CS 520 Final Question 4 Solution

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1 Solutions

1.1 Question a)

Solution:

Let the number of goat in location A denoted as $number\_A$, the number of goat in location B as $number\_B$ and the number of goat in location C as $number\_C$. To begin with, we know that one goat is in one of the three locations, therefore we have $number\_A + number\_B + number\_C = 1$. (By the way, this formulation looks unlike of what most people think of a logical formulation should look like, but I have asked prof. Cowan in his office hour and he said such type of formulation can also be accounted as a logical formulation.)

Now we express the problem in a probabilistic way. Let the event A be that the goat is in location A, we then similar have event B and C. Because any of the three events is exclusive to the other two, we have probabilities that the goat in a certain location in A, B and C as $P(A), P(B), P(C)$.

1.2 Question b)

Solution:

At this point, under the logical formulation, just take selecting A as an example. If there is a goat in A, we have notation $number\_A = 1$, since we have $number\_A + number\_B + number\_C = 1$, we can conclude that $number\_B + number\_C = 0$, in other words, no goat is in location B or C. On the other side, if there is no goat found in A, we have notation $number\_A = 1$, since we have $number\_A + number\_B + number\_C = 1$, we can conclude that $number\_B + number\_C = 1$, in other words, the goat is in location either B or C. The similar method can be applied to situations where we select B or C to check.

At this point, we can see that as we select one of the three locations, we have no way to decide which location is more likely to have the goat. In this case, we have to select one location and take a look ourselves. Even worse, if there is no goat there, we would go back to the same situation again because there is still no useful prior knowledge to help us make any decisions further. Therefore, there is no obvious choice of best action so far.

1.3 Question c)

Solution:

Based on the given statement, before taking any actions, we have $P(A) = P(B) = P(C) = 1/3$ because the goat is in exactly one of the three locations. Now we select A, if the goat is in the location, then $P(B) = P(C) = 0$. If the goat is not in the location, then it must be in either B or C and thus $P(B) = P(C) = 1/2$. Now we see that we cannot make any wise determinations here because there is no information indicating whether it is more likely that the goat is in B or vice versa, unless we go on making explorations further. And if we select B or C, the same situation would happen again. Therefore, there is no obvious choice of best action so far.

1.4 Question d)

Solution:

Based on the given information, we can conclude that $number\_B = 0$ because it is known that the goat is not in B. At this point, as we use this information in the problem represented as $number\_A + number\_B + number\_C = 1$, we can easily conclude that $number\_A + number\_C = 1$. However, we just stop here and cannot make any conclusions further because we have no information to decide if the goat is more likely in A or C.

1.5 Question e)

Solution:

This is exactly the **Monty Hall problem (or three doors problem)**. Now we select location A and it is known that the goat is not in location B. At this point, we have two decisions to choose from: 1. change our mind and choose another location C, 2. insist that the goat is in location A and make no changes. To illustrate the whole situation, **we simply assume that the bot hasn't yet told us which location has no goat and that the goat is in location A.**

Because the probability that a goat in any of the three locations is the same, as big as $1/3$, here we can see that if we take information from the bot and change our mind, different from the initial situation where the probability to find the goat is $1/3$, current probability to find the goat becomes $2/3$ (2 situations found versus 1 situation not found). **And no matter the goat is in which location, any location is selected by the user or which other location is reported by the bot, these situations are essentially the same problem that one goat is hidden in one certain unknown location and a location is known to have no goat.** Suppose the goat is in location A, there are three situations:

1. We choose location A and suppose the bot chooses location B to report the absence of the goat. **If we change our mind and choose location C instead, then we would not find the goat. However, if we insists that**

we choose A, then we would find the goat. Here we simply take the example of report location B, but the whole situation would not change if the bot chooses location C and we make one of the decisions above.

2. We choose location B and suppose the bot chooses location C to report the absence of the goat. **If we change our mind and choose location A instead, then we would find the goat. However, if we insists that we choose B, then we would not find the goat.** Here we simply take the example of report location C, but the whole situation would not change if the bot chooses location A and we make one of the decisions above.

3. We choose location C and suppose the bot chooses location B to report the absence of the goat. **If we change our mind and choose location A instead, then we would find the goat. However, if we insists that we choose C, then we would not find the goat.** Here we simply take the example of report location B, but the whole situation would not change if the bot chooses location A and we make one of the decisions above.

The whole situation is shown in the figure below.

A(has the goat)	B(empty)	C(empty)
Player is here	Bot finds no goat	
	Player is here	Bot finds no goat
	Bot finds no goat	Player is here

Now let's go back to the given situation with the assumptions that we select location A and it is known that no goat is found in location B. Based on the analysis we made above, we can conclude some new probabilities in terms of the situation, that is:

$P(C|\overline{B}) = 2/3, P(A|\overline{B}) = 1/3$.

1.6 Question f)

Solution:

Based on the logical formulation from question b) and d), currently we know that the problem becomes $number\_A + number\_C = 1$ since it is known no goat is in location B, or $number\_B = 0$. If we still select location A, then we would come into one of the two situations: 1. if the goat is there, then we know the goat cannot be in location C, which means that $number\_A = 1$ and $number\_C = 0$, this situation is satisfied; 2. if the goat is not there, then we know the goat must be in location A because we know the goat is not in B or C, in other words, $number\_A = 1$ and $number\_B = number\_C = 0$, the situation is also satisfied. And the same situation would happen if we re-select location C because we still have no prior knowledge. Of course, if we re-select location B, we would surely find no goat.

So we see that both two consequential situations of re-selecting a certain location are satisfied and thus we cannot decide which action is better than the other. In other words, there is no obvious choice of best action either.

1.7 Question g)

Solution:

Based on probability formulations from question c) and e), suppose we are still in the assumption that we select location A and it is known that no goat is found in location B. Based on information we have obtained so far, now we can calculate the probabilities that we can find the goat in location A, B and C:

$P(A) = P(A|\overline{B}) * P(\overline{B}) + P(A|B) * P(B) = 1/3 * 1 + 0 * 0 = 1/3$

$P(B) = 0$, because it is known that no goat is found in location B.

$P(C) = P(C|\overline{B}) * P(\overline{B}) + P(C|B) * P(B) = 2/3 * 1 + 0 * 0 = 2/3$

Based on the probabilities above, we know that if we re-select location A (in other words, keep sticking with the previous decision), we have possibility to find the goat as large as $1/3$, if we re-select location C, then the possibility becomes $2/3$. And if we re-select location B, we can never find the goat, which means the possibility becomes 0. Obviously, the best action is to re-select location C.

1.8 Question h)

Solution:

Based on the calculation from question d) and f), we know that we still no way to decide if we should be stick with location A or not. As we have seen in question d), even if we are now able to know that location B can be excluded from our decisions, we can not make sure if choosing location A is better than choosing location C, or vice versa. Therefore, we cannot decide which action is better here.

1.9 Question i)

Solution:

Based on the calculation from question e) and g), we know that we should probably not be stick with location A and make a change. As we have seen in question e), in form of the Monty Hall problem, as soon as we know one location where the goat is not found, the distribution of probabilities that we can find the goat in the rest two locations changes reasonably (simply from previously being equal to later then being unequal), which can provide us with rich information that if we change our mind to select another location, it would be more likely to find the goat.

1.10 Question j)

Solution:

Based on the questions above, we can make two conclusions. For the **LogicalGoatDiscoveryBot**, even if we can grasp extra information from **CBMHBot**, we can still find no best action to take, only to make sure that one location can be excluded because it is known that no goat can be found there. And for the **ProbabilisticGoatDiscoveryBot**, however, the extra information from **CBMHBot** helps us a lot because we now can decide which action is better than the other one by looking at the corresponding probabilities.

Therefore, in terms of the difficulty to make a wise decision at each step, the **ProbabilisticGoatDiscoveryBot** is better than **LogicalGoatDiscoveryBot** because that we are always able to choose a better action each step when using the former one.

1.11 Bonus

Solution:

In this case, the situation where **CBMHBot** selects location C to report is exactly the same to that where it selects location B to report.

Suppose the utility of finding the goat is 1, the utility of not finding the goat is 0, and let $U(stickingwithdecision)$ be the utility we are required to calculate, as shown in previous questions, we have $U(stickingwithdecision) = 1 * 1/3 + 0 * 2/3 = 1/3$, no matter location B or location C is chosen by the bot, because all the situations are the same problem essentially where one goat is hidden in one certain unknown location and a location is known to have no goat.

Suppose the utility of finding the goat is 1, the utility of not finding the goat is 0. As for the utility of switching to location C, we need to calculate it in different situations. In the situation where location B is chosen by the bot, as shown in question i), the utility of switching to C, in other words re-selecting C, suppose the utility of finding the goat is 1, we would have the utility of switching as $2/3 * 1 = 2/3$. And in the situation where location C is chosen by the bot, location C must have no goat. Otherwise, it would not be reported at all. Suppose the utility of not finding the goat is 0, the utility of switching to location C is 0 all the time. To sum these two situations up, the whole utility of switching to location C now is $2/3 * p + 0 * (1 - p) = 2p/3$.

Using the same method, we can calculate the utility of switching to location B is $0 * p + (1 - p) * 2/3 = 2(1 - p)/3$.

Based on the calculation we made above, the utility of switching is $U(switching\ to\ C) + U(switching\ to\ B) = 2p/3 + 2(1-p)/3 = 2/3$, suppose the utility of finding the goat is 1 and the utility of not finding the goat is 0. And we have known the utility of sticking with the initial selection is $1/3$ under the same assumptions. Based on these results, the rational choice is to switch to another unknown location each time a location is reported by the bot, and it does not depend on p at all.