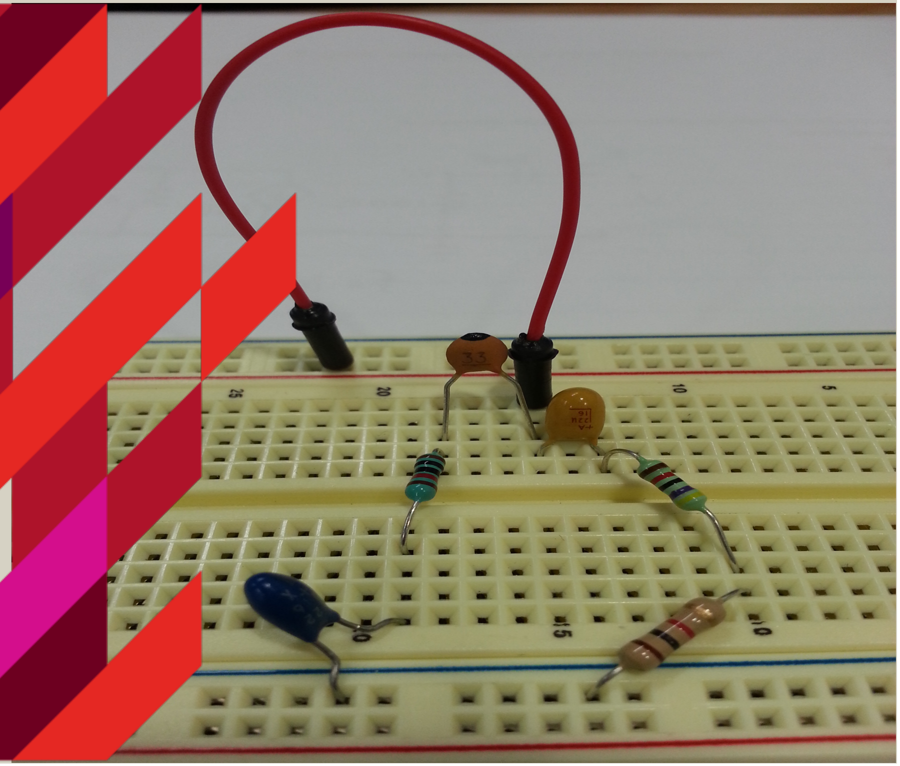
ELEC2070 Circuits and Devices

Week 7: Frequency domain analysis for steady state sinusoidal inputs

Stuart Jackson



Time domain vs frequency domain

Frequency domain

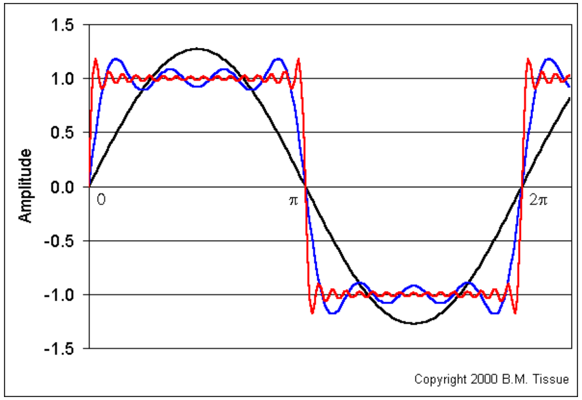
Up to now we have considered:

1. DC circuit analysis $\rightarrow$ mesh analysis, node equations, equivalent circuits, etc
2. Time domain analysis $\rightarrow$ energy storage, complete response of RL, RC and RLC circuits

Now we move to the **frequency domain**

FYI: **Fourier Transform converts time-domain functions into frequency-domain functions (Chapter 15)**

If the time-domain signal is periodic, the frequency-domain signal is a superposition of sinusoids.



Periodic signals

Typical periodic signals in time domain and frequency domain
(you will test some in the laboratory sessions)

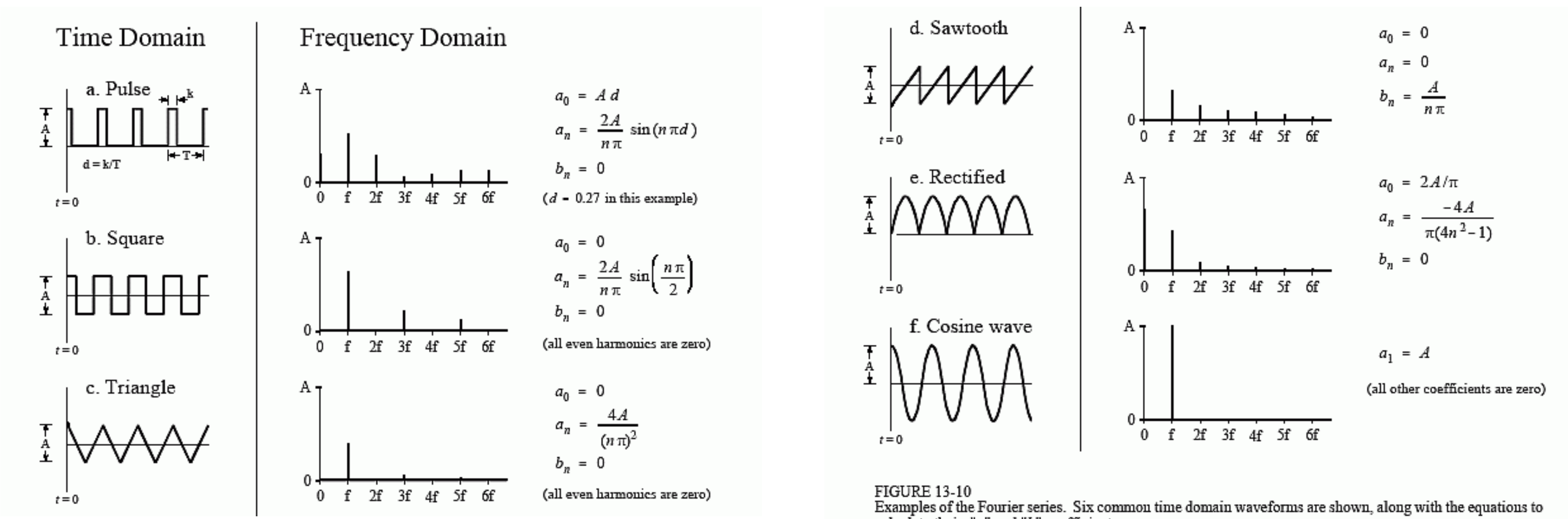


FIGURE 13-10
Examples of the Fourier series. Six common time domain waveforms are shown, along with the equations to calculate their "a" and "b" coefficients.

http://www.dspguide.com/ch13/4.htm

Analysing circuits in the frequency domain

For the next few lectures we will be looking at:

1. Steady state solutions with sinusoidal sources (i.e., **sinusoidal forcing functions**).
2. Forced response only - transients are not calculated.
3. Determine response from AC forcing functions - <u>one frequency at a time.</u>
4. Superposition is used to find the total frequency response.
5. Differential equations are not needed, **complex algebra** is used.

AC circuits and sinusoidal functions

Chapter 10 of Dorf and Svoboda

AC circuits

Consider a circuit having a sinusoidal input. The complete response will be:

$$v(t) = v_n(t) + v_f(t)$$

The natural response [$v_n(t)$] will die out leaving the forced response $v_f(t)$. When all the inputs have the **same** frequency the forced response will have the **same** frequency.

Overall when the following 2 conditions are fulfilled:

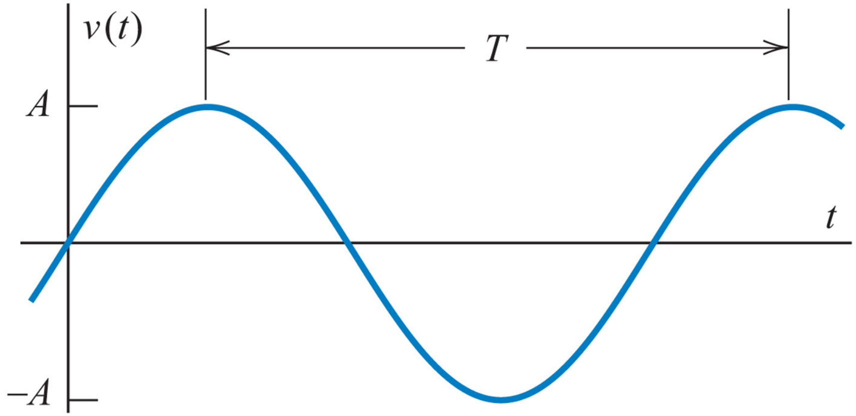
1. All of the inputs are sinusoidal and have the same frequency and
2. the circuit is at steady state,

Then all of the currents and voltages are sinusoidal and have the **same** frequency as the inputs.

Traditionally, sinusoidal currents have been called alternating currents (AC) and circuits that satisfy the above conditions are called AC circuits

Sinusoidal sources

We are going to examine all what we have covered so far in the unit when the independent forcing function is a **sinusoidal** function - AC circuit analysis (always in the steady state)



$$v(t) = A\sin(\omega t)\ \mathrm{V}$$

From RL, RC and RLC circuit analysis, we know that the forced response part of the complete response will be a sinusoid with the same frequency!

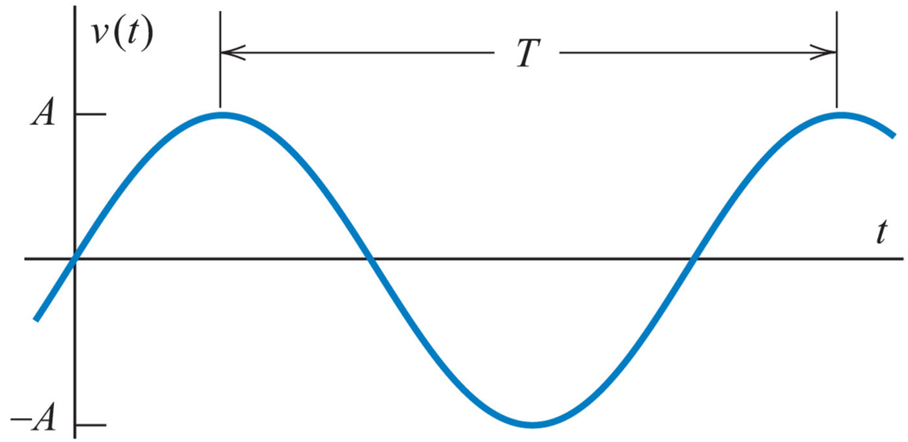
Forced responses with sinusoidal input (forcing functions) are easier dealt with in the **frequency domain**.

The important parameters relating to a sinusoidal signal

A sinusoid is a periodic function, i.e.:

$$v(t) = v(t+T)$$

Period (s)



The general functional form for a sinusoid is: $v(t) = A\sin(\omega t)$ (V)

Angular frequency (in radians/second)

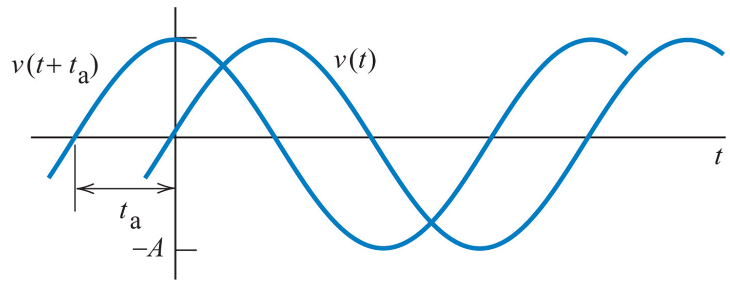
The angular frequency (rad/s) is defined as: $\omega = 2\pi f$

Where the frequency (in s^{-1}) is given by: $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$ (Hz)

There are 2π radians in a "wavelength".

Time advance, delay and phase angle

Consider the effect of replacing t by t + t_a



$$v(t+t_a) = A\sin[\omega(t+t_a)] = A\sin[\omega t + \omega t_a] = A\sin[\overbrace{\omega t + \theta}^{\text{(in rad)}}]$$

Time advance (s)

Phase angle (rad)

The phase angle (radians) is related to the time t_a: $\omega t_a = \theta = \dfrac{2\pi t_a}{T}$

Phase angle

Similarly, replacing t by $t - t_d$ produces a sinusoid that is identical to v(t) except that $v(t - t_d)$ is delayed from v(t) by time t_d.

We have $$v(t - t_d) = A\sin[\omega(t - t_d)] = A\sin[\omega t - \omega t_d] = A\sin[\omega t + \theta]$$

The phase angle here is: $\theta = -\dfrac{2\pi t_d}{T}$

NOTE:

1. An advance or delay of a full period leaves a sinusoid unchanged, that is $v(t \pm T) = v(t)$. Consequently, an **advance** by time t_a is equivalent to a **delay** by time $T - t_a$.

2. Similarly, a **delay** by time t_d is equivalent to an **advance** by time $T - t_d$

Phasors and phasor arithmetic

Phasor notation

ANY current or voltage in an AC circuit is a sinusoid at the SAME input frequency. Such a current or voltage is characterised by its amplitude and phase angle.

A **phasor** is a complex number that is used to represent the amplitude and phase angle of a sinusoid. The relationship between the sinusoid and the phasor is described by:

$$A\cos(\omega t + \theta) \quad \leftrightarrow \quad A \angle \theta$$

Note:

1. The sinusoid is represented using the **cosine** rather than the sine function. This is a convention.
2. The phasor is a complex number represented here in **polar** form.

The magnitude of the phasor is equal to the amplitude of the sinusoid, and the angle of the phasor is equal to the phase angle of the sinusoid.

Why phasors? Since all the elements in the circuit have a steady state response with an identical frequency to the source (forcing function) frequency, **only the amplitude and phase of the currents and voltages in every other element in the circuit will be different** – hence we use an abbreviation that just includes amplitude and phase angle!

Phasors

Amplitudes and angles depend on the frequency (even if they are not always shown this way)

$$A(\omega)\cos[\omega t + \theta(\omega)] \;\leftrightarrow\; A(\omega)\angle\theta(\omega)$$

Frequency Domain !

Lowercase i and v is used to indicate a sinusoidal current or voltage (often with a subscript) that is a sinusoidal function of time. Sometimes we will write i_1 instead of $i_1(t)$.

We will use bold uppercase **I** and **V** to indicate the corresponding phasor current or voltage with the same subscript.

The phasors are functions of the input frequency. In an AC circuit, the input frequency is fixed and we often shorten $I_1(\omega)$ to $\mathbf{I}_1$.

The bold capital notation of phasor means it is a vector quantity:

$$v(t) = A\cos[\omega t + \theta] \;\leftrightarrow\; \mathbf{V}(\omega) = \boldsymbol{V} = A\angle\theta$$

Example 10.3-1

Determine the phasors corresponding to the sinusoids:

$$i_1(t) = 120\cos(400t + 60^o)\,\text{mA}$$
$$i_2(t) = 100\sin(400t - 75^o)\,\text{mA}$$

Solution:

$$\begin{aligned}\sin(-\theta) &= -\sin\theta\\ \cos(-\theta) &= \cos\theta\\ \tan(-\theta) &= -\tan\theta\end{aligned}$$

$$\boldsymbol{I}_1(\omega) = 120\langle 60^o\ mA$$

And

Need to convert to cosine first

$$i_2(t) = 100\sin(400t - 75^o) = 100\cos(400t - 75^o - 90^o)$$

Hence

$$\boldsymbol{I}_2(\omega) = 100\langle -165^o\ mA$$

Important trig functions

$$\begin{aligned}\sin(-\theta) &= -\sin\theta \\ \cos(-\theta) &= \cos\theta \\ \tan(-\theta) &= -\tan\theta\end{aligned}$$

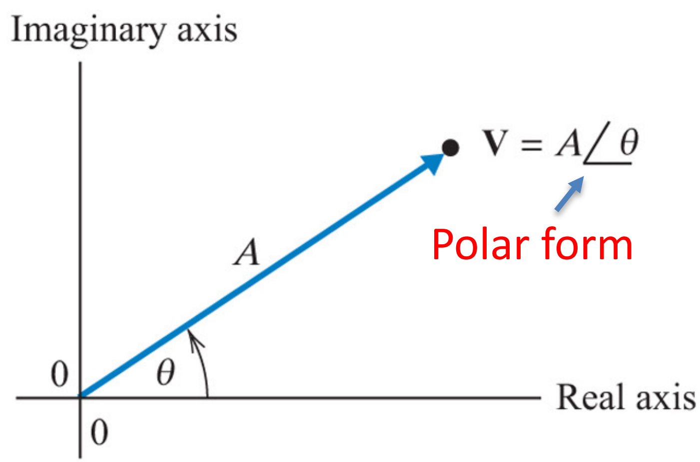
$$\cos\theta = \sin\left(\tfrac{\pi}{2} - \theta\right) \qquad \sin\theta = \cos\left(\tfrac{\pi}{2} - \theta\right)$$

$$\cot\theta = \tan\left(\tfrac{\pi}{2} - \theta\right) \qquad \tan\theta = \cot\left(\tfrac{\pi}{2} - \theta\right)$$

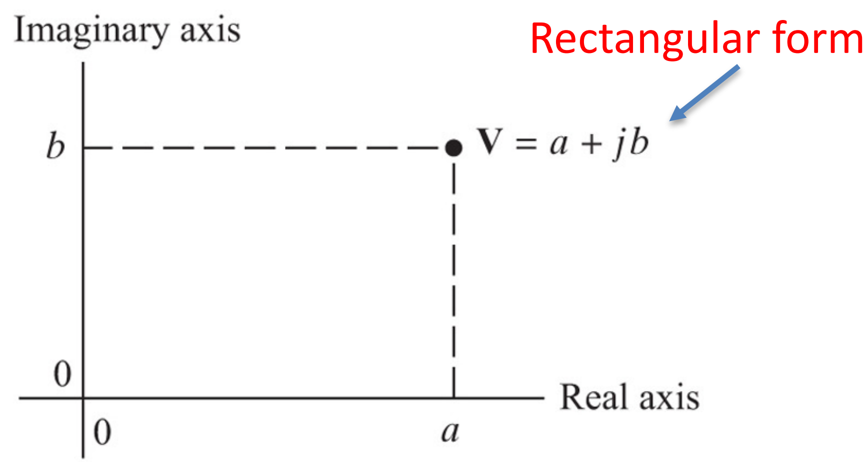
$$\csc\theta = \sec\left(\tfrac{\pi}{2} - \theta\right) \qquad \sec\theta = \csc\left(\tfrac{\pi}{2} - \theta\right)$$

Defining phasors as complex numbers

We can represent our phasor in the complex plane as:



Which has complex coordinates:



A is the magnitude and θ is the angle of the phasor: $A = |\mathbf{V}|$ and $\theta = \angle \mathbf{V}$

The phasor can be represented as a complex (with a real and imaginary components) number: $a = \mathrm{Re}\{\mathbf{V}\}$ and $b = \mathrm{Im}\{\mathbf{V}\}$

Hence we have: $a + jb = \mathbf{V} = A \angle \theta$

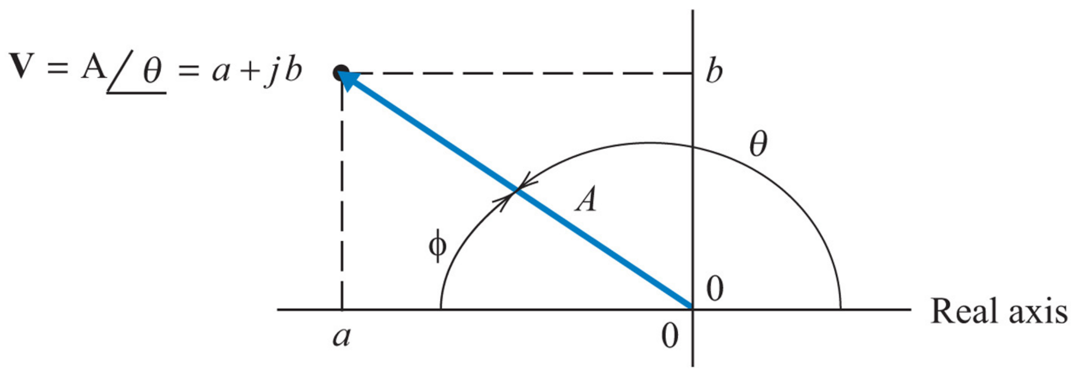
Remember $j = \sqrt{-1}$

Why the complex plane?

Phasor arithmetic can be easily accomplished using the two forms of the phasor. For the arithmetic operations addition and subtraction we use the rectangular form of the phasor **V** and for division and multiplication we use the polar form for the phasor **V**.

The phasor angle is positive in the counter clockwise direction

A phasor V with Re{V} < 0:



Two important observations

$a = a\angle 0°$

$jb = b\angle 90°$

Because $\emptyset \neq \theta$ we reference the phasor angle ALWAYS from the POSITIVE real axis

To convert from phasor to complex notation: $a = A\cos(\theta),\ b = A\sin(\theta),\ A = \sqrt{a^2 + b^2}$

with: $$\theta = \begin{cases} \tan^{-1}\left(\dfrac{b}{a}\right) & a > 0 \\ 180° - \tan^{-1}\left(\dfrac{b}{-a}\right) & a < 0 \end{cases}$$

$$1 = 1\underline{/0°},\ j = 1\underline{/90°},\ -1 = 1\underline{/\pm 180°} \text{ and } -j = 1\underline{/-90°} = 1\underline{/270°}$$

Complex algebra

$$\mathbf{V}_1 + \mathbf{V}_2 = (a + j\,b) + (c + j\,d) = (a + c) + j\,(b + d)$$

$$\mathbf{V}_1 - \mathbf{V}_2 = (a + j\,b) - (c + j\,d) = (a - c) + j\,(b - d)$$

For addition and subtraction use rectanglular notation

$$\mathbf{V}_1 \cdot \mathbf{V}_2 = \left(E \angle \theta\right)\left(F \angle \phi\right) = EF \angle(\theta + \phi)$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{A \angle \theta}{B \angle \phi} = \frac{A}{B} \angle(\theta - \phi)$$

For multiplication and division use phasor notation

The complex conjugate

$$\mathbf{V}_1{}^* = (a + jb)^* = a - jb$$
$$= \left(E \angle \theta\right)^* = E \angle -\theta$$

The exponential form of a phasor

Euler's formula is given by:

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Svoboda & Dorf: Appendix B

If

$$Ae^{j\emptyset} = A\cos\emptyset + jA\sin\emptyset$$

and

$$A\langle\emptyset = A\cos\emptyset + jA\sin\emptyset$$

then

$$A\,e^{j\,\phi} = A\,\angle\phi$$

The exponential form of a phasor

The conversion between polar and exponential forms of phasors is immediate.

Forms of a complex voltage or current

$$A\,e^{j\,(\omega\,t+\theta)} = A\,\cos(\omega\,t+\theta) + j\;A\,\sin(\omega\,t+\theta)$$

$$A\cos(\omega t+\theta) = \mathrm{Re}\left\{A\,e^{j\,(\omega t+\theta)}\right\}$$

POLAR FORM: $\boldsymbol{V} = M\langle\theta$

RECTANGULAR FORM: $\boldsymbol{V} = a + jb$

EXPONENTIAL FORM: $\boldsymbol{V} = M e^{i\theta}$

KVL and KCL with complex values

KVL:

If a set of sinusoidal voltages $v_i(t)$ satisfy KVL for an ac circuit, the corresponding phasor voltages $\mathbf{V}_i(\omega)$ satisfy the same KVL equation.

KCL:

If a set of sinusoidal currents $i_i(t)$ satisfy KCL for an AC circuit, the corresponding phasor currents $\mathbf{I}_i(\omega)$ satisfy the same KCL equation.

Circuit elements in the time and frequency domains

Voltage and current in the time and frequency domains

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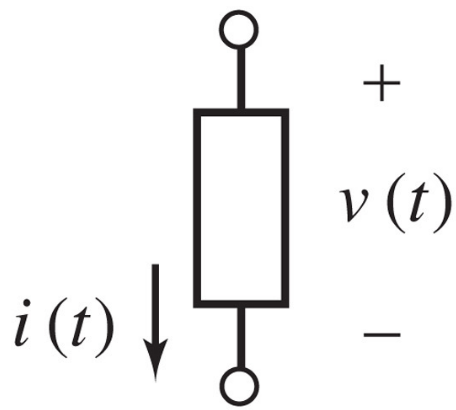
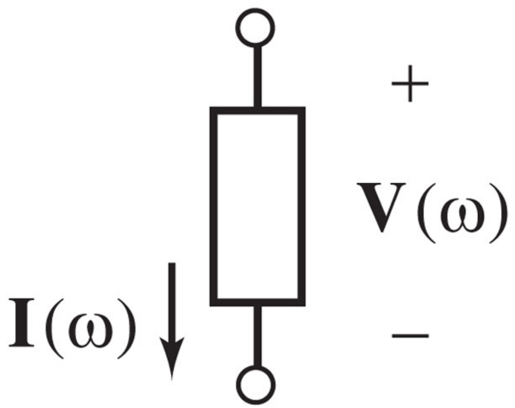
Time domain | **Frequency domain**



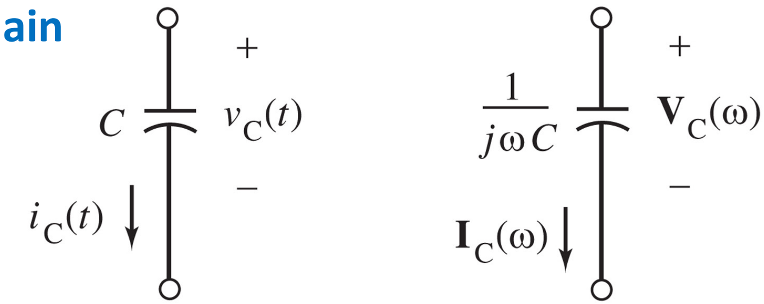


VOLTAGE: $v(t) = V_m \cos[\omega t + \theta] \;\leftrightarrow\; \mathbf{V}(\omega) = V_m \angle \theta$

CURRENT: $i(t) = I_m \cos[\omega t + \phi] \;\leftrightarrow\; \mathbf{I}(\omega) = I_m \angle \phi$

Capacitors in the time and frequency domains

Time domain **Frequency domain**



Voltage and current are not in phase

VOLTAGE: $v_C(t) = V_{Cm}\cos[\omega t + \theta] \quad \leftrightarrow \quad \mathbf{V}_C(\omega) = V_{Cm}\angle\theta$

CURRENT: $i_C(t) = C\dfrac{dv_C(t)}{dt} = -C\omega V_{Cm}\sin[\omega t + \theta] = C\omega V_{Cm}\cos[\omega t + \theta + 90°]$

CURRENT LEADS THE VOLTAGE BY 90^0

$$\mathbf{I}_C(\omega) = C\omega V_{Cm}\angle(\theta + 90°) = jC\omega\mathbf{V}_C(\omega)$$

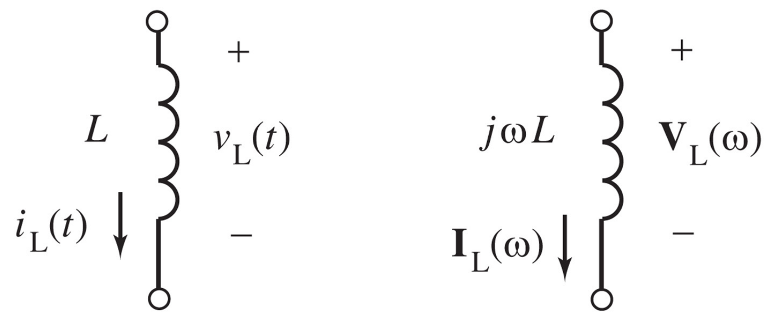
$$\mathbf{I}_C(\omega) = j\omega C\mathbf{V}_C(\omega)$$

Remember: $j = 1\angle 90°$

Inductors in the time and frequency domains

Time domain

Frequency domain



Voltage and current are not in phase

CURRENT: $i_L(t) = I_{Lm}\cos[\omega t + \phi] \quad \leftrightarrow \quad \mathbf{I}_{\boldsymbol{L}}(\omega) = I_{Lm}\angle\phi$

VOLTAGE: $v_L(t) = L\dfrac{di_L(t)}{dt} = -L\omega I_{Lm}\sin[\omega t + \phi] = L\omega I_{Lm}\cos[\omega t + \phi + 90°]$

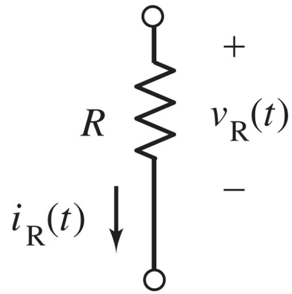
VOLTAGE LEADS THE CURRENT BY 90^0

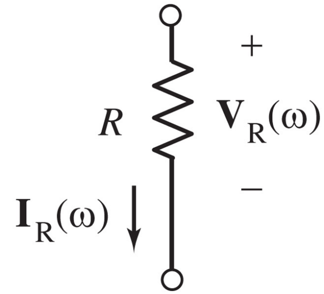
$$v_L(t) = L\frac{di_L(t)}{dt} \quad \leftrightarrow \quad \mathbf{V}_{\boldsymbol{L}}(\omega) = L\omega I_{Lm}\angle(\phi + 90°) = jL\omega\mathbf{I}_{\boldsymbol{L}}(\omega)$$

$$\mathbf{V}_{\boldsymbol{L}}(\omega) = jL\omega\mathbf{I}_{\boldsymbol{L}}(\omega)$$

Resistors in the time and frequency domains

Time domain





Frequency domain

CURRENT: $i_R(t) = I_{Rm}\cos[\omega t + \phi] \quad \leftrightarrow \quad \mathbf{I}_{\boldsymbol{R}}(\omega) = I_{Rm}\angle\phi$

VOLTAGE: $v_R(t) = Ri_R(t) = RI_{Rm}\cos[\omega t + \phi] \quad \leftrightarrow \quad \mathbf{V}_{\boldsymbol{R}}(\omega) = RI_{Rm}\angle\phi = R\mathbf{I}_{\boldsymbol{L}}(\omega)$

$$\mathbf{V}_{\boldsymbol{R}}(\omega) = R\mathbf{I}_{\boldsymbol{L}}(\omega)$$

Voltage and current **are** in phase

Voltage and Current Relationships in Time Domain

	Resistor	Capacitor	Inductor
Voltage	$R\ i_R$	v_C (no instantaneous change in time)	$L\dfrac{di_L}{dt}$
Current	i_R	$C\dfrac{dv_C}{dt}$	i_L (no instantaneous change in time)

Voltage and Current Relationships using s-operators (in Time Domain)

	Resistor	Capacitor	Inductor
Voltage	$R\ i_R$	$\dfrac{1}{sC}i_C$	$sL\ i_L$
Current	i_R	i_C	i_L

Voltage and Current Relationships in Frequency Domain

	Resistor	Capacitor	Inductor
Voltage	$R\ I_R$	$\dfrac{1}{j\omega C}I_C$	$j\omega L\ I_L$
Current	I_R	I_C	I_L

Impedance and Admittance

Ohm's Law in the frequency domain

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Impedance is similar to the resistance in the time domain

A sinusoidal voltage through a circuit element:

$$v(t) = V_m \cos[\omega t + \theta] \leftrightarrow \mathbf{V}(\omega) = V_m \angle\theta$$

Note same frequency

Different phase angle

The corresponding current:

$$i(t) = I_m \cos[\omega t + \phi] \leftrightarrow \mathbf{I}(\omega) = I_m \angle\phi$$

We define:

Impedance: $\mathbf{Z}(\omega) = \frac{\mathbf{V}(\omega)}{\mathbf{I}(\omega)} = \frac{V_m \angle\theta}{I_m \angle\phi} = \frac{V_m}{I_m} \angle(\theta - \phi)$

Ohm's Law: $\boxed{\mathbf{V}(\omega) = \mathbf{Z}(\omega) \cdot \mathbf{I}(\omega)}$

Impedance unit is Ω (ohm)

Impedance vocabulary:

(Since impedance is complex)

$$\mathbf{Z}(\omega) = \mathbf{R}(\omega) + j\mathbf{X}(\omega)$$

Resistance (real part)

Reactance (imaginary part)

Ohm's Law

In terms of admittance (similar to conductance in time domain)

Admittance:

$$\mathbf{Y}(\omega) = \frac{\mathbf{1}}{\mathbf{Z}(\omega)} = \frac{\mathbf{I}(\omega)}{\mathbf{V}(\omega)}$$

Admittance unit is 1/Ω (mho) or Siemens (S)

For an AC circuit in the frequency domain, a RLC circuit can be analysed using the impedances since the system is steady state. Remember in DC circuits, capacitors and inductors act like open and short circuits.

Impedances of circuit elements

Capacitor

$$\mathbf{Z}_C(\omega) = \frac{\mathbf{V}_C(\omega)}{\mathbf{I}_C(\omega)} = \frac{1}{j\omega C} \ \Omega$$

Inductor

$$\mathbf{Z}_L(\omega) = \frac{\mathbf{V}_L(\omega)}{\mathbf{I}_L(\omega)} = j\omega L \ \Omega$$

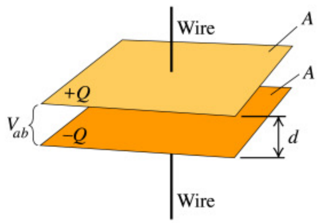
Resistor

$$\mathbf{Z}_R(\omega) = \frac{\mathbf{V}_R(\omega)}{\mathbf{I}_R(\omega)} = R \ \Omega$$

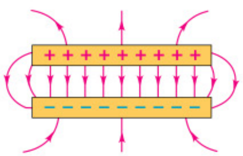
This means that the value of the impedance CHANGES with the frequency. For every source frequency applied to a circuit, the impedances of EVERY element in that circuit will change.

What does impedance mean?

It is a combination of electric and magnetic fields and resistance



(a)



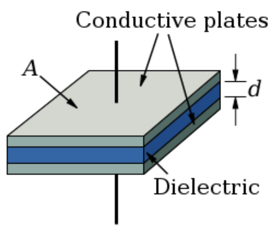
(b)

$$C = \frac{Q}{V}$$

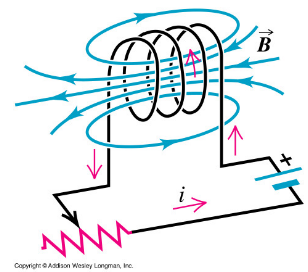
Electric charge

Voltage

http://www.physics.sjsu.edu/becker/physics51/capacitors.htm



$$C = \frac{\epsilon_r \epsilon_0 A}{d}$$

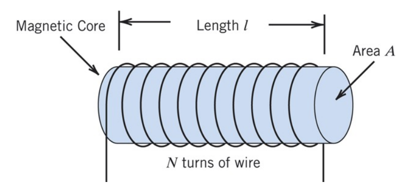


$$B = \mu \frac{N}{l} I$$

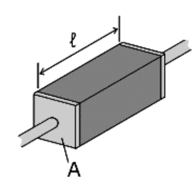
Magnetic field

Current

http://www.physics.sjsu.edu/becker/physics51/induction.htm



$$L = \frac{\mu_r \mu_0 N^2 A}{l}$$



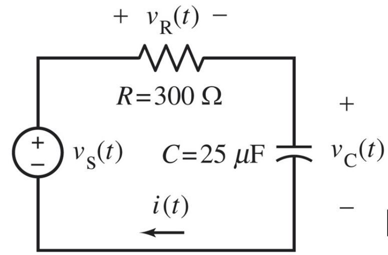
$$R = \frac{\rho l}{A}$$

- L, C, and R values don't change with frequency
- But impedance does!

Kirchhoff's Laws using phasors

Kirchhoff's Laws

Apply in the frequency domain



Example 10.3-4 (Dorf 9th ed.)

Kirchhoff's laws apply to all time domain voltages and currents.

Therefore they work for sinusoidal voltages and currents.

Hence they work in frequency domain too.

Source voltage: $v_s(t) = 25\cos(100t + 15°)\ \text{V}$

In phasor form: $\mathbf{V}_s(100) = 25\angle 15°\ \text{V}$

KVL: $\mathbf{V}_s(\omega) = \mathbf{Z}_R(\omega)\,\mathbf{I}\,(\omega) + \mathbf{Z}_C(\omega)\,\mathbf{I}\,(\omega)$

CAPACITOR: $\mathbf{Z}_C(100) = \dfrac{1}{j100C} = \dfrac{1}{j100\times 25\times 10^{-6}} = -j400\ \Omega$

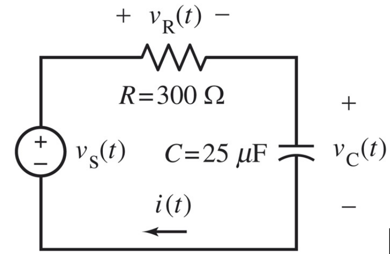
RESISTOR: $\mathbf{Z}_R(100) = 300\ \Omega$

$$\mathbf{I}\,(100) = \frac{\mathbf{V}_s(100)}{\mathbf{Z}_R(100)\ + \mathbf{Z}_C(100)} = \frac{25\angle 15°}{300 - j400} = \frac{25\angle 15°}{500\angle(-53.1°)} = 0.05\angle 68.1°\ \text{A}$$

$$i(t) = 0.05\cos(100t + 68.1°)\ \text{A}$$

Voltage division

Applies in the frequency domain



Example 10.3-4

Source voltage: $v_s(t) = 25\cos(100t + 15°)\ \text{V}$

In phasor form: $\mathbf{V}_s(100) = 25\angle 15°\ \text{V}$

Voltage division: $\mathbf{V}_R(\omega) = \frac{\mathbf{Z}_R(\omega)}{\mathbf{Z}_C(\omega)+\mathbf{Z}_R(\omega)}\mathbf{V}_S(\omega)$

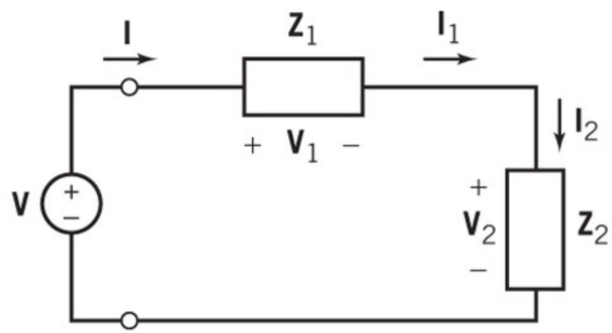
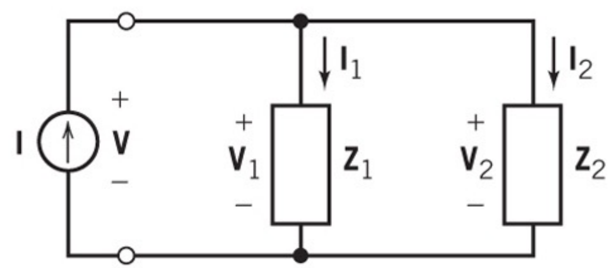
$$\mathbf{V}_R(100) = \frac{300}{300 - j400}25\angle 15° = \frac{3}{3 - j4}25\angle 15° = \frac{3}{5\angle(-53.1°)}25\angle 15° = 15\angle 68.1°$$

$$v_R(t) = 15\cos(100t + 68.1°)\ \text{V}$$

Remember to convert to radians before calculating cosine!

Current and voltage division

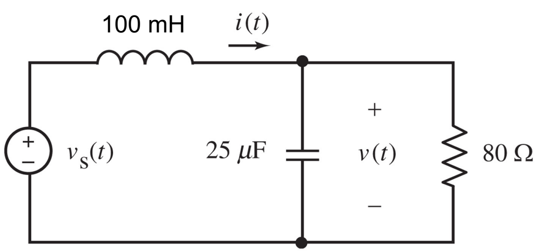
Also applies in the frequency domain

	CIRCUIT	EQUATIONS
Voltage division		$\mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I}$ $\mathbf{V}_1 = \dfrac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$ $\mathbf{V}_2 = \dfrac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$
Current division		$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V}$ $\mathbf{I}_1 = \dfrac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$ $\mathbf{I}_2 = \dfrac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$

Circuits in the frequency domain

Example 10.4-2

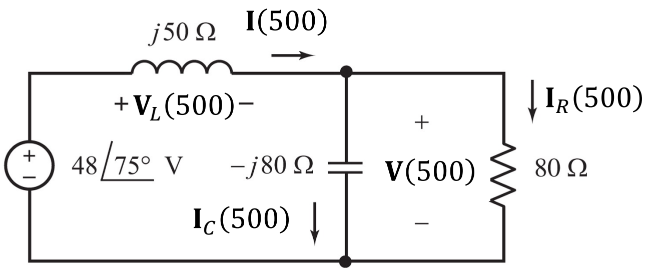
Time Domain



$$v_s(t) = 48\cos(500t + 75°)\ \text{V}$$

$$\omega = 500\ \text{rad/s}$$

Frequency Domain



$$\mathbf{V}_s(500) = 48\angle 75°\ \text{V}$$

$$\mathbf{Z}_R = 80\ \Omega$$

$$\mathbf{Z}_C(500) = \frac{1}{j500\ C} = -j80\ \Omega$$

$$\mathbf{Z}_L(500) = j500\ L = j50\ \Omega$$

Solution to Example 10.4-2

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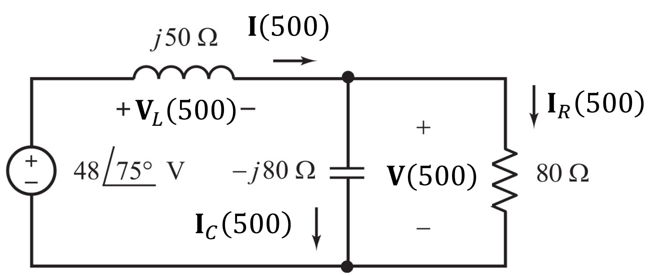
Frequency Domain



$$\mathbf{V}_s(500) = 48\angle 75°\ \mathrm{V}$$

KVL: $\mathbf{V}_s(500) = \mathbf{V}_L(500) + \mathbf{V}(500)$

KCL: $\mathbf{I}(500) = \mathbf{I}_C(500) + \mathbf{I}_R(500)$

OL: $\mathbf{V}_L(500) = j50 \cdot \mathbf{I}(500)$

$$\mathbf{I}_C(500) = \frac{\mathbf{V}(500)}{-j80}$$

$$\mathbf{I}_R(500) = \frac{\mathbf{V}(500)}{80}$$

Short notation to reduce complexity of the calculation:

KVL now becomes: $48\angle 75° = j50\left(\frac{\mathbf{V}}{-j80} + \frac{\mathbf{V}}{80}\right) + \mathbf{V}$

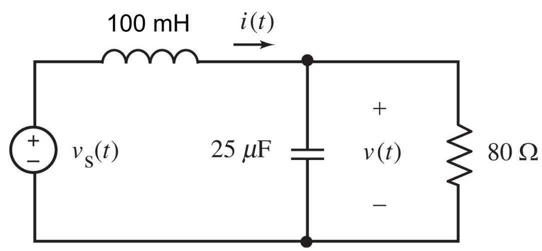
$$48\angle 75° = (-0.625 + j0.625 + 1)\mathbf{V} = (0.375 + j0.625)\mathbf{V}$$

$$\mathbf{V} = \frac{48\angle 75°}{0.375 + j0.625} = 65.9\angle 16°$$

Solution in both domains

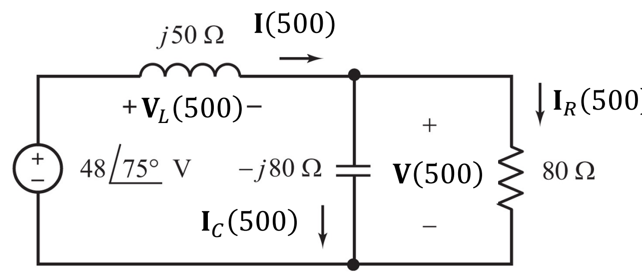
Example 10.4-2

Time Domain



$$v_s(t) = 48\cos(500t + 75°)\ \text{V}$$

Frequency Domain



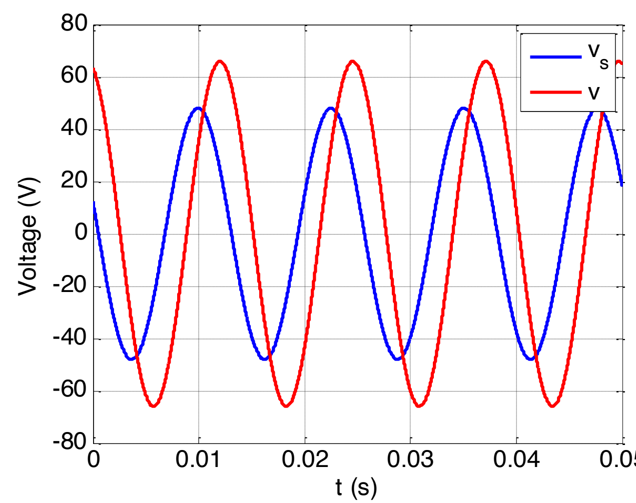
$$\mathbf{V}_s(500) = 48\angle 75°\ \text{V}$$

$$\mathbf{V}(500) = 65.9\angle 16°$$

$$v(t) = 65.9\cos(500t + 16°)\ \text{V}$$

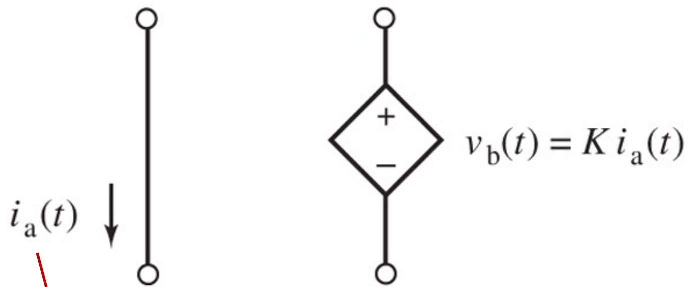
$$f = \frac{\omega}{2\pi} = \frac{500}{2\pi} = 79.6\ \text{Hz}$$

$$T = \frac{1}{f} = 12.6\ \text{ms}$$

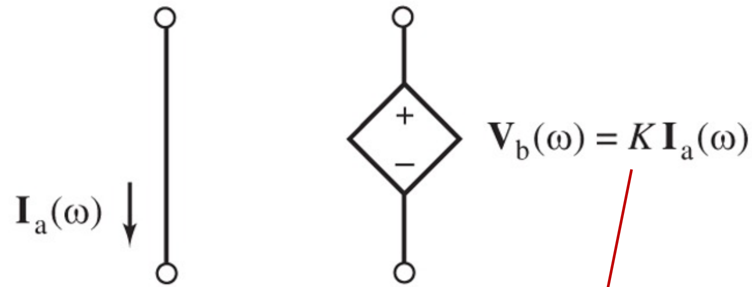


Dependent sources in the time and frequency domains

Time Domain



Frequency Domain



Controlling current: $i_a(t) = A\cos(\omega t + \theta)$

K can be a complex number

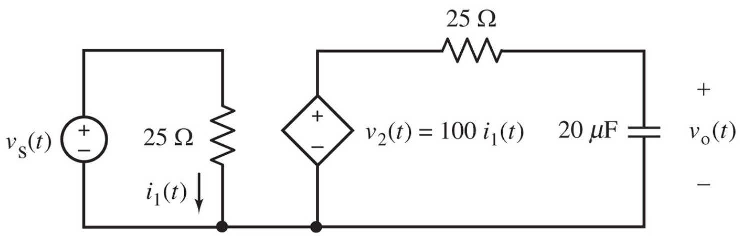
The current controlled voltage is: $v_b(t) = KA\cos(\omega t + \theta)$

Using phasor notation: $\boldsymbol{I_a}(\omega) = A\angle\theta \; and \; \boldsymbol{V}_b(\omega) = KA\angle\theta, \text{hence } \boldsymbol{V}_b(\omega) = K\boldsymbol{I_a}(\omega)$

Circuit with a dependent source

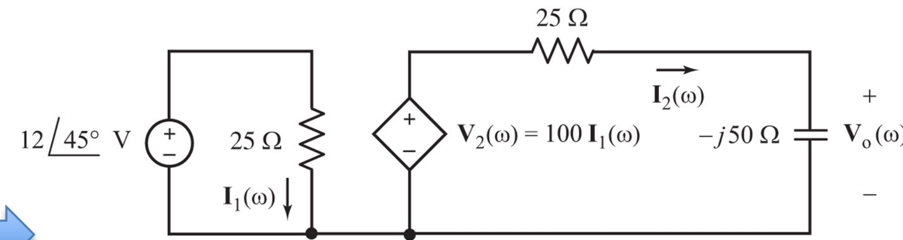
Example 10.4-3

Time Domain



$v_s(t) = 12\cos(1000t + 45°)\ \mathrm{V}$

Frequency Domain



$\omega = 1000\ \mathrm{rad/s}$

$$\mathbf{V}_s = 12\angle 45°\ \mathrm{V}$$

$$\mathbf{Z}_C = \frac{1}{j1000C} = \frac{1}{j1000\times 20\times 10^{-6}} = -j50\ \Omega$$

$$\mathbf{I}_1 = \frac{12\angle 45°}{25} = 0.48\angle 45°\ \mathrm{A} = \text{controlling current}$$

$$\mathbf{V}_2 = 100\mathbf{I}_1 = 48\angle 45°\ \mathrm{A}$$

Voltage division

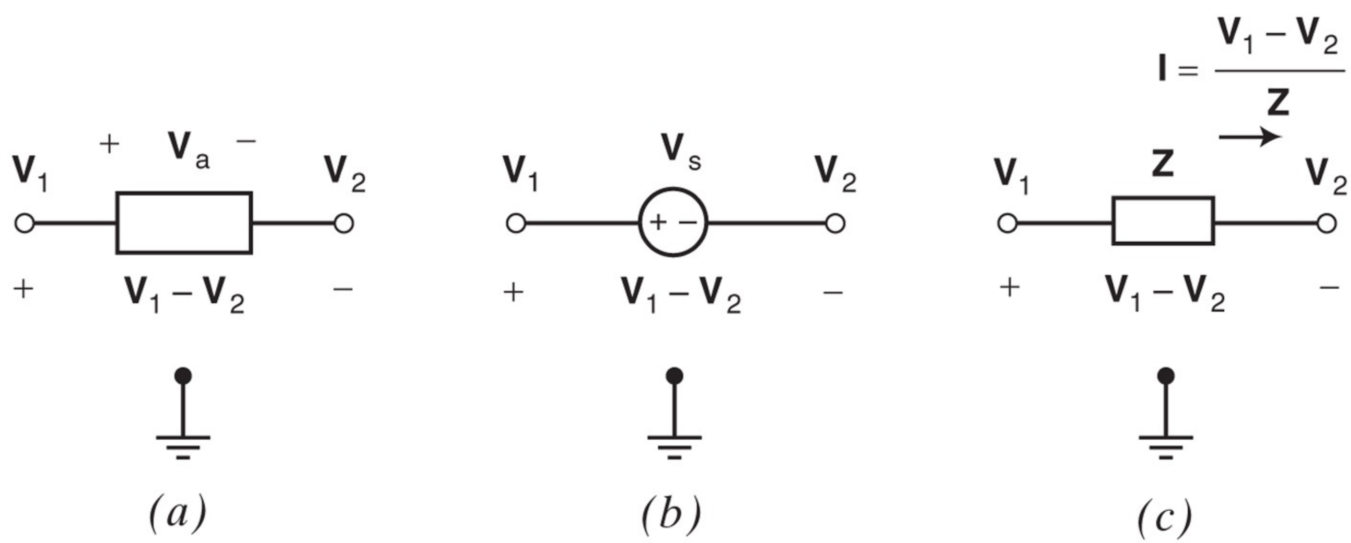
$$\mathbf{V}_o = \frac{-j50}{25-j50}\mathbf{V}_2 = \frac{-j2}{1-j2}\mathbf{V}_2 = \frac{2\angle(-90°)}{2.24\angle(-63.4°)}48\angle 45° = 42.93\angle 18.4°\ \mathrm{V}$$

Time Domain: $v_o(t) = 42.93\cos(1000t + 18.4°)\ \mathrm{V}$

Node voltage and mesh current analysis using phasors

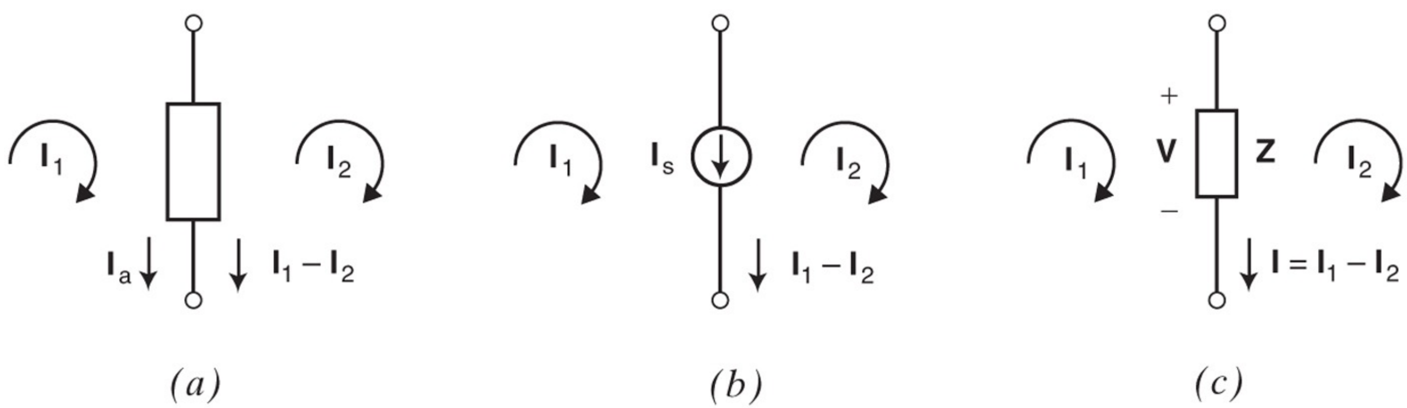
Node analysis in the frequency domain

- Express element voltages and currents in terms of node voltages.
- Same as the DC case, but using phasors.
- Apply Kirchhoff's Current Law at each node.
- Solve simultaneous equations to find each node voltage.



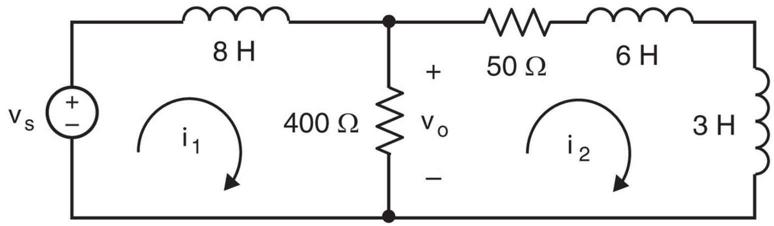
Mesh equations in the frequency domain

- Express element voltages and currents in terms of mesh currents.
- Same as the DC case, but using phasors.
- Apply Kirchhoff's Voltage Law for each mesh.
- Solve simultaneous equations to find each mesh current.



Problem 10.6-8 – Mesh analysis

Find i_1, i_2 and v_o



$$v_s(t) = 25\cos(40t + 45°)\ \text{V}$$

$$\omega = 40\ \text{rad/s}$$

Convert to frequency domain (phasors)

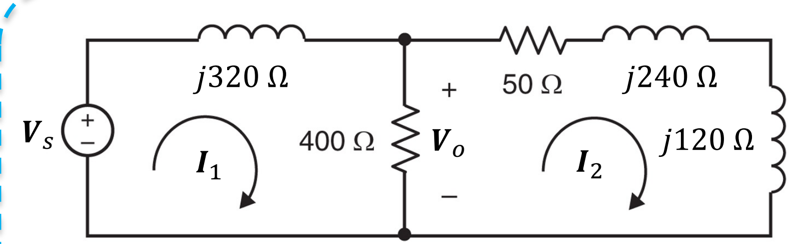
$$\boldsymbol{V}_s = 25\angle 45°\ \text{V}$$

$$\boldsymbol{Z}_L(\omega) = j\omega L$$

$$\boldsymbol{Z}_{L8} = j40 \cdot 8 = j320\ \Omega$$

$$\boldsymbol{Z}_{L6} = j40 \cdot 6 = j240\ \Omega$$

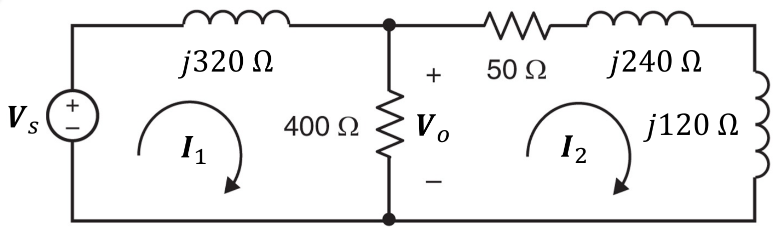
$$\boldsymbol{Z}_{L3} = j40 \cdot 3 = j120\ \Omega$$



$\boldsymbol{V}_s = 25\angle 45°\ \text{V}$ $\omega = 40\ \text{rad/s}$

Problem 10.6-8

Find i_1, i_2 and v_o



$V_s = 25\angle 45°\ \text{V}$ $\quad \omega = 40\ \text{rad/s}$

Two mesh currents makes two equations

(1) $25\angle 45° = j320\boldsymbol{I}_1 + 400(\boldsymbol{I}_1 - \boldsymbol{I}_2)$

$$25\angle 45° = (400 + j320)\boldsymbol{I}_1 - 400\boldsymbol{I}_2$$

(2) $0 = 400(\boldsymbol{I}_2 - \boldsymbol{I}_1) + 50\boldsymbol{I}_2 + j240\boldsymbol{I}_2 + j120\boldsymbol{I}_2$

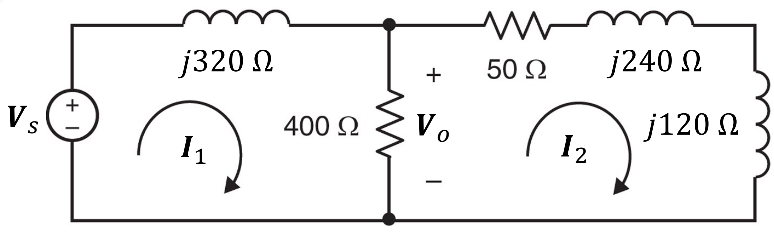
$$0 = -400\boldsymbol{I}_1 + (400 + 50 + j240 + j120)\boldsymbol{I}_2$$

$$0 = -400\boldsymbol{I}_1 + (450 + j360)\boldsymbol{I}_2$$

Solve (1) and (2) simultaneously

Problem 10.6-8

Find i_1, i_2 and v_o



$\boldsymbol{V_s} = 25\angle 45°\ \text{V}$ $\omega = 40\ \text{rad/s}$

Solve (1) and (2) simultaneously

$$25\angle 45° = (400 + j320)\boldsymbol{I}_1 - 400\boldsymbol{I}_2 \qquad (1)$$

$$0 = -400\boldsymbol{I}_1 + (450 + j360)\boldsymbol{I}_2 \qquad (2)$$

Using Matlab:

$$\begin{bmatrix} 25\ \angle 45° \\ 0 \end{bmatrix} = \begin{bmatrix} 400 + j320 & -400 \\ -400 & 450 + j360 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix}$$

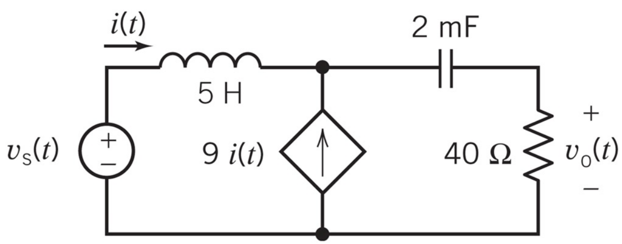
$$\begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 47.5\ \angle - 24.63° \\ 33.3\angle - 63.29° \end{bmatrix} \text{mA}$$

Solution in frequency domain: $\boldsymbol{V}_o = 400(\boldsymbol{I}_1 - \boldsymbol{I}_2) = 11.98\angle 18.8°\ \text{V}$

In time domain: $v_o(t) = 11.98\cos(40t + 18.8°)\ \text{V}$

Problem 10.6-14: find v_0

Find



$$v_s(t) = 25\cos(100t - 15°)\ \text{V}$$

$$\omega = 100\ \text{rad/s}$$

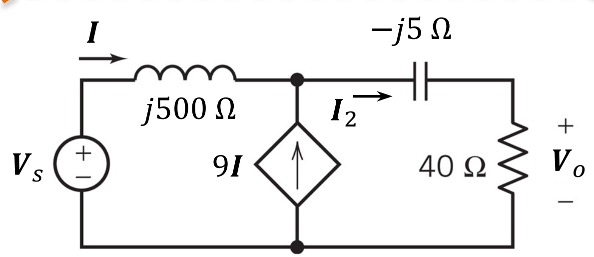
Convert to frequency domain

$$\boldsymbol{V}_s = 25\angle -15°\ \text{V}$$

$$\boldsymbol{Z}_L(\omega) = j\omega L \qquad \boldsymbol{Z}_C(\omega) = 1/j\omega C$$

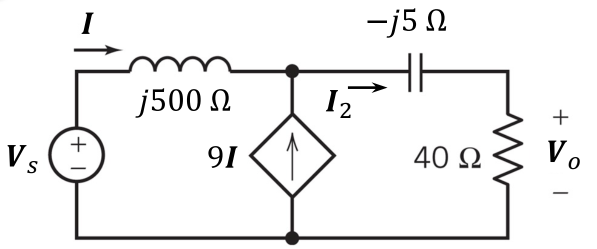
$$\boldsymbol{Z}_L = j100 \cdot 5 = j500\ \Omega$$

$$\boldsymbol{Z}_C = \frac{1}{j100\times 2\times 10^{-3}} = -j5\ \Omega$$



$\boldsymbol{V}_s = 25\angle -15°\ \text{V}$ $\qquad \omega = 100\ \text{rad/s}$

Problem 10.6-14



$V_s = 25\angle -15°$ V $\quad \omega = 100$ rad/s

KCL at top node: $I_2 = (9+1)I = 10I$

Super-mesh equation:

$$25\angle -15° = j500I + (-j5)10I + (40)10I$$

$$25\angle -15° = j500I - j50I + 400I$$

$$25\angle -15° = (j450 + 400)I$$

$$I = \frac{25\angle -15°}{j450 + 400} = (41.52\angle -63.37°)\ \text{mA}$$

Solution in frequency domain: $V_o = (40)10I = (16.61\angle -63.37°)$ V

In time domain: $v_o(t) = 16.61\cos(100t - 63.37°)$ V