Review of AC systems and Power Factor Correction

ELEC2005 Electrical and Electronic Systems - Week 8

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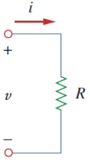
Table of Contents

Linear circuit elements - time domain

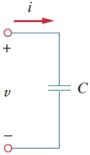
Resistor



$$\boxed{v(t) = R\, i(t)}$$

$$i(t) = \frac{v(t)}{R}$$

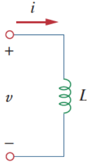
Capacitor



$$v(t) = \frac{1}{C}\int_0^t i(t)\, dt$$

$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

Inductor



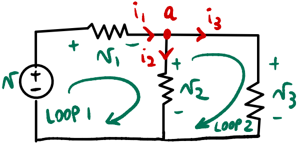
$$\boxed{v(t) = L\,\frac{di(t)}{dt}}$$

$$i(t) = \frac{1}{L}\int_0^t v(t)\, dt$$

Kirchoff current and voltage laws (KCL & KVL) - quick example

KCL: $\sum\limits_{\text{node}} \pm i_{\mathrm{j}} = 0$

KVL: $\sum\limits_{\text{loop}} \pm v_{\mathrm{k}} = 0$



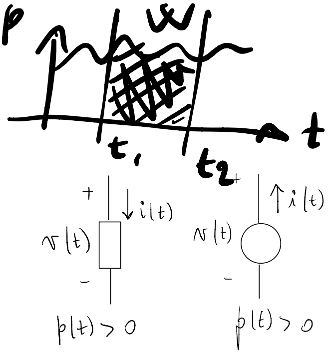
$$-i_1 + i_2 + i_3 = 0$$

$$-v + v_1 + v_2 = 0$$

$$-v_2 + v_3 = 0$$

(one eq. per branch)

Power and energy



absorbed generated

Instantaneous power

$$\boxed{p(t) = v(t)\,i(t)\ \ [\mathrm{W}]}$$

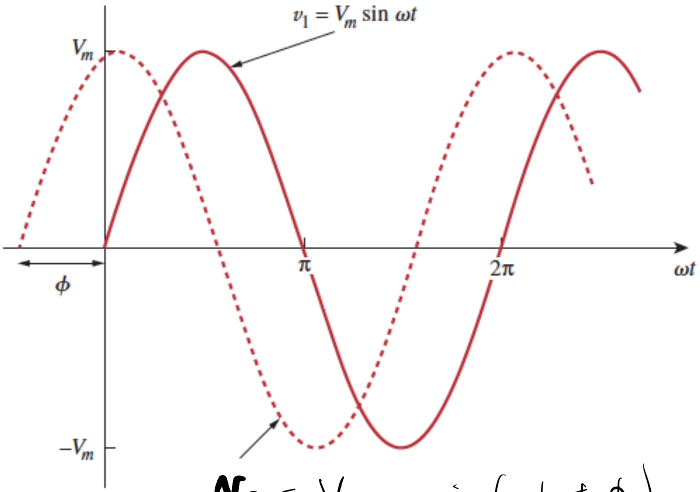
Energy

$$\boxed{W = \int_{t_1}^{t_2} p(t)\,dt\ [\mathrm{J}]}$$

Average power

$$\boxed{P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt\ [\mathrm{W}]}$$

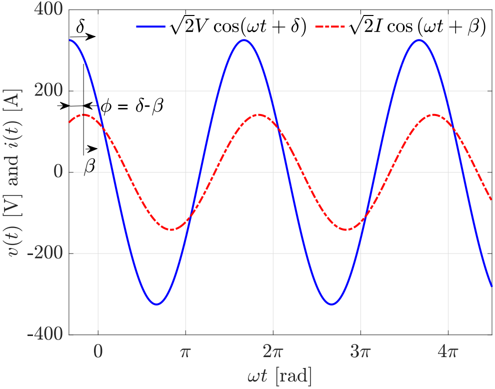
Question 1: what is the correct expression of the dashed waveform?



$v_2 = V_m \times \sin(\omega t + \phi)$

AC circuits in the time domain

$$v(t) = V_{\max}\cos(\omega t + \delta) = \sqrt{2}V\cos(\omega t + \delta)$$
$$i(t) = I_{\max}\cos(\omega t + \beta) = \sqrt{2}I\cos(\omega t + \beta)$$



rms value:

$$V = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt} = \frac{V_{\max}}{\sqrt{2}}$$

$\omega = 2\pi f$: angular frequency,
δ: angle of $v(t)$ @ $\omega t = 0$,
β: angle of $i(t)$ @ $\omega t = 0$,
$\phi = \delta - \beta$: angle difference between $v(t)$ and $i(t)$

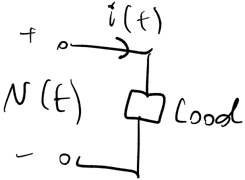
- $\phi > 0$: $i(t)$ lags $v(t)$
- $\phi < 0$: $i(t)$ leads $v(t)$

In this example:

$$\delta = \frac{\pi}{3},\ \beta = \frac{\pi}{6},\ \phi = \frac{\pi}{6}\ \text{rad}$$

$i(t)$ lags $v(t)$, $I_{\max}$ after $V_{\max}$

Instantaneous power



$$
\begin{aligned}
\boxed{p(t) = v(t)\, i(t)} & \\
&= 2VI\cos(\omega t + \delta)\cos(\omega t + \beta)^{1} \\
&= VI\{\cos(\delta - \beta) + \cos(2\omega t + \delta + \beta)\}^{2} \\
&= VI\{\cos(\delta - \beta) + \cos[2(\omega t + \delta) - (\delta - \beta)]\}^{3} \\
&= VI\cos(\phi)\{1 + \cos[2(\omega t + \delta)]\} + VI\sin(\phi)\sin\{[2(\omega t + \delta)]\} \\
&= \underbrace{VI_{\mathrm{R}}\{1 + \cos[2(\omega t + \delta)]\}}_{p_{\mathrm{R}}(t)} + \underbrace{VI_{\mathrm{X}}\sin[2(\omega t + \delta)]}_{p_{\mathrm{X}}(t)}
\end{aligned}
$$

- $I_{\mathrm{R}} = I\cos(\phi)$: current component in phase with $v(t)$
- $I_{\mathrm{X}} = I\sin(\phi)$: current component 90° out of phase from $v(t)$
- $\frac{1}{T}\int_0^T [p_{\mathrm{R}}(t)]\, dt = P$ active power; $\frac{1}{T}\int_0^T [p_{\mathrm{X}}(t)]\, dt = 0$

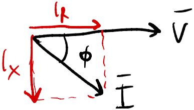
---

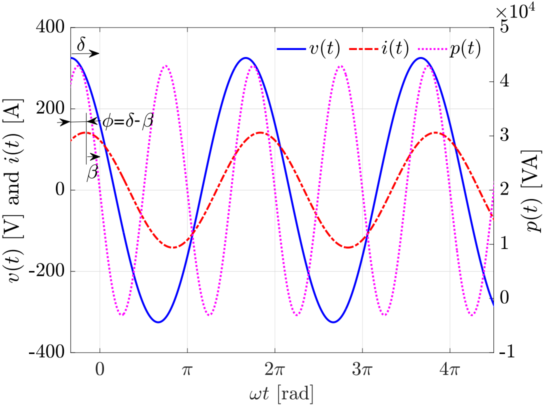
[1] $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

[2] $2\omega t + \delta + \beta = 2(\omega t + \delta) - (\delta - \beta)$

[3] $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Single phase instantaneous power on a generic load





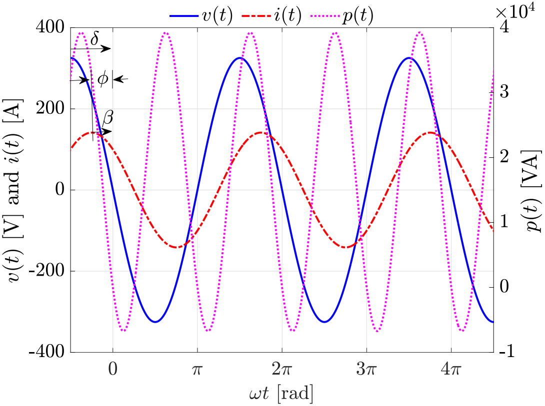
$$p(t) = VI_{\mathrm{R}}\{1 + \cos[2(\omega t + \delta)]\}$$
$$+VI_{\mathrm{X}}\sin[2(\omega t + \delta)]$$

- Instantaneous power $p(t)$ pulsates @ 2ω
- Instantaneous power can be positive and negative
- In this example:

$$\delta = \frac{\pi}{3},\ \beta = \frac{\pi}{6},$$

$$\phi = \frac{\pi}{6}\ \text{rad}\ (30°,\ \text{lagging})$$

Single phase instantaneous power on a generic load (cont'd)



- In this example:

  $$\delta = \frac{\pi}{2},\ \beta = \frac{\pi}{4},$$

  $$\phi = \frac{\pi}{4} \text{ rad } (45^\circ, \text{ lagging})$$

Active power, reactive power, power factor

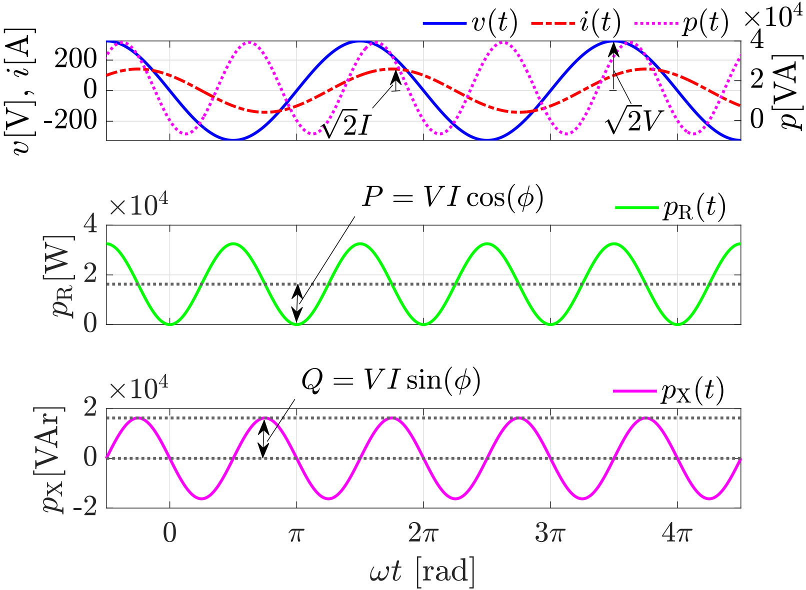
$$p(t) = v(t)\,i(t) = \ldots = \underbrace{\overbrace{V I_{\mathrm{R}}}^{\substack{\text{real}\\ \text{power P}}} \{1 + \cos[2(\omega t + \delta)]\}}_{p_{\mathrm{R}}(t)} + \underbrace{\overbrace{V I_{\mathrm{X}}}^{\substack{\text{reactive}\\ \text{power Q}}} \sin[2(\omega t + \delta)]}_{p_{\mathrm{X}}(t)}$$

Definitions:

- $P = V I_{\mathrm{R}} = V I \cos(\phi)$: real power (**active power**) **[W]**
- $Q = V I_{\mathrm{X}} = V I \sin(\phi)$: **reactive power** **[**VAr**]**
- $\phi = \delta - \beta$: phase difference [rad], [°], angle between v and i
  - ϕ lagging: $i(t)$ behind $v(t)$
  - ϕ leading: $i(t)$ ahead $v(t)$
- $\cos(\phi)$ is also called **power factor** (dimensionless quantity)

Active power, reactive power, power factor

Example: generic load



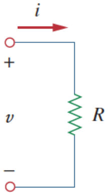
Active power, reactive power, power factor

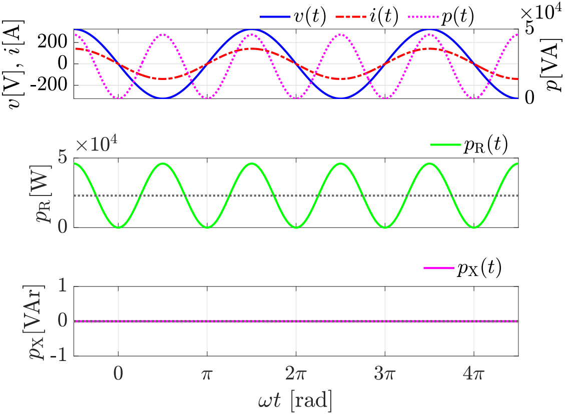
Power "exchanged" by a load is varying sinusoidally, at twice the fundamental frequency, and:

- Active power P: average value of $p_{\mathrm{R}}(t)$
- Reactive power Q: peak value of $p_{\mathrm{X}}(t)$ - does not create any work

Active power, reactive power, power factor

Example: purely resistive load





$$v(t) = \sqrt{2}V\cos(\omega t + \delta)$$

$$\boxed{i(t) = \frac{v(t)}{R}}$$

$$i(t) = \sqrt{2}\frac{V}{R}\cos(\omega t + \delta)$$

$$\phi = 0$$

$$P = \frac{V^2}{R};\quad Q = 0$$

Question 2:

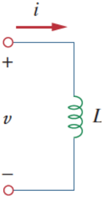
The instantaneous power consumed by a resistor is constant.
True or False?

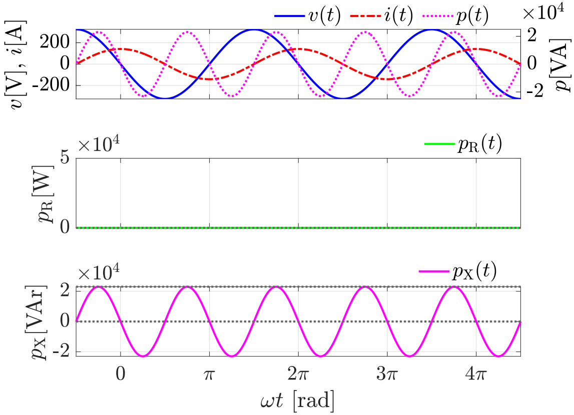
Question 3:

Find the amplitude, phase, period, and frequency of the sinusoid
$v(t) = 12\cos(50t + 10^\circ)$

Real power, reactive power, power factor

Example: purely inductive load





$$v(t) = \sqrt{2}V\cos(\omega t + \delta)$$

$$\boxed{v(t) = L\frac{di(t)}{dt}}$$

$$i(t) = \frac{1}{L}\int_0^t v(t)\,dt$$

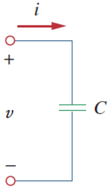
$$i(t) = \frac{\sqrt{2}V}{\omega L}\cos(\omega t + \overbrace{\delta - \frac{\pi}{2}}^{\beta})$$

$$\phi = \frac{\pi}{2} = \cancel{\delta} - \left(\cancel{\delta} - \frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$P = 0; \quad Q = \frac{V^2}{\omega L}\sin\left(\frac{\pi}{2}\right) = \frac{V^2}{\omega L}$$

Real power, reactive power, power factor

Example: purely capacitive load

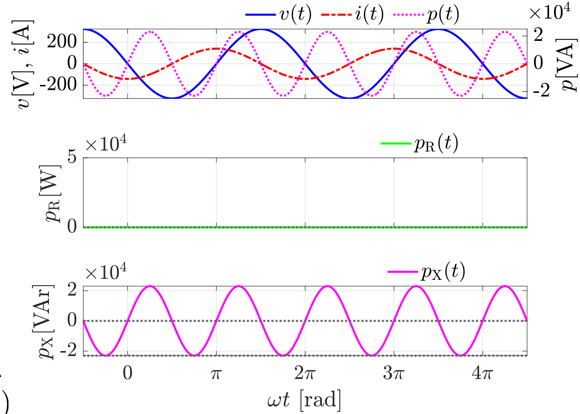


$$v(t) = \sqrt{2}V\cos(\omega t + \delta)$$

$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

$$i(t) = \sqrt{2}\omega C V \cos(\omega t + \overbrace{\delta + \frac{\pi}{2}}^{\beta})$$

$$\phi = -\frac{\pi}{2}$$



$$P = 0; \quad Q = \omega C V^2 \sin\left(-\frac{\pi}{2}\right) = -\omega C V^2$$

Phasor representation

Assumption:

- constant frequency, i.e., $\omega = 2\pi f$ is fixed

Cosinusoidal quantity $x(t)$, e.g., voltage or current, characterised by:

- maximum value X_{M}
- phase angle δ

$$\begin{aligned} x(t) &= X_{\mathrm{M}} \cos(\omega t + \delta) \\ &= \sqrt{2} X \cos(\omega t + \delta) \quad \text{where } X = \frac{X_{\mathrm{M}}}{\sqrt{2}} \text{ rms, or effective value} \end{aligned}$$

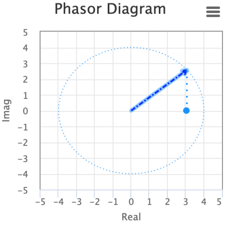
Using Euler's identity $e^{j\phi} = \cos\phi + j\sin\phi$, then:

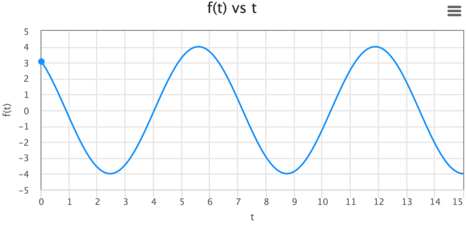
$$x(t) = \mathrm{Re}[X_{\mathrm{M}} e^{j(\omega t + \delta)}] = \mathrm{Re}[\sqrt{2}(X e^{j\delta}) e^{j\omega t}] \quad \text{where } j = \sqrt{-1}$$

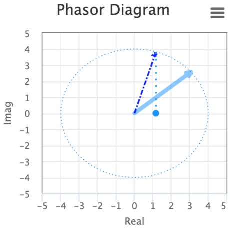
$$\boxed{\underbrace{\bar{X}}_{\substack{\text{rms}\\ \text{phasor}}} = \underbrace{X e^{j\delta}}_{\substack{\text{exponential}\\ \text{form}}} = \underbrace{X \angle \delta}_{\substack{\text{polar}\\ \text{form}}} = \underbrace{X \cos\delta + jX \sin\delta}_{\substack{\text{rectangular}\\ \text{form}}}}$$

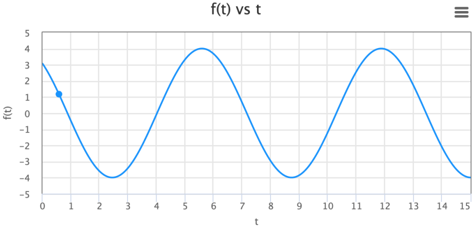
Phasor representation

E.g. given $v(t) = \sqrt{2}V\cos(\omega t + \delta)$, with $\omega = 1\,\frac{\mathrm{rad}}{\mathrm{s}}$, $\delta = 40°$, $\sqrt{2}V = 4\,\mathrm{V}$
Phasor Diagram: x-axis = *Real* axis; y-axis = *Imaginary* axis

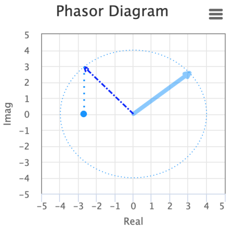


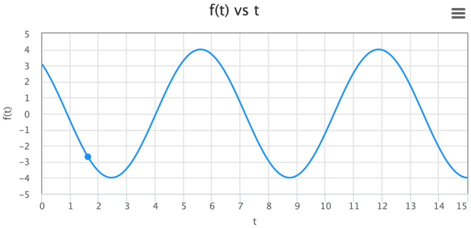


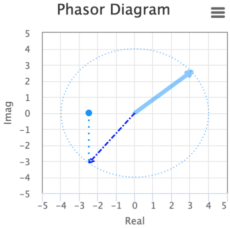


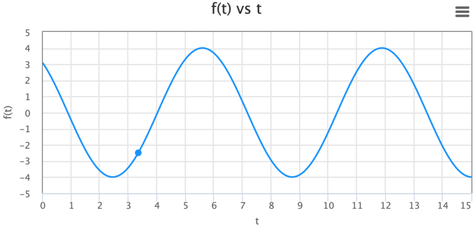


Phasor representation

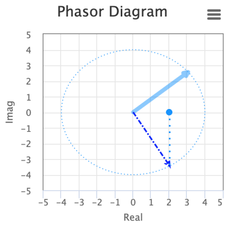


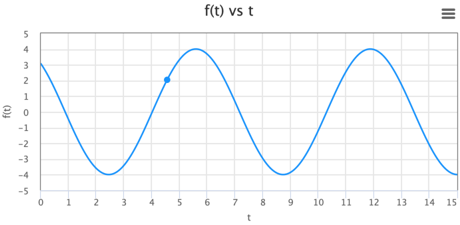






Phasor representation



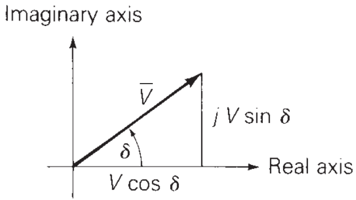


- counter clockwise rotating vector $\sqrt{2}(Ve^{j\delta})e^{j\omega t}$ projection on the x-axis is $v(t) = \sqrt{2}V\cos(\omega t + \delta)$ therefore, $v(t) = \mathrm{Re}[\sqrt{2}(Ve^{j\delta})e^{j\omega t}]$
- $\bar{V} = Ve^{j\delta}$ is defined as the rms phasor, it is a vector <u>fixed</u> at the angle of the rotating vector (δ) when $t = 0$
  - V: rms value, is the magnitude of the rms phasor, $V = {}^{V_{\max}}/\sqrt{2}$
- time $\rightarrow$ rms phasor domain: $v(t) = \sqrt{2}V\cos(\omega t + \delta) \;\rightarrow\; \bar{V} = Ve^{j\delta}$
- rms phasor $\rightarrow$ time domain: $\bar{V} = Ve^{j\delta} \rightarrow v(t) = \mathrm{Re}[\sqrt{2}(Ve^{j\delta})e^{j\omega t}]$

Figures from: `https://lpsa.swarthmore.edu/BackGround/phasor/phasor.html`

Phasor representation

$$v(t) \to \bar{V}$$

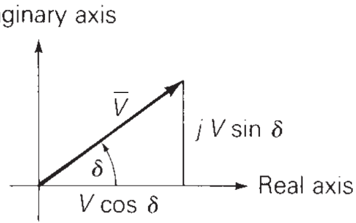


note on the transformation from rms phasor $[\bar{V}]$ to time domain $[v(t)]$:

$$\begin{aligned}\bar{V} = Ve^{j\delta} \to v(t) &= \mathrm{Re}[\sqrt{2}(Ve^{j\delta})e^{j\omega t}] \\ &= \mathrm{Re}[\sqrt{2}Ve^{j(\omega t+\delta)}] \\ &= \mathrm{Re}[\sqrt{2}V(\cos(\omega t+\delta) + j\sin(\omega t+\delta)] \\ &= \sqrt{2}V\cos(\omega t+\delta)\end{aligned}$$

Note that we consider the rms value "V" as the rms phasor magnitude, i.e., $\boxed{V = V_{\max}/\sqrt{2}}$. This convention is used in electrical engineering and may be different from conventions you have used in other units (ELEC2070)

Phasor operations



Polar form
coordinates (V, δ) useful for multiplication and division:

$$\bar{X} = \bar{Y}\bar{Z} = Ye^{j\alpha}Ze^{j\beta}$$
$$= (YZ)e^{j(\alpha+\beta)}$$

$$\bar{X} = \frac{\bar{Y}}{\bar{Z}} = \frac{Ye^{j\alpha}}{Ze^{j\beta}} = \left(\frac{Y}{Z}\right)e^{j(\alpha-\beta)}$$

Rectangular form
coordinates $(\mathrm{Re}[\bar{V}], \mathrm{Im}[\bar{V}])$ useful for addition and subtraction:

$$\bar{Y} = a + jb$$
$$\bar{Z} = c + jd$$
$$\bar{X} = \bar{Y} \pm \bar{Z}$$
$$= (a \pm c) + j(b \pm d)$$

Phasor operations (cont.'d 1)

Given, $v(t) = \sqrt{2}V\cos(\omega t + \delta)$:

- Derivation:

$$\frac{dv(t)}{dt} = -\sqrt{2}V\omega\sin(\omega t + \delta) = \sqrt{2}V\omega\cos\left(\omega t + \delta + \frac{\pi}{2}\right)$$

$$= \mathrm{Re}[\sqrt{2}V\omega e^{j(\omega t + \delta + \frac{\pi}{2})}]$$

$$\mathrm{Re}[\sqrt{2}V\omega e^{j(\omega t + \delta + \frac{\pi}{2})}] \to V\omega e^{j(\delta + \frac{\pi}{2})} = V\omega e^{j\delta}e^{j\frac{\pi}{2}}$$

$$= V\omega e^{j\delta}(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}) = j\omega V e^{j\delta}$$

$$\boxed{\frac{dv(t)}{dt} \to j\omega V e^{j\delta}}$$

Derivation in the time-domain becomes a multiplication by $j\omega$ in the rms phasor-domain

Phasor operations (cont.'d 2)

Given, $v(t) = \sqrt{2}V\cos(\omega t + \delta)$:

- Integration:

$$\int v(t)dt = \frac{\sqrt{2}V}{\omega}\sin(\omega t + \delta) = \frac{\sqrt{2}V}{\omega}\cos\left(\omega t + \delta - \frac{\pi}{2}\right)$$

$$= \mathrm{Re}[\frac{\sqrt{2}V}{\omega}e^{j(\omega t + \delta - \frac{\pi}{2})}]$$

$$\mathrm{Re}[\frac{\sqrt{2}V}{\omega}e^{j(\omega t + \delta - \frac{\pi}{2})}] \rightarrow \frac{V}{\omega}e^{j(\delta - \frac{\pi}{2})} = \frac{V}{\omega}e^{j\delta}e^{-j\frac{\pi}{2}} = \frac{V}{\omega}\frac{e^{j\delta}}{e^{j\frac{\pi}{2}}}$$

$$= \frac{Ve^{j\delta}}{\omega\left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right)} = \frac{V}{j\omega}e^{j\delta}$$

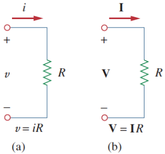
$$\boxed{\int v(t)dt \rightarrow \frac{V}{j\omega}e^{j\delta}}$$

Integration in the time-domain becomes a division by $j\omega$ in the rms phasor-domain

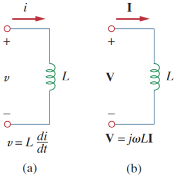
Phasor operations

$$\underbrace{\frac{dv(t)}{dt}}_{\text{time domain}} \rightarrow \underbrace{j\omega V e^{j\delta}}_{\text{rms phasor domain}} \qquad \underbrace{\int v(t)dt}_{\text{time domain}} \rightarrow \underbrace{\frac{V}{j\omega} e^{j\delta}}_{\text{rms phasor domain}}$$

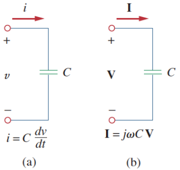
Phasor relationships for circuit elements



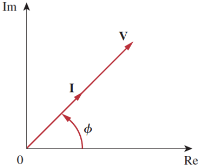
$\bar{I}$, $\bar{V}$ in phase

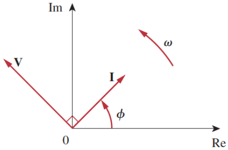


$\bar{I}$ lags $\bar{V}$



$\bar{I}$ leads $\bar{V}$





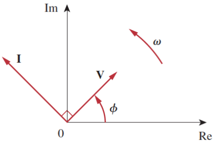
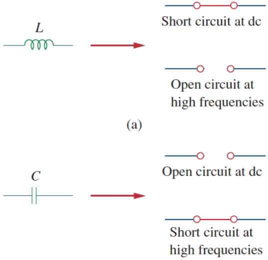


TABLE 9.2

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$



bold font indicates a rms phasor

- Impedance: $Z = \dfrac{\bar{V}}{\bar{I}}$ Admittance $Y = \dfrac{\bar{I}}{\bar{V}}$
- Ohm's law can be written in the rms phasor domain for any passive circuit element, as: $\bar{V} = Z\,\bar{I}$
- series and parallel combination of passive elements imply that the impedance has a real and imaginary component, in general $Z = R \pm jX$ and $Y = G \pm jB$
- $Z = |Z|e^{j\varphi}$ $|Z| = \sqrt{R^2 + X^2}$ $\tan\varphi = \dfrac{X}{R}$

Example:

Using the rms phasor approach, determine the time-domain expression for the current in a circuit described by the differential equation:

$$4i + 8\int i\,dt - 3\frac{d\,i}{dt} = 50\cos\left(2t + 75^\circ\right) \qquad \omega = 2$$

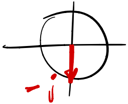
$$4I + \frac{8}{j\omega}I - 3\cdot j\omega I = \left(\frac{50}{\sqrt{2}}\right)e^{j75^\circ}$$

$$4I + \frac{8}{j2}I - j3\cdot 2\cdot I = \frac{50}{\sqrt{2}}e^{j75^\circ}$$

$$4I - j4I - j6I = \frac{50}{\sqrt{2}}e^{j75^\circ}$$

$$I(4 - j10) = \frac{50}{\sqrt{2}}e^{j75^\circ}$$

$$\frac{1}{j} = (j)^{-1} = \left(e^{j\frac{\pi}{2}}\right)^{-1} = e^{-j\frac{\pi}{2}}$$



$$I\left(\sqrt{4^2+(-10)^2}\right) e^{j\,\mathrm{atan}\left(\frac{-10}{4}\right)} = \frac{50}{\sqrt{2}} e^{j75^\circ}$$

$$I\,(10.77)\, e^{-j68.2^\circ} = \frac{50}{\sqrt{2}} e^{j75^\circ}$$

$$I = \frac{50}{(10.77)\sqrt{2}} e^{j(68.2^\circ+75^\circ)} = \frac{4.64}{\sqrt{2}} e^{j(143^\circ)} \quad A \quad \left(\begin{array}{c}\text{RMS}\\ \text{Phasor}\end{array}\right)$$

$$\boxed{i(t) = 4.64 \cdot \cos(2t + 143^\circ)\ A}$$

Example:

Using the rms phasor approach, determine the time-domain expression for the current in a circuit described by the differential equation:

$$4i + 8\int i\,dt - 3\frac{d\,i}{dt} = 50\cos\left(2t + 75^{\circ}\right)$$

Solution:

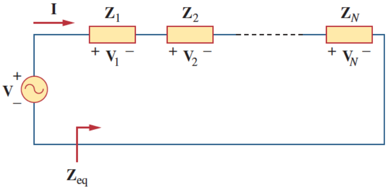
rms phasor: $\bar{I} = \left(\dfrac{4.642}{\sqrt{2}}\right) e^{j143.2^{\circ}}$

time-domain expression: $i\left(t\right) = 4.642\,\cos\left(2t + 143.2^{\circ}\right)$ A

Impedance combinations

Series

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_N = \mathbf{I}(\mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N)$$

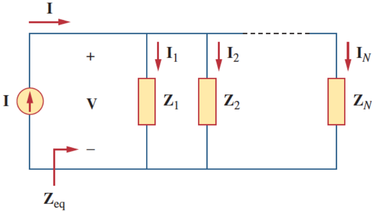


$$\mathbf{Z}_{eq} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

$$\boxed{\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N}$$

Parallel

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_N = \mathbf{V}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}\right)$$



$$\frac{1}{\mathbf{Z}_{eq}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

$$\boxed{\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N}$$

Complex power definition

Power definition in the time-domain, **instantaneous power**

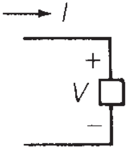
$$p(t) = v(t)\, i(t)$$

Power definition in the phasor domain, **complex power**

$$\bar{S} = \bar{V}\bar{I}^* = VIe^{j(\delta-\beta)} = VI\cos(\delta-\beta) + jVI\sin(\delta-\beta) = P + jQ$$

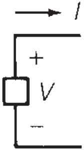
where $\bar{V} = Ve^{j\delta}$ and $\bar{I} = Ie^{j\beta}$ are the voltage and current rms phasors, and $\bar{I}^*$ is the complex conjugate of $\bar{I}$, i.e., $\bar{I}^* = Ie^{-j\beta}$

Load convention



$P > 0$: adsorbed (sinked)
$P < 0$: generated (sourced)
$Q > 0$: adsorbed (sinked)
$Q < 0$: delivered (sourced)

Generator convention



$P > 0$: generated (sourced)
$P < 0$: adsorbed (sinked)
$Q > 0$: delivered (sourced)
$Q < 0$: adsorbed (sinked)

Complex power definition and power triangle

$$\text{Resistor:} \quad \bar{S}_{\mathrm{R}} = \bar{V}\bar{I}_{\mathrm{R}}^{*} = Ve^{j\delta}\frac{V}{R}e^{-j\delta} = \frac{V^2}{R}$$

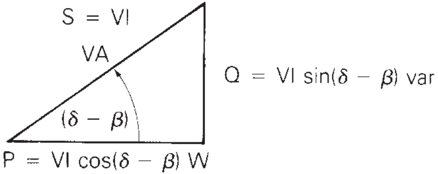
$$\text{Inductor:} \quad \bar{S}_{\mathrm{L}} = \bar{V}\bar{I}_{\mathrm{L}}^{*} = Ve^{j\delta}\frac{V}{\omega L}e^{-j(\delta-\frac{\pi}{2})} = j\frac{V^2}{\omega L} = j\frac{V^2}{X_L}$$

$$\text{Capacitor:} \quad \bar{S}_{\mathrm{C}} = \bar{V}\bar{I}_{\mathrm{C}}^{*} = Ve^{j\delta}V\omega Ce^{-j(\delta+\frac{\pi}{2})} = -j\omega C\,V^2 = -j\frac{V^2}{X_C}$$

For a general (passive) RLC circuit with the load convention: $P > 0$, $Q > 0$ for inductive loads (Q is adsorbed), $Q < 0$ for capacitive loads (Q is sourced)

Given:

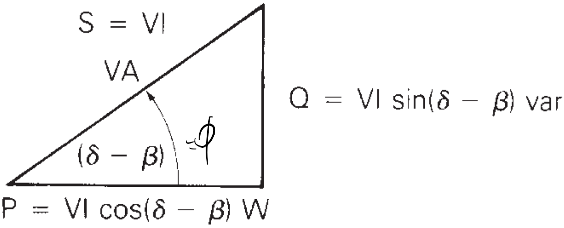
$$\bar{V} = Ve^{j\delta}$$
$$\bar{I} = Ie^{j\beta}$$



Complex power definition and power triangle

Power triangle

$$\bar{S} = P + jQ$$



$$S = \sqrt{P^2 + Q^2}$$

$$\delta - \beta = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$Q = P \tan(\delta - \beta)$$

power factor (p.f., or PF)

$$\text{p.f.} = \cos\underbrace{(\delta - \beta)}_{\phi} = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

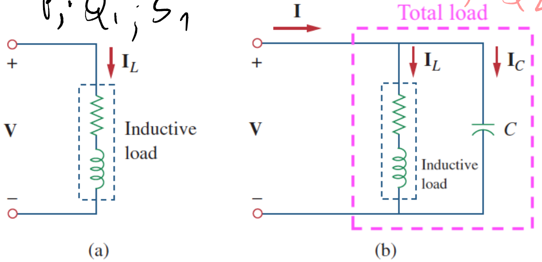
note that sometimes you may find that the symbol θ is used instead of ϕ

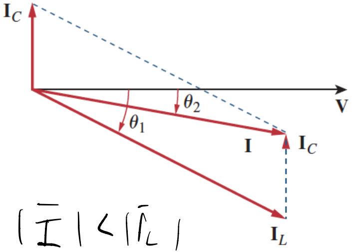
1. Linear circuit elements and AC power in the time domain
2. Phasors
   - Linear circuit elements in the frequency domain
3. Complex power
   - AC power in the frequency domain
4. Power factor correction (PFC)

Power factor correction (inductive load example)

- Leading power factor: $\bar{I}$ leads $\bar{V}$ (e.g. capacitive loads)
- Lagging power factor: $\bar{I}$ lags $\bar{V}$ (e.g. inductive loads)
- PFC: increasing PF of the overall load seen by voltage source $\bar{V}$

P_1, Q_1, S_1 $\quad$ P_1, Q_2, S_2 $\quad$ $\bar{I}_c = j\omega C \bar{V}$



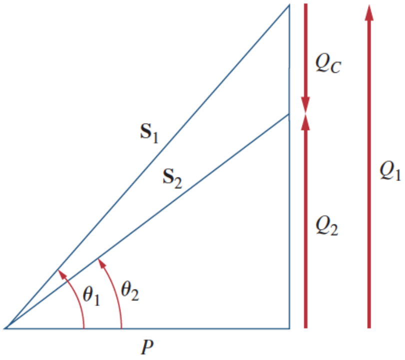


$|\bar{I}| < |\bar{I}_L|$

(a) Circuit with low p.f. (b) Circuit with increased p.f.

(a) without capacitor, angle between total load current and voltage is θ_1
(b) with capacitor, angle between total load current and voltage is $\theta_2 < \theta_1$

Power factor correction (inductive load example cont.'d)

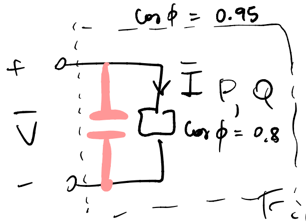


Case (a) - without capacitor

$$P = S_1 \cos\theta_1$$
$$Q_1 = S_1 \sin\theta_1 = P\tan\theta_1$$

Case (b) - with capacitor

$$Q_2 = P\tan\theta_2$$
$$|Q_C| = Q_1 - Q_2 = P\left(\tan\theta_1 - \tan\theta_2\right)$$
$$|Q_C| = \omega C V^2$$
$$\boxed{C = \frac{|Q_C|}{\omega V^2} = \frac{P\left(\tan\theta_1 - \tan\theta_2\right)}{\omega V^2}}$$

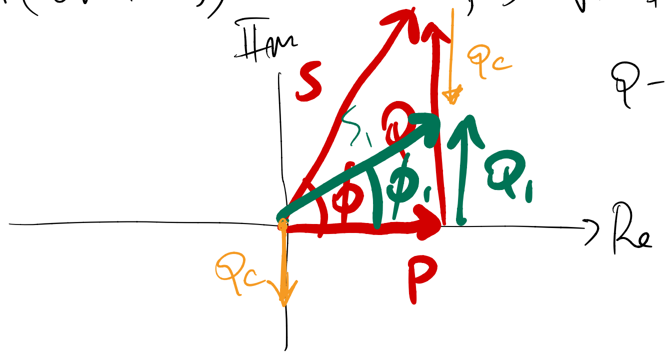


$P = 100\ kW$

$\cos\phi = 0.8$ (lagging)

Find the reactance, in order to have $\cos\phi = 0.95$ (lagging)

$$Q = 100 \cdot tg\left(\cos^{-1}(0.8)\right) = 75\ kVAr\ ; \quad S = \sqrt{100^2 + 75^2} = 125\ kVA$$



$$Q - |Q_c| = Q_1$$

$$Q_1 = P \cdot tg\,\phi_1 = 100 \cdot tg\left(\cos^{-1} 0.95\right) = 32.8\ kVAr$$

$$Q_c = Q - Q_1 = 75 - 32.8 = 42.2\ kVAr$$

$$S_1 = \sqrt{100^2 + (32.8)^2} = 105.26\ kVA$$

$V = 400V$

- without PF correction

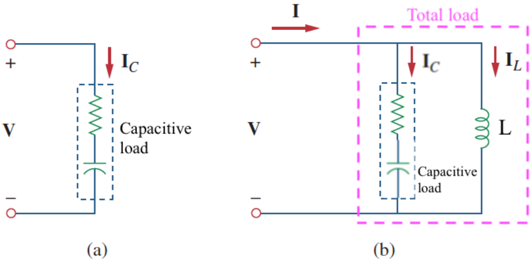
$$I = \frac{125 \, kVA}{400 V} = 312.5 \ A$$

- with PF correction

$$I = \frac{105.2 \, kVA}{400 V} = 263 \ A$$

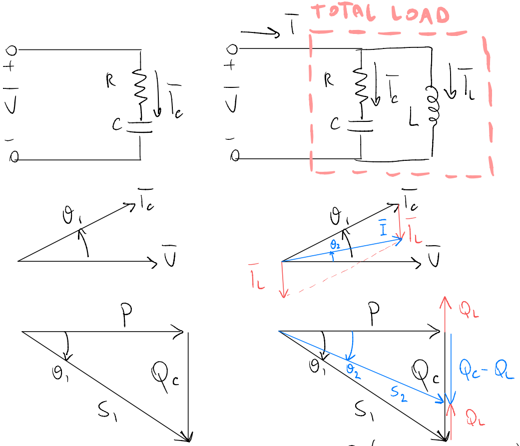
Power factor correction (capacitive load example)

If the load is capacitive, and it is desired to increase the power factor, then a shunt inductor can be used



$$\boxed{L = \frac{V^2}{\omega Q_L} = \frac{V^2}{\omega\left(Q_1 - Q_2\right)}}$$

Power factor correction (capacitive load example (cont.'d))



$$P = S_1 \cos\theta_1$$

$$Q_L = P\left(tg\,\theta_1 - tg\,\theta_2\right)$$

$$Q_L = \frac{V^2}{\omega L} \Rightarrow \boxed{L = \frac{V^2}{\omega Q_L} = \frac{V^2}{\omega P\left(tg\,\theta_1 - tg\,\theta_2\right)}}$$

References

- Glover, J. D., Overbye, T. J., & Sarma, M. S. (2017). *Power system analysis & design* (Sixth edition). Boston, MA, USA: Cengage Learning
- Sadiku, M., Alexander, C. (2013). *Fundamentals of Electric Circuits* (Fifth edition). United Kingdom: McGraw-Hill Education
- lpsa.swarthmore.edu (live phasor representation)

Readings

From *Glover (2017), Power System Analysis and Design*

- Chapter 2: Fundamentals (MQ library link)
  - 2.1: Phasors
  - 2.2: Instantaneous Power in Single-Phase ac Circuits
    - Example 2.1: Instantaneous, real and reactive power, and the power factor
  - 2.3: Complex Power (incl. Power Factor Correction)
    - Example 2.2: Real and reactive power, delivered or absorbed
    - Example 2.3: Power triangle and power factor correction

Other reminders

- Practical_2 take place at Lab G18, 44 Waterloo Road, School of Engineering Building
  - Check your student timetable
  - Labs use high voltage equipment (no at-home options available)
  - Labs must be completed/answers submitted within allotted time ($3\,\text{h}$)
  - Please read lab worksheet posted on iLearn before attending the Lab

- Assignment 2 is due on Tuesday the 26$^{\text{th}}$ of September (check iLearn)