Categorical Independent Variables

1. Models without Interactions

An *indicator* (or *binary variable*) is a variable that can have only two values, 0 or 1.
Indicator variables are used to represent *categorical* (or *nominal* or *qualitative*) variables.
A categorical variable with *k* classes is represented in the regression model by *k-1* indicators.

(Indicators are also called "dummy variables" but this usage is unfortunate as it incorrectly implies that indicators are "fake" variables in some sense, which they are not.)

Note on naming an indicator variable: consider naming an indicator variable after the category that has the value 1. For example a variable that 1 for female, and 0 for male can be named FEMALE; the meaning remains clear. If the variable is named SEX or GENDER, you *may* forget which one is 1 and which one is 0.

Ph.D. Data

In the Ph.D. example, professor rank is categorical with 3 categories: assistant, associate and full. The indicators used are

X_2 = 1 if Associate, 0 otherwise
X_3 = 1 if Full, 0 otherwise

Assistant rank does not have an indicator associated with it. It is called the *reference, baseline*, or *omitted* category or class. The reference class is the class for which every dummy variable is set to zero.

For the regression model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$$

X_{i1} is continuous and X_{i2} and X_{i3} are the two dummy variables.

The meaning of the coefficients is revealed by *examining the regression function*

$$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

There are three cases, depending on rank:

Values of Regression Function $E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$

Rank	Values of Indicators	Regression Function
Assistant	$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2(0) + \beta_3(0)$	$E\{Y\} = \beta_0 + \beta_1 X_1$
Associate	$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2(1) + \beta_3(0)$	$E\{Y\} = (\beta_0 + \beta_2) + \beta_1 X_1$
Full	$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2(0) + \beta_3(1)$	$E\{Y\} = (\beta_0 + \beta_3) + \beta_1 X_1$

The table shows that β_2 and β_3 represent the *differences in intercept* for associate and full rank, respectively, relative to assistant (the omitted category).

2. Models with Interactions

The coefficient of a continuous independent variable Y may be allowed to vary as a function of the dummy variable by using an interaction term. The response function for the interaction model becomes

$$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_1 X_2 + \beta_5 X_1 X_3$$

Again to understand the meaning of the coefficients one must examine the regression (response) function for each category of the categorical variable:

Values of Regression Function $E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_1 X_2 + \beta_5 X_1 X_3$

Rank	Values of x_2 and $x_1 x_2$	Regression function
Assistant	$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2(0) + \beta_3(0) + \beta_4(0) + \beta_5(0)$	$E\{Y\} = \beta_0 + \beta_1 X_1$
Associate	$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2(1) + \beta_3(0) + \beta_4 X_1(1) + \beta_5 X_1(0)$	$E\{Y\} = (\beta_0 + \beta_2) + (\beta_1 + \beta_4) X_1$
Full	$E\{Y\} = \beta_0 + \beta_1 X_1 + \beta_2(0) + \beta_3(1) + \beta_4 X_1(0) + \beta_5 X_1(1)$	$E\{Y\} = (\beta_0 + \beta_3) + (\beta_1 + \beta_5) X_1$