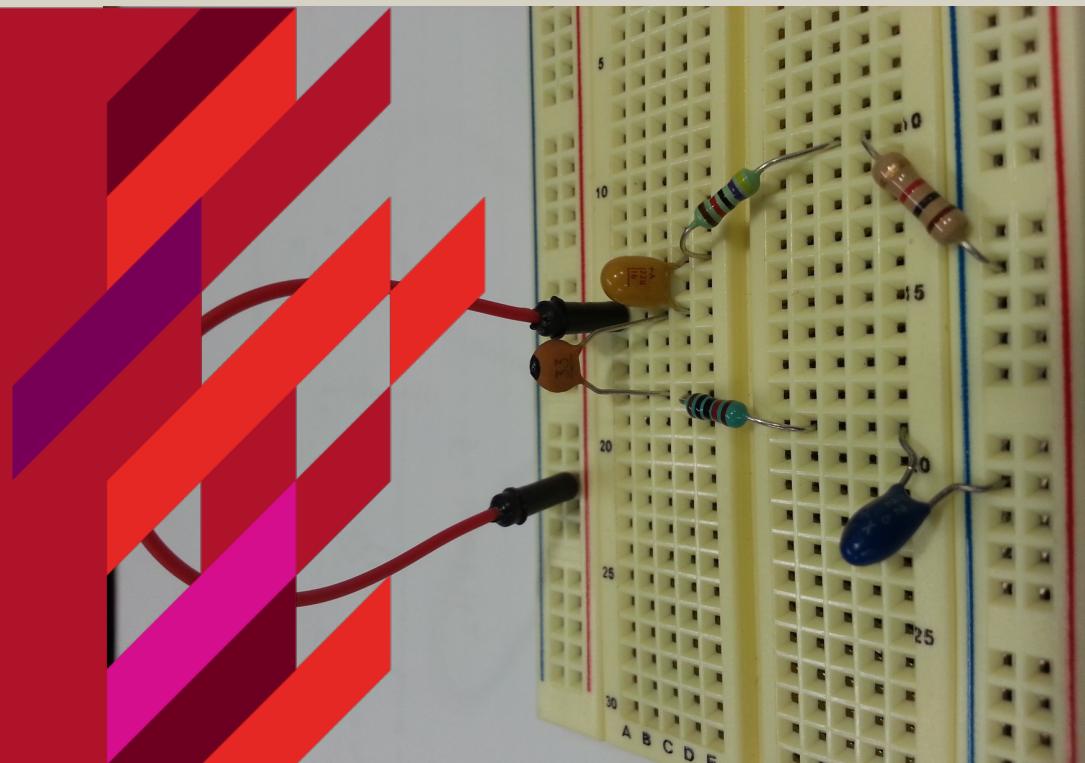




ELEC2070: Linear Circuits and Devices

Week 1: Introduction + Review + Fundamentals

Stuart Jackson



Stuart Jackson

Professor
Deputy Director MQ Photonics

Director of the Mid-Infrared Fibre Sources Group
Associate Editor Journal of Lightwave Technology

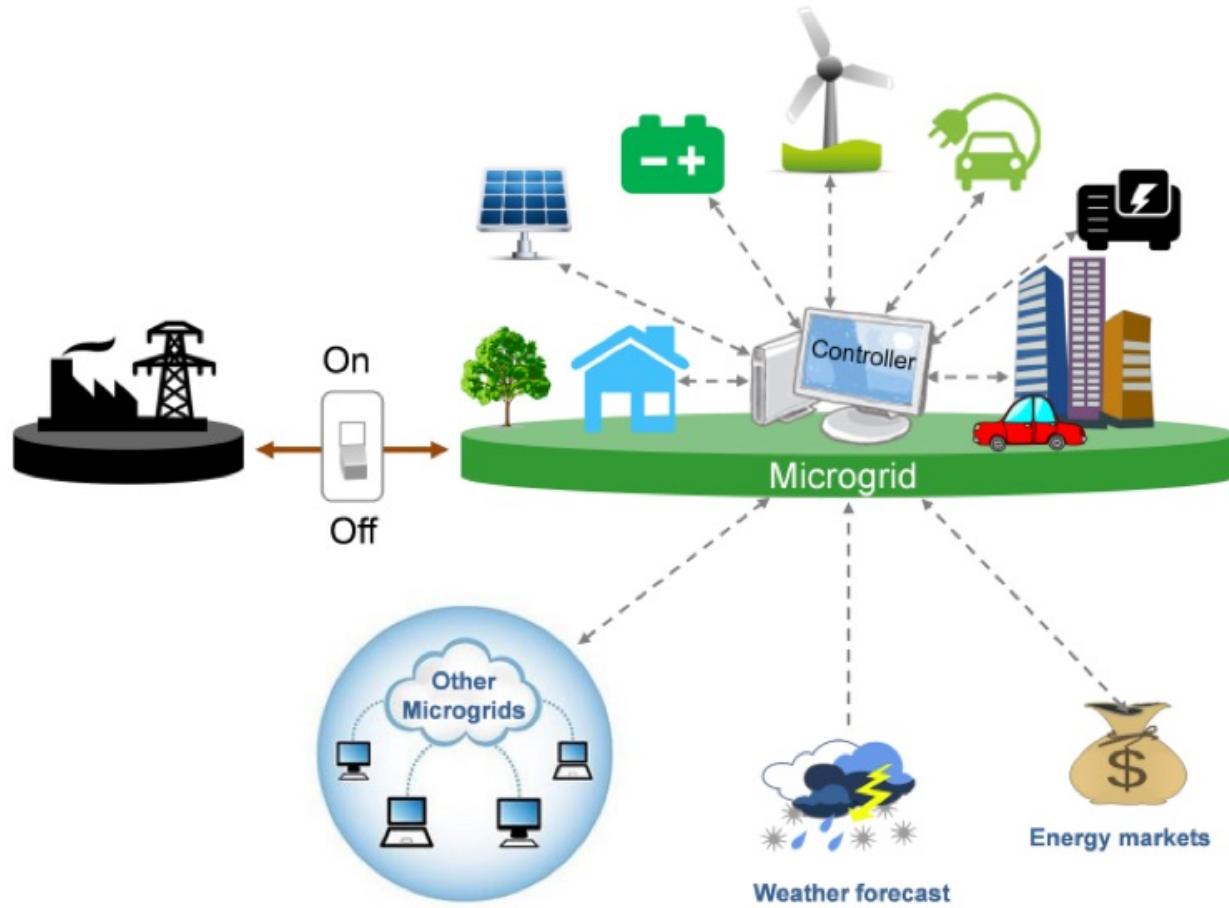
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Electrical (sub)systems



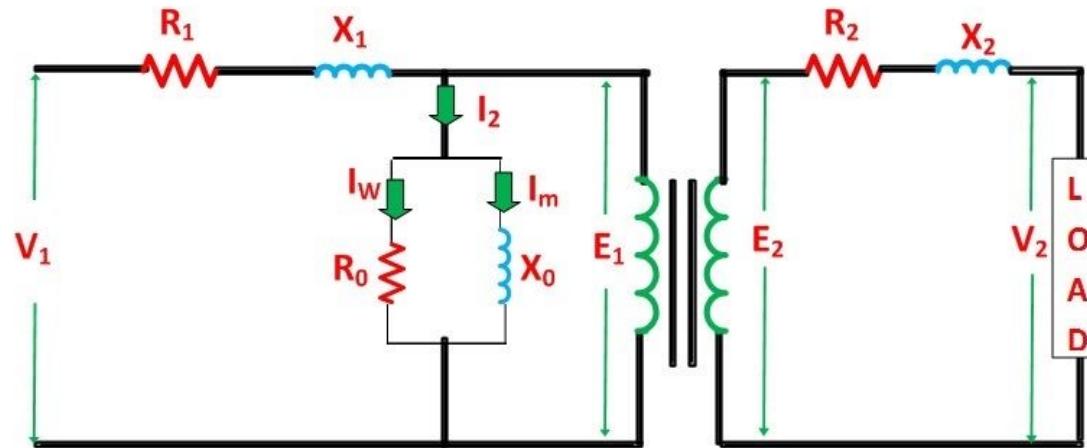
Example: energy generation and distribution



Electrical (sub)systems

Device-Subsystem-System

Example: Transformer circuit



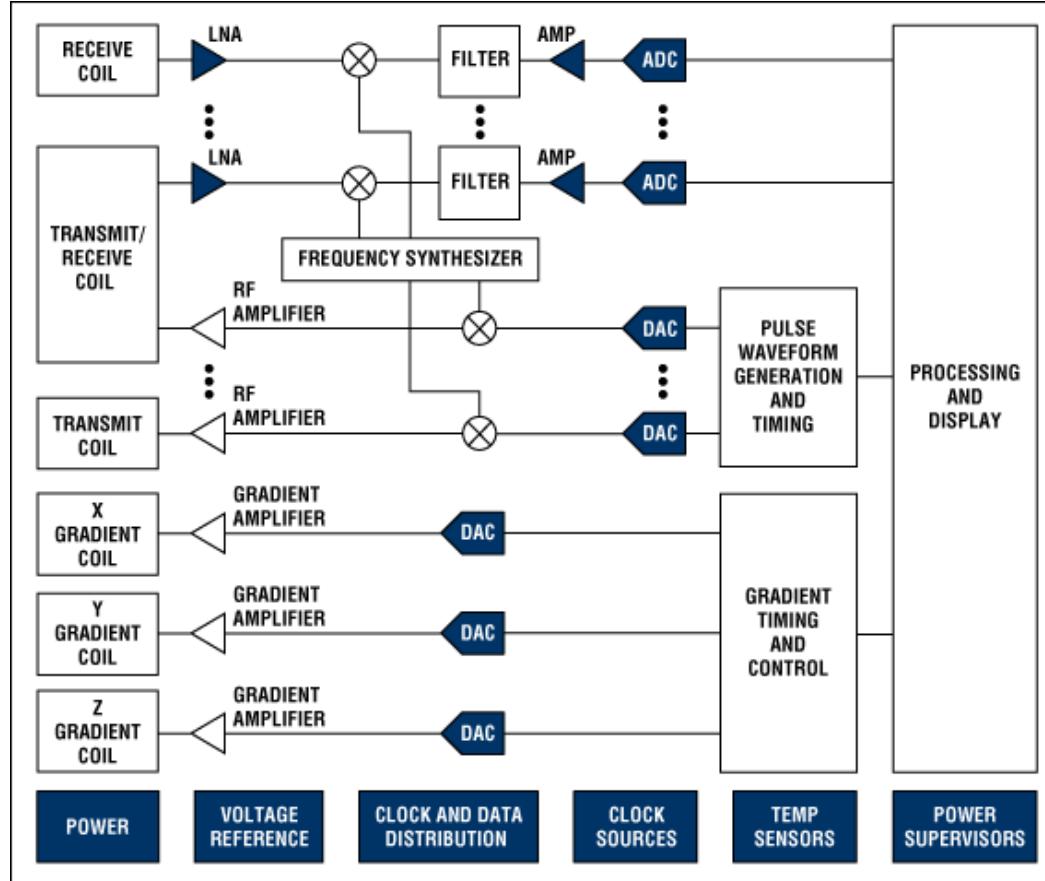
Circuit Globe

Electrical (sub)systems



Device-Subsystem-System

Example: MRI scanner





Prerequisites for ELEC2070

Critical engineering prerequisites:

Circuit theorems

KVL and KCL using node and mesh analysis

Electrical power

Critical maths prerequisites:

Simultaneous equations

Differentiation and integration

Complex number algebra



Purpose of ELEC2070

ELEC2070: overarching theme is to determine the response of a linear circuit to any time variant or invariant current or voltage input.

Time domain:

The basic current and voltage relationships for capacitors, inductors, and ideal operational amplifiers.

Determine the voltage, current and power for switched DC circuits using differential equations.

Frequency domain:

Find voltage, current and network function for sinusoidal inputs using angular frequency domain analysis.

Find voltage, current and transfer function for general inputs using complex frequency domain analysis.

Learning Outcomes

(first 4 each form a module)

Calculate the response of circuits composed of linear circuit elements with a source of constant energy to determine all currents, voltages and power.

Apply the method of time domain analysis of circuits composed of linear circuit elements with a source of constant energy to determine all currents, voltages and power when an abrupt change is applied to the circuit.

Analyse circuits composed of linear circuit elements with a source of energy with a sinusoidal time dependence to determine the circuit's response in terms of current, voltage and power.

Apply the method of frequency domain analysis and the Laplace Transform to determine the circuit response for all applied frequencies.

Physically construct and experimentally characterise circuits with one or more linear components

Write concise reports summarising methodologies used and the results obtained.



The textbooks

Introduction to Electric Circuits , James A. Svoboda, Richard C. Dorf

Engineering Circuit Analysis, W. H. Hayt, J. E. Kennedy, S. M. Durbin, MacGraw-Hill

Electrical Engineering, Principles and Practice, A. R. Hambley, (ENGG150)

ELEC2070 Unit Guide



It is important to read and understand it.

Unit Guides are prepared specifically for this semester and followed.

Assignments (12%)

- Solve questions at home
- Electronic submission
- 4 assignments

Laboratory (23%)

- Preparation
- Participation
- Logging
- Weekly

Final Exam (50%)

- 3 hours, formal
- Closed book

In-class Tests (15%)

- Based on each module
- 3 tests



ELEC2070 Weekly Plan

Check on iLearn.

There will be updates during semester.

W	Lecture date	Lecture Topics	Lecturer	Learning Outcomes	Book chapters	Home Assignment	In-class test	Laboratory session
1	21/2/23	Introduction and Review of Thévenin and Norton circuits. Maximum available power	SJ	1 (DC circuits and DC power transfer)	Dorf 1-5	Assignment 1		No laboratory session
2	28/2/23	Energy Storage	SJ		Dorf 7			Workshop: DC circuits
3	7/3/23	The Operational Amplifier	SJ		Dorf 6		Test 1	Two-week lab session: Instrumentation
4	14/3/23	Complete Response: First order circuits	SJ	2 (transient response)	Dorf 8	Assignment 2		Workshop: Transient response
5	21/3/23	Complete Response: Second order circuits	SJ		Dorf 9			
6	28/3/23	Second order circuits: Problems and examples	SJ					Two-week lab session: Inductor and capacitor circuits, Pt. 1
7	4/4/23	Steady state sinusoidal analysis. Concept of impedance.	SJ	3 (sinusoidal response)	Dorf 10,11	Assignment 3	Test 2	Two-week lab session: Inductor and capacitor circuits, Pt. 2
	11/4/23	No lecture						
	18/4/23	No lecture						



ELEC2070 Weekly Plan

Check on iLearn.

There will be updates during semester.

8	25/4/23	AC circuit analysis. AC Thévenin and Norton circuits.	SJ	4 (frequency response)	Dorf 13, 14	Assignment 4		Two-week lab session: AC circuits and AC power transfer
9	2/5/23	Complex power: Maximum available, average power	SJ					
10	9/5/23	Time domain vs frequency domain. Frequency response. Simple filters.	SJ					Workshop: AC circuits
11	16/5/23	The Laplace Transform I	SJ					Test 3
12	23/5/23	The Laplace Transform II	SJ					Two-week lab and workshop: Frequency response
13	30/5/23	Review and Q & A session	SJ					Workshop: Practice for final exam, final logbook marking

Assignments and Tests



- Assignment questions will be posted on iLearn at least two weeks before the submission date.
- There will instructions on iLearn on how to submit electronically.
- Solutions will be posted 2-3 days after the submission date.
- Submissions will not be accepted after the due date.
- Assignments are for you to learn each learning outcome and prepare for the exam. There will also be practice sessions (workshops) in the laboratory for each learning outcome.

Absences



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- Late notices or absences from assignments tests, workshops and laboratories will be considered under extenuating circumstances upon lodgement and approval of a **formal notice of disruption of studies**.



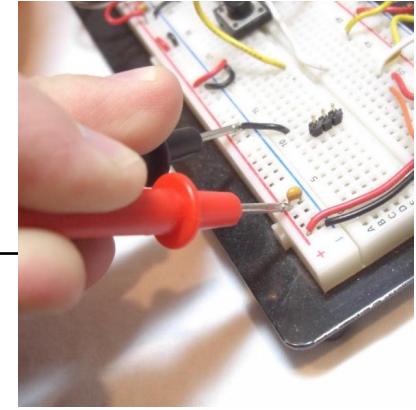


Assignment 1

- Questions are posted on iLearn already (Week 1)
- Due end of Week 3, Sunday mid-night
- Solutions will be posted in Week 4
- 20 min invigilated test will be in lecture in Week 3

Assignment 1 Questions on iLearn

Laboratory



Aligned with learning outcomes.

1. Laboratory sessions for constructing and measuring circuits.
2. Problem solving workshops for practicing for tests.

Start in Week 2 (check Weekly Plan)



Arrive on time.
Use your time
efficiently.



Revise lecture
notes.
Complete
preliminary
work.



Bound
notebook for
the whole
semester.



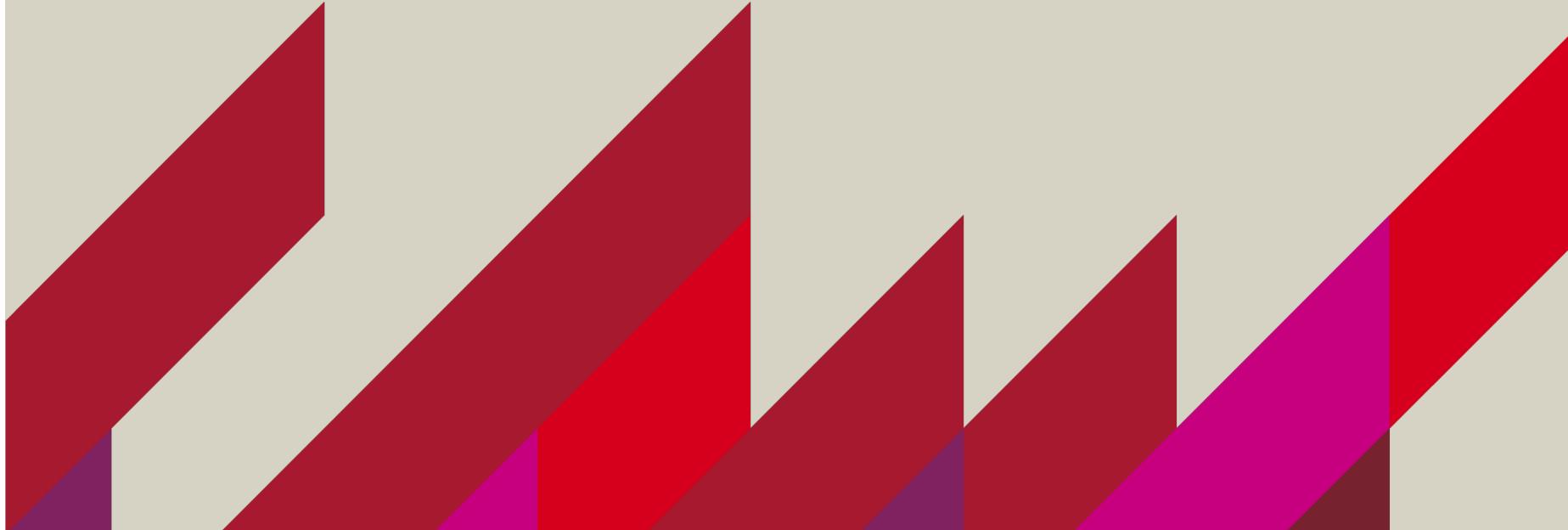
Apply for
“Disruption to
Studies” if you
can’t attend
your session.

You will get marks for preparation, participation and recording.



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Lecture 1: Fundamental concepts





Current, power, voltage

$$i = \frac{dq}{dt}$$

current charge time

$$q = \int_{-\infty}^t i d\tau = \int_0^t i d\tau + q(0)$$

$$p = \frac{dw}{dt}$$

power energy time

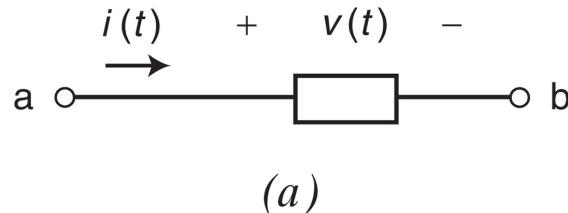
$$w = \int_{-\infty}^t p d\tau$$

$$v = \frac{dw}{dq}$$

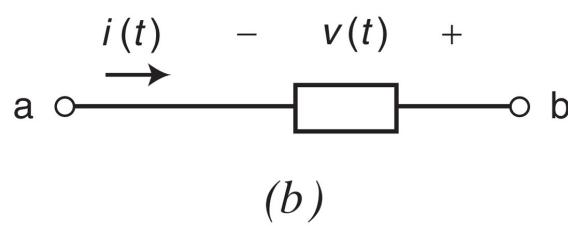
voltage energy charge



Polarity, resistance



(a) absorbs (dissipates) energy.
It is a “passive” element
(for example a resistor).

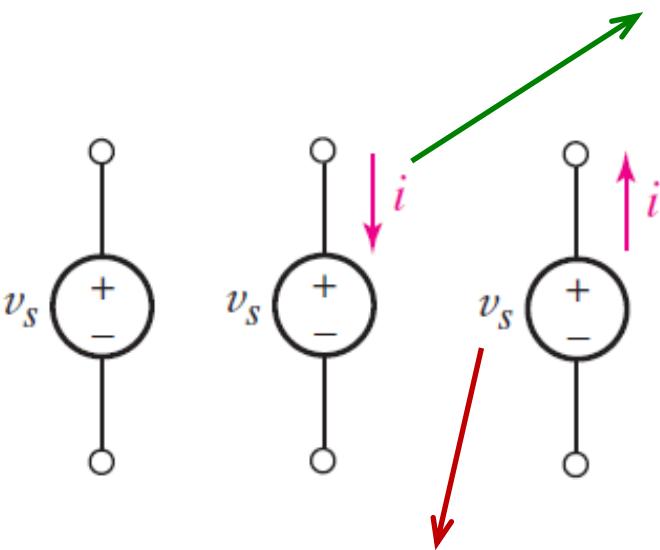


(b) supplies (generates) energy.
It is an “active” element
(for example a battery).

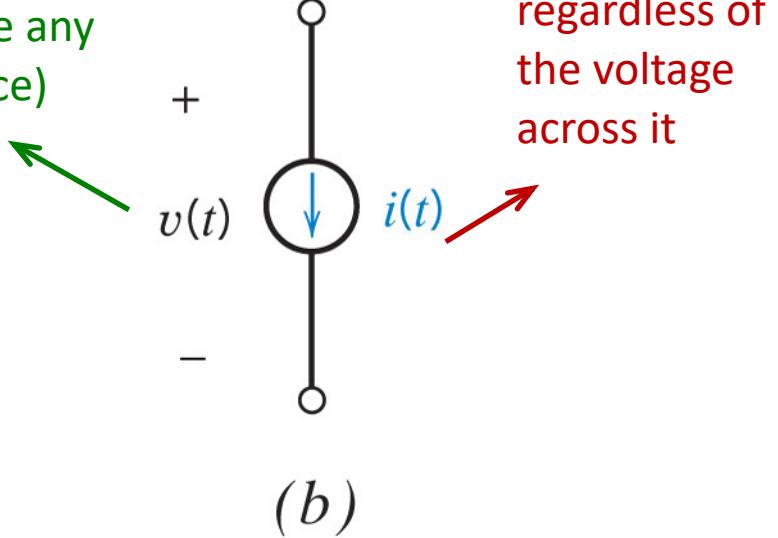
We are going to use resistors in their linear region

$v = Ri$ → Mathematical model of an ideal (linear) resistor

Ideal independent sources



These are unknown values until we calculate in a circuit (they can be any value if ideal source)

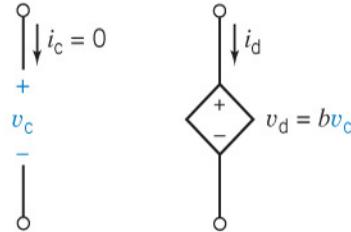


This is FIXED regardless of the current passing through

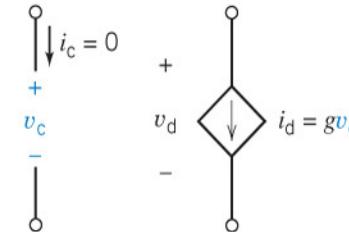
Symbol of ideal independent voltage source

Symbol of ideal independent current source

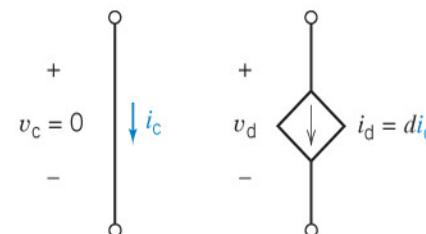
Dependent sources



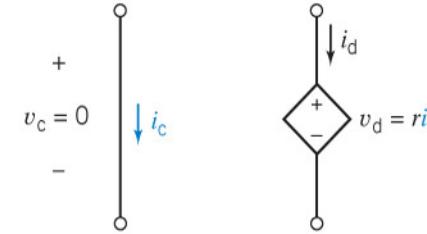
Voltage-Controlled
Voltage Source (VCVS)



Voltage-Controlled
Current Source (VCCS)



Current-Controlled
Current Source (CCCS)



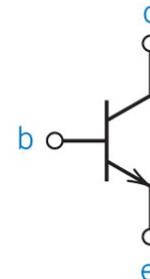
Current-Controlled
Voltage Source (CCVS)

There are no simple dependent sources in practice! But they are very useful for modelling transistors, amplifiers, etc.

Dependent source example

Transistor action:

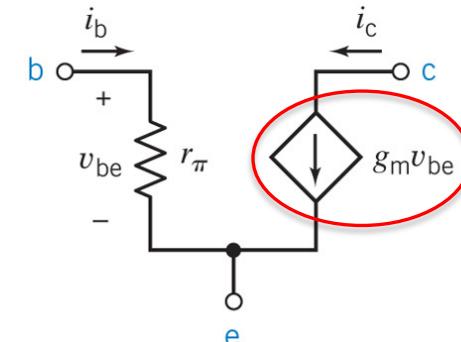
Current between **c** and **e** depends on voltage between **b** and **e**



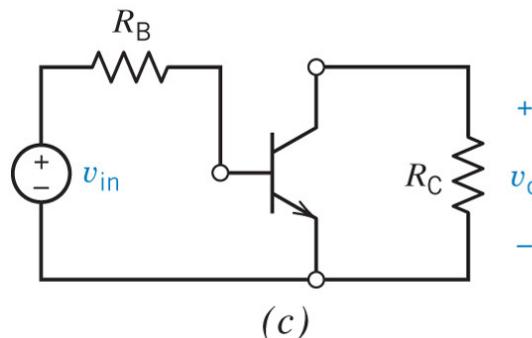
(a)

$$i_c = g_m v_{be}$$

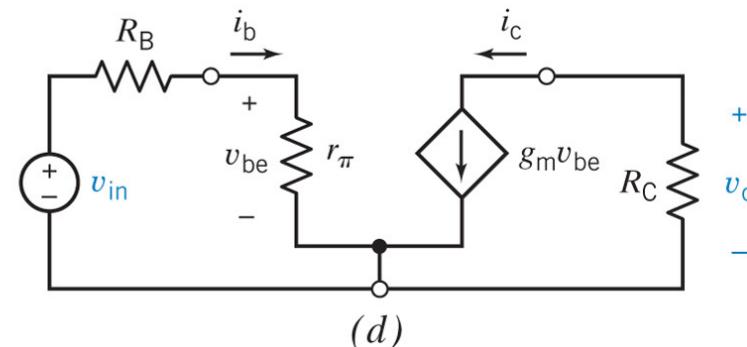
↑
constant



(b)



Actual circuit



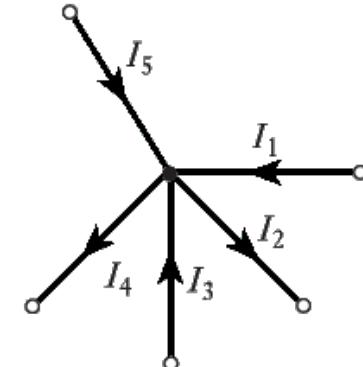
Equivalent circuit

- easier to analyse
- describes behavior of original circuit



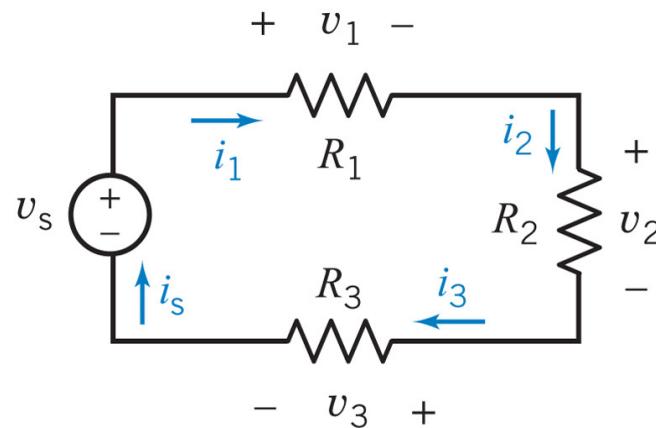
Kirchoffs laws, (valid ALWAYS)

The algebraic sum of all currents entering (or leaving) a node is zero at any time instant.



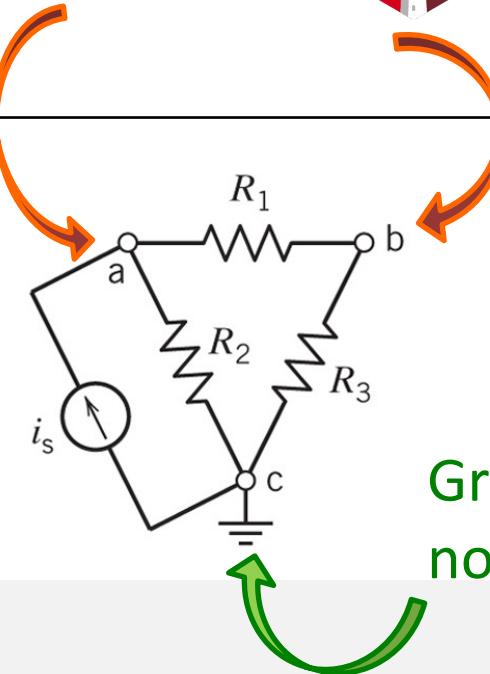
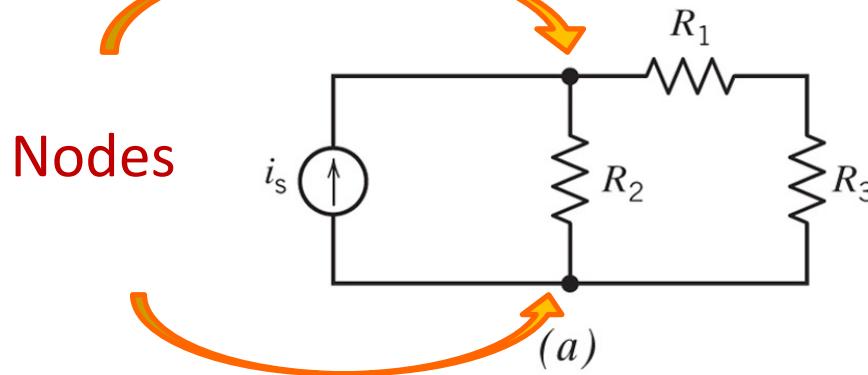
$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

The algebraic sum of all voltages around a loop is zero at any time instant.



$$v_1 + v_2 + v_3 - v_s = 0$$

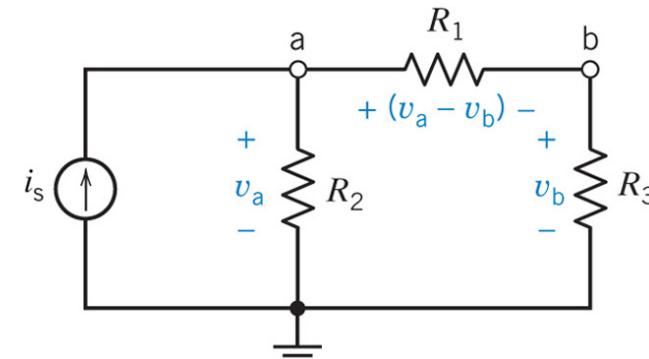
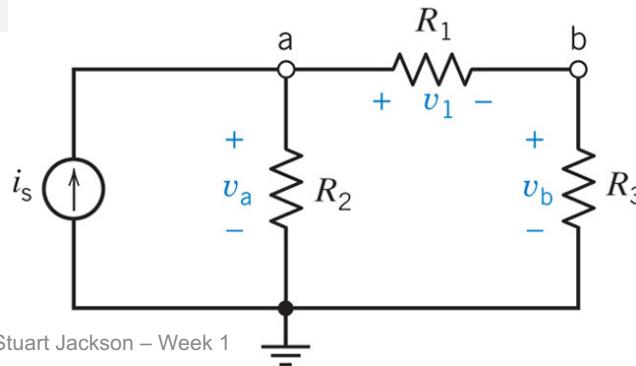
Nodes



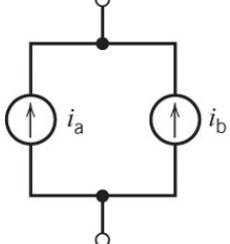
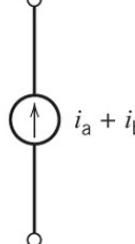
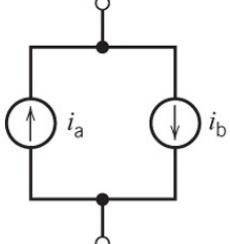
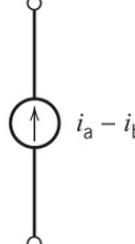
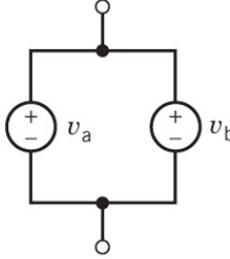
Ground node potential is zero volts.

All other nodes can be measured relative to ground.

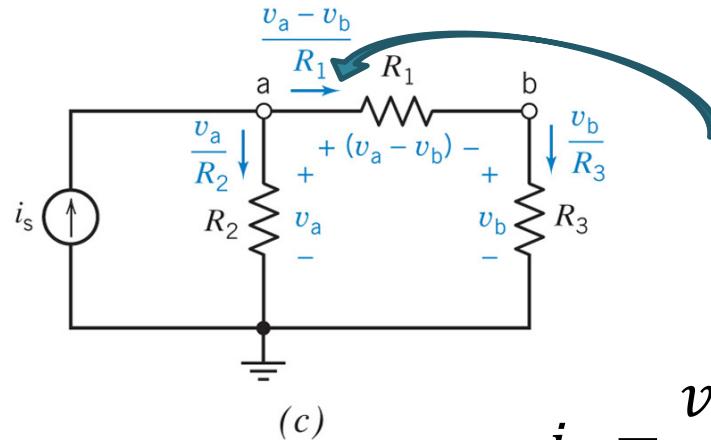
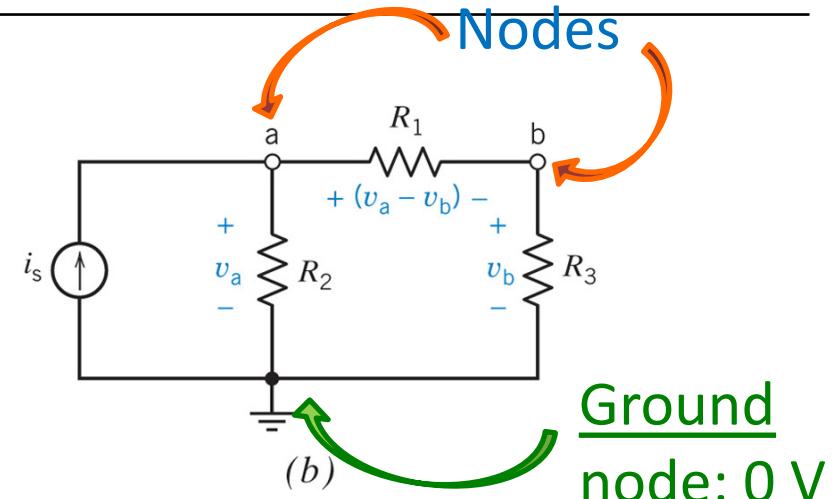
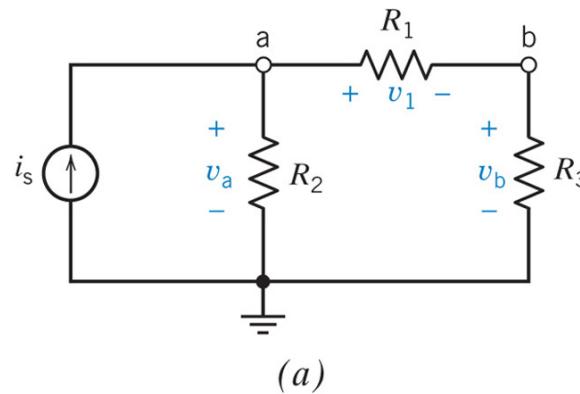
Each node corresponds to a unique voltage (relative to ground).



Parallel and series voltage and current sources

CIRCUIT	EQUIVALENT CIRCUIT	CIRCUIT	EQUIVALENT CIRCUIT
	$v_a + v_b$		
	$v_a - v_b$		
			
Not allowed		Not allowed	
<p>In practice it is possible to do this because they will not be ideal sources. The source with higher current may take over. But not good practice!</p>		<p>This is like connecting two batteries with different voltages in parallel. Not good practice.</p>	
		Not allowed	

Node voltage analysis: Step 1



First, write element currents in terms of node voltages:

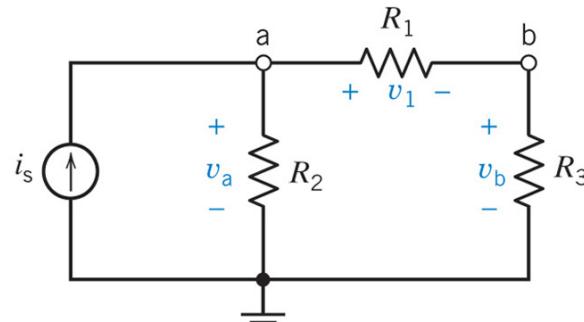
Pay attention to the direction of current!

$$i_1 = \frac{v_a - v_b}{R_1}$$

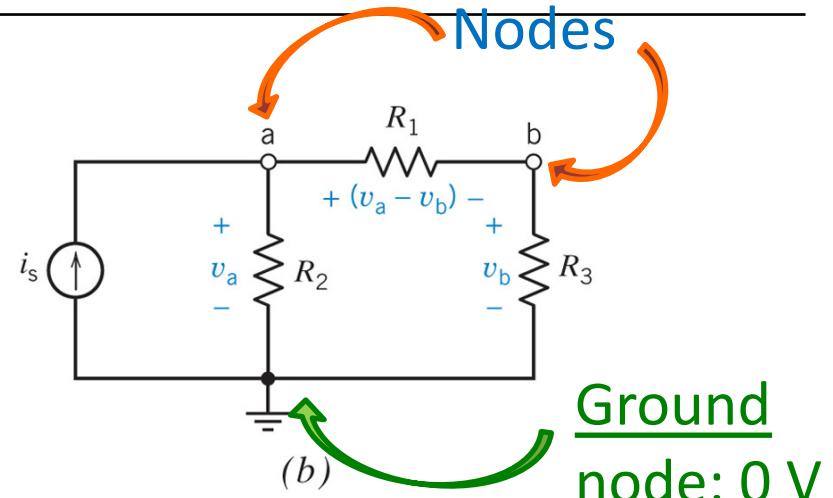
$$i_2 = \frac{v_a}{R_2}$$

$$i_3 = \frac{v_b}{R_3}$$

Node voltage analysis: Step 1

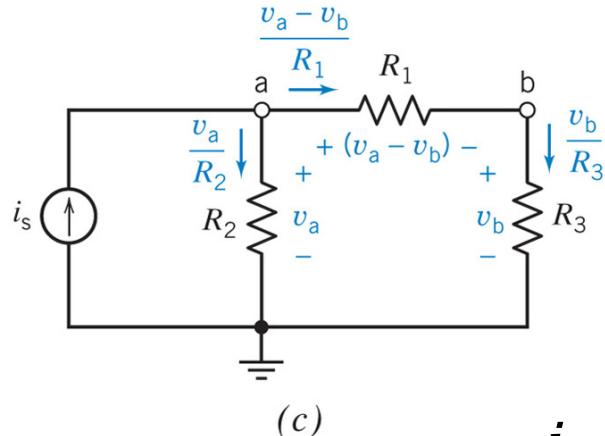


(a)



(b)

Ground
node: 0 V



(c)

Label everything clearly!!!

$$i_1 = \frac{v_a - v_b}{R_1}$$

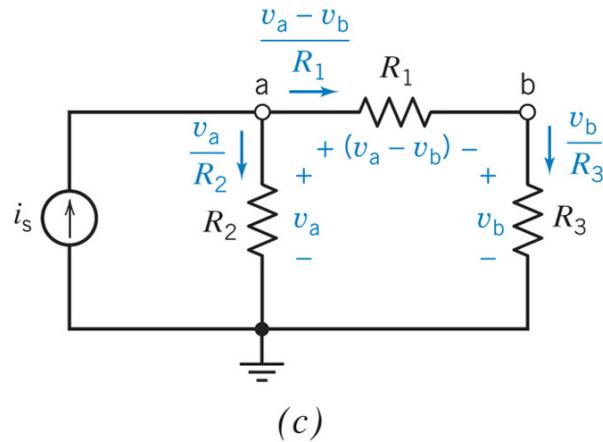
$$i_2 = \frac{v_a}{R_2}$$

$$i_3 = \frac{v_b}{R_3}$$

Node voltage analysis: Step 2



Second, apply Kirchhoff's Current Law at **each node**, except for the ground node:



Node a: $i_s = i_1 + i_2$

$$i_s = \frac{v_a - v_b}{R_1} + \frac{v_a}{R_2}$$

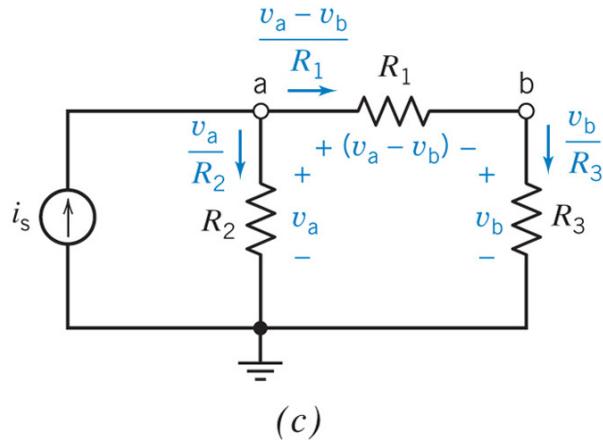
Node b: $i_1 = i_3$

The variables i_1 and i_2 are not needed if we want to find only the node voltages. The source current i_s is normally given.

$$\frac{v_a - v_b}{R_1} = \frac{v_b}{R_3}$$

Node voltage analysis: Step 3

Third, order the equations with respect to the unknowns (such as you would do for the matrix method) and solve them:



Node
a:

$$i_s = \frac{v_a - v_b}{R_1} + \frac{v_a}{R_2}$$

$$i_s = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_a - \frac{1}{R_1} v_b$$

Node
b:

$$\frac{v_a - v_b}{R_1} = \frac{v_b}{R_3}$$

$$0 = \left(\frac{1}{R_1} + \frac{1}{R_3} \right) v_b - \frac{1}{R_1} v_a$$

Node voltage analysis with dependent sources



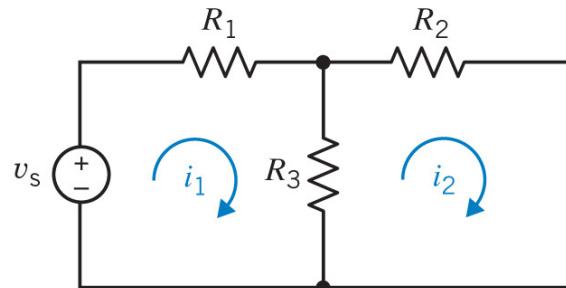
1. Write the controlling currents and voltages of the dependent sources in terms of node voltages.
2. Write the remaining node equations the usual way.
3. Solve the simultaneous equations.

Node voltage analysis: Summary

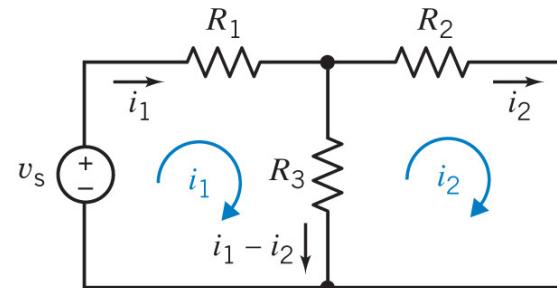
For solving node voltages:

1. Element currents are expressed in terms of node voltages (pay attention to current direction).
2. KCL is applied to each node (except ground).
3. If there are only current sources there are nice symmetries in equations.
4. If there are dependent sources, controlling voltage or current is expressed in terms of node voltages → if there is an additional unknown another equation is needed.

Mesh current analysis with independent voltage sources : Step 1

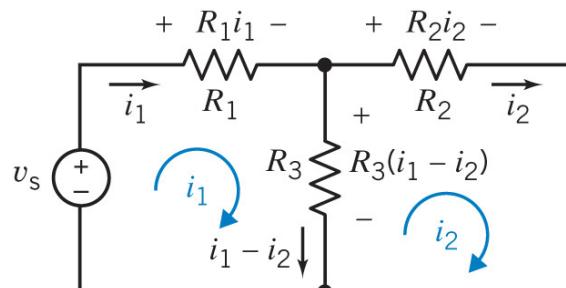


(a)



(b)

Write element voltages in terms of mesh currents:



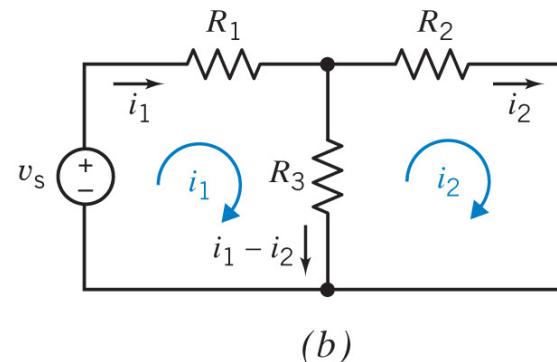
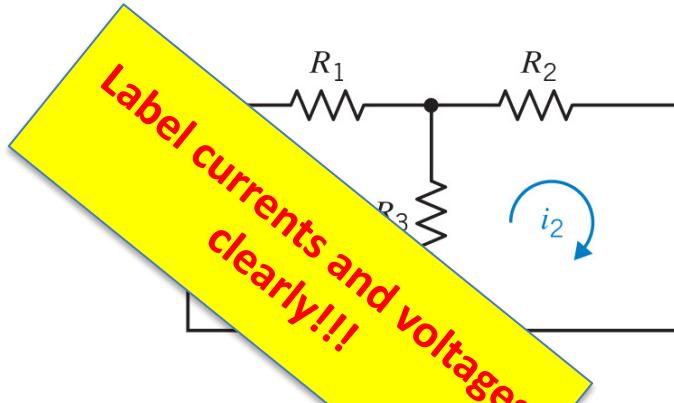
(c)

$$v_1 = R_1 i_1$$

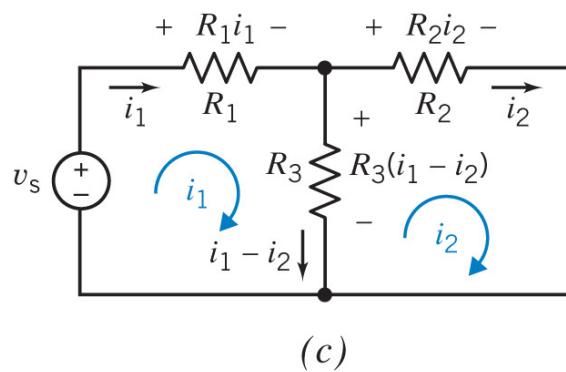
$$v_2 = R_2 i_2$$

$$v_3 = R_3 i_3 = R_3(i_1 - i_2)$$

Mesh current analysis with independent voltage sources : Step 1



Write element voltages in terms of mesh currents:



$$v_1 = R_1 i_1$$

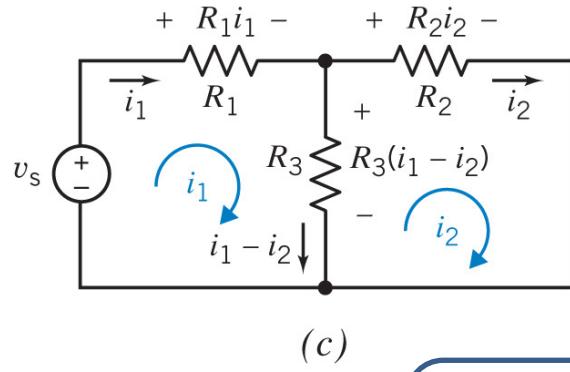
$$v_2 = R_2 i_2$$

$$v_3 = R_3 i_3 = R_3(i_1 - i_2)$$

Mesh current analysis with independent voltage sources : Step 2



Apply Kirchhoff's Voltage Law to write loop equations for each mesh.



Mesh 1:

$$-v_s + R_1 i_1 + R_3(i_1 - i_2) = 0$$

(c)

Mesh 2:

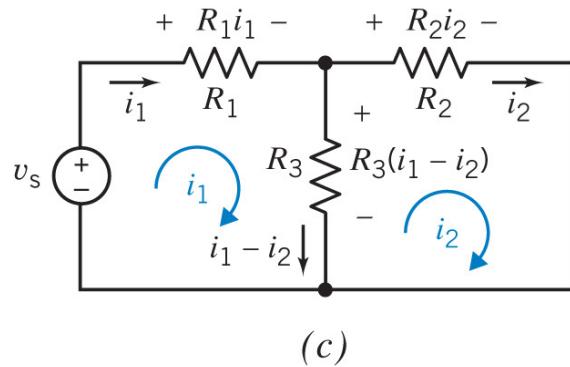
$$-R_3(i_1 - i_2) + R_2 i_2 = 0$$

Alternative: $R_3(i_2 - i_1) + R_2 i_2 = 0$

Mesh current analysis with independent voltage sources : Step 3



Reorder the equations with respect to the unknowns and solve them:



Mesh 1:

$$R_1 i_1 + R_3(i_1 - i_2) = v_s$$

$$(R_1 + R_3)i_1 - R_3 i_2 = v_s$$

Mesh 2:

$$-R_3(i_1 - i_2) + R_2 i_2 = 0$$

$$(R_2 + R_3)i_2 - R_3 i_1 = 0$$

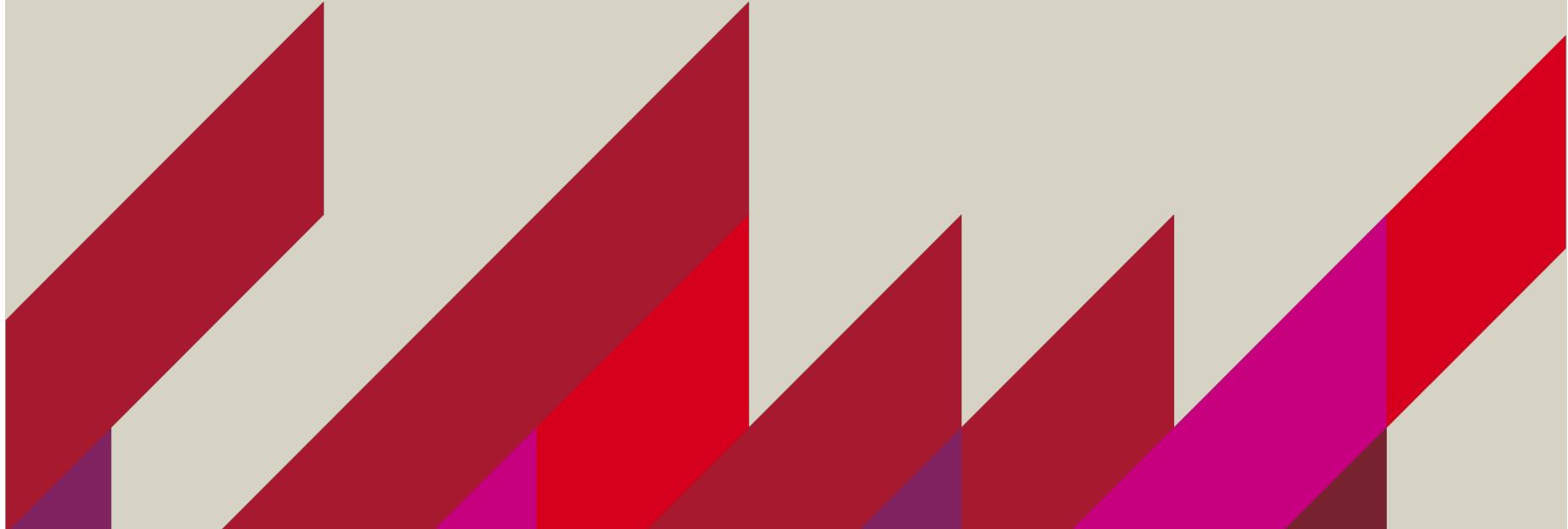
Mesh current analysis with dependent sources

1. Write the controlling currents and voltages of the dependent sources in terms of mesh currents.
2. Write the remaining mesh equations the usual way.
3. Solve the simultaneous equations.



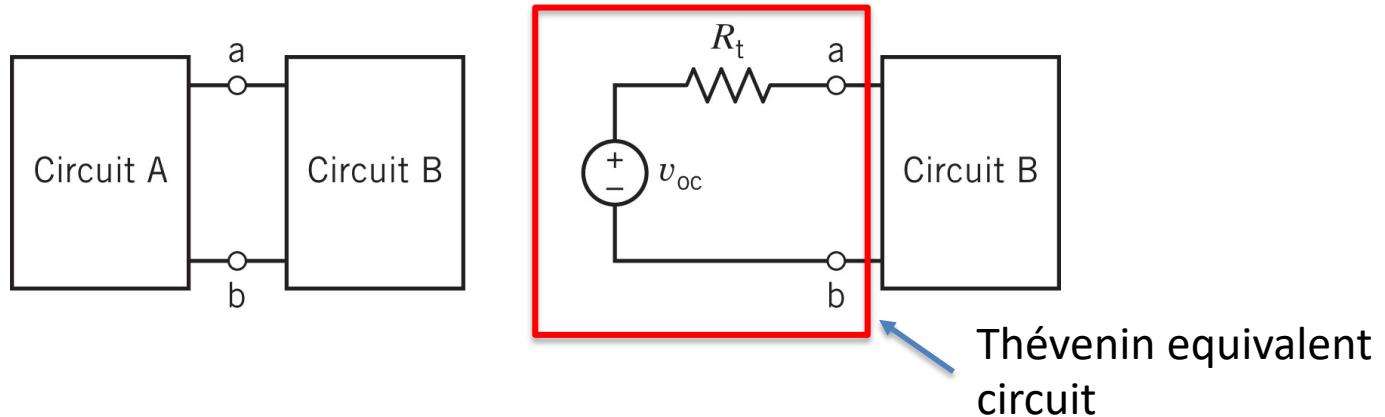
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Thévenin's Theorem





Thévenin's Theorem

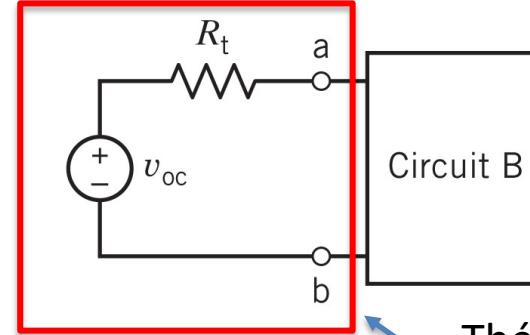
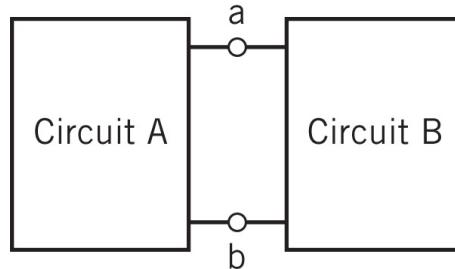


“Any combination of voltage sources and resistors with two terminals can be replaced by a single voltage source and a single series resistor.”

Circuit A can be replaced with a simpler Thévenin equivalent circuit.



Thévenin's Theorem



To find the Thévenin equivalent circuit of Circuit A:

1. Measure (or calculate) the **open-circuit voltage** and the **short-circuit current** of Circuit A.
2. Then calculate Thévenin resistance using the formula:

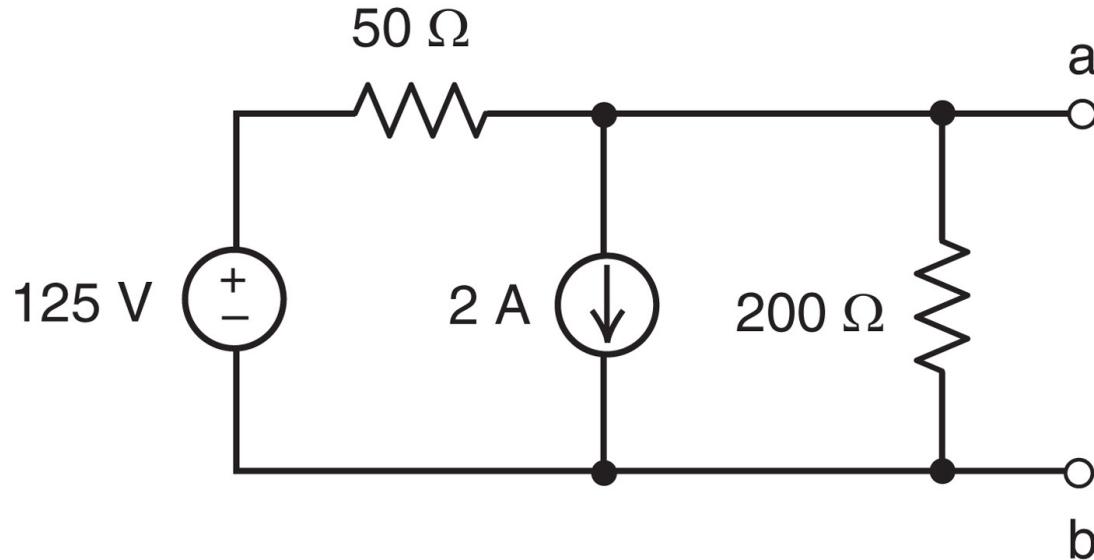
$$R_t = \frac{v_{oc}}{i_{sc}}$$

- **This method works well for calculations but may not be good in practice.**
- **For example: if you short circuit a battery, you will discharge it.**



Independent sources example

Find the Thévenin equivalent of the circuit below

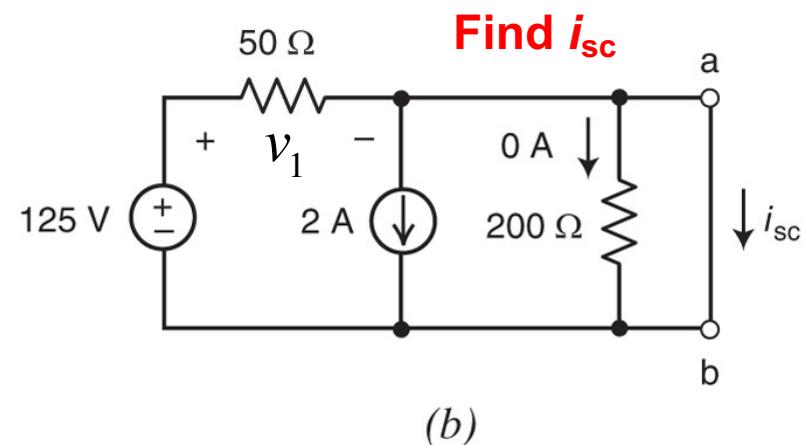
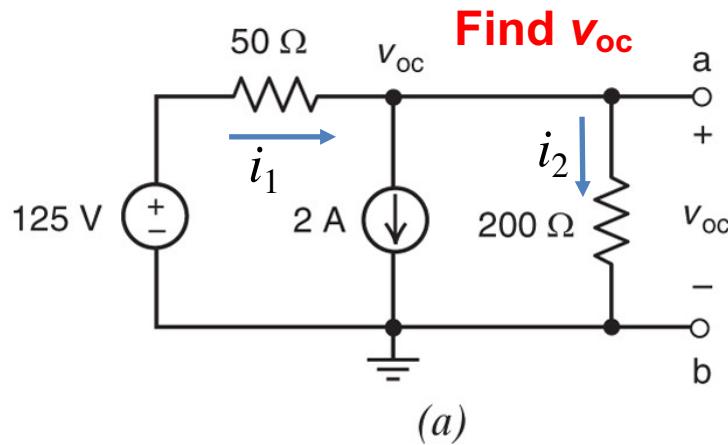


Method 1:

We now need to:

1. Find the open circuit voltage
2. Find the short circuit current
3. Calculate Thévenin resistance using the relation: $R_t = \frac{v_{oc}}{i_{sc}}$

Independent sources example



$$i_1 = \frac{125 - v_{oc}}{50}$$

$$i_2 = \frac{v_{oc}}{200}$$

$$\text{KCL: } i_1 = 2 + i_2$$

$$v_{oc} = 20 \text{ V}$$

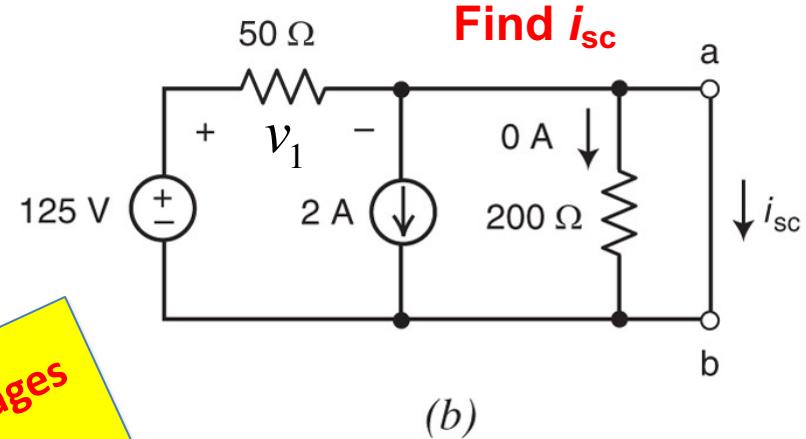
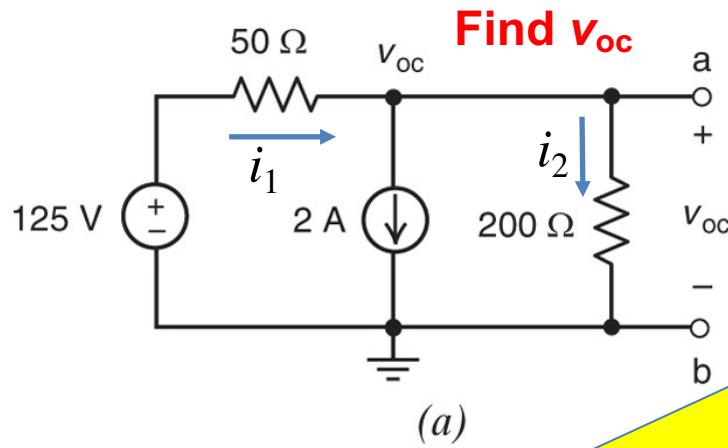
$$\text{KVL: } -125 + v_1 = 0$$

$$\text{KCL: } \frac{125}{50} = 2 + i_{sc}$$

$$i_{sc} = 0.5 \text{ A}$$

$$\text{Find } R_t: R_t = \frac{v_{oc}}{i_{sc}} = 40 \Omega$$

Independent sources example



$$i_1 = \frac{125 - v_{oc}}{50}$$

$$i_2 = \frac{v_{oc}}{200}$$

$$\text{KCL: } i_1 = 2 + i_2$$

$$v_{oc} = 20 \text{ V}$$

Label currents and voltages
clearly!!!

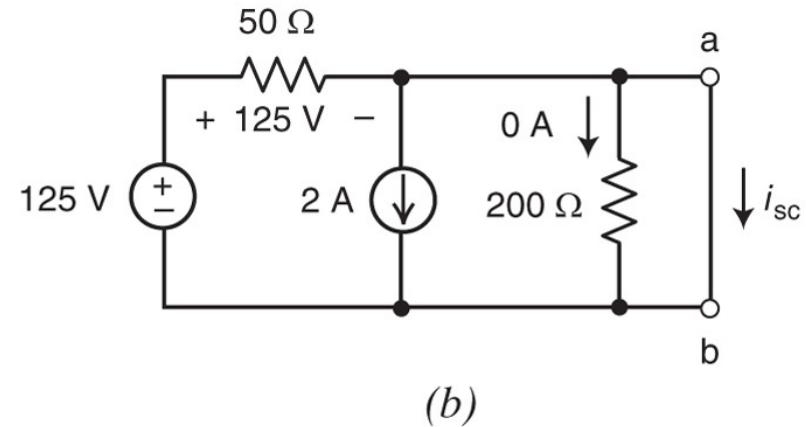
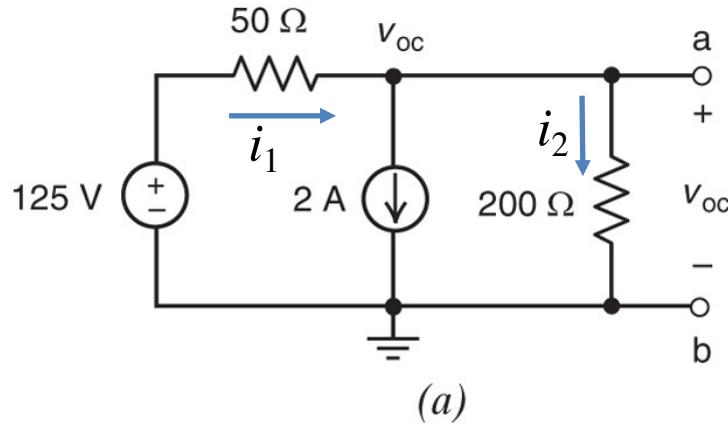
$$\text{KVL: } -125 + v_1 = 0$$

$$\text{KCL: } \frac{125}{50} = 2 + i_{sc}$$

$$i_{sc} = 0.5 \text{ A}$$

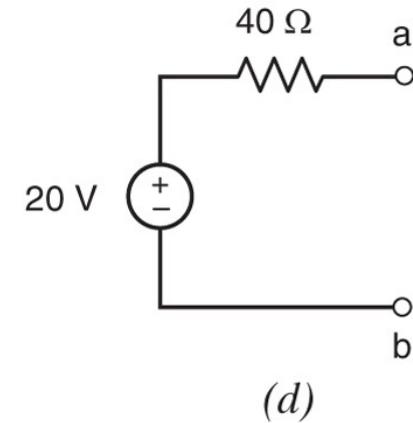
Find R_t : $R_t = \frac{v_{oc}}{i_{sc}} = 40 \Omega$

Independent sources example



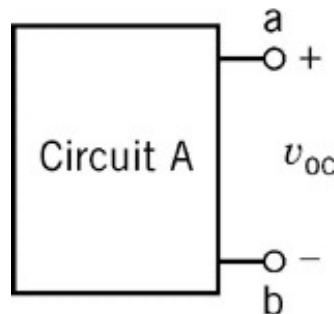
Thévenin equivalent circuit:

(Example 5.4-1 in Dorf)

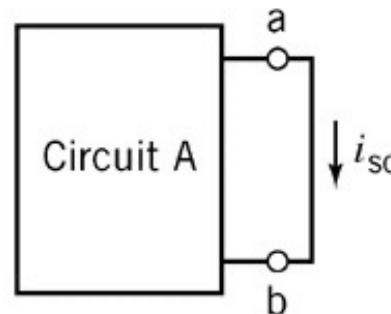




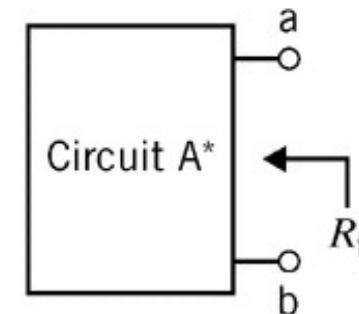
The Thévenin parameters



(a)



(b)



(c)

(a) The **open circuit voltage**: v_{oc}

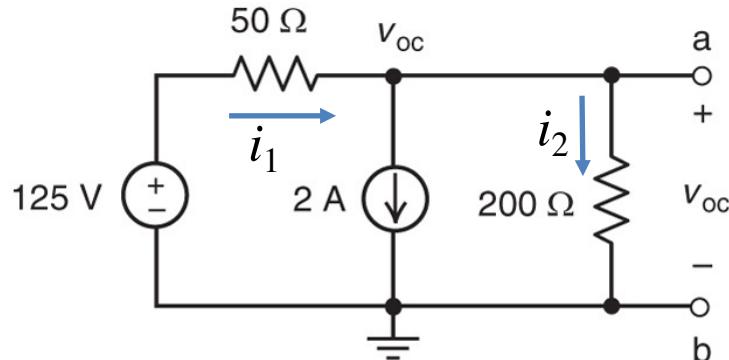
(b) The **short circuit current**: i_{sc}

(c) The **Thévenin resistance**: R_t This can be found by:

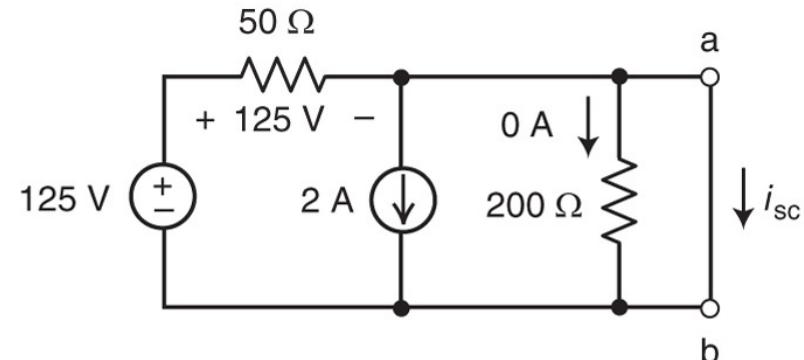
i. Finding $R_t = \frac{v_{oc}}{i_{sc}}$.

- ii. Replace all independent voltage sources with short circuits and all independent current sources with open circuits. Then calculate final resistance – R_t . **Dependent sources cannot replaced by open and short circuits. Need to use $R_t = \frac{v_{oc}}{i_{sc}}$ or a third method!**

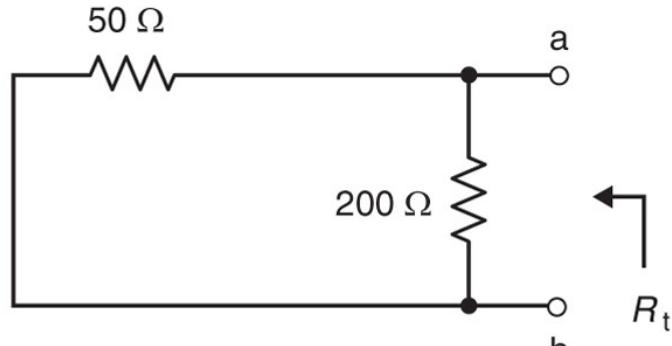
Simple method to find R_t (only for independent sources)



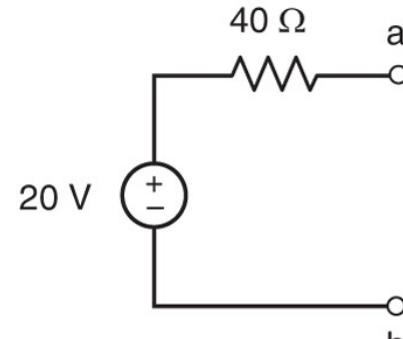
(a)



(b)



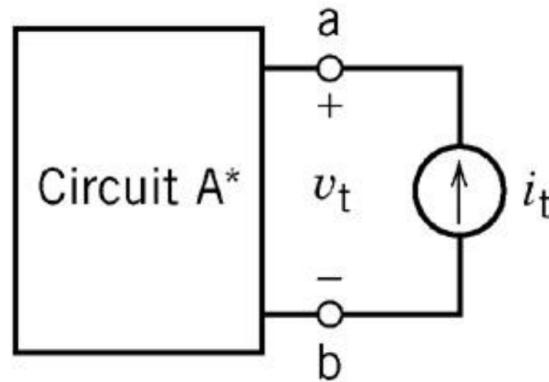
(c)



(d)



A third method to calculate R_t



The Thévenin resistance can be found by connecting a current source to the terminals of the circuit. The voltage across the current source is measured or calculated. The Thévenin resistance is found from $R_t = \frac{v_t}{i_t}$

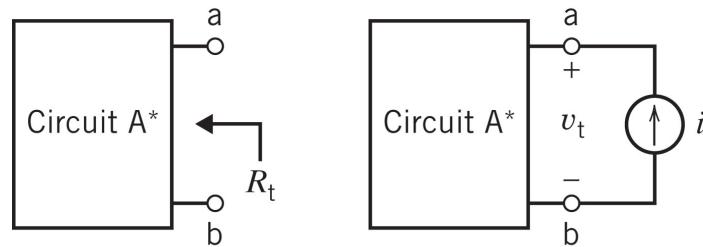
(This could be carried out a few times to check linearity)

Particularly useful for complicated circuits with dependent sources.

Calculating the Thévenin resistance

METHOD 1: Find v_{oc} and i_{sc} then apply $R_t = v_{oc} / i_{sc}$. Useful for simple circuits and when doing paper calculations.

METHOD 2: Make a new Circuit A* by “zeroing” the independent sources of Circuit A. Find equivalent resistance of Circuit A*. This is the Thévenin resistance of A.



METHOD 3: Apply a current source i_t to the terminals of Circuit A*; measure v_t ; calculate R_t from v_t and i_t :

$$R_t = \frac{v_t}{i_t}$$

* You should not connect a current supply to the output of a circuit in practice without knowing what is inside, you may damage it!



Dependent sources

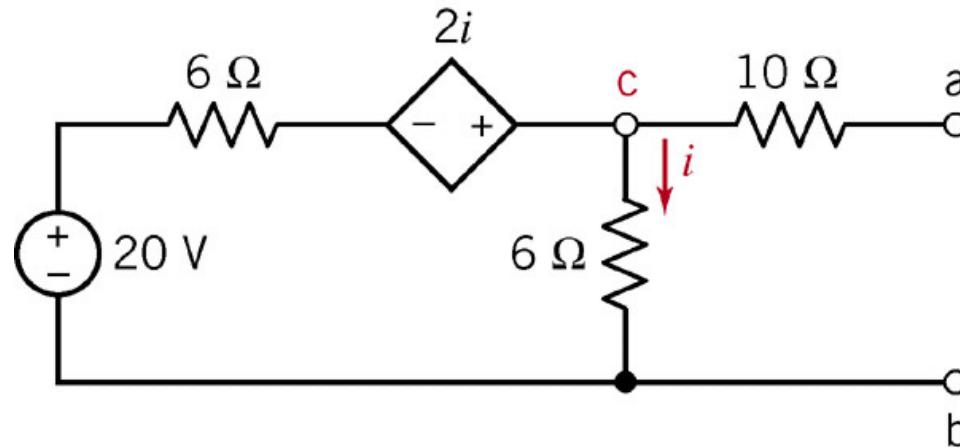
The presence of dependent sources means we cannot find R_t using the normal “switching off independent sources” technique.

We now need to:

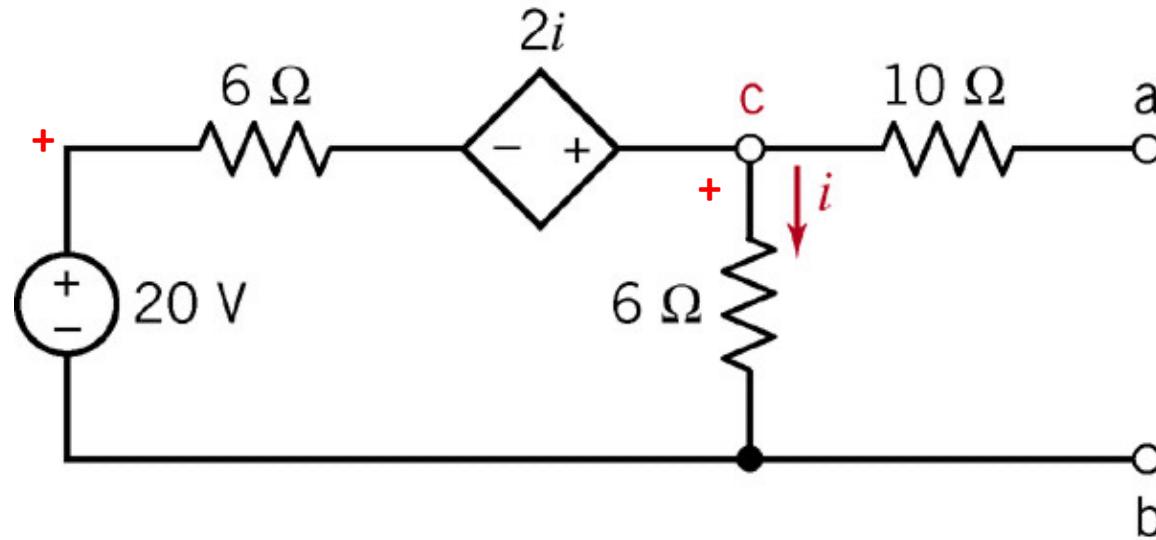
1. Find the open circuit voltage
2. Find the short circuit current
3. Calculate Thévenin resistance using the relation: $R_t = \frac{v_{oc}}{i_{sc}}$

Example:

This circuit involves a dependent source



Solution process: find v_{oc}



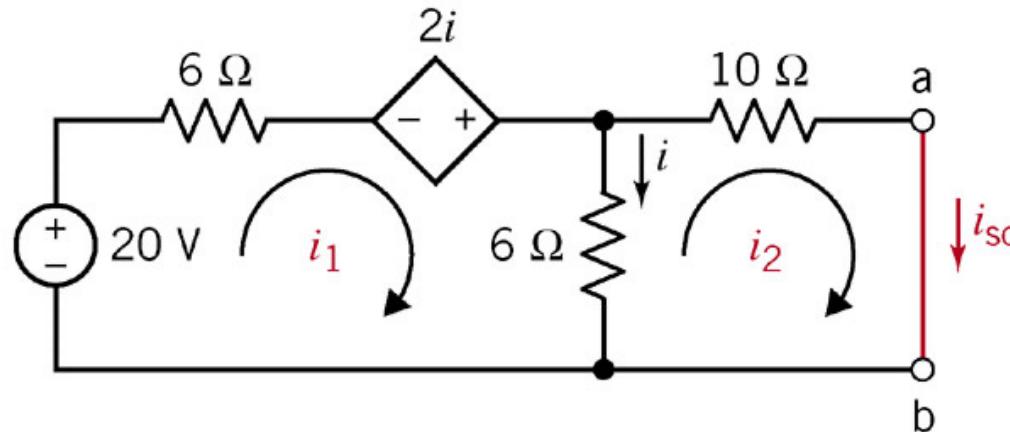
Find the open-circuit voltage

Since we have one closed loop we can apply KVL to this loop – remember the polarities

$$\begin{aligned} \text{KVL: } & -20 + 6i - 2i + 6i = 0 \\ & i = 2 \text{ A} \end{aligned}$$

Therefore the open-circuit voltage: $v_{oc} = 6i = 12 \text{ V}$

Solution process: find i_{sc}



Find the short-circuit current by connecting the terminals at a-b

Now we have a simple mesh equation problem

$$\text{Mesh 1: } -20 + 6i_1 - 2(i_1 - i_2) + 6(i_1 - i_2) = 0$$

$$\text{Mesh 2: } 6(i_2 - i_1) + 10i_2 = 0 \text{ this equation gives } i_1 = \frac{8}{3}i_2 = 2.67i_2$$

$$\text{Put this value for } i_1 \text{ into Mesh 1 equation gives: } i_2 = \frac{60}{68} = 0.88 = i_{sc}$$

$$\text{Therefore: } R_t = \frac{v_{oc}}{i_{sc}} = \frac{12 \times 68}{60} = \frac{68}{5} = 13.6 \Omega$$

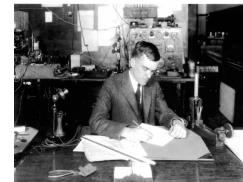


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Norton's equivalent circuit

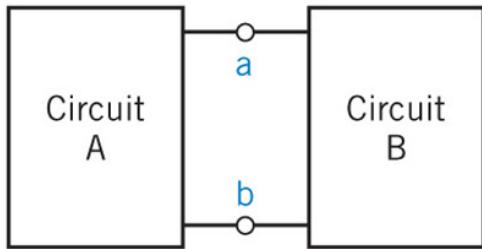
Norton equivalent circuit

Is found the same way

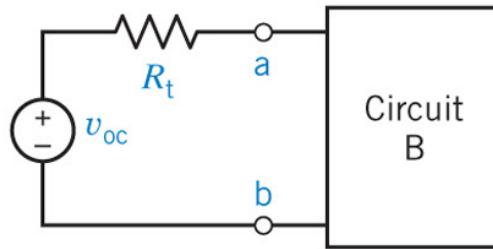


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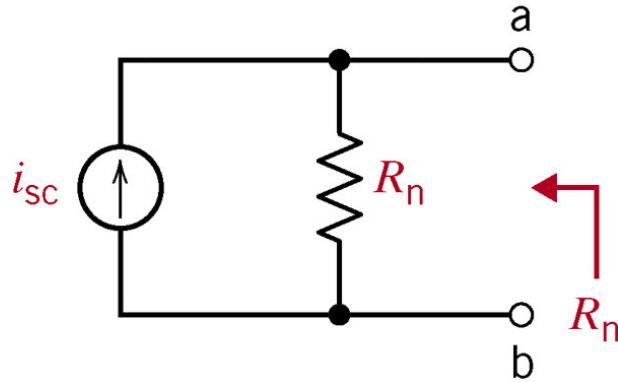
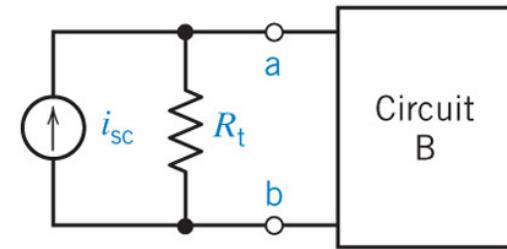
ORIGINAL CIRCUIT



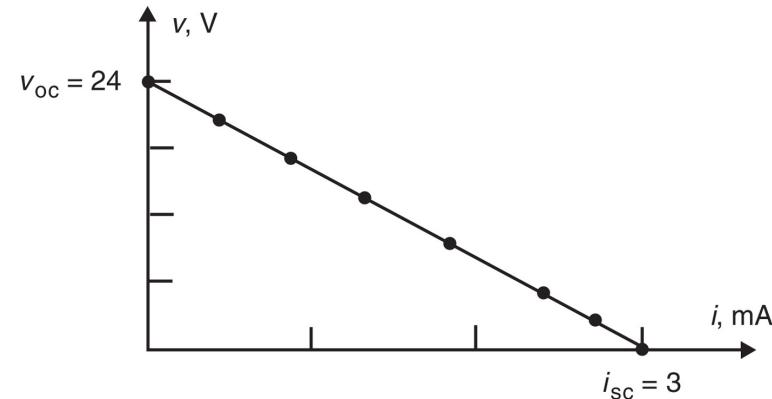
THÉVENIN CIRCUIT



NORTON EQUIVALENT CIRCUIT



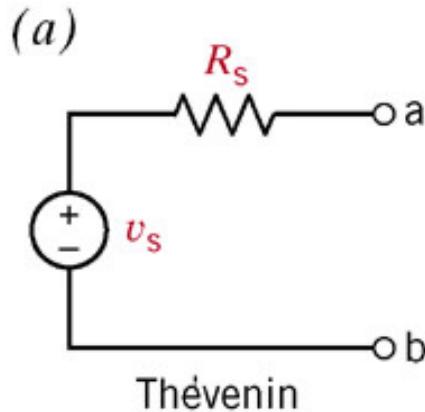
$$R_n = R_t = \frac{v_{oc}}{i_{sc}}$$



Thévenin \leftrightarrow Norton = Source transformation



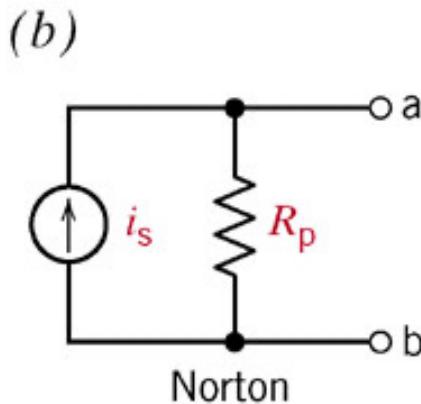
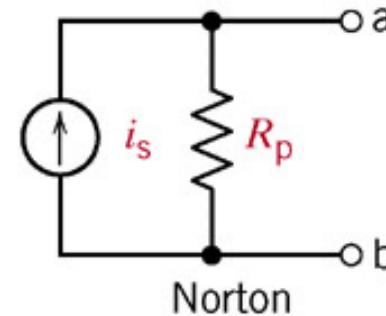
Method



Set $i_s = \frac{v_s}{R_s}$



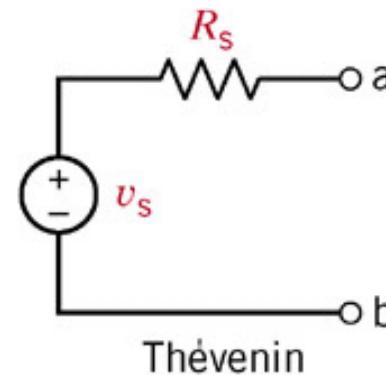
Set $R_p = R_s$



Set $v_s = i_s R_p$



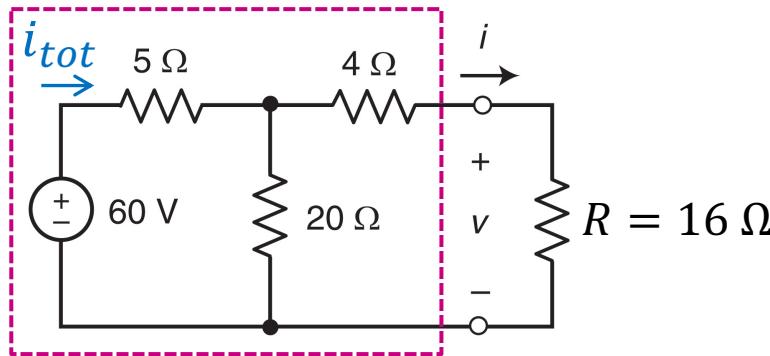
Set $R_s = R_p$





Thévenin or Norton Circuit?

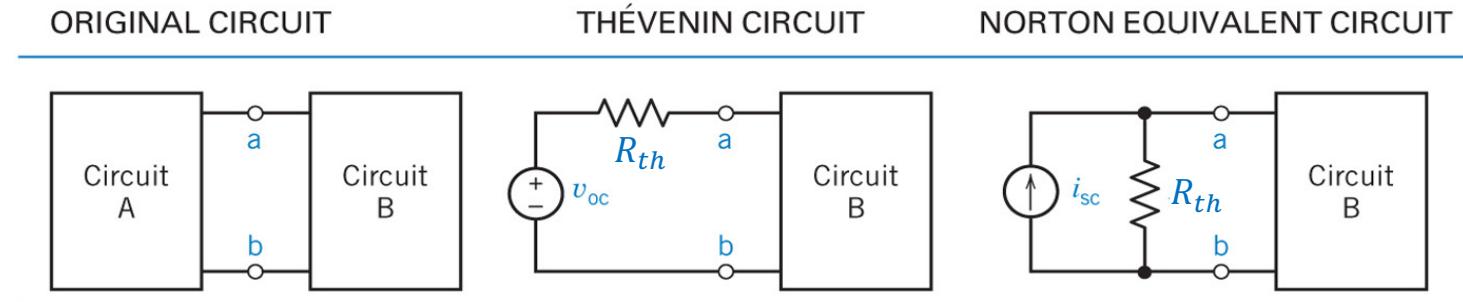
We have:



$$v_{oc} = 48 \text{ V}$$

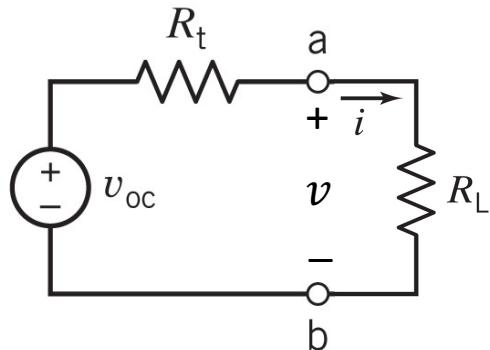
$$R_t = 8 \Omega$$

$$i_{sc} = 6 \text{ A}$$



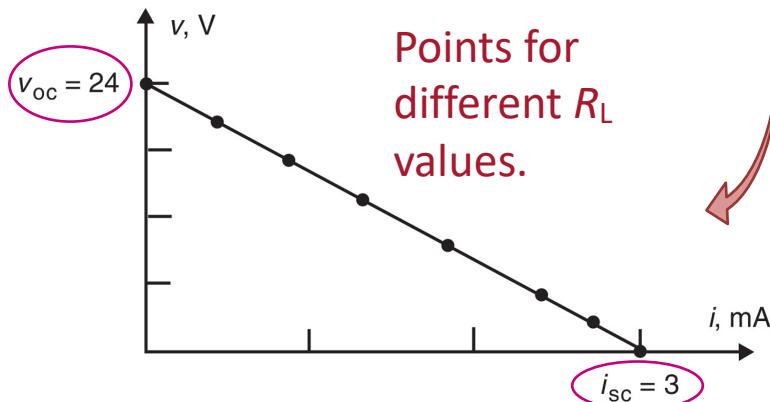
We choose the equivalent circuit depending on the overall problem we are trying to solve.

Graphical analysis to determine Thévenin and Norton circuits



From KVL we have

$$v = v_{oc} - R_t i$$

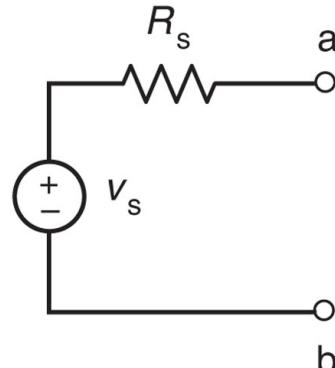


Only v_{oc} and one R_L measurement is needed to make a linear graph. This is easy.

GOOD WAY TO DO IT WHEN WE ARE IN THE LAB!

A plot of terminal voltage and currents.

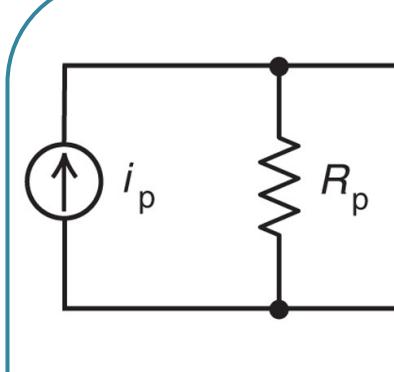
Our two options



$$v_s = R_p i_p \text{ and } R_s = R_p$$

Thévenin
equivalent circuit

≡



$$i_p = \frac{V_s}{R_s} \text{ and } R_p = R_s$$

Norton
equivalent circuit

- These are only models (=equivalent circuits) !!!
- In practice, a voltage source is always a voltage source.
- The real circuit with its internal nodes and branches is still there.



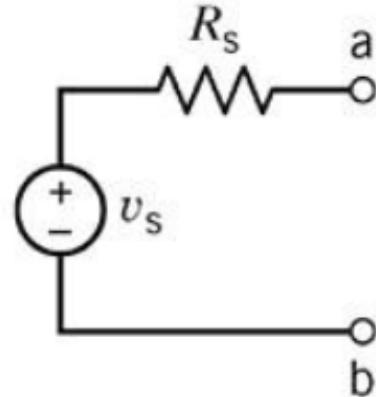
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Source transformation

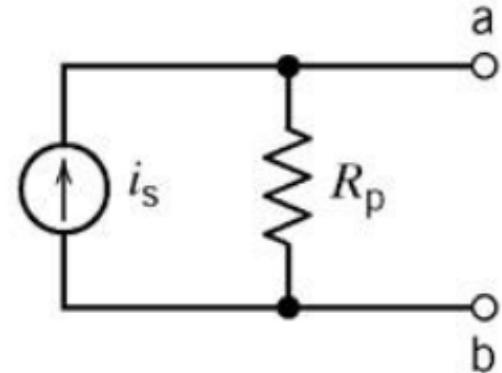




Non-ideal sources



Non-ideal voltage source



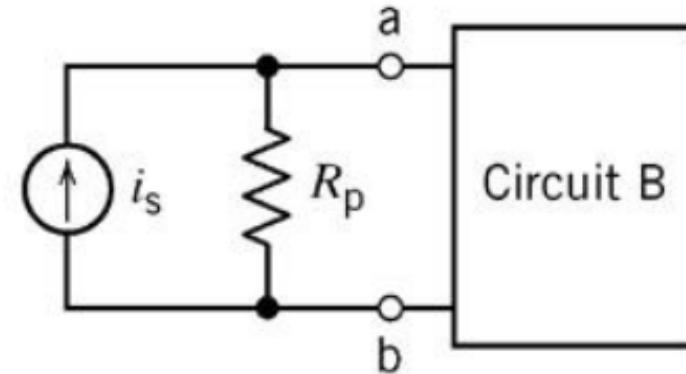
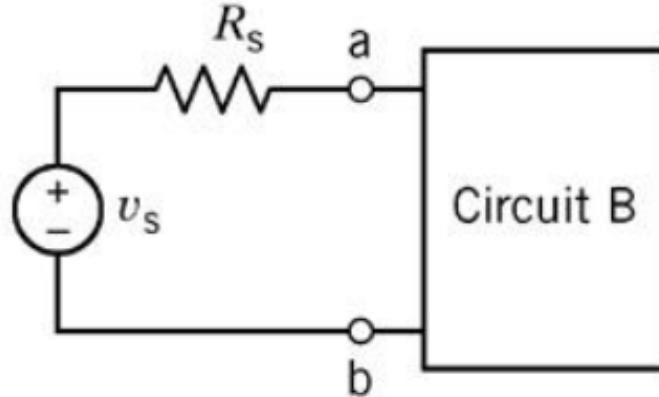
Non-ideal current source

Note:

The current is unchanged between the ideal and non-ideal voltage source.
The voltage is unchanged between the ideal and non-ideal current source.



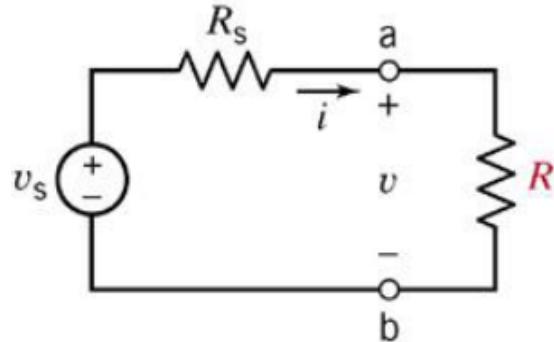
Equivalence



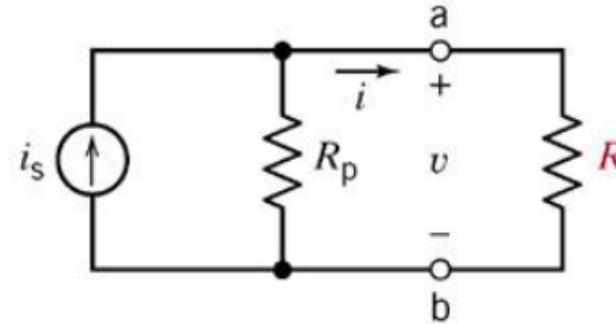
If we connect the non-ideal sources to the circuit B, then all the currents and voltages in B will be unchanged. This is called *equivalence*.

The process of transforming between these two non-ideal sources is called a *source transformation*.

Source transformation: conditions



(a)



(b)

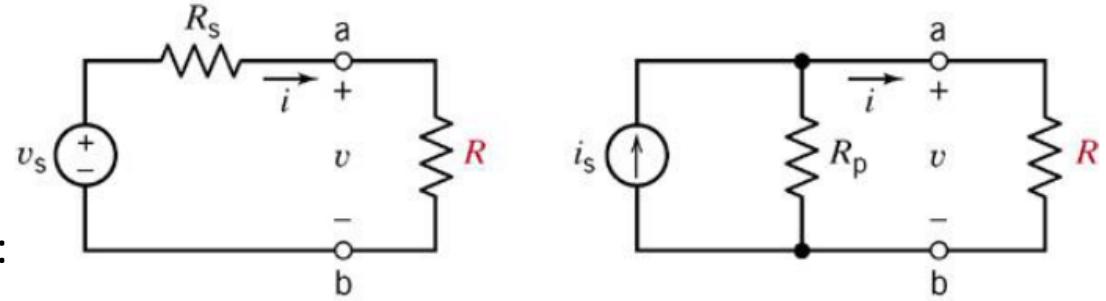
If the transformed sources are equivalent, then the current and voltage in R will be identical.

So we start the analysis with 2 extreme examples: $R = 0$ and $R = \infty$

1) When $R = 0$, we have a short circuit: Hence for (a) $i = \frac{v_s}{R_s}$ and for (b) $i = i_s$ (since no current flows through R_p) **therefore:** $i = \frac{v_s}{R_s} = i_s$

2) When $R = \infty$, we have an open circuit, for (a) no current flows ($i=0$) therefore $v=v_s$
 (b) $v = i_s R_p$ the voltage across the resistor **therefore:** $v = v_s = i_s R_p$

Source transformation: process



So now we have 2 conditions:

$$i_s = \frac{v_s}{R_s} \text{ and } v_s = i_s R_p = \left(\frac{v_s}{R_s} \right) R_p \quad (a)$$

Therefore $R_p = R_s$

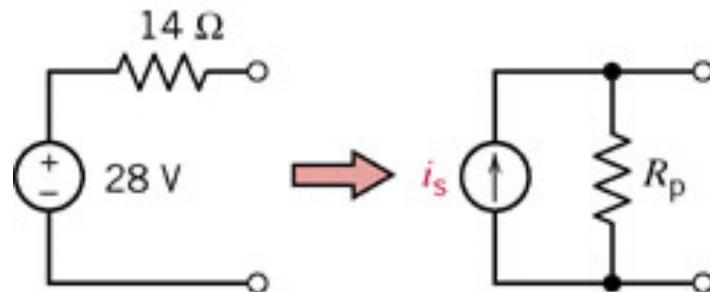
The source transformation theorem:

A voltage source v_s connected in series with a resistor R_s and a current source i_s connected in parallel with a resistor R_p are equivalent provided that

$$R_p = R_s \text{ and } v_s = R_s i_s$$

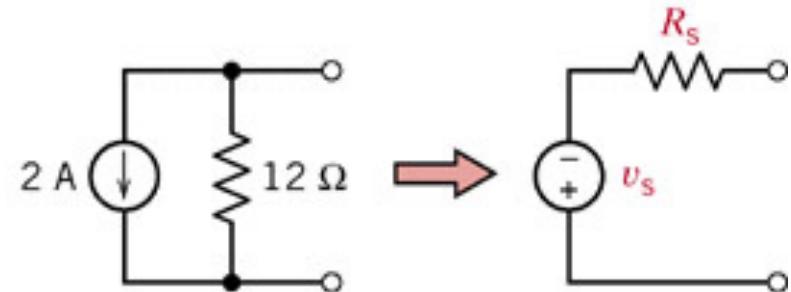


Example problem 1



(a)

$$i_s = 2 \text{ A}, R_p = 14 \Omega$$



(b)

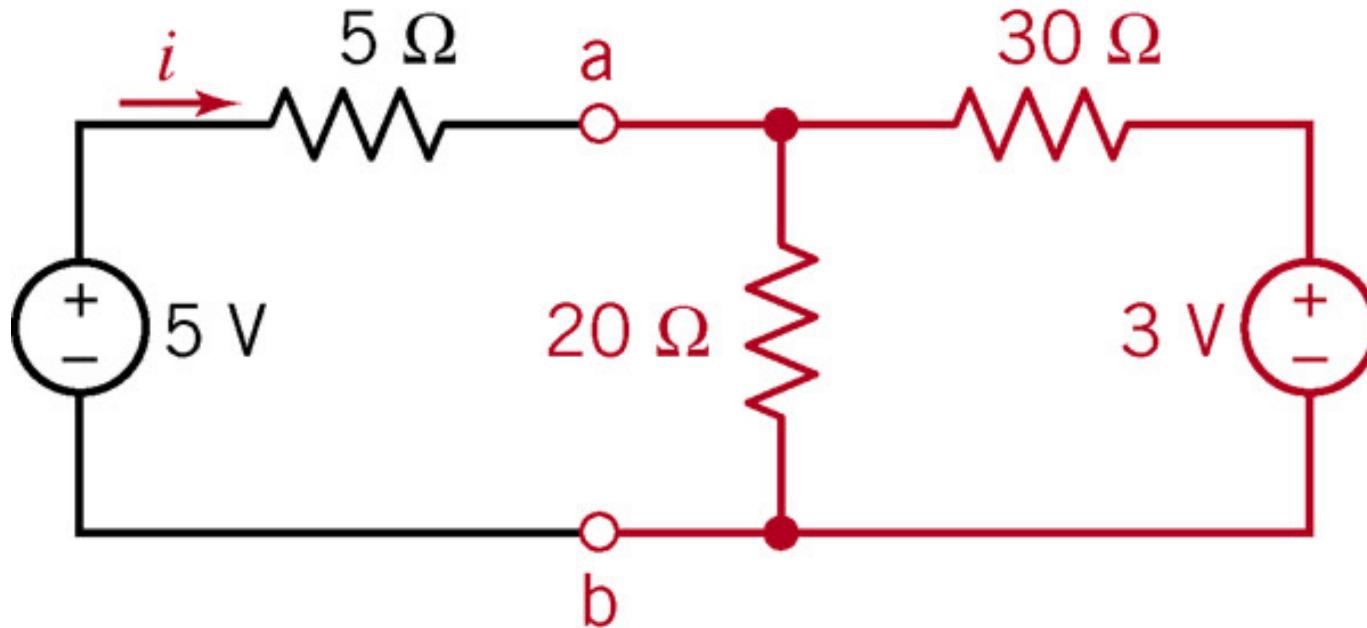
$$v_s = 24 \text{ V}, R_s = 12 \Omega$$

Warning: current direction (polarity) is important.

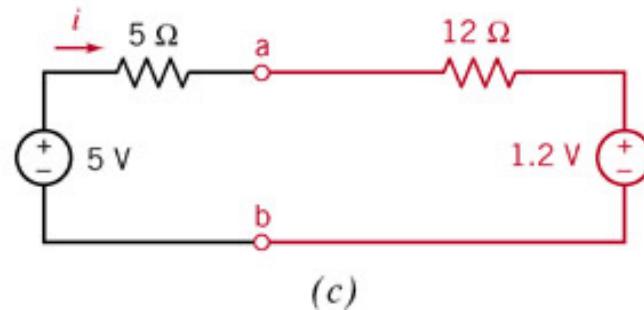
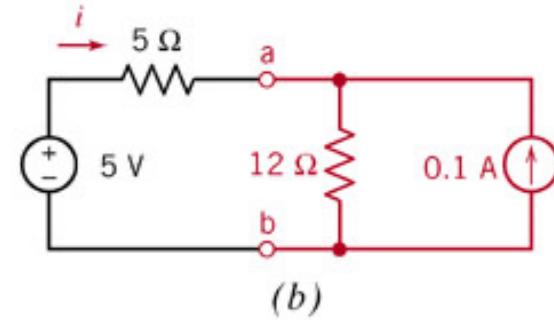
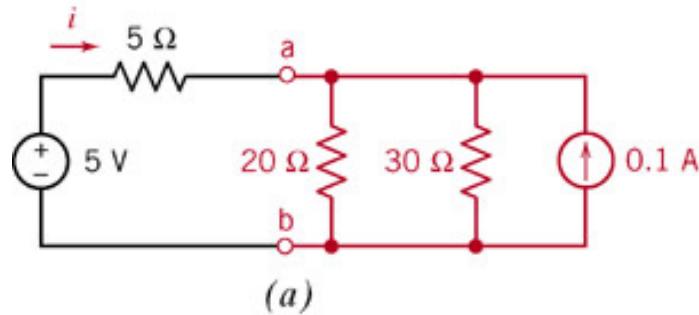


Example problem 2

Find the current i by reducing the circuit on the right (red) to its simplest form.



Answer



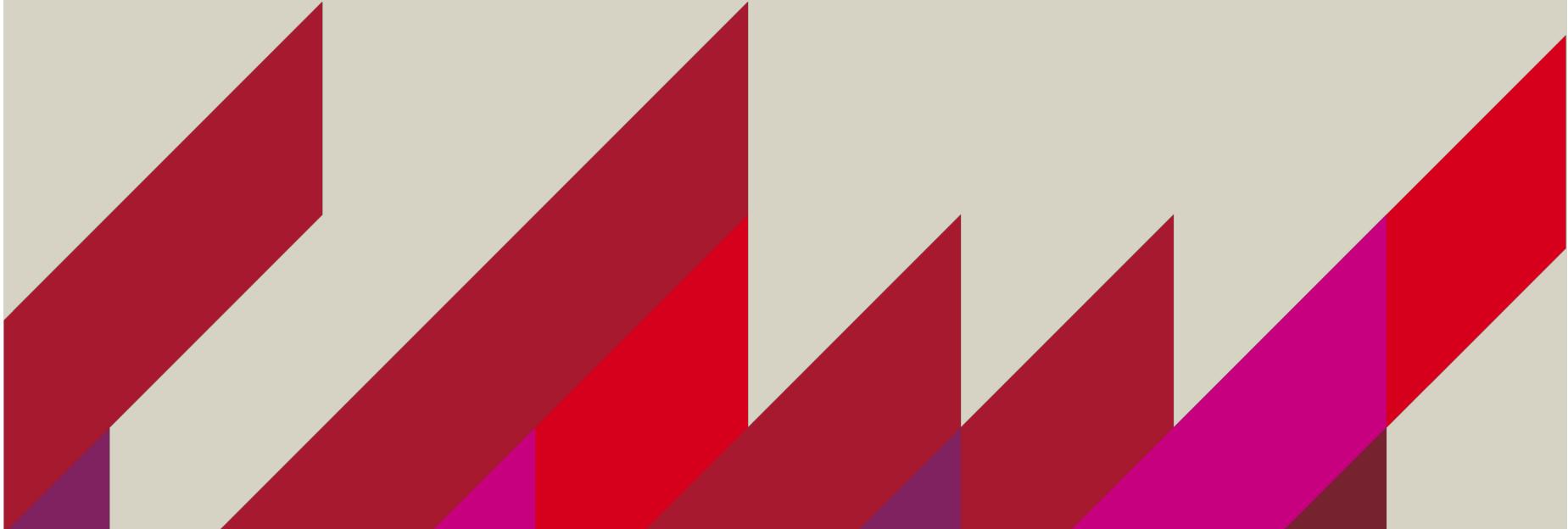
- (a) Transform far-RHS voltage source to current source
- (b) Find equivalent resistance of parallel resistances
- (c) Transform current source on RHS to voltage source

Now we add the resistances and voltages, and apply Ohm's Law i.e., $i = (3.8/17) \text{ A}$



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Superposition



Superposition: the principle

Useful when there are several independent sources in a circuit

- All the circuits we study in this unit are linear.
- If the circuit is linear, you can calculate the output voltages and currents by considering one independent source at a time, then add up all solutions to get the final solution.
- To consider one source at a time, you need to “zero” the other sources.
- To zero the independent voltage sources you need to replace them by short-circuits.
- To zero the independent current sources you need to replace them with open-circuits.



Linear circuits

The overall output from a linear circuit can be defined by:

$$v_0 = a_1 v_1 + a_2 v_2 + a_3 v_3 + \dots$$

Where:

v_0 = is the overall output of the circuit (it could be voltage or current)

a_i = are the linear coefficients, which are REAL constants and they are called **gains**

v_i = are the separate inputs to the circuit (current or voltage)

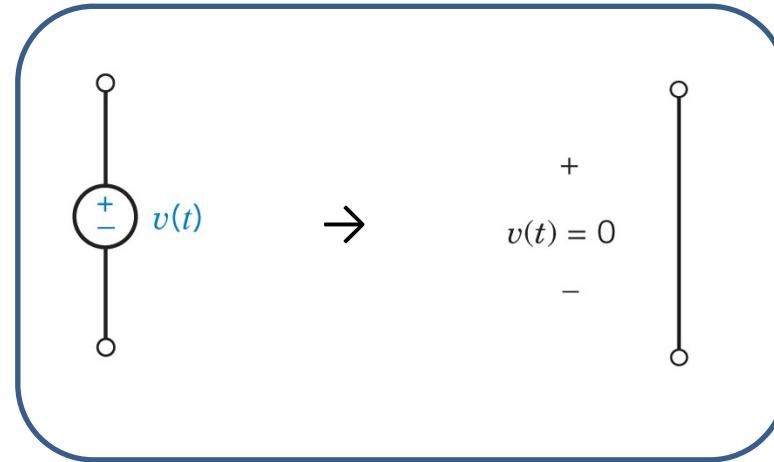
If we turn off all inputs instead of v_1 we get $v_{01} = a_1 v_1$

Do this process for all inputs we can see that we can sum all the v_{0i} to get the final v_0

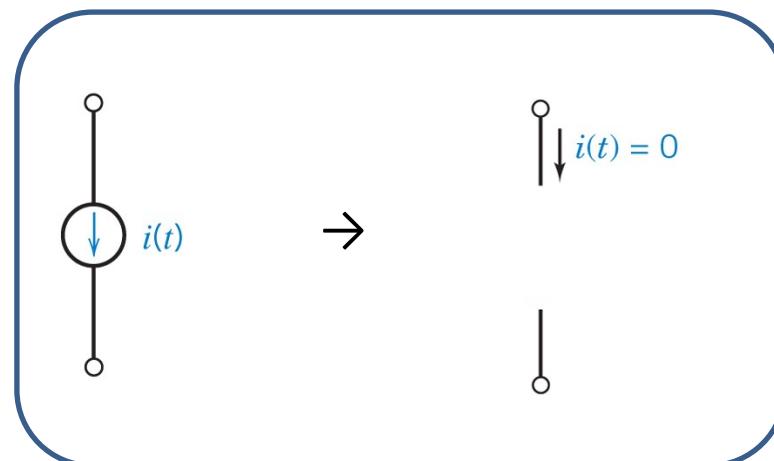


To “Zero” Independent Sources

Independent voltage source:

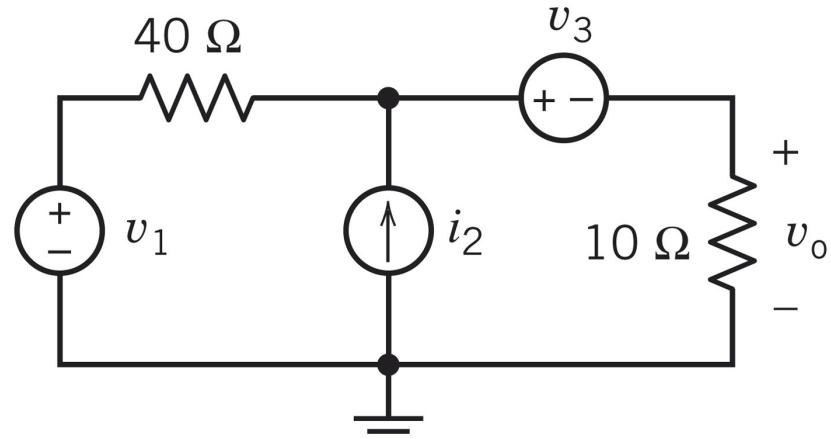


Independent current source:



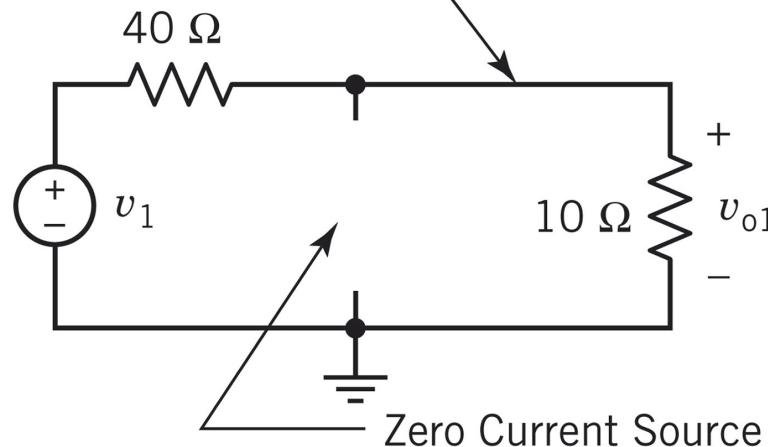
Example 5.3 -1 (1)

$$v_o = v_{o1} + v_{o2} + v_{o3}$$



Switch off all except v_1

Zero Voltage Source



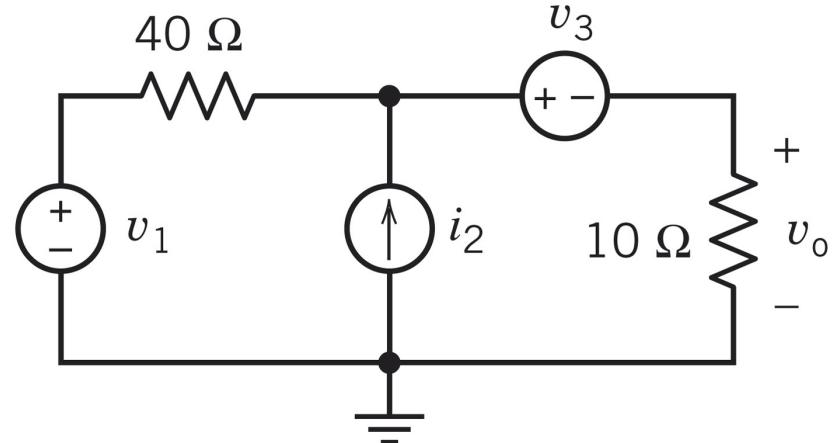
(voltage division)

$$v_{o1} = \frac{10}{10 + 40} v_1 = \frac{1}{5} v_1$$

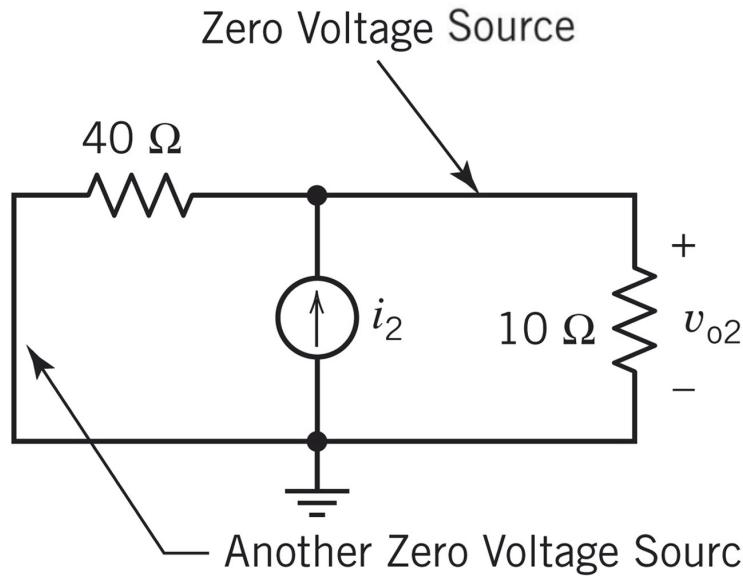
Example 5.3 -1 (2)



$$v_o = v_{o1} + v_{o2} + v_{o3}$$



Switch off all except i_2



$$\frac{v_{o2}}{40} + \frac{v_{o2}}{10} - i_2 = 0 \text{ (node equation)}$$

$$5v_{o2} = 40i_2$$

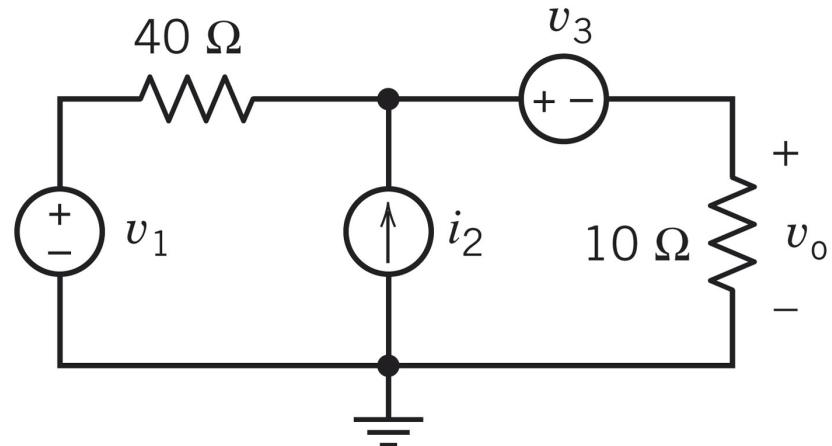
$$v_{o2} = 8i_2$$

Example 5.3 -1 (3)

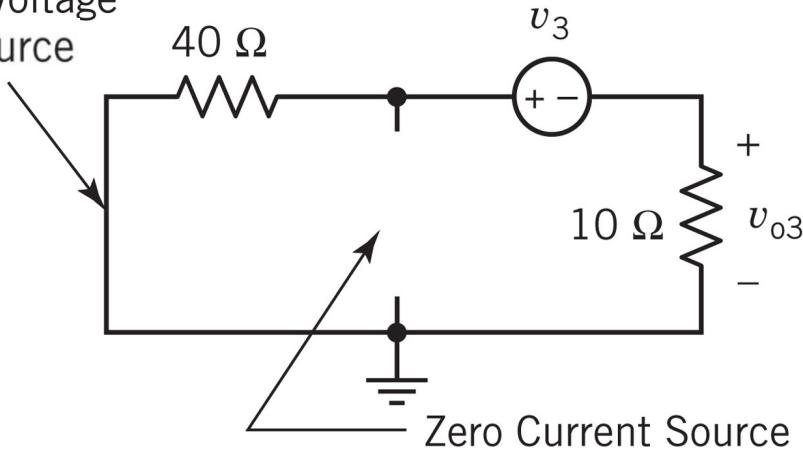


$$v_o = v_{o1} + v_{o2} + v_{o3}$$

Switch off all except v_3



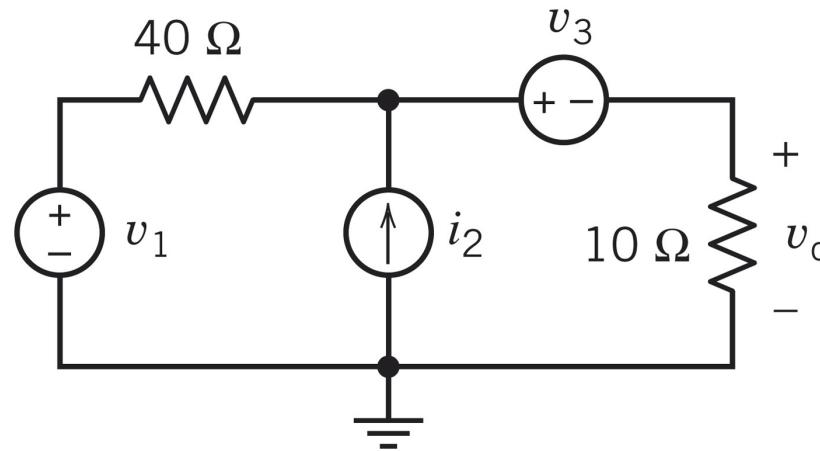
Zero Voltage Source



$$\begin{aligned}
 v_{o3} &= -\frac{10}{10 + 40} v_3 \\
 &= -\frac{1}{5} v_3
 \end{aligned}
 \quad (\text{voltage division})$$



Example 5.3 -1 - answer



$$v_o = v_{o1} + v_{o2} + v_{o3}$$

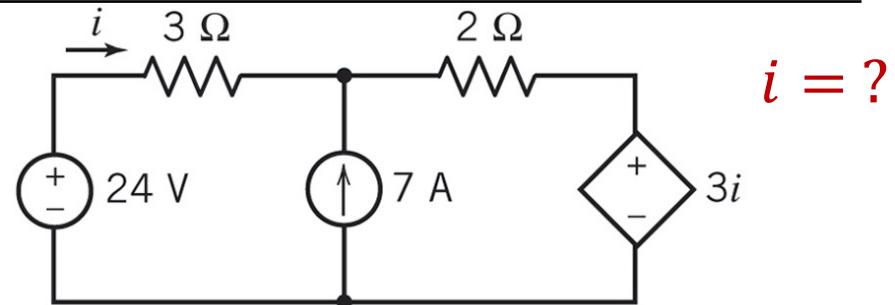
$$v_o = \frac{1}{5}v_1 + 8i_2 - \frac{1}{5}v_3$$



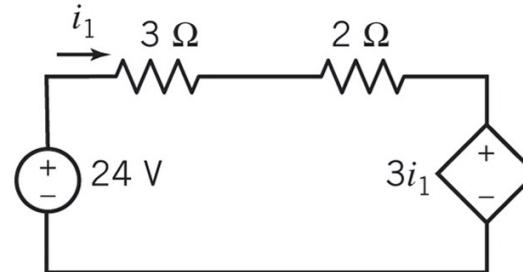
Example 5.3-2 (1)

Want to find the current passing through the $3\ \Omega$ resistor.

The inputs are the independent sources only.



Switch off (open) the independent current source:



$$(\text{KVL}) \quad -24 + (3 + 2)i_1 + 3i_1 = 0$$

$$8i_1 = 24$$

$$i_1 = 3 \text{ A}$$

Example 5.3 -2 (2)

Zero (short) the independent voltage source:

(looks like we need to apply
node analysis)

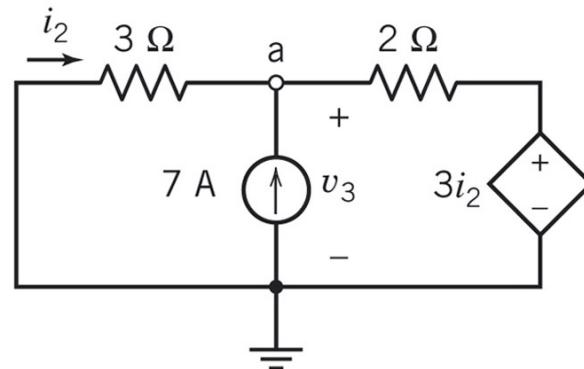
Find controlling current: $i_2 = -\frac{v_a}{3} \rightarrow v_a = -3i_2$ (note polarity)

Apply KCL at node a: $i_2 + 7 - \frac{v_a - 3i_2}{2} = 0$

$$i_2 + 7 - \frac{-3i_2 - 3i_2}{2} = 0$$

$$4i_2 + 7 = 0$$

$$i_2 = -\frac{7}{4} \text{ A}$$



Add the 2 currents for answer:

$$i = i_1 + i_2 = 3 - \frac{7}{4} = \frac{5}{4} \text{ A}$$