Tyler Pranger

Professor Arias

Language Study

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1. a) Variable

    b) Constant

    c) Variable

    d) Constant

    e) Non-ground atomic formulas

    f) Atomic formula

    g) Atomic formula

2. (csg("CMP T220", S, G) AND snap(S, "L. Van Pelt", A, P)) → answer(G)

3. a) $\forall X(\exists Y(\text{NOT } (p(X) \text{ OR } (P(Y) \text{ AND } q(X)))))$

    b) $\exists X((\text{NOT } P(X)) \text{ AND } (\exists Y \, P(Y)) \text{ OR } (\exists X \, q(X,Z)))$

4. See picture on the back

5. $(\exists X)((\text{NOT}p(X) \text{ AND } q(X)) \text{ AND } ((\exists Y)(p(Y)) \text{ OR } (\exists Z)(q(Z))))$

6. a) $\forall(\text{csg}(C, S, A) \text{ AND } \text{snap}(S, \text{"C.Brown"}, A, P))$

    b) $\text{NOT}(\forall(\text{csg}(C, S, A) \text{ AND } \text{snap}(S, \text{"C.Brown"}, A, P)))$

7. a) $(\forall X)(\exists Y)(\text{loves}(X, Y))$

This instance is true if X is a husband and Y is a wife. All husbands love their wives, if they have them. This could be false, however,

b) p(X) → NOTp(X) This would be false in almost all instances, such as "if it is raining, then it is not raining."

c) (∃X)p(X) → (∀X)p(X) This is always true, such as the situation "If someone chair, then there is a chair."

d) (p(X, Y) AND p(Y, Z)) → p(X, Z) This would be true if someone said "If it is raining with a chance of snow, and if there is a chance of snow, and you bring an umbrella, then if it is raining, you bring an umbrella." However, it would be false if you said "If I am doing homework because I have a class, and I have a class because I am failing, I am doing homework because I am failing." You do not only do homework if you are failing, so this would be an untrue statement.

8. a) This statement is a tautology because no matter what side one object is put in an OR statement, One of them must exist, which means they will have the same truth tables. Therefore, they will both be equal, thus being all true, or a tautology

b) This statement is a tautology because they both boil down to the statements. p(X,Y) has the same truth statements as (p(X, Y) AND p(X, Y)), since they both deal with ANDs for X=x and Y=y. Thus, them together would produce all true values, being equal, and be a tautology.

c)This is a tautology because the first statement is saying that if there is an instance of p(X), then it is false. Which would essentially reverse the truth table output of p(X). This

is the same thing that NOTp(X) would do, so by having them both be equal, these statements together produce a truth table output of all Trues, making it a tautology.

9. a) $(\exists X)(NOT\ p(X))\ AND\ ((\exists X)q(X, Z))\ OR\ (\exists Y)p(Y)))$

    b) $(\exists X)(p(X)\ OR\ (X)q(X)\ OR\ r(X))$

10. a) $\forall X(p(X)\ AND\ (\exists Y)q(Y)p(Y)$

    b) $(\forall X, Y)(p(X, Y)\ OR\ p(Y, X))$

11. Yes, because in both instances, p(X,Y) and q(X) are being shown to exist, meaning that because of the law, they would be tautologies, meaning that they are equivalent when together.

12. a) $NOT\ \forall(X)(p(X))\ OR(NOT\ (\exists Y)p(Y)))\ OR\ ((\exists X)q(X, Z))))$

    b) $(\forall X)(p(X)\ AND\ (X)q(X)\ AND\ r(X))$

13. $NOT((Q1x)x)\ OR\ (Q2y)y)$

14. $((\forall X)(\forall Y)NOTp(X, Y))$

    $(\forall X)p(X) -> (\exists Y)q(X, Y)))$

15. It is true that E would be a tautology whenever $(\exists X)E$ is a tautology because if the latter is a tautology, that means there is a true case of E that exists, and only a true case. SO therefore, E would have to always be true.

a)

∀x
│
∃y
│
NOT
│
OR
/ \
P(x)   AND
/   \
p(x)   q(x)

b)

∃y
│
NOT P(x)
│
AND
/   \
OR
/   \
∃y   ∃x
│   / \
pY   q(x) q(z)