EN4610 Lab Report 1: Nyquist Stability Analysis

Tye Cameron-Robson (Student No: C2038216)
MEng in Mechanical Engineering, Cardiff School of Engineering, Cardiff University.

1. The Forward Loop Transfer Function

$$L(s) = C(s)P(s)\left(\frac{100}{s+100}\right)e^{-h\cdot s}$$

$$= \left(\frac{100Kk_p s + 100Kk_i}{\tau s^3 + (1+100\tau)s^2 + 100s}\right)e^{-h\cdot s}$$

$$= \left(\frac{226s+2260}{0.12s^3+13s^2+100s}\right)e^{-0.01\cdot s}$$

2. Nyquist Plot of L(s)

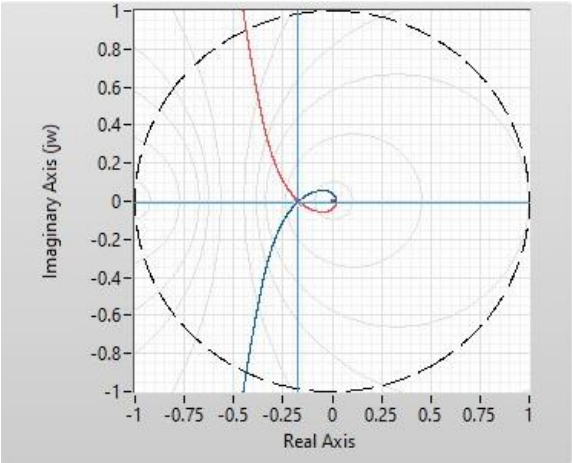

Figure 1: Nyquist Plot of Transfer Function L(s)

3. Theoretical Values Calculated

Gain Margin	Gain Margin Frequency (rad/s)	Phase Margin (rad)	Phase Margin Frequency (rad/s)	Delay Margin (s)	Delay Margin Frequency (rad/s)	nDelay
5.9	84.86	1.12	19.14	0.059	19.14	5

Table 1: Theoretical Features of L(s)

4. Graphical Estimations

$\left(\frac{1}{g_m}\right) = 0.17$ $\quad g_m = 5.88$

$\varphi_m = 63°$ $\quad \varphi_m = 1.1\text{rad}$

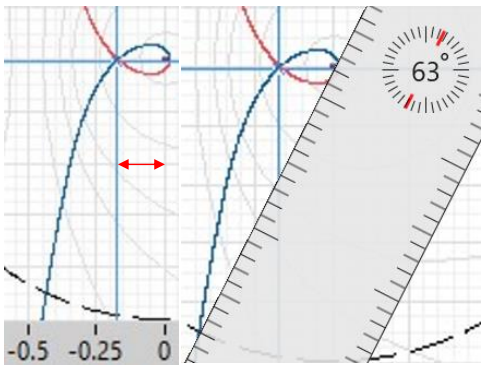

Figure 2: Graphical Gain and Phase Margins of the Nyquist Plot

5: Experimental Gain Margin and Gain Margin Frequency

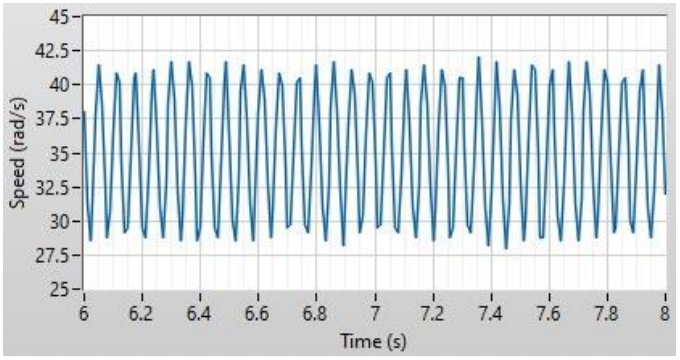

Figure 3: Marginally Stable Motor Speed Graph (Due to Gain)

Loop Rate (s)	Set Point (rad/s)	kp	ki	Loop Gain	nDelay
0.01	35	0.1	1	3.37	0

Table 2: Experimental Setpoints for the Gain Margin

$$g_m = 3.37$$
$$\omega_{gm} = \sim \frac{32}{2} Hz$$
$$16Hz = 100.5 rad/s$$

6: Experimental Phase Margin and Phase Margin Frequency

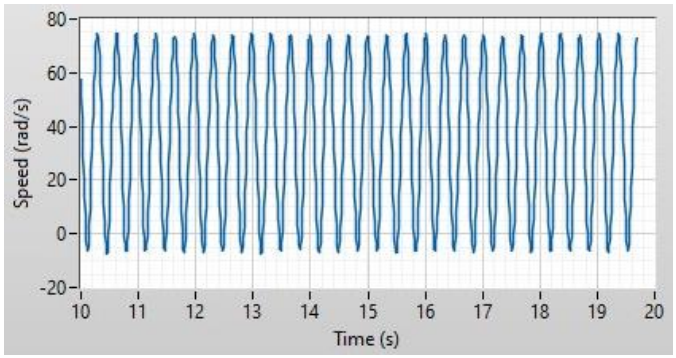

Figure 4: Marginally Stable Motor Speed Graph (Due to Time Delay)

Loop Rate (s)	Set Point (rad/s)	kp	ki	Loop Gain	nDelay
0.01	35	0.1	1	1	7

Table 3: Experimental Setpoints for the Phase Margin

$$T_{dm} = nDelay \cdot Loop\ Rate = 0.07s$$
$$\omega_{pm} = \frac{24}{8} Hz = 18.85 rad/s$$
$$\phi_m = \omega_{pm} \cdot T_{dm} = 1.32 rad$$

7: Comments

- The graphically estimated values were very close to the theoretical Nyquist values but lacked a degree of accuracy.
- The experimental gain margin and gain margin frequency were dissimilar to the theorised values. This could be caused by inconsistencies between the model and the real system, i.e. nonlinearities or system parameter uncertainty.
- The experimental phase margin and phase margin frequency were similar to the theorised values, suggesting the model captures the phase characteristics. Slight discrepancies still arise, likely due to the whole integer increments of nDelay, but could be caused by sampling rates or aforementioned parameters.

_____