1 Generic Model from Literature

A Simple Dynamical System - Economic Dynamics: Phase Diagrams and Their Economic Application by Ronald Shone

$$q_d = a - bp \qquad b > 0 \tag{1}$$
$$q_s = c + fp \qquad f > 0 \tag{2}$$
$$\frac{\mathrm{d}p}{\mathrm{d}t} = \alpha(q_d - q_s) = \alpha(b+f)p - \alpha(a-c) \qquad \alpha > 0 \tag{3}$$

There is an equilibrium at $p(t) = \frac{a-c}{b+f} + \left[p_0 - \left(\frac{a-c}{b+f}\right)\right] e^{-\alpha(b+f)t}$ $q_d,\ q_s,\ \&\ p$ are continuous functions of time.

Analyzing the Jacobian of this system for q_d and p we get

$$\begin{matrix} 0 & -b \\ 0 & \alpha(b+f) \end{matrix}.$$

From here I calculated the eigenvalues to find $\lambda_1 = \alpha(b+f)$ and $\lambda_2 = 0$. $\lambda_2 = 0$ could be a problem if we were dealing with a nonlinear system, however this is a linear system so the behavior of the fixed points is determined by λ_1. This means that if $\lambda_1 < 0$ the system is stable, and if $\lambda_1 > 0$ then it is unstable. However we know that $\lambda_1 > 0$, since the restraints on our system require that $\alpha > 0$, $b > 0$, and $f > 0$. For b and f, this is to account for how we know demand decreases and supply increases as price increases, which are well established results in economics except in extreme outliers. $\alpha > 0$ comes from examining $(q_d - q_s)$. If $\alpha < 0$, then we could rewrite it as $|\alpha|(q_s - q_d)$, but demand does not shift to meet supply, supply and price both shift to meet demand in an economy. Thus we see that for all possible versions of this system, we have an unstable equilibrium.

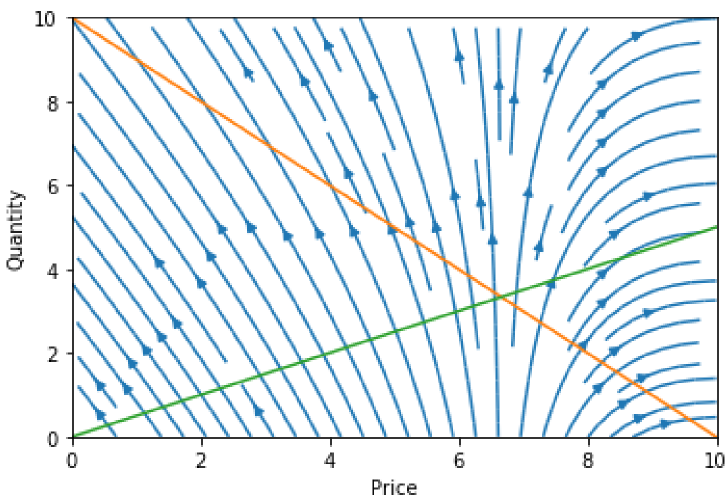

Figure 1: The vector field plot of the system described in (1)

Numerically I analyzed the system with $a = 10,\ b = 1,\ c = 0,\ f = 0.5$, and $alpha = 0.5$. The orange line is the quantity demanded that I assumed in equation (1). The green line is the quantity supplied that I assumed in equation (2). Consumer utility for the product is assumed to not change over time, and so these demand lines are both held constant over time. We see in Figure 1 the equilibrium point of $(6\frac{2}{3}, 3\frac{1}{3})$ in the classic supply and demand system, and also how for this system a price of $6\frac{2}{3}$ does not change. Further, the quantity demanded can also be thought of as the maximum quantity sold at any given price, so only the dynamics in the bottom left half of the plot should be analyzed.

As expected from the analysis, in figure 1 we see that prices do not flow towards the price equilibrium, but instead to move away from it. This is made even more clear in figure 2, which is the time evolution

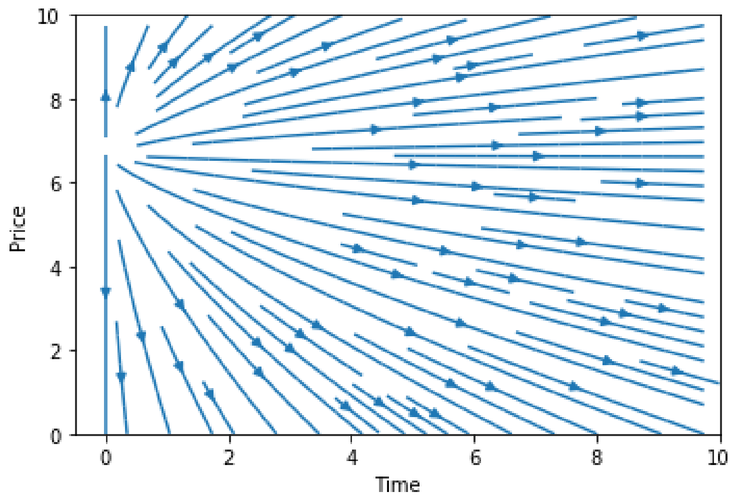

Figure 2: The evolution of price over time according to (3)

of price. In this system we see that the price can only reach the equilibrium of $6\frac{2}{3}$ by already starting at that value. This is in direct conflict to current theory which states that prices will converge to equilibrium. To achieve a dynamical system that fits a stable equilibrium, we must break one of our main underlying economic assumptions.

There are many problems with this model. First the dynamics don't make sense. Then, this model does not indicate at all how things evolve if the demand or supply shifts. With the underlying issues in the model, it is not clear that any conclusions drawn would be meaningful.

2 My Model

Supply and Demand are positively correlated. As demand increases supply increases to match, but by a smaller amount. This smaller amount allows the price to also increase, creating greater profit margins. This is in an idealized market of course, since if there are competitors, that could keep a company from raising their prices. However, a company will never purposefully supply more than the amount demanded since that will drive down prices and profit margin, while also increasing stocks keeping profit down in the future. When demand quantity decreases we would then expect the supply quantity to decrease more than this. The reasoning is similar to why it would increase less than with demand increase. We can call this amount of disconnect in change δ. δ is then approximately $(\frac{dS}{dq} - \frac{dD}{dq})^{-1}$ according to the linear supply and demand model taught in macroeconomics textbooks.

Thus we have that

$$\dot{q}_S = (\frac{dS}{dq} - \frac{dD}{dq})^{-1}(q_D + \dot{q}_D) \tag{4}$$

$$+ \epsilon \ddot{q}_D \tag{5}$$

From equation (4) we come to the big question of what does $\frac{dS}{dq}$ and $\frac{dD}{dq}$ mean? These are the slopes of the linear lines in the model displayed in Figure 3. As written though, we are missing the dependence on time, which means that we just shift the intercepts as shown. This means that we need to allow a time dependence in $\frac{dS}{dq}$ and $\frac{dD}{dq}$. Even if we do that though, I have still only thought about them as linear so that we can calculate a constant coefficient out front of $(q_D + \dot{q}_D)$. What happens if the slope is non-linear so that it matters what the current market quantity/prices are?

Another aspect to the model is that it is possible that to an attempt to predict the necessary supply in the future, companies add on a second derivative term such as (5). It would be optimizing in that if they expect things to change quickly, they can try not to run out of stock in stores. This may be especially

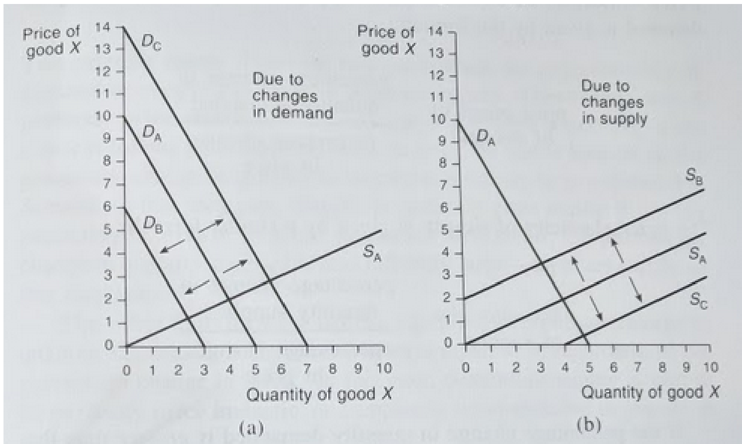

Figure 3: Supply and Demand as a classic linear system.

relevant for the case of perishable goods where rather than always being behind demand until it reaches a equilibrium, companies are trying to be ahead and always have supplied what the consumers will purchase. This would likely need an unknown constant in front of the term that is less than one (it should have less cost associated to under-perform in this term).

We also want to look into how the quantity demanded changes changes over time. I believe as a consumer, that this will be based mostly on the price, where p is the transaction price taking place in the market. There is also something to be said for ease of access and rarity of the goods, so I believe that a positive quadratic term makes sense for the supply available effecting the quantity demanded. I argue that change in quantity supplied is not noticeable to a consumer. Either the store has the good or it does not, I have no idea what others are up to. Thus the change in quantity supplied term is left out.

Something interesting that I always thought was left out is rarity. It is really obvious in trading cards, but also in Grey Poupon mustard. Something can be more expensive and that attracts people to it perhaps because of ideas of luxury. Because of this sort of thing, I do not believe that the interaction between supply and demand is linear, so then the question is what kind of non-linearity makes sense.

$$\dot{q}_D = -c_1\dot{p} - c_2(q_S - c_3)^3 \tag{6}$$

In this system we can assume that companies may change their supply without a large effect on consumption, as long as the consumers can still easily access a good (or highly value the rarity of it which is handled in c_3).

2.1 Taking Stocks of Goods into Account

The stock of excess supplied goods also needs to be taken into account. This is only relevant for the case of non-perishable goods, just as I would guess that the second derivative term is only relevant for the case of perishable goods. Do companies want at least a certain amount stockpiled? The only difference in the model is that companies would make less money, so we can ignore this in terms of understanding the dynamics. One major difference is that the as stockpiles grow, prices tend to drop as a way of clearing them out. So it's not just about the amount supplied, but the amount in total available that determines changes in price. We definitely know that

$$\dot{b} = q_s - q_d$$

3 Problems

- It seems like it would be less relevant/useful to model q, the surplus quantity supplied, than changes in the quantity demanded and the quantity supplied. This is partially because then when we see changes (since presumable preferences do have the possibility of changing over time) we can relate this to the model better.

- I don't think S and D should be rates but functions that are greater than or equal to 0. This way aggregate supply and aggregate demand could also be modeled in the same system, and these are generally not considered to be linear.