

# Introduction and Set Theory

AMC-12

September 20, 2021

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1 Intensive Lecture

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1 Intensive Lecture

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# Introduction to AMC12 I

## 1 Algebra:

- ▶ Systems of Linear Equations;
- ▶ Quadratics & Polynomials;
- ▶ Exponents & Logarithms;
- ▶ Inequalities;
- ▶ Sequences & Series;
- ▶ Trigonometric Functions;
- ▶ Complex Number;
- ▶ ...

# Introduction to AMC12 II

## 2 Geometry:

- ▶ Triangle;
- ▶ Circle;
- ▶ Quadrilateral;
- ▶ Analytical Geometry;
- ▶ Solid Geometry;
- ▶ ...

# Introduction to AMC12 III

## 3 Number Theory:

- ▶ Divisibility;
- ▶ Prime and Factorization;
- ▶ Greatest Common Divisor & Least Common multiple;
- ▶ Congruence;
- ▶ Fermat's little theorem & Euler Theorem;
- ▶ ...

# Introduction to AMC12 IV

## 4 Combinatorics:

- ▶ Fundamental Counting Principles;
- ▶ Permutation and Combination;
- ▶ Probability;
- ▶ Binomial Theorem;
- ▶ ...

# Set Theory

## Definition 1 (Set)

A **set** is a collection of distinct objects, called elements of the set. A set can be defined by describing the contents, or by listing the elements of the set, enclosed in curly brackets.

### Example:

- The set of all even numbers;
- The set of all books written about travel to Chile;
- $\{1, 3, 9, 12\}$ ;
- $\{\text{red, orange, yellow, green, blue, indigo, purple}\}$ .



# Notations

- Commonly, we will use a variable to represent a set:  
 $A = \{1, 2, 3, 4\}$ ;
- The symbol  $\in$  means "is an element of":  $2 \in A, 5 \notin A$ ;
- A set that contains no elements,  $\{\}$ , is called the empty set and is notated  $\emptyset$ ;
- $|A|$  or  $\text{Card}(A)$  means the cardinal of  $A$ , i.e. the number of elements in  $A$ ;
- A subset of a set  $A$  is another set that contains only elements from the set  $A$ , but may not contain all the elements of  $A$ :  
 $\{1\} \subseteq \{1, 2\}, \{3\} \not\subseteq \{1, 2\}$ ;
- A proper subset is a subset not identical to the original set:  
 $\{1\} \subsetneq \{1, 2\}$ .

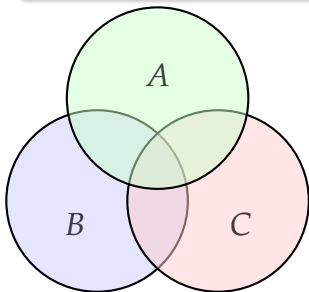
# Operations

- **Union:** The union of two sets contains all the elements contained in either set (or both sets). The union is notated  $A \cup B$ . More formally,  $x \in A \cup B$  if  $x \in A$  or  $x \in B$  (or both);
- **Intersection:** The intersection of two sets only the elements that are in both sets. The intersection is notated  $A \cap B$ . More formally,  $x \in A \cap B$  if  $x \in A$  and  $x \in B$ ;
- **Complement:** The complement of a set  $A$  contains everything that is in the **universal set**  $U$  but not in the set  $A$ . The complement is notated  $\bar{A}$  or  $U \setminus A$ .

# Venn Diagram

## Definition 2 (Venn diagram)

A Venn diagram represents each set by a circle, usually drawn inside of a containing box representing the universal set. Overlapping area indicate elements common to both sets.



Principle of Inclusion-Exclusion:

- $|A \cup B| = |A| + |B| - |A \cap B|;$
- $|A \cup B \cup C| =$   
 $|A| + |B| + |C| - |A \cap B| -$   
 $|B \cap C| - |C \cap A| + |A \cap B \cap C|.$

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# A Short Review

- Definition;
- Notations: in, subset, proper subset;
- Operations: union, intersection, complement;
- Venn diagram and the inclusion-exclusion principle.