Exploring Bounds

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1 Introduction

An interesting problem in coding theory is determining the maximum number of codewords in a code with certain parameters. For an n-dimensional code, we are interested in the maximum number of codewords such that the code has minimum distance d. Formally, we use the notation $A_q(n, d)$ and $B_q(n, d)$ to define the maximum of number of codewords in a code over $\mathbb{F}_q$ of length n and minimum distance d for an arbitrary (linear on non-linear) code and linear code, respectively.

For arbitrary n and d, it is difficult to find $A_q(n, d)$ and $B_q(n, d)$ exactly. In lecture, we have considered upper bounds—Sphere Packing, Singleton, and Greismer—as well as lower bounds—Gilbert and Varshamov—on these values. The purpose of this paper is to take a survey of other well-known bounds in the literature that we have not discussed. In the first part of the paper, we will consider the Plotkin Upper bound, the Elias Upper bound, and the Linear Programming Upper Bound. In the second part of the paper, we will consider asymptotic versions of Singleton, Plotkin, Hamming, and Elias. (I am very certain that I won't get to discuss all of these bounds, but I am currently unsure how much space each discussion will take up so I've listed all that I am considering.) For each bound, we will consider its proof as well as related examples. Lastly, in the final part we will consider lexicodes, an interesting subset of linear codes that meet the Gilbert Bound.

Some implementation specific details: for the Linear Programming bound, I will introduce basic linear programming concepts and as well as the Krawtchouck polynomials. For the Asymptotic Hamming bound I will introduce the Hilbert entropy function.