

Figure 1: Vertexes in red, faces in parentheses in blue. Faces are numbered by the vertex opposite from the face. Outwardly directed right-hand rule for vertex numbering on a face.

The fluxes  $\psi_i$  is the discontinuous spatial degree of freedom at vertex i. The fluxes  $\psi_j^{\mathrm{b}(k)}$  are the values  $\psi_j$  if  $\Omega \cdot \hat{n}_k > 0$  where  $\hat{n}_k$  the outward normal on face k, otherwise they are the corresponding values at across the face k at vertex j in the neighboring cell sharing the face k (or the prescribed boundary condition if it is a boundary face).

$$\begin{split} \frac{1}{12}\overline{A}_{1} \left(2\psi_{0}^{\text{b}(1)} + \psi_{2}^{\text{b}(1)} + \psi_{3}^{\text{b}(1)}\right) + \frac{1}{12}\overline{A}_{2} \left(2\psi_{0}^{\text{b}(2)} + \psi_{3}^{\text{b}(2)} + \psi_{1}^{\text{b}(3)}\right) + \frac{1}{12}\overline{A}_{3} \left(2\psi_{0}^{\text{b}(3)} + \psi_{1}^{\text{b}(3)} + \psi_{2}^{\text{b}(3)}\right) \\ + \frac{1}{12}\overline{A}_{0} \left(\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) + \frac{\sigma_{t} V}{20} \left(2\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) = \frac{V}{20} \left(2Q_{0} + Q_{1} + Q_{2} + Q_{3}\right) \\ \frac{1}{12}\overline{A}_{0} \left(2\psi_{1}^{\text{b}(0)} + \psi_{3}^{\text{b}(0)} + \psi_{2}^{\text{b}(0)}\right) + \frac{1}{12}\overline{A}_{2} \left(2\psi_{1}^{\text{b}(2)} + \psi_{0}^{\text{b}(2)} + \psi_{3}^{\text{b}(2)}\right) + \frac{1}{12}\overline{A}_{3} \left(2\psi_{1}^{\text{b}(3)} + \psi_{0}^{\text{b}(3)} + \psi_{2}^{\text{b}(3)}\right) \\ + \frac{1}{12}\overline{A}_{1} \left(\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) + \frac{\sigma_{t} V}{20} \left(\psi_{0} + 2\psi_{1} + \psi_{2} + \psi_{3}\right) = \frac{V}{20} \left(Q_{0} + 2Q_{1} + Q_{2} + Q_{3}\right) \\ \frac{1}{12}\overline{A}_{0} \left(2\psi_{2}^{\text{b}(0)} + \psi_{1}^{\text{b}(0)} + \psi_{3}^{\text{b}(0)}\right) + \frac{1}{12}\overline{A}_{1} \left(2\psi_{2}^{\text{b}(1)} + \psi_{3}^{\text{b}(1)} + \psi_{0}^{\text{b}(1)}\right) + \frac{1}{12}\overline{A}_{3} \left(2\psi_{2}^{\text{b}(3)} + \psi_{1}^{\text{b}(3)} + \psi_{0}^{\text{b}(3)}\right) \\ + \frac{1}{12}\overline{A}_{2} \left(\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) + \frac{\sigma_{t} V}{20} \left(\psi_{0} + \psi_{1} + 2\psi_{2} + \psi_{3}\right) = \frac{V}{20} \left(Q_{0} + Q_{1} + 2Q_{2} + Q_{3}\right) \\ \frac{1}{12}\overline{A}_{0} \left(2\psi_{3}^{\text{b}(0)} + \psi_{1}^{\text{b}(0)} + \psi_{1}^{\text{b}(0)}\right) + \frac{1}{12}\overline{A}_{1} \left(2\psi_{3}^{\text{b}(1)} + \psi_{0}^{\text{b}(1)} + \psi_{0}^{\text{b}(1)}\right) + \frac{1}{12}\overline{A}_{2} \left(2\psi_{3}^{\text{b}(2)} + \psi_{1}^{\text{b}(2)} + \psi_{0}^{\text{b}(2)}\right) \\ + \frac{1}{12}\overline{A}_{3} \left(\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) + \frac{\sigma_{t} V}{20} \left(\psi_{0} + \psi_{1} + 2\psi_{2} + \psi_{3}\right) = \frac{V}{20} \left(Q_{0} + Q_{1} + Q_{2} + Q_{3}\right) \\ + \frac{1}{12}\overline{A}_{3} \left(\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) + \frac{\sigma_{t} V}{20} \left(\psi_{0} + \psi_{1} + \psi_{2}^{\text{b}(1)}\right) + \frac{1}{12}\overline{A}_{2} \left(2\psi_{3}^{\text{b}(2)} + \psi_{1}^{\text{b}(2)}\right) \\ + \frac{1}{12}\overline{A}_{3} \left(\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) + \frac{\sigma_{t} V}{20} \left(\psi_{0} + \psi_{1} + \psi_{2}^{\text{b}(1)}\right) + \frac{1}{12}\overline{A}_{2} \left(2\psi_{3}^{\text{b}(2)} + \psi_{1}^{\text{b}(2)}\right) \\ + \frac{1}{12}\overline{A}_{3} \left(\psi_{0} + \psi_{1} + \psi_{2} + \psi_{3}\right) + \frac{\sigma_{t} V}{20} \left(\psi_{0} + \psi_{1} + \psi_{2}^{\text{b}(1)}\right) + \frac{1}{12}\overline{A}_{2} \left(2\psi_{3}^{\text{b}(2)} + \psi_{1}^{\text{b}(2)}\right) \\ + \frac{1}{12}\overline{A}_{3} \left($$