

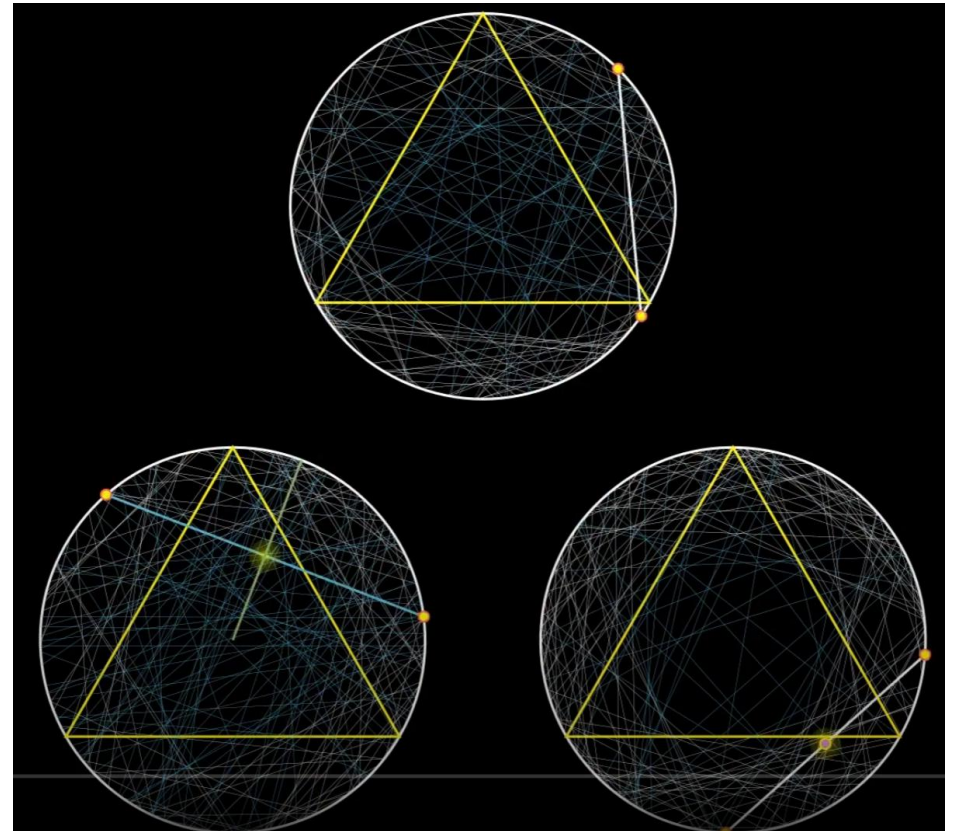
Bertrand's Paradox

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What is Bertrand's Paradox?

- Consider an equilateral triangle that is inscribed in a circle. Suppose a chord of the circle is chosen at random. What is the probability that the chord is longer than a side of the triangle?

$$\frac{1}{2} ? \quad \frac{1}{3} ? \quad \frac{1}{4} ?$$

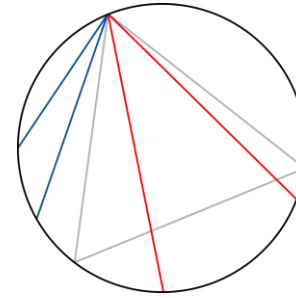


Three different way

- Bertrand gave three arguments (each using the principle of indifference), all apparently valid yet yielding different results:

- The "**random endpoints**" method:

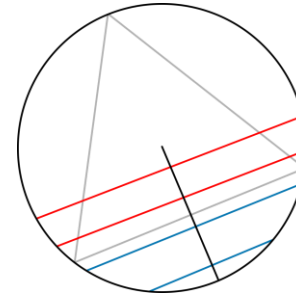
- Choose two random points on the circumference of the circle and draw the chord joining them.



$$\frac{1}{2}$$

- The "**random radial point**" method:

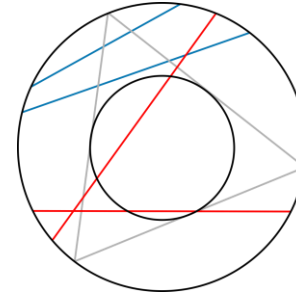
- Choose a radius of the circle, choose a point on the radius and construct the chord through this point and perpendicular to the radius.



$$\frac{1}{3}$$

- The "**random midpoint**" method:

- Choose a point anywhere within the circle and construct a chord with the chosen point as its midpoint.

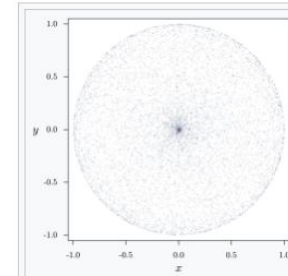


$$\frac{1}{4}$$

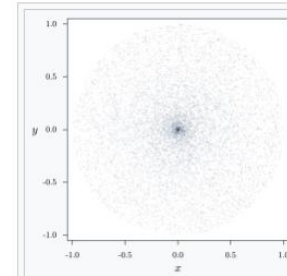
Why & Conclusion

- These three selection methods differ as to the weight they give to chords which are diameters. This issue can be avoided by "regularizing" the problem so as to exclude diameters, without affecting the resulting probabilities. But as presented above, in method 1, each chord can be chosen in exactly one way, regardless of whether or not it is a diameter; in method 2, each diameter can be chosen in two ways, whereas each other chord can be chosen in only one way; and in method 3, each choice of midpoint corresponds to a single chord, except the center of the circle, which is the midpoint of all the diameters.

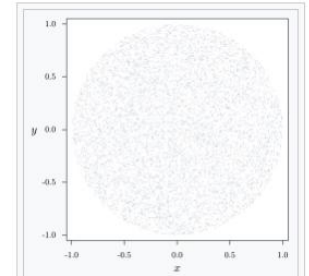
Scatterplots showing simulated Bertrand distributions, midpoints/chords chosen at random using the above methods.



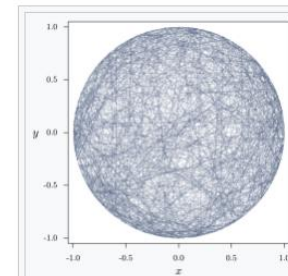
Midpoints of the chords chosen at random using method 1



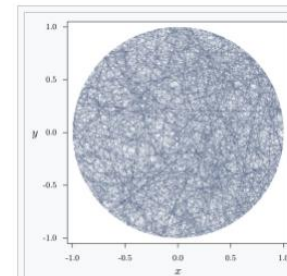
Midpoints of the chords chosen at random using method 2



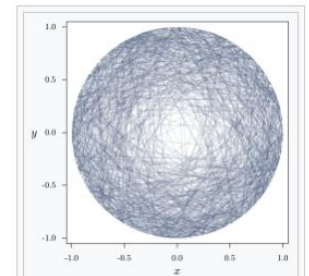
Midpoints of the chords chosen at random using method 3



Chords chosen at random, method 1



Chords chosen at random, method 2



Chords chosen at random, method 3

Reference

- Wikipedia contributors. (n.d.). Bertrand paradox (probability). Wikipedia, The Free Encyclopedia. Retrieved March 5, 2025, from [https://en.wikipedia.org/wiki/Bertrand_paradox_\(probability\)](https://en.wikipedia.org/wiki/Bertrand_paradox_(probability))
- 3Blue1Brown. (2020, May 18). Bertrand's paradox || Expected values and probabilities [Video]. YouTube. <https://www.youtube.com/watch?v=mOwTedomFzQ>