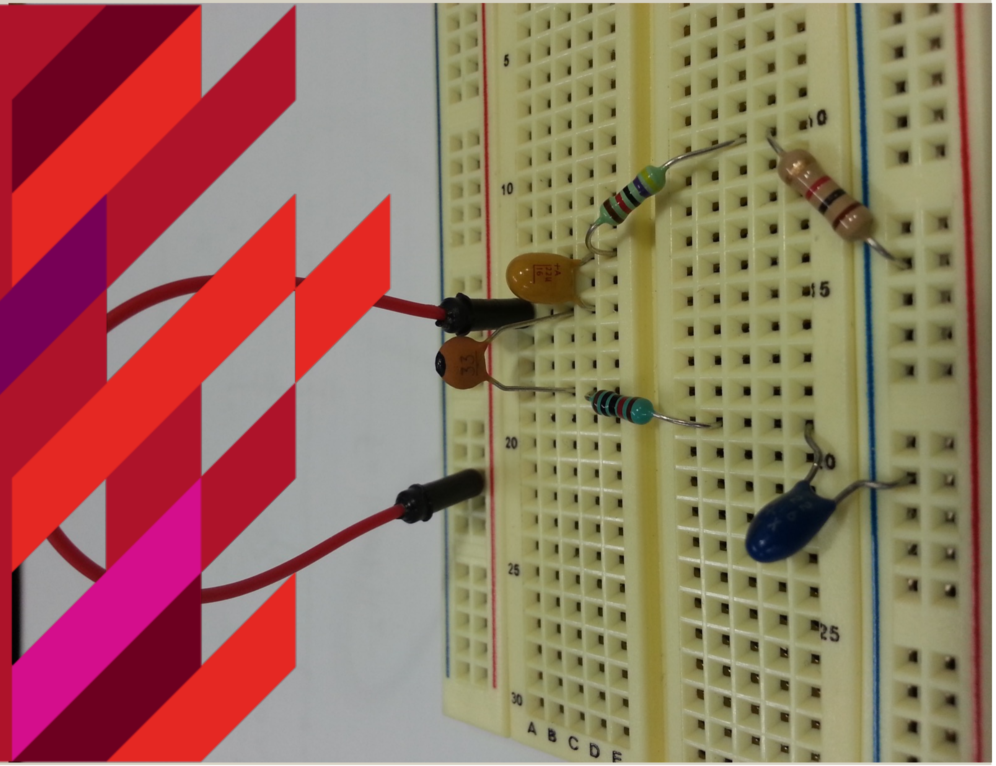
ELEC2070 Linear Circuits and Devices

Week 10: Frequency response of simple circuits + Network Function

Stuart Jackson



Frequency response

In many real systems, the forcing function (and hence the response) is **not** pure sinusoidal.

In systems in which information is to be transmitted, a single sinusoid is useless since it repeats itself every cycle. More complex functions are more useful.

We begin with the frequency response for sinusoids because close to all periodic functions can be decomposed into linear summation of sinusoidal functions.

The frequency response of circuit to a sinusoidal forcing function depends on the angular frequency, ω. Why, because the impedances are functions of ω. Hence the amplitudes and phases angles change with ω.

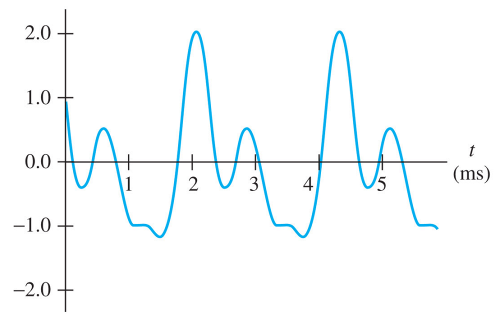
Now, we are going to examine the frequency response of circuit for a sinusoidal forcing function as a function of ω.

Frequency response and Bode plots

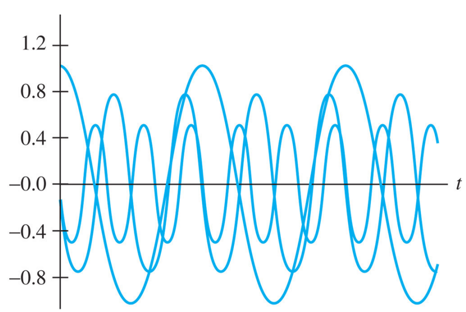
Chapter 13 in Dorf and Chapter 16 in Hayt, Kemmerly and Durbin

Signals in the frequency domain

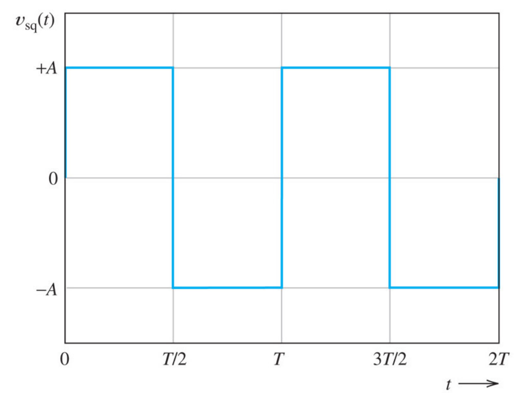
Signals can be decomposed into sinusoidal components using the Fourier transform.



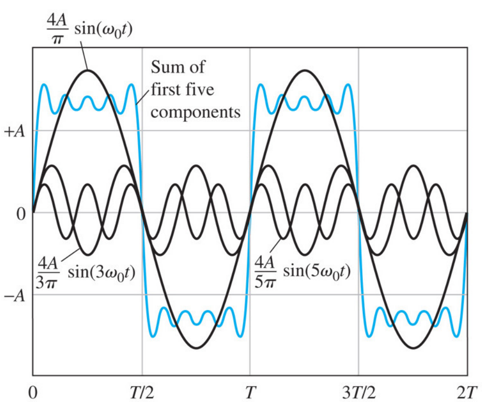
(a) Music waveform



(b) Sinusoidal components



(a) Periodic square wave



(b) Several of the sinusoidal components and the sum of the first five components

Each frequency component can be shown separately on the frequency axis.

This is called the frequency spectrum of the signal.

Bandwidth

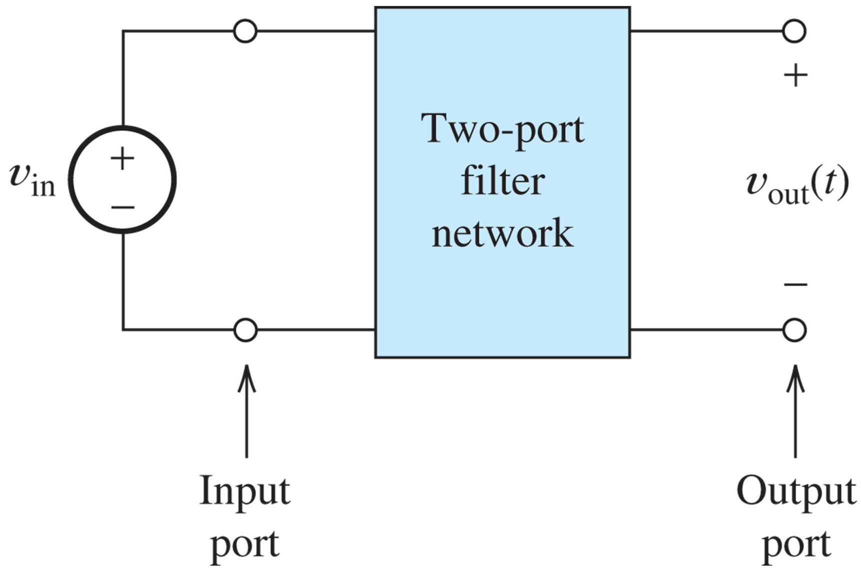
All real signals occupy a limited range (or bandwidth) in the frequency domain.

Table 6.1. Frequency Ranges of Selected Signals

Electrocardiogram	0.05 to 100 Hz
Audible sounds	20 Hz to 15 kHz
AM radio broadcasting	540 to 1600 kHz
HD component video signals	Dc to 25 MHz
FM radio broadcasting	88 to 108 MHz
Cellular phone	824 to 894 MHz and 1850 to 1990 MHz
Satellite television downlinks (C-band)	3.7 to 4.2 GHz
Digital satellite television	12.2 to 12.7 GHz

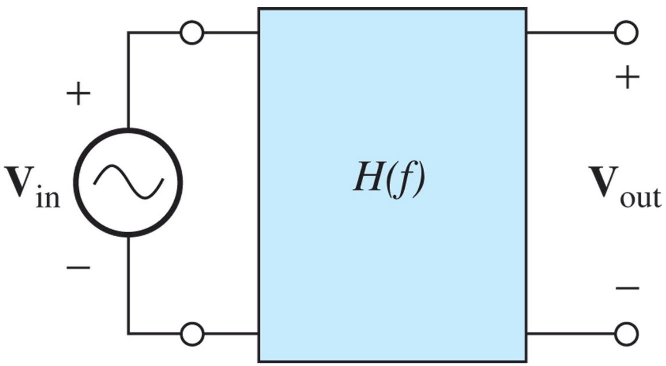
Filters (in the frequency domain)

Filters are usually two port circuits. They are designed to filter a portion of the frequency components of the input signal.



The filter circuit responds to each frequency component differently because impedance values differ at each frequency.

Network function in the frequency domain



The network function is the frequency response of a circuit

$$H(f) = \frac{V_{out}(f)}{V_{in}(f)}$$

OR

The transfer function is H(s)

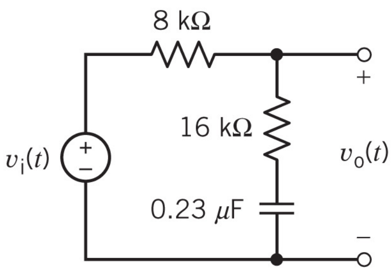
It can also be represented as a function of ω

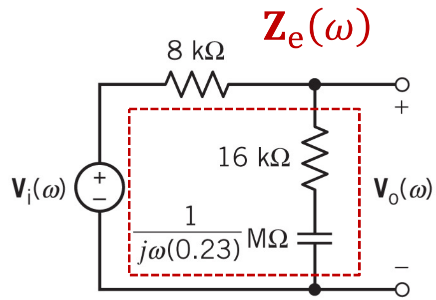
$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$$

Where: $\omega = 2\pi f$

We need two plots: one for **amplitude**, one for **angle**.

Example



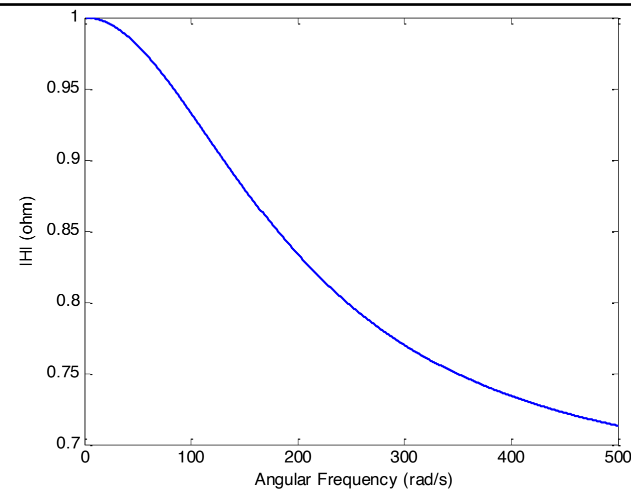


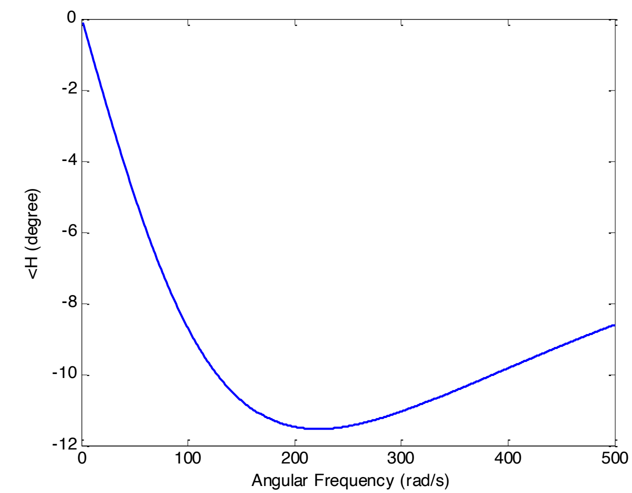
$$\mathbf{Z}_e(\omega) = 16000 + \frac{10^6}{j(0.23)\omega}$$

Voltage division

$$\mathbf{V}_o(\omega) = \frac{\mathbf{Z}_e(\omega)}{8000 + \mathbf{Z}_e(\omega)} \mathbf{V}_i(\omega)$$

$$\mathbf{H}(\omega) = \frac{1 + j\dfrac{\omega}{272}}{1 + j\dfrac{\omega}{181}}$$





A low-pass filter!

Input and output signals in the time domain

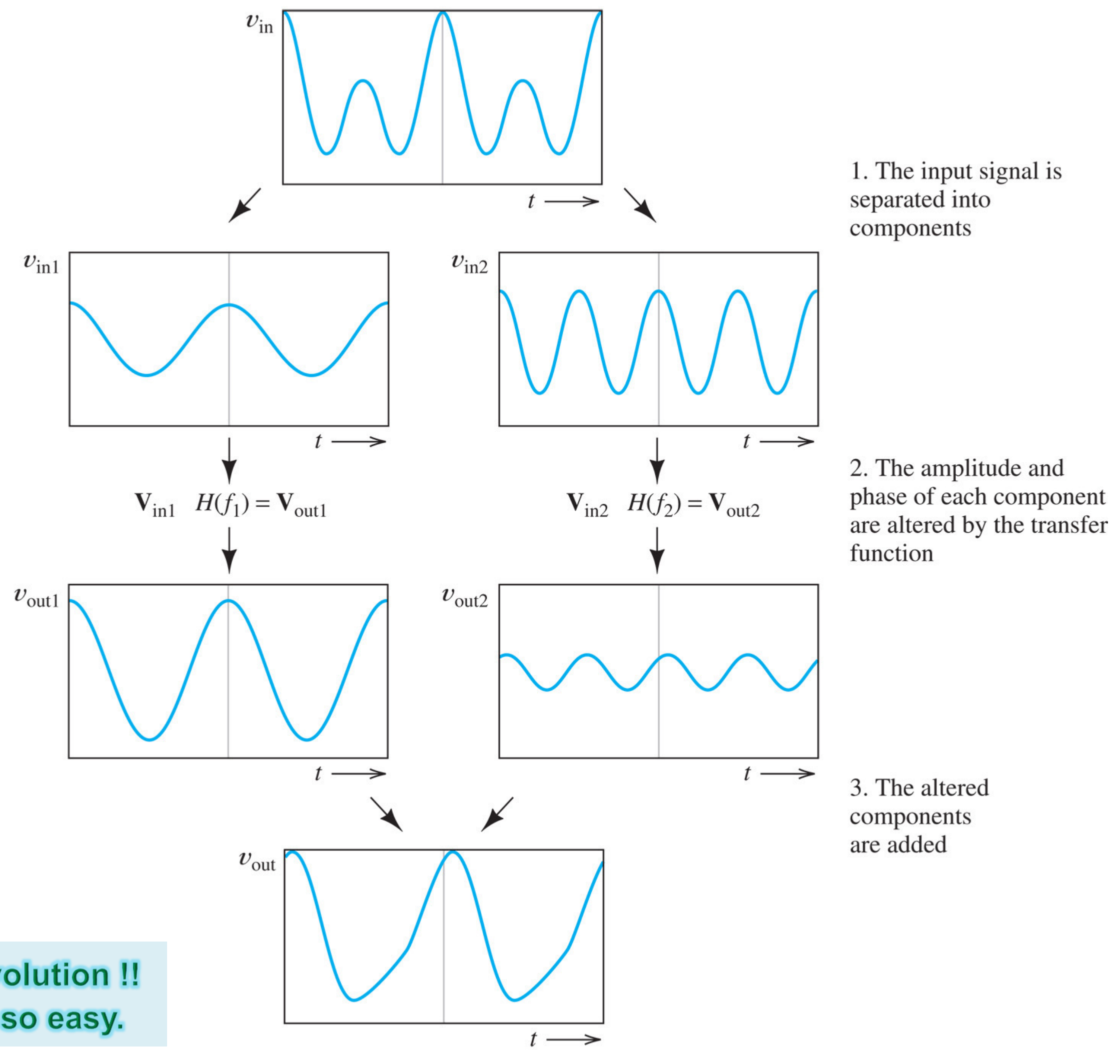
In the frequency domain, the output signal is found by multiplying the input signal with the network function

$$V_{out}(f) = V_{in}(f) \cdot H(f)$$

What do you do in the time domain?

$$v_{out}(t) = v_{in}(t) * h(t)$$

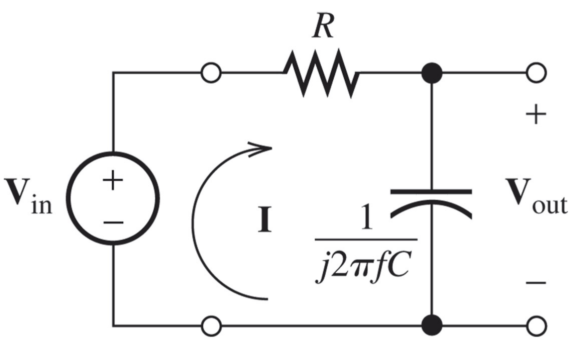
Convolution !!
Not so easy.



First order filters

Involve one capacitor or one inductor

First order low pass filters using one capacitor



This is a simple circuit. The voltage network function can be found by voltage division:

$$H(f) = \frac{V_{out}(f)}{V_{in}(f)} = \frac{\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} = \frac{1}{1 + j2\pi fRC}$$

Define the Corner (or Break) frequency:

$$f_c = \frac{1}{2\pi RC} \quad \Rightarrow \quad H(f) = \frac{1}{1 + j\frac{f}{f_c}}$$

This form of the network function is very useful.

This form is easier to interpret and plot.

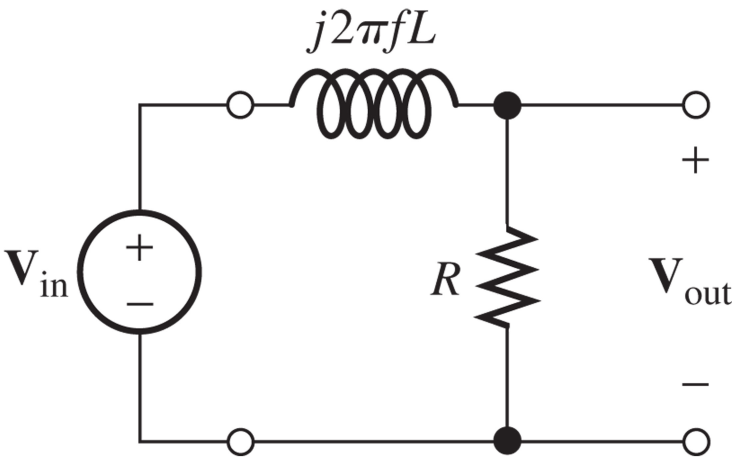
(In Hambley, the corner (or break) frequency is shown as f_B)

(In Dorf, the corner frequency is shown as f_o which is confusing. You should use f_c or f_B)

We use "B" in this lecture

Notice the corner frequency and the natural time constant of the circuit are related: $\omega_c = \frac{1}{RC} = \frac{1}{\tau}$

First order low pass filters using one inductor



Another type of low pass filter.

Derive its voltage network function at home.

Find its corner frequency.

Corner or frequency: $$f_B = \frac{R}{2\pi L}$$

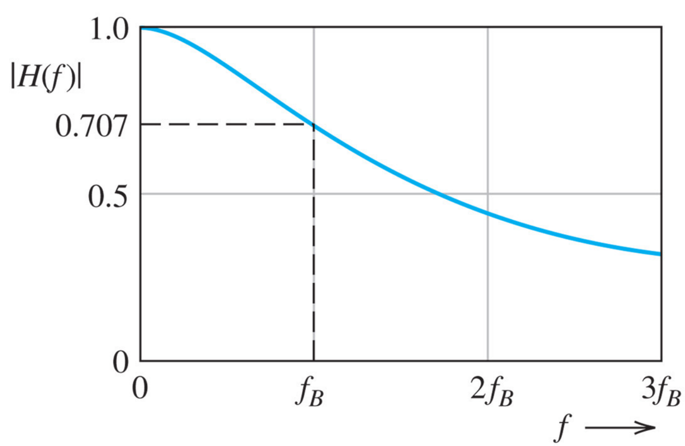
Network function: $$H(f) = \frac{1}{1 + j\dfrac{f}{f_B}}$$

Magnitude and phase angle plots of the voltage network function

Magnitude of the network function

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_B)^2}}$$

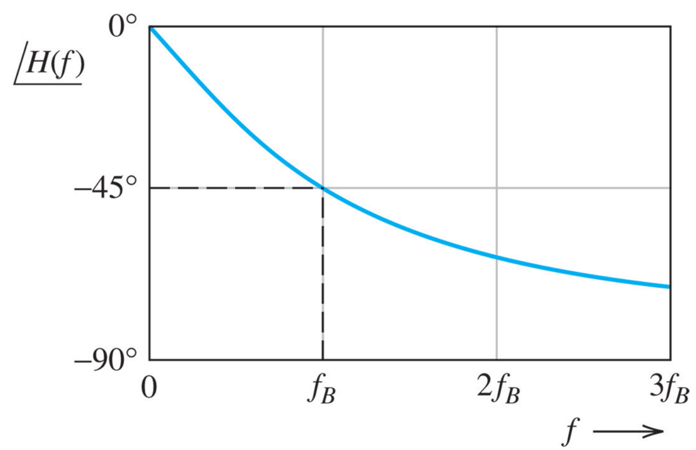
Remember: the magnitude of H(f) is the magnitude of the numerator (unity) divided by the magnitude of the denominator!



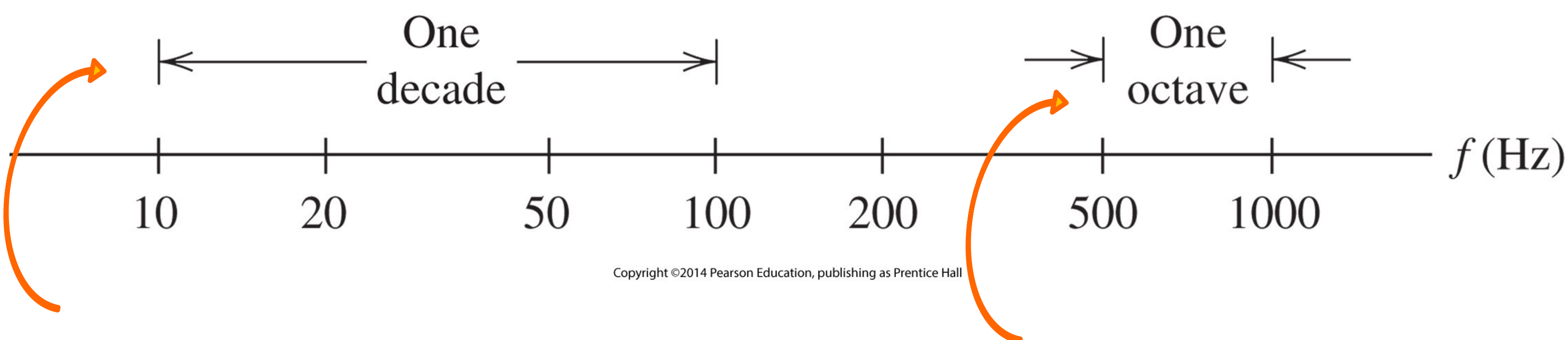
Phase angle of the network function

$$\angle H(f) = -\arctan\left(\frac{f}{f_B}\right)$$

Remember: the phase angle of H(f) is the phase angle of numerator (zero) MINUS the phase angle of the denominator!



Logarithmic scales



Ten-to-one change in frequency

two-to-one change in frequency

Log-log plots

Only the magnitude is plotted on the log-log scale.

$$|H(f)|_{dB} = 20\log_{10}|H(f)|$$

Table 6.2. **Network-Function Magnitudes and Their Decibel Equivalents**

$\lvert H(f)\rvert$	$\lvert H(f)\rvert_{\mathrm{dB}}$
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	−3
1/2	−6
0.1	−20
0.01	−40

Decibel (dB) scale for voltage and power

These are useful in practice when using equipment with dB scales

Voltage (rms-volts): $\mathrm{dBV}(V_{rms}) = 20 \cdot \log_{10}\left(\frac{V_{rms}}{1\ \mathrm{V_{rms}}}\right)$

Definition of dBV is not always consistent. It may mean $20 \cdot \log_{10}\left(\frac{|V|}{1\mathrm{V}}\right)$ You need to watch out.

Power (watts): $\mathrm{dB}(P_a) = 10 \cdot \log_{10}\left(\frac{P_a}{1\ \mathrm{W}}\right)$

Definition of dBm and dB (of power) are always consistent.

Power (milliwatts): $\mathrm{dBm}(P_a) = 10 \cdot \log_{10}\left(\frac{P_a}{1\ \mathrm{mW}}\right)$

$$\mathrm{dBm}(P_a) = \mathrm{dB}(P_a) + 30$$

Converting back from the dB scale

Voltage (rms-volts): $$\frac{V_{rms}}{1\ \mathrm{V_{rms}}} = 10^{\left(\frac{\mathrm{dB}(V_{rms})}{20}\right)}$$

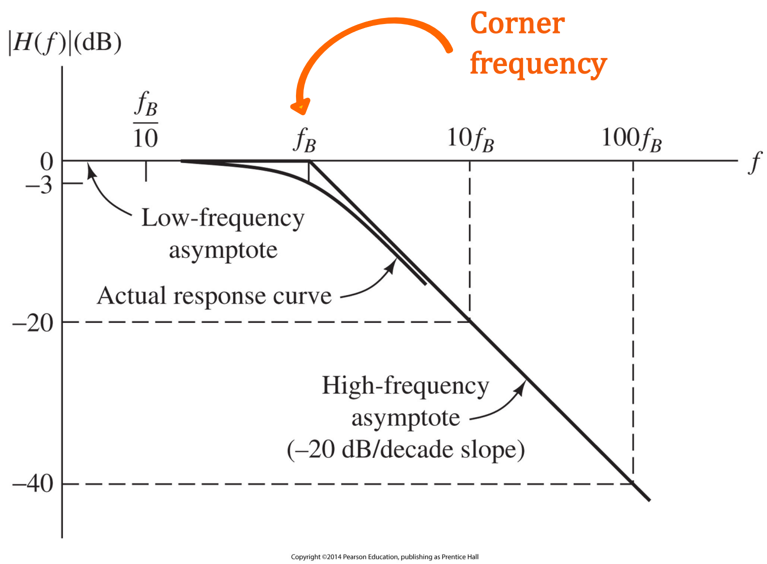
Power (watts): $$\frac{P_a}{1\ \mathrm{W}} = 10^{\left(\frac{\mathrm{dB}(P_a)}{10}\right)}$$

Power (milliwatts): $$\frac{P_a}{1\ \mathrm{mW}} = 10^{\left(\frac{\mathrm{dBm}(P_a)}{10}\right)}$$

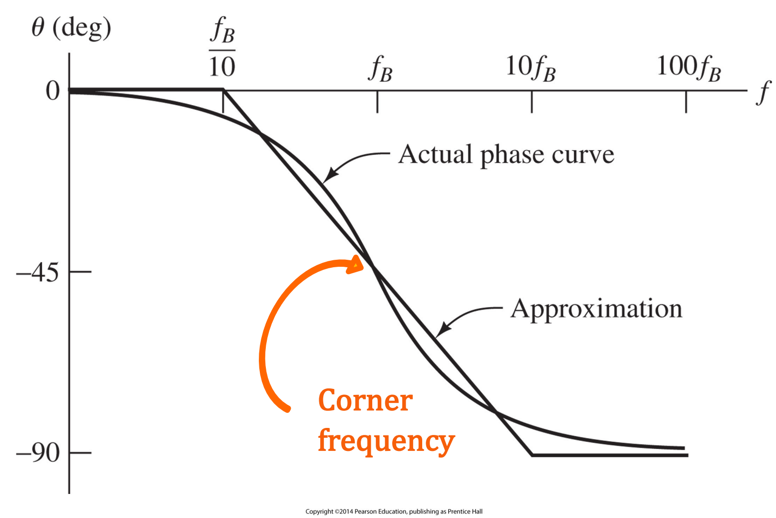
Check out other dB types
https://en.wikipedia.org/wiki/Decibel
http://www.sengpielaudio.com/calculator-db-volt.htm

Bode plots

Bode plot of a low pass filter



Magnitude of the network function



Phase angle of the network function

Gain and phase shift

Gain is the ratio of the amplitude of the output sinusoid to the amplitude of the input sinusoid.

Phase shift is the difference between the phase angle of the output sinusoid to the phase angle of the input sinusoid.

		$\lvert \boldsymbol{H}(\omega) \rvert$ (V/V)	$\angle \boldsymbol{H}(\omega)$ (deg)
f (Hz)	ω (rad/s)	GAIN	PHASE SHIFT
100	628.3	9.98	176°
500	3,141.6	9.54	163°
1,000	6,283	8.47	148°
5,000	31,416	3.03	108°
10,000	62,830	1.57	99°

dB values of a simple low-pass filter

$$H(f) = \frac{1}{1 + jf/f_B}$$

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_B)^2}}$$

$$|H(f)|_{\text{dB}} = 20\log\frac{1}{\sqrt{1+(f/f_B)^2}}$$

Using the properties of the logarithm, we obtain

$$|H(f)|_{\text{dB}} = 20\log(1) - 20\log\sqrt{1+\left(\frac{f}{f_B}\right)^2}$$

Of course, the logarithm of unity is zero. Therefore,

$$|H(f)|_{\text{dB}} = -20\log\sqrt{1+\left(\frac{f}{f_B}\right)^2}$$

Finally, since $\log(\sqrt{x}) = \frac{1}{2}\log(x)$, we have

$$|H(f)|_{\text{dB}} = -10\log\left[1+\left(\frac{f}{f_B}\right)^2\right]$$

Table 6.3. Values of the Approximate Expression (Equation 6.20) for Selected Frequencies

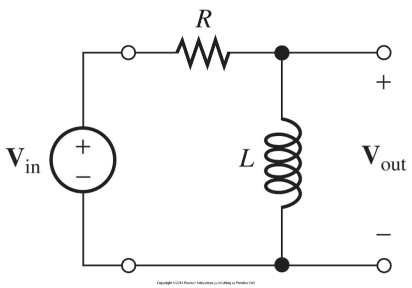
f	$\lvert H(f)\rvert_{\text{dB}}$
f_B	0
$2f_B$	−6
$10f_B$	−20
$100f_B$	−40
$1000f_B$	−60

−6dB slope per octave

−20dB slope per decade change in frequency

Approximate value: $|H(f)|_{dB} \cong -20log_{10}\left(\frac{f}{f_B}\right)$

First order high pass filters



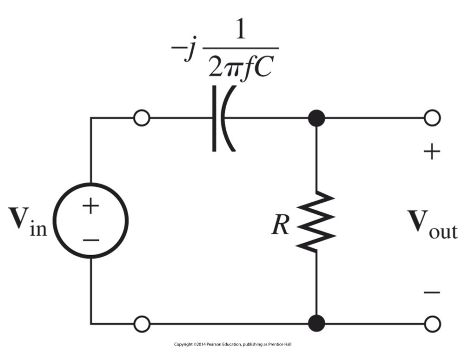
Reordered for easier plotting:

$$H(f) = \frac{j2\pi fL}{R + j2\pi fL} = \frac{j2\pi fL/R}{1 + j2\pi fL/R} = \frac{jf/f_B}{1 + jf/f_B}$$

$$f_B = \frac{R}{2\pi L}$$

Notice again that the corner frequencies and the natural time constants of the circuits are related

Another high pass filter:



This one is more popular.
Can you guess why?

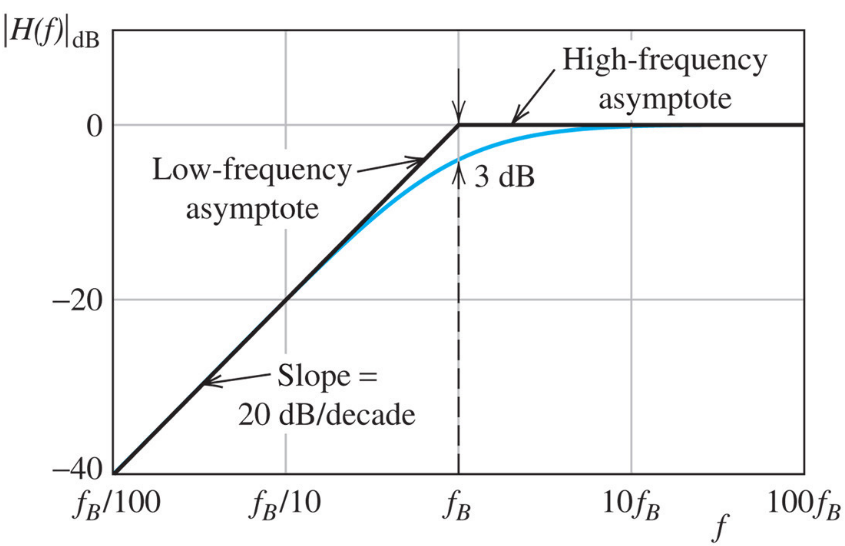
Capacitors are cheaper and easier to make.

Wider range of capacitor values are available.

Derive the voltage network function at home

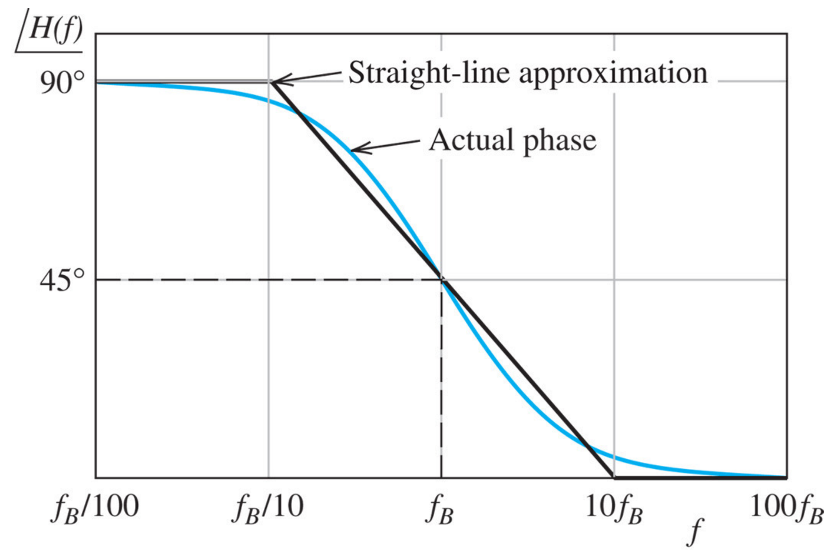
Bode plot of a high pass filter

Magnitude of the network function



(a) Magnitude

Phase angle of the network function



(b) Phase

$$|H(f)| = \frac{\frac{f}{f_B}}{\sqrt{1 + \left(\frac{f}{f_B}\right)^2}}$$

$$phase\ angle = 90^0 - \arctan\left(\frac{f}{f_B}\right)$$

dB values of a simple high pass filter

$$H(f) = \frac{j\,f/f_B}{1 + j\,f/f_B}$$

$$|H(f)|_{\text{dB}} = 20\log \frac{f/f_B}{\sqrt{1+(f/f_B)^2}}$$

This can be written as

$$|H(f)|_{\text{dB}} = 20\log\left(\frac{f}{f_B}\right) - 10\log\left[1+\left(\frac{f}{f_B}\right)^2\right] \quad (6.25)$$

For $f << f_B$, the second term on the right-hand side of Equation 6.25 is approximately zero. Thus, for $f << f_B$, we have

$$|H(f)|_{\text{dB}} \cong 20\log\left(\frac{f}{f_B}\right) \quad \text{for } f << f_B \quad (6.26)$$

Table 6.4. Values of the Approximate Expression Given in Equation 6.26 for Selected Frequencies

f	$\lvert H(f)\rvert_{\text{dB}}$	
f_B	0	6dB slope per octave
$f_B/2$	−6	
$f_B/10$	−20	20dB slope per decade
$f_B/100$	−40	

Approximate value: $|H(f)|_{dB} \cong 20log_{10}\left(\frac{f}{f_B}\right)$

How to guess the type of filter?

Capacitors at low and high frequencies

Impedance of a capacitor: $$Z_c(\omega) = \frac{1}{j\omega C}$$

In polar notation: $$Z_c(\omega) = \frac{1}{\omega C}\angle -90°$$

Phasor diagram: $\downarrow \frac{1}{\omega C}$

At very low frequencies: $\omega \approx 0$

$$Z_c \approx \frac{1}{j0} \approx -j\infty$$

$$Z_c \approx \infty\angle -90°$$

$\downarrow \approx \infty$

≈ open circuit with a −90° phase shift

At very high frequencies: $\omega \approx \infty$

$$Z_c \approx \frac{1}{j\infty} \approx -j0$$

$$Z_c \approx 0\angle -90°$$

$\downarrow \approx 0$

≈ short circuit with a −90° phase shift

Inductors at low and high frequencies

Impedance of an inductor: $Z_L(\omega) = j\omega L$

In polar notation: $Z_L(\omega) = \omega L\angle 90°$

Phasor diagram: $\uparrow \; \omega L$

At very low frequencies: $\omega \approx 0$

$Z_L \approx j0$

$Z_L \approx 0\angle 90°$

$\uparrow \approx 0$

$\approx$ short circuit with a 90° phase shift

At very high frequencies: $\omega \approx \infty$

$Z_L \approx j\infty$

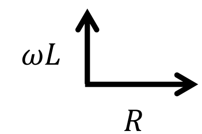
$Z_L \approx \infty\angle 90°$

$\uparrow \approx \infty$

$\approx$ open circuit with a 90° phase shift

Series RL combination at low and high frequencies

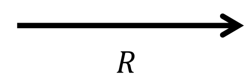
$$Z(\omega) = R + j\omega L$$



At very low frequencies: $\omega \approx 0$

$$Z \approx R + j0 \approx R$$

$$Z \approx R$$

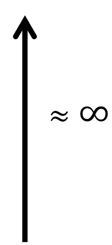


$\approx$ resistor

At very high frequencies: $\omega \approx \infty$

$$Z \approx R + j\infty \approx j\infty$$

$$Z \approx \infty\angle 90°$$



$\approx$ open circuit with a 90° phase shift

Series RC combination at low and high frequencies

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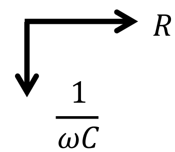
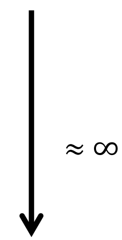
$$Z(\omega) = R + \frac{1}{j\omega C}$$



At very low frequencies: $\omega \approx 0$

$$Z \approx R - j\infty \approx -j\infty$$

$$Z \approx \infty\angle - 90°$$

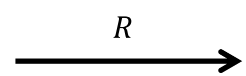


≈ open circuit with a $-$ 90° phase shift

At very high frequencies: $\omega \approx \infty$

$$Z \approx R + j0 \approx R$$

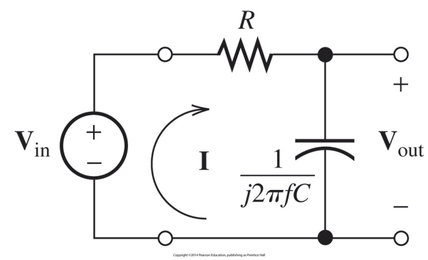
$$Z_L \approx R$$



≈ resistor

First order low pass filter at low and high frequencies

$$H(\omega) = \frac{1}{1 + j\,\omega/\omega_c}$$



CAPACITOR CIRCUIT

At very low frequencies: $\omega \approx 0$ or: $\omega \ll \omega_c$

$$1 + j\,\omega/\omega_c \approx 1 + j0 \approx 1 \quad \Rightarrow \quad H \approx \frac{1}{1 + j0} \approx 1$$

pass band

At very high frequencies: $\omega \approx \infty$ or: $\omega \gg \omega_c$

$$1 + j\,\omega/\omega_c \approx 1 + j\infty \approx j\infty \quad \Rightarrow \quad H \approx \frac{1}{j\infty} \approx \frac{-j}{\infty} \approx 0\angle - 90°$$

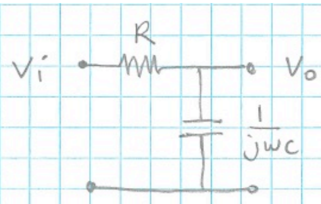
stop band

At the corner frequency: $\omega = \omega_c$

$$1 + j\,\omega/\omega_c = 1 + j1 = \sqrt{2}\angle 45° \quad \Rightarrow \quad H = \frac{1}{1 + j1} = \frac{1}{\sqrt{2}}\angle - 45°$$

The magnitude indicates the pass or stop band

Plotting by hand

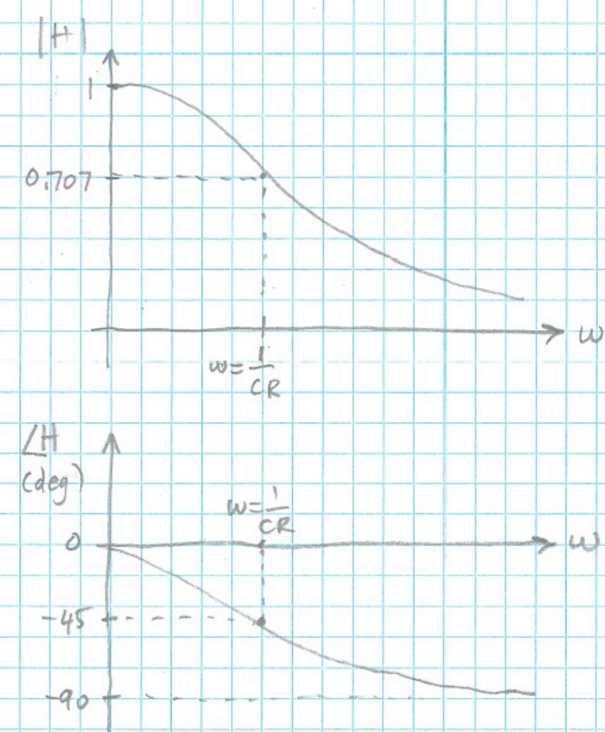


$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1} = H(\omega)$$

$\omega \approx 0 \rightarrow H \approx 1$

$\omega \approx \infty \rightarrow H \approx \frac{1}{j\omega CR} \approx 0 \angle -90^\circ$

$\omega = \frac{1}{CR} \rightarrow H = \frac{1}{j+1} = \frac{1}{\sqrt{2}} \angle -45^\circ$



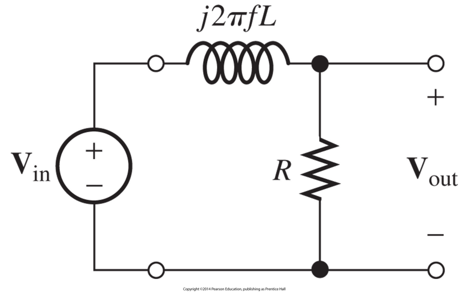
Remember natural time constant $\tau = RC$

$\omega = \frac{1}{\tau} = \frac{1}{RC}$

$\omega = \frac{1}{RC}$ is the band-stop frequency of a filter.

First order high pass filter at low and high frequencies

$$H(\omega) = \frac{j\,\omega/\omega_c}{1 + j\,\omega/\omega_c}$$



INDUCTOR CIRCUIT

At very low frequencies: $\omega \approx 0$ or: $\omega \ll \omega_c$

$$1 + j\,\omega/\omega_c \approx 1 + j0 \approx 1 \quad \Rightarrow \quad H \approx \frac{j0}{1} \approx j0 \approx 0\angle 90^\circ$$

stop band

At very high frequencies: $\omega \approx \infty$ or: $\omega \gg \omega_c$

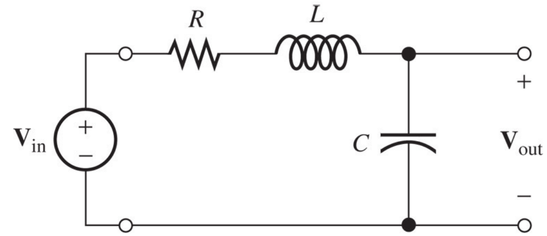
$$1 + j\,\omega/\omega_c \approx 1 + j\infty \approx j\infty \quad \Rightarrow \quad H \approx \frac{j\infty}{j\infty} \approx 1$$

pass band

At the corner frequency: $\omega = \omega_c$

$$1 + j\,\omega/\omega_c = 1 + j1 = \sqrt{2}\angle 45^\circ \quad \Rightarrow \quad H = \frac{j1}{1 + j1} = \frac{1\angle 90^\circ}{\sqrt{2}\angle 45^\circ} = \frac{1}{\sqrt{2}}\angle 45^\circ$$

Another filter to guess



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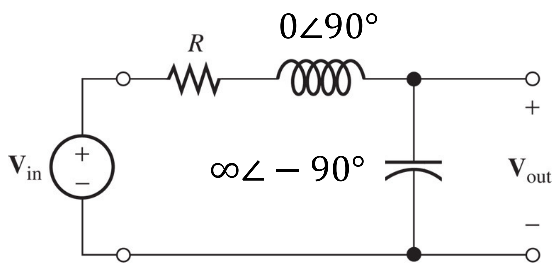
At low frequencies: $\omega \approx 0$



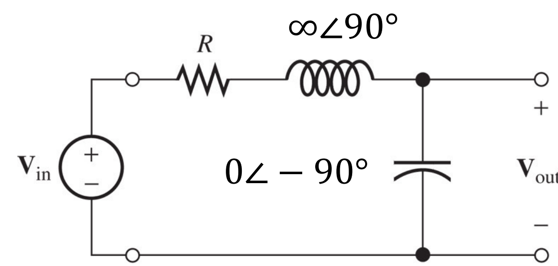
Voltage division:

$$H \approx \frac{\infty\angle -90°}{\infty\angle -90° + R + 0\angle 90°}$$

$$H \approx \frac{\infty\angle -90°}{\infty\angle -90°} \approx 1\angle 0°$$

pass band

At high frequencies: $\omega \approx \infty$



Voltage division:

$$H \approx \frac{0\angle -90°}{0\angle -90° + R + \infty\angle 90°}$$

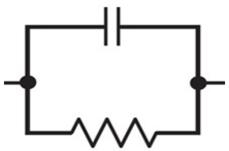
$$H \approx \frac{0\angle -90°}{\infty\angle 90°} \approx 0\angle -180°$$

stop band

The magnitude indicates the pass or stop band

This is a second order low pass filter

Parallel RC combination at low and high frequencies

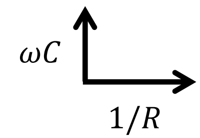


In the case of parallel combinations it is easier to add the admittances:

Admittance: $Y(\omega) = \dfrac{1}{Z(\omega)}$

$Y_c(\omega) = j\omega C$

$Y_R(\omega) = \dfrac{1}{R}$



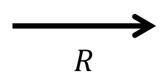
Admittance phasor diagram

$$Y_{eq}(\omega) = \frac{1}{R} + j\omega C$$

If you are using admittances make sure you write this next to your diagram otherwise it gets too confusing !!!

At very low frequencies: $\omega \approx 0$

$Y \approx \dfrac{1}{R} + j0 \approx \dfrac{1}{R}$ $\qquad Z \approx R$



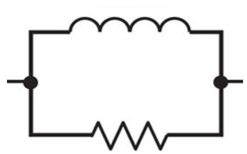
$\approx$ resistor

At very high frequencies: $\omega \approx \infty$

$Y \approx \dfrac{1}{R} + j\infty \approx j\infty$ $\qquad Z \approx \dfrac{1}{j\infty} \approx -j0 \approx 0\angle -90°$

$\approx$ short circuit with a $-90°$ phase shift

Parallel RL combination at low and high frequencies

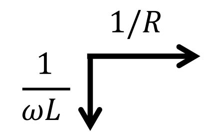


In case of the parallel combinations it is easier to add the admittances:

Admittance: $Y(\omega) = \dfrac{1}{Z(\omega)}$

$$Y_L(\omega) = \frac{1}{j\omega L} = \frac{-j}{\omega L}$$

$$Y_R(\omega) = \frac{1}{R}$$



Admittance phasor diagram

$$Y_{eq}(\omega) = \frac{1}{R} - j\frac{1}{\omega L}$$

If you are using admittances make sure you write this next to your diagram otherwise it gets too confusing !!!

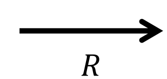
At very low frequencies: $\omega \approx 0$

$$Y \approx \frac{1}{R} - j\frac{1}{0} \approx \infty\angle -90^\circ \qquad Z \approx \frac{1}{\infty\angle -90^\circ} \approx 0\angle 90^\circ$$

≈ 0 $\approx$ short circuit with a 90° phase shift

At very high frequencies: $\omega \approx \infty$

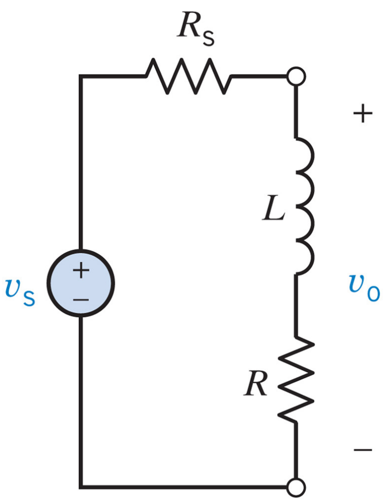
$$Y \approx \frac{1}{R} - j0 \approx \frac{1}{R} \qquad Z \approx R$$

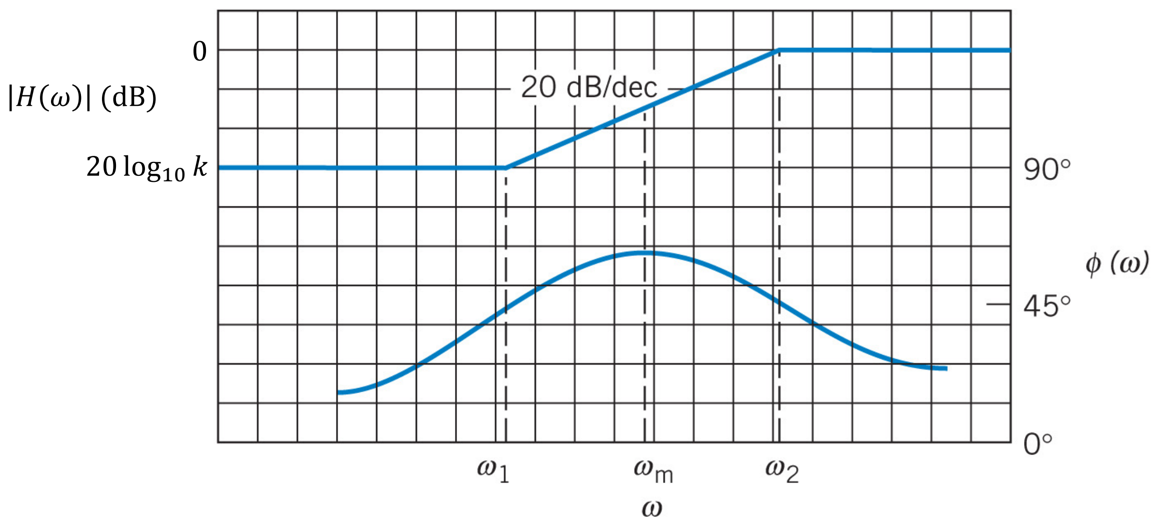


$\approx$ resistor

First order circuits with two corner frequencies

Dorf Section 13.3



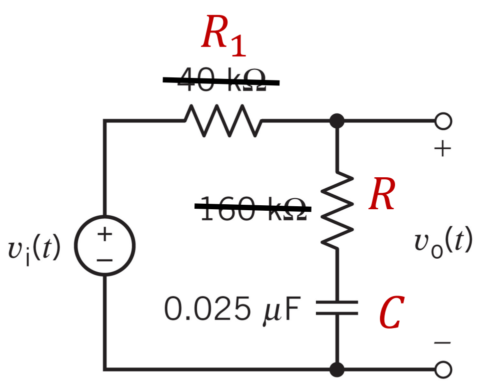


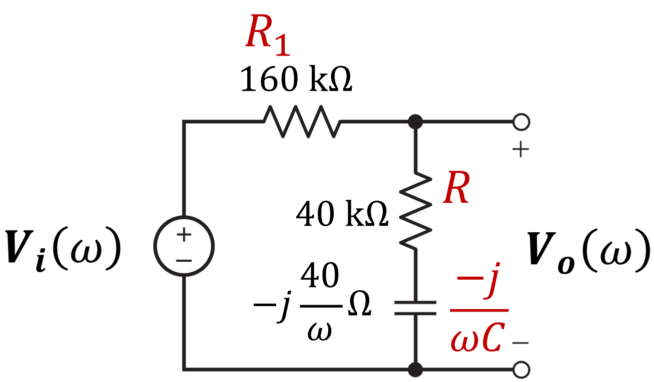
$$H(\omega) = k \cdot \frac{\left(1 + j\dfrac{\omega}{\omega_1}\right)}{\left(1 + j\dfrac{\omega}{\omega_2}\right)}$$

$$\mathbf{H} = \left(\frac{R}{R + R_s}\right) \frac{1 + j\dfrac{\omega L}{R}}{1 + j\dfrac{\omega L}{R + R_s}}$$

Problem 13.2-2 (Modified) (end of chapter 13)

Plot the network function





$$R_1 = 160\ \text{k}\Omega$$

$$R = 40\ \text{k}\Omega$$

$$\frac{-j}{\omega C} = -j\,\frac{40}{\omega}$$

Problem 13.2-2

Plot the network function

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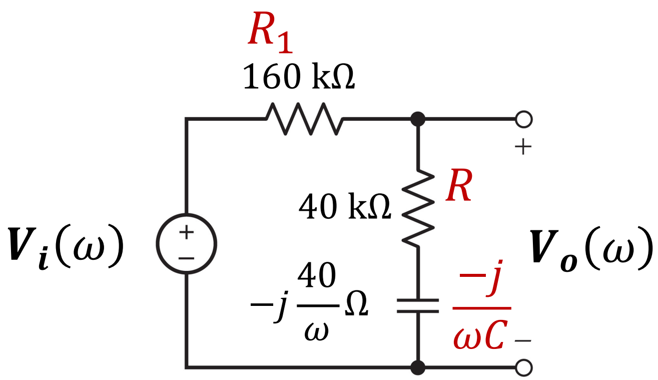


$$\boldsymbol{H}(\omega) = \frac{\boldsymbol{V_o}(\omega)}{\boldsymbol{V_i}(\omega)} = \frac{R + \frac{1}{j\omega C}}{R_1 + R + \frac{1}{j\omega C}}$$

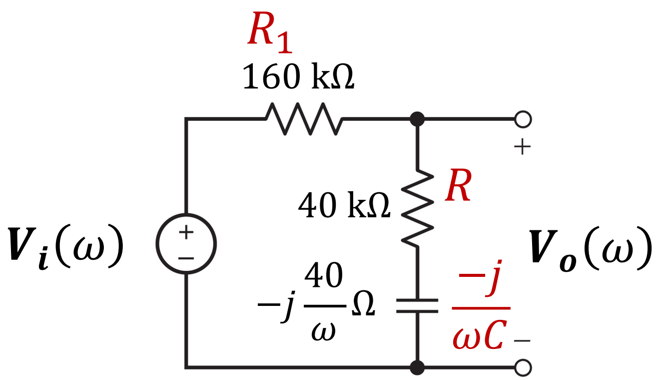
Standard form: $$\boldsymbol{H}(\omega) = \frac{1 + j\omega RC}{1 + j\omega(R_1 + R)C}$$

Extreme frequencies: $\boldsymbol{H}(\approx 0) = 1$ $\quad \boldsymbol{H}(\approx \infty) = \frac{R}{R_1 + R} = 0.2$

Two corner frequencies: $\omega_1 = \frac{1}{(R_1 + R)C}$ $\quad \omega_2 = \frac{1}{RC}$

Problem 13.2-2

Plot the network function



Standard form

$$\boldsymbol{H}(\omega) = \frac{1 + j\omega RC}{1 + j\omega(R_1 + R)C}$$

Corner frequencies:

$$\omega_1 = \frac{1}{(R_1 + R)C} = \frac{1}{200\times10^3\times0.025\times10^{-6}} = 200 \text{ rad/s}$$

$$\omega_2 = \frac{1}{RC} = \frac{1}{40\times10^3\times0.025\times10^{-6}} = 1000 \text{ rad/s}$$

The network function can be simplified to:

$$\boldsymbol{H}(\omega) = \frac{1 + j\dfrac{\omega}{1000}}{1 + j\dfrac{\omega}{200}}$$

Problem 13.2-2

Plot network function

$$\boldsymbol{H}(\omega) = \frac{1 + j\dfrac{\omega}{1000}}{1 + j\dfrac{\omega}{200}}$$

Extreme frequencies: $\boldsymbol{H}(\approx 0) = 1$ $\boldsymbol{H}(\approx \infty) = 0.2$

Corner frequencies:

$$\boldsymbol{H}(200) = \frac{1 + j\dfrac{200}{1000}}{1 + j\dfrac{200}{200}} = \frac{1 + j0.2}{1 + j} = \frac{1.02\angle 11.3°}{\sqrt{2}\angle 45°} = 0.72\angle - 33.7°$$

$$\boldsymbol{H}(1000) = \frac{1 + j\dfrac{1000}{1000}}{1 + j\dfrac{1000}{200}} = \frac{1 + j}{1 + j5} = \frac{\sqrt{2}\angle 45°}{5.1\angle 78.7°} = 0.28\angle - 33.7°$$

Problem 13.2-2

Plot network function

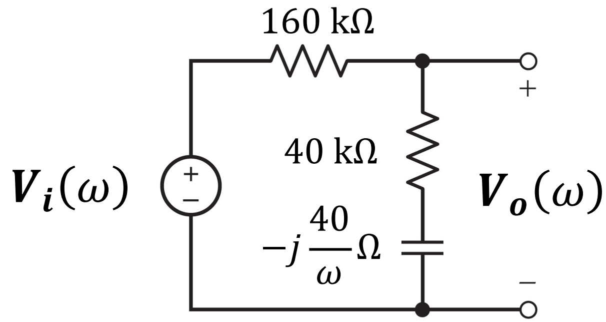
$$\boldsymbol{H}(\omega) = \frac{1 + j\dfrac{\omega}{1000}}{1 + j\dfrac{\omega}{200}}$$

We can also define a new frequency:

Median frequency: $\sqrt{\omega_1\omega_2} = \sqrt{200}\sqrt{1000} = 447.2 \text{ rad/s}$

$$\boldsymbol{H}(447.2) = \frac{1 + j\sqrt{0.2}}{1 + j\sqrt{5}} = 0.45\angle - 41.8°$$

Bode Plot



$$\boldsymbol{H}(\omega) = \frac{1 + j\frac{\omega}{1000}}{1 + j\frac{\omega}{200}}$$

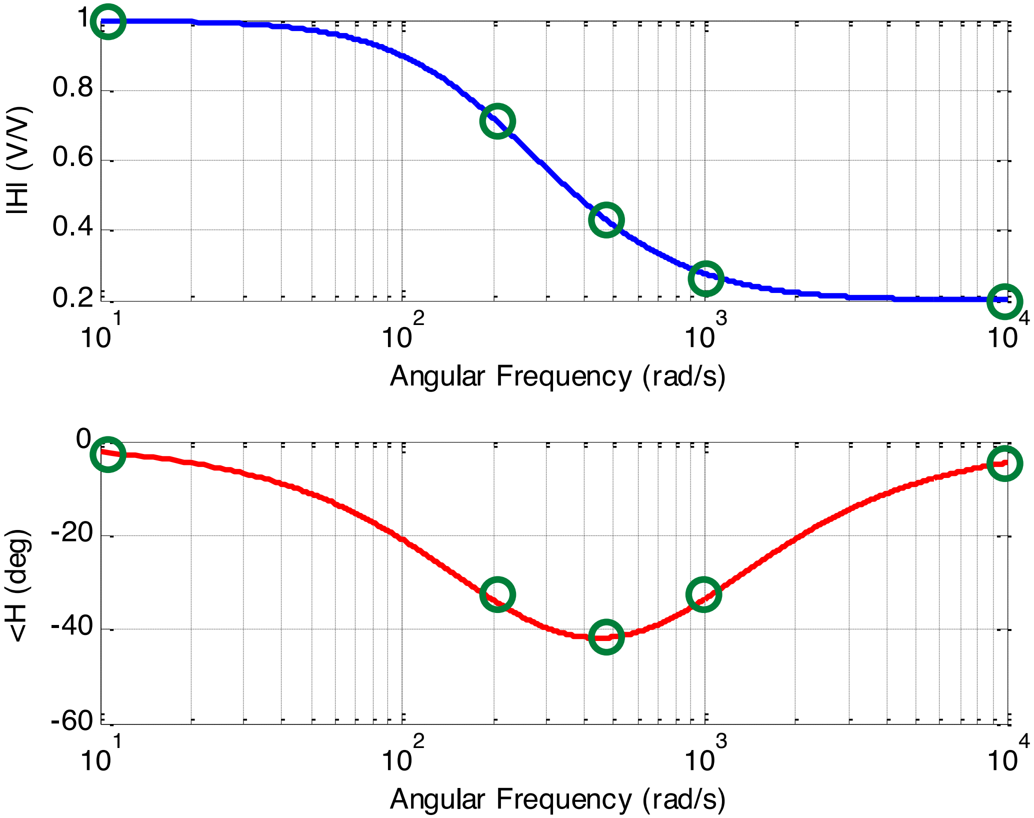
$$\boldsymbol{H}(\approx 0) = 1$$

$$\boldsymbol{H}(200) = 0.72\angle - 33.7°$$

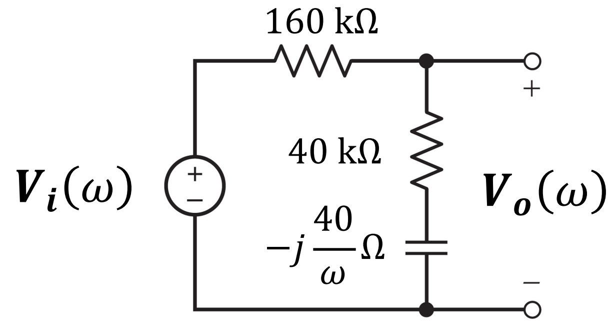
$$\boldsymbol{H}(447.2) = 0.45\angle - 41.8°$$

$$\boldsymbol{H}(1000) = 0.28\angle - 33.7°$$

$$\boldsymbol{H}(\approx \infty) = 0.2$$



Bode Plot on the dB scale



$$H(\omega) = \frac{1 + j\frac{\omega}{1000}}{1 + j\frac{\omega}{200}}$$

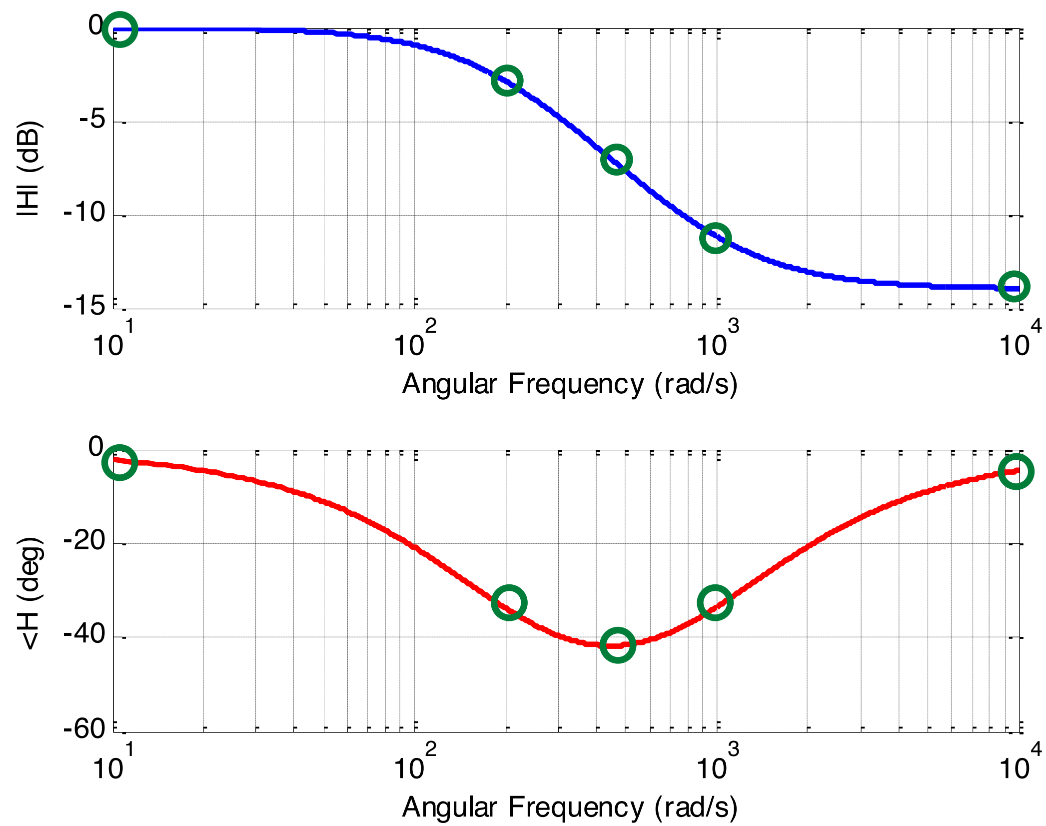
$$|H(\approx 0)| = 20\log(1) = 0 \text{ dB}$$

$$|H(200)| = 20\log(0.72) = -2.84 \text{ dB}$$

$$|H(447.2)| = 20\log(0.45) = -7 \text{ dB}$$

$$|H(1000)| = 20\log(0.28) = -11.14 \text{ dB}$$

$$|H(\approx \infty)| = 20\log(0.2) = -14 \text{ dB}$$



Second order filters

Involve two energy storage elements

Resonance

Resonance in ac circuits

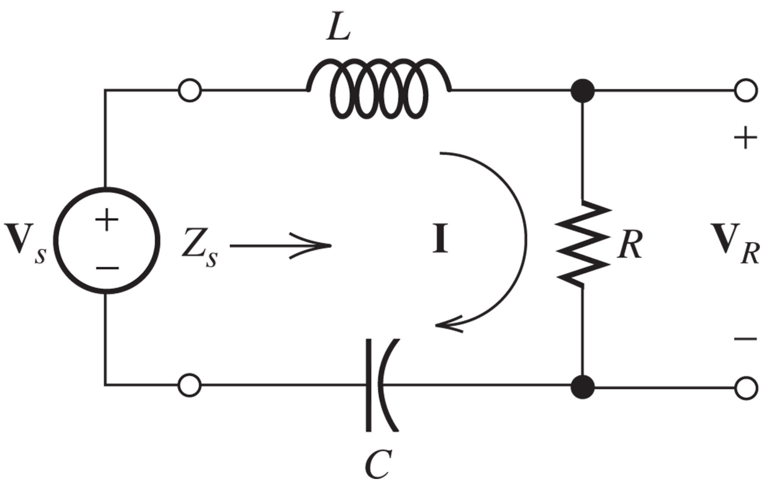
In ac circuits resonances happen when values of the capacitive and inductive reactances are equal in value and cancel each other.

This happens at a specific frequency called the resonant frequency f_0 (Hz) or ω_0 (rad/s)

Series resonance

Series resonance

The total impedance is given by: $Z_s(\omega) = j\omega L + R - \frac{j}{\omega C}$



The reactances cancel each other (making the circuit purely resistive at the **resonant frequency**:

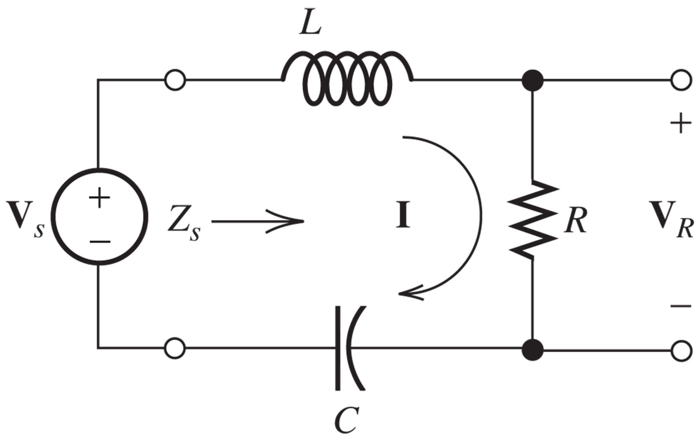
$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{CL}$$

Hence the resonant frequency is given by: $\omega_0 = \frac{1}{\sqrt{LC}}$

This is the same natural undamped resonant frequency for a series second order circuit in the time domain (transient response)

Bandwidth and quality factor, Q



Recall:

$$Z_s(\omega) = j\omega L + R - \frac{j}{\omega C}$$

Reactances cancel each other at a specific frequency (called the resonant frequency)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The quality factor of a **series circuit** in the frequency domain is defined as the ratio of the reactance of the inductor at the resonant frequency to the resistance:

$$Q_s = \frac{\omega_0 L}{R} = \frac{\omega_0}{B_\omega}$$

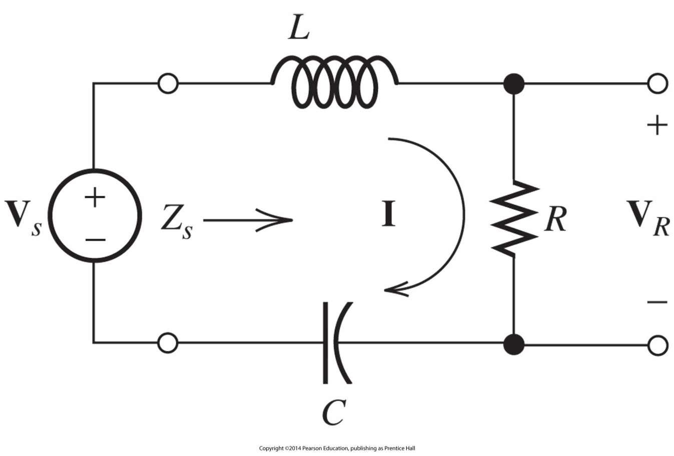
The **bandwidth** (in rad/s) in the frequency domain is defined as the resonant frequency divided by the quality factor

$$B_\omega = \frac{\omega_0}{Q_s} = \frac{R}{L}$$

$$B = \frac{f_0}{Q_s} = \frac{R}{2\pi L} \text{ in Hambley}$$

There is no direct correspondence of the quality factor to the time domain

Series impedance



The total impedance: $Z_s(\omega) = j\omega L + R - \dfrac{j}{\omega C}$

Knowing the resonant frequency and Q factor relations:

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q_s = \frac{\omega_0 L}{R}$$

We get the impedance in terms of Q factor and resonant frequency:

$$Z_s(\omega) = R\left[1 + jQ_s\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

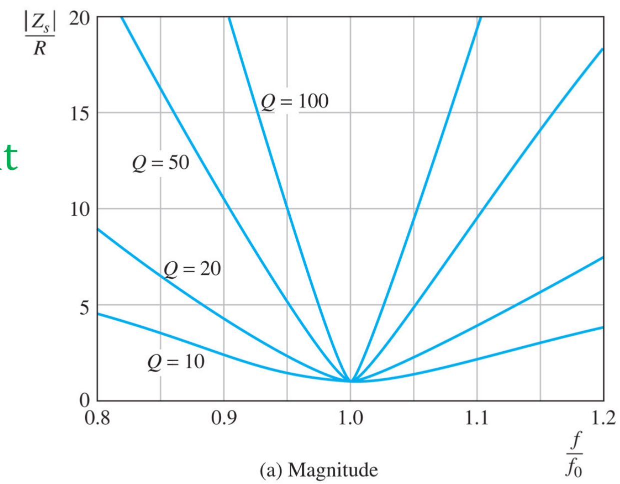
What values do you expect $Z_s(\omega)$ be equal to at very low and very high frequencies and at the resonant frequency?

The band pass filter

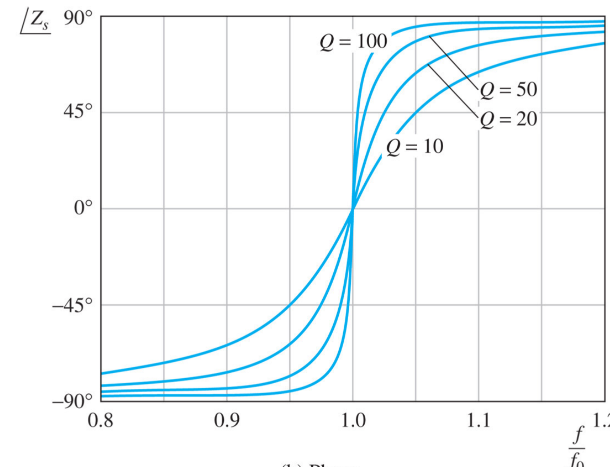
$$Z_s(\omega) = R\left[1 + jQ_s\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

Notice the impedance magnitude is minimum at the resonant frequency. As the quality factor increases the minimum becomes sharper

Normalised impedance plots with respect to frequency



(a) Magnitude

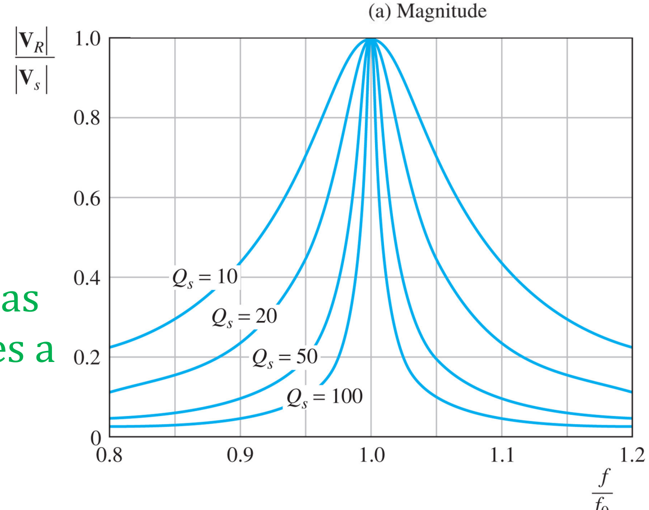


(b) Phase

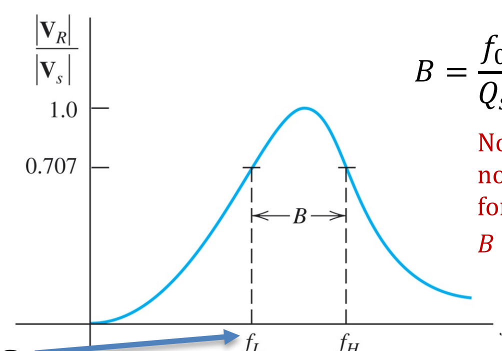
Voltage network function

$$H(\omega) = \frac{V_R}{V_s} = \frac{R}{Z_s(\omega)}$$

Taking the voltage across the resistor as the output produces a bandpass filter!



Bandwidth is the difference between the half-power frequencies (in Hz)



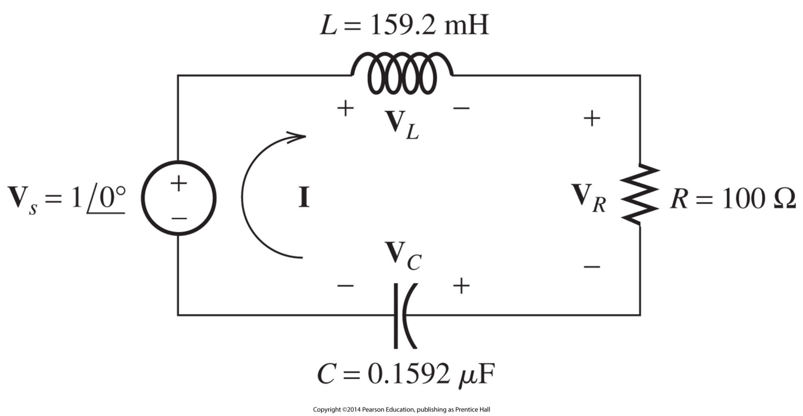
$$B = \frac{f_0}{Q_s} \text{ (Hz)}$$

Notice this definition is not always consistent for example in Dorf $B = \frac{\omega_0}{Q_s}$ (rad/s)

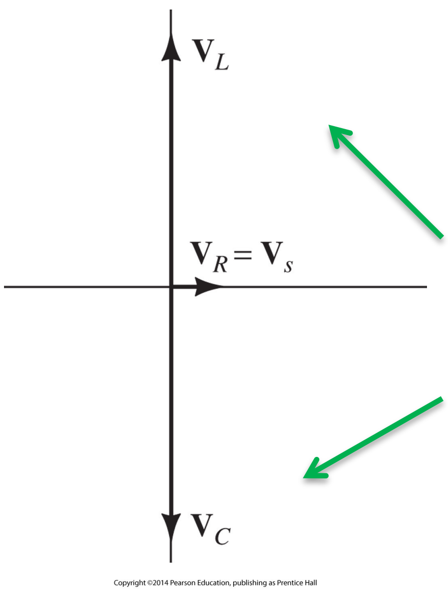
Half power frequencies

Series resonance example

Example 6.5 in Hambley



Phasor diagram of the voltages at the resonant frequency:



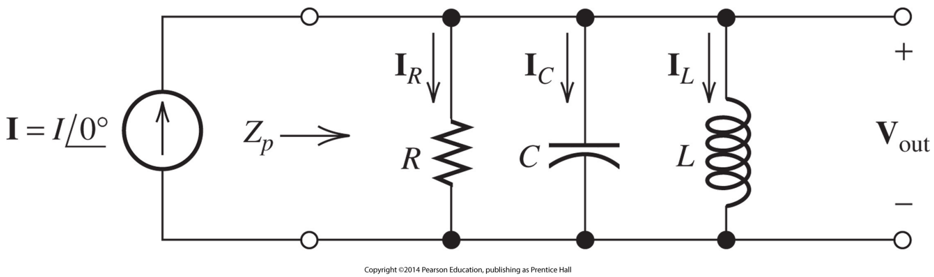
V_L and V_C cancel each other if $f = f_0$ then $V_R = V_s$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1 \text{ kHz} \qquad Q_s = \frac{2\pi f_0 L}{R} = 10$$

$$B = \frac{f_0}{Q_s} = 100 \text{ Hz}$$

Parallel resonance

Parallel resonance



If you use admittances, you can apply the previous method (of adding phasors) for guessing the low and high frequency approximations

If you don't remember how to use admittances it is perfectly OK to use impedances. But in that case you should note that V_{out} is common to all elements and the currents can simply be added.

The admittance is given by: $$Y_p(\omega) = \frac{1}{R} + j\omega C - \frac{j}{\omega L}$$

The impedance is given by: $$Z_p(\omega) = \frac{1}{\frac{1}{R} + j\omega C - \frac{j}{\omega L}}$$

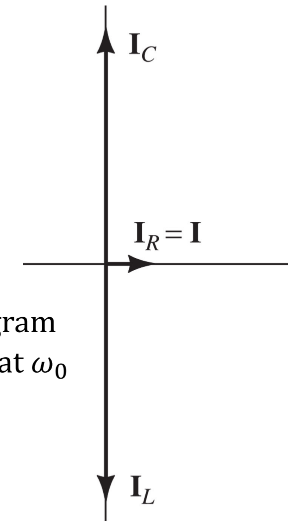
Remembering that: $$Z_p(\omega) = \frac{1}{Y_p(\omega)}$$

Parallel resonance

Again, the resonant frequency is the frequency when the circuit is purely resistive:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Same as for the series circuit



Phasor diagram of currents at ω_0

The quality factor of a **parallel circuit** in the frequency domain is defined as the ratio of the resistance to the reactance of the inductor at the resonant frequency :

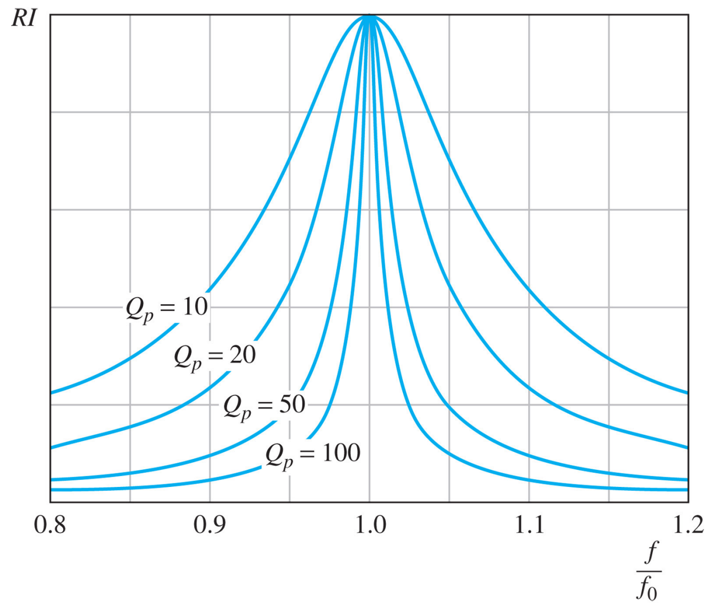
$$Q_p = \frac{R}{\omega_0 L}$$

Hence: $$Q_p = \frac{1}{Q_s}$$

The impedance can now be expressed as:

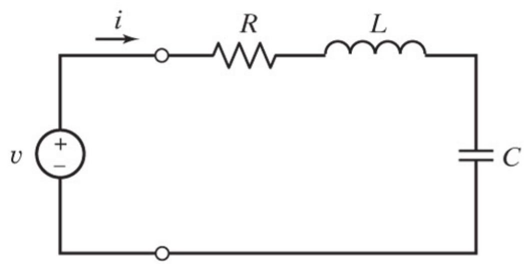
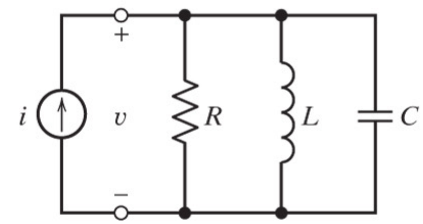
$$Z_p(\omega) = \frac{R}{1 + jQ_p\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Output voltage $V_{out}(f)$ for the parallel resonant circuit

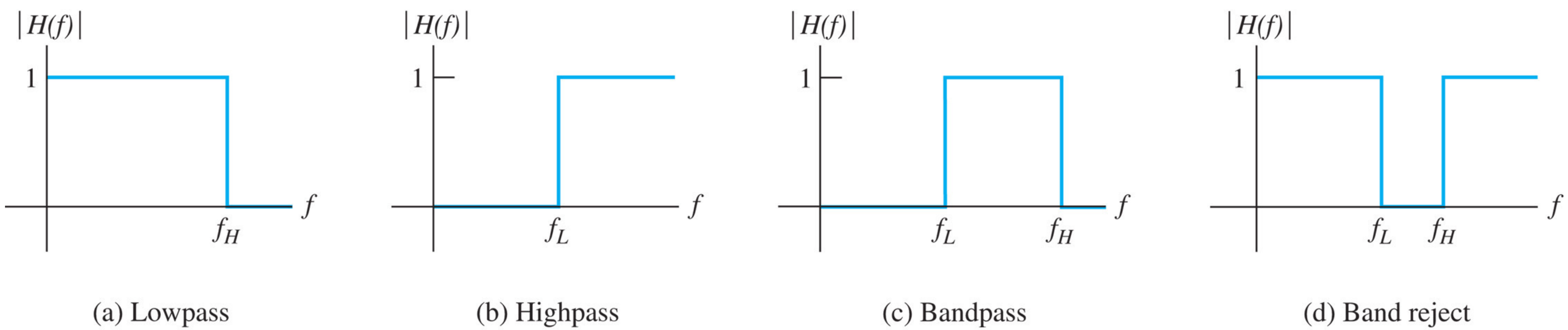


$$B = \frac{f_0}{Q_p} \text{ (Hz)}$$

Summary

	SERIES RESONANT CIRCUIT	PARALLEL RESONANT CIRCUIT
Circuit		
Network function	$\mathbf{Y} = \dfrac{k}{1 + jQ\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)}$	$\mathbf{Z} = \dfrac{k}{1 + jQ\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)}$
Resonant frequency	$\omega_0 = \dfrac{1}{\sqrt{LC}}$	$\omega_0 = \dfrac{1}{\sqrt{LC}}$
Maximum magnitude	$k = \dfrac{1}{R}$	$k = R$
Quality factor	$Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$	$Q = R\sqrt{\dfrac{C}{L}}$
Bandwidth	$BW = \dfrac{R}{L}$	$BW = \dfrac{1}{RC}$

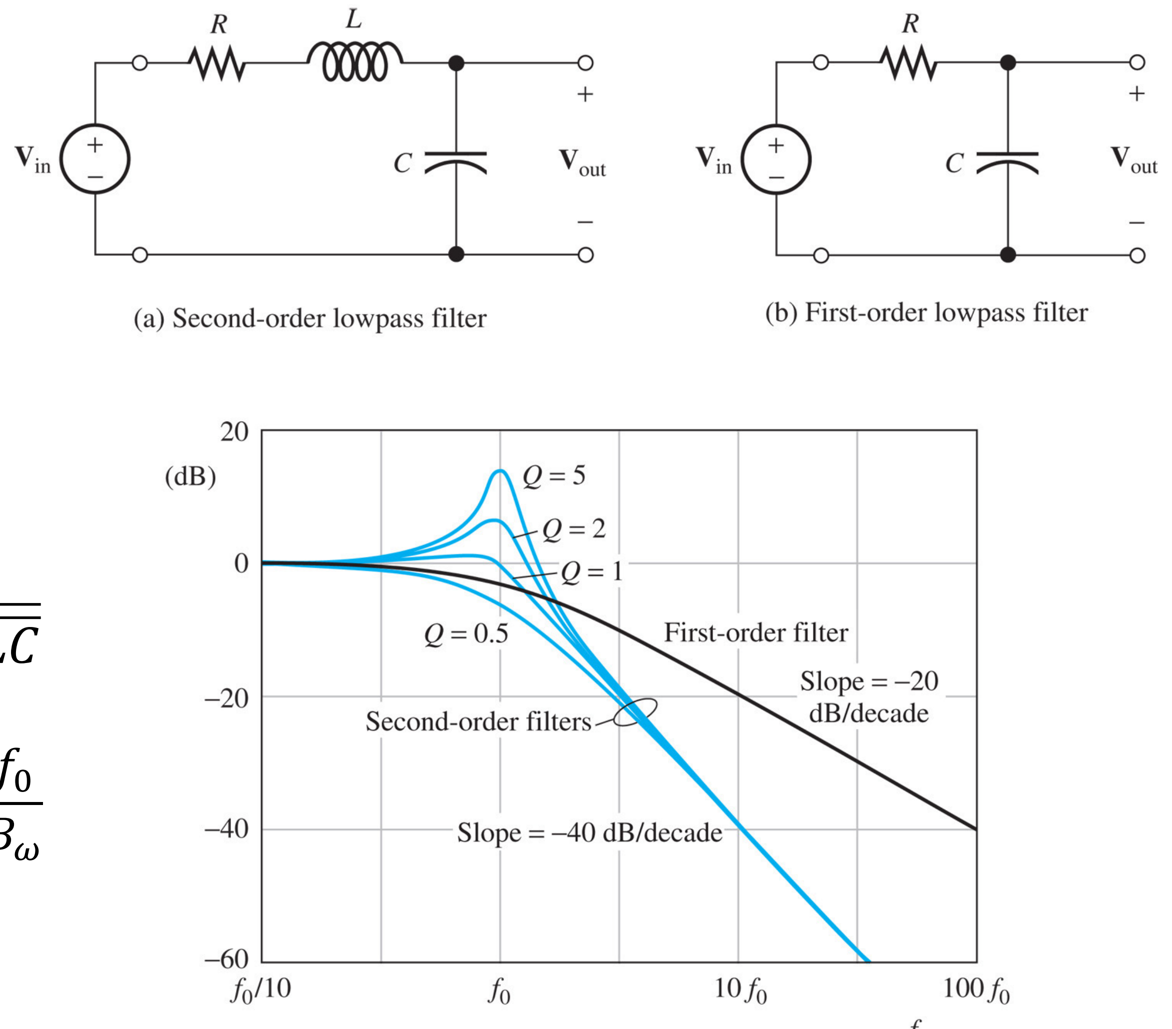
Types of ideal filters



The real filter circuits using resistors, inductors and capacitors are only approximations of these ideal filters

You can make (approximately) any of these types of filters using one resistor, one inductor and/or one capacitor only

Second order low pass filter

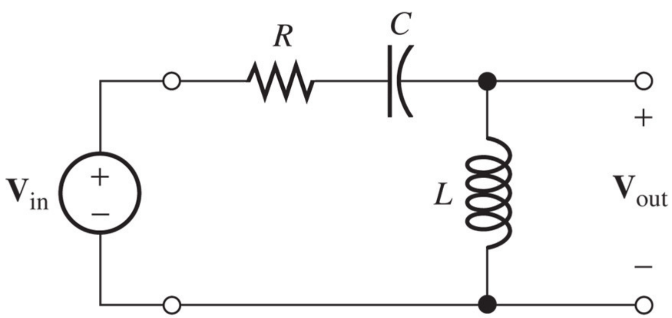


$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

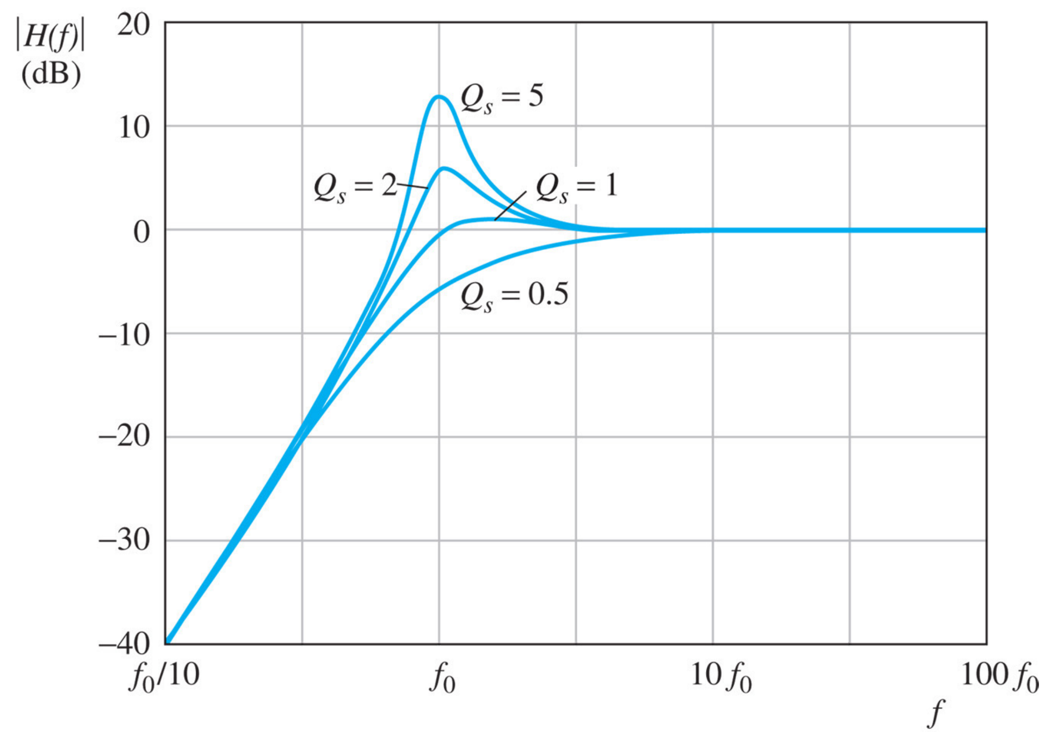
$$Q_s = \frac{f_0 L}{R} = \frac{f_0}{B_\omega}$$

(c) Transfer-function magnitudes

Second order high pass filter

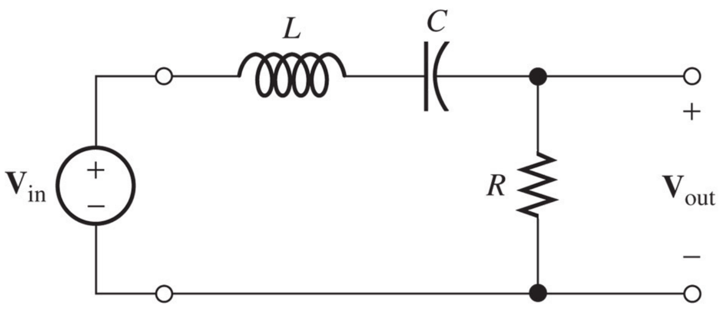


(a) Circuit diagram

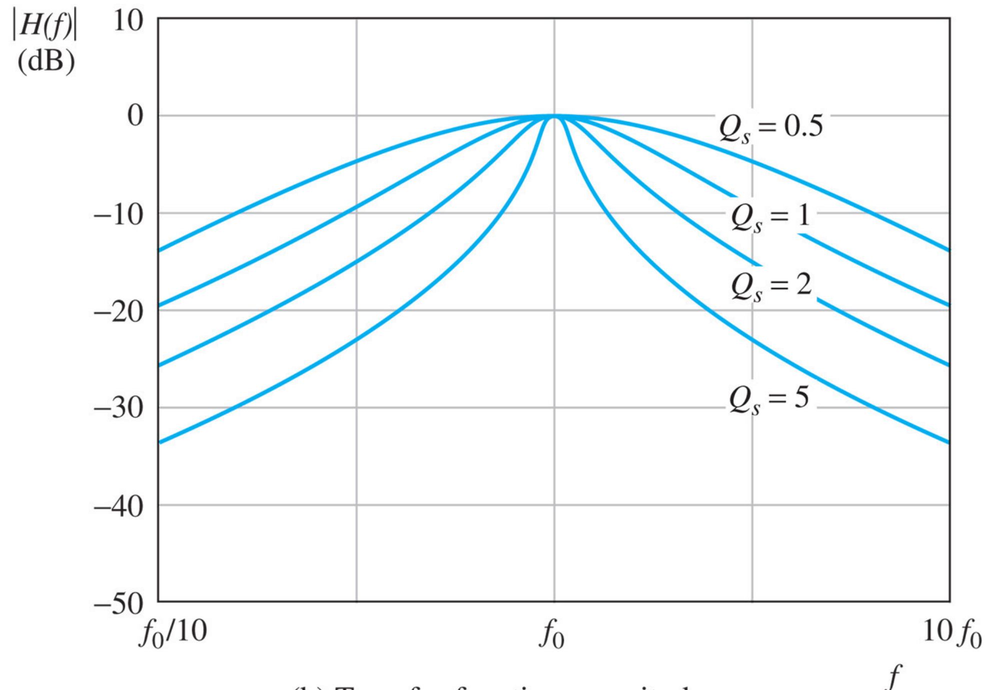


(b) Transfer-function magnitude

Second order band pass filter

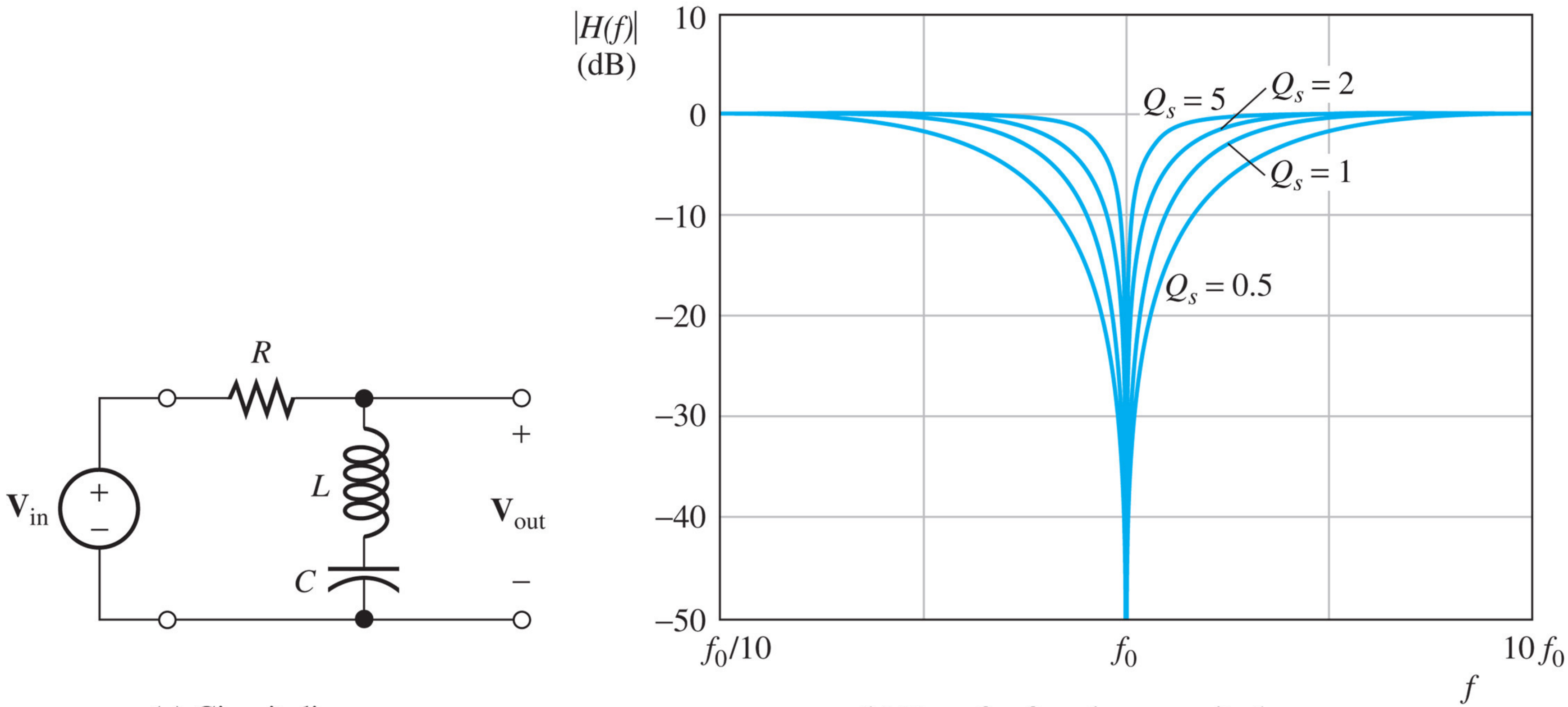


(a) Circuit diagram



(b) Transfer-function magnitude

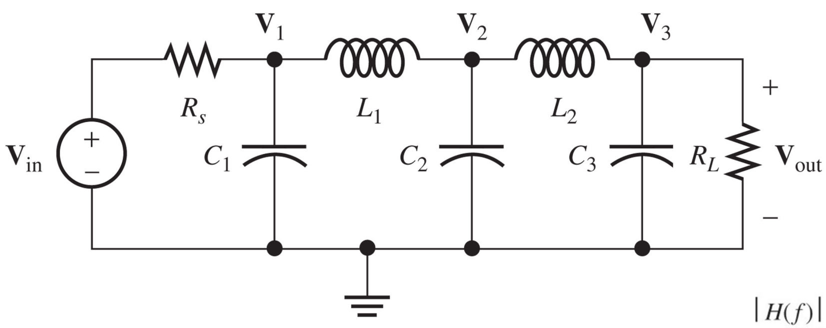
Second order band stop filter



(a) Circuit diagram (b) Transfer-function magnitude

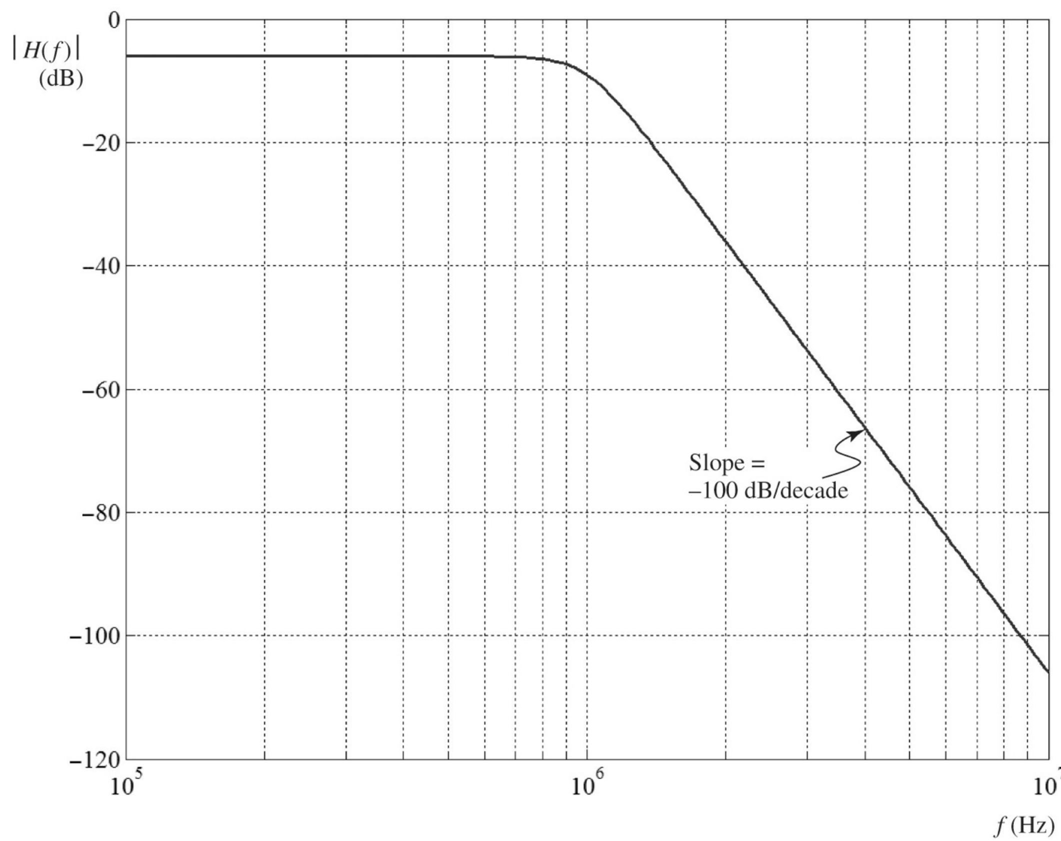
Fifth order Butterworth lowpass filter

Example 6.9 in Hambley



$R_s = R_L = 50\ \Omega$ $\quad$ $L_1 = L_2 = 12.88\ \mu H$

$C_1 = C_3 = 1967\ pF$ $\quad$ $C_2 = 6366\ pF$



Measurements in the frequency domain

The oscilloscope measures the signal voltages in the time domain

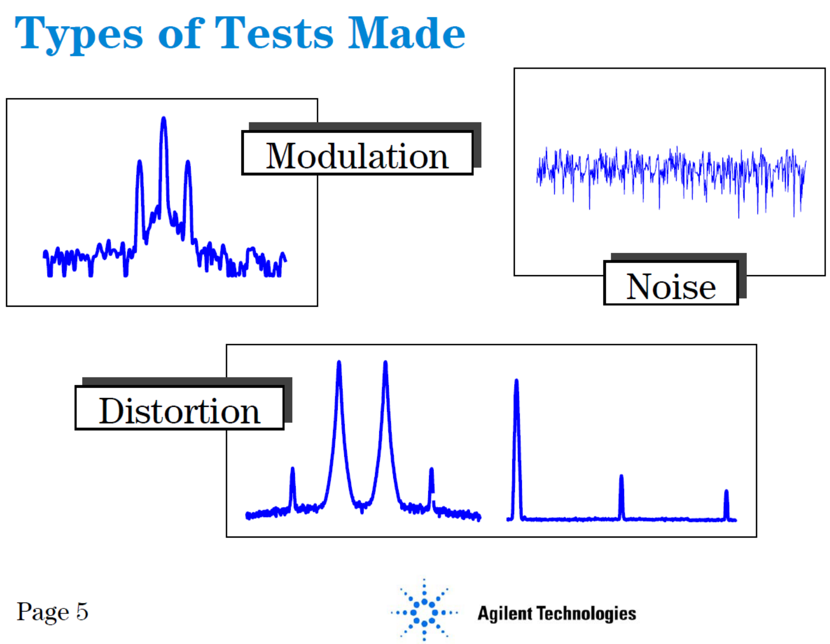
- The oscilloscope is good for displaying and measuring periodic voltages with respect to time.
- The periodic voltages can have any shape.
- Multiple voltage waveforms can be measured at once (on separate channels) and relative time delays or phase can be measured.

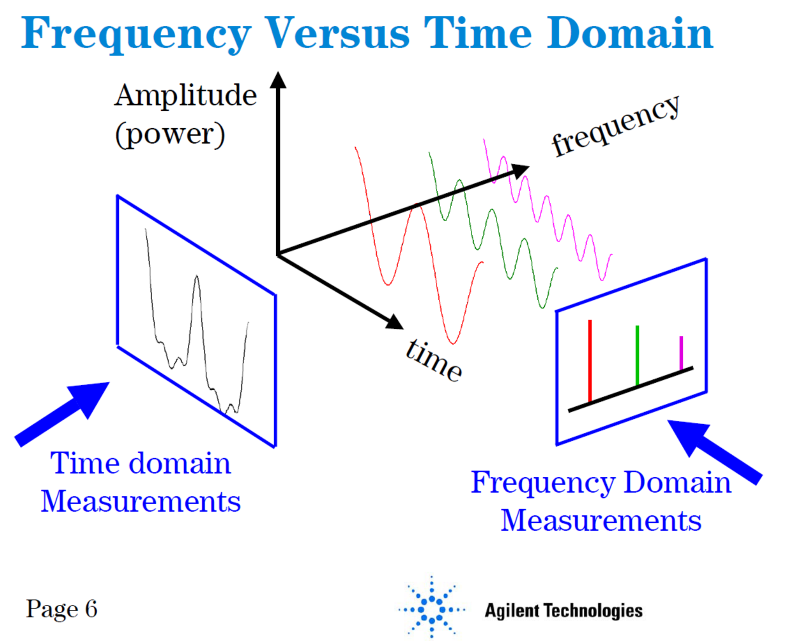
The spectrum analyser measures the signals with respect to the frequency

- If a signal has multiple frequency components it is easy to display and measure that with a spectrum analyser.
- Spectrum analyser measures only the amplitudes at each frequency.
- Spectrum analysers usually have 50 Ω input impedances.

Spectrum analyser basics

http://www.keysight.com/.../02-18-03-B2B-RF-SpectrumAnalysis-Thomas-Holmes-Hightower-839.pdf





Basics of amplitude and distortion measurements on a scope and spectrum analyser
https://www.youtube.com/watch?v=XHA4c82msqo