're given 2x the dose, you are expected to gain 2x the weight.

g) only 4 rats total in the experiment
one rat at each dose, and the data given are monthly wt gains over 5 months.

Cumulative wt gain? or use the values new weight each month?

smaller sample size
could still run the same way.
Just be cautious.

glycemic control = feeding prac. + TI/II + TI/II * feed

↑ higher is worse

↑ longer is prob better

↑ ind. Var. = 1 if TII

a) $\vec{y} = \vec{X}\vec{\beta} + \vec{\varepsilon}$

y = n x 1 = vector of HbA1c %

X = n x 4 = ~~matrix~~ of predictor vars (int, duration, diab, dur*diab

β = 4 x 1 = ~~matrix~~ of param est $(\beta_0\ \beta_1)'$

ε = n x 1 = vector of random error

where n = sample size

β_0 = intercept
β_1 = duration breast
β_2 = diabetes (ind var)
β_3 = interaction

b) Provide C + θ_0 matrices for testing the following hyp.

$H_0: C\beta = \theta_0$

i. $C = (0\ 0\ 1\ 0)$ $\theta_0 = 0$ ($\beta_D = 0$ = no relationship)

ii. $C = (0\ 1\ 0\ 0)$ $\theta_0 = 0$ ($\beta_B = 0$ = no relation)

iii. $(\beta_0 + 6\beta_B + \beta_{TII} + \beta_{int})0.5 = (\beta_0 + 6\beta_D)$

$(\beta_0 + 6\beta_B + \beta_{TII} + \beta_{int})$ HBA1c in TII

0.5 half as high

$(\beta_0 + 6\beta_D)$ HbA1c in TI

$-0.5\beta_0 - 3\beta_B + \beta_{TII} + \beta_{int} = 0$

$C = (-0.5\ -3\ 1\ 1) = 0 = \theta_0$

The question is poorly written. makes sense it should be 1.5x higher.
↳ but c makes it seem right sooo...

IV. $C = (0\ 0\ 0\ 1)$ $\theta_0 = 0$ (diabetes does not depend on length of breastfeeding)

(no int. effect)

V. $\beta_{0,} + 0\beta_B = \beta_0 + 3\beta_B \quad -3\beta_B = 0$

$(0\ -3\ 0\ 0) = 0 = \theta_0$

c) Within TII participants, is duration meaningful

$y = int + duration$

i. compute $\hat{\beta} = (x'x)^{-1}(x'y)$

$(x'x)^{-1} = \frac{1}{|A|}\begin{pmatrix} -d & b \\ c & -a \end{pmatrix}$ ← formula on slides by

$(x'x)^{-1}(x'y) = 2 \times 1$ matrix of $\hat{\beta}_0, \hat{\beta}_1$

ii. $\beta_D = 0$

$$\frac{\hat{\beta}_1}{\sqrt{Var\hat{\beta}_1}}$$

$Var\hat{\beta}_1 = \sigma^2 (x'x)^{-1}$ (cell 22)

$\sigma^2 = \frac{277.999}{n-2}$

iii. prediction interval:

$H = x(x'x)^{-1}x'$

$\hat{y} \sim N((x\beta)\ \sigma^2(H_i + I))$

③ 456 → biracial, men/women, 65-89
(b/w)

diagnosed as normal, mild, or dementia
⇒ cognitive impairment

a) MCI = INT RACE GENDER AGE

Where RACE = 1 if black; 0 if white (white is ref)
GENDER = 1 if female; 0 if female (females ref)

RACE = diff in MCI prev. for black vs. white participants
GENDER = diff in MCI prev. for females vs. females
AGE = inc in MCI prev. for every 1 add'l yr of age
INT = exp. MCI prev. for a white female age = 0
(could center so this is useful)

change slightly if logistic reg.

b) MCI = INT RACE GENDER AGE RACE*GENDER
adds effect of race (being black) on MCI if the participant is male

c) P(dementia) = race gender age

'algebraic expression' logit(p_dementia) = $\beta_0 + \beta_1(\text{race}) + \beta_2(\text{gender}) + \beta_3(\text{age})$

d) $\text{prob} = \frac{\exp(X\beta)}{1+\exp(X\beta)} = \frac{(-14.5105 + 1.0890 + 85(0.1476)}{1 + \text{''}} = \text{prob}$

e) $\text{Exp}(\hat{\beta} \pm 1.96\,SE(\hat{\beta}))$

OR for black female 5 yr. inc = $5(\hat{\beta})$

f) yes, use MCI = 1 if cog. imp, 0 otherwise? how is this diff. from a?

20 rats; 4 doses (0, 1, 2, 3 mg/day); 5 rats/dose BALANCED!

$d_i = 1$ for dose i, 0 else $\quad i = 0, 1, 2, 3$

data + comp. output provided.

a) mean wt gain $= \alpha_0 d_0 + \alpha_1 d_1 + \alpha_2 d_2 + \alpha_3 d_3$ CELL MEANS

α_3 = estimated wt gain of a mouse on dose 3 mg/day. (161.40)

SE = 3.28177

p-val = <0.0001 ($H_0: \alpha_3 = 0$, so 161.4 is diff. from 0... ok...) not super relevant

---

$\widehat{cov}(\hat{\beta}) = \sigma^2 (X'X)^{-1}$ $\quad X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ etc.

$n \times 4$

$(X'X) = \begin{pmatrix} n_0 & 0 & 0 & 0 \\ 0 & n_1 & 0 & 0 \\ 0 & 0 & n_2 & 0 \\ 0 & 0 & 0 & n_3 \end{pmatrix}_{4\times4}$ an aside derivation

$(X'X)^{-1} = \begin{pmatrix} 1/n_0 & 0 & 0 & 0 \\ 0 & 1/n_1 & 0 & 0 \\ 0 & 0 & 1/n_2 & 0 \\ 0 & 0 & 0 & 1/n_3 \end{pmatrix}_{4\times4}$

$\sigma^2 (X'X)^{-1} = cov(\hat{\beta})$

So $SE(\alpha_3) = \sqrt{\sigma^2 \left(\frac{1}{n_3}\right)} = 3.28$:)

b) consider

$$\text{wt gain} = \underbrace{\mu}_{\text{mean of ref group}} + \overbrace{\gamma_1 d_1 + \gamma_2 d_2}^{\text{diffs}} + \gamma_3 d_3 \qquad \text{REF CELL}$$

γ_1 : estimated diff in wt gain in group 1 compared to group 2.

estimate γ_1 and its SE

$\gamma_1 = 120.2 - 102.2 = 18$ mg/day

$SE(\gamma_1) = 4.64$

$\text{cov}(\hat{\gamma}) = \sigma^2 (X'X)^{-1}$ $\qquad X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}_{n \times 4}$

$$(X'X) = \begin{pmatrix} N & n_1 & n_2 & n_3 \\ n_1 & n_1 & 0 & 0 \\ n_2 & 0 & n_2 & 0 \\ n_3 & 0 & 0 & n_3 \end{pmatrix}_{4 \times 4}$$

aside useful derivations

$$(X'X)^{-1} = \begin{pmatrix} 1/n & 1/n & 1/n & 1/n \\ 1/n & 2(\frac{1}{n}) & 1/n & 1/n \\ 1/n & 1/n & 2(\frac{1}{n}) & 1/n \\ 1/n & 1/n & 1/n & 2(\frac{1}{n}) \end{pmatrix}$$

σ^2 is the same as cell means!

So $SE(\gamma_1) = \sqrt{53.85(2/5)} = 4.64$

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c) Test that the 4 doses have the same mean wt gain.
Report - test stat, null distn, p-val, report the results

$H_0: \alpha_0 = \alpha_1 = \alpha_2 = \alpha_3$

$\Rightarrow \alpha_0 - \alpha_1 = 0$
$\alpha_0 - \alpha_2 = 0$
$\alpha_0 - \alpha_3 = 0$

$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$ $\theta_0 = 0$

$\hat{\theta} = C\hat{\alpha} = \begin{pmatrix} -18 \\ -37.6 \\ -59.2 \end{pmatrix}$

$F = \dfrac{\{(\hat{\theta} - \theta_0)' M^{-1} (\hat{\theta} - \theta_0)\}/a}{MSE}$

$M^{-1} = (C(X'X)^{-1}C')^{-1}$ $\begin{pmatrix} 1/n_0 & 1/n_1 & 0 & 0 \\ 0 & 1/n_2 & 1/n_3 \end{pmatrix}$

$(3\times4)(4\times4)(4\times3) = 3\times3$

num = $(1\times3)(3\times3)(3\times1)$

$= \dfrac{3246.067}{53.85} = 60.28$ * SAME F-STAT from overall test in ref cell*

$F \sim df_n = 3 \quad df_d = 16$

p-val: < 0.0001

There is evidence that the weight gains among doses are not the same.

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d) wt gain $= \beta_0 + \beta_1 \text{dose}$

β_1 = increase in wt gain w/ a 1 mg increase in the drug

p-val = < 0.0001

The relationship is significant (↑ dose ⇒ ↑ wt gain)
Also means that the jump from dose 0, 1-2, 2-3, 3-4 are the same

e) Using the replicates, test the linearity assumption in the last part. Report t-stat, null, p-val + interpret

If linear

$2\gamma_1 = \gamma_2$

$3\gamma_1 = \gamma_3$

$H_0: \gamma_1 = \frac{1}{2}\gamma_2 = \frac{1}{3}\gamma_3$

$6\gamma_1 = 3\gamma_2 = 2\gamma_3$

$\equiv \quad 2\gamma_1 - \gamma_2 = 0$

$3\gamma_1 - \gamma_3 = 0$

$\gamma_1 = 18$

$\gamma_2 = 37.6$

$\gamma_3 = 59.2$

$C = \begin{pmatrix} 2 & -1 & 0 \\ 3 & 0 & -1 \end{pmatrix}$ $\quad \theta_0 = 0$ $\quad \hat{\theta} = \begin{pmatrix} -1.6 \\ -5.2 \end{pmatrix}$ $\quad M^{-1} = (C(X'X)^{-1}C')^{-1}$

$=$

$$F = \frac{(\hat{\theta}-\theta_0)' M^{-1} (\hat{\theta}-\theta_0) / a}{MSE} = 0.1507$$

$F \sim 2, 16$

p-val $\sim 0.86 \Rightarrow$ FAIL TO REJECT

no evidence the relationship is linear based on this test

f) g) are discussion / synthesizing questions