

# The Currency Problem

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## 1 Problem

Bobby has four dimes. Amy has 30 pennies. Which child has more money?

## 2 Proof

We know that the value of a dime is equal to ten pennies. We can then say that the value of a dime is equal to  $10x$ , where  $x$  is equal to the number of dimes. We can model possible combinations of the dime and penny with the equation

$$S = \{x, y | y, i, x \in N \{ \forall x \{ \forall y < 10x \{ \forall i = 10x \{ y < i \} \} \} \} \}$$

where  $S$  is the set of possible combinations where the value of the dimes is greater than the value of the pennies,  $x$  is the number of dimes,  $i$  is the value of the dimes and  $y$  is the number and value of the pennies. We can prove this statement is true using induction.

### 2.1 Basis Step

Let  $x$  be equal to 1. The value of  $i$  is thus 10. By definition,  $y$  must then be a number less than 10. Clearly,  $y < i$ .

### 2.2 Induction Step

Assume that  $x = k$ . The value of  $i$  is thus  $10k$ . By definition,  $y$  must then be a number less than  $10k$ . Therefore,  $y < i$

## 3 Solution

Let the number of Bobby's dimes in section 1 be  $x$ , and the value of Amy's pennies be  $y$ . If  $x, y \in S$ , then  $i > y$  where  $i$  is the value of Bobby's currency. Therefore, Bobby has more money if  $x, y \in S$ . Let  $x$  be equal to 4 and  $y$  be equal to 30.  $i$  is thus equal to  $10 * 4 = 40$ .  $30 < 40$ , so clearly  $y < i$ . Therefore, Bobby has more money.