MACQUARIE University SYDNEY·AUSTRALIA

PHYS2010

Classical and Quantum Oscillations and Waves

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School of Mathematical and Physical Sciences

Unit guide - executive summary

- **PHYS2010** is a core second year physics unit
- 1st half text : *The Physics of Vibrations and Waves*, Pain, 6th Edn, Wiley. (see unit guide for recommended reading for 2nd half)
- Lecturers: David Spence and Alexei Gilchrist
- Delivery is on-campus
  - Lectures: audio and slides will be available on iLearn
  - Compulsory laboratory work
    - Computer Labs – Python (Weeks 2–5, 11–13)
    - Practical Labs – Mechanical/Electrical Oscillators (Weeks 6-7, 9-10)
  - SGTAs (Weeks 1-13) include quizzes from week 2 onwards!!
- Assessment
  - 15% Python Labs – *hurdle* = 40% for *each* write-up
  - 10% Practical Labs – *hurdle* = 40% for *each* write-up
  - 30% In-tutorial tests (~15 mins, based on previous week's tutorial content)
    hurdle: sit at least 8 of the 12 tests
  - 45% Final exam – *hurdle* = 40% on the final exam
  - To pass need mark of 50% overall

Laboratory activities

- Laboratory work (weeks 2 – 13) in 14SCO 209 and 217
  - 4 weeks introductory Python (weeks 2–5)
  - 4 weeks hands-on experiments (weeks 6-7, 9-10)
  - 3 weeks Python applied to quantum mechanics (weeks 11–13)
- Closed shoes please (even for computer labs)
- You must make up any missed labs.

Self-guided study is the key

- 75% of your final mark is for problem-solving
  - 30 % tutorial tests, 45% final exam
- SGTAs are the most efficient way to get help, so prepare!
  — attend/watch the lecture; review the slides *before* the tutorial
  - Particularly focus on proofs, and key concepts
- Attempt exercises and some tutorial problems *before* the tutorial
- Chase up things you didn't understand
- Read the textbook etc
- Work through the published exercises and sample quiz problems
- Talk to your colleagues. Ask questions!
- Take stock of your understanding, and take steps to overcome any roadblocks

- Systems disturbed from a stable equilibrium will oscillate, e.g.
  - old bridges

Tacoma Narrows Bridge
https://youtu.be/qbOjxPCfaFk

- Systems disturbed from a stable equilibrium will oscillate, e.g.
  - old bridges
  - new bridges

Millennium Bridge

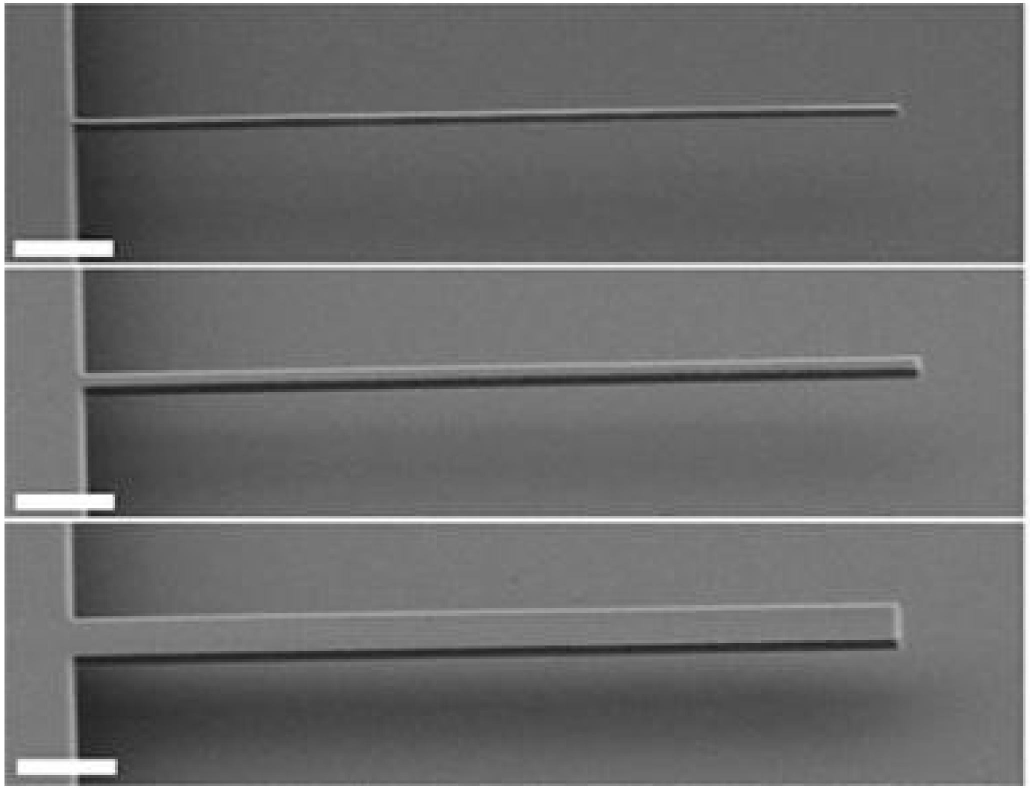
https://youtu.be/eAXVa__XWZ8

- Systems disturbed from a stable equilibrium will oscillate
  - old bridges
  - new bridges
  - cars

Car suspension at work

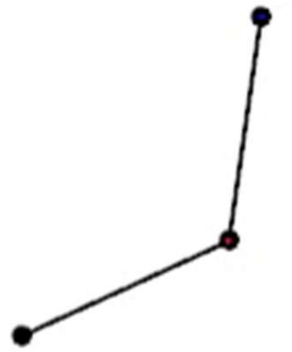
https://youtu.be/JenlKKwHmtk

- Systems disturbed from a stable equilibrium will oscillate, e.g.
  - – old bridges
  - – new bridges
  - – cars
  - – nanoscale cantilevers



Vibrations and oscillations are everywhere…

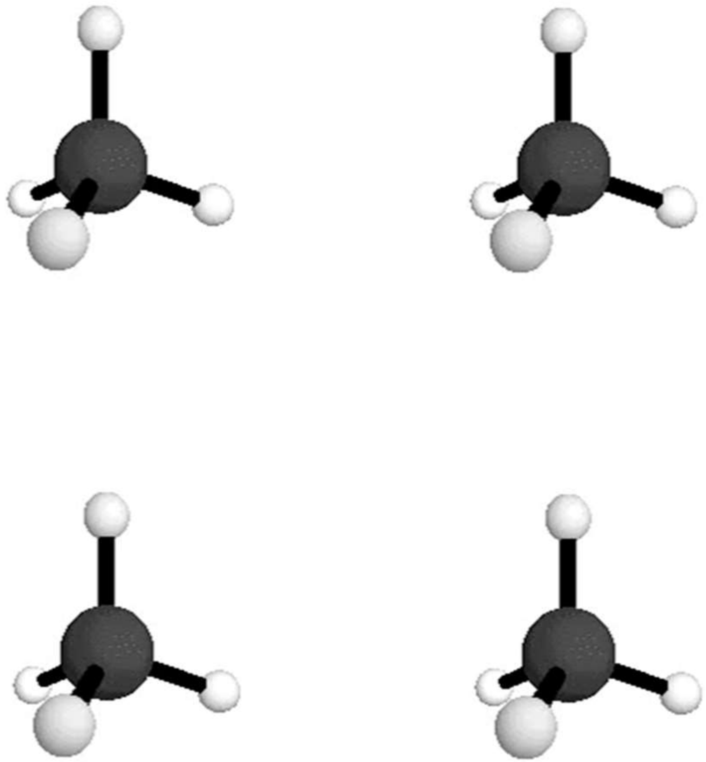
- Systems disturbed from a stable equilibrium will oscillate, e.g.
  - old bridges
  - new bridges
  - cars
  - nanoscale cantilevers
  - coupled oscillators



Double pendulum

https://commons.wikimedia.org/wiki/File:Trajektorie_eines_Doppelpendels.gif

- Systems disturbed from a stable equilibrium will oscillate, e.g.
  - old bridges
  - new bridges
  - cars
  - nanoscale cantilevers
  - coupled oscillators
  - molecules

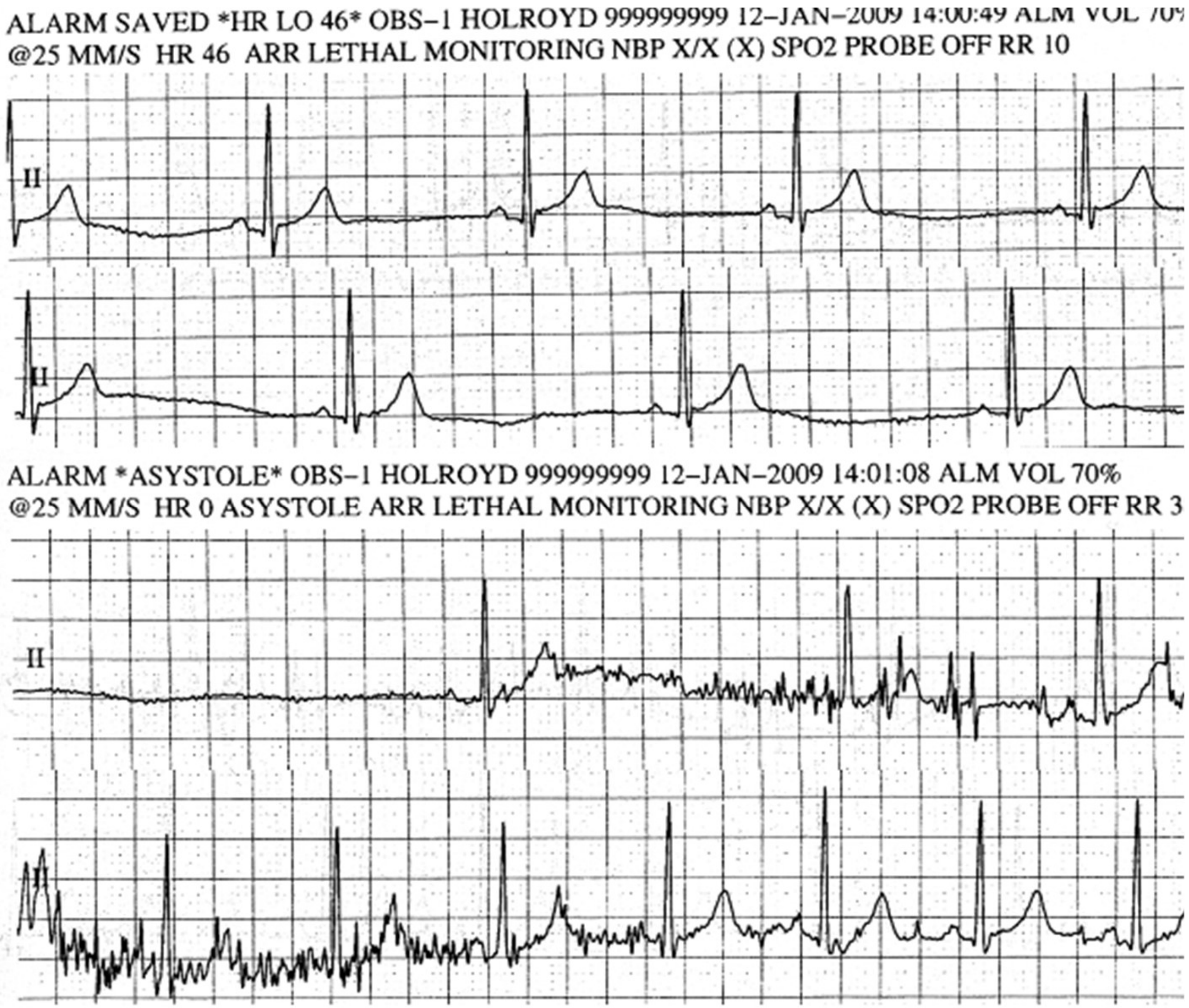


Methane molecule, CH_4

http://www2.ess.ucla.edu/~schauble/MoleculeHTML/CH4_html/CH4_page.html

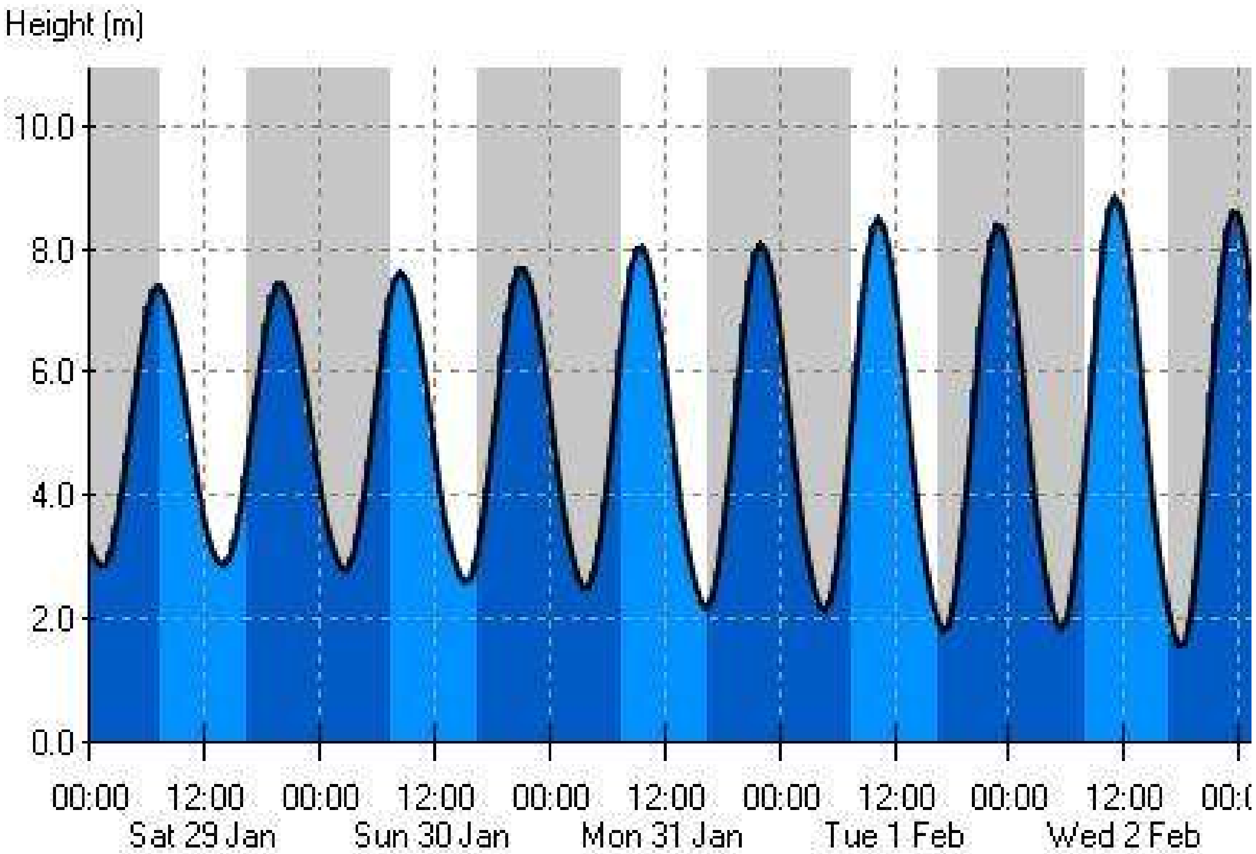
Many systems show oscillatory behaviour

- Heartbeats, breathing



Many systems show oscillatory behaviour

- Heartbeats, breathing
- Comets, sunspots, planetary motion, tides

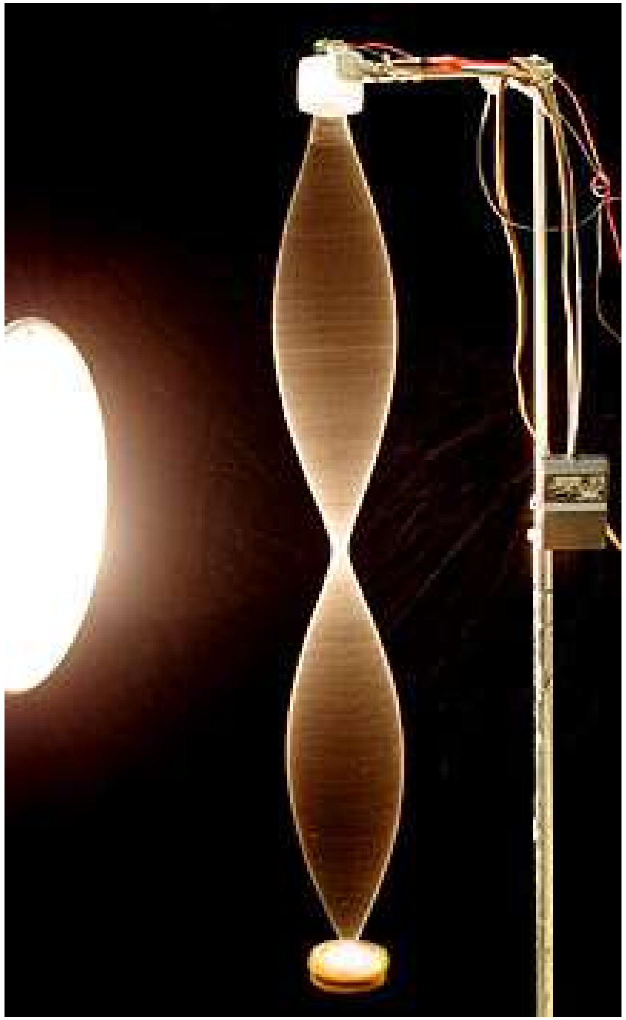


Many systems show oscillatory behaviour

- Heartbeats, breathing
- Comets, sunspots, planetary motion, tides
- Quartz crystal oscillators

Many systems show oscillatory behaviour

- Heartbeats, breathing
- Comets, sunspots, planetary motion, tides
- Quartz crystal oscillators
- String on a guitar

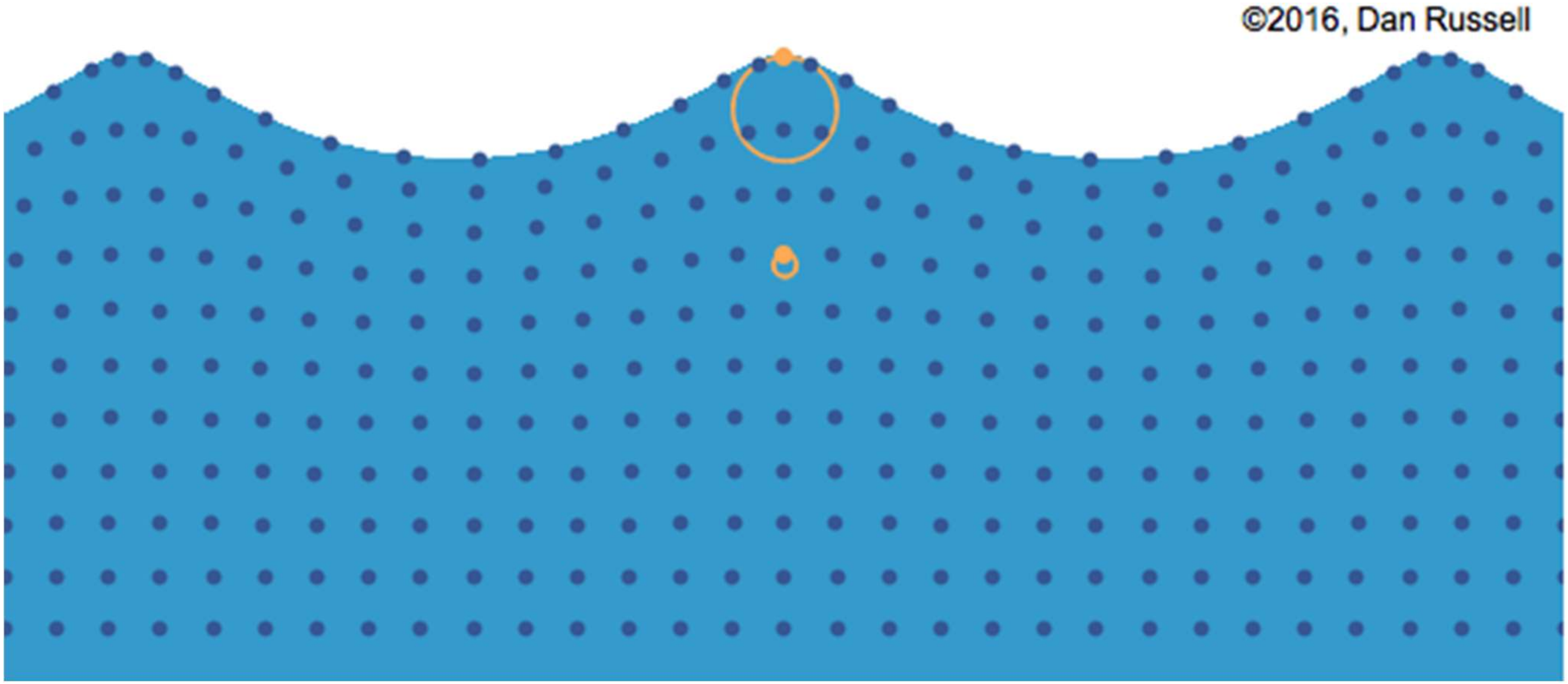


Many systems show oscillatory behaviour

- Heartbeats, breathing
- Comets, sunspots, planetary motion, tides
- Quartz crystal oscillators
- String on a guitar
- Electrical circuits

and *coordinated* oscillations are *waves*

- Water

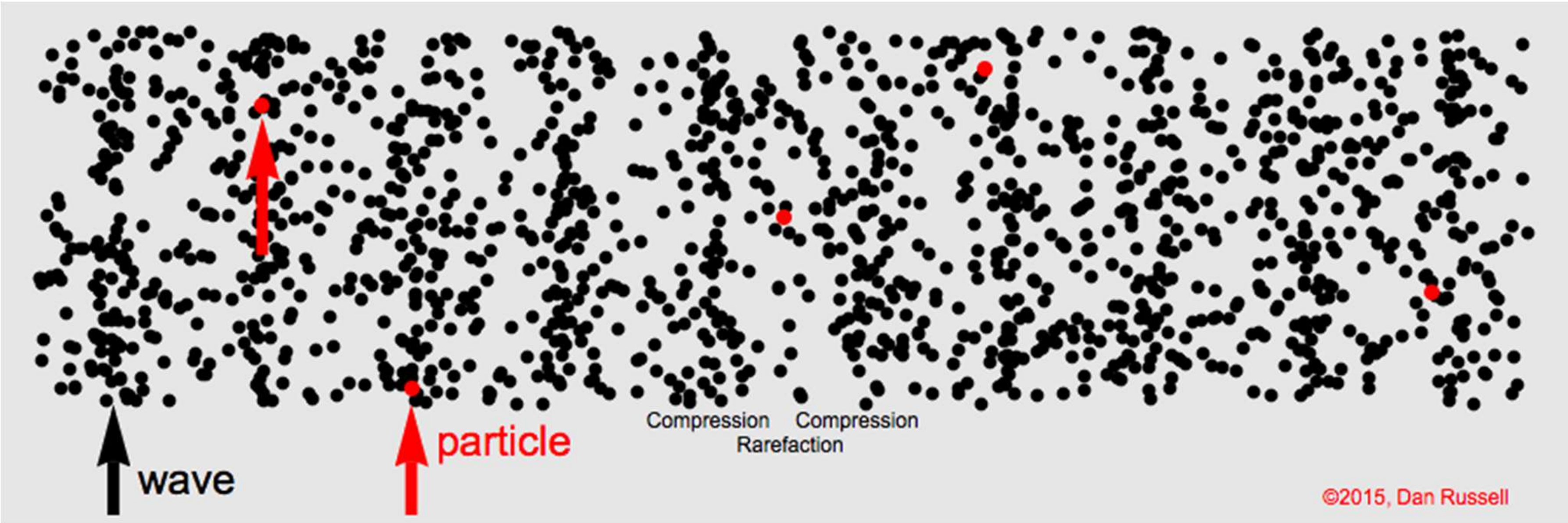


Animations courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State

http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

and *coordinated* oscillations are *waves*

- Water
- Earthquakes
- Sound



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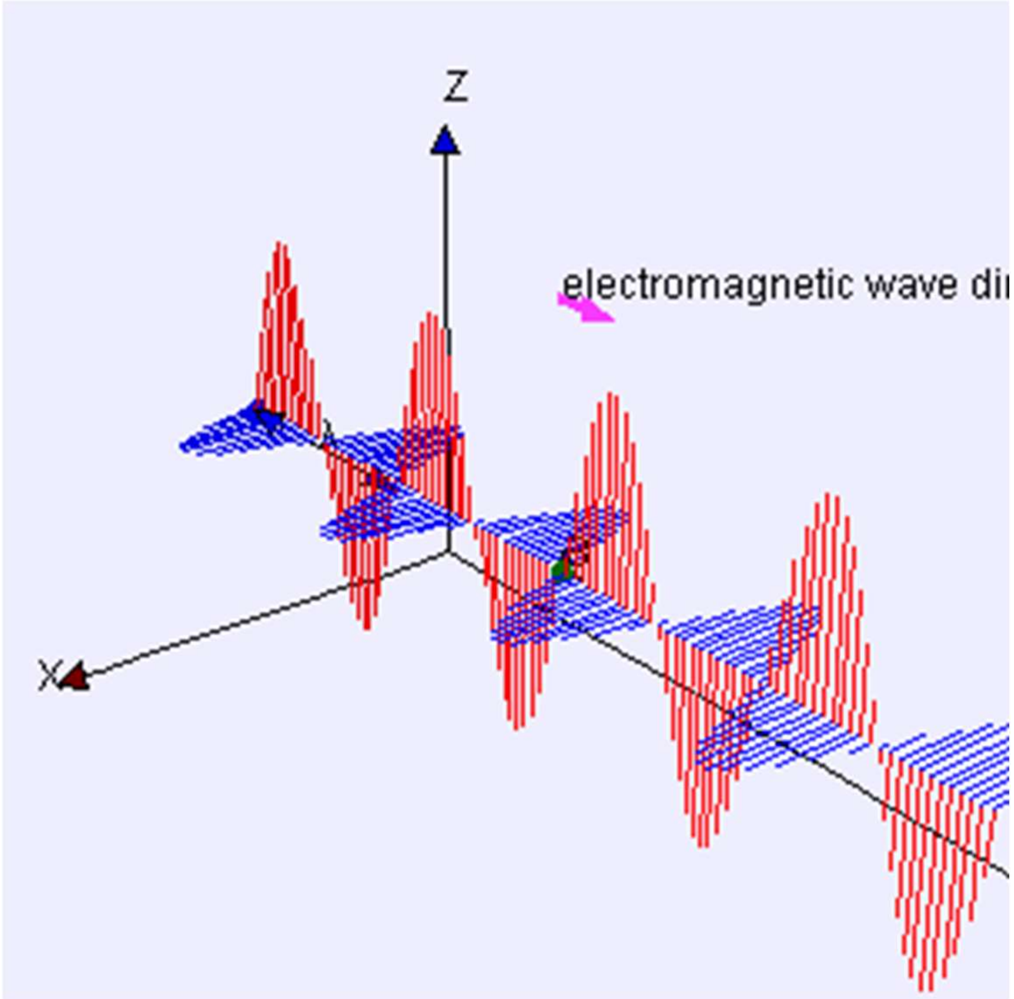
and *coordinated* oscillations are *waves*

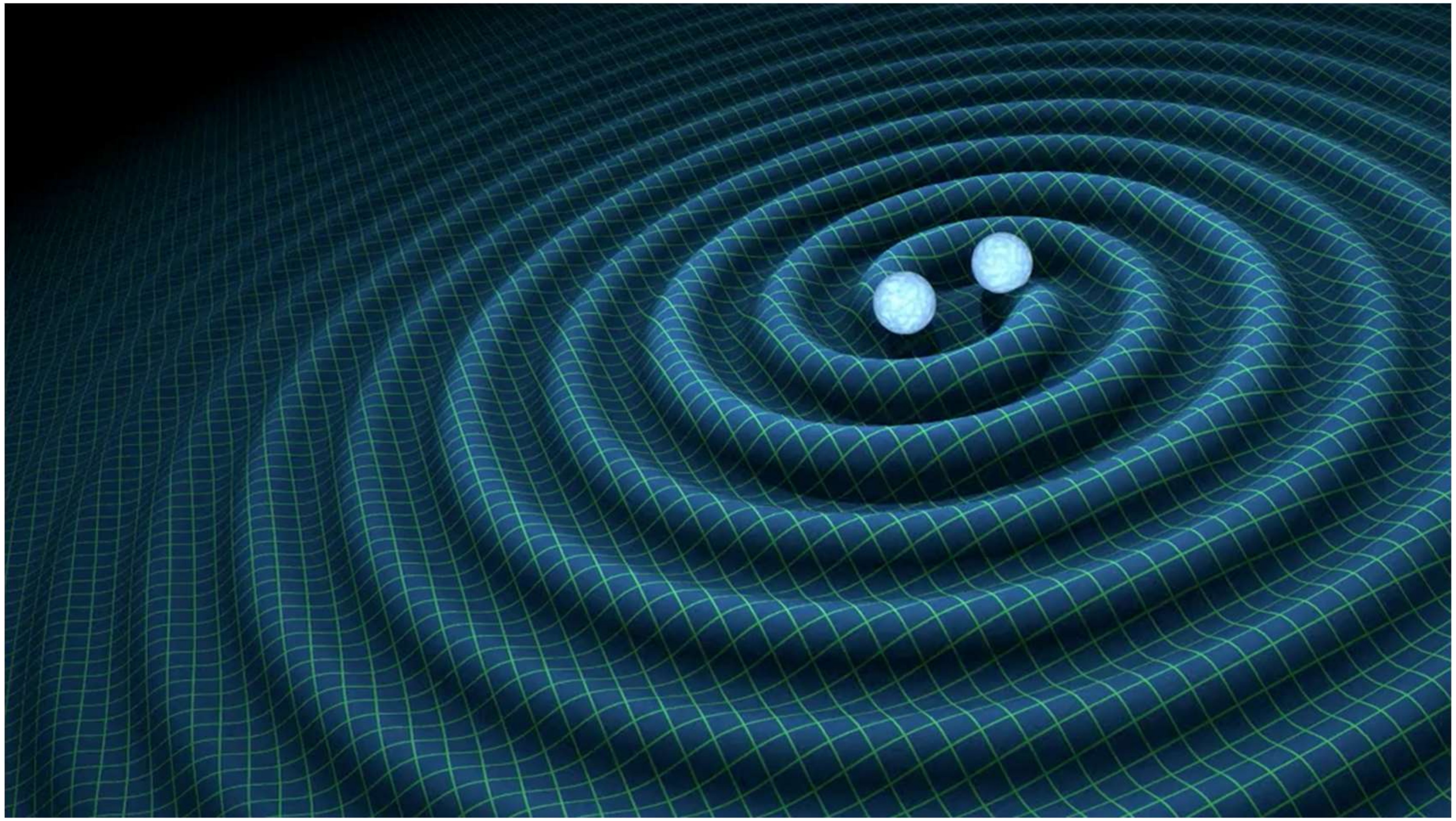
- Water
- Earthquakes
- Sound
- E/M



https://commons.wikimedia.org/wiki/File:Electromagneticwave3D.gif

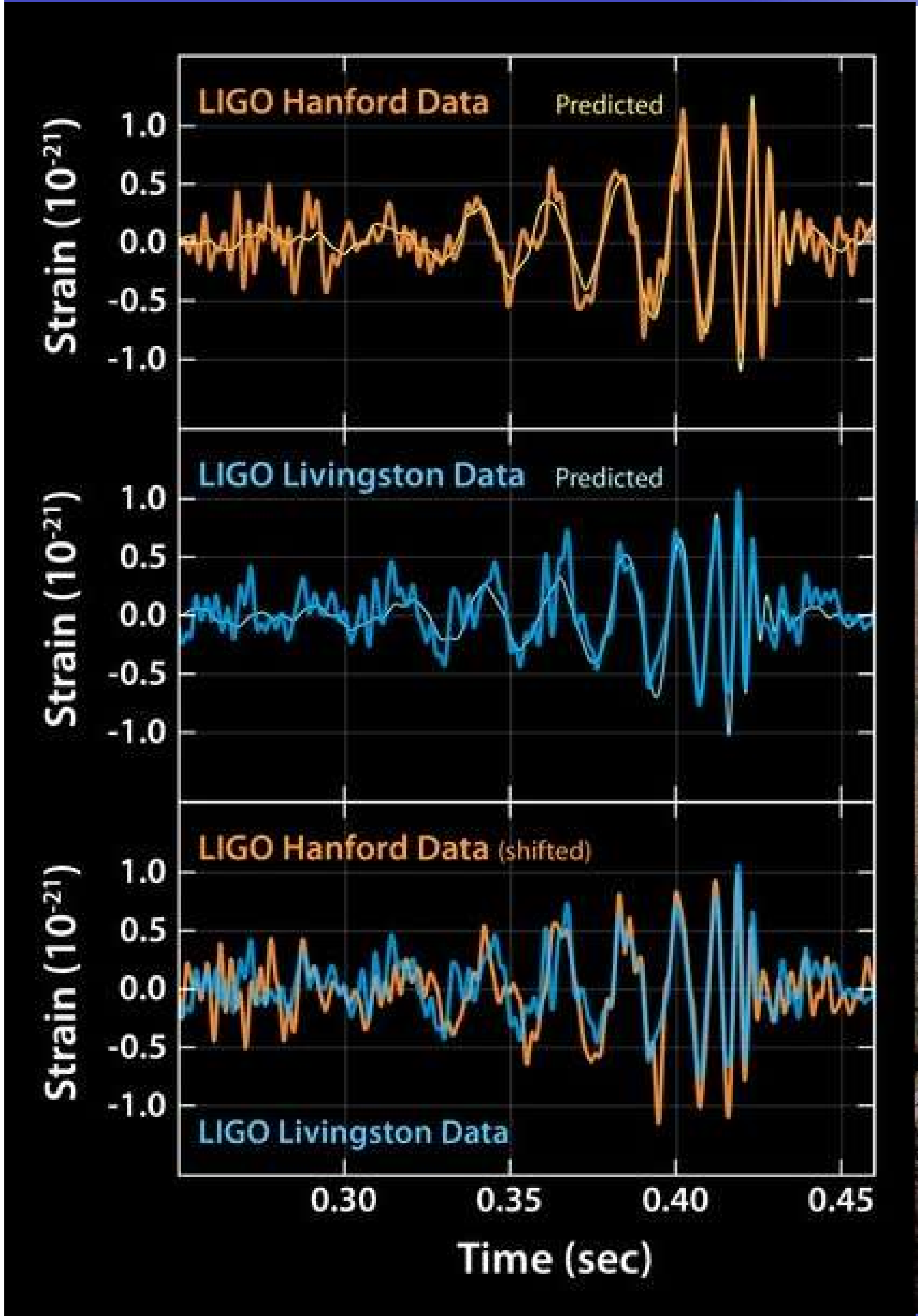
and *coordinated* oscillations are *waves*

- Water
- Earthquakes
- Sound
- E/M
- Gravitational



https://youtu.be/B4XzLDM3Py8

LIGO detects gravitational waves



https://youtu.be/tQ_teIUb3tE

https://youtu.be/I_88S8DWbcU

https://youtu.be/QyDcTbR-kEA

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla u$$

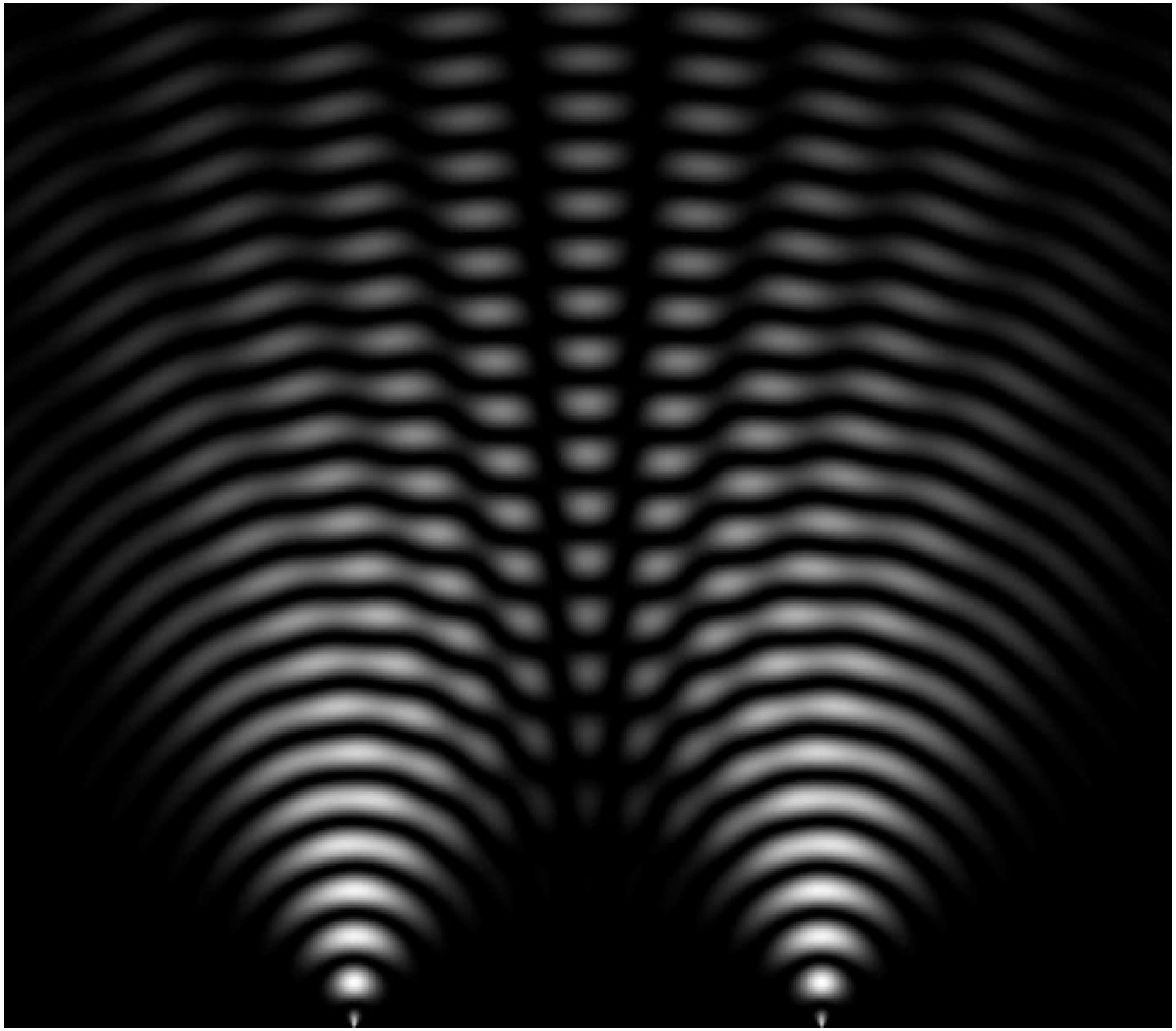
Wave equation

- diffraction

Waves, diffraction, and interference

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla u$$

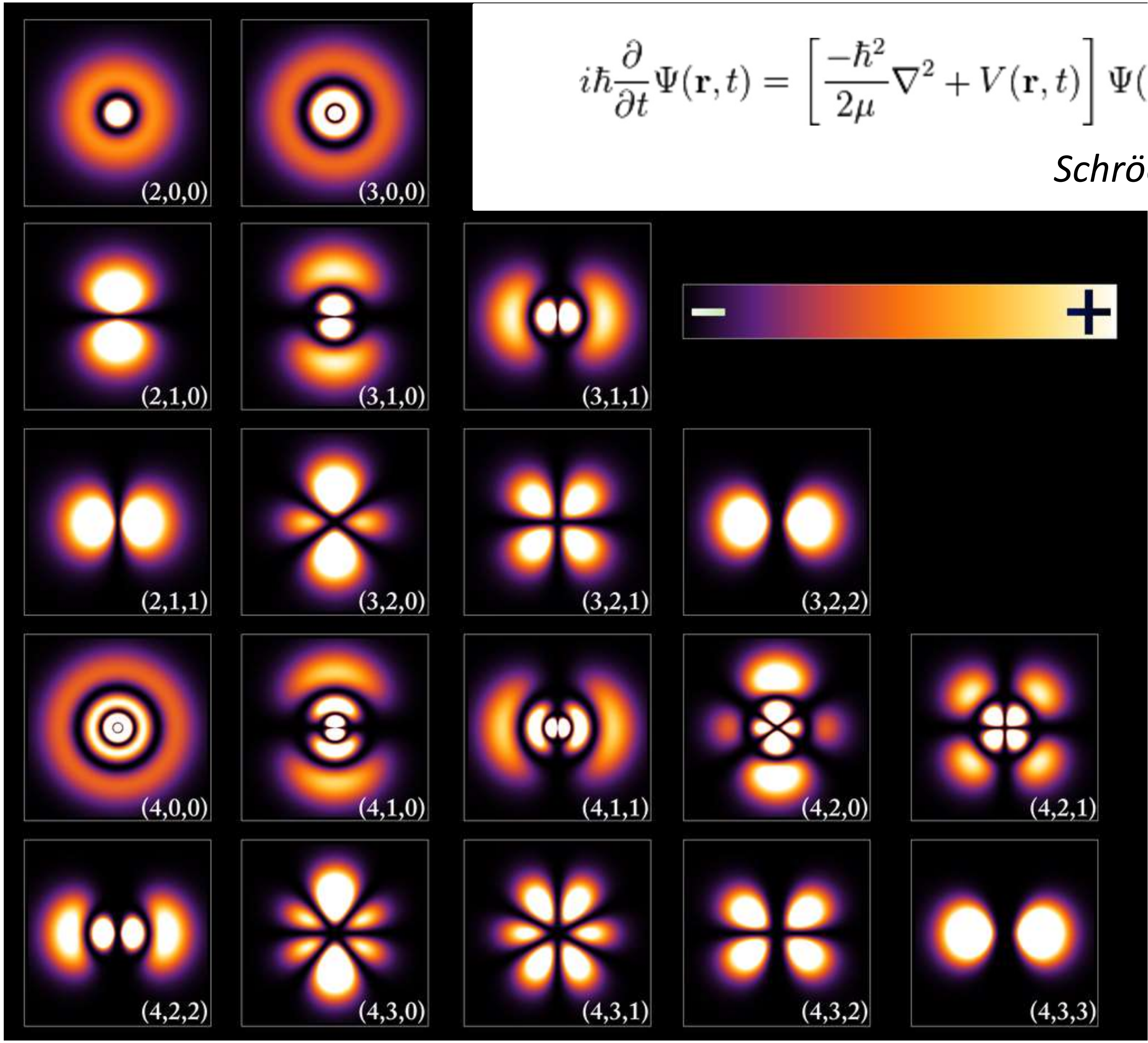
- diffraction
- interference
- beating



Hydrogen atom - wave functions

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2}{2\mu}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

Schrödinger equation



http://en.wikipedia.org/wiki/Wave_function#mediaviewer/File:Hydrogen_Density_Plots.png

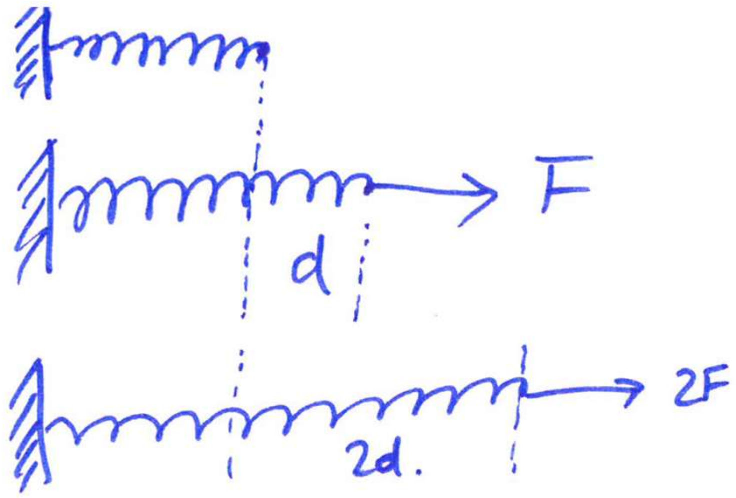
Schedule of topics

Week	Topic
1	General overview, simple harmonic motion.
2	Superposition and damped harmonic motion.
3	Forced oscillation, resonance.
4	Coupled simple harmonic motion.
5	Coupled SHM and loaded strings.
6	Waves.
7	Interference and diffraction.
8	Schrödinger’s equation.
9	Infinite well.
10	Particle on a circle.
11	Free particle and observables.
12	Potential step.
13	Potential barrier.

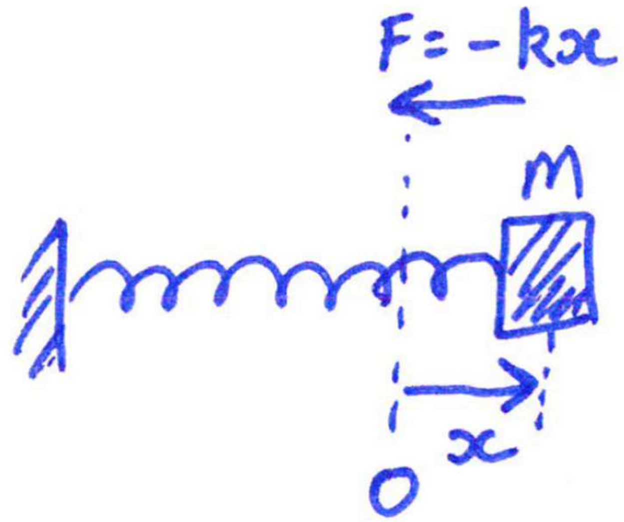
Simple harmonic motion

Mass on a spring

- Hooke's law $F = -kd$
  - d is extension of spring from relaxed length
  - Linear restoring force characterised by stiffness k

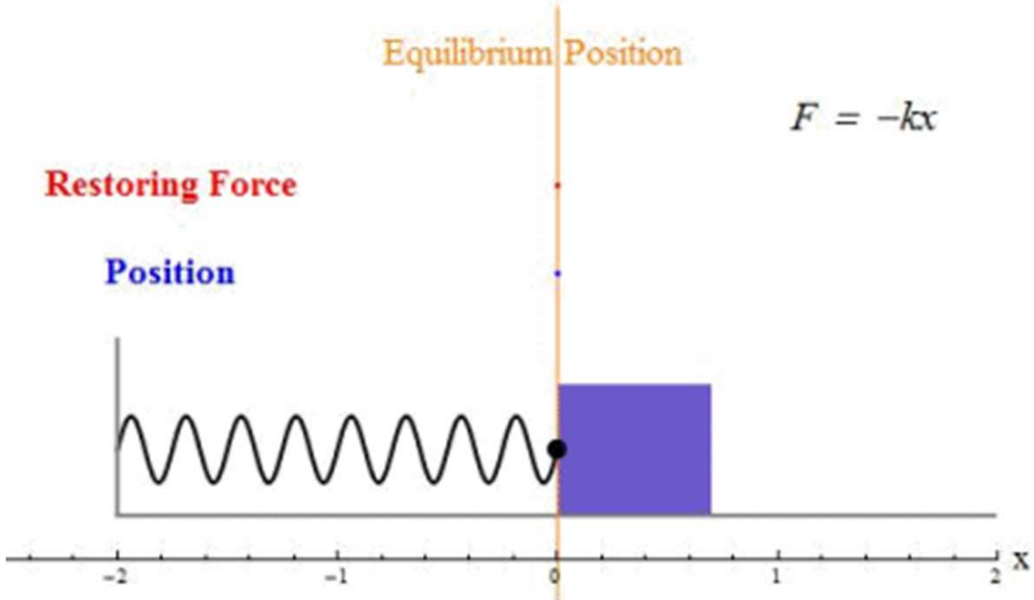


- Consider the system at some non-equilibrium position
  - Use $F = ma$
    - $-kx = ma = m\ddot{x}$
    - $\ddot{x} = -(k/m)x$
  - $\ddot{\boldsymbol{x}} = -\boldsymbol{\omega}^2\boldsymbol{x}$ with $\omega = \sqrt{k/m}$



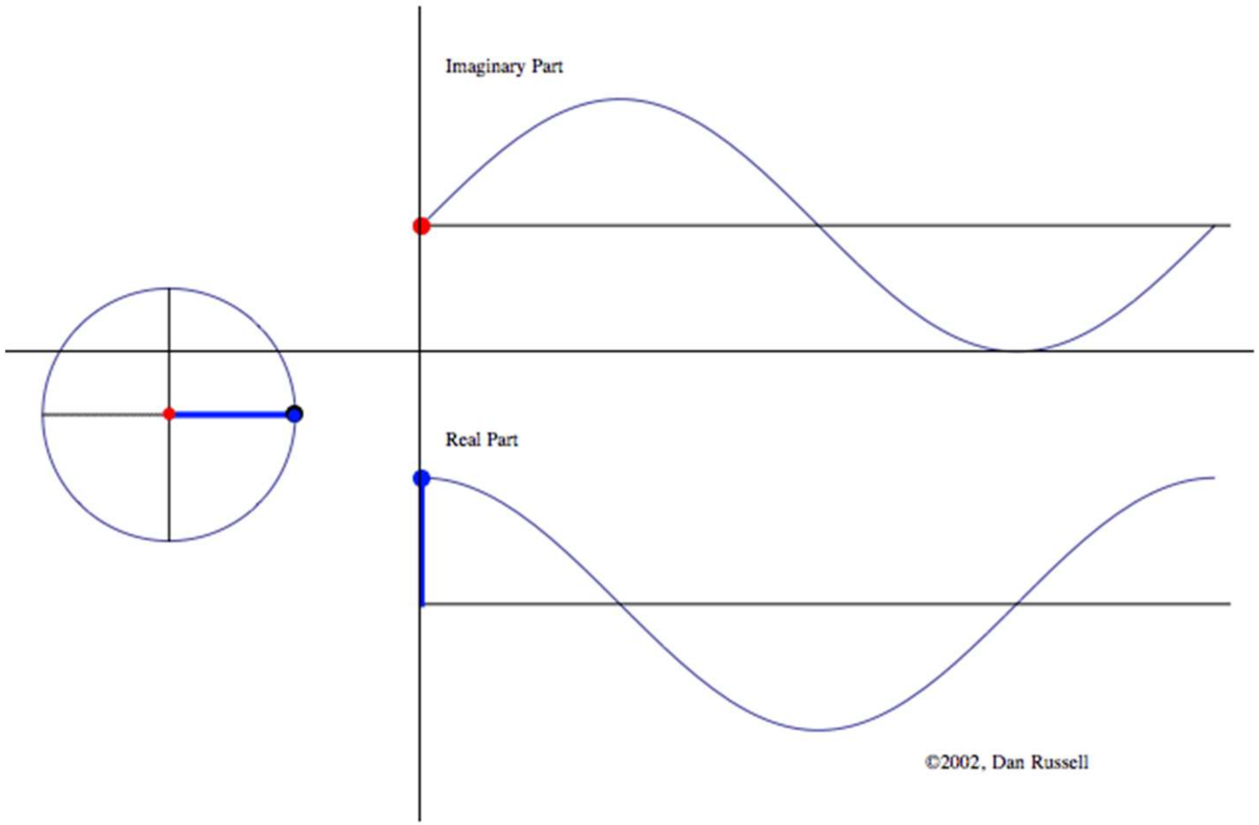
- Try (one possible) solution to $\ddot{x} = -\omega^2 x$
  - $x = A\sin\omega t$
  - $\frac{dx}{dt} = \dot{x} = A\omega\cos\omega t$
  - $\frac{d^2x}{dt^2} = \ddot{x} = -A\omega^2 \sin\omega t = -\omega^2 x$ as required ☺

- Position of mass will oscillate with time with
  - Angular frequency ω
  - Period $T = \frac{2\pi}{\omega}$
  - Frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$
  - Amplitude A



Circular motion vs SHM

One component of uniform circular motion looks like SHM



….. motivates representation by complex numbers (see later)

Spring with gravity?

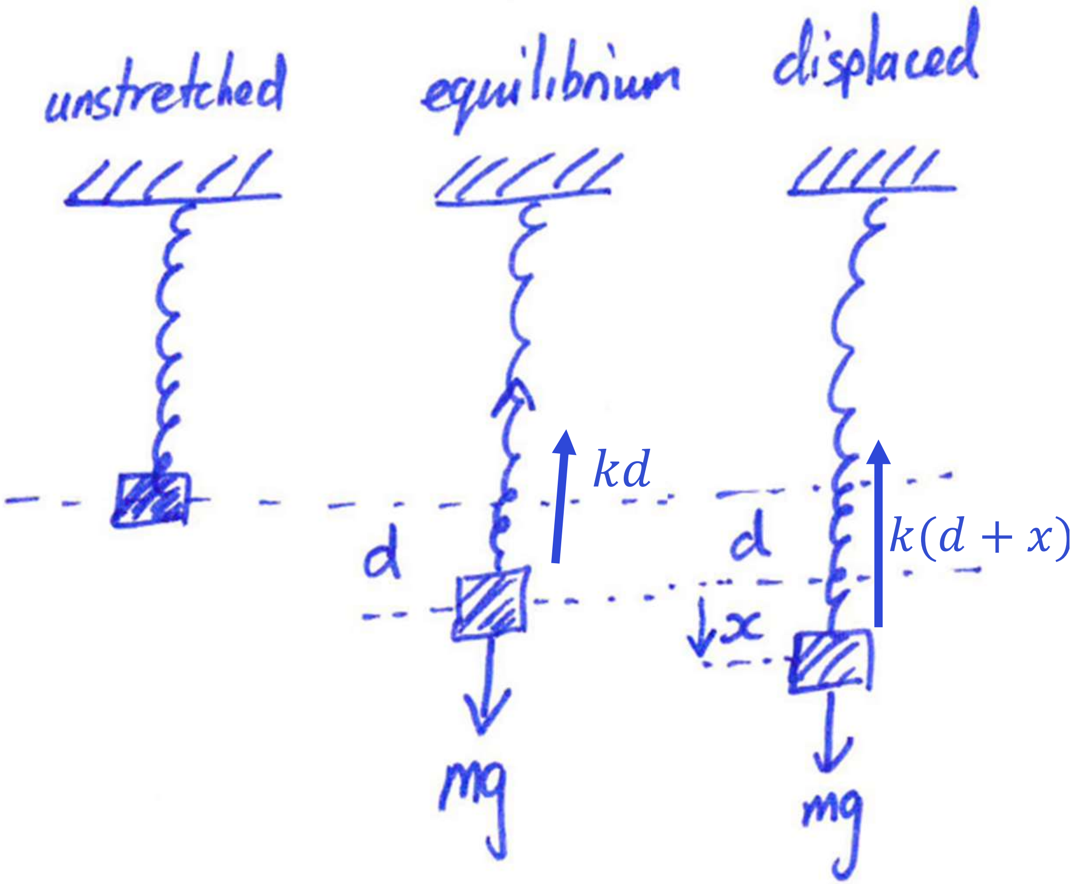
- $mg = kd$ at equilibrium

When displaced,

$$\begin{aligned} ma &= F \\ m\ddot{x} &= mg - k(d+x) \\ &= (mg - kd) - kx \\ &= -kx \end{aligned}$$

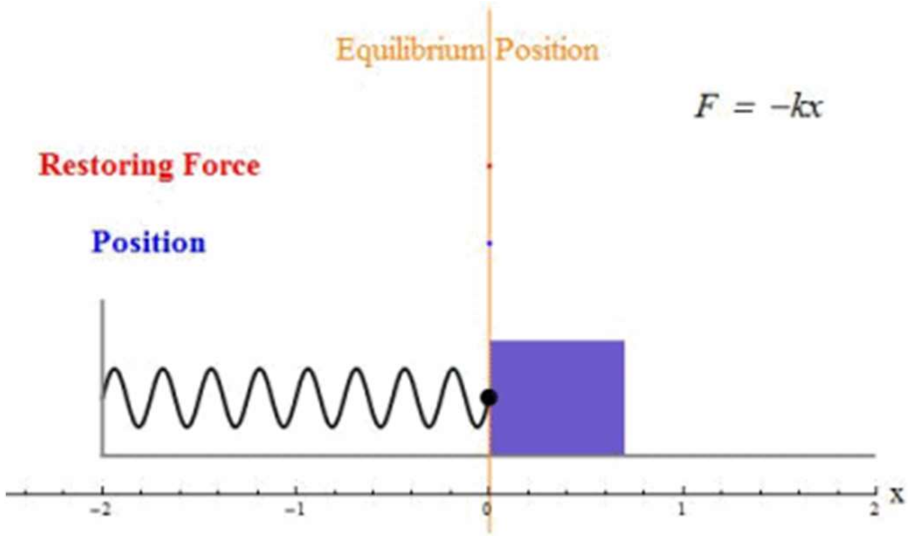
$\ddot{x} = -\omega^2 x$ as before

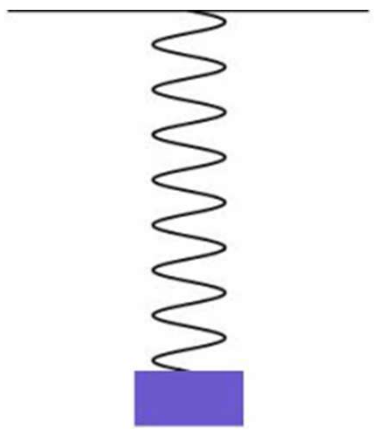
- Oscillates about equilibrium position

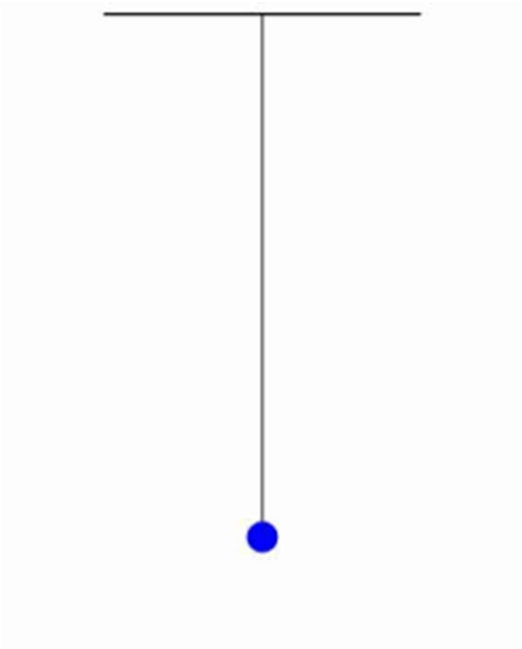


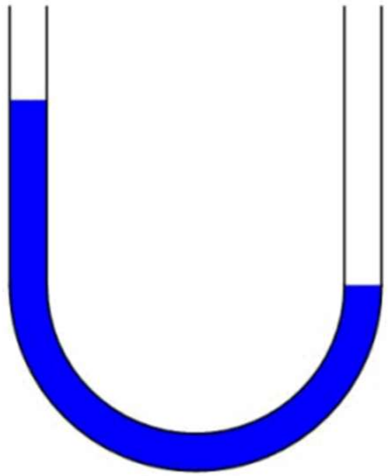
NB: we chose downwards to be the "+" direction

Generalise to 1D SHM









http://mathforum.org/mathimages/index.php/Simple_Harmonic_Motion

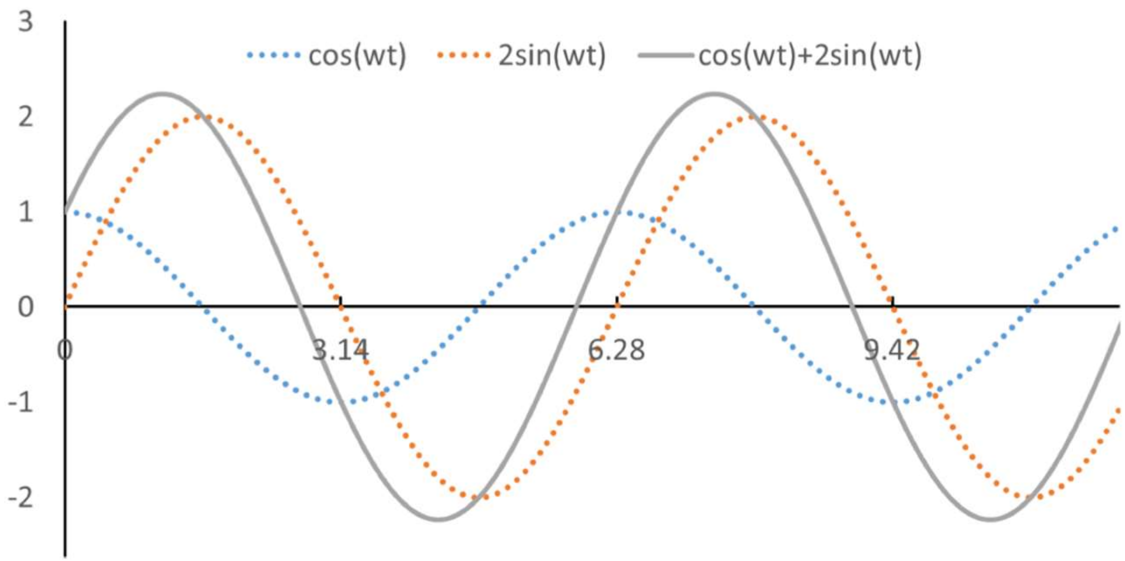
SHM general solution

- SHM occurs when a system experiences a linear restoring force with displacement from equilibrium
    - $F = -kx$
- $\ddot{x} + \omega^2 x = 0$
    - Second-order linear differential equation with constant coefficients

- We know $x = B \sin \omega t$ is a solution. So too is $x = A \cos \omega t$
- General solution is $\boldsymbol{x = A\cos\omega t + B\sin\omega t}$

- Constants A and B are determined by initial conditions

- Sum is still ‘sinusoidal’



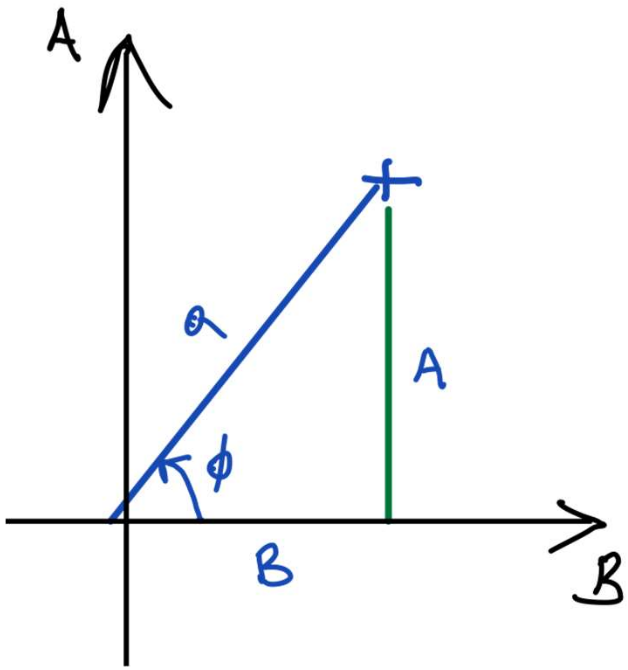
Amplitude and phase

- General solution is $x = A\cos\omega t + B\sin\omega t$

Write $A = a\sin\phi$ and $B = a\cos\phi$

$$a^2 = A^2 + B^2$$

$$\phi = \sin^{-1}(A/a) \quad \text{or} \quad \cos^{-1}(B/a)$$



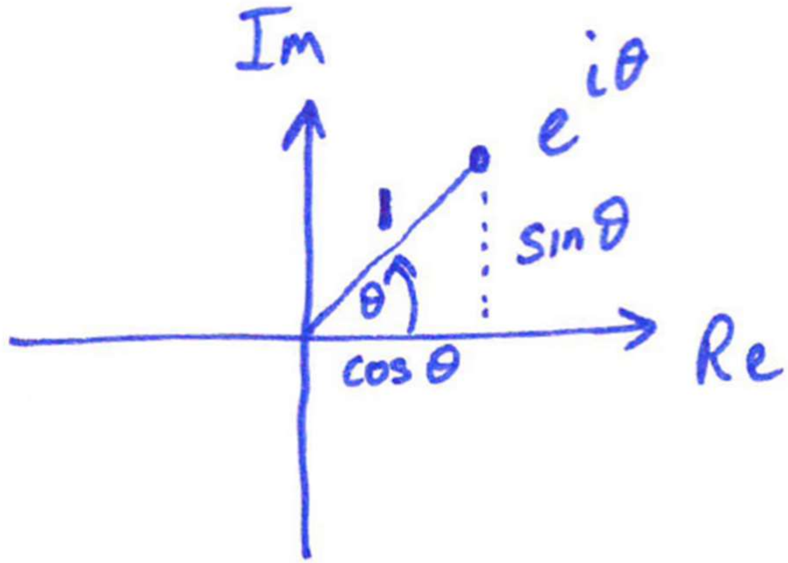
$$x = a\sin\phi\cos\omega t + a\cos\phi\sin\omega t$$

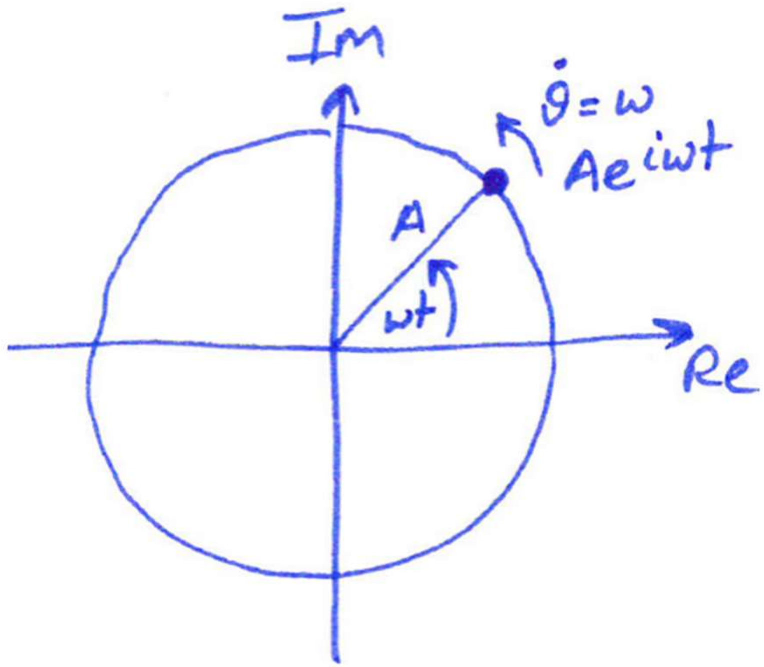
$$\boldsymbol{x = a\sin(\omega t + \phi)}$$

- Summing cos and sin creates a sinusoid with shifted phase

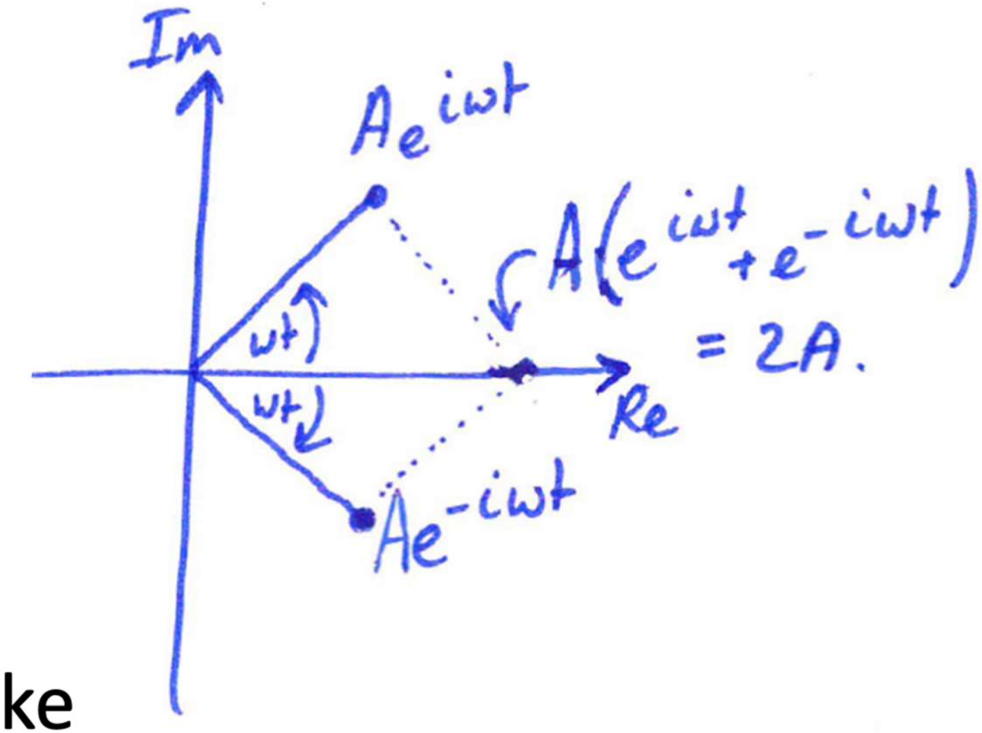
Complex exponentials

- Recall that $e^{i\omega t} = \cos\omega t + i\sin\omega t$
- Thus we can write
  - $\mathrm{Re}\left[Ae^{i\omega t}\right] = A\cos\omega t$
  - $\mathrm{Re}\left[Ae^{i(\omega t+\phi)}\right] = A\cos(\omega t+\phi)$
  - $\mathrm{Re}\left[Ae^{i\phi}e^{i\omega t}\right] = A\cos(\omega t+\phi)$
  - $\mathrm{Re}\left[Ze^{i\omega t}\right] = A\cos(\omega t+\phi)$
- $Ze^{i\omega t}$ indeed a solution to $\ddot{x} + \omega^2 x = 0$
- We can use $x = Ze^{i\omega t}$ as our trial solution, and take the real part whenever we want a physical value





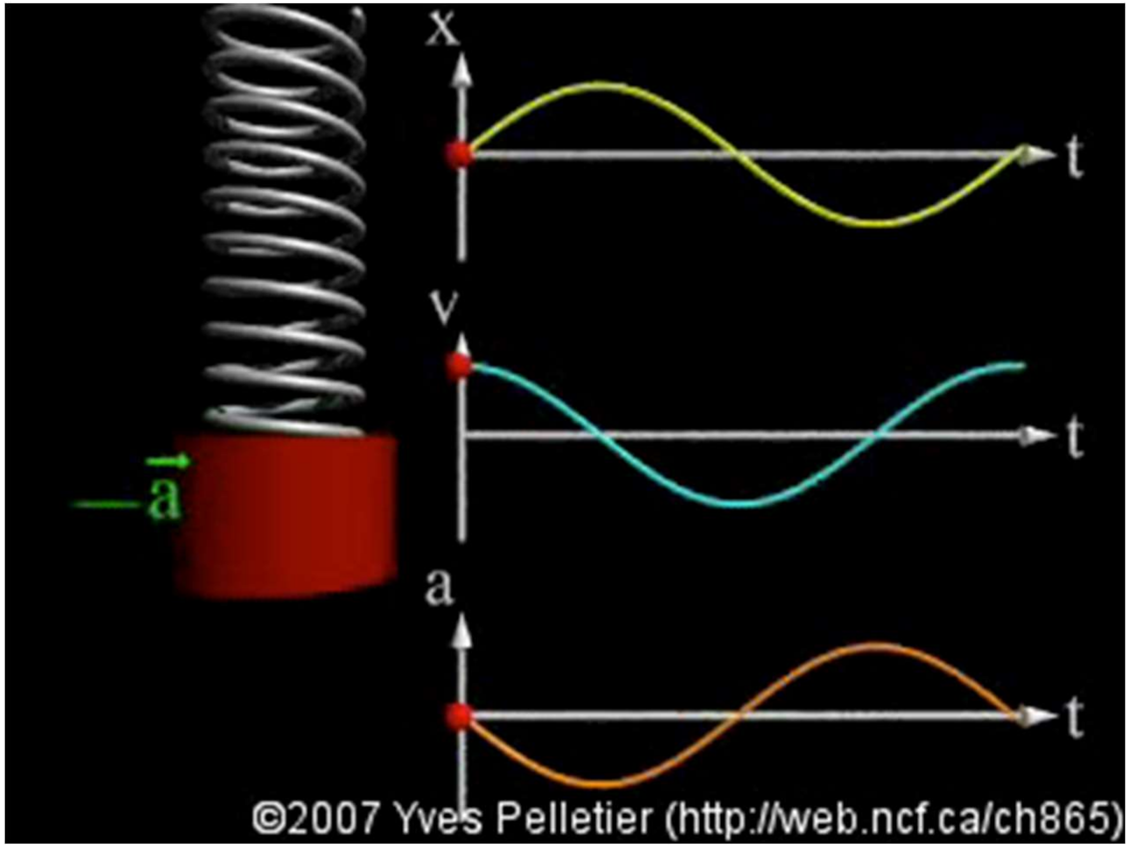
- Alternatively, we can construct a real-valued combination:
  - $Ae^{i\omega t} + Ae^{-i\omega t} = 2A\cos\omega t$
  - $Ae^{i\omega t} + c.c. = 2A\cos\omega t$



- This is also a solution to $\ddot{x} + \omega^2 x = 0$

- Has advantages over the 'take the real part', particularly when multiplying solutions in nonlinear problems

Displacement, velocity, and time



https://youtu.be.com/eeYRkW8V7Vg

Position, velocity, and acceleration

- Displacement $x(t) = A\sin(\omega t + \phi)$

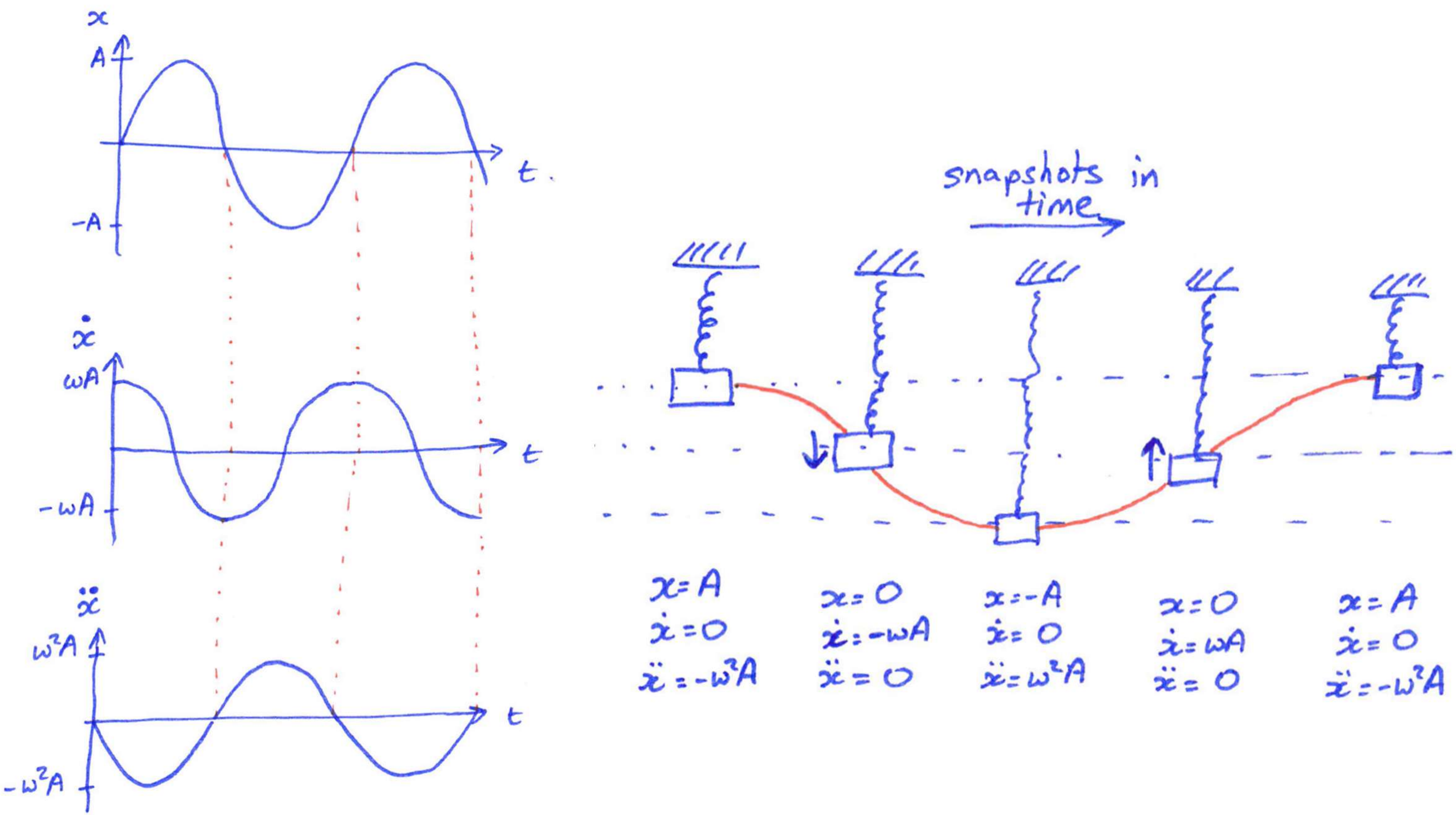
- Velocity
$$\begin{aligned}\dot{x}(t) &= A\omega\cos(\omega t + \phi)\\ &= A\omega\sin(\omega t + \phi + \pi/2)\end{aligned}$$

- Acceleration
$$\begin{aligned}\ddot{x}(t) &= -A\omega^2\sin(\omega t + \phi)\\ &= A\omega^2\sin(\omega t + \phi + \pi)\end{aligned}$$

- Velocity leads position by $\pi/2$ or 90°
  - e.g. maximum *positive* velocity occurs a quarter of a cycle before maximum *positive* displacement

- Acceleration out of phase by π or 180° (we knew that..)
  - e.g. maximum *positive* acceleration half a cycle before maximum *positive* displacement, or at the same time as maximum *negative* displacement

Displacement, velocity, and time

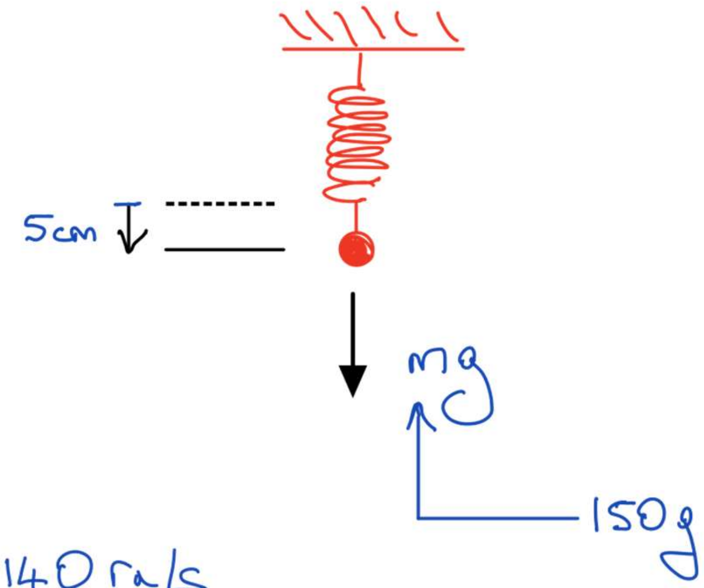


Example problem

A 150g potato is attached to the free end of a spring hanging from the ceiling. The spring extends by 5.0 cm then comes to rest. The potato is then pulled down a little, extending the spring further, and is then released, and the potato executes vertical, undamped, simple harmonic motion.

What is the period of oscillation?

The maximum speed of the potato is 2.5 cm/s. What is the amplitude of the oscillation?



$$F = -kx \Rightarrow k = \frac{mg}{5.0\,\text{cm}} = \frac{0.15 \times 9.8}{0.05} = 29.4\ \text{N/m}$$

$$\omega = \sqrt{\frac{k}{m}} = 14.0\ \text{rad/s}$$

$$T = \frac{2\pi}{\omega} = \underline{\underline{0.45\,\text{s}}}$$

$$x = A\sin(\omega t + \phi)$$

$$\dot{x} = \omega A\cos(\omega t + \phi) \Rightarrow \omega A = 2.5\ \text{cm/s}$$

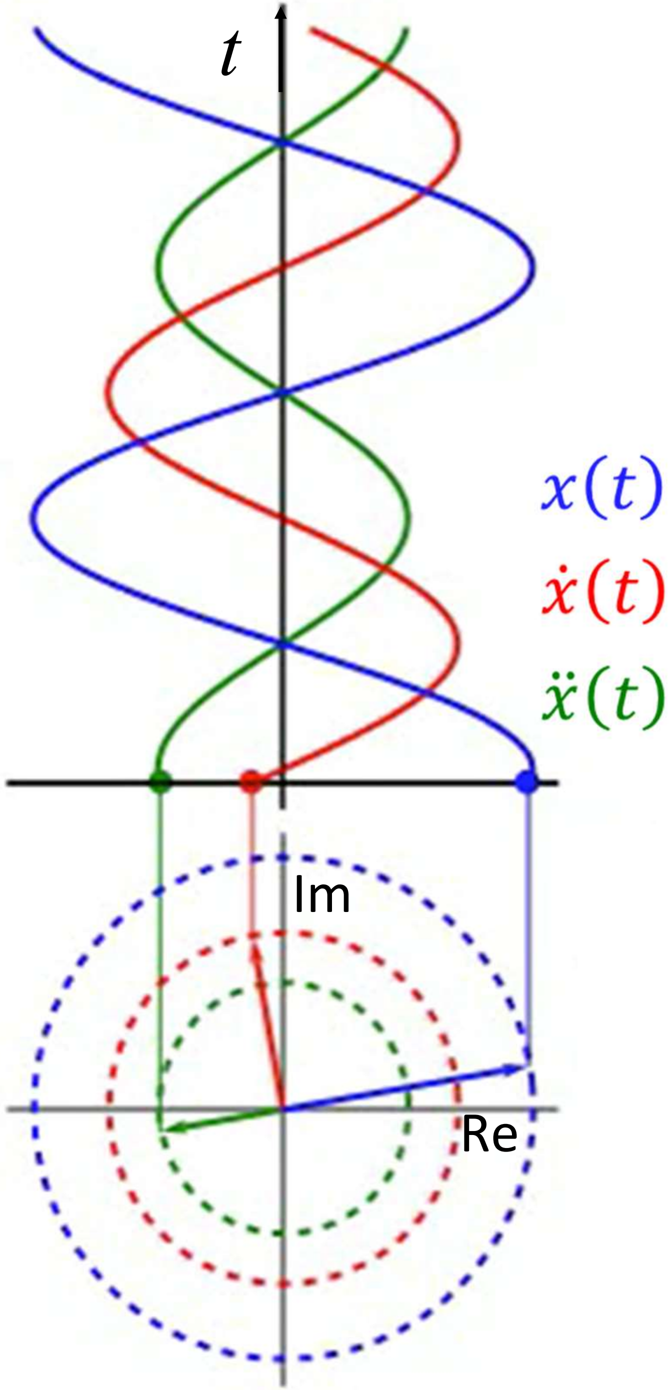
$$\Rightarrow A = \frac{2.5\,\text{cm/s}}{14\ \text{rad/s}} = \underline{\underline{1.8\,\text{mm}}}$$

Displacement, velocity, and time: Phasors

$$x(t) = A\sin(\omega t + \phi)$$

$$\dot{x}(t) = A\omega\cos(\omega t + \phi)$$
$$= A\omega\sin(\omega t + \phi + \pi/2)$$

$$\ddot{x}(t) = -A\omega^2\sin(\omega t + \phi)$$
$$= A\omega^2\sin(\omega t + \phi + \pi)$$

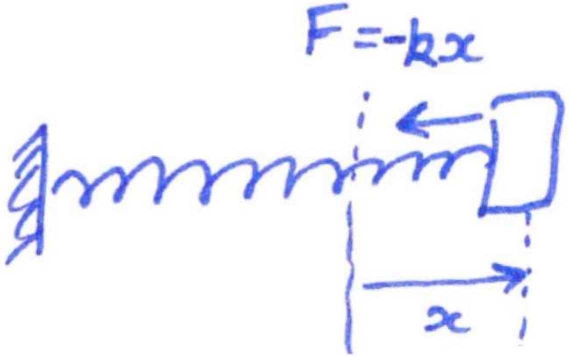


http://es.wikipedia.org/wiki/Movimiento_arm%C3%B3nico_simple#mediaviewer/File:Fasorxva.gif

Kinetic and potential energy

- Consider SHM with $x(t) = A\sin(\omega t + \phi)$

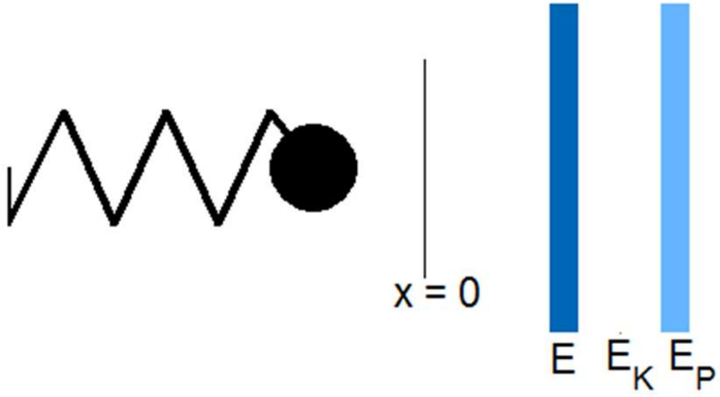
$$\dot{x}(t) = \omega A\cos(\omega t + \phi)$$



- Kinetic energy depends on velocity:
  - $KE = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega^2 A^2\cos^2(\omega t + \phi)$

$$= \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

$$= \frac{1}{2}k(A^2 - x^2)$$



- Potential energy is work done against a conservative force:
  - $PE = -\int F dl = \int_0^x kx dx$

$$= \frac{1}{2}kx^2 = \frac{1}{2}A^2\sin^2(\omega t + \phi)$$

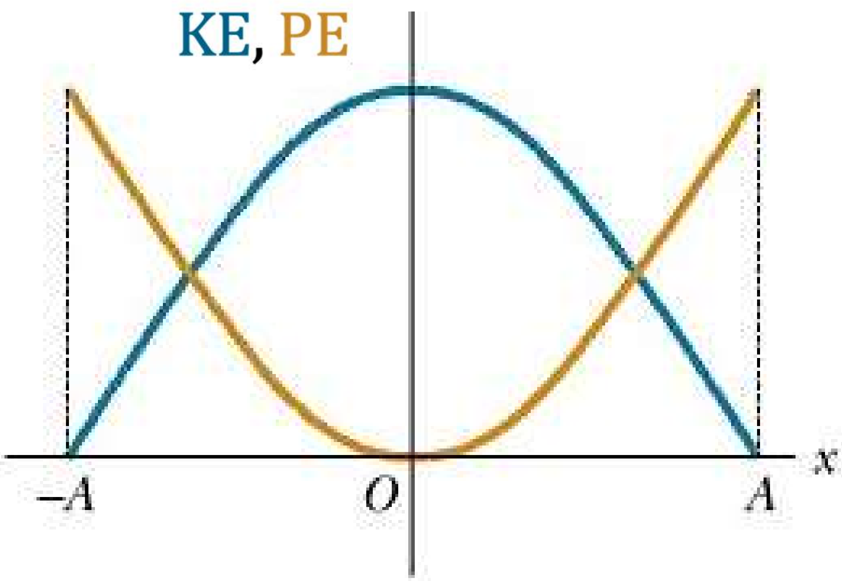
- Total energy is constant: $PE + KE = \frac{1}{2}kA^2$

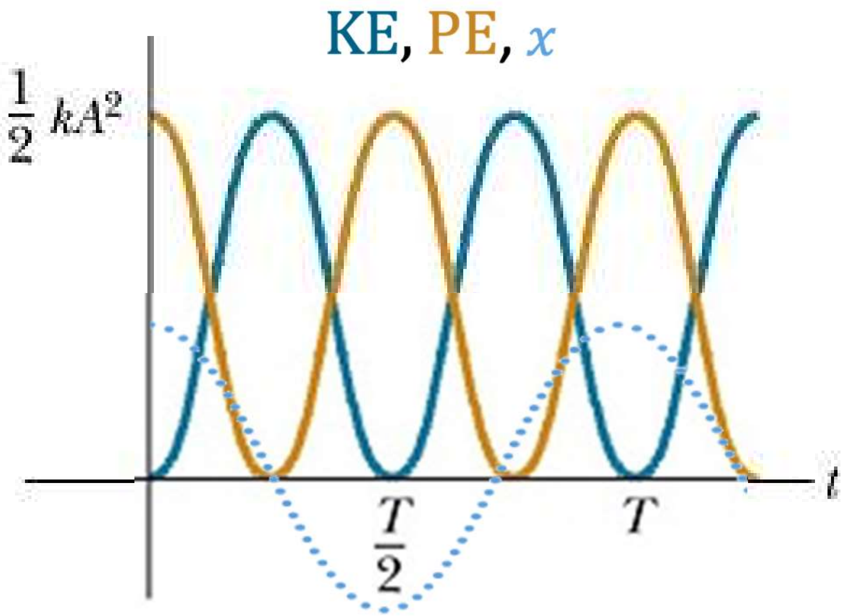
Kinetic and potential energy

$$\mathrm{KE} = \frac{1}{2}kA^2\cos^2(\omega t + \phi) = \frac{1}{2}kA^2(A^2 - x^2)$$

$$\mathrm{PE} = \frac{1}{2}kA^2\sin^2(\omega t + \phi) = \frac{1}{2}kx^2$$

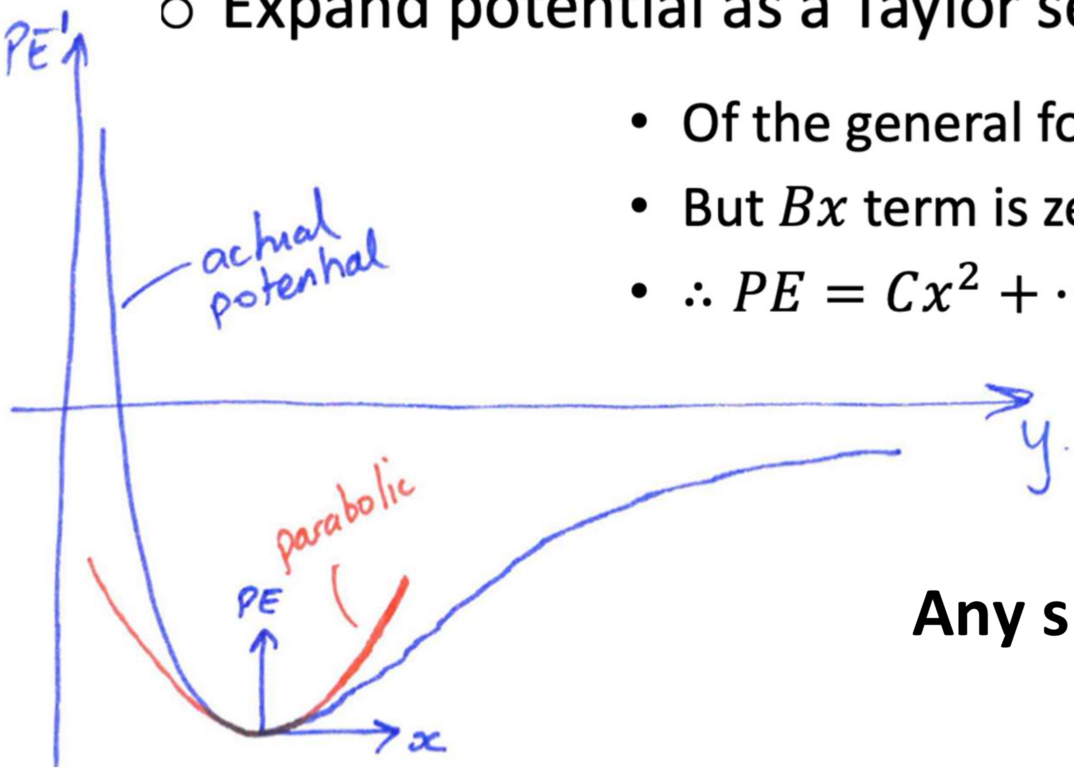
Total energy is $\frac{1}{2}kA^2$





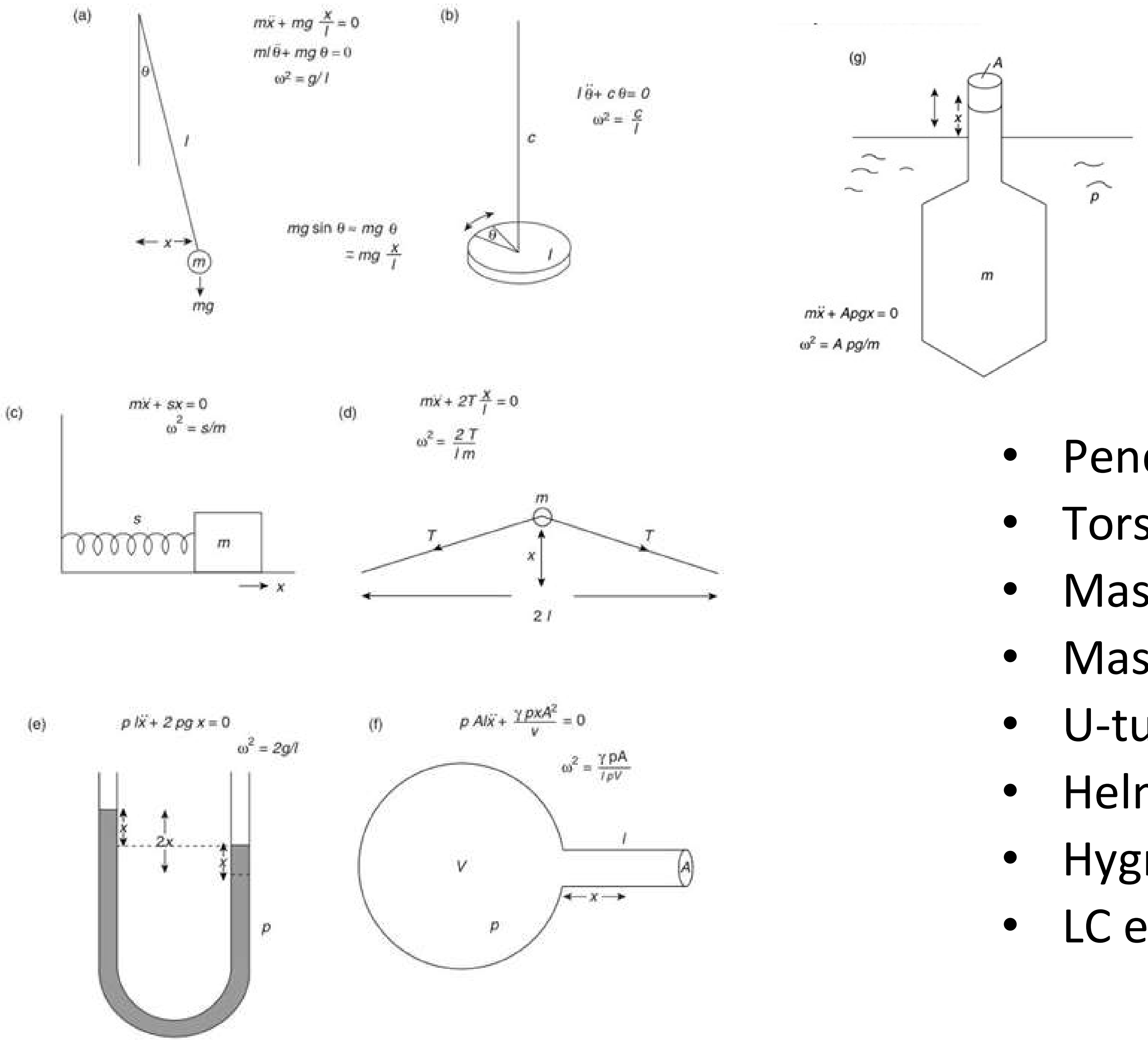
http://www.kshitij-iitjee.com/energy-of-a-simple-harmonic-oscillator

- SHM associated with a linear restoring force $F = -kx$
  - Equivalently, a parabolic potential well, $PE = \frac{1}{2}kx^2$
- Consider an arbitrary potential well
  - e.g. Electric potential energy of molecular bonding
  - Expand potential as a Taylor series around the stable point:
    - Of the general form $PE = A + Bx + Cx^2 + Dx^3 + \cdots$
    - But Bx term is zero at a stable point
    - $\therefore PE = Cx^2 + \cdots$ and so $F = -2Cx + \cdots$



Any small oscillation about a stable point is SHM!

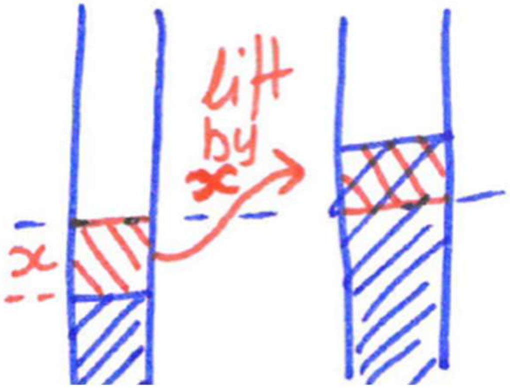
SHM is everywhere…

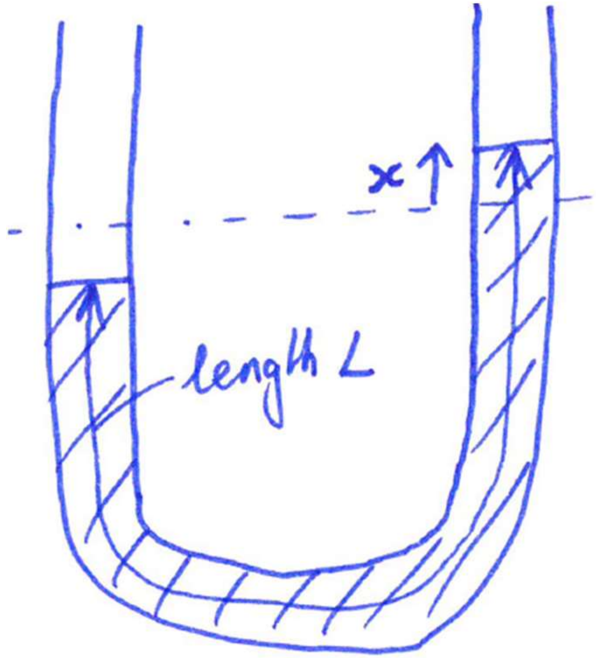


- Pendulum
- Torsional pendulum
- Mass and spring
- Mass on string
- U-tube
- Helmholtz resonator
- Hygrometer
- LC electrical circuit

U-tube SHM

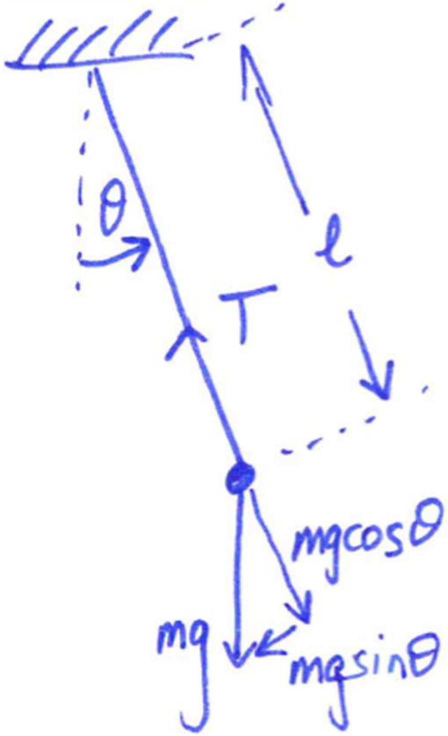
- Consider a u-tube
  - o uniform cross section A
  - o fluid density ρ
  - o displaced by $\pm x$
- Derive total energy:
  - o $KE = \frac{1}{2}(\rho LA)\dot{x}^2$
  - o $PE = (\rho x A)gx$
  - o $KE + PE = \frac{1}{2}\rho LA\dot{x}^2 + \rho g A x^2$
- Differentiate wrt time:
  - o $\rho LA\dot{x}\ddot{x} + 2\rho g A x\dot{x} = 0$ (chain rule!)
  - o $\ddot{x} = -\frac{2g}{L}x$ and $\boldsymbol{\omega} = \sqrt{\frac{2g}{L}}$





Pendulum

- Using $\tau = I\ddot{\theta}$ for a point mass
  - $-mgl\sin\theta = ml^2\ddot{\theta}$
  - $\ddot{\theta} = -(g/l)\theta$ for small angles
  - SHM with $\boldsymbol{\omega} = \sqrt{\boldsymbol{g/l}}$
  - $\theta = A\cos(\omega t)$
  - Independent of mass



$I = ml^2$

- For arbitrary shapes, we need to know:
  - mass m
  - distance from pivot to centre of mass l_{CM}
  - moment of inertia about the pivot I
  - $\boldsymbol{\omega} = \sqrt{\dfrac{\boldsymbol{mgl_{CM}}}{\boldsymbol{I}}}$