Assignment 1

Problem 1

Solution:

Given:

- Probability of motherboard (MB) problems, $P(MB) = 0.4$
- Probability of hard drive (HD) problems, $P(HD) = 0.3$
- Probability of both MB and HD problems, $P(MB \cap HD) = 0.15$

To find the probability that a computer has **no MB or HD problems**, we first calculate the probability of having **at least one problem** using the principle of inclusion-exclusion:

$$P(MB \cup HD) = P(MB) + P(HD) - P(MB \cap HD)$$

$$P(MB \cup HD) = 0.4 + 0.3 - 0.15 = 0.55$$

The probability of **no problems** is the complement of the union:

$$P(\text{No MB/HD problems}) = 1 - P(MB \cup HD)$$

$$P(\text{No MB/HD problems}) = 1 - 0.55 = 0.45$$

Answer: The probability of a fully functioning MB and HD is $\boxed{0.45}$.

Problem 2

Solution:

Given:

- Probability a programmer knows Java, $P(J) = 0.7$
- Probability a programmer knows Python, $P(P) = 0.6$
- Probability a programmer knows both, $P(J \cap P) = 0.5$

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1. Probability of not knowing Python and not knowing Java:

First, calculate the probability of knowing *at least one language* using inclusion-exclusion:

$$P(J \cup P) = P(J) + P(P) - P(J \cap P)$$

$$P(J \cup P) = 0.7 + 0.6 - 0.5 = 0.8$$

The probability of knowing **neither** language is the complement:

$$P(\text{Neither}) = 1 - P(J \cup P) = 1 - 0.8 = 0.2$$

Answer: $\boxed{0.20}$

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2. Probability of knowing Java but not Python:

This is the probability of knowing Java *minus* the overlap with Python:

$$P(J \setminus P) = P(J) - P(J \cap P)$$

$$P(J \setminus P) = 0.7 - 0.5 = 0.2$$

Answer: $\boxed{0.20}$

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3. Probability of knowing Java given they know Python:

Use the definition of conditional probability:

$$P(J \mid P) = \frac{P(J \cap P)}{P(P)}$$

$$P(J \mid P) = \frac{0.5}{0.6} \approx 0.8333$$

Answer: $\boxed{\frac{5}{6}} \quad \text{(or approximately 0.8333)}$

Problem 3

Solution:

When selecting k elements from n distinct elements **with replacement**, the number of permutations and combinations are derived as follows:

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1. Permutations (Order Matters):

Each selection is ordered, and elements can be repeated. For each of the k positions, there are n choices. Thus:

$$\text{Number of permutations} = n \times n \times \cdots \times n = n^k$$

Example: For $n = 3$, $k = 2$: Permutations = $3^2 = 9$ (e.g., AA, AB, AC, BA, BB, BC, CA, CB, CC).

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2. Combinations (Order Does Not Matter):

Elements can repeat, but order is irrelevant. This is equivalent to distributing k identical items into n distinct bins (stars and bars method). The formula is:

$$\text{Number of combinations} = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!\,(n-1)!}$$

Example: For $n = 3$, $k = 2$: Combinations = $\binom{3+2-1}{2} = \binom{4}{2} = 6$ (e.g., {AA, AB, AC, BB, BC, CC}).

Final Answer:

- Permutations with replacement: $\boxed{n^k}$
- Combinations with replacement: $\boxed{\binom{n+k-1}{k}}$

Problem 4

Solution:

We need to compute the probability that **at least one couple** is paired together when 4 male-female couples are randomly paired for a dance.

Total Pairings: There are $4! = 24$ ways to pair 4 males with 4 females.

Using the Principle of Inclusion-Exclusion:

Let A_i be the event that the i-th couple is paired together. The probability of at least one A_i is:

$$P\left(\bigcup_{i=1}^{4} A_i\right) = \sum P(A_i) - \sum P(A_i \cap A_j) + \sum P(A_i \cap A_j \cap A_k) - P(A_1 \cap A_2 \cap A_3 \cap A_4)$$

1. **Single Couple Pairing:**
   - $\binom{4}{1}$ ways to choose 1 couple.
   - Probability: $\frac{3!}{4!} = \frac{1}{4}$.
   - Total: $4 \cdot \frac{1}{4} = 1$.
2. **Two Couples Paired:**
   - $\binom{4}{2} = 6$ ways to choose 2 couples.
   - Probability: $\frac{2!}{4!} = \frac{1}{12}$.
   - Total: $6 \cdot \frac{1}{12} = 0.5$.
3. **Three Couples Paired:**
   - $\binom{4}{3} = 4$ ways to choose 3 couples.
   - Probability: $\frac{1!}{4!} = \frac{1}{24}$.
   - Total: $4 \cdot \frac{1}{24} = \frac{1}{6}$.
4. **All Four Couples Paired:**
   - Probability: $\frac{1}{4!} = \frac{1}{24}$.

Combining these:

$$P(\text{At least one couple}) = 1 - 0.5 + \frac{1}{6} - \frac{1}{24} = \frac{15}{24} = \frac{5}{8}.$$

Alternative Derangement Approach: The number of derangements (no couple paired) for $n = 4$ is $!4 = 9$. Thus:

$$P(\text{No couples}) = \frac{9}{24} = \frac{3}{8} \implies P(\text{At least one couple}) = 1 - \frac{3}{8} = \frac{5}{8}.$$

Final Answer: $\boxed{\frac{5}{8}}$

Problem 5

Solution:

We need to find the probability that the sum of two randomly chosen numbers X and Y from the interval $[0, 1]$ is less than $\frac{7}{5} = 1.4$.

Geometric Interpretation: The problem is equivalent to finding the area of the region $X + Y < \frac{7}{5}$ within the unit square $[0, 1] \times [0, 1]$.

1. **Breakdown of the Region:**
   - For $0 \leq X \leq 0.4$, Y can range from 0 to 1 (since $\frac{7}{5} - X \geq 1$).
   - For $0.4 < X \leq 1$, Y can range from 0 to $\frac{7}{5} - X$.

2. **Area Calculation:**
   - **First Region (Rectangle):**

$$\text{Area}_1 = 0.4 \times 1 = 0.4$$

   - **Second Region (Integral):** $$ \text{Area}2 = \int{0.4}^1 \left(\frac{7}{5} - X\right) dX $$

$$= \left[\frac{7}{5}X - \frac{1}{2}X^2\right]_{0.4}^{1}$$

$$= \left(\frac{7}{5}(1) - \frac{1}{2}(1)^2\right) - \left(\frac{7}{5}(0.4) - \frac{1}{2}(0.4)^2\right)$$

$$= (1.4 - 0.5) - (0.56 - 0.08) = 0.9 - 0.48 = 0.42$$

3. **Total Area:**

$$\text{Total Area} = 0.4 + 0.42 = 0.82$$

Probability: The probability is the total area divided by the unit square area (1):

$$P\left(X + Y < \frac{7}{5}\right) = \frac{41}{50} = 0.82$$

Final Answer: $\boxed{\frac{41}{50}}$

Problem 6

See screenshot after the `Problem 7` solution.

Problem 7

1. Satisfy the requirements of the `Introductory Course`
2. Review the theroy of `Probability` and `Combinatorics`
3. Our teacher is beautiful and kind !!!