Homework 6

5.1. The Lyapunov function of a Hopfield network is written as

$$V = -\frac{1}{2}(7x_1^2 + 12x_1x_2 - 2x_2^2)$$

Point out the matrix of synaptic weight of the network.

$$W = \begin{bmatrix} 7 & 6 \\ 6 & -2 \end{bmatrix}$$

5.5. Consider a Hopfield network made up two neurons. The synaptic weight matrix of the network is

$$W = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

The bias applied to each neuron is zero. The following are the four possible states of the network:

$x^1 = [+1, +1]^T$
$x^2 = [-1, +1]^T$
$x^3 = [-1, -1]^T$
$x^4 = [+1, -1]^T$

1. Demonstrate that states x^2 and x^4 are stable, whereas states x^1 and x^3 exhibit a limit cycle. Do this demonstration by using the stability condition and energy function, respectively.
2. Confirm the length of the limit cycle characterizing the states x^1 and x^3.

1. ① $x^2 = [-1, +1]^T$ ①

$$Wx^2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ +1 \end{bmatrix} = [-1 \quad +1]^T = x^2$$

Stable.

② $x^4 = [+1, -1]^T$

$$Wx^4 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} +1 \\ -1 \end{bmatrix} = [+1 \quad -1]^T = x^4$$

Stable.

② $E = -\frac{1}{2}x^T W x = -\frac{1}{2}(-2x_1x_2) = x_1x_2$

$E(x^2) = E(x^4) = -1$.

It is a minima of energy.

$E(x^1) = E(x^3) = 1$

It's a higher energy.

2. $Wx^1 = x^3$, $Wx^3 = x^1$

$x^1 \rightleftharpoons x^3$

so the length of limit cycle is 2.