
ECON 424 FINAL PROJECT

Analysis of six assets

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Econ 424 Final Project

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1. Introduction

In this project, we analyze 5 years of monthly closing price data from the end of January 2014 to the end of January 2019 of the following six funds and index: S&P 500 index (vfinx), European stock index (veurx), Emerging markets fund (veiex), Long-term bond fund (vbltx), Short-term bond fund (vbisx) and Pacific stock index (vpacx). The data used in this project is downloaded from Yahoo Finance. Excel and R studio are the main tools of analyzing the data and CER model is applied for these six assets. Also, an assumption is made that the market has a risk free asset with monthly return of 0.0004167 (corresponding to annual return of 0.005).

1.1 data description

a. S&P 500 index (vfinx)

S&P 500 index is one of the best representation of U.S. stock market which records the stock prices of 500 large companies having common stock listed on the New York Stock Exchange, Nasdaq stock market and Cboe BZX Exchange.

b. European stock index (veurx)

This index tracks the performance of investment on the stocks issued by companies located in European stock market. It is a market-capitalization-weighted index.

c. Emerging market fund (veiex)

A benchmark index tracks the return of stocks issued by companies located in emerging market countries. The fund invests by sampling the index, meaning that it holds a broadly diversified collection of securities that, in the aggregate, approximates the index in terms of key characteristics.

d. Long-term bond fund (vbltx)

This index tracks performance of the Bloomberg Barclays U.S. Long Government/Credit Float Adjusted Index, including all medium and larger issues of U.S. government, investment-grade corporate, and investment-grade international dollar-denominated bonds that have maturities of greater than 10 years and are publicly issued.

e. Short-term bond fund (vbisx)

This index tracks the performance of Bloomberg Barclays U.S. 1-5 Year Government/Credit Float Adjusted Index including all medium and larger issues of U.S. government, investment-grade corporate, and investment-grade international dollar-denominated bonds that have maturities between 1 to 5 years and are publicly issued.

f. Pacific stock index (vpacx)

A benchmark index that measures the investment return of stocks issued by companies located in the major market of the Pacific region.

2. Executive summary

- The prices of all six assets increase during the past five years from the end of January 2014 to the end of January 2019.
- Vfinx has the highest starting and ending price during this period.
- Vfinx has the highest expected return and vbisx has the lowest expected return.
- The return of all six assets fluctuates around zero, a significant drop is observed in vbtlx and vbisx in 2016.
- The future value of vfinx is the largest and that of vbisx is the smallest given the same level of initial investment.
- Returns of six assets are time dependent.
- Return of veieix is the most normally distributed while vfinx is the least normally distributed.
- Veieix has the highest standard deviation (volatility) and vbisx has the lowest standard deviation.
- Vfinx is the most preferred for investors among these six assets with the highest Sharpe Ratio.
- The estimation of the expected return and volatility for vbisx is the most precise.

- On average, the estimation of the volatility is more precise than the estimation of expected return.
- Veidx and vpcx are the most highly correlated with each other.
- The estimation of covariance between these six assets supports that diversification in investment will reduce the risk.
- Under same level of probability and initial investment, veidx has the highest level of potential loss while vbix has the lowest.
- The global minimum variance portfolio is the best portfolio for the most risk averse investors with an expected return of 0.000897508 and a volatility of 0.002513957.
- Investors would put most of their wealth in vbix and short sell veidx and vblx to achieve the global minimum variance portfolio.
- With an \$100,000 initial investment in the global minimum variance portfolio, investors would face a monthly loss of \$323.2903 with 5% probability and a monthly loss of \$493.9148 with 1% probability.
- If short sale is prevented, the most risk averse investors would form the global minimum variance portfolio by only buying vfinx and vbix. Most of their wealth is still allocated in vbix.
- The return and volatility of global minimum variance portfolio without short sale is higher than those of global minimum variance portfolio with short sale.
- At the same level of probability, investors would have a higher potential loss when short sale is prevented.
- With the efficient portfolio which is a convex combination of the global minimum variance portfolio and the target max average return portfolio, investors would achieve a lower level of return with the same level of volatility when short sale is not allowed. More specifically, at a volatility level of 0.02, the cost when short sale is not allowed is 0.003048799 in monthly expected return.

- Investors would short sell veurx, veiex and vpacx and buy vfinx, vbltx and vbisx long to form a tangency portfolio.
- Tangency portfolio also weights vbisx the most but less than the global minimum variance portfolio does.
- Tangency portfolio has a monthly expected return of 0.002883225 and volatility of 0.005697101. The annual Sharpe Ratio of the portfolio is 1.499781671.
- The return of the efficient portfolio consisting of tangency portfolio and risk free asset (T-bill) is linear related with the volatility of the efficient portfolio.
- If short sale is prevented, investors would buy 0.14487923 share of vfinx and 0.855120797 share of vbisx.
- The no short sale tangency portfolio has a monthly expected return of 0.002049779 and volatility of 0.005265483. The Sharpe Ratio of the portfolio is 1.074406001. All estimates are small than that of tangency portfolio which allows short sale.
- To achieve a monthly return of 0.005, investors would buy 0.410550885 share of vfinx, 0.310319183 share of vbltx and 0.279129932 share of vbisx when short sale is not allowed.
- Investors cannot achieve a portfolio with monthly expected return of 0.01 when short sale is not allowed.

3. Return Calculation and Sample statistics

3.1 Time plot, monthly continuously compounded returns and equity curve

Figure 1 Time plot of monthly closing price

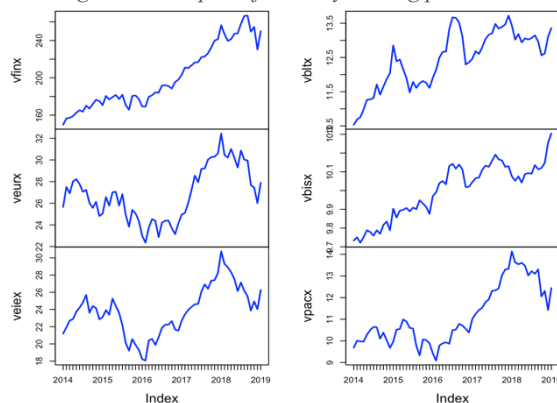


Figure 2 Time plot of monthly closing price



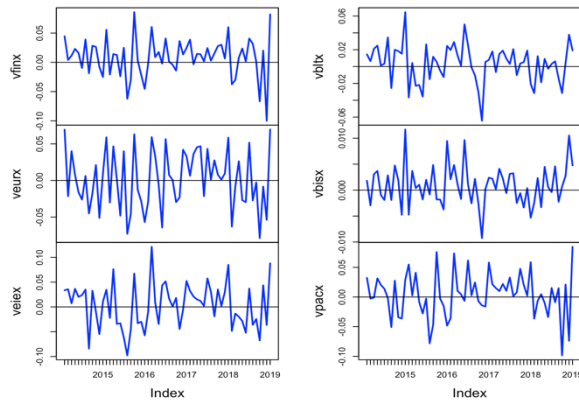


Figure 3 Time plot of monthly continuously compounded return

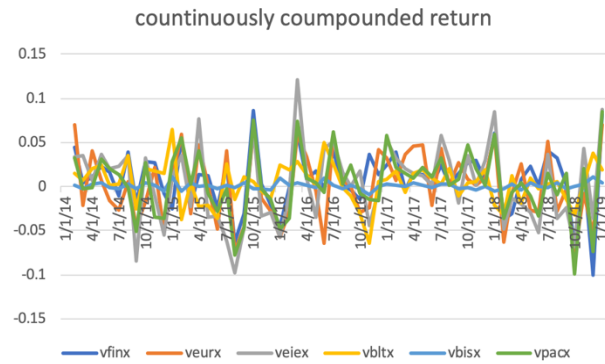


Figure 4 Time plot of monthly continuously compounded return

It can be observed that the price of all six assets are increasing during the past 5 years (Figure 1). Putting curves of monthly closing price of six assets together, one can observe that the price of S&P 500 is much higher than other five assets (Figure 2). The closing price of veurx reached its bottom over the past five years, which might be influenced by the UK European Union membership referendum.

Comparing the range of monthly continuously compounded return, the return of vbisx is much smaller than return of other assets with the lowest level of fluctuation (Figure 4). In general, the returns of six assets fluctuate around zero. The process of the fluctuation of the return can be regarded as a stationary process because it fluctuates around its expected value, zero, and its variation seems unchanged over time. Moreover, from the middle of 2016 through the beginning of 2017, a significant drop of returns of vbisx and vbtx is observed (Figure 3), which might be influenced by the U.S presidential election.

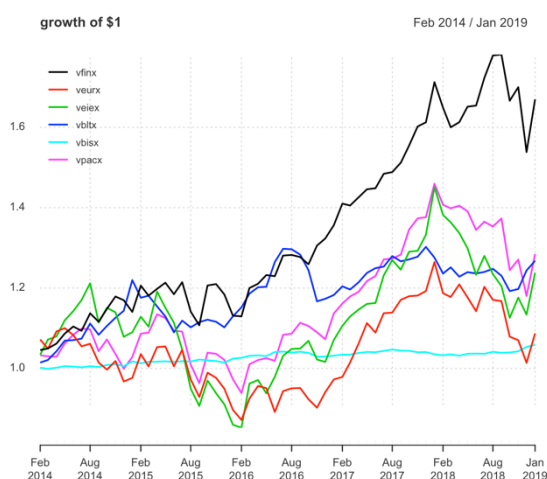


Figure 5 equity curve (future value of \$1)

The equity curve which representing the future value of one dollar invested at the base year shows that S&P 500 would have the highest future value about 1.67 at the end of January 2019. The result is not surprising because the S&P 500 measures the performance of the stocks of 500 large

companies in U.S. market which are believed to perform well in the market by the general public.

3.2 distribution of asset return

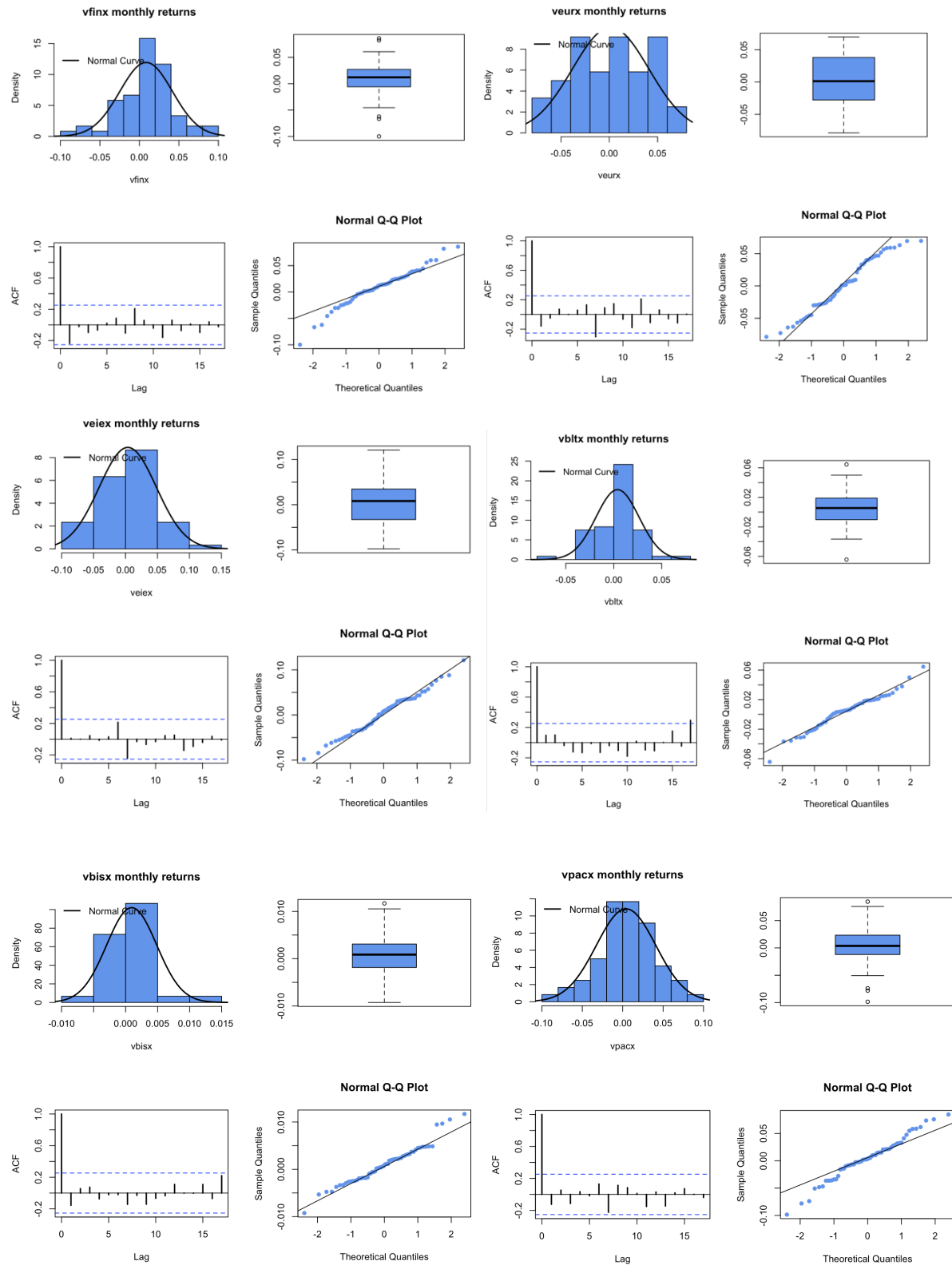


Figure 6 Four panel plot of all assets

By analysing the four panel diagnostic plot (Figure 6), it can be observed that generally all assets except veurx are likely to be regarded as normally distributed. Though the histogram of return of veurx is symmetric, the qq-plot does not look like a straight line and has heavy fat tails on both sides. Therefore, veurx is the least likely to be normally distributed. It also can be observed that the histogram of other five assets have different levels of skewness and the qq-plot have fat tails, but overall they can be treated as being normal distributed. The boxplots show that outliers exist in vfinx, vbtlx, vbisx and vpacx.

The ACF (autocorrelation function) plot which shows the relationship of correlation verses j-lag (distance between time) provides evidence about linear time dependence. For all six assets, the ACF functions only show a significantly large value at lag equal to zero and the rest correlation being around zero, suggesting that these assets are time independent.

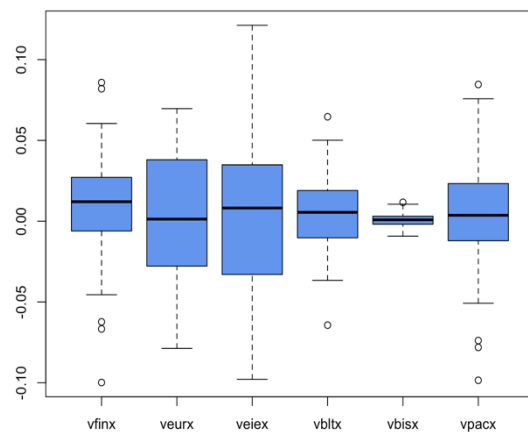


Figure 7 boxplot of return

Putting all boxplot together (Figure 7), one can observe that the return of these assets are nearly centered at zero and spread symmetricly around their center. However, the levels of spread differs and outliers exist. The range of return of veieix is the largest among these six assets.

3.3 univariate descriptive statistics

	vfinx	veurx	veieix	vbtlx	vbisx	vpacx
mean	0.008544553	0.00139288	0.003562833	0.003954066	0.000949399	0.004174592
variance	0.001116613	0.001569717	0.002013214	0.000505209	0.0000152895	0.001359217
sd	0.033415757	0.039619649	0.04486885	0.022476862	0.003910175	0.036867565
skewness	-0.506759603	-0.0474817	0.085538577	-0.31153785	0.429146103	-0.275533695
excess kurtosis	1.277212989	-0.95175878	0.255219601	0.81169526	0.776145239	0.41607213
1% quantile	-0.080282115	-0.07532182	-0.089804703	-0.04798624	-0.006929985	-0.086458773
5% quantile	-0.046365964	-0.06314931	-0.062044525	-0.03181575	-0.004740783	-0.051918556

Figure 8 univariate statistics

The univariate descriptive statistics suggests that vfinx has the highest average return while vbisx the lowest. Meanwhile, veieix has the highest standard deviation and vbisx the lowest. The ranking of return for vbisx, veurx, veieix and vfinx follows the ranking of standard

deviation for these four assets, convincing investors' expectation that the risk positively correlates with return. High return comes with high risk while low return implies low risk.

The value of skewness indicates the level of symmetry with a positive value of skewness suggesting the data is right skewed. The value of kurtosis suggests the degree of fat tails. the larger kurtosis is, the heavier fat tails data has. Moreover, one can obtain excess kurtosis by applying the formula,

$$\text{Excess Kurtosis} = \text{Kurt}(X) - 3,$$

to check the likeness of being normally distributed. Having an excess Kurtosis larger than zero implies that the distribution of data has a fatter tail than the normal distribution.

The result shows that veix has tails which is close to that of the normal distribution. Comparing all statistical result, it can be concluded that veix is the most normally distributed as it have a relative lower level of skewness and tails close to normal distribution. Vfinx can be regarded as the least normally distributed with the highest level of skewness and excess Kurtosis.

3.4 Sharpe Ratio analysis and Bootstrapping

Sharpe Ratio is a statistics measuring the performance of an investment by adjusting for its risk. Generally the Sharpe Ratio is used to rank assets. Assets with high Sharpe Ratios are preferred to those with low Sharpe Ratio. A risk free asset with fixed and known rate of return over investment horizon is introduced to compute the Sharpe Ration. In the analysis of Sharpe Ratio of these six assets, a risk free asset with monthly rate of return of 0.0004167 which corresponds to an annual continuously compounded rate of return of 0.5% is applied.

	vfinx	veurx	veix	vbltx	vbisx	vpacx
Sharpe Ratio	0.243235131	0.024639623	0.070119157	0.157379585	0.136242574	0.101930385
Bootstrapping bias	0.01254939	-0.002021986	0.000510483	0.005841051	-0.007119167	0.000955979
Bootstrapping Standard error	0.1383285	0.1324511	0.1317395	0.1361262	0.1330476	0.1334762
MSE	0.019292261	0.017547382	0.017355556	0.01856446	0.017752346	0.01781681

Figure 9 Sharpe Ratio

The result shows that vfinx has the highest Sharpe Ratio among these six assets, indicating that vfinx is preferred than other assets. Also, the bootstrapping standard error of the estimation of

Sharpe Ratio is about 0.13 to 0.14. MSE (mean squared error) can be computed by applying the following formula,

$$\text{MSE} = \text{bias}^2 + \text{Se}^2,$$

to measure the precision of the estimation. Statistics above show small MSE of the estimation of the Sharpe Ratio about the six assets, implying that the estimation is precise and the precision among the estimation of six assets are at the same level.

3.5 Precision of estimation of parameters

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
se of mean	0.004313956	0.005114875	0.005792544	0.00290175	0.000504801	0.004759582
95%CI	(-0.00008335867, 0.017172464)	(-0.00883687, 0.011622629)	(-0.008022255, 0.015147920)	(-0.001849437, 0.009757558)	(-0.00006020041, 0.001959000)	(-0.005344573, 0.013693756)
se of sd	0.003050427	0.003616763	0.004095947	0.002051847	0.000356948	0.003365533
95%CI	(0.02731490, 0.039516612)	(0.03238612, 0.046853174)	(0.03667696, 0.053060744)	(0.01837316, 0.026580542)	(0.00319627, 0.004624061)	(0.03013650, 0.043598631)

Figure 10 standard error and 95% Confidence Interval

The estimated standard error of estimations of mean and standard deviation suggests the precision of the estimations. The standard error of estimation of standard deviation is slightly lower than that of the estimation of mean, suggesting that the estimation of standard deviation is more precise. Therefore, the confidence interval of mean is wider than that of the standard deviation. Overall, the estimated standard errors are at a very low level (comparing to the standard error of Sharpe Ratio estimation), so the estimations are precise for mean and standard deviation.

3.6 Annualization

annualization	vfinx	veurx	veiex	vbltx	vbisx	vpacx
mean	0.102534633	0.016714559	0.042753991	0.047448791	0.011392787	0.050095101
sd	0.115755579	0.137246491	0.155430255	0.077862135	0.013545242	0.127712992
sharpe ratio	0.842591209	0.085354157	0.242899883	0.545178874	0.471958122	0.353097211
simple annual return	0.107975674	0.016855028	0.043681108	0.048592503	0.011457932	0.051371078
5-year Future value	1.66974875	1.08716435	1.238337748	1.267750356	1.05861763	1.284636124

Figure 11 annual statistics

After the annualization of the mean and standard deviation of return, the ranking of the values still follows the ranking of the monthly values. Vfinx has the highest annual return and veiex has the highest annual standard deviation. Also, the ranking of the annual Sharpe Ratio follows the ranking of the monthly Sharpe Ratio: veurx, veiex, vpacx, vbisx, vbltx, vfinx from low to high. Also, vfinx has the highest 5-year future value at 1.66974875 dollar for 1 dollar invested

in the base year. The ranking of the future value also follows the ranking of mean of return: vbisx, veurx, veieix, vbltx, vpacx, vfinx from low to high.

3.7 Covariance

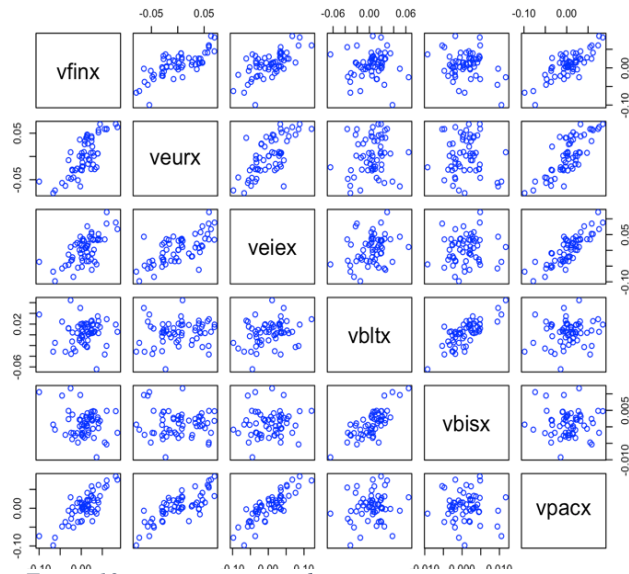


Figure 12 pair-wise scatter plot

Covariance provides evidence about the direction of the relationship between two assets. The pair-wise scatter plot suggests a strong and positive relationship between the following pairs: (vfinx, veurx), (vfinx, vpacx), (veurx, veieix), (veieix, vpacx) and (vbltx, vbisx), while other pairs look independent.

Covariance	vfinx	veurx	veieix	vbltx	vbisx	vpacx
vfinx	0.001116613	0.001000597	0.000991834	0.000382366	-0.0000278171	0.000990412
veurx	0.001000597	0.001569717	0.001208787	0.00013205	-0.00000911664	0.001150217
veieix	0.000991834	0.001208787	0.002013214	0.000292651	0.0000209575	0.001391868
vbltx	0.000382366	0.00013205	0.000292651	0.000505209	0.0000674856	0.000135026
vbisx	-0.0000278171	-0.0000091166	0.0000209575	0.0000674856	0.0000152894	-0.00000610621
vpacx	0.000990412	0.001150217	0.001391868	0.000135026	-0.00000610621	0.001359217

Figure 13 covariance

The covariance matrix is symmetric because the covariance between asset A and asset B is a mutual statistics. The value on the diagonal line suggests the variance of the asset. A positive value in the covariance matrix suggests a positive relationship between two assets. The matrix shows subtle negative relationship between these pairs because of negative values of covariance: (vfinx, vbisx), (veurx, vbisx), (vbltx, vbisx) and (vbisx, vpacx). Meanwhile other pairs of assets shows a positive relationship. However, the strength of the relationship is difficult to interpret with covariance along. For this reason, correlation is introduced in the next section to proceed the analysis of the relationship.

3.8 Correlation

Correlation	vfinx	veurx	veiex	vbltx	vbisx	vpacx
vfinx	1	0.75578361	0.6615196	0.05090869	-0.21289456	0.8039331
veurx	0.75578361	1	0.6799771	0.1482829	-0.05884761	0.7874532
veiex	0.6615196	0.6799771	1	0.290181	0.1194538	0.8414117
vbltx	0.05090869	0.1482829	0.290181	1	0.76785672	0.1629441
vbisx	-0.21289456	-0.05884761	0.1194538	0.76785672	1	-0.0423577
vpacx	0.8039331	0.7874532	0.8414117	0.1629441	-0.0423577	1

Figure 14 Correlation

The sign of correlation corresponds to the direction of relationship. A large absolute value of correlation suggests a strong relationship between two assets. The correlation matrix shows that veiex and vpacx has the highest correlation, 0.8414117, among all pairs of assets indicating that veiex is positive and the closest related with vpacx. The correlation matrix is also symmetric because the correlation of asset A verses asset B is the same as the correlation of asset B verses asset A.

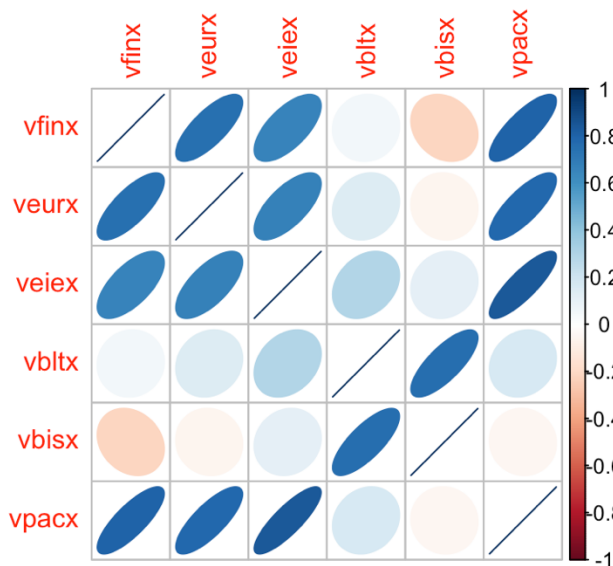


Figure 15 correlation plot

The correlation matrix plot (Figure 16) gives a visual display of the correlation. The size of ellipse in correlation plot shows the level of correlation. The direction as well as the color of the ellipse suggest the direction of correlation. The thinner ellipse is, the closer correlation there is. Blue and upward sloping mean a positive

correlation and red and downward sloping show a negative correlation. Also, a darker color suggests a closer correlation. The result visually shows that (veiex, vpacx) are the most highly correlated while (vbisx, vpacx) are the least.

Based on the estimation, diversification would reduce the risk. From the calculation in the previous section, investors can observe that vbltx and vbisx are two of the asset of the lowest risk with lowest standard deviation (Figure 8). Analysis in section 3.7 and 3.8 show that these two assets are least correlated with other assets, i.e. the most diverse from other assets.

4. Value-at-Risk (VaR) Calculations

Value-at-risk is a measure of the risk of loss for investment. In this section, an assumption is made that investors intend to invest \$100,000 starting at June 30, 2016.

4.1 Monthly 1% and 5% VaR and Bootstrapping standard error

Monthly	vfinx	veurx	veiox	vbltx	vbisx	vpacx
1% VaR	-6685.2616	-8677.7939	-9590.2184	-4718.5378	-811.3908	-7835.2261
Bootstrapping	1050.063	772.2792	924.4334	642.4633	94.36817	953.0024
Standard error						
95% CI	(-8857, -4741)	(-10269, -7241)	(-11599, -7975)	(-6040, -3522)	(-1009.2, -639.3)	(-9768, -6033)
5% VaR	-4535.857	-6178.453	-6782.98	-3247.795	-546.725	-5490.248
Bootstrapping	852.6412	641.7638	770.3901	515.3851	71.79868	798.8267
Standard error						
95% CI	(-6282, -2939)	(-7487, -4971)	(-8369, -5349)	(-4300, -2279)	(-698.2, -416.7)	(-7107, -3976)

Figure 16 Monthly Value-at-risk

The interpretation of VaR is the loss at a given level of probability and at a given initial investment. For example the first cell with the value of -6685.2616 can be interpreted that if one invests \$100,000 in vfinx, there is a 5% probability that he would lose \$6685.2616 monthly. Veiox has the highest level of VaR while the vbisx has the lowset at both 1% level and 5% level. The bootstrapping standard error evaluates the precision of the VaR. It can be concluded that the estimation of VaR of vbisx is the most precise while the estimation of VaR of vfinx the least precise. Therefore, the 95% confidence interval of VaR of vbisx is the narrowest and that of VaR of vfinx is the widest.

4.2 Annually 1% and 5% VaR

Annually	vfinx	veurx	veiox	vbltx	vbisx	vpacx
1% VaR	-15359.145	-26108.215	-27300.149	-12513.494	-1991.714	-21886.692
5% VaR	-8411.672	-18863.384	-19176.791	-7745.86	-1082.81	-14783.412

Figure 17 annual Value-at-Risk

Annual VaR can be computed by annualize mean and standard deviation of estimated return. The ranking of potential loss follows the ranking of potential loss estimated by monthly VaR. Under 1% probability, the worst case is that investors may loss 27300.149 annually with an investment of \$100,000 dollar in veiox.

4.3 Empirical 1% and 5% VaR

empirical monthly	vfinx	veurx	veiox	vbltx	vbisx	vpacx
1% VaR	-7714.4042	-7255.5031	-8589.031	-4685.3096	-690.6028	-8282.664
5% VaR	-4530.7484	-6119.6708	-6015.8961	-3131.4953	-472.9563	-5059.3813

Figure 18 Empirical VaR (monthly)

Under 5% probability, all assets except vfinx have lower level of VaR using the empirical quantiles of return distributions. That is, the potential loss under empirical VaR is less than those under normal VaR for those five assets. However, the investment in vfinx would be expected to loss more with empirical VaR than with normal VaR. At 1% probability, all assets are expected to have less loss under empirical value-at-risk on average, and the difference between estimated potential loss of vfinx is the smallest.

5. Portfolio Theory

In this section, CER model estimates (mean/return, standard deviation/velocity, covariance and etc.) computed in the previous section (Figure 8) will be used for building portfolios.

Also, assume that investors have \$100,000 to invest at January 31 2019.

5.1 global minimum variance portfolio

Global minimum variance portfolio is a portfolio with the smallest possible variance/ lowest level of risk. This portfolio is chosen by the most risk averse individuals.

global min var portfolio	weight
vfinx	0.048399807
veurx	0.0003
veiex	-0.015100019
vbltx	-0.129101373
vbisx	1.093101585
vpacx	0.0024

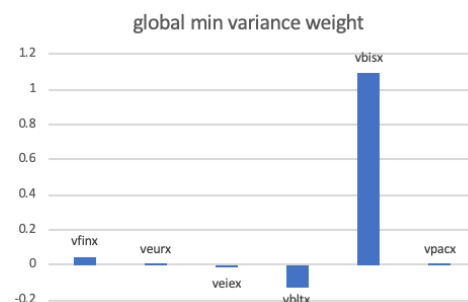


Figure 20 global minimum variance portfolio weight

The result suggests that the weight of veiex and vbltx are negative, implying that investors should perform short sale for these two assets to achieve the global minimum variance portfolio. The bar chart of the weight shows that vbisx weights the most in the global minimum variance portfolio, which is reasonable with the fact that vbisx has the lowest variance among these six assets.

global minimum variance portfolio					
R[p]	0.000897508	annual return	0.010770092	monthly VaR	
Var[p]	0.00000631998	annual volatility	0.008708601	1%	-493.9148
sd[p]	0.002513957	annual Sharpe ratio	0.662573856	5%	-323.2903

Figure 9 global minimum variance portfolio statistics

Comparing to the single asset monthly statistics, it can be observed that this portfolio achieves a lower variance than all six assets individually but meanwhile the portfolio has a lower expected return. The annualized estimates agree with this statement. As a result, the global minimum variance portfolio has the smallest VaR, implying that investors can minimize their potential loss by applying the global minimum variance portfolio. Specifically, investors may face a loss of \$323.2903 monthly under 5% probability.

5.2 global minimum variance portfolio (no short sale)

There are some cases that short sale is not allowed. For example, Exchanges may prohibit short sale some assets or some institutions like pension funds are prevented from short sale. More specifically, investors cannot short sell mutual funds. Therefore, we intend to create a global minimum variance portfolio for the most risk averse investors in these situations.

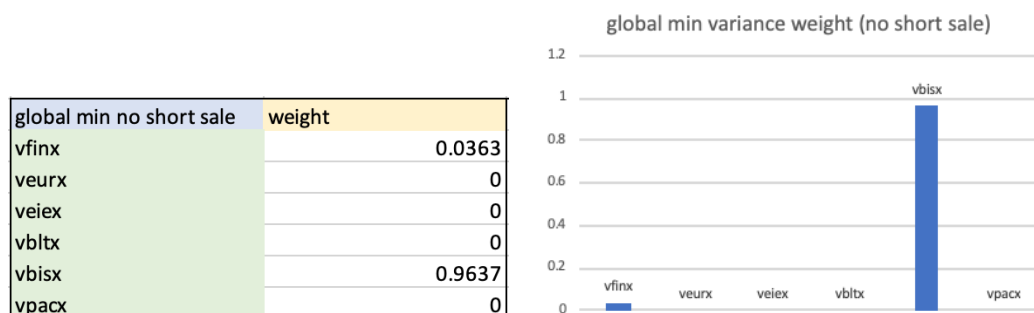


Figure 22 global minimum variance portfolio with no short sale

When the short sale is not allowed, the global minimum variance portfolio only consists of vfinx and veiex and the portfolio still highly weights vbisx.

global minimum variance portfolio (no short sale)					
R[p]	0.001225103	annual return	0.014701236	monthly VaR	
Var[p]	0.0000137247	annual volatility	0.012833394	1%	-736.602
sd[p]	0.003704682	annual Sharpe ratio	0.755936873	5%	-485.6731

Figure 23 global minimum variance portfolio with no short sale statistics

Comparing to the global minimum variance portfolio allowing short sale, this portfolio has slightly higher expected return and expected variance. Accordingly, this portfolio would have a higher VaR. That is, the investors would face a higher level of potential loss. However, when short sale is not allowed, the global minimum variance portfolio is more preferred than the global minimum variance portfolio with short sale. A larger Sharpe Ratio supports this statement.

5.3 efficient portfolio frontier

An efficient portfolio can be represented by a convex combination of other two efficient portfolios. In section 5.3 and section 5.4, we would consider combinations of the global minimum variance portfolio and the efficient minimum variance portfolio with a target return equal to the max average returns among the six assets.

First of all, a minimum variance portfolio with the target return equal to the average return of vfinx can be computed as the following.

efficient portfolio		target return		0.008544553
	weight	R[p]		0.008544553
vfinx	0.940098134	Var[p]		0.00039378
veurx	-0.452079027	sd[p]		0.019843895
veiex	-0.124934311			
vbltx	0.421484079			
vblsx	0.303629538			
vpacx	-0.088198413			

Figure 24 efficient portfolio weight and statistics

To achieve the minimum variance portfolio with the target return, investors would short sale veurx,

veiex and vpacx. Vfinx dominates other assets in this portfolio. Comparing to the single asset statistics, this portfolio have the same level of expected return with vfinx but a much lower level of risk than vfinx.

efficient portfolio with global min and target return efficient portfolio					
weight of global min var	weight of target return	R[p]	Var[p]	SD[p]	
-1.5	2.5	0.020015121	0.00242773361	0.049272037	
-1.4	2.4	0.019250416	0.00223790022	0.04730645	
-1.3	2.3	0.018485712	0.00205581490	0.045341095	
-1.2	2.2	0.017721007	0.00188147765	0.043376003	
-1.1	2.1	0.016956303	0.00171488847	0.041411212	
-1	2	0.016191598	0.00155604737	0.039446766	
-0.9	1.9	0.015426893	0.00140495433	0.03748272	
-0.8	1.8	0.014662189	0.00126160936	0.035519141	
-0.7	1.7	0.013897484	0.00112601246	0.033556109	
-0.6	1.6	0.01313278	0.00099816363	0.031593728	
-0.5	1.5	0.012368075	0.00087806288	0.029632126	
-0.4	1.4	0.011603371	0.00076571019	0.027671469	
-0.3	1.3	0.010838666	0.00066110558	0.025711973	
-0.2	1.2	0.010073962	0.00056424903	0.023753927	
-0.1	1.1	0.009309257	0.00047514056	0.021797719	
0	1	0.008544553	0.00039378015	0.019843895	
0.1	0.9	0.007779848	0.00032016782	0.017893234	
0.2	0.8	0.007015144	0.00025430355	0.015946898	
0.3	0.7	0.006250439	0.00019618736	0.01400669	
0.4	0.6	0.005485735	0.00014581924	0.012075564	
0.5	0.5	0.00472103	0.00010319918	0.0101587	
0.6	0.4	0.003956326	0.00006832720	0.008266027	
0.7	0.3	0.003191621	0.00004120329	0.006418979	
0.8	0.2	0.002426917	0.00002182745	0.004671986	
0.9	0.1	0.001662212	0.00001019968	0.003193694	
1	0	0.000897508	0.00000631998	0.002513957	
1.1	-0.1	0.000132803	0.00001018835	0.003191919	
1.2	-0.2	-0.000631901	0.00002180479	0.00466956	
1.3	-0.3	-0.001396606	0.00004116930	0.006416331	
1.4	-0.4	-0.00216131	0.00006828188	0.008263285	
1.5	-0.5	-0.002926015	0.00010314253	0.010155911	

Figure 25 efficient portfolio frontier

The table of efficient frontier(Figure 25) shows that the more investors weight on global min variance portfolio, the less return they receive and the lower risk they take. However, if investors start short selling the target return portfolio, they are expected to face a loss.

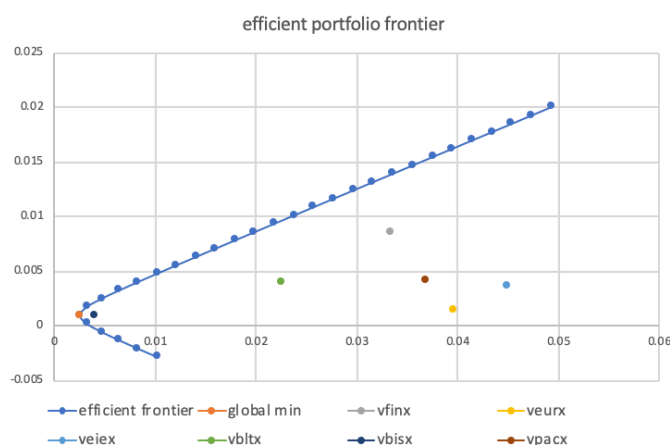


Figure 26 efficient portfolio frontier

Graphically, any portfolio below the global minimum variance portfolio is considered insufficient because investors can receive higher return at the same level of risk. The insufficient portfolio corresponds to the short sale of global minimum variance portfolio.

5.4 efficient portfolio frontier (no short sale)

When the short sale is prohibited, we would combine the global minimum variance portfolio with no short sale and the efficient minimum variance portfolio with expected return and with no short sale to form the no short sale efficient portfolio frontier.

efficient portfolio with no short sale		target return	0.008544553
	weight	R[p]	0.008544553
vfinx	1	Var[p]	0.001116613
veurx	0	sd[p]	0.03341576
velex	0		
vbltx	0		
vbisx	0		
vpacx	0		

Figure 27 efficient portfolio with target return and without short sale

Following the same procedure, an efficient minimum variance portfolio with expected return and

with no short sale is obtained. When the short sale is not allowed, the efficient minimum variance portfolio with expected return is formed by a whole share of vfinx to achieve the same level of return with vfinx because any weight on other five assets would lead to a lower return for the portfolio.

efficient portfolio with global min and target return			efficient portfolio (no short sale)		
weight of global min var	weight of target return	R[p]	Var[p]	SD[p]	
0	1	0.008544553	0.001116613	0.03341576	
0.1	0.9	0.007812608	0.000907064	0.03011751	
0.2	0.8	0.007080663	0.000719574	0.026824868	
0.3	0.7	0.006348718	0.00055414	0.023540187	
0.4	0.6	0.005616773	0.000410765	0.020267338	
0.5	0.5	0.004884828	0.000289447	0.017013151	
0.6	0.4	0.004152883	0.000190187	0.013790842	
0.7	0.3	0.003420938	0.000112985	0.010629444	
0.8	0.2	0.002688993	0.0000578406	0.007605298	
0.9	0.1	0.001957048	0.0000247537	0.004975314	
1	0	0.001225103	0.0000137247	0.003704682	

Figure 28 efficient portfolio frontier (no short sale)

The table of efficient portfolio frontier with no short sale shows that the more weighted on global minimum variance portfolio with no short sale, the lower the expected return is and the lower risk investors would take.

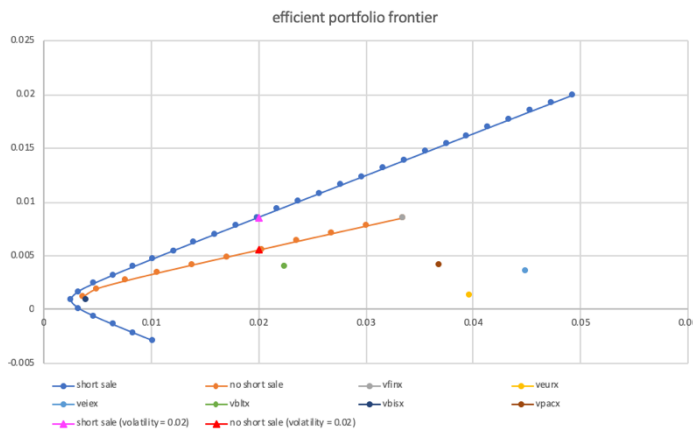


Figure 29 efficient portfolio frontier (no short sale)

Plotting the efficient portfolio frontier for portfolio with or without short sale, it can be observed that with the same level of risk, the efficient portfolio would have a higher level of return with short sale.

Target volatility	0.02				
With short sale					
weight of global min var	weight of target return	R[p]	Var[p]	SD[p]	
-0.007998739	1.007998739	0.00860572	0.000400003	0.020000072	
no short sale					
weight of global min var	weight of target return	R[p]	Var[p]	SD[p]	
0.408178329	0.591822671	0.005556921	0.000400016	0.020000409	
cost in return		0.003048799			

Figure 30 target volatility statistics

More specifically, if investors intend to invest in a portfolio with a target volatility of 0.02, two portfolios for with and without short sale can be computed as above. The cost of not choosing short sale is a loss of 0.003048799 of expected return.

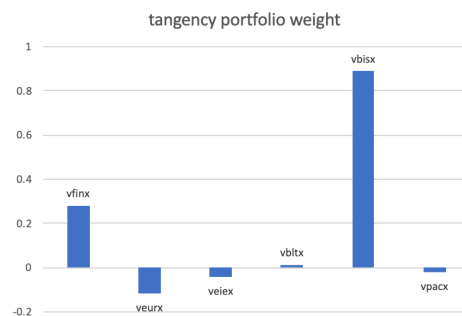
5.5 Tangency portfolio

Tangency portfolio is the portfolio with the maximized Sharpe Ratio. This portfolio is the most preferred to the general investors regarding to the best risk-return trade-off. In section 5.5 and

5.6, a risk-free asset with annual return of 0.005 (monthly return of 0.0004166667 correspondingly) is used. First assume a market allowing short sale.

tangency portfolio	weight
vfinx	0.27997105
veurx	-0.117174151
veiex	-0.043643488
vbltx	0.013872688
vbisx	0.8881374
vpacx	-0.021163499

Figure 31 tangency portfolio weight



The weights of veurx, veiex and vpacex are negative, so investors would short sell these three assets to obtain a tangency portfolio. Vbisx dominates other five assets but is weighted less than the global minimum variance portfolio does.

tangency portfolio			
R[p]	0.002883225	annual return	0.034598695
Var[p]	0.0000324570	annual volatility	0.019735336
sd[p]	0.005697101	annual Sharpe ratio	1.499781671
Sharpe Ratio	0.432949676		

Figure 32 tangency portfolio statistics

Comparing to the single asset statistics, the tangency portfolio achieves a higher Sharpe Ration than those of all assets, so investing in tangency portfolio is preferred to investing in any single assets with a whole basket. Meanwhile tangency portfolio achieves a extremely small variance, implying that the tangency portfolio has a low level of risk. An annualized Sharpe Ratio exceeding 1 shows that the tangency portfolio is a good choice for investors to have a relatively high return with a relatively low risk.

portfolio with T-bill and Tangency portfolio				
weight of tangency p	R[p]	Var[p]	sd[p]	
-1.5	-0.003283171	0.00007302825	0.008545657	
-1.4	-0.003036515	0.00006361572	0.007975946	
-1.3	-0.002789859	0.00005485233	0.007406236	
-1.2	-0.002543203	0.00004673808	0.006836525	
-1.1	-0.002296548	0.00003927297	0.006266815	
-1	-0.002049892	0.00003245700	0.005697105	
-0.9	-0.001803236	0.00002629017	0.005127394	
-0.8	-0.00155658	0.00002077248	0.004557684	
-0.7	-0.001309924	0.00001590393	0.003987973	
-0.6	-0.001063268	0.00001168452	0.003418263	
-0.5	-0.000816613	0.00000811425	0.002848552	
-0.4	-0.000569957	0.00000519312	0.002278842	
-0.3	-0.000323301	0.00000292113	0.001709131	
-0.2	-0.000076645	0.00000129828	0.001139421	
-0.1	0.000170011	0.00000032457	0.00056971	
0	0.000416667	0.00000000000	0	
0.1	0.000663323	0.00000032457	0.00056971	
0.2	0.000909978	0.00000129828	0.001139421	
0.3	0.001156634	0.00000292113	0.001709131	
0.4	0.00140329	0.00000519312	0.002278842	
0.5	0.001649946	0.00000811425	0.002848552	
0.6	0.001896602	0.00001168452	0.003418263	
0.7	0.002143258	0.00001590393	0.003987973	
0.8	0.002389913	0.00002077248	0.004557684	
0.9	0.002636569	0.00002629017	0.005127394	
1	0.002883225	0.00003245700	0.005697105	
1.1	0.003129881	0.00003927297	0.006266815	
1.2	0.003376537	0.00004673808	0.006836525	
1.3	0.003623193	0.00005485233	0.007406236	
1.4	0.003869848	0.00006361572	0.007975946	
1.5	0.004116504	0.00007302825	0.008545657	

Figure 33 efficient portfolio with tangency portfolio and T-bill

The table above shows an option that make an efficient portfolio with tangency portfolio and T-bill, a short-term debt obligation backed by the Treasury Department of the U.S. government with a maturity of less than one year. With this characteristics, the T-bill with a monthly expected return of 0.000416667 becomes the risk free assets in the efficient portfolio.

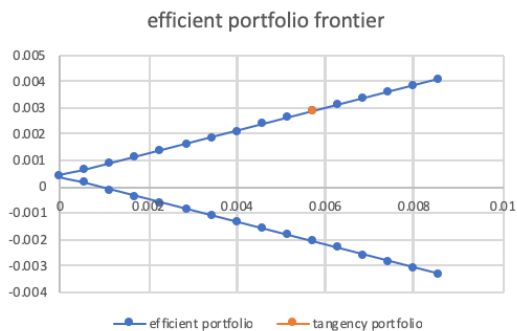


Figure 34 efficient portfolio frontier

The efficient portfolio frontier shows a linear relationship between the risk and return of the portfolio consisting of tangency portfolio and T-bill. Theoretically, any portfolio having an expected return above the tangency portfolio is achieved by short selling the risk free asset (T-bill).

5.6 tangency portfolio (no short sale)

If the short sale is not allowed, investors can also build a tangency portfolio adding a constraint that weight of assets in the portfolio should be a positive number.

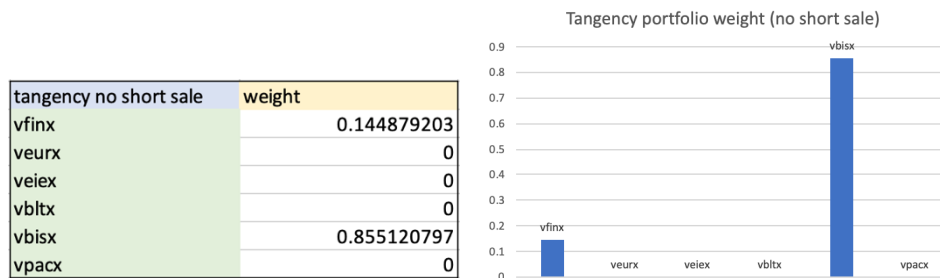


Figure 35 tangency portfolio weight (no short sale)

The result of weight shows that when short sale is prevented, the tangency portfolio only consists of vfinx and vbisx with the dominance of vbisx.

tangency portfolio (no short sale)			
R[p]	0.002049779	annual return	0.024597345
Var[p]	0.0000277253	annual volatility	0.018240167
sd[p]	0.005265483	annual Sharpe ratio	1.074406001
Sharpe Ratio	0.310154297		

Figure 36 tangency portfolio statistics (no short sale)

Comparing to the tangency portfolio allowing short sale, tangency portfolio without short sale has smaller standard deviation as well as smaller expected return. Meanwhile, tangency portfolio achieves a smaller Sharpe Ratio, indicating that this portfolio is less preferred to tangency portfolio allowing short sale. However, this portfolio is still preferred to investments in a single asset with investors' whole wealth and is a good choice for investors with an annualized Sharpe Ratio close to 1.

6. Asset Allocation

In this section we assume that investors want to achieve an efficient portfolio with a target return when short sale is not allowed for the six Vanguard funds while facing the lowest level of risk in investment.

6.1 target expected return: 6% (annual)/0.5% (monthly) (no short sale)

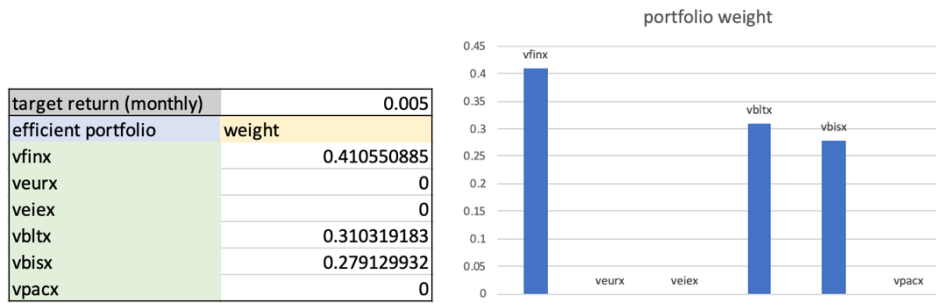


Figure 37 target return portfolio weight

To achieve a monthly return of 0.005, investors would perform a portfolio with 0.41 share of vfinx, 0.31 share of vbltx and 0.28 share of vbisx.

target return (monthly)	0.005			
R[p]	0.005	annual return	0.06	Value at Risk (monthly)
Var[p]	0.000253108	annual volatility	0.055111631	1%
sd[p]	0.015909357	annual Sharpe ratio	0.997974469	5%
				-3150.378
				-2094.608

Figure 38 target return portfolio statistics

This portfolio has annual Sharpe Ratio about 1, indicating that the portfolio is a good one with relatively low risk versing high return. Assuming investors invest \$100,000 initially, they would face a monthly potential loss of \$2094.608 at 1% probability and a monthly potential loss of \$3150.378 at 5% probability.

6.2 target expected return: 12%(annual)/1% (monthly) (no short sale)

If investors intend to achieve a monthly expected return of 0.01 (corresponding to an annual return of 0.12), there is no feasible portfolio when short sale is prevented for the mutual fund. To explore the reason, some possible portfolios with six assets are plotted in the graph below. Any feasible portfolio can be represented by a convex combination of six assets used in this project with positive share.

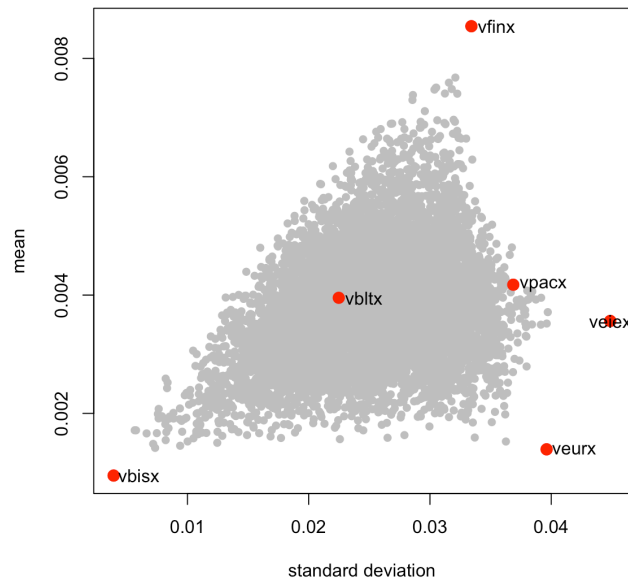


Figure 39 efficient portfolio frontier

It can be observed that all possible portfolios lay within the area between these six assets. Therefore, investors cannot achieve a monthly expected return exceeding the max average return when short sale is not allowed. That is, the return of vfinx of 0.008544553 monthly.