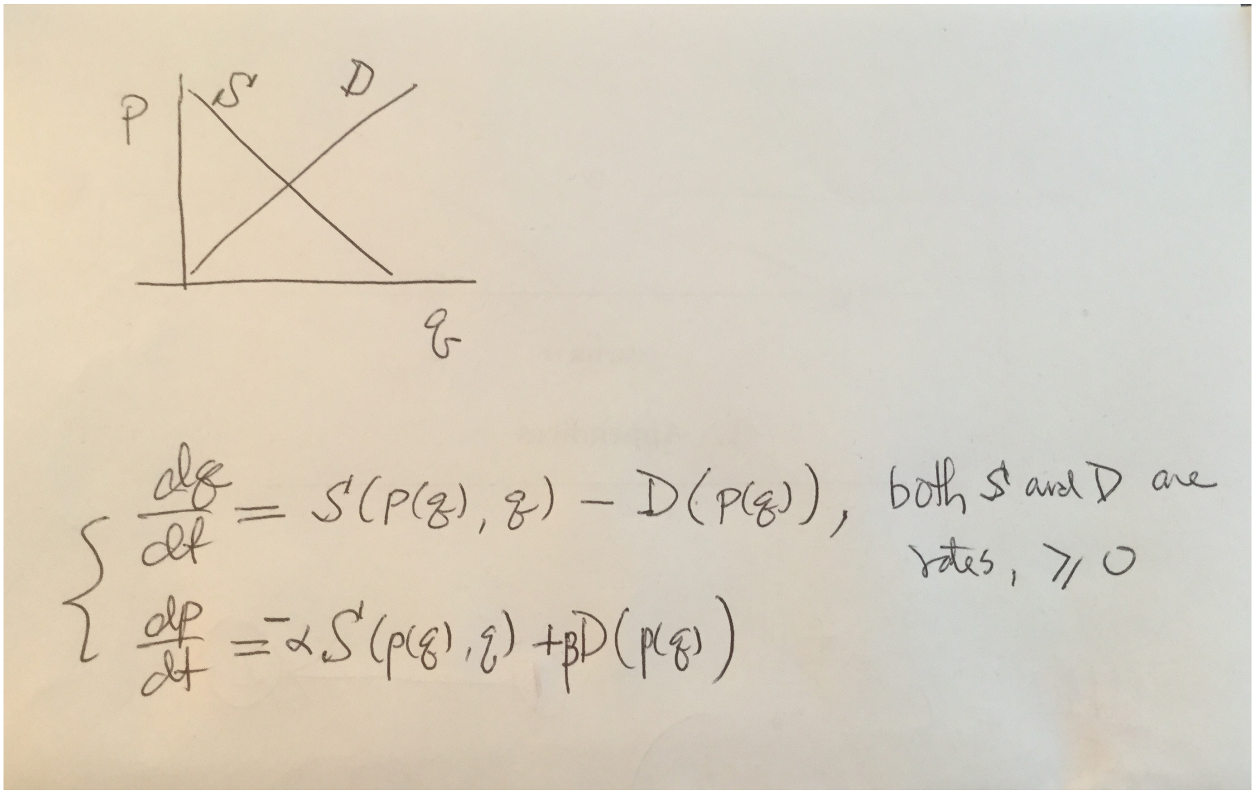

Figure 1: The System of ODE's that Hong created to describe supply and demand

"Does it make sense? If not why? If yes, has anyone already done that? "

I also want to know if economics is weak chaos (bounded).

In which α and β are two positive constants that convert the rate of change in q to that or rate of change in p.

1 Has anyone already done that?

So I answered this question first. I had a meeting with Matt and Hong 02/27/20 where they made clear in person that I was supposed to be doing this at the same time as answering the other two questions. Now, rereading what Hong sent me before Winter Break, I think it also implied that if not at the same time then after.

The first order simple ODE has been studied extensively by economists, and I looked at and presented one form of this already. Clower wrote a paper analyzing the meaning of the first order dynamical system in regards to the basic supply and demand curves, and disequilibrium theory has done some work with this.

What has not been done, is the analysis of a second order description of the system that describes the state of depression in the economy as an equilibrium as well. This is the model that I want to work on.

First thing suggested to check is whether this reproduce the figure in `https://en.wikipedia.org/wiki/Supply_and_demand`," which is more or less Fig 1. What I found in the literature is that this picture is exactly how the current dynamical systems models were made. Rather than checking in retrospect, they defined the calculus analysis already inherent there as the dynamics. My goal is to start from scratch as best I can and then see how that compares to previous models

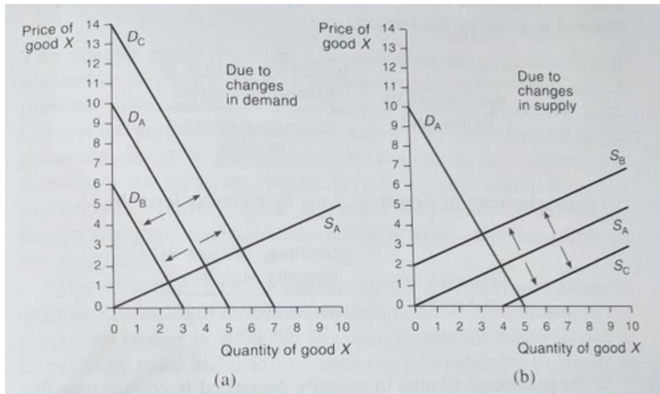

Figure 2: Walrasian supply and demand

2 First Model from Lit

A Simple Dynamical System - Economic Dynamics: Phase Diagrams and Their Economic Application by Ronald Shone

- In a typical economy the price is set by producers, and then adjusted based on observations
- This means typically what is observed in the economy is *dynamic adjustment process*
- To not just discuss that price will increase or decrease so that supply matches demand, we can examine the underlying dynamics behind price movement

Model 1

$$\begin{aligned} q_d &= a - bp & b > && 0 \\ q_s &= c + fp & f > && 0 \\ \frac{\mathrm{d}p}{\mathrm{d}t} &= \alpha(q_d - q_s) = \alpha(b+f)p - \alpha(a-c) & \alpha > && 0 \end{aligned}$$

q_d, q_s, & p are continuous functions of time.

Supply and Demand

At equilibrium we have that $\frac{\mathrm{d}p}{\mathrm{d}t} = \alpha(q_d - q_s) = 0 \;\Rightarrow\; p^* = \frac{a-c}{b+f}$ and $q^* = \frac{af+bc}{b+f}$

Without Stocks

Assumption 2.1:
In disequilibrium the short side of the market is transacted. In other words only current supply is available to fill demand and there is no stock of goods.

- Excess demand gives a signal for producers to increase supply and price
- Excess supply gives signal for producers to decrease supply and price
  - It says nothing about what happens to excess goods...

With Stocks

Assumption 2.2:
Stocks are sufficient to meet demand at any price, and price adjusts to changes in stock levels.

Model 2

$$
\begin{aligned}
i =& i_0 + \int_0^t q_s - q_d \mathrm{d}t \\
\frac{\mathrm{d}i}{\mathrm{d}t} =& q_s - q_d \\
\frac{\mathrm{d}p}{\mathrm{d}t} =& -\alpha \frac{\mathrm{d}i}{\mathrm{d}t} = \alpha(q_d - q_s) \qquad\qquad \alpha > 0
\end{aligned}
$$

- The price path solution is the same
- However now we allow for the quantity traded to increase s.t. $q(t) = q_d(t) \; \forall \; p$

Further systems

- This approach can be applied to labor markets with and without flexible wages
- These labor models are then related to supply and demand in that decreases in price indicate decreases in labor demand
  - In sticky wage models we make assumptions about how demand once again determines dynamics
  - There are also assumptions accounting for how there are asymmetric adjustments: wages increase faster than they decrease
- Discretizing the time flow leads to what is called the cobweb model

3 My Model

Could also be a markov chain that describes the system.

3.1 Things that go into supply and demand

- supply
- demand
- price
- quantity
- current stock
- expiration of stock (rate of decay of item, often idealized to infinity or zero)
- utility - useful, time it takes to acquire, ease of use, etc.
- available substitutions
- budget, including savings (so expected much less than income because housing and so on)
- coolness factor
- elasticity (assumed linear or is it non-linear?)

3.2 How do supply and demand interact?

Something interesting that I always thought was left out is rarity. It is really obvious in trading cards, but also in Grey Poupon mustard. Something can be more expensive and that attracts people to it perhaps because of ideas of luxury. Because of this sort of thing, I do not believe that the interaction between supply and demand is linear, so then the question is what kind of non-linearity makes sense.

I do not believe it would be oscillatory, so the question is what order polynomial and how does it relate? Thinking about AMATH 568 for non-linear ODE's, we work almost entirely with non-linear second order ODE's. Here we have been working with second order ODE's, so I need to think more about how this translates into real world systems. Also sometimes one of our terms has a very small constant out in front. For example what we have below is a second order ODE with a very small damping term.

$$-y''[q] - \epsilon y'[q] + y[q] = 0$$

Since $y''[q]$ has an opposing sign to $y[q]$ we may think of the exponential function as our ansatz. The inclusion of the damping term would keep our solution from shooting off to ∞.

Supply and Demand would also be positively correlated. As demand increases supply will increase to match, but less than the quantity demanded, since the price should also be able to increase. This is in an idealized market of course, since if there are competitors, that could keep a company from raising there prices. However, a company will never supply more than the amount demanded since that will drive down prices and thus profit margin, while also increasing stocks leading to reduced profit in the future. When demand quantity decreases we would then expect the supply quantity to decrease more than this, for similar reasons to why it would increase less than with demand increase. We can call this amount of disconnect in change δ so that we have $\frac{\mathrm{d}q_S}{\mathrm{d}t} \approx (1-\delta)q_d$.

From macroeconomic textbooks we have that

$$\dot{q}_S = c_1 q_D + \left(\frac{dS}{dq} - \frac{dD}{dq}\right)^{-1}\dot{q}_D \tag{1}$$

$$+ \epsilon \ddot{q}_D \tag{2}$$

But from this equation we come to the big question of what $\frac{dS}{dq}$ and $\frac{dD}{dq}$ mean. So perhaps it is still useful to call this term $(1-\delta)$ where we can theoretically understand δ as the inverse of the spread between what the producers and the consumers theoretically expect for that quantity of goods in the market. An interesting note is that for linear systems the amount that $\dot{q}_S$ changes does not depend on if the change in q_D was positive or negative.

Further, is it possible that in an attempt to predict the necessary supply in the future the company adds on a second derivative term such as (**??**)? It would be optimizing in that if they expect things to change quickly, they can try not to run out of stock in stores. This may be especially relevant for the case of perishable goods where rather than always being behind demand until it reaches a equilibrium, trying to be ahead and always have supplied what the consumers will purchase. This would likely need an unknown constant in front of the term that is less than one (it probably has less cost associated to under-perform still).

We also want to look into how the quantity demanded changes changes over time. I believe as a consumer, that this will be based mostly on the price, where p is the transaction price taking place in the market. There is also something to be said for ease of access and rarity of the goods, so I believe that a positive quadratic term makes sense for the supply available effecting the quantity demanded. I argue that change in quantity supplied is not noticeable to a consumer. Either the store has the good or it does not, I have no idea what others are up to. Thus the change in quantity supplied term is left out.

$$\dot{q}_D = -c_1\dot{p} - c_2(q_S - c_3)^3 \tag{3}$$

In this system we can assume that companies may change their supply without a large effect on consumption, as long as the consumers can still easily access a good (or highly value the rarity of it which is handled in c_3).

Something else that I want to pay attention to is not only the constant shifts in supply and demand curves (think shifting the intercept), but also the non-linear shifts in slope. I have not analyzed a case like that yet.

3.2.1 Taking Stocks of Goods into Account

The stock of excess supplied goods also needs to be taken into account. This is only relevant for the case of non-perishable goods, just as I would guess that the second derivative term is only relevant for the case of perishable goods. Do companies want at least a certain amount stockpiled? The only difference in the model is that companies would make less money, so we can ignore this in terms of understanding the dynamics. One major difference is that the as stockpiles grow, prices tend to drop as a way of clearing them out. So it's not just about the amount supplied, but the amount in total available that determines changes in price. We definitely know that

$$\dot{b} = q_s - q_d$$

4 Demand Functions

5 Problems

- It seems like it would be less relevant/useful to model q, the surplus quantity supplied, than changes in the quantity demanded and the quantity supplied. This is partially because then when we see changes (since presumable preferences do have the possibility of changing over time) we can relate this to the model better.
- I don't think S and D should be rates but functions that are greater than or equal to 0. This way aggregate supply and aggregate demand could also be modeled in the same system, and these are generally not considered to be linear.

5.1 First Meeting with Phillip

- Is the model of a single good or combination of many goods? How does this manifest?
- Do you need the supply and demand curves to intersect twice for there to be two stable equilibria?
- How are we going to aggregate across many goods in a way that makes sense? Currently the concept is adding a bunch of linear lines, can this be made to look like a dynamical system with two stable equilibrium?
- What kinds of exotic arguments would I have to come up with to explain this system with two stable equilibrium?
- Are the consumers/suppliers forward looking?
- Does it make sense for for demand to be based on past expectations and supply to be forward looking so that they have two different behaviors?
- Does it make sense to explore a field that was really popular for ten years and then fizzled out in what he assumed was complete frustration?
- Are the curves going to be made of individual agents and be an agent based model, or are they going to remain deterministic? - I feel like the goal was always to start with something deterministic, and then build up to stochastic. Right now I am envisioning describing a single good and then having the coefficients be instances of random variables/a stochastic process and treating each individual good market as an agent in the system to be aggregated as one potential path.
- There was a comment about how I need to start with something well beyond the system that I showed him today if I want to design a model that makes sense, so then what would be a good starting point? the two quadratic curves are what I have seen for an aggregate system. I don't think this is what he was referring to though.
- Should I take the micro courses as well as the macro in Econ?

- What am I really asking of a committee member?
- Do I want to have someone on my general committee who wants to wait to decide if he will be on the defense committee

5.2 First Meeting with Jing

This meeting went well in my opinion. Jing had a lot to suggest and questions that dug into some issues with my current way of presenting and thinking about the material, but did so in a non-confrontational way. She also brought up some important things to take into consideration:

- The field of industrial organization works on estimating demand, and so they would hve some new and interesting models that go beyond the basic linear one for single markets
- Demand comes from people's utility function, so it absolutely must be monotonic (personal note: this still allows for some people to have a sudden increase in demand for rare goods)
- Demand is often estimated using splines because there is a huge interest in discovering price elasticity. Suppliers want to know this, so there is decent research into this topic (i.e. industrial organization).
- This demand estimation is where Jing works
- Engle curves are the relationship between income and consumption.
  Note to self: This is very important to understand if I want to start talking about cubic consumption aka the goods that are purchased in a marketplace
- I need to work on understanding where current economic research is, which has a lot to do with knowing the terminology
- I was sent five resources by Jing directly after the meeting: 3 on Engel Curves, and 2 more on demand in differentiated markets aka demand at a macro level and how demand of individual products interact

6 Advisor Meetings

6.1 April 17, 2020

My next steps are

1. Read Chapter 1 of Murray
   (a) Try and translate this into economics
   (b) Be prepared to present this to the group
   (c) I'm looking for how to think about dynamical systems and how they are presented
   (d) Afterwards I will read Chapter 3 and probably Chapter 10
2. Use Mathematica to manipulate the streamplot from section 1 of SupplyAndDemandMth.pdf. The goal here is to play with how different parameters change the system.
3. create a supply and demand logistic equation, where supply is the birth and demand is the death. This is another way of thinking about the system in a new way.
4. fluid limit is something that I may want to understand. It relates to queueing thoery, and how random systems behave in the limit (kind of like a smoothing I think)

6.2 April 30, 2020: Group Meeting

I presented, Murray_Economics_Style.pdf, the slides I made based on reading Murray Mathematical Biology: An Introduction Chapter 1.

- I would like the quote by "Boo-show": "A fit is not a theory"
- In order to get a demand/consumption curve that is not monotonic, we need to have some kind of positive feedback
- We will almost definitely need multiple goods to achieve any kind of positive feedback
- money/investments is an example of an economic item that may follow logistic??? or maybe just exponential growth. It is something that grows based on how much you already have
- What happens when we add an ϵ times Brownian Motion term to our economic model? What is the difference between $\lim_{\epsilon \to 0} \lim_{t \to \infty}$ and $\lim_{t \to \infty} \lim_{\epsilon \to 0}$ on the growth model (i.e. logistic growth)

6.3 May 1, 2020

1. economics systems should not be understood from discontinuity, but in terms of limits
   i.e. how do we get price to be zero (or would it be infinite, like a assymptote?) when quantity is zero
2. we need to think about how instantaneously differs from frozen time supply and demand models. Are they really the same?
3. Is there a different local demand curve that we need to focus on? Think multiple scales.
4. Imagine the most reasonable economic situation and try to write an equation to describe that
5. Always be thinking where does this come from. Question everything you read and think about how to modify it. $\leftarrow$ especially when taking econ courses
6. we need some kind of history dependent term that can handle people's awareness of the product. It's probably not going to be a function of price/quantity though
7. Have people in economics looked at hysteresis? It should be built into the model, but we may expect to see this in what we do

To Do:

1. Check-out the basic model plot? Why is quantity not being taken into account? Everything should converge to the point, not just the line in 2-D
2. Read the intro that Hong sends out regarding different types of models. I need to be able to explain the importance of mechanistic models and convince others that this is a meaningful pursuit
3. Read about the different aggregation models in economics for micro to macro scales, what do they have, where does it come from?

6.4 May 08, 2020

- partial derivative of utility is assuming everyone maximizes at the same time
- utility relates to random behavior? - aka difference preferences for buying things
- what does it mean to have a stochastic model to describe when people buy what?
  - and to have utility theory to describe this behavior
  - i.e. Markov or Poisson process...
  - at given price level people buy this product at a certain rate

- utility is a fundamental part of what is happening as part of mathematical theory so we can back out the (one) "utility" function from the dynamics - this has mathematical basis i.e. a landscape like Gibbs function
  - it should be a distribution because it describes the entire community
- But isn't a utility function only defined as such when it is concave and increasing?
- So show theory of utility function in connection to agent based dynamics
- Some kind of conservation of mass in terms of quantities supplied and demanded

To Do List:

1. Understand how to use utility function to understand equilibrium - type up a doc to show in meeting next week
2. Ask Yuchen about paper of supply and demand functions
3. Work on the streamline plot to ensure quantity is being plotted (supply and demand not price and quantity?)
4. I need to research how multiple competing goods and marginal utility combine

6.5 May 15, 2020

Today we looked over the notes I wrote-up for how marginal-utility plays a role in the demand function (at least for Marshallian Demand). It was clear that all of the current assumptions about how utility functions should look (monotonic and strictly convex) were chosen purely out of mathematical convenience so that the optimization problem set-up would have a unique solution that is relatively simple to compute. Hong came up with the idea to explore "utility" not as an ordering of preferences, but as a probability distribution. Moving forward it is my job to make sure I fully understand what this means. Here are my notes on what is going on:

- We are working with a joint probability distribution for all of the goods in our system. Starting with two goods, this is a joint distribution on (x_1, x_2).
- We maintain the constraint that $p_1x_1 + p_2x_2 = y$ where $\mathbf{p}$ is the vector of market prices and y is the budget of the consumer.
  - The goal here is to collapse the distribution onto a single line in the x_1, x_2 plane.
  - This also means that choosing a value for either x_1 or x_2 gives us the exact value of the other, so it is like reducing down to one variable.
  - This should lead to a marginal on y. $\leftarrow$ don't understand this yet (wouldn't it be x_1?)
- It would take massive surveying and strong statistical analysis to practically use this model. "With \$$y$ how much of x_1 and how much of x_2 would you buy?"
- Since we don't have the statistics (maybe I should do a literature search on this), we will instead work with the Strong Law of Large Numbers and Central Limit Theorem.
- We want to assume a large enough scale so that it is a normal distribution of "preferences"
- This whole time our probability will have to be conditioned on price. aka how can I encode the constraint into the system?
- If the population goes to ∞ what does the distribution look like? It should become a distribution of the averages of the normal distributions because it was conditioned on price?

6.6 May 22, 2020

- we don't need to actually define $(\Omega, \mathcal{A}, \mathbb{P})$. Just stating it exists is enough. Leave it general because we can never really define all of the parameters that are technically in Ω, we can only estimate it with the use of random variables
  `https://arxiv.org/pdf/1902.09536.pdf`
- This means that we assume that μ exists s.t. we can state that X_1 & X_2 exist and have a probability measure
- we may be interested in examining the log of the joint distribution function $f_{X_1 X_2}$ (log is already consistently used as a utility function in econ)
- Write the conditional(marginal?) distribution of X_1 and explore using it in the optimization problem put forth in micro
- We want to just start with i.i.d.

- What is the difference between maxing the probability distribution for X_1 and for X_2? How does it compare to optimizing both at the same time?
  - Does optimizing with the condition of being $\leq$ instead of $= y$ lead to a different conclusion?
  - For the joint distribution we would probably want to apply the CLT and LLN in multiple dimensions. Also there would be two possibilities, the max is in the interior of B, or on the edge.
  - note that the log of a Gaussian function is quadratic s.t. the max is at the mean of the function
  - Why do we need to subtract the means when applying the CLT?
  - when we fix y to get a marginal distribution of X_1, we can then apply the SLLN to show that $\exists X_1^*$
- What does the Cauchy Distribution look like, when is there no generalized CLT? i.e. what is a Cauchy Distribution
- budget is a complicating factor and we don't actually need it, it tells the story but we can start without it
- We need some scaling factor on the utility function. $\leftarrow$ aka the probability distribution
- We should come to the conclusion that it is not a unique utility function, just something that maintains the relationship of X_1, X_2
- The whole time I am going through this, especially applying the CLT, think about how it relates to perturbation theory and taking limits for each order of convergence separately.

6.7 June 05, 2020

Notes on writing style

- Be clear when and where Ω plays a role
- When are you talking about a set of events i.e. $B = b$? One needs to be very careful about saying that a constant is equal to a random variable. In continuous space this is more like $B \in [b, b + db]$, aka the infinitesimal set of events near $B = b$.
- Probability is **NOT** a simple change of variables, always best to write out the probability density functions and cumulative density functions.

Notes about progressing in the research

- Regarding why in the section on supply as a probability space $\mathbf{y}^* = \mathbf{x}^*$, we are assuming equilibrium while discussing the maximum values. This means that all of the same arguments that economists employ should apply here too for why this would be true.
  - Why should supply exceed demand? This leads to money loss.
  - If demand exceeds supply, this is acknowledging lost money (can raise price OR make more).
  - These arguments are essentially the dynamics of how we get to $\mathbf{y}^* = \mathbf{x}^*$
- Stochastic means that our $\overline{\mathbb{P}}$ distribution will come out of the dynamics
- Without treating this as a stochastic process we certainly are missing a piece of the explanation, so we need to be at equilibrium when we examine supply or demand individually
- Perhaps a ratio of suppliers to consumers would be good to examine (aka more than one supplier in a market)
- There needs to be no differentiation between the goods supplied by different suppliers. As soon as consumers can tell a difference, they are treated as separate things X_1 and X_2
- Suppliers dynamics are always trying to match consumer purchasing, but consumers do their own thing... but is there any consumer dynamic that tries to match the supplier's supply?
- We want to add stochasticity to the supply and demand story, aka a B_t term
- Is it reasonable to combine both uncertainties into one term? When wouldn't it be, it it was dependent B_t?
- Moving into a dynamical systems framework means that we are working in supply/time and consumption/time, so would this be memoryless?