Principles of Magnetics and Transformers

ELEC2005 Electrical and Electronic Systems - Week 10

Dr. Leonardo Callegaro

School of Engineering
Faculty of Science and Engineering

MACQUARIE
University

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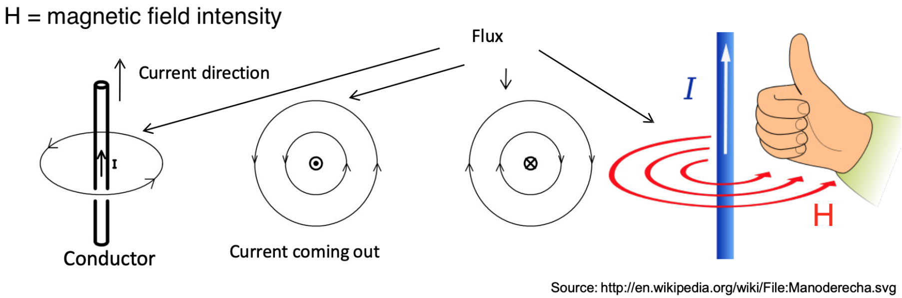
Right hand thumb rule

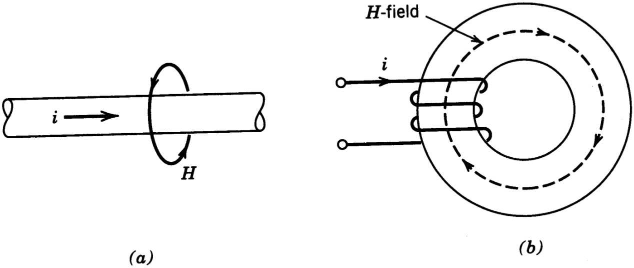


If a conductor is held with the right hand with the thumb indicating the current direction, then the fingertips will indicate the direction of the magnetic field intensity $\mathbf{H}$

Also consult: www.khanacademy.org (click here)

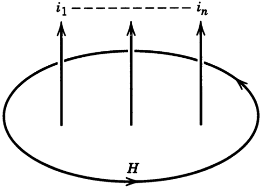
Right-hand thumb rule (cont'd.)

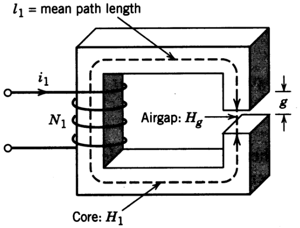
Direction of $\mathbf{H}$ around wire (a) and inside magnetic core (b) depends on the current direction and the way the coil is wound



Basic magnetic relations

- Ampere's law: since a current produces and magnetic field, the line integral of magnetic field intensity $\mathbf{H}$ equals total enclosed current



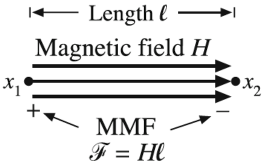


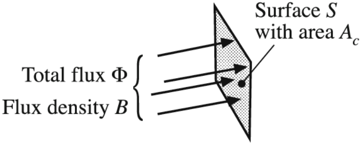
$$\underbrace{\oint \mathbf{H}\cdot\mathbf{d}\boldsymbol{\ell}}_{\text{line integral of } \mathbf{H}} = \sum i \qquad \sum_k H_k \ell_k = \sum_m N_m i_m$$

$$\underbrace{H_1\ell_1 + H_\mathrm{g}\ell_\mathrm{g}}_{\mathscr{F} \text{ Magneto Motive Force (MMF)}} = N_1 i_1$$

Basic magnetic relations (cont'd 1)

Magnetic field intensity $\mathbf{H}$, field density $\mathbf{B}$, flux Φ

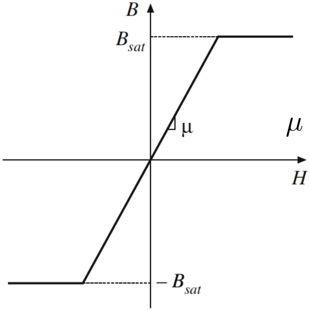




- Flux density (aka magnetic induction) or $\mathbf{B}$-field:

$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mu_r\mathbf{H}$$

μ magnetic permeability (material)
μ_0 air permeability
μ_r relative permeability

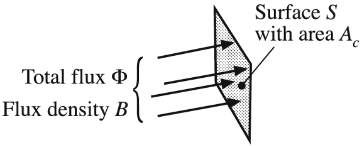


$$\mu = \frac{\Delta B}{\Delta H}$$

Basic magnetic relationships (cont'd 2)

Magnetic field intensity $\mathbf{H}$, field density $\mathbf{B}$, flux Φ

- Flux Φ, is the amount of $\mathbf{B}$ passing through area A_c



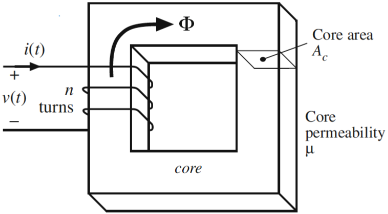
$$\Phi = \iint_{A_c} \mathbf{B} \cdot \mathbf{dA}$$

- Faraday's law: $v(t)$ induced by *change in flux* $\Phi(t)$
  - in one turn:

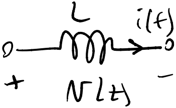
$$v_{\text{turn}}(t) = \frac{d\Phi}{dt}$$

  - in n-turns:

$$v(t) = n\frac{d\Phi}{dt}$$



Basic magnetic relationships (cont'd 3)

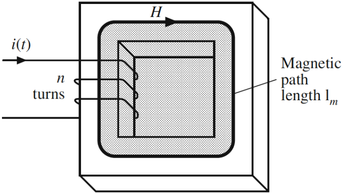


$v(t) = L\frac{di}{dt}$

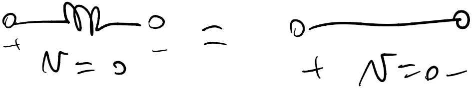
Inductance definition

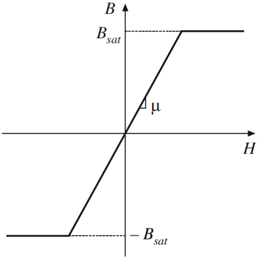
$$v(t) = n\frac{d\Phi}{dt} = nA_c\frac{dB}{dt} = nA_c\mu\frac{dH}{dt}$$
$$= \frac{n^2 A_c \mu}{\ell_m}\frac{di}{dt} = L\frac{di}{dt}$$

$$n\,i = H\,\ell_m \rightarrow H = \frac{n\,i}{\ell_m}$$



- Inductance, L, depends on
  - number of turns: n
  - geometry: A_c, ℓ_m
  - material: μ
- if the core is saturated $\frac{dB}{dt} = 0$, hence $v(t) = 0$ (inductance behaves like a *short circuit*)





Basic magnetic relations (cont'd 4)

Inductance definition

We have derived: $\boxed{v = L\dfrac{d\,i}{dt}}$ where L is a constant

$$\int v\,dt = \int L\frac{d\,i}{dt}\,dt \rightarrow \int v\,dt = L\,i$$

$$v = n\frac{d\,\Phi}{dt} \rightarrow \int v\,dt = n\Phi = \lambda$$

Inductance definition

$$\boxed{\lambda = L\,i}$$

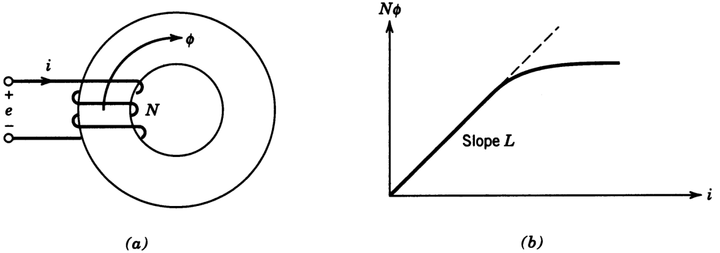


Figure 3-17 Self-inductance L.

Basic magnetic relations (cont'd 5)

Magnetic energy stored

- Magnetic energy stored in an inductor, in J (joules)

$$W = \frac{1}{2}Li^2$$

- is energy stored due to magnetic field inside a material (μ is a material-dependent constant)

$$W = \frac{1}{2}\frac{B^2}{\mu}\textit{Volume}$$

- magnetic energy density, in J/m^3

$$w = \frac{1}{2}\frac{B^2}{\mu}$$

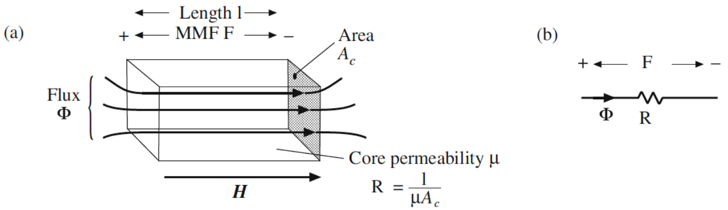
Magnetic units (International System)

$\mu_0, \mu \quad \left[\frac{H}{m}\right]$

Quantity	MKS
Core material equation	$B = \mu_0 \mu_r H$
B	Tesla
H	Ampere/meter
Φ	Weber

adapted from: Erickson (2020), Fundmanetals of Power Electronics, 3rd edn

Magnetic circuits



(Magneto-motive force) $\mathscr{F} = \mathscr{R}\,\Phi \;\leftrightarrow\;$ (Electro-motive force) $V = R\,i$

$$\mathscr{F} = H\,\ell$$

$$H = \frac{B}{\mu}$$

$$B = \frac{\Phi}{A_c}$$

$$\mathscr{F} = \mathscr{R}\,\Phi = \frac{\ell}{\mu A_c}\,\Phi$$

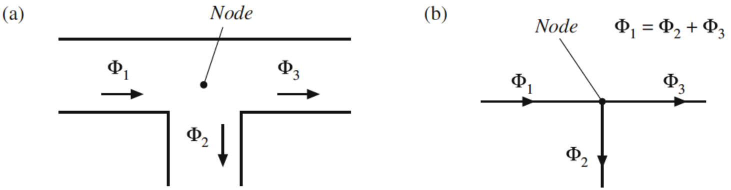
Remember

The concept of magnetic reluctance, $\mathscr{R}$, is similar to the electric resistance, R

elec. $R = \rho \cdot \frac{\ell}{A_c} = \frac{1}{\mu}\frac{\ell}{A_c}$ mag. $\mathscr{R} = \frac{\ell}{\mu A_c}$

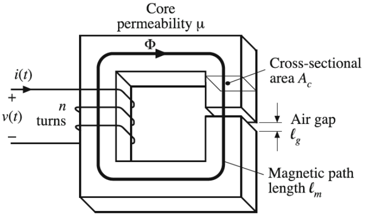
Magnetic circuits (cont.'d)

Kirchhoff's current law, for magnetic circuits, in a node: $\boxed{\sum_{\text{Node}} \Phi_{\mathrm{k}} = 0}$

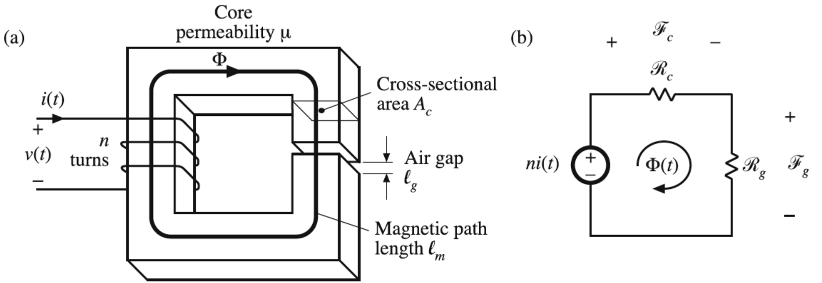


Kirchhoff's voltage law, for magnetic circuits

$$H_m \ell_m + H_{\mathrm{g}} \ell_{\mathrm{g}} = n\, i$$
$$\mathscr{F}_m + \mathscr{F}_g = n\, i$$
$$(\mathscr{R}_m + \mathscr{R}_g)\, \Phi = n\, i$$
$$\boxed{\sum_{\text{loop}} \mathscr{R}\, \Phi = \sum_{\text{loop}} n\, i}$$



Magnetic circuits (cont.'d 2)



$$\mathscr{R}_c = \frac{\ell_m}{\mu A_c}$$

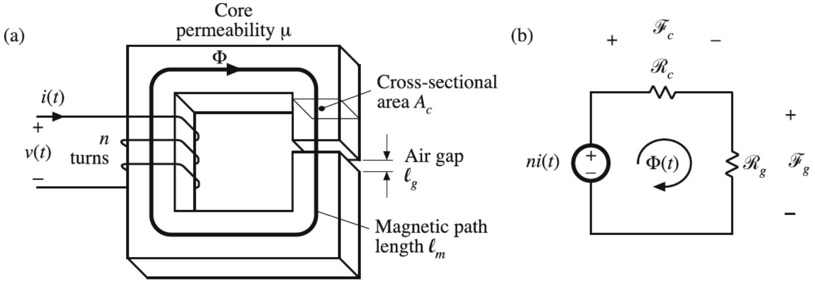
$$\mathscr{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

$$\sum_{\text{loop}} \mathscr{R}\,\Phi = \sum_{\text{loop}} n\,i$$

$$(\mathscr{R}_c + \mathscr{R}_g)\,\Phi = n\,i$$

Magnetic circuits (cont.'d 3)

Inductance and magnetic core reluctance



We can determine the inductance using the magnetic circuit relations

$$(\mathscr{R}_c + \mathscr{R}_g)\,\Phi = n\,i(t)$$

$$v(t) = n\frac{d\Phi(t)}{dt}$$

$$v(t) = L\frac{di(t)}{dt}$$

$$v(t) = \left(\frac{n^2}{\mathscr{R}_c + \mathscr{R}_g}\right)\frac{di(t)}{dt}$$

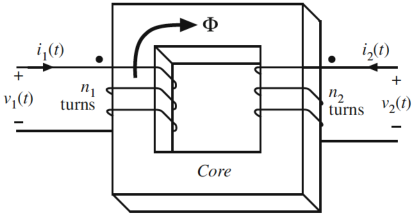
Electrical and magnetic circuits analogies

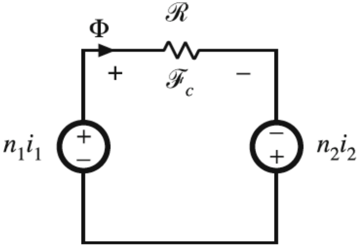
Table 3-3 Magnetic–Electrical Circuit Equation Analogy

Magnetic	*Electrical (dc)*
$\frac{Ni}{\phi} = \mathscr{R} = \frac{l}{\mu A}$	Ohm's law: $\frac{v}{i} = R = \frac{l}{A/\rho}$
$\phi \sum_k \mathscr{R}_k = \sum_m N_m i_m$	Kirchhoff's voltage law: $i \sum_k R_k = \sum_m v_m$
$\Sigma\, \phi_k = 0$	Kirchhoff's current law: $\sum_k i_k = 0$

Source: Mohan, Undeland, Robbins (2002) *Power Electronics: Converters, Applications, and Design*, 3rd Edition, Chapter 3

Transformer modelling





$$\boxed{\sum_{\text{loop}} \mathscr{R}\,\Phi = \sum_{\text{loop}} n\,i}$$

$$\boxed{H_m\,\ell_m = n_1\,i_1 + n_2\,i_2}$$

$$\mathscr{R}\,\Phi = n_1\,i_1 + n_2\,i_2$$

$$0 = n_1\,i_1 + n_2\,i_2$$

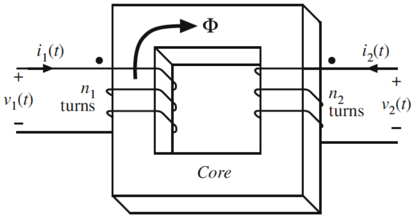
(ideal transformer $\mathscr{R} = 0$)

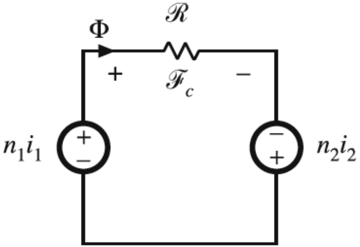
dot (•) convention (on transformer windings):

(1) current entering the • produces flux in the direction shown (right hand thumb rule)

(2) current entering/leaving the • in one winding, induces +ve/-ve voltage at the • of the other winding

Ideal transformer $\mathscr{R} = 0$

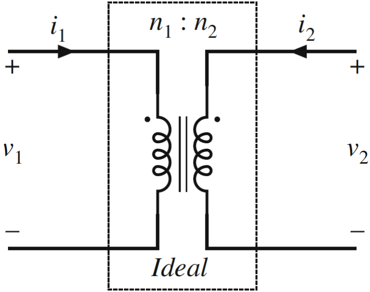




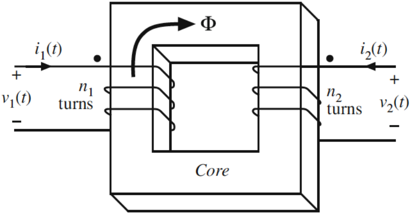
$$\boxed{0 = n_1\, i_1 + n_2\, i_2}$$

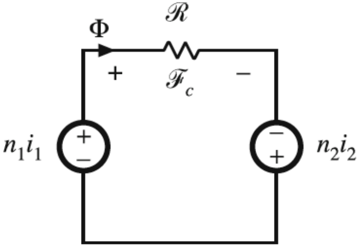
Faraday's law: $\begin{cases} v_1 = n_1 \dfrac{d\Phi}{dt} \\ v_2 = n_2 \dfrac{d\Phi}{dt} \end{cases}$

$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2} \rightarrow \boxed{\frac{v_1}{v_2} = \frac{n_1}{n_2}}$$



Actual transformer $\mathscr{R} > 0$, magnetizing inductance L_M



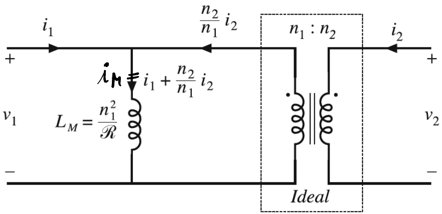


$$\boxed{\mathscr{R}\Phi = n_1\, i_1 + n_2\, i_2}$$

$$\Phi = \frac{n_1\, i_1 + n_2\, i_2}{\mathscr{R}}$$

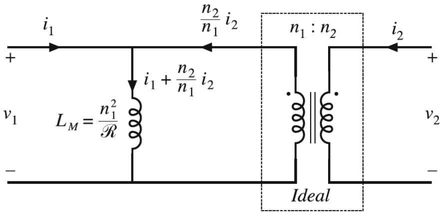
Faraday's law:

$$v_1 = n_1 \frac{d\Phi}{dt} = \underbrace{\frac{n_1^2}{\mathscr{R}}}_{L_M} \frac{d}{dt} \underbrace{\left(i_1 + \frac{n_2}{n_1} i_2 \right)}_{i_M}$$



Actual transformer (cont'd)

$$v_1 = \underbrace{\frac{n_1^2}{\mathscr{R}}}_{L_M} \frac{d}{dt} \underbrace{\left(i_1 + \frac{n_2}{n_1} i_2\right)}_{i_M}$$

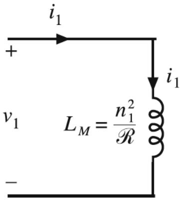


- dc input voltage
  $v_1 = V_{\text{dc}} = \text{constant}$

$$\frac{d}{dt} \underbrace{\left(i_1 + \frac{n_2}{n_1} i_2\right)}_{i_M} = \text{constant}$$

i_M grows at constant rate, i.e. the primary sees a short-circuit!

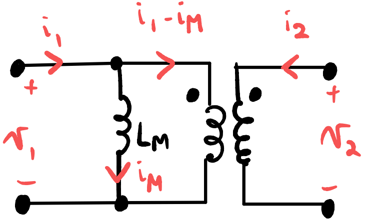
- Secondary disconnected $(i_2 = 0)$



Actual transformer (cont'd 2)

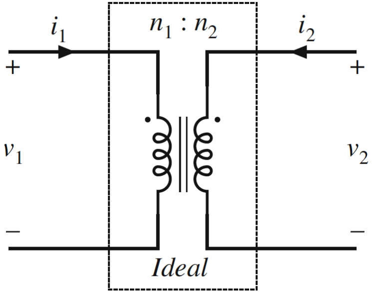
key takeaways:

- transformer changes voltage (and current) level between two circuits
- ideal transformer ($\mathscr{R} = 0$): $n_1\, i_1 + n_2\, i_2 = 0$
- actual transformer ($\mathscr{R} > 0$): $n_1\, i_1 + n_2\, i_2 = \mathscr{R}\,\Phi \rightarrow \mathscr{R}\,\Phi = n_1\, i_M$
  - $n_1\, i_M$ is the magneto motive force establishing flux Φ in the core
  - $n_1\, i_M$ is the amount by which $n_1\, i_1 + n_2\, i_2 \neq 0$
- alternative way to represent the actual transformer, based on $n_1\, i_1 + n_2\, i_2 = n_1\, i_M \rightarrow n_1\,(i_1 - i_M) + n_2\, i_2 = 0$



- often, it is posed: $i_1 - i_M = i_1'$

Ideal transformer in AC circuits



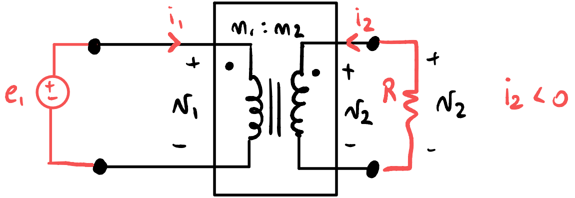
$$\bar{V}_1 = n_1 \frac{d\bar{\Phi}}{dt} = j\,\omega\,n_1\,\bar{\Phi}$$

$$\bar{V}_2 = n_2 \frac{d\bar{\Phi}}{dt} = j\,\omega\,n_2\,\bar{\Phi}$$

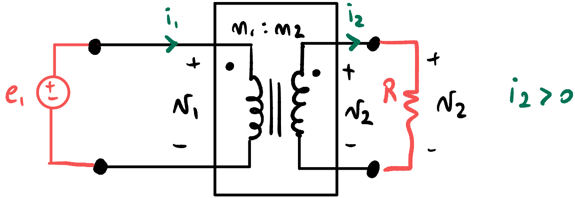
$$\boxed{\frac{\bar{V}_1}{\bar{V}_2} = \frac{n_1}{n_2}} \qquad \frac{n_1}{n_2} = a_t\text{: turns ratio}$$

Ideal transformer in AC circuits (cont'd 1)

Assuming power flow from left to right (from voltage source to R-load), then secondary current is +ve if the convention is as per the bottom figure



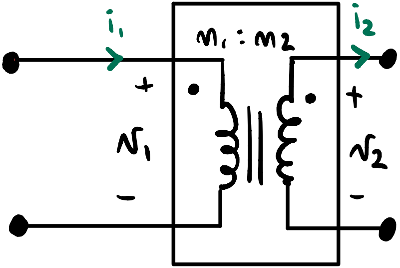
$$n_1 i_1 + n_2 i_2 = 0$$



$$n_1 i_1 + n_2 (-i_2) = 0$$
$$n_1 i_1 - n_2 i_2 = 0$$

dot convention → since i_1 enters primary •, the voltage induced in the secondary, v_2, is +ve at the •

Ideal transformer in AC circuits (cont'd 2)



$$V = n\frac{d\Phi}{dt} \rightarrow \boxed{\frac{\bar{V}_1}{\bar{V}_2} = \frac{n_1}{n_2} = a_t}$$

$$n_1\bar{I}_1 = n_2\bar{I}_2 \rightarrow \boxed{\frac{\bar{I}_1}{\bar{I}_2} = \frac{n_2}{n_1} = \frac{1}{a_t}}$$

- Complex power entering winding 1 equals the complex power leaving winding 2:

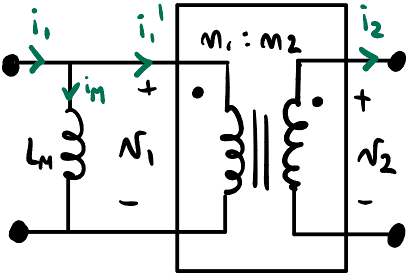
$$\bar{S}_1 = \bar{S}_2$$
$$\bar{V}_1\bar{I}_1^* = \bar{V}_2\bar{I}_2^*$$
$$\bar{S}_1 = \bar{V}_1\bar{I}_1^* = a_t\bar{V}_2\frac{\bar{I}_2^*}{a_t} = \bar{S}_2$$

- impedance $Z_2 = \dfrac{\bar{V}_2}{\bar{I}_2}$ seen from winding 1

$$Z_2{}' = \frac{\bar{V}_1}{\bar{I}_1} = \frac{a_t\bar{V}_2}{\bar{I}_2/a_t} = a_t{}^2 Z_2$$

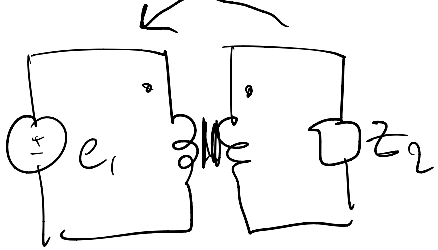
Practical transformer in AC circuits

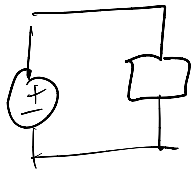
Magnetic core reluctance $\mathscr{R} > 0$



$$\bar{V} = n\frac{d\bar{\Phi}}{dt} \rightarrow \boxed{\frac{\bar{V}_1}{\bar{V}_2} = \frac{n_1}{n_2} = a_t}$$

$$n_1\bar{I}_1' = n_2\bar{I}_2 \rightarrow \boxed{\frac{\bar{I}_1'}{\bar{I}_2} = \frac{n_2}{n_1} = \frac{1}{a_t}}$$

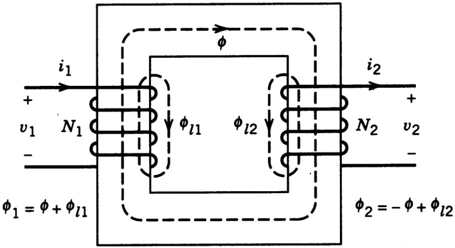




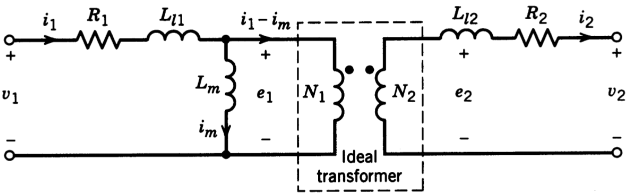
$z_2' = ?$

Practical transformer in AC circuits (cont'd 1)

Magnetic core reluctance $\mathscr{R} > 0$, and windings leak some flux outside the core

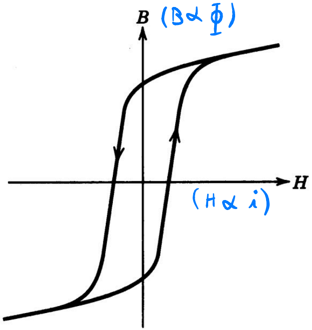


- Flux is "leaked" outside the core by each winding Φ_{l1}, Φ_{l2}
- these leaked flux is modelled by leakage inductance L_{l1}, L_{l2}
- R_1 and R_2 represent windings' copper (ohmic) losses

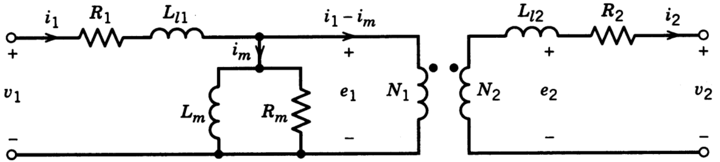


Practical transformer in AC circuits (cont'd 2)

Magnetic core reluctance $\mathscr{R} > 0$, windings leak some flux outside the core, and core dissipates active power (hysteresis and other losses)



- due to hyteresis of the B-H curve, energy is dissipated inside the core to create the magnetizing flux
- this energy loss in the magnetization process is represented by an additional shunt resistance R_m in parallel with L_m



Readings

From *Mohan (2011), Power Electronics - A First Course*

- Chapter 7: Magnetic Circuit Concepts Fundamentals (MQ library link)
  - 7.1: Ampere-Turns and Flux
  - 7.2: Inductance L
  - 7.3: Faraday's Law: Induced Voltage in a Coil Due to Time-Rate of Change of Flux Linkage
  - 7.5: Transformers

From *Glover (2017), Power System Analysis & Design*

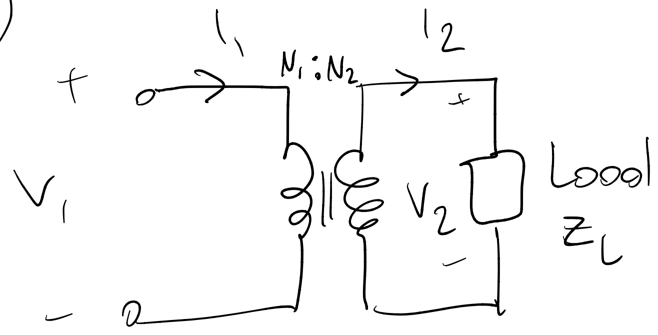
- Chapter 3: Power Transformers (MQ library link)
  - 3.1: The Ideal Transformer
    - Example 3.1: Ideal, single-phase two windings transformer
  - 3.2: Equivalent Circuits for Practical Transformers
    - Example 3.2 (optional): Transformer short-circuit and open-circuit tests

References

1. Ned Mohan, Tore M. Undeland, William P. Robbins (2002), *Power Electronics: Converters, Applications, and Design*, 3rd Edition, Wiley
2. Erickson, R. W., Maksimovic, D. (2020), *Fundamentals of Power Electronics*, Springer International Publishing

Example 3.1 p. 99 (Glover 2017)

- 1ϕ transformer (ideal)



$\bar{V}_2 = 118\, e^{j0^\circ}\ V$

$S = 20\,kVA$

$V_1 = 480\ V$
$V_2 = 120\ V$ } rated values

$f = 60\ Hz$

- Load

$S_L = 15\ kVA$

$\cos\phi = 0.8$ (lagging)

$V_2 = 118\ V$ (assume $\bar{V}_2 = 118\, e^{j0^\circ}\ V$)

a) $V_1 = ?$

b) $Z_L = ?$

c) $Z_L' = ?$ (Z_L seen by the primary winding)

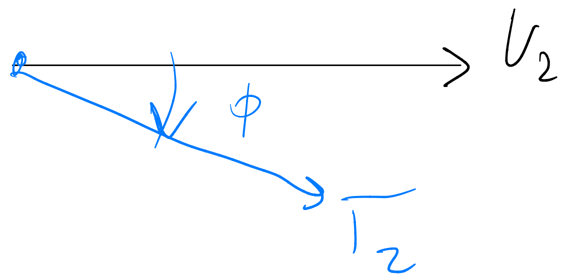
d) P, Q supplied by the transformer

a) $\dfrac{V_1}{V_2} = \dfrac{480 \text{ V}}{120 \text{ V}} = a_t = \dfrac{N_1}{N_2} = 4$

if $V_2 = 118 \text{ V}$ then $V_1 = \dfrac{N_1}{N_2} V_2 = 4 \cdot 118$
$= 472 \text{ V}$

b) $Z_L = ?$ $\qquad \cos\phi = 0.8$ lagging

$|\bar{S}_2| = |\bar{V}_2| \cdot |\bar{I}_2^*|$



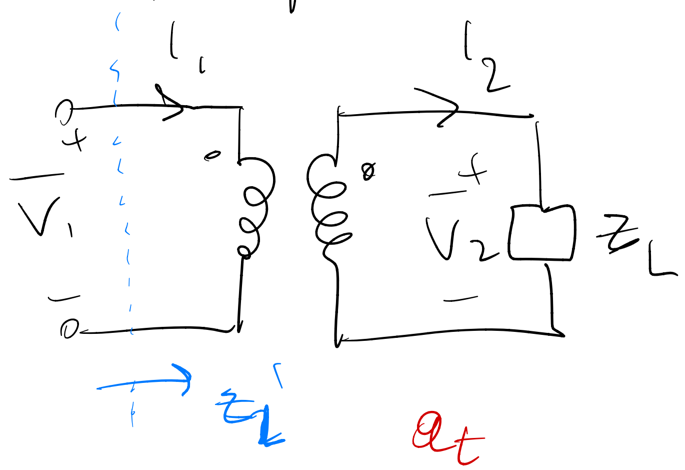
$S_2 = V_2 \cdot I_2$

$I_2 = \dfrac{S_2}{V_2} = \dfrac{15 \text{ kVA}}{118 \text{ V}} = 127.12 \text{ A}$ $\qquad \phi = \cos^{-1}(0.8)$

$\bar{I}_2 = 127.12\, e^{-j\phi} = 127.12\, e^{-j36.87^\circ} \text{ A}$

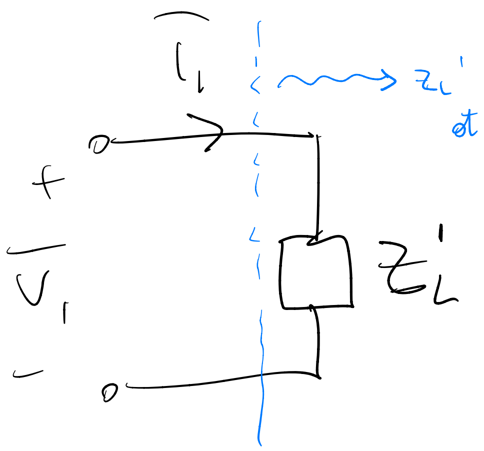
$$Z_L = \frac{\bar{V}_2}{\bar{I}_2} = \frac{118}{127.12\, e^{-j36.87^\circ}} = 0.928\, e^{+j36.87^\circ}\ \Omega$$

(c) load impedance seen at the primary



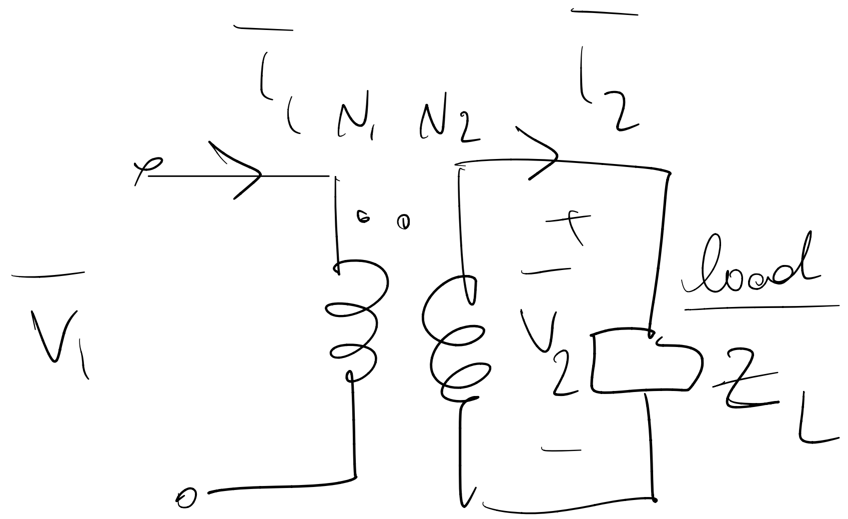
$$Z_L' = \frac{\bar{V}_1}{\bar{I}_1} = \frac{\overbrace{\frac{N_1}{N_2}}^{a_t}\bar{V}_2}{\underbrace{\frac{N_2}{N_1}}_{\frac{1}{a_t}}\bar{I}_2} = (a_t)^2\,\frac{\bar{V}_2}{\bar{I}_2} = (a_t)^2\, Z_L$$

$$= (4)^2 \cdot 0.928\, e^{+j36.87^\circ} = 14.8528\, e^{+j36.87^\circ}\ \Omega$$



Z_L' is the impedance seen at the primary terminals

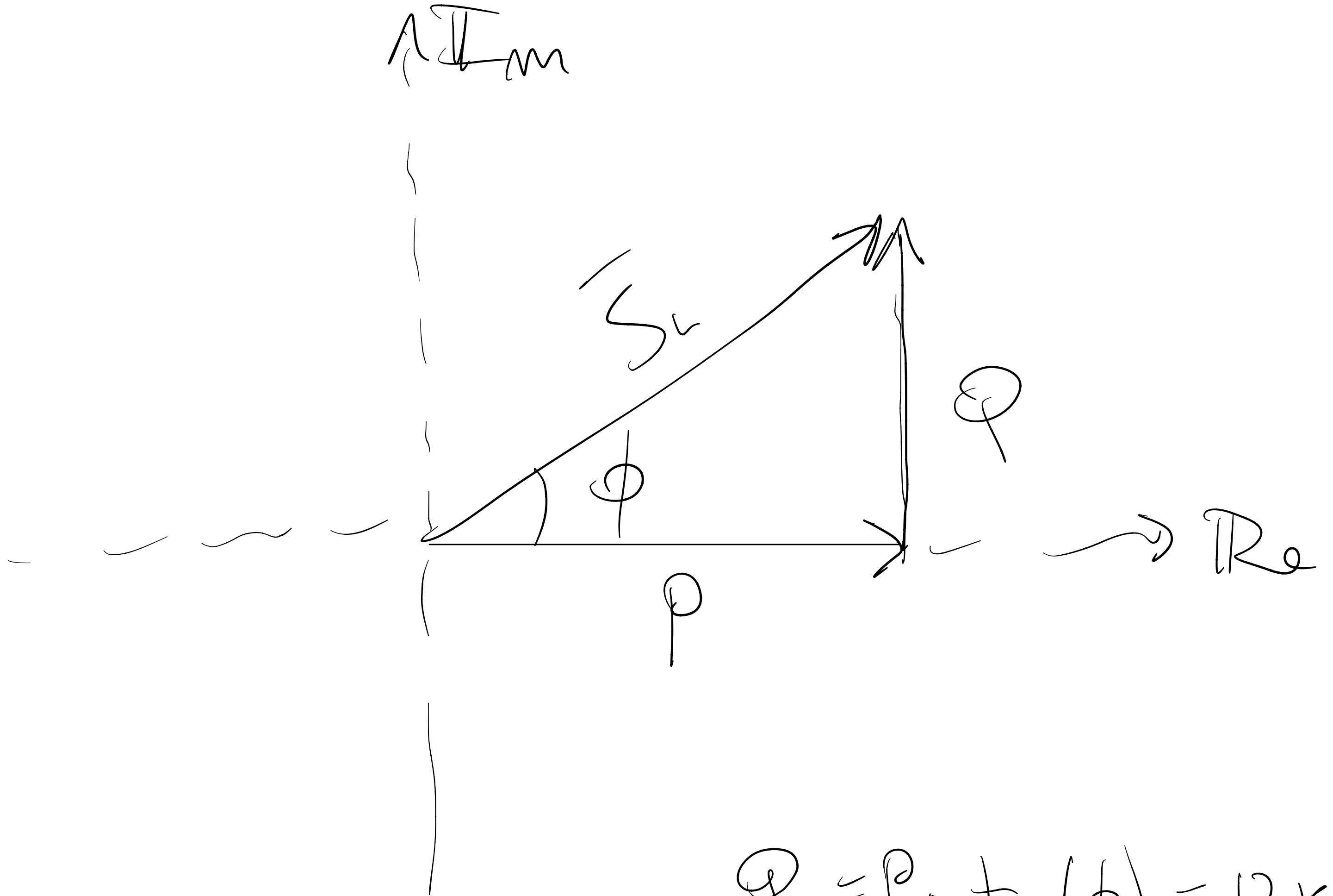
(d) P and Q supplied by the primary



load

$S_L = 15\ kVA$

$\cos\phi = 0.8$ (lagging)



$P = S_L \cdot \cos\phi$

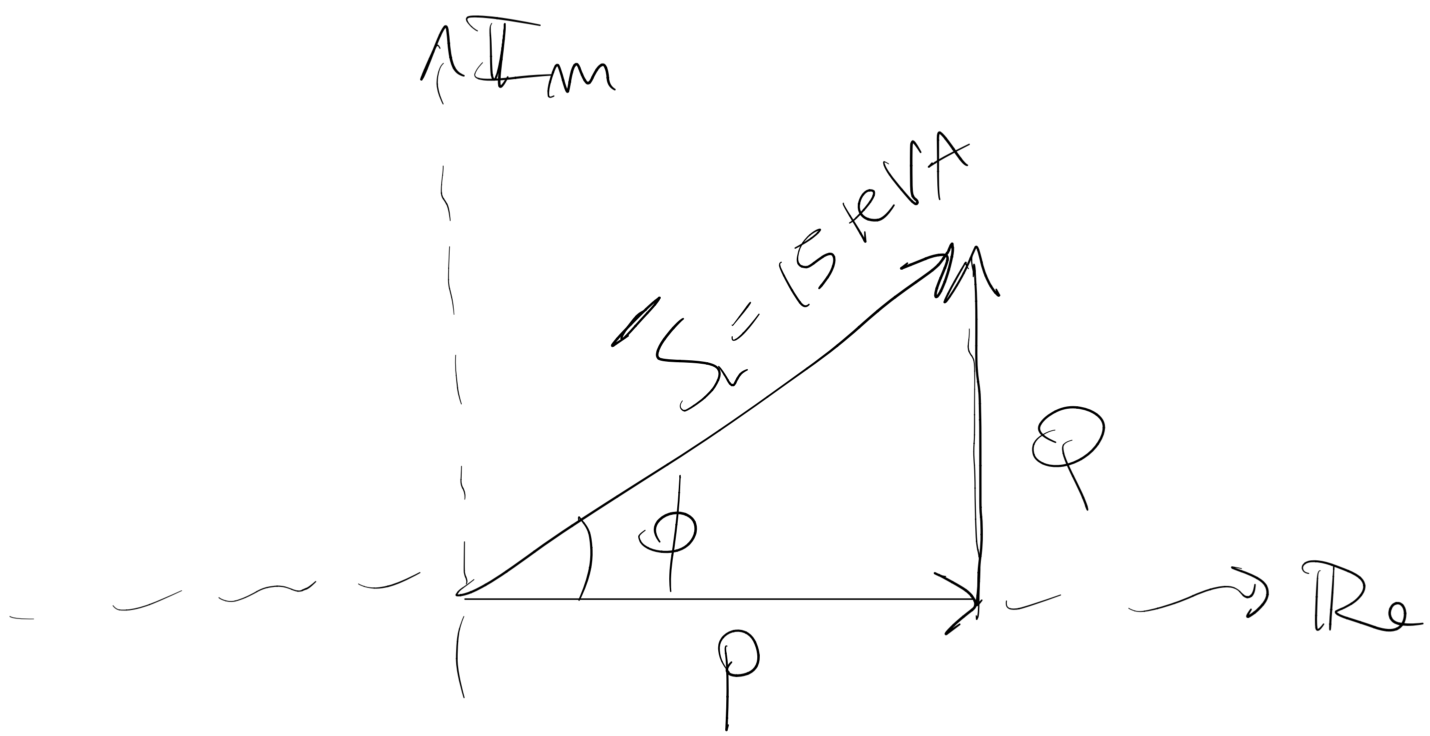
$= 15\ kVA \cdot 0.8$

$= 12\ kW$

$\frac{Q}{P} = tg\,\phi$

$Q = P \cdot tg(\phi) = 12\ kW \cdot tg(\cos^{-1}(0.8)) = 9\ kVAr$

$$S_L = 12\,kW + j\,9\,kVAr = \bar{S}_1$$



Q

Alternative way to calculate P, and Q using the complex power:

$$\bar{S}_L = S_L e^{j\phi} = 15\,KVA\; e^{j\overbrace{\cos^{-1}(0.8)}^{36.87^\circ}}$$

$$= 15\,kVA\left(\cos(36.87^\circ) + j\sin(36.87^\circ)\right)$$

$$= 12\,kW + j\,9\,kVAr$$