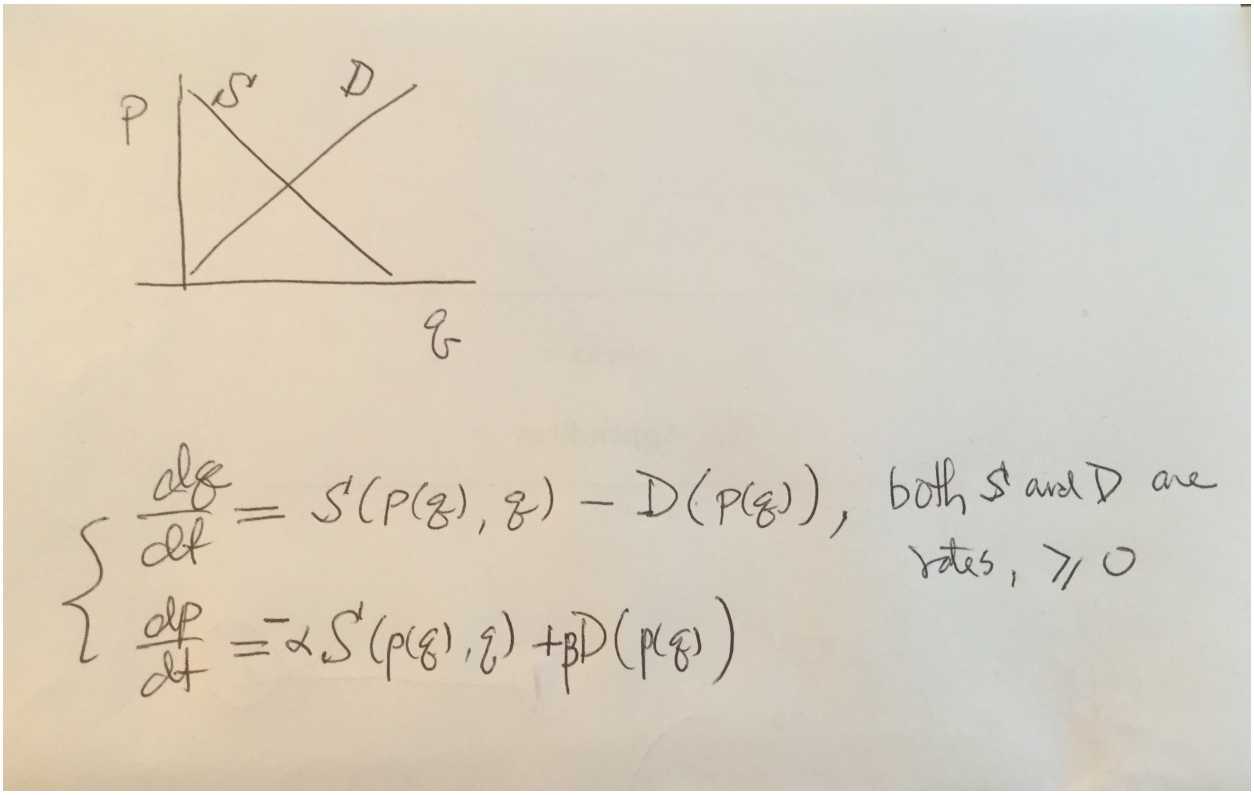

Figure 1: The System of ODE's that Hong created to describe supply and demand

"Does it make sense? If not why? If yes, has anyone already done that?

In which α and β are two positive constants that convert the rate of change in q to that or rate of change in p.

First thing we like to check is whether this reproduce the figure in `https://en.wikipedia.org/wiki/Supply_and_demand`"

1 Problems

- D should also depend on the quantity supplied.
- It seems like it would be less relevant/useful to model q, the surplus quantity supplied, than changes in the quantity demanded and the quantity supplied. This is partially because then when we see changes (since presumable preferences do have the possibility of changing over time) we can relate this to the model better.
- I don't think S and D should be rates but functions that are greater than or equal to 0. This way aggregate supply and aggregate demand could also be modeled in the same system, and these are generally not considered to be linear.

2 Basic Model

- Smale 1976
- Dynamic Economics textbook

A Simple Dynamical System

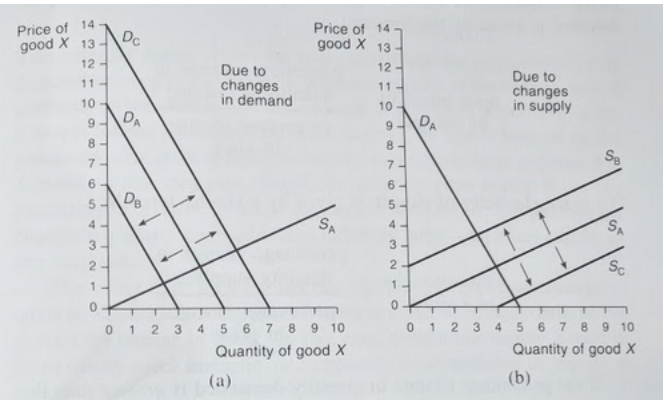

Figure 2: Walrasian supply and demand

- In a typical economy the price is set by producers, and then adjusted based on observations
- This means typically what is observed in the economy is *dynamic adjustment process*
- To not just discuss that price will increase or decrease so that supply matches demand, we can examine the underlying dynamics behind price movement

Model 1

$$\begin{aligned} q_d =& a - bp & b >& \quad 0 \\ q_s =& c + fp & f >& \quad 0 \\ \frac{\mathrm{d}p}{\mathrm{d}t} =& \alpha(q_d - q_s) = \alpha(b+f)p - \alpha(a-c) & \alpha >& \quad 0 \end{aligned}$$

q_d, q_s, & p are continuous functions of time.

Supply and Demand

At equilibrium we have that $\frac{\mathrm{d}p}{\mathrm{d}t} = \alpha(q_d - q_s) = 0 \;\Rightarrow\; p^* = \frac{a-c}{b+f}$ and $q^* = \frac{af+bc}{b+f}$

Without Stocks

Assumption 1. *In disequilibrium the short side of the market is transacted. In other words only current supply is available to fill demand and there is no stock of goods.*

- Excess demand gives a signal for producers to increase supply and price
- Excess supply gives signal for producers to decrease supply and price
  - It says nothing about what happens to excess goods...

With Stocks

Assumption 2. *Stocks are sufficient to meet demand at any price, and price adjusts to changes in stock levels.*

Model 2

$$\begin{aligned} i =& i_0 + \int_0^t q_s - q_d \mathrm{d}t \\ \frac{\mathrm{d}i}{\mathrm{d}t} =& q_s - q_d \\ \frac{\mathrm{d}p}{\mathrm{d}t} =& -\alpha\frac{\mathrm{d}i}{\mathrm{d}t} = \alpha(q_d - q_s) \qquad \alpha > 0 \end{aligned}$$

- The price path solution is the same

- However now we allow for the quantity traded to increase s.t. $q(t) = q_d(t) \; \forall \; p$

Further systems

- This approach can be applied to labor markets with and without flexible wages
- These labor models are then related to supply and demand in that decreases in price indicate decreases in labor demand
  - In sticky wage models we make assumptions about how demand once again determines dynamics
  - There are also assumptions accounting for how there are asymmetric adjustments: wages increase faster than they decrease
- Discretizing the time flow leads to what is called the cobweb model