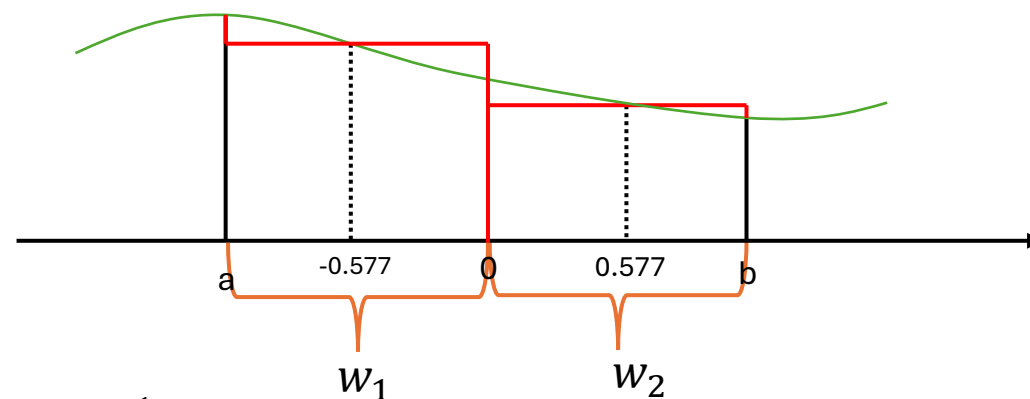
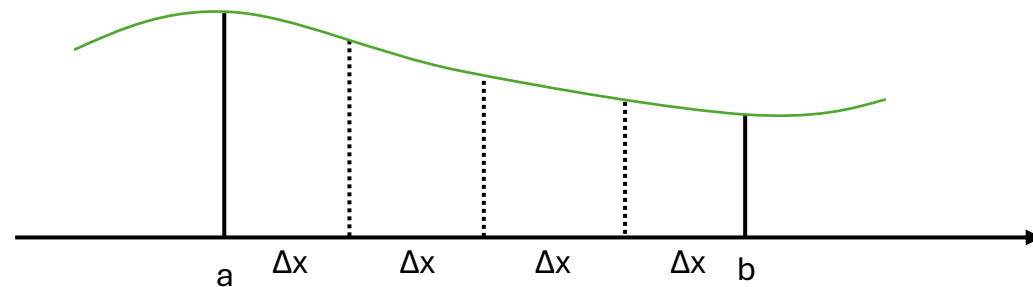


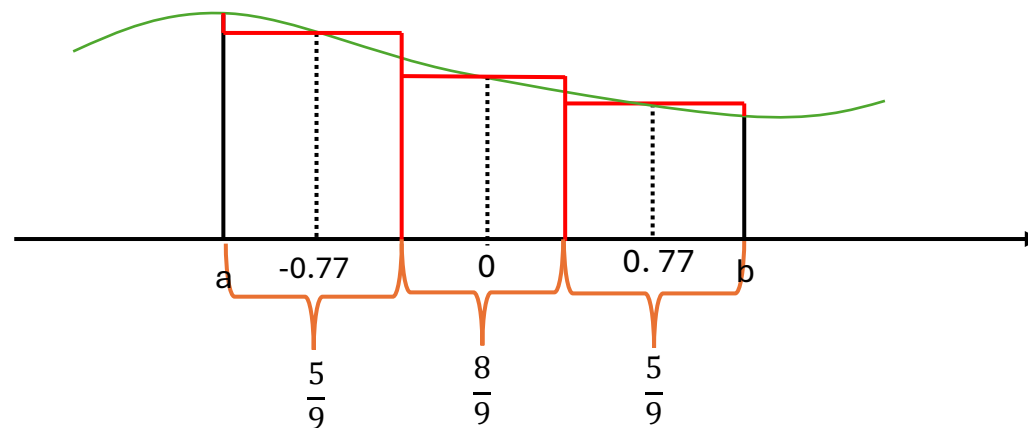


MES – całkowanie numeryczne

Number of points, n	Points, x_i	Weights, w_i
1	0	2
2	$\pm\sqrt{\frac{1}{3}}$	1
3	0	$\frac{8}{9}$
	$\pm\sqrt{\frac{3}{5}}$	$\frac{5}{9}$
4	$\pm\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}}$	$\frac{18+\sqrt{30}}{36}$
	$\pm\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}}$	$\frac{18-\sqrt{30}}{36}$
5	0	$\frac{128}{225}$
	$\pm\frac{1}{3}\sqrt{5 - 2\sqrt{\frac{10}{7}}}$	$\frac{322+13\sqrt{70}}{900}$
	$\pm\frac{1}{3}\sqrt{5 + 2\sqrt{\frac{10}{7}}}$	$\frac{322-13\sqrt{70}}{900}$



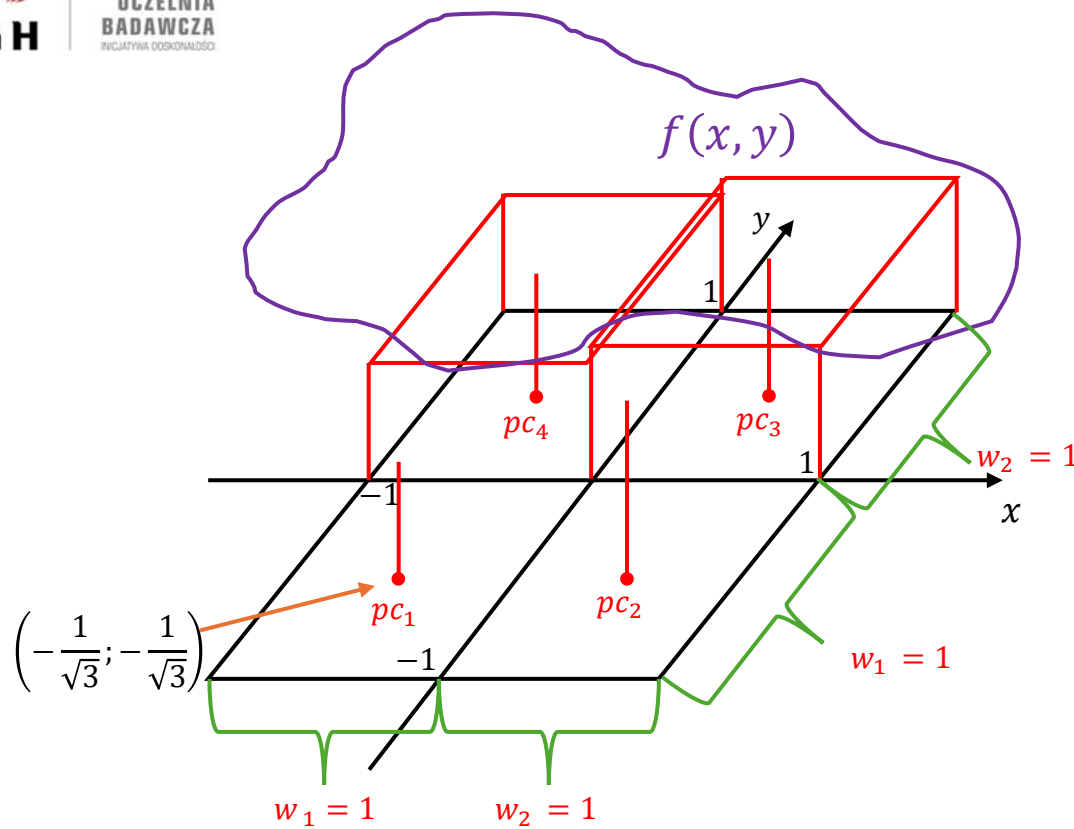
$$\int_{-1}^1 f(x)dx = f(p_{c1}) * w_1 + f(p_{c2}) * w_2$$



$$\int_{-1}^1 f(x)dx = f(pc_1) * w_1 + f(pc_2) * w_2 + f(pc_3) * w_3$$

$$f(x) = 2x^2 + 0.1x + 3$$

$$\int_{-1}^1 f(x)dx = ?$$

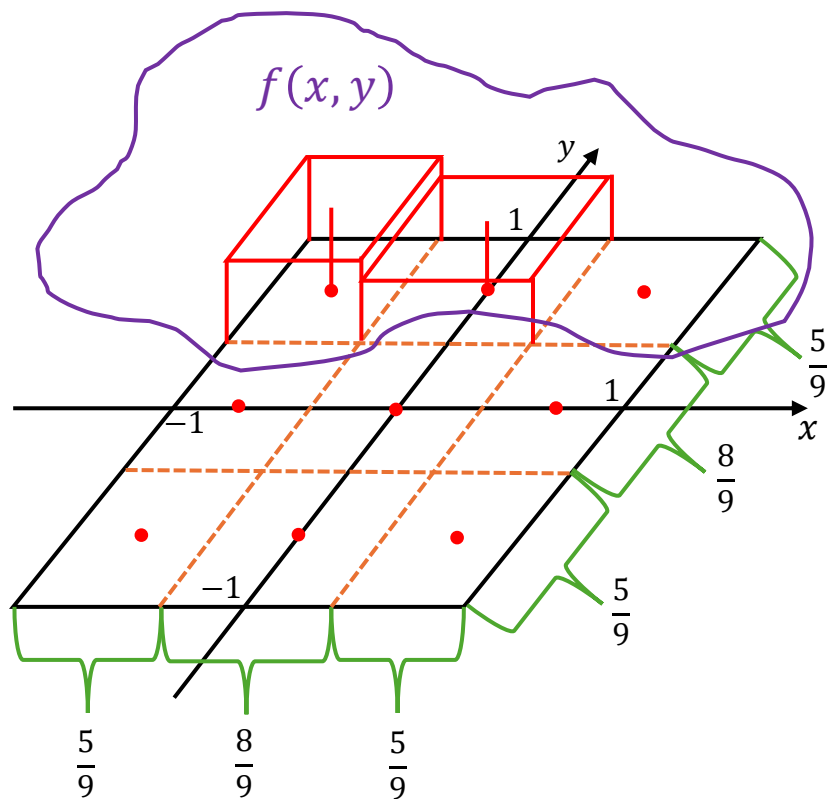


$$\int_{-1}^1 \int_{-1}^1 f(x, y) dx dy = f(pc_1) * w_1 w_1 + f(pc_2) * w_2 w_1 + f(pc_3) * w_2 w_2 + f(pc_4) * w_1 w_2$$

$$\sum_{i=1}^n \sum_{j=1}^n f(x_i, y_j) * w_i w_j$$

$$f(x, y) = -2x^2y + 2xy + 4 \quad \int_{-1}^1 \int_{-1}^1 f(x, y) dx dy = ?$$

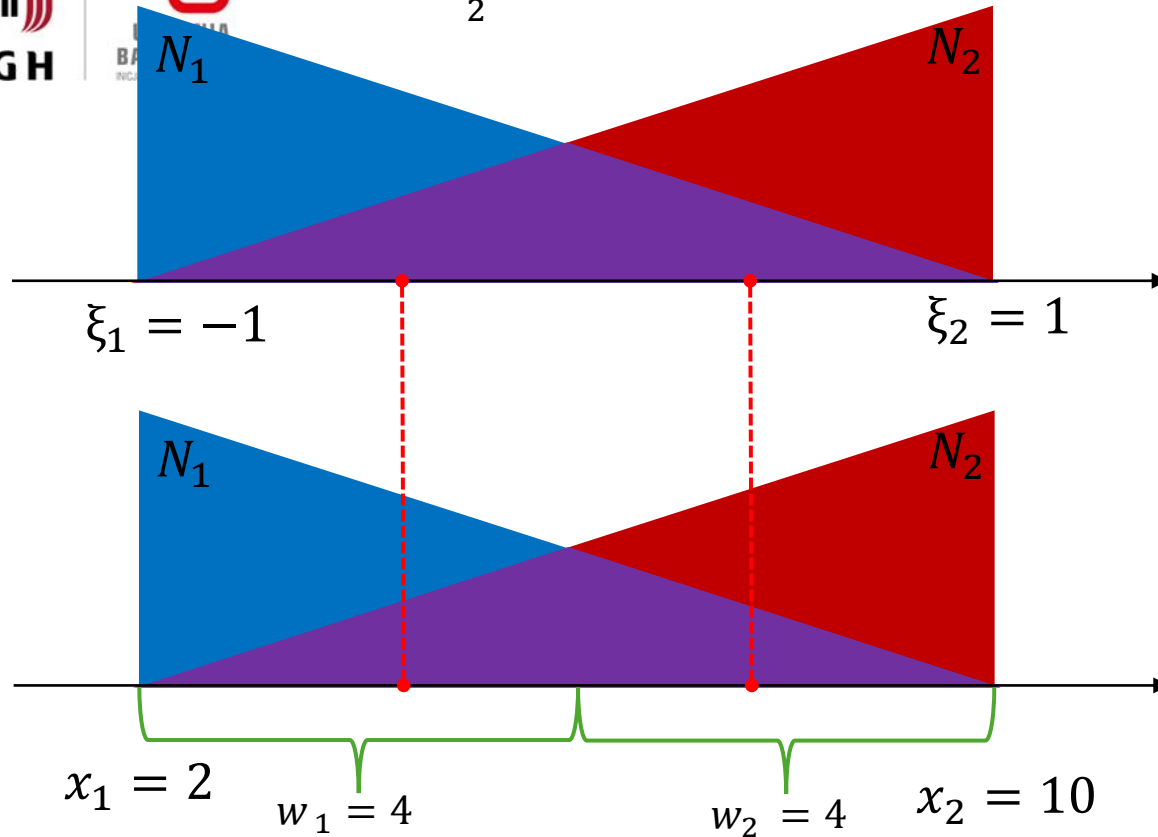
	pc_1	pc_2	pc_3	pc_4
x	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$
y	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
w_1	1	1	1	1
w_2	1	1	1	1



$$\sum_{i=1}^n \sum_{j=1}^n f(x_i, y_j) * w_i w_j$$

$$f(x, y) = -5x^2y + 2xy + 10 \quad \int_{-1}^1 \int_{-1}^1 f(x, y) dx dy = ?$$

$$\int_2^{10} f(x) dx = ?$$



$$x_{pc1} = N_1 x_1 + N_2 x_2 = 0.788 * 2 + 0.212 * 10 = 3.696$$

$$x_{pc2} = N_1 x_1 + N_2 x_2 = 0.212 * 2 + 0.788 * 10 = 8.304$$

$$N_1 = \frac{x_2 - x}{x_2 - x_1} = \left| \begin{matrix} x_1 = -1 \\ x_2 = 1 \end{matrix} \right| = \frac{1 - x}{1 - (-1)} = \frac{1}{2}(1 - x)$$

$$N_2 = \frac{x - x_1}{x_2 - x_1} = \left| \begin{matrix} x_1 = -1 \\ x_2 = 1 \end{matrix} \right| = \frac{x - (-1)}{1 - (-1)} = \frac{1}{2}(1 + x)$$

$$N_1(-0.577) = \frac{1}{2}(1 - (-0.577)) = 0.788$$

$$N_2(-0.577) = \frac{1}{2}(1 + (-0.577)) = 0.212$$

$$\det J = \frac{\partial x}{\partial \xi} = \frac{\Delta x}{\Delta \xi} = \frac{x_2 - x_1}{\xi_2 - \xi_1} = \frac{10 - 2}{1 - (-1)} = 4$$

$$\int_2^{10} f(x) dx = (f(x_{pc1}) * w_1 + f(x_{pc2}) * w_2) * \det J$$

$$\int_2^{10} (2x^2 + 0.1x + 3) dx = ?$$

Praca domowa



Zaimplementowanie funkcji, która w układzie $(-1;1)$ realizuje całkowanie 2 i 3 punktowe. Funkcja otrzymuje schemat całkowania i całkuje dowolną funkcję 2D.