Exploring Bounds

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1 Introduction

An interesting problem in coding theory is how to determine the maximum possible number of codewords in a code. For an n-dimensional code, we are interested in the maximum number of codewords such that the code has weight d. Formally, we use the notation $A_q(n,d)$ and $B_q(n,d)$, defined as the maximum of number of codewords in a code over $\mathbb{F}_q$ of length n and minimum distance d for an arbitrary (linear on non-linear) code and linear code, respectively. It is not always possible to In lecture, we have considered upper bounds—Sphere Packing, Singleton, and Greismer—as well as lower bounds—Gilbert and Varshamov—on these values. The purpose of this paper is to consider other well-known bounds in the literature.

This paper is broken up into x sections.

2 Asymptotic Bounds

We now wish to consider these $A_q(n,d)$ values as n goes to infinity. To do so, we must more formally define two terms.

In class, we have considered the *rate* of a linear code, k/n, as one measure of the goodness of a code. That is, the rate tells us how much information relative to redundancy that our codewords provide. The concept of rate can be generalized to non-linear codes as well. For a possibly nonlinear code over $\mathbb{F}_q$ with M codewords, the *rate* is defined to be $n^{-1}\log_q M$. Notice that for an $[n,k,d]$ linear code, $M = q^k$ and hence the rate is k/n as we expect.

A second notion of goodness that we have discussed, but I think not formally defined, is the *relative distance* of a code. For a linear or nonlinear code of length n has minimum distance d, this value is the ratio d/n.

Consequently, for our asymptotic bounds, we are interested in the largest possible rate for a family of codes over $\mathbb{F}_q$ of lengths going to infinity with relative distances approaching some constant δ. In other words, we consider the equation:

$$\alpha_q(\delta) = \limsup_{n\to\infty} n^{-1}\log_q A_q(n,\delta n) \tag{1}$$

[What does this imply?]

2.1 Asymptotic Singleton Bound

Recall the Singleton Bound from lecture:

Theorem 2.1.1. *For $d \leq n$, $A_q(n, d) \leq q^{n-d+1}$.*

[explain what this means, what are the implications]

Theorem 2.1.2. *If $0 \leq \delta \leq 1$, then $\alpha_q(\delta) \leq 1 - \delta$.*

This theorem follows directly from the Singleton bound.

Proof.

$$\begin{aligned}\alpha_q(\delta) &= \limsup_{n\to\infty} n^{-1} \log_q A_q(n, \delta n) \\ &\leq \limsup_{n\to\infty} n^{-1} \log_q q^{n-\delta n+1} \\ &\leq \limsup_{n\to\infty} \frac{n - \delta n + 1}{n} \\ &\leq 1 - \delta\end{aligned} \tag{2}$$

□

2.2 Asymptotic Plotkin Bound

Asymptotic singleton, include 2 examples Asymptotic plotkin, some examples draw graph Asymptotic hamming define hilbert, prove 135,136, define vq(n,a), black box 2.10.3, prove 2.10.5

Table of common$a_q(n, d)$ values Recall basic theory: thm 2.1.2, 2.1.6

Sections

Plotkin talk about importance define it prove it work out some examples

Elias talk about importance define it prove two lemmas that do the heavy lifting easy proof to finish it off work out some examples

Linear Programming Bound talk about importance define it prove it work out some examples

Note existence of two lower bounds, say we talked about them in class