Three-phase systems

ELEC2005 Electrical and Electronic Systems - Week 9

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Table of Contents

Advantages of three-phase (3ϕ) systems

- Can transmit more power for same amount of wire (twice as much as 1ϕ)
  - Better infrastructure utilization
- Torque produced by 3ϕ machines is constant (because, contrarily to 1ϕ systems, instantaneous-3ϕ-power is constant)
  - No mechanical vibration
  - Longer machine lifetime
- 3ϕ machines use less material for same power rating
  - Cost savings
- 3ϕ machines start-up more easily than 1ϕ machines
  - Simple operation

Balanced three-phase (3ϕ) circuits

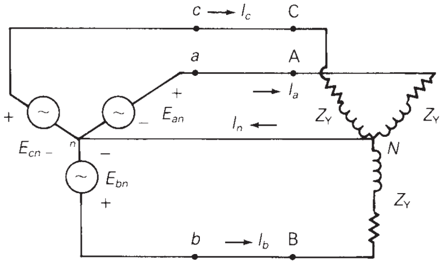
- A balanced 3 phase (3ϕ) circuit has
  - 3 voltage sources with equal magnitude, but with an angle shift of $120°$
  - equal loads on each phase
  - equal impedance on the lines connecting the generators to the loads
- Bulk power systems are almost exclusively 3ϕ
  - Bulk means "high-voltage" system, e.g. to transfer power between different geographical areas
- 1ϕ is used primarily in low voltage, low power settings, such as residential and some commercial settings

$$\bar{E}_{an} = Ee^{j0}$$

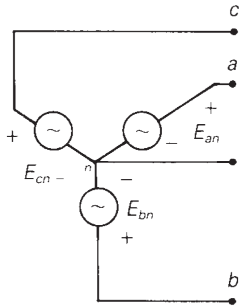
$$\bar{E}_{bn} = Ee^{-j\frac{2}{3}\pi}$$

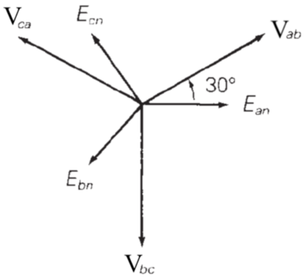
$$\bar{E}_{cn} = Ee^{j\frac{2}{3}\pi}$$

$$Z_{AN} = Z_{BN} = Z_{CN} = Z_Y$$

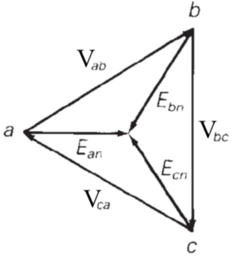


Line to neutral and line to line voltages





(a) Phasor diagram



(b) Voltage triangle

Balanced line to neutral voltages

$$\bar{E}_{an} = Ee^{j0}$$
$$\bar{E}_{bn} = Ee^{-j\frac{2}{3}\pi} = \bar{E}_{an} e^{-j\frac{2}{3}\pi}$$
$$\bar{E}_{cn} = Ee^{j\frac{2}{3}\pi}$$

Balanced line to line voltages

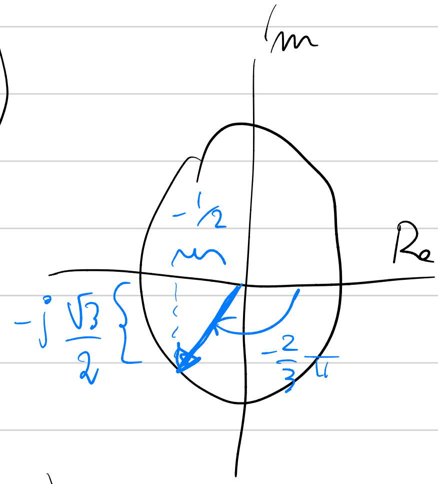
$$\bar{V}_{ab} = \bar{E}_{an} - \bar{E}_{bn} = \sqrt{3}\bar{E}_{an}e^{j\frac{\pi}{6}}$$
$$\bar{V}_{bc} = \bar{E}_{bn} - \bar{E}_{cn} = \sqrt{3}\bar{E}_{bn}e^{j\frac{\pi}{6}}$$
$$\bar{V}_{ca} = \bar{E}_{cn} - \bar{E}_{an} = \sqrt{3}\bar{E}_{cn}e^{j\frac{\pi}{6}}$$

Note: often the notation $\bar{V}_{ln}$ or $\bar{V}_{LN}$ (rather than $\bar{E}_{ln}$, with $l = a, b, c$) is used to indicate the line to neutral voltage

$$\bar{V}_{ab} = \bar{E}_{an} - \bar{E}_{bn} = \bar{E}_{an} - \bar{E}_{an} e^{-j\frac{2}{3}\pi}$$

$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$= \bar{E}_{an}\left(1 - \left[\cos\left(-\frac{2}{3}\pi\right) + j\sin\left(-\frac{2}{3}\pi\right)\right]\right)$$

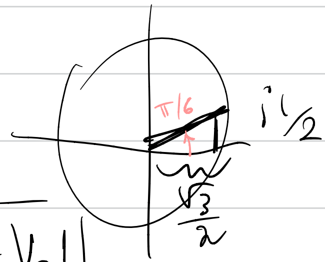


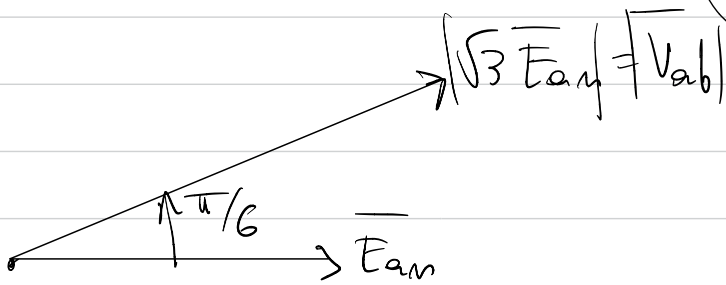
$$= \bar{E}_{an}\left(1 - \left[\left(-\frac{1}{2}\right) + j\left(-\frac{\sqrt{3}}{2}\right)\right]\right)$$

$$= \bar{E}_{an}\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

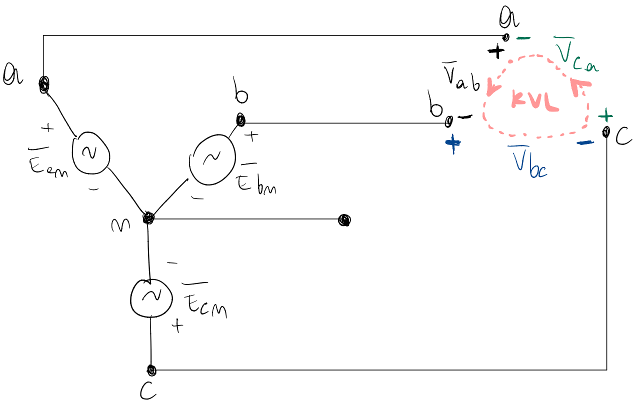
$$= \bar{E}_{an}\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,\bar{E}_{an}\underbrace{\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)}$$

$$= \sqrt{3}\,\bar{E}_{an}\, e^{j\frac{\pi}{6}}$$





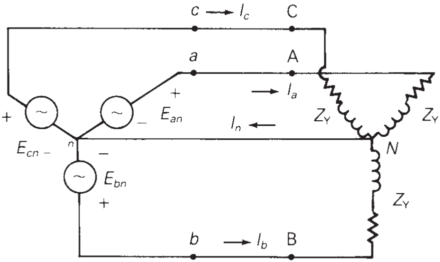
Balanced line to neutral voltages

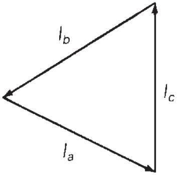


Balanced conditions:
$\bar{E}_{an} + \bar{E}_{bn} + \bar{E}_{cn} = 0$

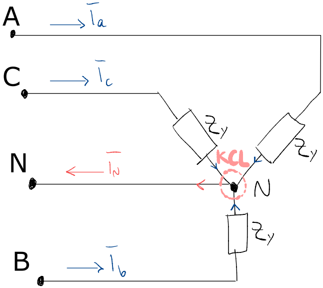
KVL:
$\bar{V}_{ab} + \bar{V}_{bc} + \bar{V}_{ca} = 0$
(sum of line to line voltages is always 0)

Balanced line currents





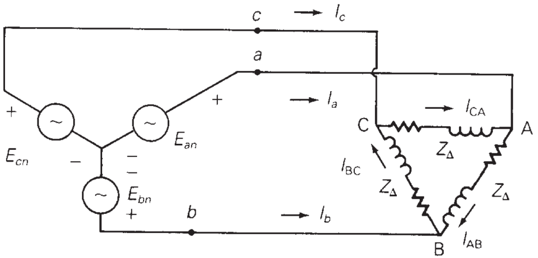
$$\bar{I}_a = \frac{\bar{E}_{an}}{Z_Y} \qquad \bar{I}_b = \frac{\bar{E}_{bn}}{Z_Y} \qquad \bar{I}_c = \frac{\bar{E}_{cn}}{Z_Y}$$



KCL (node N): $\bar{I}_a + \bar{I}_b + \bar{I}_c - \bar{I}_N = 0$

Balanced conditions: $\bar{I}_N = 0$

Balanced-Δ loads



Load currents under balanced conditions:

$$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{Z_\Delta}$$

$$\bar{I}_{BC} = \frac{\bar{V}_{BC}}{Z_\Delta}$$

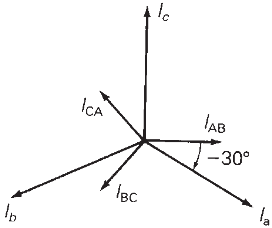
$$\bar{I}_{CA} = \frac{\bar{V}_{CA}}{Z_\Delta}$$

Line current equations:

KCL (A): $\bar{I}_a = \bar{I}_{AB} - \bar{I}_{CA} = \sqrt{3}\,\bar{I}_{AB}e^{-j\frac{\pi}{6}}$

KCL (B): $\bar{I}_b = \bar{I}_{BC} - \bar{I}_{AB} = \sqrt{3}\,\bar{I}_{BC}e^{-j\frac{\pi}{6}}$

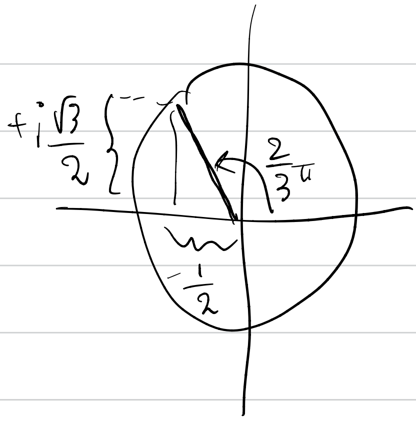
KCL (C): $\bar{I}_c = \bar{I}_{CA} - \bar{I}_{BC} = \sqrt{3}\,\bar{I}_{CA}e^{-j\frac{\pi}{6}}$



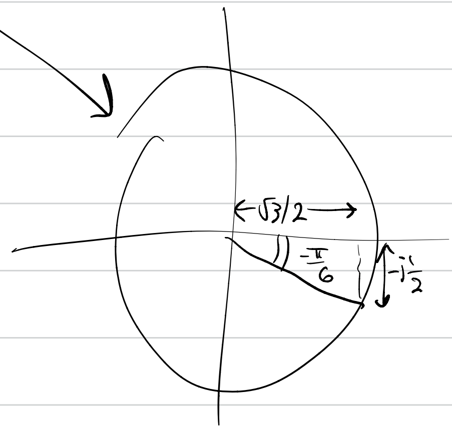
$$\bar{I}_a = \bar{I}_{AB} - \bar{I}_{CA} = \bar{I}_{AB} - \bar{I}_{AB}\, e^{j\frac{2}{3}\pi}$$

$$= \bar{I}_{AB}\left(1 - \left[\cos\left(\frac{2}{3}\pi\right) + j\sin\left(\frac{2}{3}\pi\right)\right]\right)$$

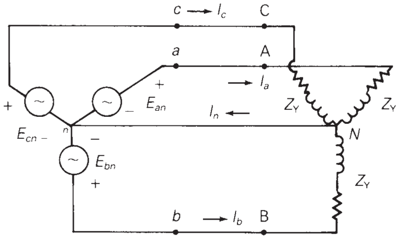
$$= \bar{I}_{AB}\left(1 - \left(-\frac{1}{2}\right) - j\left(\frac{\sqrt{3}}{2}\right)\right)$$

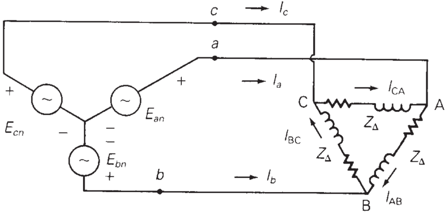


$$= \bar{I}_{AB}\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,\bar{I}_{AB}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = \sqrt{3}\,\bar{I}_{AB}\, e^{-j\frac{\pi}{6}}$$



Δ-Y conversion for balanced loads





Posing equality of **line** currents for the circuit on the left and on the right:

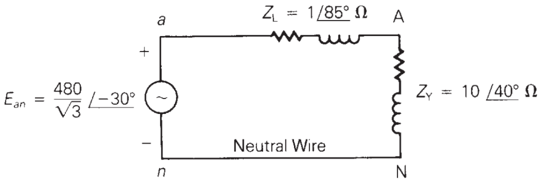
$$\bar{I}_a = \frac{\bar{E}_{an}}{Z_Y} = \frac{\bar{V}_{AB}}{\sqrt{3}\, Z_Y} e^{-j\frac{\pi}{6}} \qquad \bar{I}_a = \sqrt{3}\, \bar{I}_{AB} e^{-j\frac{\pi}{6}} = \sqrt{3}\, \frac{\bar{V}_{AB}}{Z_\Delta} e^{-j\frac{\pi}{6}}$$

$\bar{I}_a$ is the same in both circuits if: $\boxed{Z_Y = \frac{Z_\Delta}{3}}$

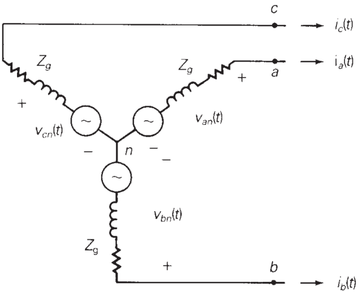
Equivalent line-to-neutral Y diagrams

With balanced three-phase circuits, only one phase needs to be analyzed.

- Δ loads can be converted to Y loads
- all source and load neutrals connected with a zero-ohm neutral wire without changing the solution
- one phase of the circuit can be solved
  - voltage and current in remaining phases are equal in magnitude and $\pm 120°$ out of phase with those of the phase solved



Instantaneous power in balanced 3ϕ circuits



Instantaneous power for phase *a*:

$$\begin{aligned} v_{an}(t) &= \sqrt{2}V_{LN}\cos(\omega t + \delta) \\ i_a(t) &= \sqrt{2}I\cos(\omega t + \beta) \\ p_a(t) &= v_{an}(t)\, i_a(t) \\ &= V_{LN}I\cos(\delta - \beta) \\ &\quad + V_{LN}I\cos(2\omega t + \delta + \beta) \end{aligned}$$

In the same way, the power for phase *b* and *c*, can be calculated.

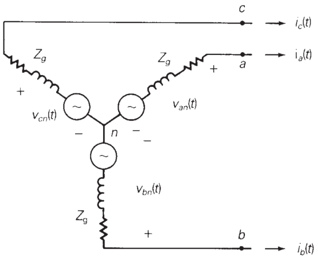
The instantaneous 3ϕ power is: V_L

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t) = 3V_{LN}I\cos\overbrace{(\delta - \beta)}^{\phi} = \sqrt{3}V_{LL}I\cos\overbrace{(\delta - \beta)}^{\phi}$$

3ϕ instant. power is not a function of time: $\boxed{p_{3\phi}(t) = \text{constant} = P_{3\phi}}$

See: Glover (2017), pp. 63-64, for derivations (click here)

Complex power in balanced 3ϕ circuits



Complex power for phase a:

$$\bar{V}_{an} = V_{LN}\, e^{j\delta}$$

$$\bar{I}_a = I\, e^{j\beta}$$

$$\bar{S}_a = \bar{V}_{an}\, \bar{I}_a^* = V_{LN}\, e^{j\delta}\, I\, e^{-j\beta} = V_{LN} I e^{j(\delta-\beta)}$$

$$= \underbrace{V_{LN} I \cos(\delta-\beta)}_{P_a} + j\, \underbrace{V_{LN} I \sin(\delta-\beta)}_{Q_a}$$

$$\bar{S}_{3\phi} = S_a + S_b + S_c = \underbrace{3V_{LN} I \cos(\delta-\beta)}_{P_{3\phi}} + j\, \underbrace{3V_{LN} I \sin(\delta-\beta)}_{Q_{3\phi}}$$

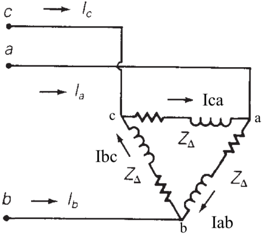
$$P_{3\phi} = \mathrm{Re}[\bar{S}_{3\phi}] = 3V_{LN} I \cos(\delta-\beta) = \sqrt{3} V_{LL} I \cos(\delta-\beta)$$ active power

$$Q_{3\phi} = \mathrm{Im}[\bar{S}_{3\phi}] = 3V_{LN} I \sin(\delta-\beta) = \sqrt{3} V_{LL} I \sin(\delta-\beta)$$ reactive power

$$|\bar{S}_{3\phi}| = S_{3\phi} = \sqrt{{P_{3\phi}}^2 + {P_{3\phi}}^2} = 3V_{LN} I = \sqrt{3} V_{LL} I$$ apparent power

Complex power in balanced 3ϕ Δ-connected circuits

For the load connected between *a* and *b*:



$$\bar{V}_{ab} = V_{LL}\, e^{j\delta}$$

$$\bar{I}_{ab} = I_\Delta\, e^{j\beta}$$

$$\bar{S}_{ab} = \bar{V}_{ab}\, \bar{I}^*_{ab} = V_{LL}\, e^{j\delta}\, I_\Delta\, e^{-j\beta} = V_{LL} I_\Delta e^{j(\delta-\beta)}$$

$$= \underbrace{V_{LL} I_\Delta \cos(\delta-\beta)}_{P_{ab}} + j\, \underbrace{V_{LL} I_\Delta \sin(\delta-\beta)}_{Q_{ab}}$$

For the entire load:

$$\bar{S}_{3\phi} = \bar{S}_{ab} + \bar{S}_{bc} + \bar{S}_{ca} = 3\bar{S}_{ab} = 3V_{LL} I_\Delta e^{j(\delta-\beta)}$$

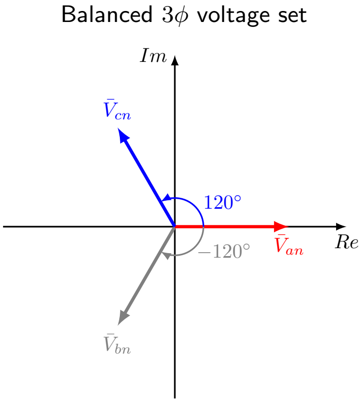
$$P_{3\phi} = \mathrm{Re}[\bar{S}_{3\phi}] = 3V_{LL} I_\Delta \cos(\delta-\beta) = \sqrt{3} V_{LL} I \cos(\delta-\beta) \text{ active power}$$

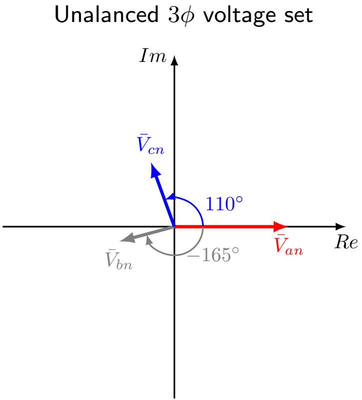
$$Q_{3\phi} = \mathrm{Im}[\bar{S}_{3\phi}] = 3V_{LL} I_\Delta \sin(\delta-\beta) = \sqrt{3} V_{LL} I \sin(\delta-\beta) \text{ reactive power}$$

$$|\bar{S}_{3\phi}| = S_{3\phi} = \sqrt{{P_{3\phi}}^2 + {P_{3\phi}}^2} = 3V_{LL} I_\Delta = \sqrt{3} V_{LL} I \qquad \text{apparent power}$$

Unbalanced 3ϕ circuits

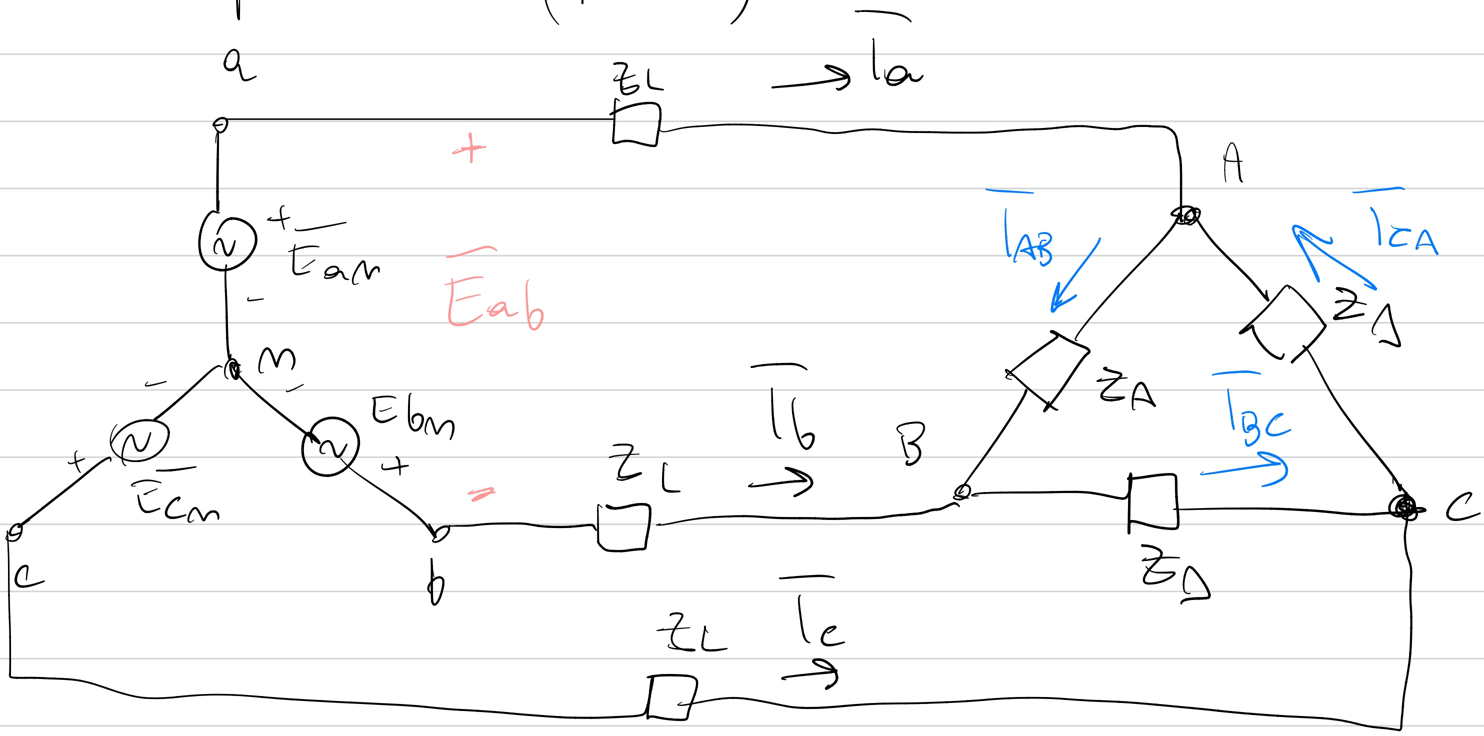
An unbalanced system is due to unbalanced voltage sources or an unbalanced load. Example:





Symmetrical Components (Fortescue, 1909): any 3ϕ voltage/current set can be resolved into three sets of *sequence* components: zero (0), positive ($+$) and negative (-)

Example 2.4 (p.61)

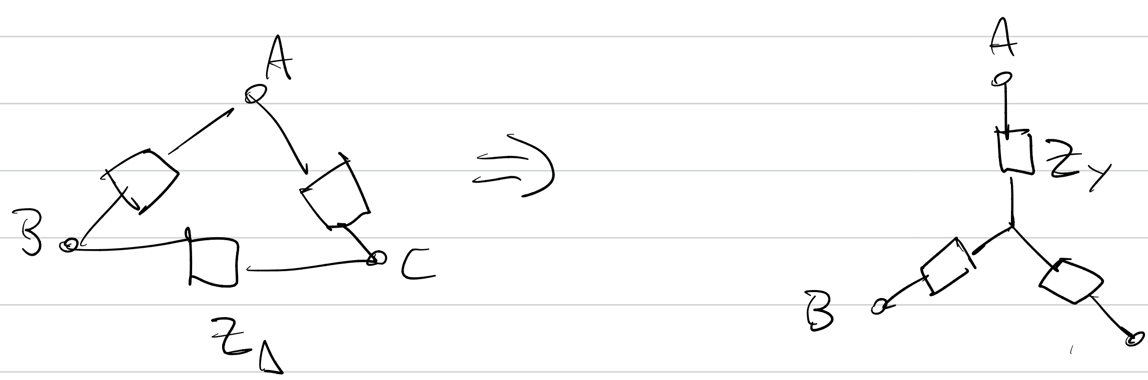


$Z_\Delta = 30\angle 40^\circ\ \Omega \qquad Z_L = 1\angle 85^\circ\ \Omega$

$\bar{E}_{ab} = 480\angle 0^\circ$ V (line to line)

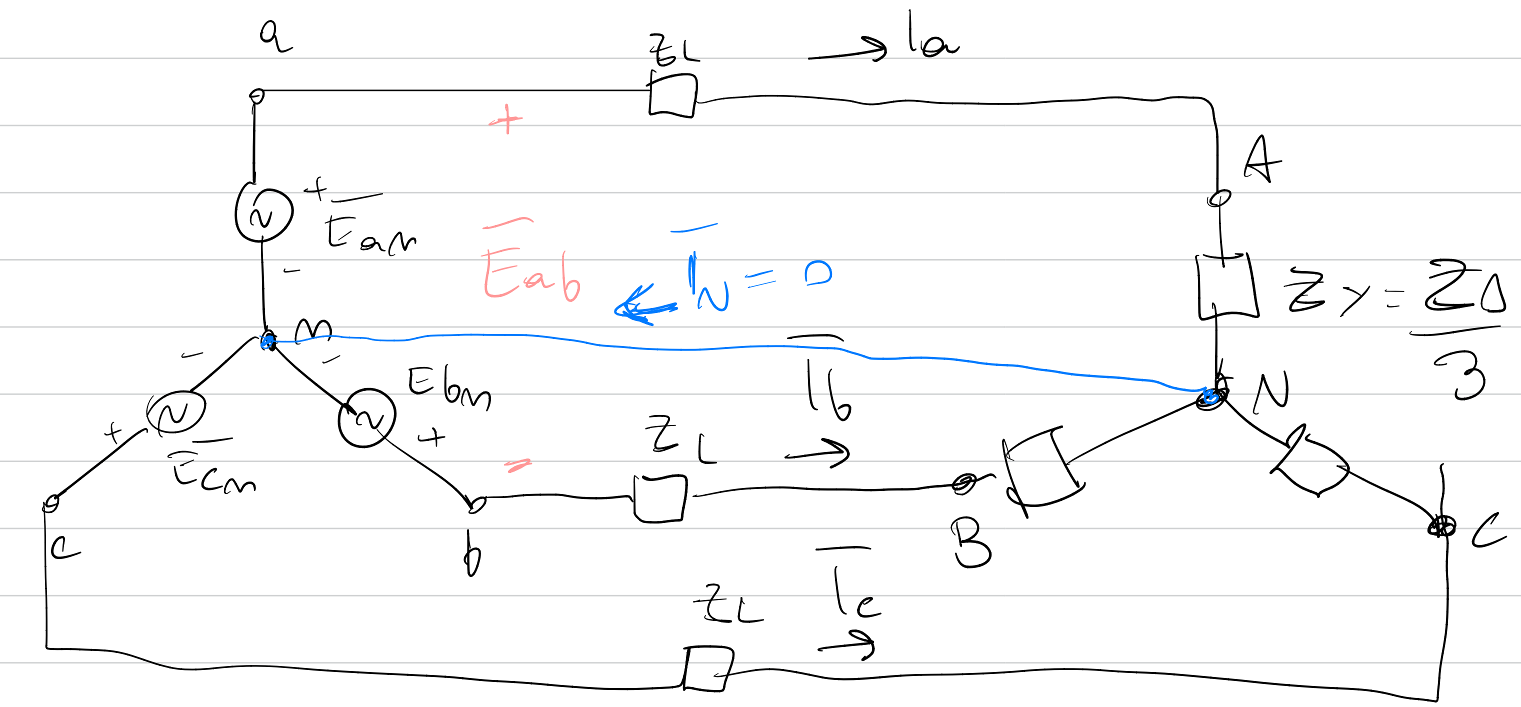
- $\bar{I}_a, \bar{I}_b, \bar{I}_c = ?$ • $\bar{V}_{AB}, \bar{V}_{BC}, \bar{V}_{CA}$
- $\bar{I}_{AB}, \bar{I}_{BC}, \bar{I}_{CA} = ?$

(1) Transform the Δ load into Y load

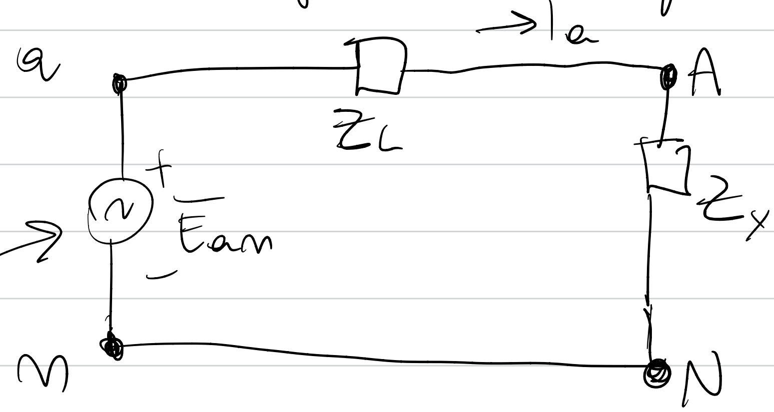


$$Z_Y = \frac{Z_\Delta}{3} = \frac{30\angle 40^\circ}{3} = 10\angle 40^\circ\ \Omega$$

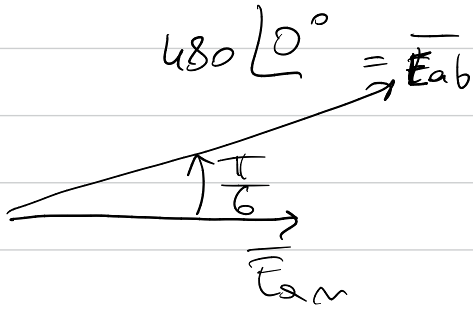
(2) Study simple phase (Y) equivalent



Study simple phase equivalent (phase a)



$\bar{E}_{an} = ?$



$$\bar{E}_{ab} = \sqrt{3}\,\bar{E}_{an}\, e^{+j\frac{\pi}{6}}$$

$$\boxed{\bar{E}_{an} = \frac{(480\angle 0^\circ - 30^\circ)}{\sqrt{3}} = \frac{480}{\sqrt{3}}\angle -30^\circ \ \text{V}}$$

$$\bar{E}_{an} = \frac{\bar{E}_{ab}}{\sqrt{3}} e^{-j\pi/6}$$

$$\bar{I}_a = \frac{\bar{E}_{an}}{Z_L + Z_y} = \frac{\left(\frac{480}{\sqrt{3}}\right)\angle -30^\circ}{1\angle 85^\circ + 10\angle 40^\circ} = 25.83\angle -73.8^\circ \text{ A}$$

$$\bar{I}_b = 25.83\angle -73.8^\circ - 120^\circ = 25.83\angle 167^\circ$$

$$\bar{I}_c = 25.83\angle -73.8^\circ + 120^\circ = 25.83\angle 46.2^\circ$$

Currents on the Δ - load?

$$\bar{I}_{AB} = \frac{\bar{I}_a}{\sqrt{3}} e^{j\frac{\pi}{6}} = \frac{\bar{I}_a}{\sqrt{3}}\angle -73.8^\circ + 30^\circ = 14.91\angle -43.8^\circ \text{ A}$$

$$\bar{I}_{BC} = 14.91\angle -43.8^\circ - 120^\circ = 14.91\angle -163.8^\circ \text{ A}$$

$$\bar{I}_{CA} = 14.91\angle -43.8^\circ + 120^\circ = 14.91\angle +76.2^\circ \text{ A}$$

load voltages

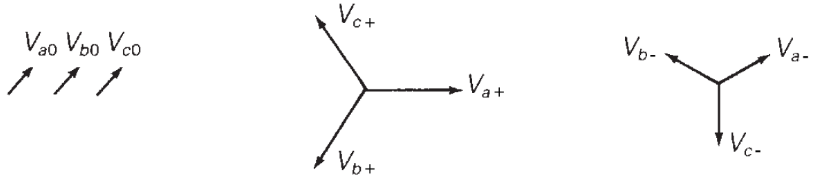
$$\bar{V}_{AB} = \bar{I}_{AB} \cdot Z_\Delta = 447.33\angle -3.78^\circ \text{ V}$$

$$\bar{V}_{BC} = 447.3\angle -123.78^\circ \text{ V}$$

$$\bar{V}_{CA} = 447.3\angle 116.22^\circ \text{ V}$$

Symmetrical components method

Represent any 3ϕ vector set as the superposition of 3 balanced, 3ϕ sets:

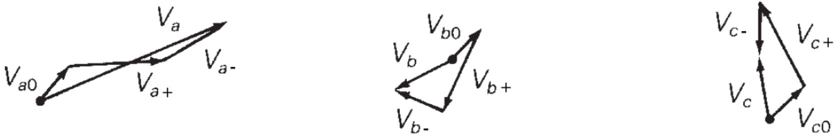


(a) Zero-sequence components

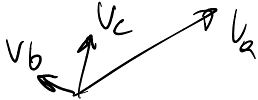
(b) Positive-sequence components

(c) Negative-sequence components

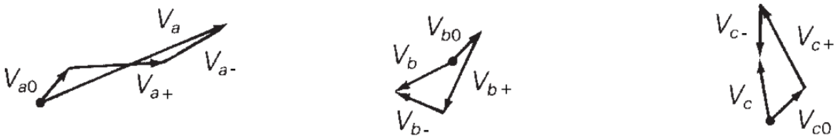
Example of a three-phase unbalanced set $\bar{V}_a, \bar{V}_b, \bar{V}_c$, where each voltage phasor is built using sequence components:



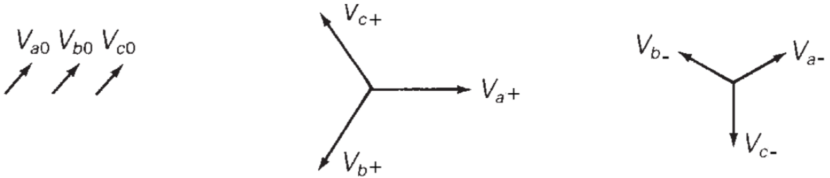
Symmetrical components method (cont'd)



Example of a three-phase unbalanced set $\bar{V}_a, \bar{V}_b, \bar{V}_c$, where each voltage phasor is built using sequence components:



$$\overbrace{\overline{V}_a = \bar{V}_{a0} + \overline{V}_{a+} + \overline{V}_{a-}} \quad \overbrace{\overline{V}_b = \bar{V}_{b0} + \overline{V}_{b+} + \overline{V}_{b-}} \quad \overbrace{\overline{V}_c = \bar{V}_{c0} + \overline{V}_{c+} + \overline{V}_{c-}}$$



(a) Zero-sequence components

(b) Positive-sequence components

(c) Negative-sequence components

$$\bar{V}_{a0}$$
$$\bar{V}_{b0} = \bar{V}_{a0}$$
$$\bar{V}_{c0} = \bar{V}_{a0}$$

$$\bar{V}_{a+}$$
$$\bar{V}_{b+} = (\bar{V}_{a+})e^{-j\frac{2\pi}{3}}$$
$$\bar{V}_{c+} = (\bar{V}_{a+})e^{j\frac{2\pi}{3}}$$

$$\bar{V}_{a-}$$
$$\bar{V}_{b-} = (\bar{V}_{a-})e^{j\frac{2\pi}{3}}$$
$$\bar{V}_{c-} = (\bar{V}_{a-})e^{-j\frac{2\pi}{3}}$$

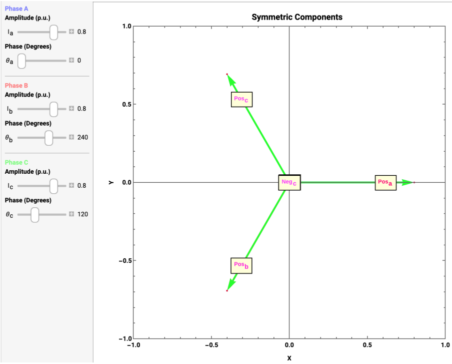
$$\boxed{\begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} \overline{V}_{a0} \\ \overline{V}_{a+} \\ \overline{V}_{a-} \end{bmatrix}}$$

$$\boxed{\begin{bmatrix} \overline{V}_{a0} \\ \overline{V}_{a+} \\ \overline{V}_{a-} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix}}$$

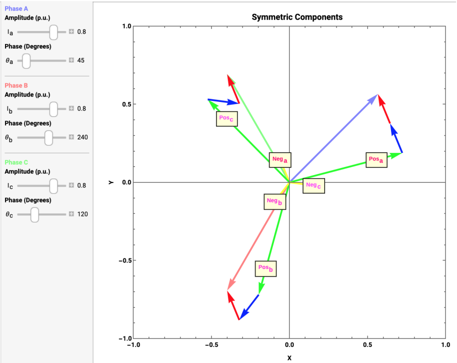
where $\alpha = e^{j\frac{2\pi}{3}}$, $\alpha^2 = e^{-j\frac{2\pi}{3}}$ and $\bar{V}_a, \bar{V}_b, \bar{V}_c$ the 3ϕ unbalanced phasor set

Symmetrical components method (continued)

Balanced 3ϕ voltage set

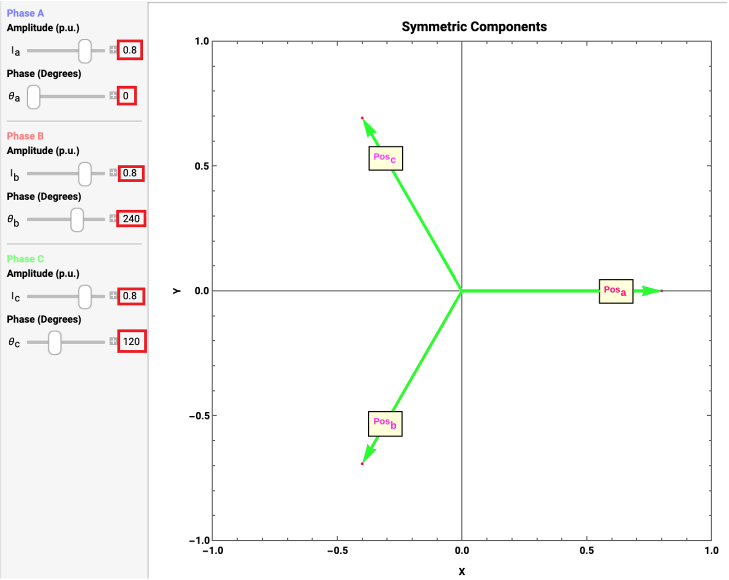


Unalanced 3ϕ voltage set (phase *a* angle is not zero)

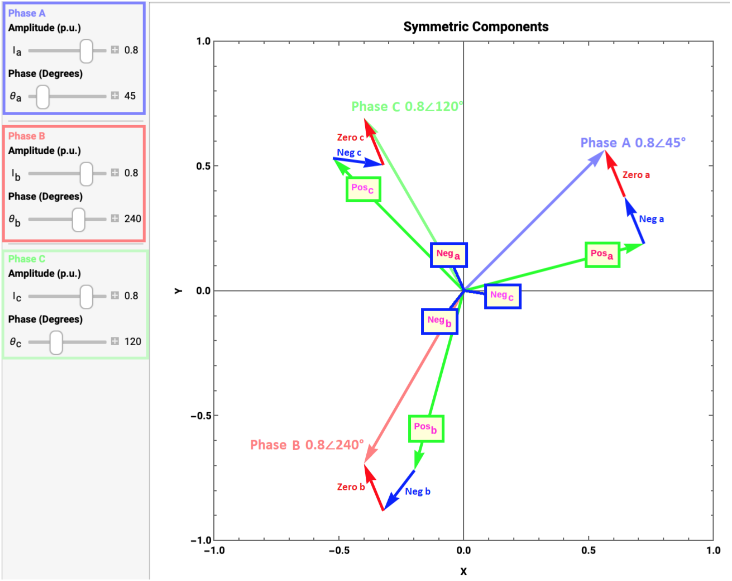


Figures from: https://demonstrations.wolfram.com/FortescuesTheoremForAThreePhaseUnbalancedSystem

Balanced 3ϕ voltage set: has only positive-sequence components



Unbalanced 3ϕ voltage set: has positive, negative and zero-sequence components (in this example the unbalance is due *Phase A* angle not being 0° (but 45°)



Readings

From *Glover (2017), Power System Analysis and Design*

- Chapter 2: Fundamentals (MQ library link)
  - 2.5: Balanced Three-Phase Circuits
    - Example 2.4: Balanced-Δ and Y-Loads
  - 2.6: Power in Balanced Three-Phase Circuits
    - Example 2.5: Power in a balanced three-phase system
  - 2.7: Advantages of Balanced Three-Phase versus Single-Phase Systems

References

- Glover, J. D., Overbye, T. J., & Sarma, M. S. (2017). Chapter 2 in *Power system analysis & design* (Sixth edition). Boston, MA, USA: Cengage Learning