

# Revision

- 1 Introduction to Mechanical Engineering Design
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- 9 Belt and Pulleys
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# Manufacturing drawings

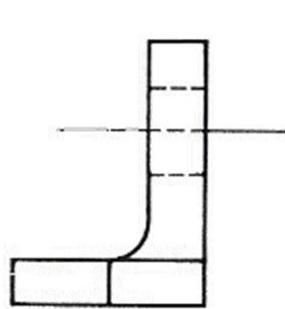
Important things to note in a manufacturing drawing.

1. Different types of views (cross section,)
2. Different types of projections (third angle projection, first angle projections)
3. Appropriate scale
4. Dimensions and tolerances
5. Title block
6. Surface finish
7. Different types of lines (centre lines, hidden lines)

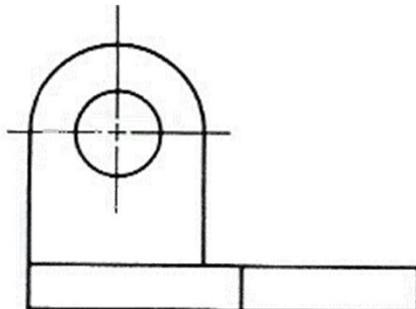
All these features for a manufacturing drawing are dictated by the Australian Standard AS1100.

# Manufacturing drawings

Projection	Symbol
First angle	 
Third angle	 



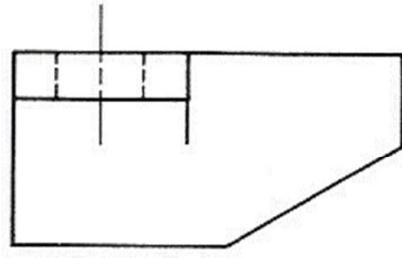
RIGHT-SIDE  
VIEW



FRONT VIEW

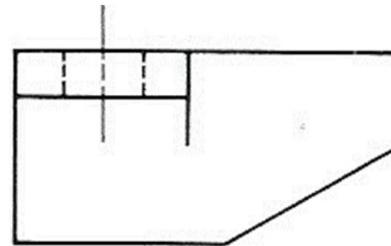


SYMBOL

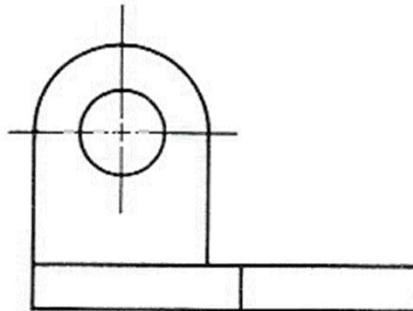


TOP VIEW

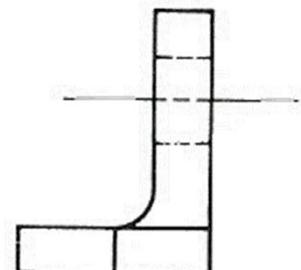
(a) FIRST-ANGLE PROJECTION



TOP VIEW



FRONT VIEW



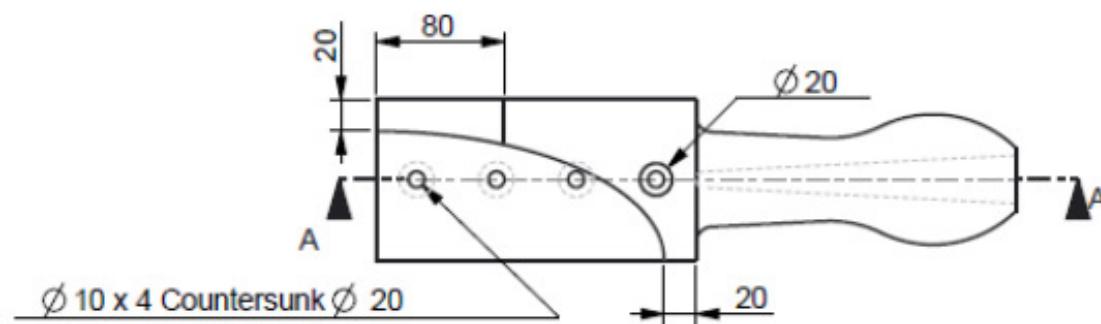
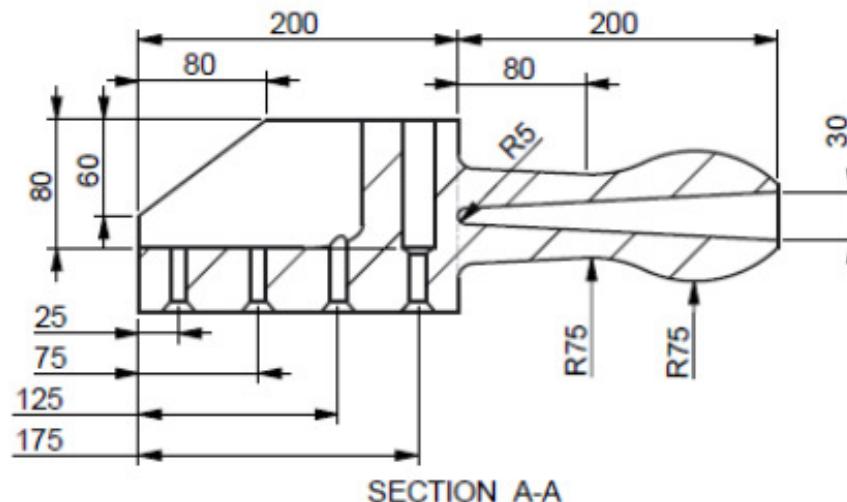
RIGHT-SIDE  
VIEW

(b) THIRD-ANGLE PROJECTION

# Manufacturing drawings

Other types of views that are useful in manufacturing drawing

1. Cross section view
2. Detailed views
3. Isometric views



# Manufacturing drawings

## Different types of lines used in manufacturing drawing

Different types of lines are used to depict different details in a drawing. These may be:

- Visible edge \_\_\_\_\_
- Hidden edge \_\_\_\_\_
- Tangents or \_\_\_\_\_
- Dimensions \_\_\_\_\_
- Centre lines \_\_\_\_\_

## Information Contained in Drawing Block

- What standard is the drawing drawn to
- What projection has been used
- Name or title of drawing
- Drawing number and/or revision number
- Name of company or organisation
- Material intended to be used for manufacture
- Names of engineers responsible and date drawn or approved

UNLESS OTHERWISE STATED ALL DIMENSIONS IN MILLIMETRES TOLERANCES: LINEAR: ANGULAR:	DRN AWB 22.3.00 CKD MJM 25.3.00 APPO T.J.L. 26.3.00 ISSUED L.J.B 30.3.00	A. SHAMBLES LTD
MATERIAL CAST STEEL		WALL BRACKET
DRAFTING STANDARD AS 1100	FINISH PAINT RED OXIDE	SIZE A: DRG N° 837624 SCALE 1:1 SHEET 1 of 1

# Manufacturing drawings

## Miscellaneous features of drawings

Similarly the scale should be selected from the following to best utilize the sheet size.

For enlargement: 2:1, 5:1, 10:1, 20:1, 50:1

Full size: 1:1

For reduction: 1:2, 1:2.5, 1:5, 1:10, 1:20, 1:50, 1:100, 1:200, 1:500, 1:1000, 1:2000, 1:5000, 1:10000

# Tolerances

Table 1.24(a) Selection of fits—hole-basis system

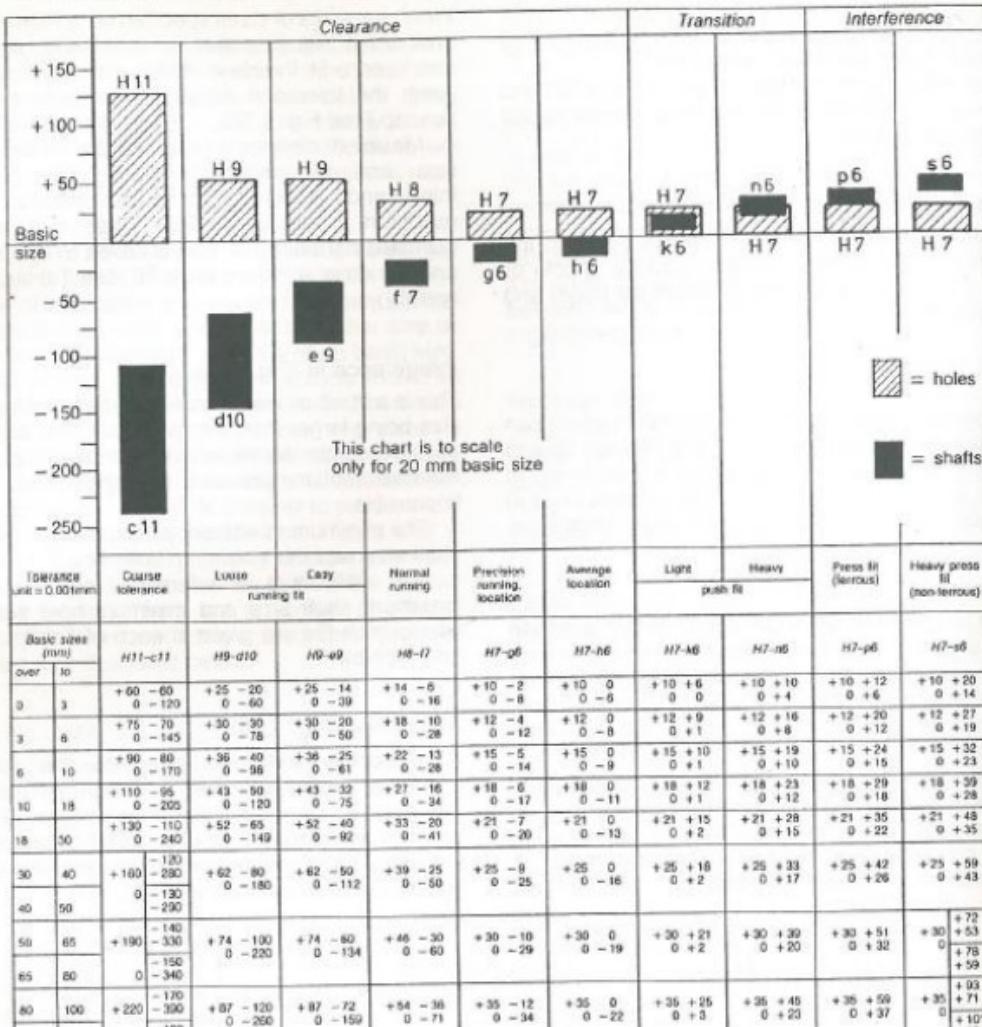
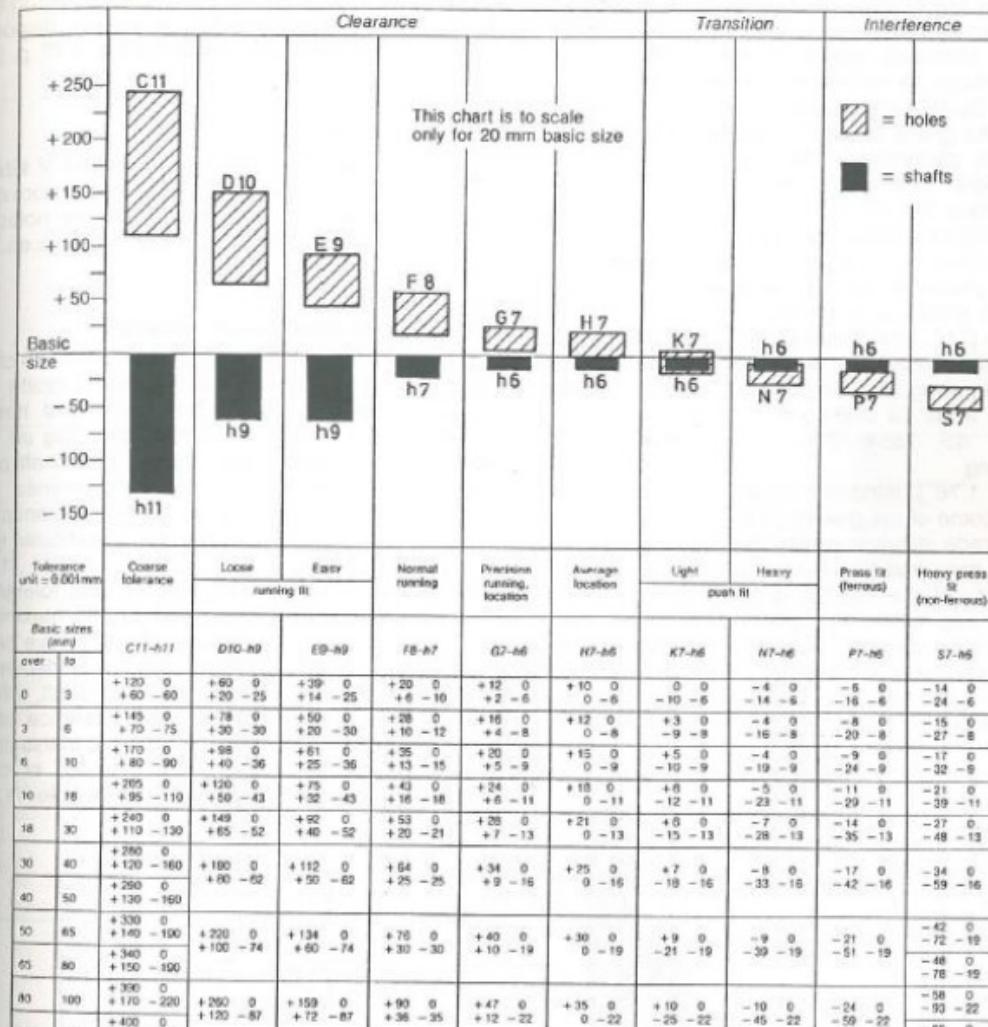
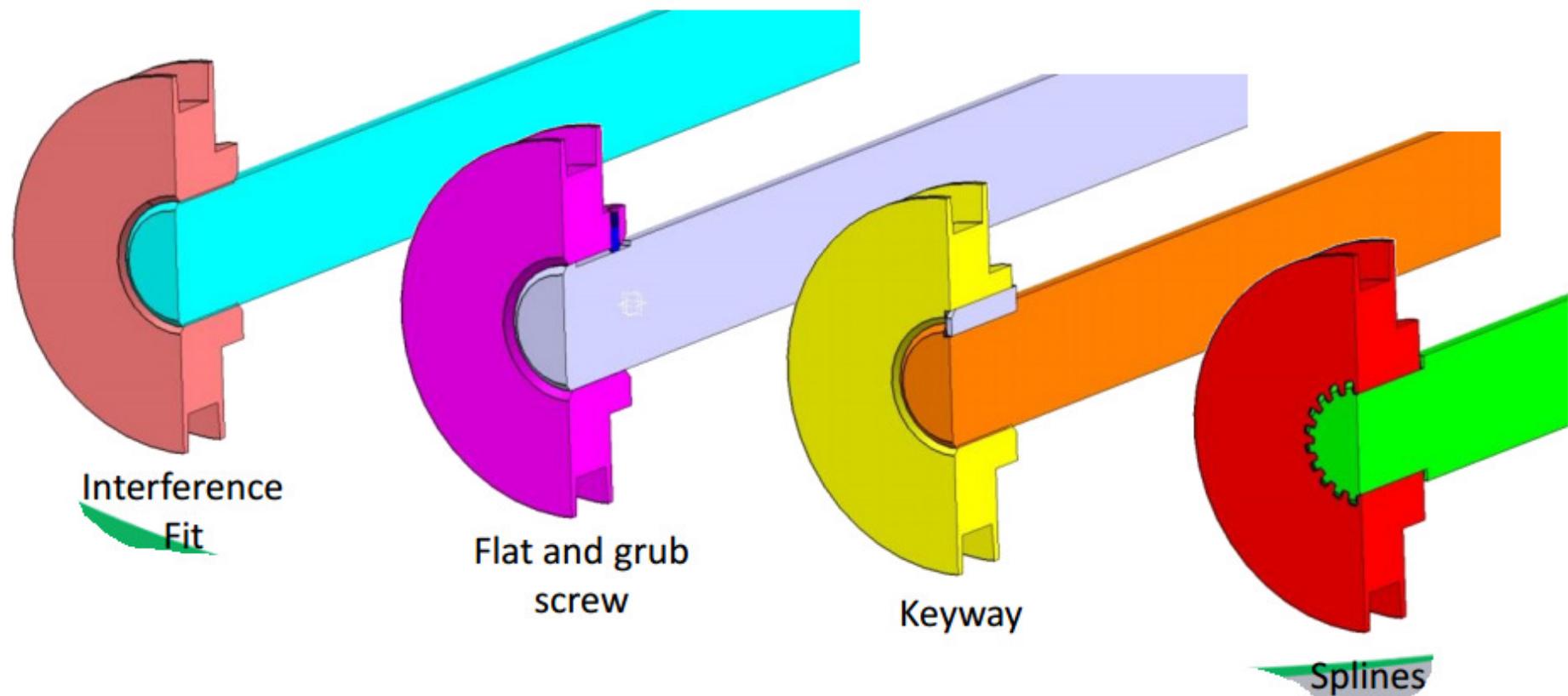


Table 1.24(b) Selection of fits—shaft-basis system



# Transferring torque between components

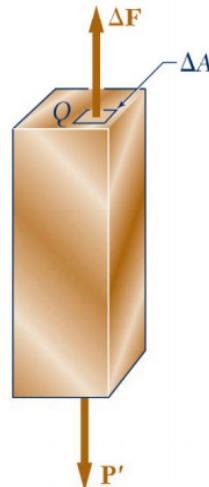


# Mechanical Engineering Design

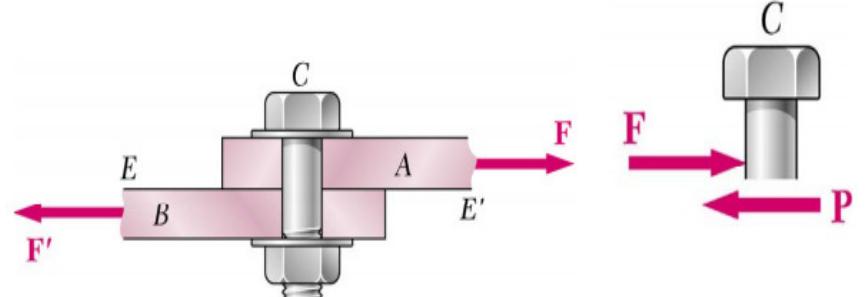
## Load and stress analysis

### Four main types of stresses

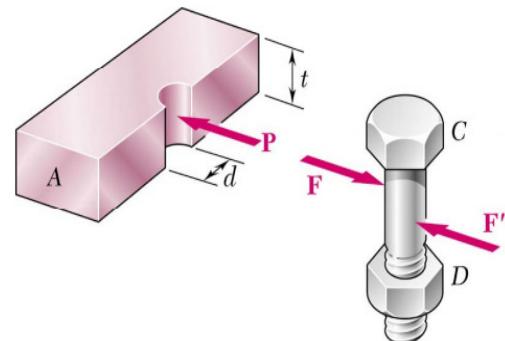
1. Normal stress



2. Shear stress

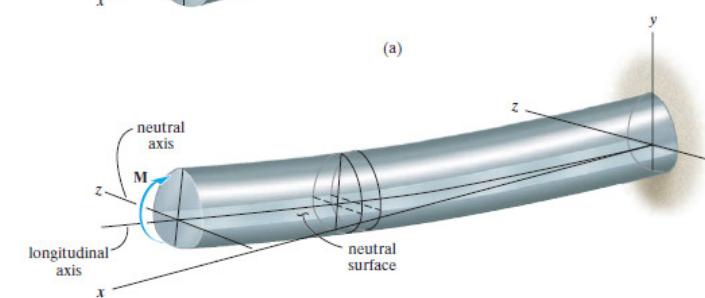
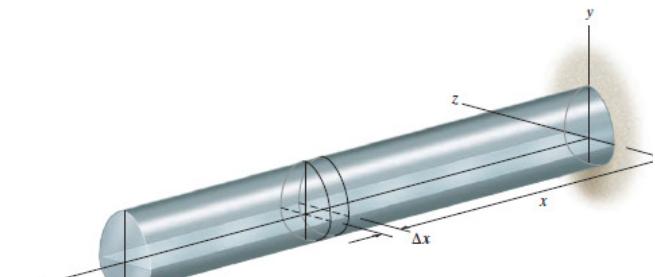


3. Bearing stress



$$\sigma_b = \frac{P}{A_b} = \frac{P}{td}$$

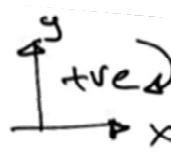
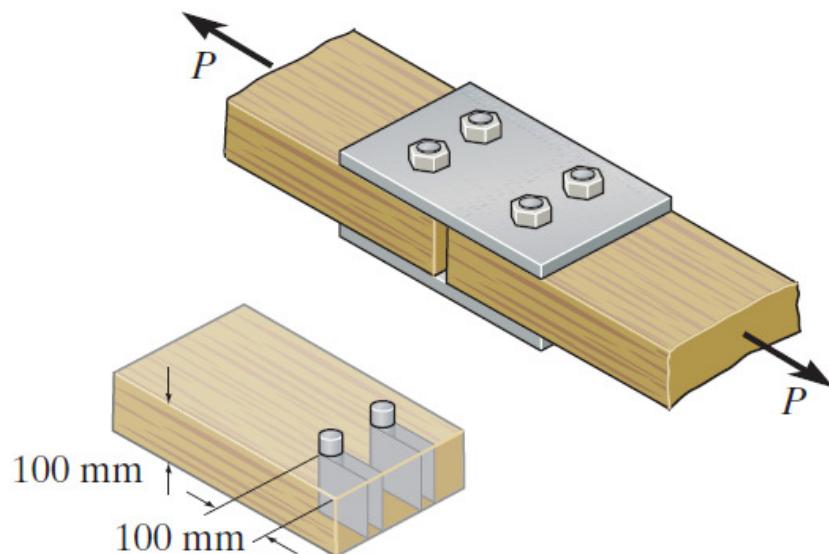
4. Bending stress



# Mechanical Engineering Design

## Example.

If the joint is subjected to an axial force of 9kN, determine the average shear stress developed in each of the 6-mm diameter bolts between the plates and the members and along each of the four shaded shear planes.



Find the force in each bolt,  
 $F = 9/2 = 4.5 \text{ kN}$

Consider the shear stress in each bolt, is it a single shear or double shear?

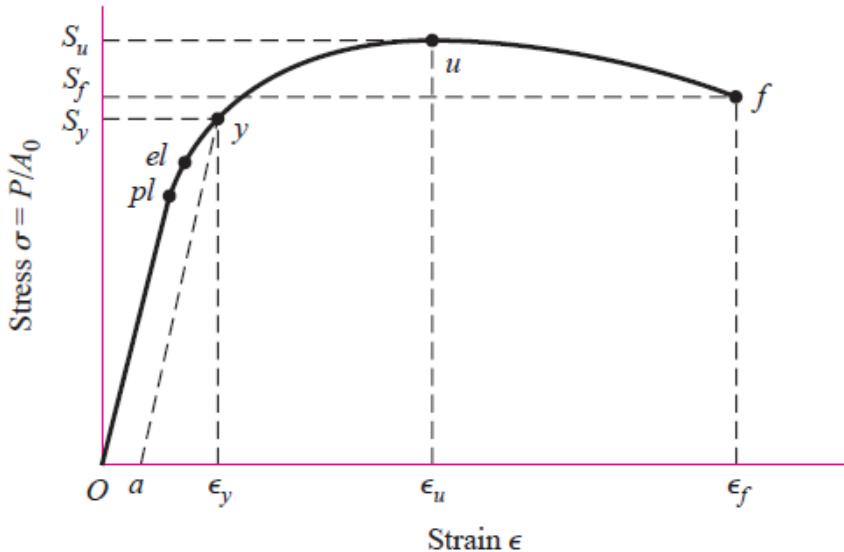
$$\begin{aligned} \text{Area of cross sectional area of bolt} &= 3.14 \times r^2 \\ &= 28.274 \times 10^{-6} \text{ m}^2 \end{aligned}$$

$$\text{Therefore shear stress} = F/2A = 79.6 \text{ MPa}$$

$$\begin{aligned} \text{Stress along the shaded shear planes} &= F/2A \\ &= 225 \text{ kPa} \end{aligned}$$

# Mechanical Engineering Design

## Mechanical properties of material



The linear range represents the elasticity of the material. Stress is proportional to strain. The modulus of elasticity ( $E$ ), also known as the Young's modulus is *a measure of the stiffness of a material*, and since strain is dimensionless, the units of  $E$  are *the same as stress*.

*pl - the proportional limit. This is the point at which the curve first begins to deviate from a straight line. Any deformation beyond P is permanent.*

*El - elastic limit. Beyond this point, the deformation is known to be plastic.*

*Y - yield point. Beyond this point, strain in the material increase very rapidly without a corresponding increase in stress and the material is deformed permanently.*

*U – ultimate tensile strength,  $S_u$  or  $S_{ut}$ . Maximum stress reached on the stress-strain diagram.*

*F – failure, when material fails. The corresponding stress is known as the failure stress,  $S_f$ .*

# Mechanical Engineering Design

## Allowable stress

For various reasons, we will specify the allowable load for a member by introducing the factor of safety in design.

$$\text{F.S.} = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}}$$

$\sigma_{\text{fail}}$  is found from the experimental testing of the material.

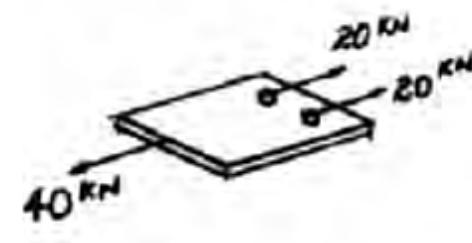
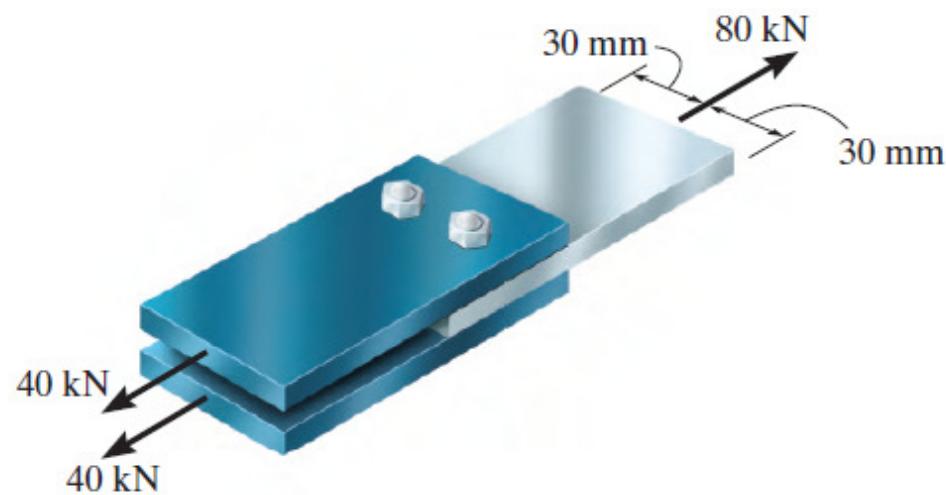
***factor of safety*** (F.S.) is  $> 1$  in order to avoid the potential of failure.

So, this means that we need to find a material that has failure stress higher than what is allowed/required. Therefore, it is common to design mechanical components based on the allowable stress.

# Mechanical Engineering Design

## Example.

The joint is fastened together using two bolts. Determine the required diameter of the bolts if the failure shear stress for the bolts is  $\tau_{\text{fail}} = 350 \text{ MPa}$ . Use a factor of safety for shear of F.S. = 2.5.



$$\text{F.S.} = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}}$$

$$\frac{350(10^6)}{2.5} = 140(10^5)$$

$$\tau_{\text{allow}} = 140(10^6) = \frac{20(10^3)}{\frac{\pi}{4} d^2}$$

$$d = 0.0135 \text{ m} = 13.5 \text{ mm}$$

# GEARS

## **Spur Gear design**

Spur gear is usually used in high speed transmission and has an operating efficiency of 98 - 99%. Other gear types are used if silent low-vibration operation is required. The pinion should be made from a harder material than the wheel. A gear pair should be selected to have the highest number of teeth consistent with a suitable safety margin in strength and wear. The pressure angle for spur gear is usually standardised as 20 degrees and the recommended minimum number of teeth on a gear with a normal pressure angle of 20 degrees is 18.

The preferred number of teeth are as follows

18 20 22 24 25 28 30 32 34 38 40 45 50 54 60  
64 70 72 75 80 84 90 96 100 120 140 150 180 200 220 250

The effects of increasing the pressure angle are:

1. Reduction in the danger of **interference**
2. Reduction of slipping speeds
3. Increased loading capacity in contact, seizure and wear
4. Increased rigidity of the tooth
5. Increased noise and radial forces

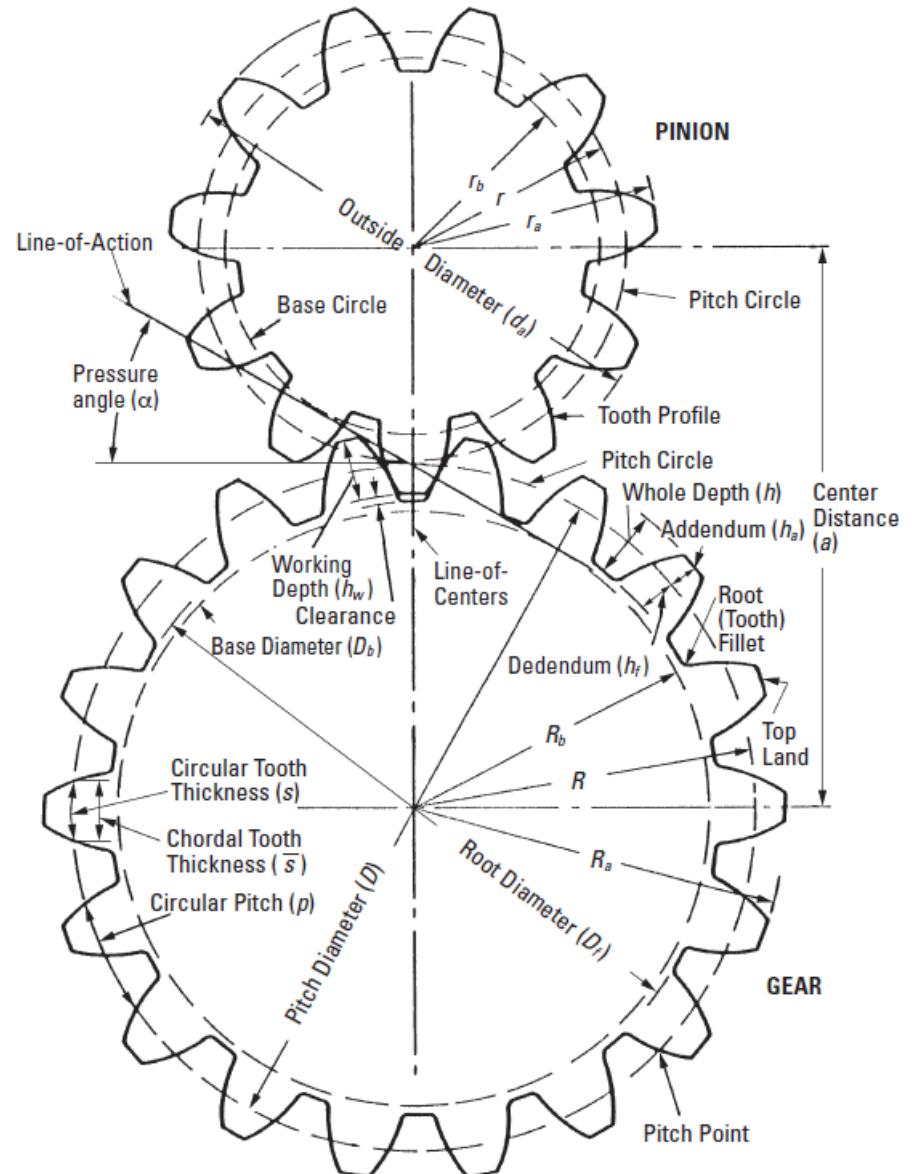
# GEARS

## Spur Gear design

GEAR PROPERTIES	FORMULAS
Pitch diameter	$D = mN$
Number of teeth	$N = D/m$
Module	$m = 25.4 / P_d$
Addendum	$a = m$
Dedendum	$b = 1.25m$
Outside diameter	$D_o = D + 2m = m(N+2)$
Centre distance	$C = m(N_1 + N_2)/2$
Circular pitch	$P_c = m * \pi() = D * \pi() / N$
Base circle diameter	$D_b = D \cos \varphi$

The two important properties of gears are:

1. The pitch diameters
2. Number of teeth



# GEARS

## Spur Gear design

### NORMAL MODULES

1st choice	millimetres
	2nd choice*
1	1.125
1.25	1.375
1.5	1.75
2	2.25
2.5	2.75
3	3.5
4	4.5
5	5.5
6	(6.5)
	7
8	9
10	11
12	14
16	18
20	22
25	28
32	36
40	45
50	

Recommended module for gears (Australian standards)

# GEARS

## Spur gear trains

Since the pitch of two mating gears must be the same;

$$d_1/N_1 = d_2/N_2$$

Where  $d$  is the pitch diameter and  
 $N$  is the number of teeth

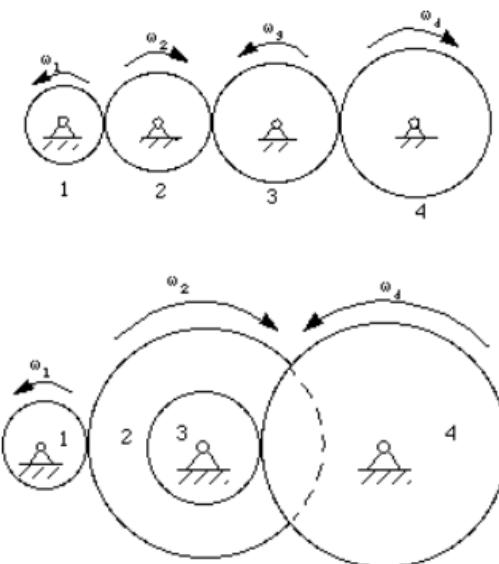
In addition, we have also established that

$$r_1/r_2 = w_2/w_1$$

$$\text{Therefore } d_1/d_2 = w_2/w_1 = N_1/N_2$$

Where  $w$  is the angular velocity. Note that multiply  $w$  by  $60/2\pi$  will give us the speed in revolution per minute (RPM),  $n$ .

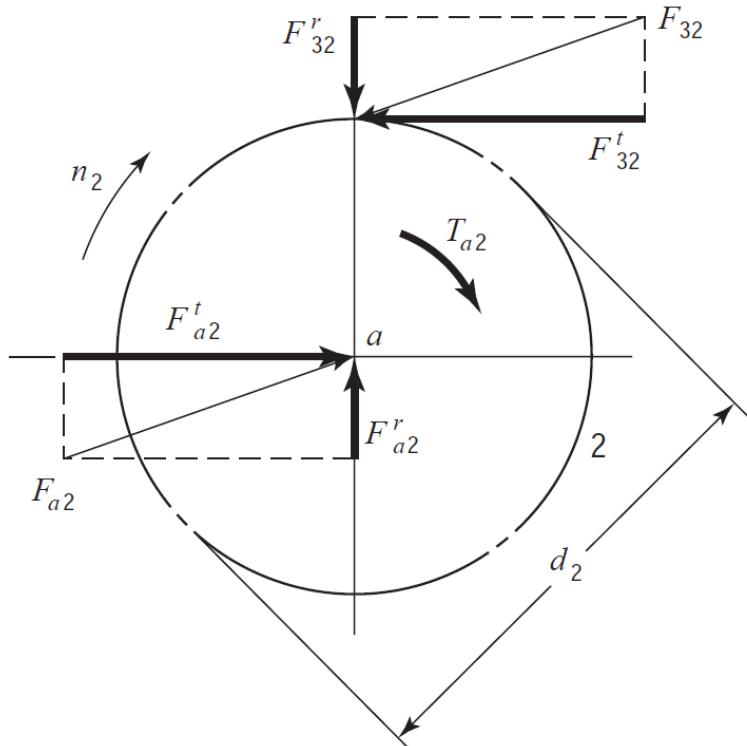
**Gear trains** consist of two or more gears for the purpose of transmitting motion from one axis to another. There are two main types of gear trains and they are **simple gear trains and compound gear train**.



# GEARS

## Force analysis in gears

It is also necessary to use superscripts to indicate directions. the radial and tangential directions by superscripts  $r$  and  $t$ . For example, with this notation,  $F^{t43}$  is the tangential component of the force of gear 4 acting against gear 3. A typical free body diagram of a gear (Gear 2) is as shown.



$$W_t = F_{32}^t$$

The force component  $F^{t32}$  (tangential load) is the component that transmits load from gear 3.  $F^{r32}$  does not transmit power.

The applied torque and the transmitted load can be analysed as:

$$T = \frac{d}{2} W_t$$

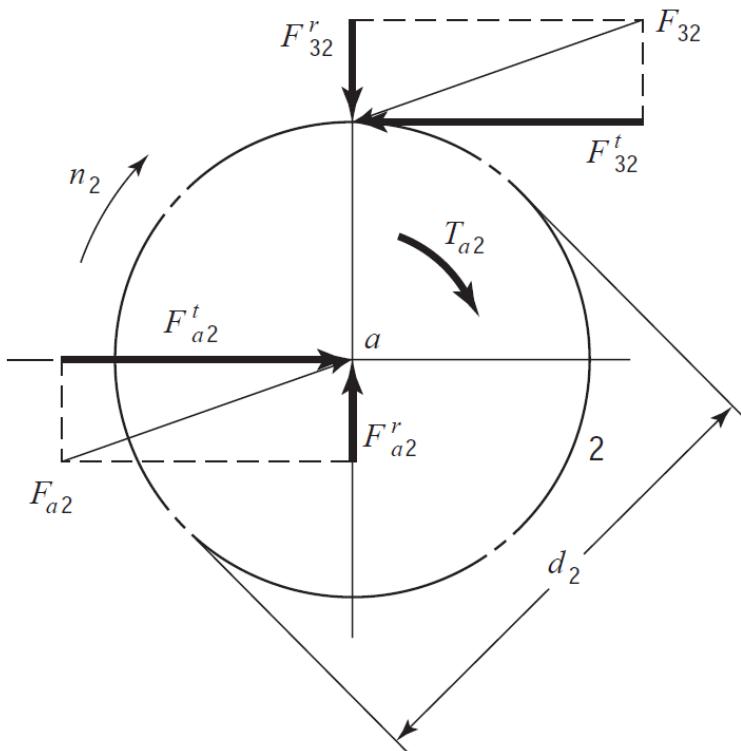
$$H = T\omega = (W_t d/2) \omega$$

Where  $H$  is the power.

Note that meshed gears are quite efficient, with losses of less than 2 percent. Therefore, a pair of meshed gears should give approximately the same power .

# GEARS

## Force analysis in gears



Gear data is often tabulated using *pitch-line velocity*, which is the linear velocity of a point on the gear at the radius of the pitch circle. Therefore

$$V = (d/2) \omega.$$

Many gear design problems will specify the power and speed, and the relationship between power, speed and  $W_t$  is:

$$W_t = \frac{60\,000 H}{\pi d n}$$

$W_t$  = transmitted load, kN

$H$  = power, kW

$d$  = gear diameter, mm

$n$  = speed, rev/min

# GEARS

## Force analysis on bevel gears

To determine shaft and bearing loads for bevel-gear applications, the usual practice is to use the tangential or transmitted load that would occur if all the forces were concentrated at the midpoint of the tooth.

While the actual resultant occurs somewhere between the midpoint and the large end of the tooth, there is only a small error in making this assumption. For the transmitted load, this gives

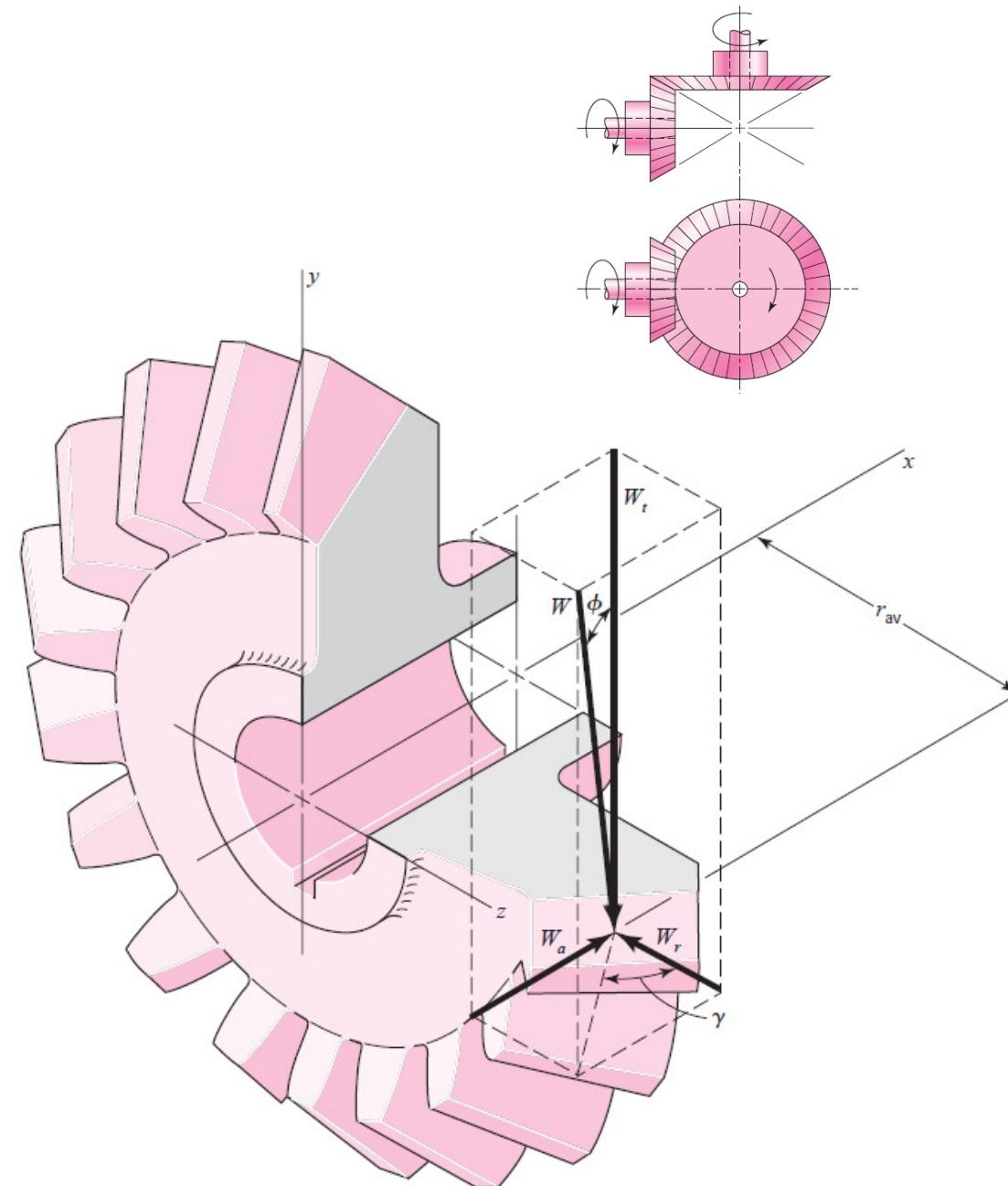
$$W_t = T / r_{av}$$

where  $T$  is the torque and  
 $r_{av}$  is the pitch radius at the midpoint of the tooth for the gear

The forces acting at the center of the tooth are shown:

$$W_r = W_t \tan \phi \cos \gamma$$

$$W_a = W_t \tan \phi \sin \gamma$$



# GEARS

## Force analysis on helical gears

The three-dimensional view of the forces acting against a helical-gear tooth is shown in the figure on the right. The point of application of the forces is in the pitch plane and in the center of the gear face. From the geometry of the figure, the three components of the tooth force  $W$  are

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi$$

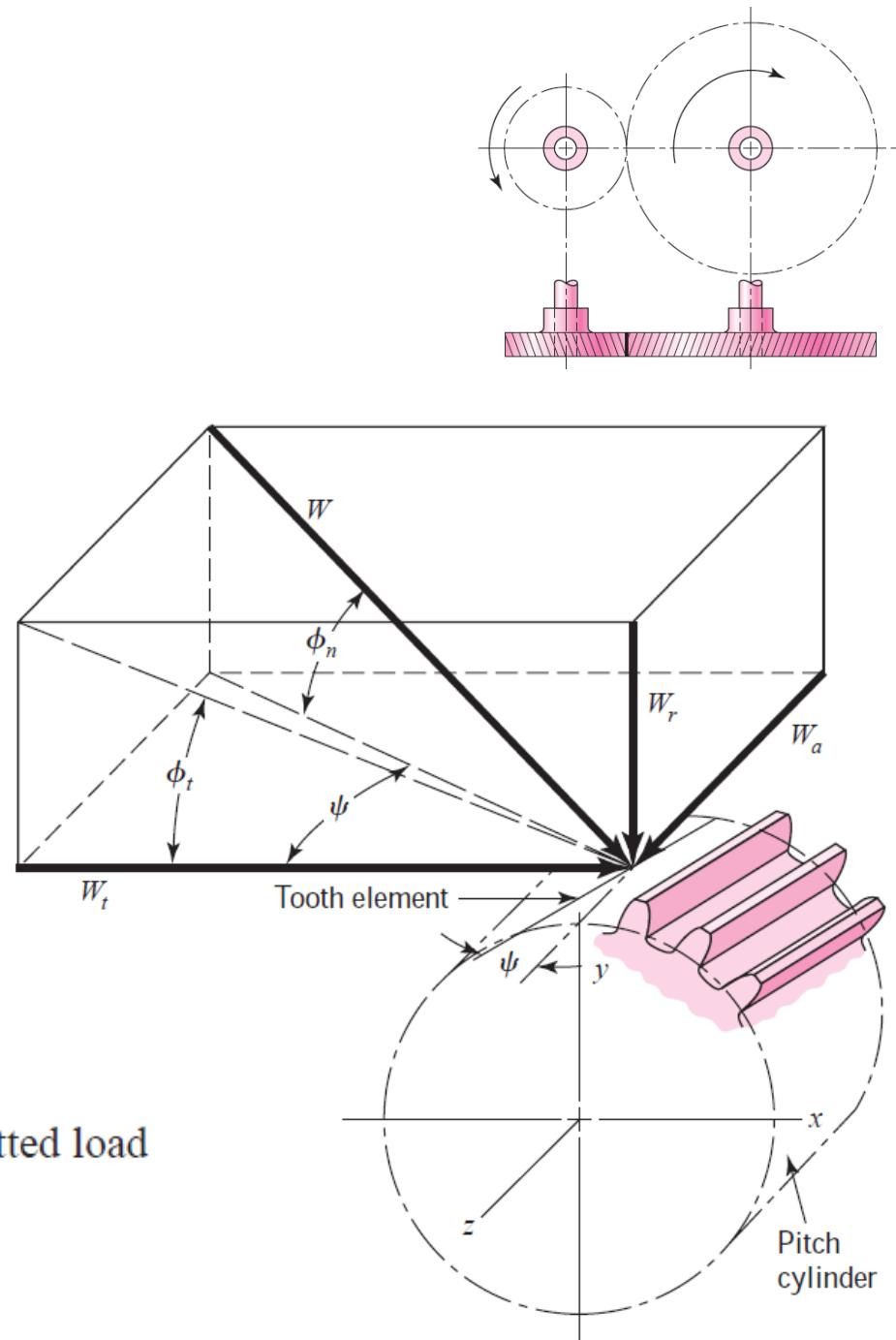
$$W_a = W \cos \phi_n \sin \psi$$

where  $W$  = total force

$W_r$  = radial component

$W_t$  = tangential component, also called transmitted load

$W_a$  = axial component, also called thrust load



# GEARS

## Force analysis on worm gears

If friction is neglected, then the only force exerted by the gear will be the force  $W$ , having the three orthogonal components  $W_x$ ,  $W_y$ , and  $W_z$ . From the geometry of the figure,

$$W^x = W \cos \phi_n \sin \lambda$$

$$W^y = W \sin \phi_n$$

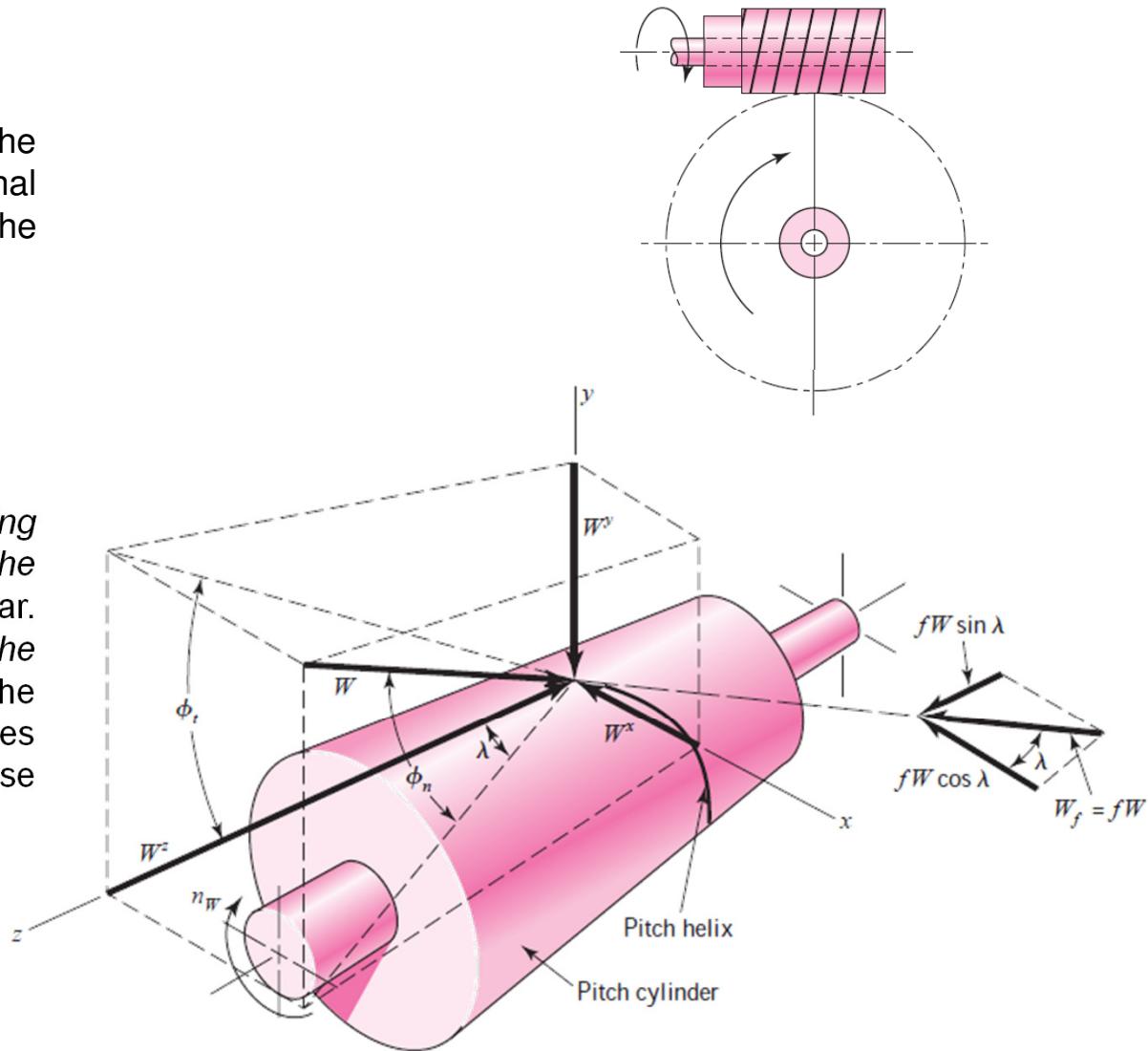
$$W^z = W \cos \phi_n \cos \lambda$$

By using subscripts  $W$  and  $G$  to indicate forces acting against the worm and gear, respectively.  $W_y$  is the separating, or radial, force for both the worm and the gear. The tangential force on the worm is  $W_x$  and is  $W_z$  on the gear, assuming a  $90^\circ$  shaft angle. The axial force on the worm is  $W_z$ , and on the gear,  $W_x$ . Since the gear forces are opposite to the worm forces, we can summarize these relations by writing

$$W_{Wt} = -W_{Ga} = W^x$$

$$W_{Wr} = -W_{Gr} = W^y$$

$$W_{Wa} = -W_{Gt} = W^z$$



# GEARS

## Force analysis on worm gears

In spur gears, the motion of one tooth relative to the mating tooth is primarily a rolling motion. The contact occurs at the pitch point and the motion is pure rolling.

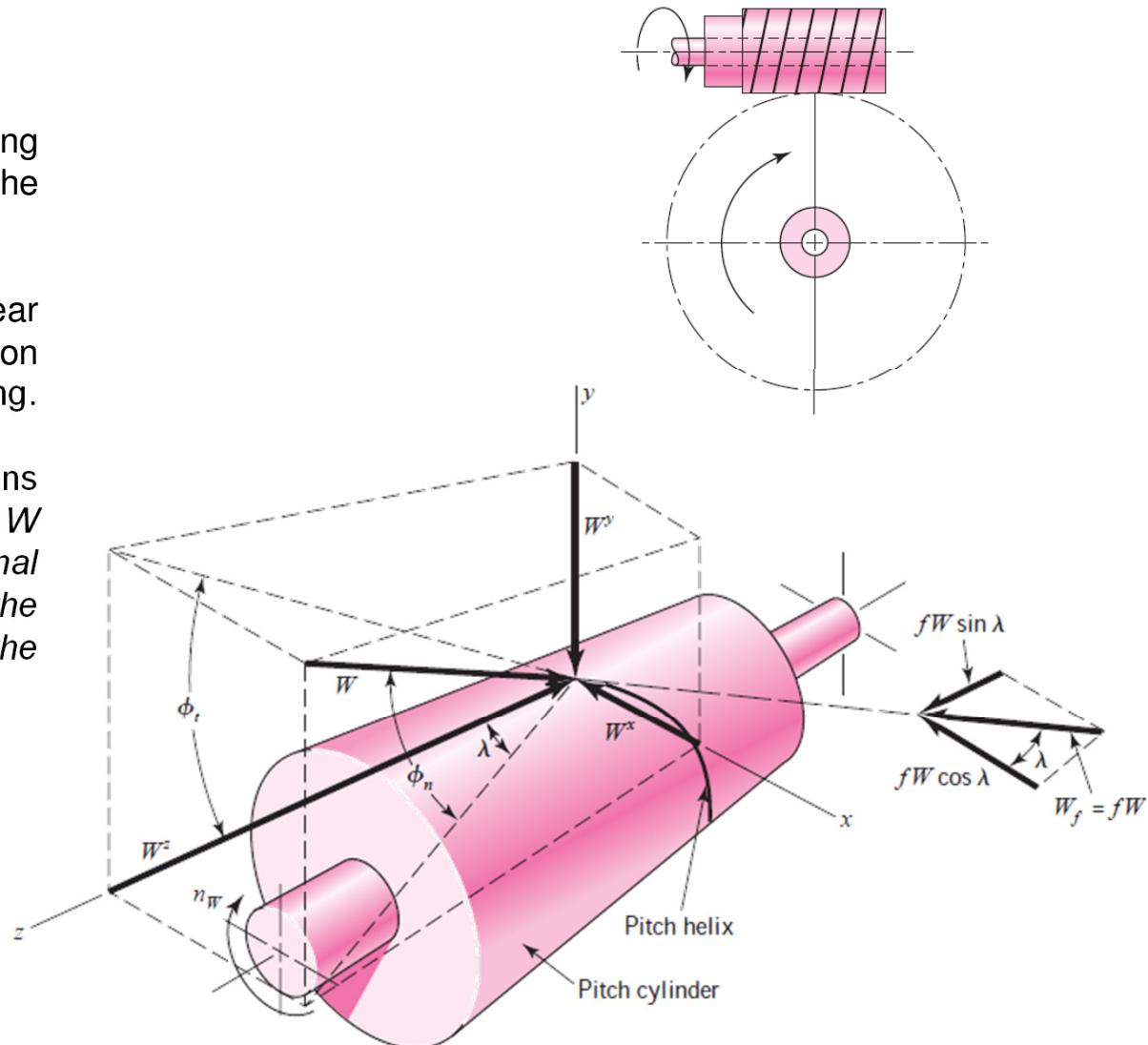
However, the relative motion between worm and worm-gear teeth is pure sliding, and so we must expect that friction plays an important role in the performance of worm gearing.

By introducing a coefficient of friction  $f$ , a set of equations on the force on worm gears can be developed. The force  $W$  acting normal to the worm-tooth profile produces a frictional force  $W_f = fW$ , having a component  $f W \cos \lambda$  in the negative  $x$  direction and another component  $f W \sin \lambda$  in the positive  $z$  direction.

$$W^x = W(\cos \phi_n \sin \lambda + f \cos \lambda)$$

$$W^y = W \sin \phi_n$$

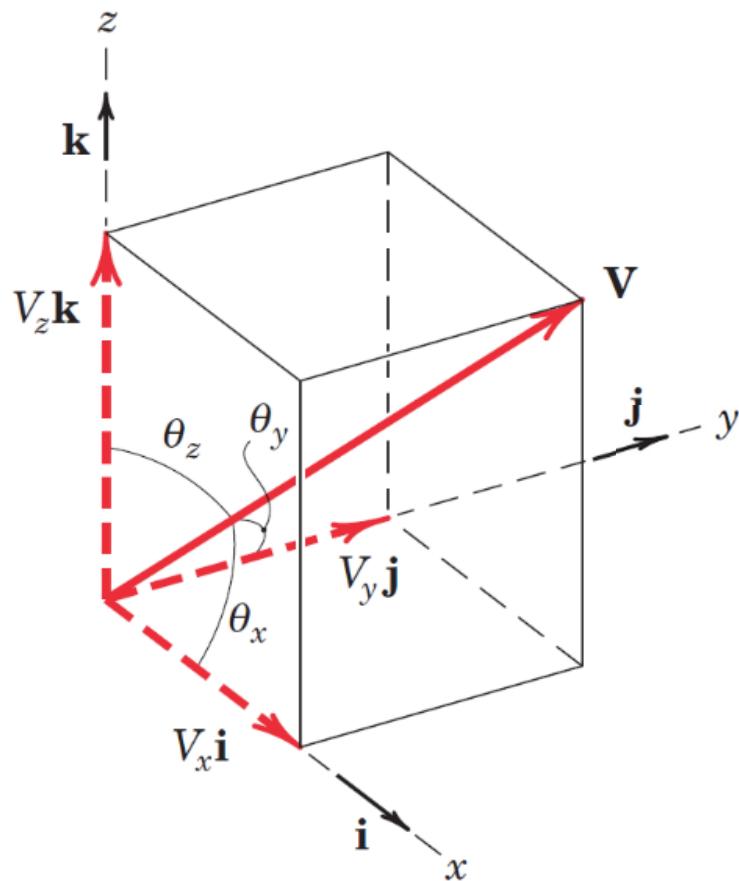
$$W^z = W(\cos \phi_n \cos \lambda - f \sin \lambda)$$



# GEARS

## Three-dimensional force system

A vector  $\mathbf{V}$  may have three rectangular components along the  $x$ ,  $y$ ,  $z$  coordinate axes and is represented by the vector sum of its three rectangular components.



In three dimensions, the set of Cartesian unit  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  is used to represent the directions of the  $x$ ,  $y$ ,  $z$  axes, respectively.

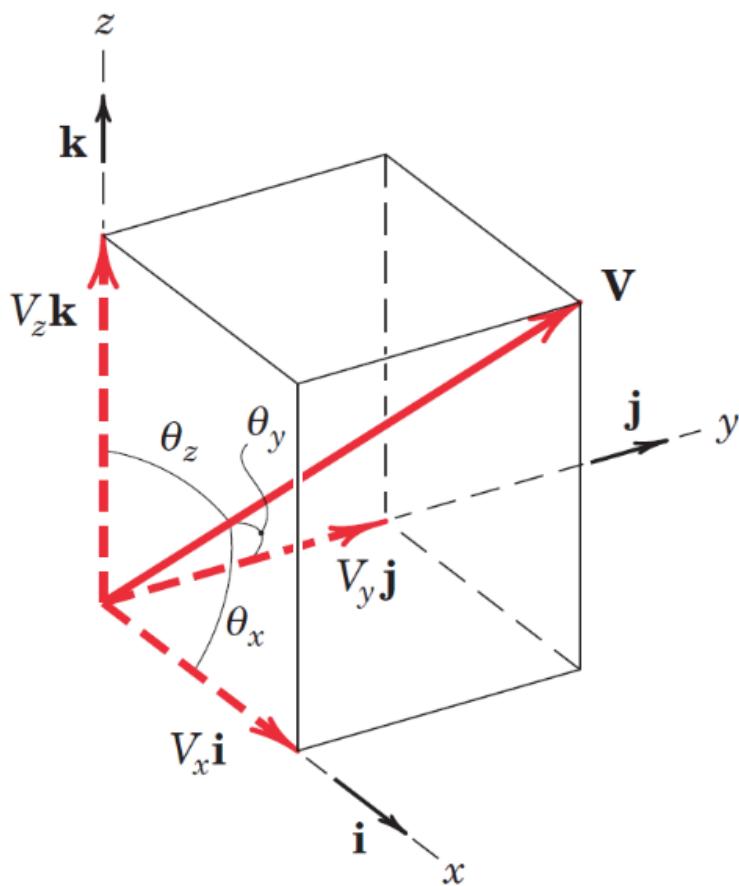
We can write  $\mathbf{V}$  in the cartesian/ unit vector form.

$$\mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j} + V_z \mathbf{k}$$

$V_x$ ,  $V_y$ , and  $V_z$  are scalar quantities.

# GEARS

## Three important relationships in a three-dimensional force system



1.  $\cos \theta_x = \frac{V_x}{V}$        $\cos \theta_y = \frac{V_y}{V}$        $\cos \theta_z = \frac{V_z}{V}$   
 $V \cos \theta_x = V_x$        $V \cos \theta_y = V_y$        $V \cos \theta_z = V_z$
2.  $\cos^2 \Theta_x + \cos^2 \Theta_y + \cos^2 \Theta_z = 1$

Remember that,

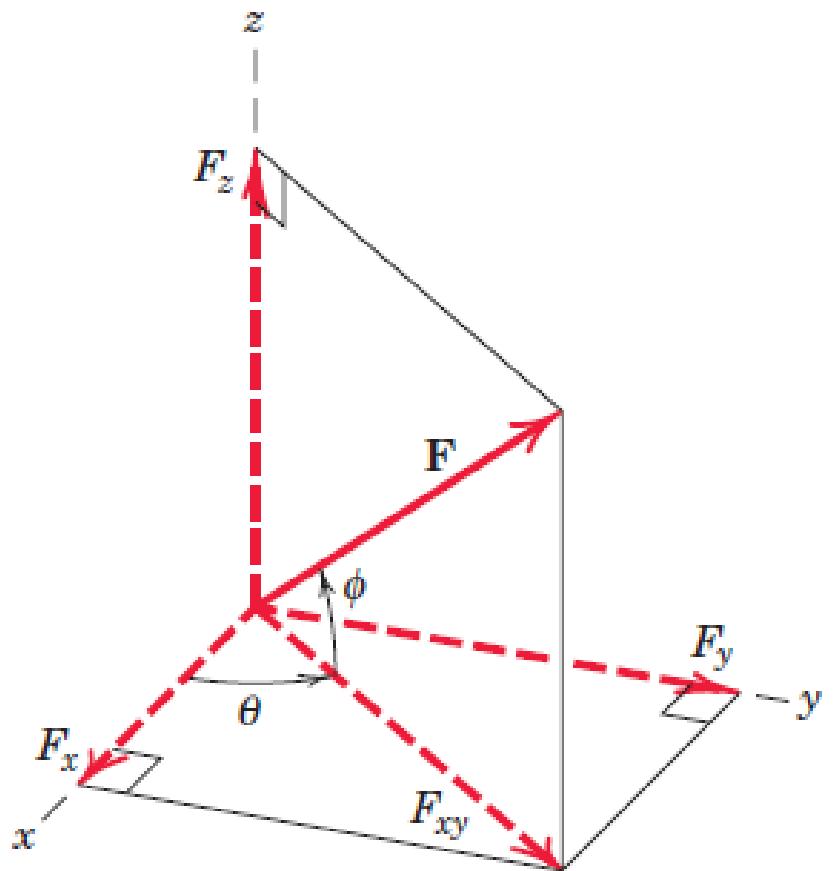
$$\cos^2 \Theta = (\cos \Theta)^2$$

e.g.  $\cos^2 60^\circ = (\cos 60^\circ)^2 = (0.5)^2 = 0.25$

3. from the Pythagorean theorem,

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

# GEARS



# Shaft and bearings

## Shaft design for stress

### The Endurance Limit modifying factors

In the laboratory, endurance limits is determined using samples that were prepared very carefully and tested under closely controlled conditions. It is unrealistic to expect the endurance limit of a mechanical or structural member to match the values obtained in the laboratory. Some differences include

- *Material: composition, basis of failure, variability*
- *Manufacturing: method, heat treatment, fretting corrosion, surface condition, stress concentration*
- *Environment: corrosion, temperature, stress state, relaxation times*
- *Design: size, shape, life, stress state, speed, fretting, galling*

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

where  $k_a$  = surface condition modification factor

$k_b$  = size modification factor

$k_c$  = load modification factor

$k_d$  = temperature modification factor

$k_e$  = reliability factor<sup>13</sup>

$k_f$  = miscellaneous-effects modification factor

$S'_e$  = rotary-beam test specimen endurance limit

$S_e$  = endurance limit at the critical location of a machine part in the geometry and condition of use

# Shaft and bearings

## Shaft design for stress

### **Surface Factor, $ka$**

The surface modification factor depends on the quality of the finish of the actual part surface and on the tensile strength of the part material.

$$ka = aSut^b$$

where  $Sut$  is the tensile strength and  $a$  and  $b$  are to be found in the table below.

Surface Finish	<b>Factor <math>a</math></b> $S_{ut}, \text{kpsi}$	<b>Exponent <math>b</math></b>
Ground	1.34	-0.085
Machined or cold-drawn	2.70	-0.265
Hot-rolled	14.4	-0.718
As-forged	39.9	-0.995

# Shaft and bearings

## Shaft design for stress

### ***Size Factor, kb***

The size of a shaft may affect its bending and torsion. For axial loading, Kb = 1

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases}$$

### ***Loading Factor, kc***

When fatigue tests are carried out with rotating bending, axial (push-pull), and torsional loading, the endurance limits differ with *Sut*.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases}$$

# Shaft and bearings

## Shaft design for stress

### ***Temperature Factor, kd***

When operating temperatures are below room temperature, brittle fracture is a strong possibility and should be investigated first. When the operating temperatures are higher than room temperature, yielding should be investigated first because the yield strength drops rapidly with temperature.

If the rotating beam endurance limit is known at room temperature, then use

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 \\ + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

If the rotating-beam endurance limit is not given, then obtain kd from the table (right) . Determine the temperature-corrected tensile strength.

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Temperature, °C	$S_T/S_{RT}$	Temperature, °F	$S_T/S_{RT}$
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

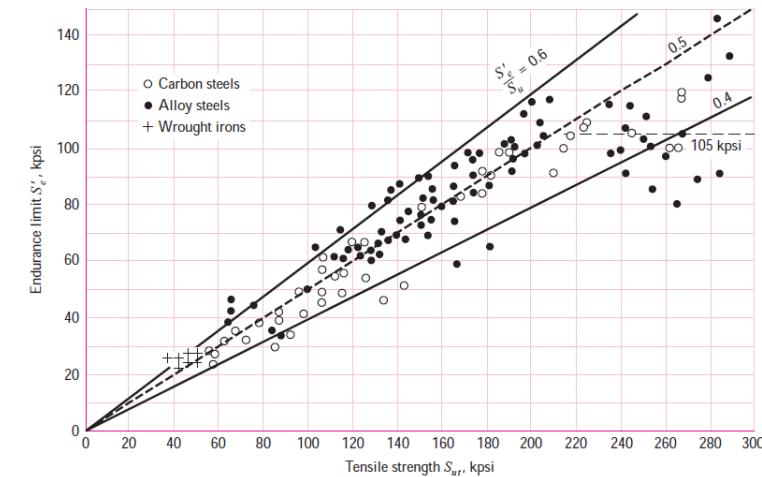
# Shaft and bearings

## Shaft design for stress

### **Reliability Factor, $k_e$**

This factor accounts for the scatter of data such as shown on the right.

Obtain reliability factor in the table below.



Reliability, %	Transformation Variate $z_a$	Reliability Factor $k_e$
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

# Shaft and bearings

## Shaft design for stress

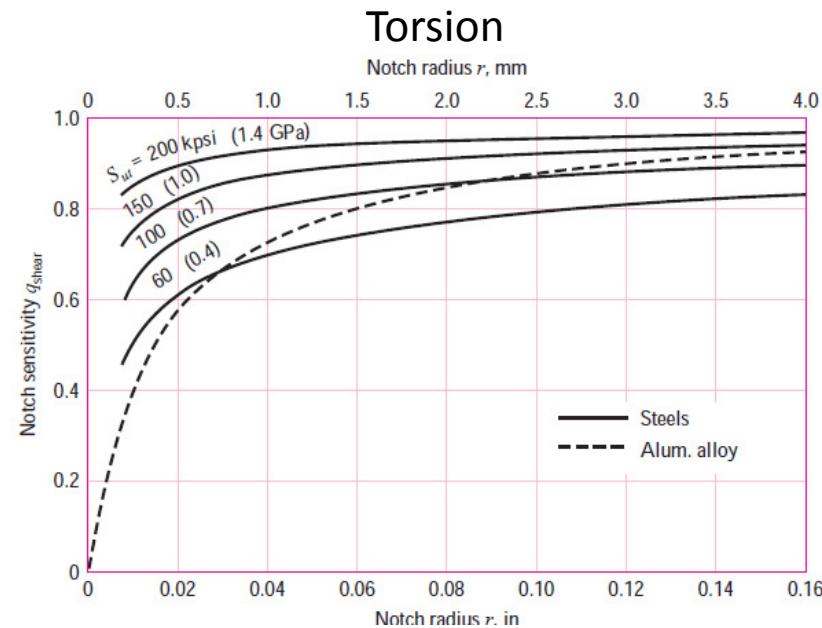
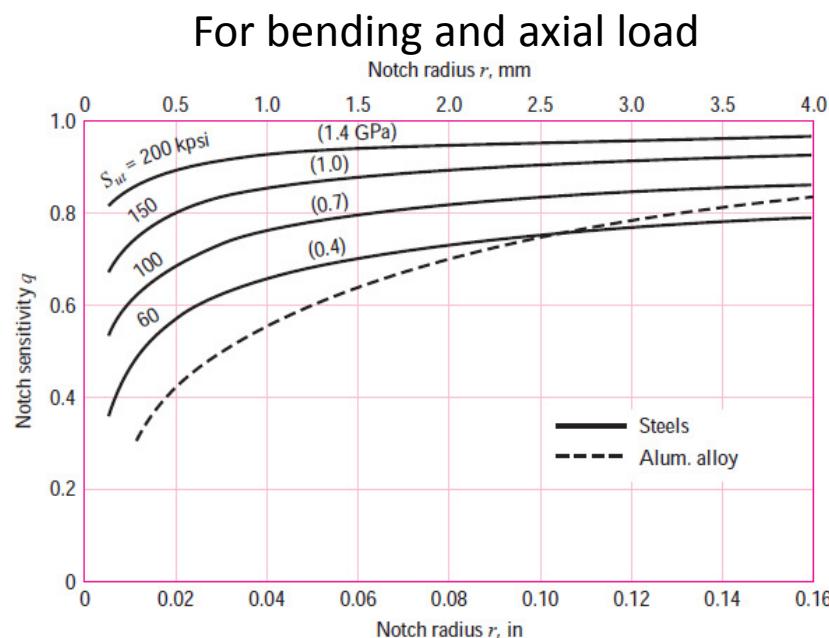
### Miscellaneous Factor Effects, $k_f$

The factor  $K_f$  is commonly called a *fatigue stress-concentration factor*. It is the stress that is related to the existence of irregularities or discontinuities, such as holes, grooves, or notches in the shaft which increases the theoretical stresses significantly.

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$

Notch sensitivity  $q$  is defined by the equation

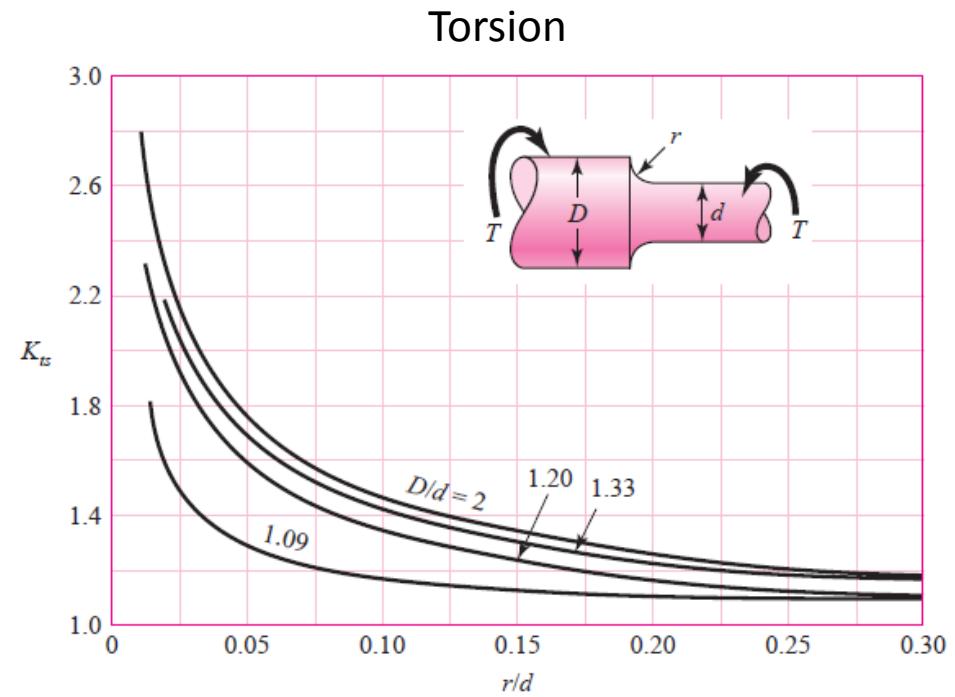
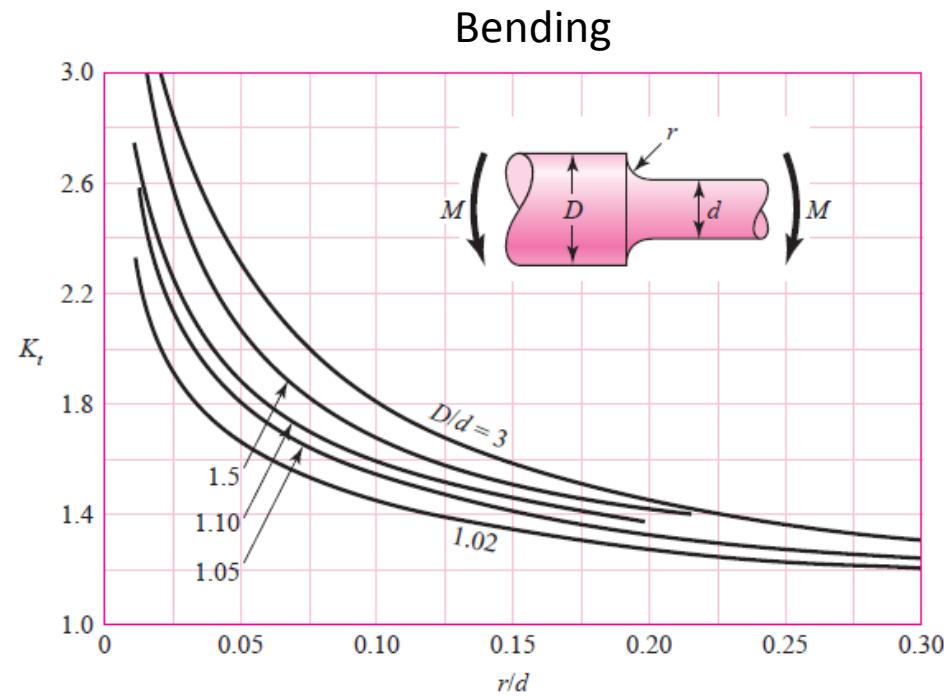
$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1}$$



# Shaft and bearings

## Shaft design for stress

*Miscellaneous Factor Effects, kf*



# Shaft and bearings

## Example

A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 550°F. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70 000 cycles. Given that  $S_{ut} = 50$  kpsi at 70°F.

To determine  $k_a$ ,

Surface Finish	Factor $a$ $S_{ut}$ , kpsi	Factor $a$ $S_{ut}$ , MPa	Exponent $b$
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272.	-0.995

$$k_a = a S_{ut}^b = 2.70(49^{-0.265}) = 0.963$$

To determine  $k_b$ ,

Since the bar is under axial loading,  $k_b = 1$ .

# Shaft and bearings

To determine  $k_c$ ,

$$k_c = 0.85$$

To determine  $k_e$ ,

For 99% reliability,  $k_e = 0.814$

Reliability, %	Transformation Variate $z_\alpha$	Reliability Factor $k_e$
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Since no other conditions are given,  $k_f = 1$

$$\begin{aligned} S_e &= k_a k_b k_c k_d k_e k_f S'_e \\ &= 0.963(1)(0.85)(1)(0.814)(1)24.5 = 16.3 \text{ kpsi} \end{aligned}$$

# Shaft and bearings

Design factor or factor of safety

$$\text{Soderberg} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

$$\text{mod-Goodman} \quad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

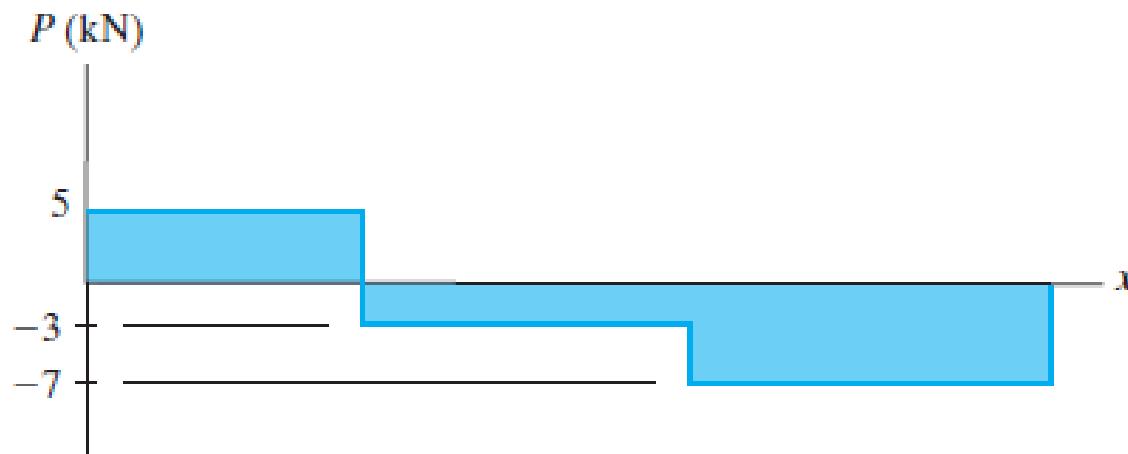
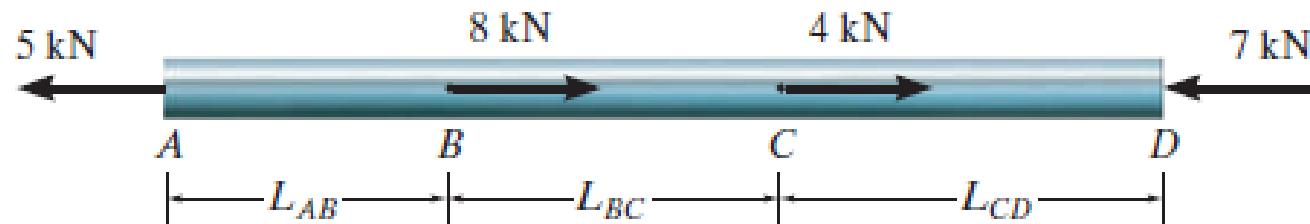
$$\text{Gerber} \quad \frac{n\sigma_a}{S_e} + \left( \frac{n\sigma_m}{S_{ut}} \right)^2 = 1$$

$$\text{ASME-elliptic} \quad \left( \frac{n\sigma_a}{S_e} \right)^2 + \left( \frac{n\sigma_m}{S_y} \right)^2 = 1$$

Where n is the design factor/ fatigue factor of safety.

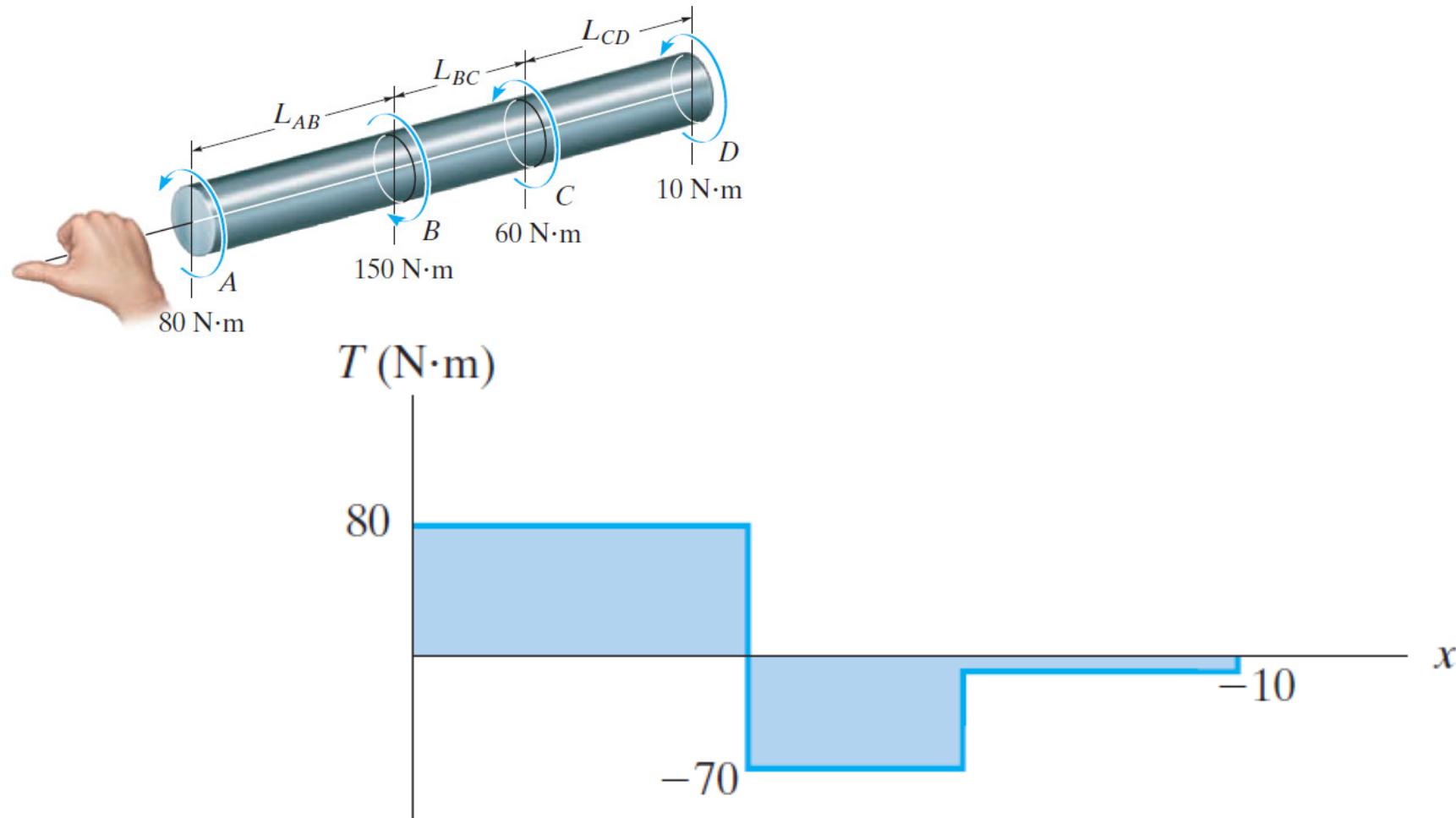
# Shaft and bearings

## Axial load diagram



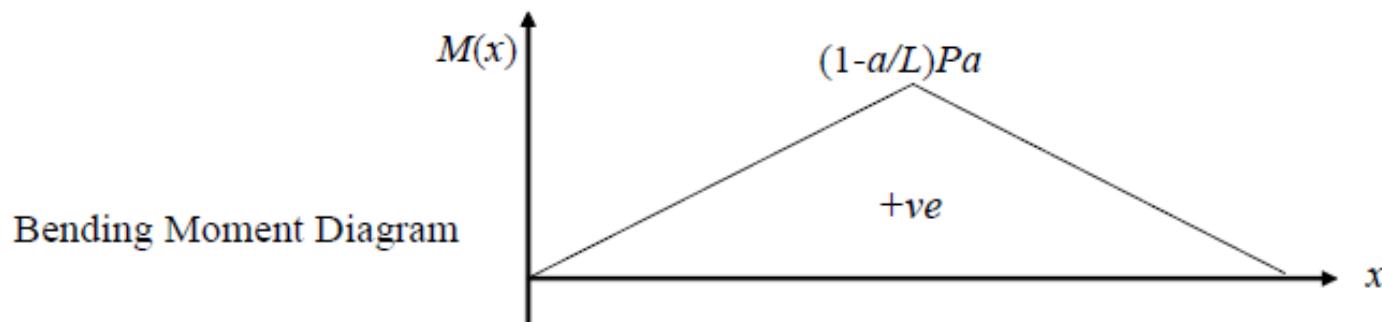
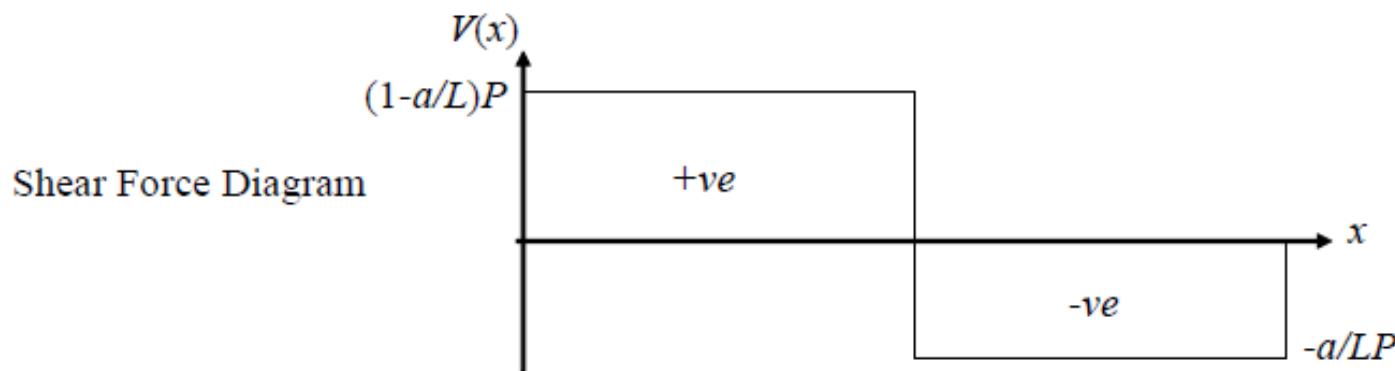
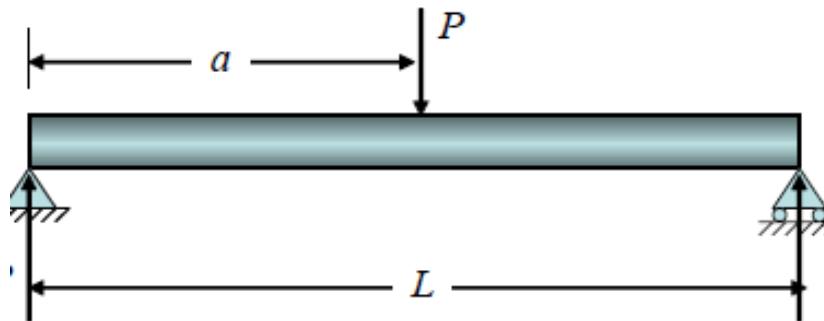
# Shaft and bearings

## Torque diagram



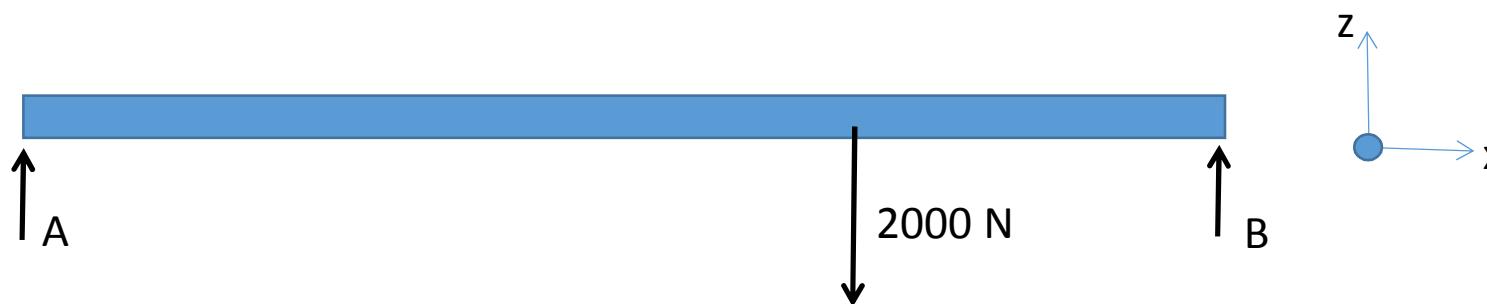
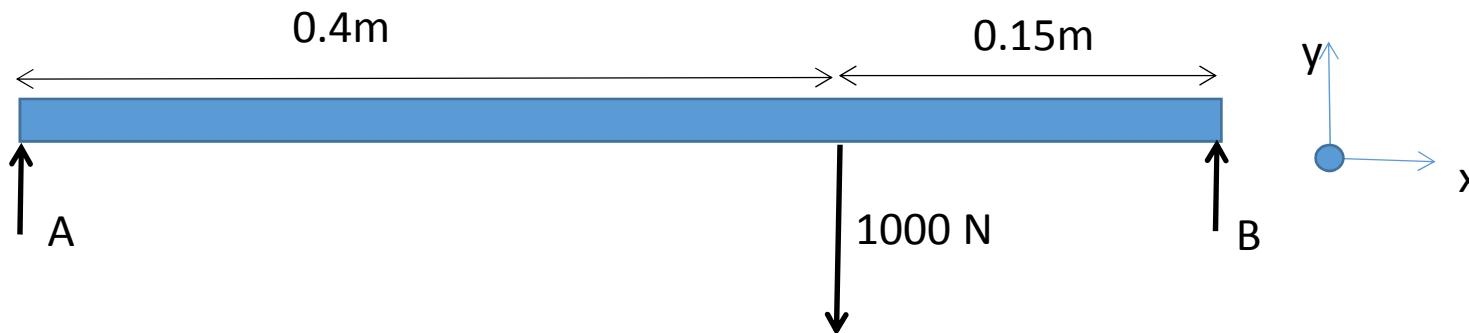
# Shaft and bearings

## Shear force and bending moment diagram



# Shaft and bearings

**Shear force and bending moment diagram – important example**



# Shaft and bearings

How to find  $\sigma_a$  and  $\sigma_m$ ?

$$\frac{1}{n} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[ \left( \frac{32K_f M_a}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_a}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[ \left( \frac{32K_f M_m}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

where  $M_m$  and  $M_a$  are the midrange and alternating bending moments,  $T_m$  and  $T_a$  are the midrange and alternating torques. **For a rotating shaft with constant bending and torsion,  $M_m$  and  $T_a$  equal to 0, which simply drops out from the above equations.**

# Shaft and bearings

## Steps to identify and design the critical diameter of a rotating shaft with constant bending and torsion

Step 1. Based on the requirements of the shaft operating environment, determine the endurance limit of the shaft ( $S_e$ ). Determine all endurance limit modifying factors (eg.  $K_a$ ,  $K_b$ ...).

Step 2. Draw the torque, axial and shear force bending moment diagram from the front view and the top view. Find the total moment bending diagram ( $M_a$ ) to determine the most critical section.

Step 3. Get the first estimate of the diameter. Remember that for a rotating shaft with constant bending and torsion,  $M_m$  and  $T_a$  equal to 0, which simply drops out from the above equations. Note:  $K_t \sim K_f$  and  $K_{ts} \sim K_{fs}$  from table below. Assume a fatigue factor of safety,  $n$  if not provided. (e.g. 1.5)

$$d = \left( \frac{16n}{\pi} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \right)^{1/3}$$

Table 7-1

First Iteration Estimates for Stress-Concentration Factors  $K_t$  and  $K_{ts}$ .

*Warning:* These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

	Bending	Torsional	Axial
Shoulder fillet—sharp ( $r/d = 0.02$ )	2.7	2.2	3.0
Shoulder fillet—well rounded ( $r/d = 0.1$ )	1.7	1.5	1.9
End-mill keyseat ( $r/d = 0.02$ )	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

# Shaft and bearings

**Steps to identify and design the critical diameter of a rotating shaft with constant bending and torsion**

Step 4. Design an approximate layout of the shaft.

Step 5. With the estimated diameter d, find actual  $k_t$  and  $k_f/k_{ts}$  and  $k_{fs}$  to obtain  $q$  and  $q_{shear}$  (notch sensitivity). See slides 13 and 14.

Step 6. Calculate the new  $S_e$ , as  $K_b$  may be affected.

Step 7. Determine whether the fatigue factor of safety,  $n$  pass the goodman criterion. Determine the actual  $d$ .

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[ \left( \frac{32K_f M_a}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_a}{\pi d^3} \right)^2 \right]^{1/2}$$

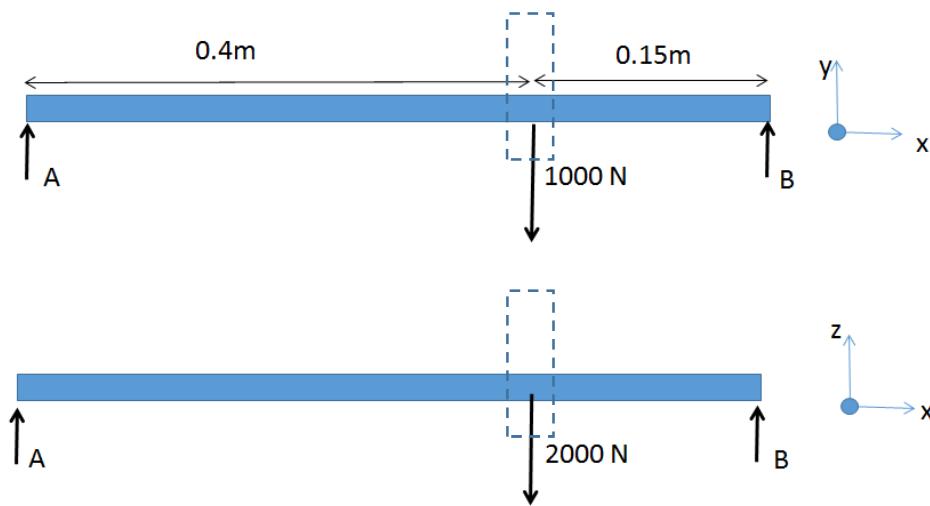
$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[ \left( \frac{32K_f M_m}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\frac{1}{n} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$

# Shaft and bearings

## Example

The figure below shows a shaft supported by two bearings at A and B. A belt pulley is located at 0.4m away from one end of the shaft and weighs 1000N. The belt pull force is 2000N. The maximum resultant bending moment related to the shaft below is 243.82 Nm. Assume that the stress endurance limit,  $S_e$  is 36 MPa and  $S_{ut}$  (aluminium) is 80 MPa. Design and specify appropriate diameters for the shaft.



Step 1: Given

Step 2: Given

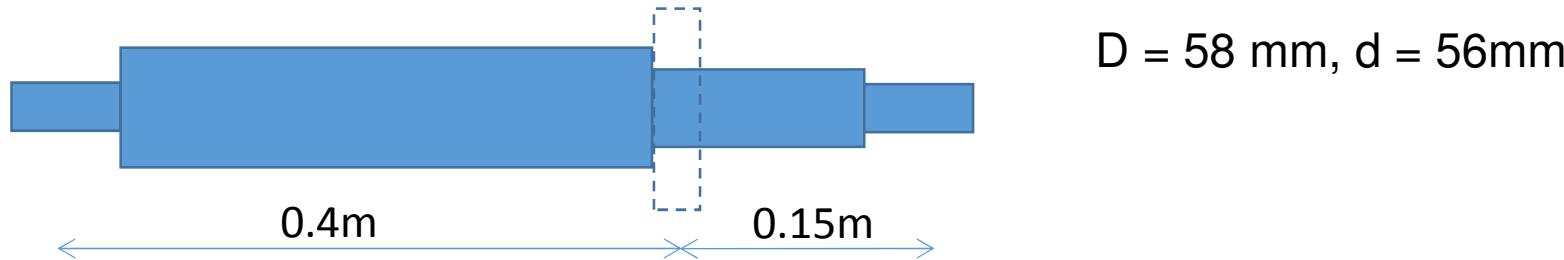
Step 3: Determine the first estimate of the diameter  $d$ ,  
As  $n$  is not given, assume it as 1.5  
From table,  $K_f = 1.7$ ,  $K_{fs} = 1.5$

$$d = \left( \frac{16n}{\pi} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \right)^{1/3}$$

$$d = 56 \text{ mm}$$

# Shaft and bearings

Step 4: Design an appropriate layout of the shaft.



Step 5: Determine actual  $k_t$  and  $k_f/ k_{ts}$  and  $k_{fs}$  to obtain  $q$  and  $q$  shear (notch sensitivity).  
 $D/d = 1.04$ ,  $r/d = 0.018$ .  $K_t = 2.23$ ,  $K_{ts} = ?$ .  $q = 0.55$ .

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{\text{shear}} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

Therefore  $k_f = 1.68$

Step 6: Calculate the new  $S_e$ , as  $K_b$  may be affected.

As  $S_e$  is given, this will not be considered in this example.

Step 7: Determine whether the fatigue factor of safety,  $n$  pass the goodman criterion.  
Determine the actual  $d$ .

# Shaft and bearings

## Bearings

The *rating life of bearing is a term used by most manufacturers*. The rating life of a group of nominally identical ball or roller bearings is defined as the number of revolutions (or hours at a constant speed) that 90 percent of a group of bearings will achieve or exceed before the failure criterion develops. The terms *minimum life, L10 life, and B10 life are also used as synonyms for rating life*. The rating life is the 10th percentile location of the bearing group's revolutions-to-failure distribution.

C<sub>0</sub> – basic static load - the load that will produce a total permanent deformation in the raceway and rolling element.

C<sub>10</sub> – dynamic load rating

# Shaft and bearings

## Bearings

### Combined Radial and Thrust Loading

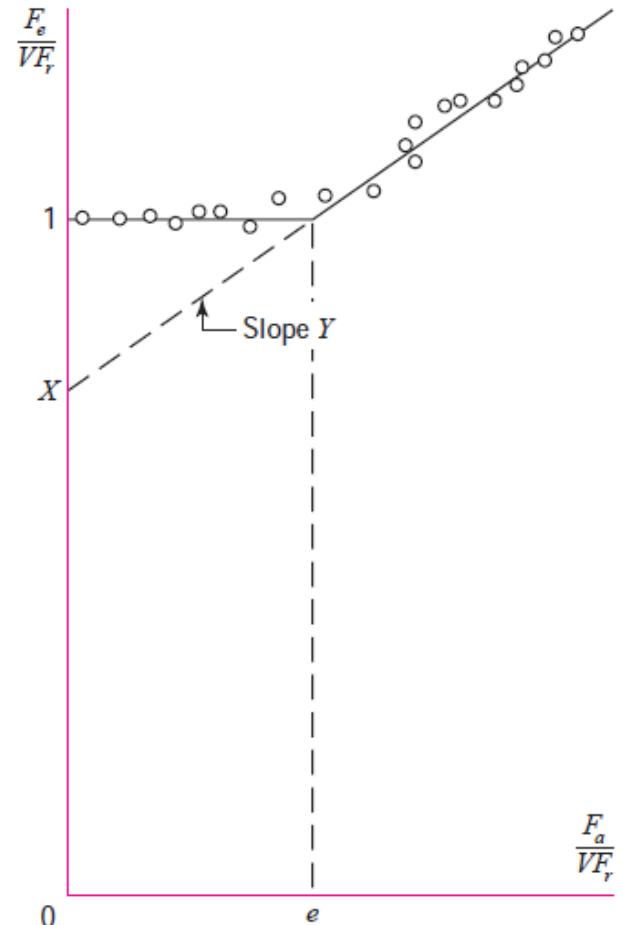
A ball bearing is capable of resisting radial loading and thrust loading. Furthermore, these can be combined.

Consider  $F_a$  and  $F_r$  to be the axial thrust and radial loads, respectively, and  $F_e$  to be the equivalent radial load that does the same damage as the combined radial and thrust loads together.

A rotation factor  $V$  is defined such that  $V = 1$  when the inner ring rotates and  $V = 1.2$  when the outer ring rotates. Two dimensionless groups can now be formed:  $F_e/V F_r$  and  $F_a/V F_r$ . When these two dimensionless groups are plotted, the data fall in a gentle curve that is well approximated by two straight-line segments. The abscissa  $e$  is defined by the intersection of the two lines. The equations for the two lines are

$$\frac{F_e}{V F_r} = 1 \quad \text{when } \frac{F_a}{V F_r} \leq e$$

$$\frac{F_e}{V F_r} = X + Y \frac{F_a}{V F_r} \quad \text{when } \frac{F_a}{V F_r} > e$$



# Shaft and bearings

## Bearings: Steps to select bearing

Step 1: Determine  $F_e$ .

If  $C_0$  – Basic static load rating **is given**.

Determine  $e$  and analyse using the criterion.

If  $C_0$  – Basic static load rating **is not given**.

$$\frac{F_e}{VF_r} = 1 \quad \text{when } \frac{F_a}{VF_r} \leq e$$

$$\frac{F_e}{VF_r} = X + Y \frac{F_a}{VF_r} \quad \text{when } \frac{F_a}{VF_r} > e$$

Determine the radial force and axial force on bearings. Does the inner or outer ring rotate?

Assume  $F_a/VF_r < e$  or  $> e$ , determine  $X_2$  and  $Y_2$  and substitute in the following equation to find  $F_e$ .

$$F_e = X_i VF_r + Y_i F_a$$

$F_a/C_0$	$e$	$F_a/(VF_r) \leq e$		$F_a/(VF_r) > e$	
		$X_1$	$Y_1$	$X_2$	$Y_2$
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

# Shaft and bearings

## Bearings: Steps to select bearing

Step 2: Calculate the usage required for the bearings, L in terms of revolutions (cycles).

Step 3: Determine the basic load rating,  $C_{10}$ .

$$C_{10} = F_e \times L^{1/3}$$

Where  $F_e$  is the equivalent radial load and L is the usage required per million cycles.

Step 4: Determine a suitable bearing from the catalogue. Make sure that  $C_{10}$  and  $C_0$  both satisfy the requirements.

Step 5: Note that if  $C_0$  is not given and you have assumed that  $F_a/VF_r < e$  or  $> e$ , you need to use the  $C_0$  selected from the table and re-do your analysis till you arrive at the same conclusion/ bearing selection.

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm		Load Ratings, kN			
				$d_s$	$d_H$	Deep Groove $C_{10}$	Angular Contact $C_0$	Deep Groove $C_0$	Angular Contact $C_{10}$
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

# Shaft and bearings

## Example

The axial and radial load on a bearing is 0.75kN and 2.5kN respectively. If the outer ring of the bearing is running at 3250 rpm and the duration required is 16 hours per day for a 6 days per week, 48 weeks and 5 years. Recommend a suitable bearing for this purpose.

Step 1: Determine  $F_e$ ,

$$F_a = 0.75 \text{ kN}, \quad F_r = 2.5 \text{ kN}, \quad V = 1.2.$$

Assume that  $F_a/F_rV < e$ ,  $X_1 = 1$ ,  $Y_1 = 0$

Therefore,

$$F_e = X_i V F_r + Y_i F_a$$

$$\begin{aligned} F_e &= 1 (1.2) 2.5 + 0 (0.75) \\ &= 3 \text{ kN} \end{aligned}$$

Step 2: Determine the total number of life cycles.

$$\begin{aligned} L &= 3250 \times 60 \times 16 \times 6 \times 48 \times 5 \\ &= 4492.8 \text{ million cycles.} \end{aligned}$$

Step 3: Determine the basic load rating,  $C_{10}$

$$\begin{aligned} C_{10} &= F_e \times L^{1/3} \\ &= 3 \times 4492.8^{1/3} \\ &= 49.5 \text{ kN} \end{aligned}$$

# Shaft and bearings

Step 4: Determine a suitable deep groove bearing.

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm		Load Ratings, kN			
				$d_s$	$d_H$	Deep Groove $C_{10}$	$C_0$	Angular Contact $C_{10}$	$C_0$
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

# Shaft and bearings

Step 5: Iterate

From step 4:  $C_{10} = 55.9\text{kN}$ ,  $C_0 = 34 \text{ kN}$

$$F_a/C_0 = 0.75/34 = 0.022$$

Therefore,  $e = 0.21$ .

$$\begin{aligned} F_a/VF_r &= 0.75/(1.2)(2.5) \\ &= 0.25 > e (0.21) \end{aligned}$$

$F_a/C_0$	$e$	$F_a/(VF_r) \leq e$		$F_a/(VF_r) > e$	
		$X_1$	$Y_1$	$X_2$	$Y_2$
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

Therefore, we have made the wrong assumption!  $X_2 = 0.56$ ,  $Y_2 = 2.15$

We have to check whether the chosen bearing is appropriate.

Go back to Step 1:

$$F_e = X_i VF_r + Y_i F_a$$

$$\begin{aligned} F_e &= 0.56(1.2)2.5 + 2.15 (0.75) \\ &= 1.68 + 1.6125 \\ &= 3.29 \text{ kN} \end{aligned}$$

Step 3: Determine the basic load rating,  $C_{10}$

$$\begin{aligned} C_{10} &= F_e \times L^{1/3} \\ &= 3.29 \times 4492.8^{1/3} \\ &= 54.29 \text{ kN} \end{aligned}$$

# Belt and pulleys

## Steps to design and select V-belt drives

Step 1. Determine the speed reduction ratio for the belt drive system.

Step 2. Calculate input power (power at the driver pulley)

$$P_i = (2\pi w T) / 60$$

Where  $\pi = 22/7$  or 3.142

w = speed of driver pulley (rev/min)

T = Torque (Nm)

What is the SI unit of power?

# Belt and pulleys

## Steps to design and select V-belt drives

Step 3. Calculate the design power,  $P_d$

$$P_d = P_i \times k_s$$

Where  $k_s$  is the service factor.

Service factor is also known as safety factor and is the overload capacity commonly built into mechanical systems such as mechanical components, and motors etc. Service factor is greater than 1. A  $k_s$  of 1.2 means that the belt drive can take 20% more load than its rated capacity before breakdown.  $k_s$  depends on the type of load conditions the belt drive is operating in.

**Table 17-15**

Suggested Service  
Factors  $K_s$  for V-Belt  
Drives

<b>Driven Machinery</b>	<b>Source of Power</b>	
	<b>Normal Torque Characteristic</b>	<b>High or Nonuniform Torque</b>
Uniform	1.0 to 1.2	1.1 to 1.3
Light shock	1.1 to 1.3	1.2 to 1.4
Medium shock	1.2 to 1.4	1.4 to 1.6
Heavy shock	1.3 to 1.5	1.5 to 1.8

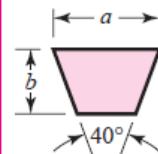
# Belt and pulleys

## Steps to design and select V-belt drives

Step 4. Based on the design power, choose an appropriate V-belt type from pg 899.

**Table 17-9**

Standard V-Belt Sections



Belt Section	Width <i>a</i> , in	Thickness <i>b</i> , in	Minimum Sheave Diameter, in	hp Range, One or More Belts
A	$\frac{1}{2}$	$\frac{11}{32}$	3.0	$\frac{1}{4}$ –10
B	$\frac{21}{32}$	$\frac{7}{16}$	5.4	1–25
C	$\frac{7}{8}$	$\frac{17}{32}$	9.0	15–100
D	$1\frac{1}{4}$	$\frac{3}{4}$	13.0	50–250
E	$1\frac{1}{2}$	1	21.6	100 and up

# Belt and pulleys

## Steps to design and select V-belt drives

Step 5. Find diameter of driver pulley.

There are two equations we can analyse here depending on what variables we are given.

1.  $d + D = C$
2.  $dw_1 = Dw_2$

Where  $d$  is the diameter of the driver pulley.

$D$  is the diameter of the driven pulley.

$C$  is the approximate minimum centre distance between the pulleys.

$w_1, w_2$  are the speed of the driver pulley and driven pulley respectively.

Note that in most cases, a certain centre distance is required in a belt system. If no preference of pulley diameters are given, approach the problem by suggesting a reasonable speed reduction ratio for the belt system if the variable is also not a critical requirement for the design.

# Belt and pulleys

## Steps to design and select V-belt drives

Step 6. Determine the pitch length,  $L_p$

$$L_p = 2C + \pi(D + d)/2 + (D - d)^2/(4C)$$

Where C is the centre distance

D is the diameter of big pulley

d is the diameter of small pulley

If the centre distance between the pulleys is not given, we can use C as the approximate minimum centre distance between the pulleys.

**Note that information in many belt design tables/ catalogues are in inches.**

# Belt and pulleys

## Steps to design and select V-belt drives

Step 7. Determine the inside circumference,  $L_i$

Calculations involving the belt length are usually based on the pitch length. For any given belt section, the pitch length is obtained by adding a quantity to the inside circumference

$$L_i = L_p - q$$

Length Conversion Dimensions (Add the listed quantity to the inside circumference to obtain the pitch length in inches).

Belt section	A	B	C	D	E
Quantity to be added	1.3	1.8	2.9	3.3	4.5

**We need to find inner circumference of v belts as they come in standards.**

# Belt and pulleys

## Steps to design and select V-belt drives

Step 7. Determine the inside circumference,  $L_i$

Section	Circumference, in
Inside Circumferences of Standard V Belts	
A	26, 31, 33, 35, 38, 42, 46, 48, 51, 53, 55, 57, 60, 62, 64, 66, 68, 71, 75, 78, 80, 85, 90, 96, 105, 112, 120, 128
B	35, 38, 42, 46, 48, 51, 53, 55, 57, 60, 62, 64, 65, 66, 68, 71, 75, 78, 79, 81, 83, 85, 90, 93, 97, 100, 103, 105, 112, 120, 128, 131, 136, 144, 158, 173, 180, 195, 210, 240, 270, 300
C	51, 60, 68, 75, 81, 85, 90, 96, 105, 112, 120, 128, 136, 144, 158, 162, 173, 180, 195, 210, 240, 270, 300, 330, 360, 390, 420
D	120, 128, 144, 158, 162, 173, 180, 195, 210, 240, 270, 300, 330, 360, 390, 420, 480, 540, 600, 660
E	180, 195, 210, 240, 270, 300, 330, 360, 390, 420, 480, 540, 600, 660

# Belt and pulleys

## Steps to design and select V-belt drives

Step 8. Determine the actual pitch length,  $L_p$

$$L_p = L_i + q$$

Step 9. Determine the actual centre distance,  $C$

$$L_p = 2C + \pi(D + d)/2 + (D - d)^2/(4C)$$

$$C = A + (A^2 - B)^{1/2}$$

$$\text{Where } A = L_p/4 - \pi/8(D + d)$$

$$B = (D - d)^2/8$$

# Belt and pulleys

## Steps to design and select V-belt drives

Step 10. Determine the correction factor for angle of contact,  $k_1$

**Table 17-13**

Angle of Contact	$\frac{D-d}{C}$	$\theta$ , deg	VV	$K_1$
Correction Factor $K_1$ for VV* and V-Flat Drives	0.00	180	1.00	0.75
	0.10	174.3	0.99	0.76
	0.20	166.5	0.97	0.78
	0.30	162.7	0.96	0.79
	0.40	156.9	0.94	0.80
	0.50	151.0	0.93	0.81
	0.60	145.1	0.91	0.83
	0.70	139.0	0.89	0.84
	0.80	132.8	0.87	0.85
	0.90	126.5	0.85	0.85
	1.00	120.0	0.82	0.82
	1.10	113.3	0.80	0.80
	1.20	106.3	0.77	0.77
	1.30	98.9	0.73	0.73
	1.40	91.1	0.70	0.70
	1.50	82.8	0.65	0.65

# Belt and pulleys

## Steps to design and select V-belt drives

Step 11. Determine the belt length correction factor,  $k_2$

**Table 17-14**

Belt-Length Correction  
Factor  $K_2^*$

Belt-Length Correction Factor $K_2^*$	Length Factor	Nominal Belt Length, in			
		A Belts	B Belts	C Belts	D Belts
	0.85	Up to 35	Up to 46	Up to 75	Up to 128
	0.90	38–46	48–60	81–96	144–162
	0.95	48–55	62–75	105–120	173–210
	1.00	60–75	78–97	128–158	240
	1.05	78–90	105–120	162–195	270–330
	1.10	96–112	128–144	210–240	360–420
	1.15	120 and up	158–180	270–300	480
	1.20		195 and up	330 and up	540 and up
					660

# Belt and pulleys

## Steps to design and select V-belt drives

Step 12. Determine the horsepower ratings of standard V-belts,  $P_r$ .

Table 17-12

Horsepower Ratings of Standard V Belts

Belt Section	Sheave Pitch Diameter, in	Belt Speed, ft/min				
		1000	2000	3000	4000	5000
A	2.6	0.47	0.62	0.53	0.15	
	3.0	0.66	1.01	1.12	0.93	0.38
	3.4	0.81	1.31	1.57	1.53	1.12
	3.8	0.93	1.55	1.92	2.00	1.71
	4.2	1.03	1.74	2.20	2.38	2.19
	4.6	1.11	1.89	2.44	2.69	2.58
	5.0 and up	1.17	2.03	2.64	2.96	2.89
B	4.2	1.07	1.58	1.68	1.26	0.22
	4.6	1.27	1.99	2.29	2.08	1.24
	5.0	1.44	2.33	2.80	2.76	2.10
	5.4	1.59	2.62	3.24	3.34	2.82
	5.8	1.72	2.87	3.61	3.85	3.45
	6.2	1.82	3.09	3.94	4.28	4.00
	6.6	1.92	3.29	4.23	4.67	4.48
C	7.0 and up	2.01	3.46	4.49	5.01	4.90
	6.0	1.84	2.66	2.72	1.87	
	7.0	2.48	3.94	4.64	4.44	3.12
	8.0	2.96	4.90	6.09	6.36	5.52
	9.0	3.34	5.65	7.21	7.86	7.39
	10.0	3.64	6.25	8.11	9.06	8.89
	11.0	3.88	6.74	8.84	10.0	10.1
D	12.0 and up	4.09	7.15	9.46	10.9	11.1
	10.0	4.14	6.13	6.55	5.09	1.35
	11.0	5.00	7.83	9.11	8.50	5.62
	12.0	5.71	9.26	11.2	11.4	9.18
	13.0	6.31	10.5	13.0	13.8	12.2
	14.0	6.82	11.5	14.6	15.8	14.8
	15.0	7.27	12.4	15.9	17.6	17.0
E	16.0	7.66	13.2	17.1	19.2	19.0
	17.0 and up	8.01	13.9	18.1	20.6	20.7
	16.0	8.68	14.0	17.5	18.1	15.3
	18.0	9.92	16.7	21.2	23.0	21.5
	20.0	10.9	18.7	24.2	26.9	26.4
	22.0	11.7	20.3	26.6	30.2	30.5
	24.0	12.4	21.6	28.6	32.9	33.8
	26.0	13.0	22.8	30.3	35.1	36.7
	28.0 and up	13.4	23.7	31.8	37.1	39.1

$$\begin{aligned}V &= wr \\&= w(d/2)\end{aligned}$$

Make sure that the belt speed is in ft/min if you want to use this catalogue.

# Belt and pulleys

## Steps to design and select V-belt drives

Step 13. Determine the corrected power,  $P_c$

$$P_c = P_r k_1 k_2$$

Step 14. Determine the minimum number of belts,  $N$ .

$$N = P_d / P_c$$

Step 15. Determine the tension in the belts,

$$\begin{aligned} T_1/T_2 &= \text{number of belts} \\ P_i &= (T_1 - T_2) v \end{aligned}$$

Where  $v$  is the linear speed of belt in m/s

# Belt and pulleys

## Summary

### **General information required to design belts and pulleys**

1. Input power.
2. RPM of the driving and driven sprocket (speed ratio).
3. Load classification.
4. Space limitations if any.

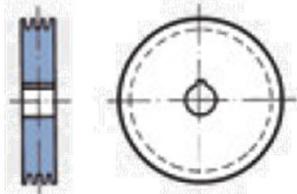
### **General information you need to specify for your belt pulley system.**

1. Diameter of pulleys.
2. Centre distance.
3. Pitch length of belt.
4. Number of belts

# Belt and pulleys

## Important design considerations

1. Long center-to-center distances are not recommended for V belts because the excessive vibration of the slack side will shorten the belt life materially. In general, the center to-center distance should not be greater than 3 times the sum of the sheave diameters and no less than the diameter of the larger sheave.
2. The different types of pulley design are as shown.



Solid pulley

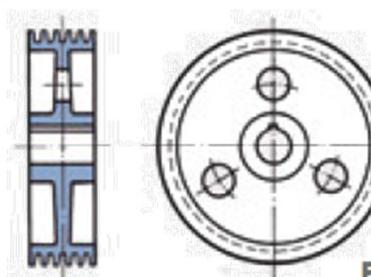
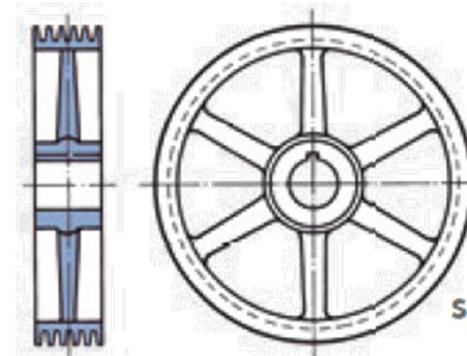


Plate pulley



Spoked pulley

# Belt and pulleys

## Question example 2.

Design a belt drive with the following specifications:

$P_i = 13.3\text{kW}$ ,  $w_1 = 1440\text{rpm}$ ,  $w_2 = 1150\text{rpm}$ , minimum centre distance of the belt drive system =  $0.6\text{m}$ ,  $K_s = 1.3$ .

# Belt and pulleys

Step 1: Determine the speed reduction ratio of the belt system.

Let D1 be 11 inch

$$w_1 D_1 = w_2 D_2$$

$$1440 / 1150 = D_2 / 11$$

$$D_2 = 13.77$$

$$D_1 + D_2 = 11 + 13.77 = 24.77 \text{ inch} > 23.62 \text{ inches (0.6m)}$$

Step 2 and 3: Calculate the design power.

$$P_i = 13.3 \text{ kW}, K_s = 1.3$$

$$P_d = 13.3 \times 1.3$$

$$= 17.29 \text{ kW} = 23.2 \text{ HP}$$

Note that 0.745 kW = 1 HP

Step 4: Choose appropriate v-belt.

Belt section C

# Belt and pulleys

Step 5: Determine the diameter of pulleys and approximate minimum centre distance.

Let D1 be 11 inch

$$\begin{aligned} w_1 D_1 &= w_2 D_2 \\ 1440 / 1150 &= D_2 / 11 \\ D_2 &= 13.77 \end{aligned}$$

$$D_1 + D_2 = 11 + 13.77 = 24.77 \text{ in} > 23.62 \text{ in (0.6m)}$$

$$\begin{aligned} C_{\min} &= D_1 + D_2 \\ &= 24.77 \text{ in} \end{aligned}$$

Step 6: Determine the L<sub>p</sub>.

$$L_p = 2C + \pi(D + d)/2 + (D - d)^2/(4C)$$

$$L_p = 88.53 \text{ in}$$

# Belt and pulleys

Step 7: Determine the inside circumference of v belts.

$$\begin{aligned}L_i &= L_p - q \\&= 88.53 - 2.9 \\&= 85.63 \text{ in}\end{aligned}$$

Step 8: Determine the standard inside circumference of v belts and the actual pitch length.

$$L_i (\text{actual}) = 85 \text{ in}$$

$$\begin{aligned}L_p (\text{actual}) &= 85 + 2.9 \\&= 87.9 \text{ in}\end{aligned}$$

Step 9: Determine the actual centre distance.

$$C = A + (A^2 - B)^{1/2}$$

$$C = 24.46 \text{ in} > 23.62 \text{ (0.6m)}$$

$$\begin{aligned}\text{Where } A &= L_p/4 - \pi/8 (D + d) \\B &= (D - d)^2/8\end{aligned}$$

# Belt and pulleys

Step 10: Determine the angle correction factor, k1.

$$\begin{aligned}(D-d)/c &= 13.77-11/24.46 \\ &= 0.113 \\ \text{Therefore, } k1 &= 0.987\end{aligned}$$

Step 11: Determine the belt length correction factor, k2.

$$K2 = 0.9$$

Step 12: Determine the horse power rating Pr.

$$\begin{aligned}W1 &= 1440 \text{ rpm (multiple by } 2\pi)/60 \\ &= 75.36 \text{ rad/s}\end{aligned}$$

$$\begin{aligned}V &= wr \\ &= 75.36 \times 11/2 \\ &= 829.4 \text{ in/s} \\ &= 4147 \text{ ft/min}\end{aligned}$$

$$Pr = 10.015 \text{ hp}$$

Note: 12 in = 1 ft

# Chain and sprockets

## Steps to design and select chain drives

Step 1. Determine number of teeth in the driving and driven sprockets.

The relationship between the number of teeth in the driving and driven sprocket and their speed of rotation is:  $w_1n_1 = w_2n_2$

Where  $w$  = speed of sprockets (rev/min)

$n$  = number of teeth

Step 2. Calculate input power (power at the driving sprocket)

$$P_i = (2\pi w T) / 60$$

Where  $\pi = 22/7$  or 3.142

$w$  = speed of sprocket (rev/min)

$T$  = Torque (Nm)

What is the SI unit of power?

# Chain and sprockets

## Steps to design and select chain drives

Step 3. Calculate the design power,  $P_d$

$$P_d = P_i \times k_s$$

Where  $k_s$  is the service factor.

Service factor is also known as safety factor and is the overload capacity commonly built into mechanical systems such as mechanical components, and motors etc. Service factor is greater than 1. A  $k_s$  of 1.2 means that the chain drive can take 20% more load than its rated capacity before breakdown.  $k_s$  depends on the type of load conditions the chain drive is operating in.

Load classifications		
UNIFORM LOAD	MODERATE SHOCK LOAD	HEAVY SHOCK LOAD
Centrifugal pumps, Agitator for liquids, Conveyors, Fans - Uniform Load	Reciprocating pumps, Wood working M/c's Grinders, Conveyors - irregular Load	Presses, Earth moving equipment Shears, Cranes & Hoists, Reciprocating and Shaker type conveyors, Crushers, Reciprocating feeders
Generators, M/c's all types with uniform non-reversing loads	Mixers and Machines all types with moderate shock and non-reversing loads	Machines – all types with severe impact shock loads or variation and reversing service

Type of Driven Load	SERVICE FACTOR		
	TYPE OF INPUT POWER	Internal Combustion Engine with Hydraulic Drive	Electric Motor or Turbine
Uniform Moderate Shock Heavy Shock	1.0 1.3 1.2	1.0 1.3 1.4	1.2 1.4 1.5 1.7

# Chain and sprockets

## Steps to design and select chain drives

Step 4. Calculate the required power,  $P_r$

$$P_d = P_r \times K_1 \times K_2$$
$$P_r = P_d / K_1 K_2$$

Definitions of  $K_1$ .

- $K_1$ : tooth correction factors related to power fluctuations.
- Obtain  $K_1$  from pg 913.

**Table 17-22**

Tooth Correction Factors,  $K_1$

Number of Teeth on Driving Sprocket	$K_1$ Pre-extreme Horsepower	$K_1$ Post-extreme Horsepower
11	0.62	0.52
12	0.69	0.59
13	0.75	0.67
14	0.81	0.75
15	0.87	0.83
16	0.94	0.91
17	1.00	1.00
18	1.06	1.09
19	1.13	1.18
20	1.19	1.28
$N$	$(N_1/17)^{1.08}$	$(N_1/17)^{1.5}$

- Pre extremes – chain power is limited by link-plate fatigue (failure due to duration)
- Post extremes – chain power performance is limited by impact fatigue.

# Chain and sprockets

## Steps to design and select chain drives

Definitions of  $K_2$ .

- $K_2$ : Multiple strands factor.

**Table 17-23**

Multiple-Strand  
Factors,  $K_2$

	<b>Number of Strands</b>	<b><math>K_2</math></b>
	1	1.0
	2	1.7
	3	2.5
	4	3.3
	5	3.9
	6	4.6
	8	6.0

- With more strands, the designed or required power is reduced. What is increased?

# Chain and sprockets

## Steps to design and select chain drives

Step 5. Based on the required power,  $P_r$ , select the right chain drive from the ANSI catalogue (Table 17-20). Choose appropriate ANSI chain number and lubrication type.

Sprocket Speed, rev/min	ANSI Chain Number				
	25	35	40	41	50
50	0.05	0.16	0.37	0.20	0.72
100	0.09	0.29	0.69	0.38	1.34
150	0.13*	0.41*	0.99*	0.55*	1.92*
200	0.16*	0.54*	1.29	0.71	2.50
300	0.23	0.78	1.85	1.02	3.61
400	0.30*	1.01*	2.40	1.32	4.67
500	0.37	1.24	2.93	1.61	5.71
600	0.44*	1.46*	3.45*	1.90*	6.72*
700	0.50	1.68	3.97	2.18	7.73
800	0.56*	1.89*	4.48*	2.46*	8.71*
900	0.62	2.10	4.98	2.74	9.69
1000	0.68*	2.31*	5.48	3.01	10.7
1200	0.81	2.73	6.45	3.29	12.6
1400	0.93*	3.13*	7.41	2.61	14.4
1600	1.05*	3.53*	8.36	2.14	12.8
1800	1.16	3.93	8.96	1.79	10.7
2000	1.27*	4.32*	7.72*	1.52*	9.23*
2500	1.56	5.28	5.51*	1.10*	6.58*
3000	1.84	5.64	4.17	0.83	4.98

Sprocket Speed, rev/min	ANSI Chain Number							
	80	100	120	140	160	180	200	240
50	2.88	5.52	9.33	14.4	20.9	28.9	38.4	61.8
100	5.38	10.3	17.4	26.9	39.1	54.0	71.6	115
150	7.75	14.8	25.1	38.8	56.3	77.7	103	166
200	10.0	19.2	32.5	50.3	72.9	101	134	215
300	14.5	27.7	46.8	72.4	105	145	193	310
400	18.7	35.9	60.6	93.8	136	188	249	359
500	22.9	43.9	74.1	115	166	204	222	0
600	27.0	51.7	87.3	127	141	155	169	
700	31.0	59.4	89.0	101	112	123	0	
800	35.0	63.0	72.8	82.4	91.7	101		
900	39.9	52.8	61.0	69.1	76.8	84.4		
1000	37.7	45.0	52.1	59.0	65.6	72.1		
1200	28.7	34.3	39.6	44.9	49.9	0		
1400	22.7	27.2	31.5	35.6	0			
1600	18.6	22.3	25.8	0				
1800	15.6	18.7	21.6					
2000	13.3	15.9	0					
2500	9.56	0.40						
3000	7.25	0						

Type A

Type B

Type C

Type C

Type C'

\*Estimated from ANSI tables by linear interpolation.

Note: Type A—manual or drip lubrication; type B—bath or disk lubrication; type C—oil-stream lubrication.

Note: Type A—manual or drip lubrication; type B—bath or disk lubrication; type C—oil-stream lubrication; type C'—type C, but this is a galling region; submit design to manufacturer for evaluation.

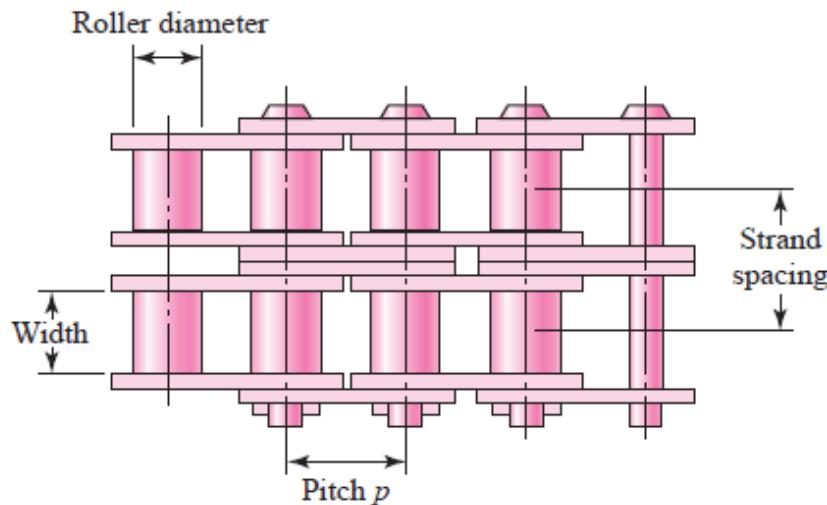
**Note: Power is in horsepower.  $0.746 \text{ kW} = 1 \text{ hp}$**

# Chain and sprockets

## Steps to design and select chain drives

Step 6. Look up for the pitch of chain, width of chain and roller diameter using the ANSI chain number.

ANSI Chain Number	Pitch, in (mm)	Width, in (mm)	Minimum Tensile Strength, lbf (N)	Average Weight, lbf/ft (N/m)	Roller Diameter, in (mm)	Multiple-Strand Spacing, in (mm)
25	0.250 (6.35)	0.125 (3.18)	780 (3 470)	0.09 (1.31)	0.130 (3.30)	0.252 (6.40)
35	0.375 (9.52)	0.188 (4.76)	1 760 (7 830)	0.21 (3.06)	0.200 (5.08)	0.399 (10.13)
41	0.500 (12.70)	0.25 (6.35)	1 500 (6 670)	0.25 (3.65)	0.306 (7.77)	—
40	0.500 (12.70)	0.312 (7.94)	3 130 (13 920)	0.42 (6.13)	0.312 (7.92)	0.566 (14.38)
50	0.625 (15.88)	0.375 (9.52)	4 880 (21 700)	0.69 (10.1)	0.400 (10.16)	0.713 (18.11)
60	0.750 (19.05)	0.500 (12.7)	7 030 (31 300)	1.00 (14.6)	0.469 (11.91)	0.897 (22.78)
80	1.000 (25.40)	0.625 (15.88)	12 500 (55 600)	1.71 (25.0)	0.625 (15.87)	1.153 (29.29)
100	1.250 (31.75)	0.750 (19.05)	19 500 (86 700)	2.58 (37.7)	0.750 (19.05)	1.409 (35.76)
120	1.500 (38.10)	1.000 (25.40)	28 000 (124 500)	3.87 (56.5)	0.875 (22.22)	1.789 (45.44)
140	1.750 (44.45)	1.000 (25.40)	38 000 (169 000)	4.95 (72.2)	1.000 (25.40)	1.924 (48.87)
160	2.000 (50.80)	1.250 (31.75)	50 000 (222 000)	6.61 (96.5)	1.125 (28.57)	2.305 (58.55)
180	2.250 (57.15)	1.406 (35.71)	63 000 (280 000)	9.06 (132.2)	1.406 (35.71)	2.592 (65.84)
200	2.500 (63.50)	1.500 (38.10)	78 000 (347 000)	10.96 (159.9)	1.562 (39.67)	2.817 (71.55)
240	3.00 (76.70)	1.875 (47.63)	112 000 (498 000)	16.4 (239)	1.875 (47.62)	3.458 (87.83)



# Chain and sprockets

## Steps to design and select chain drives

Step 7. Determine diameters of driving sprocket and driven sprocket,

$$d = N_1 \times p$$

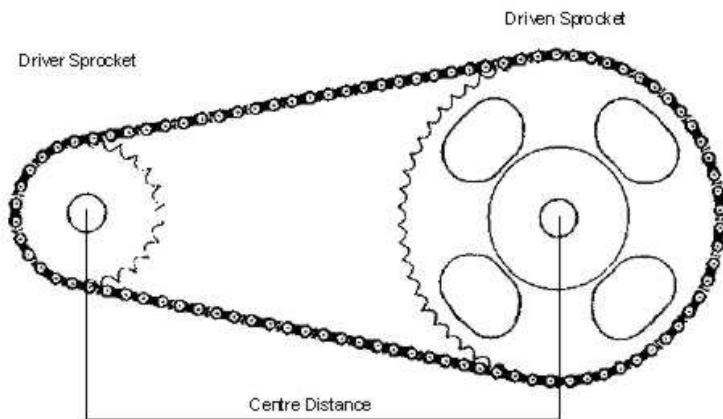
$$D = N_2 \times p$$

Where D is diameter of driver sprocket and  
d is diameter of driven sprocket  
 $d_{\text{roller}}$  is diameter of roller (Obtain from 17-19)

# Chain and sprockets

## Steps to design and select chain drives

Step 8. Calculate minimum centre distance,  $C_{min}$  between the sprockets.



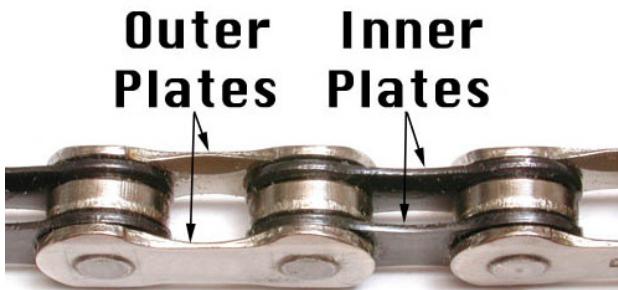
$$C_{min} = (D + d)/2 + d_{\text{roller}}$$

Where  $d$  is diameter of driver sprocket and  
 $D$  is diameter of driven sprocket  
 $d_{\text{roller}}$  is diameter of roller (Obtain from 17-19)

# Chain and sprockets

## Steps to design and select chain drives

Step 9. To find the number of links base on centre distance.



$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$

Where L/p is chain length in pitches – number of links.

C is the centre distance.

p is the pitch. Table 17.19.

N<sub>1</sub> is the number of teeth on the driving sprocket

N<sub>2</sub> is the number of teeth in driven sprocket

# Chain and sprockets

## Steps to design and select chain drives

Step 10. To find actual centre distance.

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p}$$

$$C = \frac{p}{4} \left[ -A + \sqrt{A^2 - 8 \left( \frac{N_2 - N_1}{2\pi} \right)^2} \right]$$

Where L/p is chain length in pitches – number of links.

C is the centre distance.

N1 is the number of teeth on the driving sprocket

N2 is the number of teeth in driven sprocket

# Chain and sprockets

## Optimisation

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p}$$

$$C = \frac{p}{4} \left[ -A + \sqrt{A^2 - 8 \left( \frac{N_2 - N_1}{2\pi} \right)^2} \right]$$

What to optimise???

Number of links in chain is dependent on centre distance between sprockets.

# Chain and sprockets

## Summary

### **General information required to design chains and sprockets**

1. Input power.
2. RPM of the driving and driven sprocket (speed ratio).
3. Load classification.
4. Space limitations if any.

### **General information you need to specify for your chain drive system.**

1. Number of teeth in driving and driven sprockets.
2. Specify ANSI number for chain.
3. Pitch, width and roller diameter.
4. Number of links.
5. Centre distance between sprockets.
6. Type of lubrication required for chain drives.

# Chain and sprockets

## Important design considerations

For maximum service life, smooth operation and optimum performance, the following points should be considered while determining the number of teeth in the **driver sprocket**.

- a) As most drives have an **even number of pitches** in the chain, the use of a driver sprocket with **an odd number of teeth** ensures even distribution of chain and wheel tooth wear.
- b) For normal, steady drives, driver sprockets should generally have minimum 17 teeth. When the sprocket speed is constant, the chain speed is subjected to regular cyclic variation and this can be reduced when number of teeth increases.
- c) For moderate and heavy shock drives, driver sprockets should have minimum 23 teeth. Note: sprockets with 67 teeth and above will reduce the life-span of the chain system.
- d) Consider the use of multi – strand chains when space available is limited or when chain drives are required to run at high speed but with a low pitch.
- e) Recommended maximum speed reduction ratio for chain drive is 6:1.
- f) Guards must be installed with chain drives if possible.

# Chain and sprockets

## Question example.

Design a chain drives with the following specifications:

$P_i = 27\text{kW}$ ,  $w_1 = 1500\text{rpm}$ ,  $w_2 = 700\text{rpm}$ , centre distance of the chain drive system  $> 500\text{mm}$ ,  $K_s = 1.2$ ,  $N_1 = 19$ , duplex strand.

- a) Recommend a suitable number of teeth for the driven sprocket and the actual speed of the sprockets.
- b) Determine the design power and recommend a suitable ANSI chain number.
- c) Determine the diameters of the sprockets, diameter of the roller and the pitch of the chain drive system.
- d) Determine the minimum centre distance of the chain drive system and suggest whether this is suitable.
- e) Find the number of links required for the chain drive system.

# Chain and sprockets

Design a chain drive with the following specifications:

$P_i = 20\text{kW}$ ,  $w_1 = 1500\text{rpm}$ ,  $w_2 = 700\text{rpm}$ , centre distance of the chain drive system  $> 500\text{mm}$ ,  $K_s = 1.2$ ,  $N_1 = 19$ , duplex strand.

Step 1. Determine number of teeth in the driving and driven sprockets.

$$\begin{aligned} w_1 N_1 &= w_2 N_2 \\ 1500/700 &= N_2 / 19 \\ N_2 &= 40.71 \text{ teeth} \\ N_2 &= 41 \text{ teeth} \end{aligned}$$

Step 2. Calculate input power (power at the driving sprocket)

$$P_i = 20\text{kW}$$

Step 3. Calculate the design power.

$$\begin{aligned} P_d &= P_i \times k_s \\ &= 20 \times 1.2 \\ &= 24 \text{ kW} \end{aligned}$$

# Chain and sprockets

Design a chain drive with the following specifications:

$P_i = 20\text{kW}$ ,  $w_1 = 1500\text{rpm}$ ,  $w_2 = 700\text{rpm}$ , centre distance of the chain drive system  $> 500\text{mm}$ ,

$K_s = 1.2$ ,  $N_1 = 19$ , duplex strand.

Step 4. Calculate the required power.  $P_r$

$$K_1 = 1.13$$

$$K_2 = 1.7$$

$$\begin{aligned} P_r &= 24/1.13 \times 1.7 \\ &= 12.49\text{kW} \text{ (16.74 Hp, 1500rpm)} \end{aligned}$$

Step 5. Choose appropriate ANSI number

Choose ANSI 80

# Chain and sprockets

Design a chain drive with the following specifications:

$P_i = 20\text{kW}$ ,  $w_1 = 1500\text{rpm}$ ,  $w_2 = 700\text{rpm}$ , centre distance of the chain drive system  $> 500\text{mm}$ ,  $K_s = 1.2$ ,  $N_1 = 19$ , duplex strand.

Step 6. Pitch of chain, width of chain and roller diameter,

$$\text{Pitch of chain} = 25.4\text{mm}$$

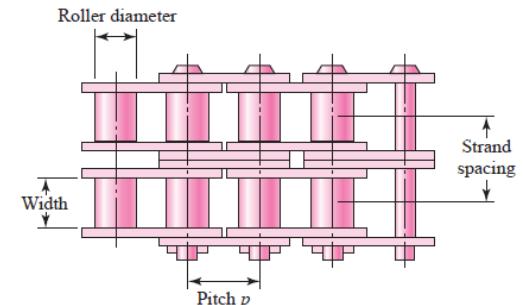
$$\text{Width of chain} = 15.88\text{mm}$$

$$\text{Roller diameter} = 15.87\text{mm}$$

Step 7. Determine sprocket diameter,

$$d = 19 \times 25.4 = 482.6\text{mm}$$

$$D = 41 \times 25.4 = 1041.4\text{mm}$$



Step 8. Minimum centre distance,

$$C_{\min} = (D + d)/2 + d_{\text{roller}}$$

$$C_{\min} = 777.87 \text{ mm} > 500 \text{ mm}$$

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$

# Chain and sprockets

Design a chain drive with the following specifications:

P<sub>i</sub> = 20kW, w<sub>1</sub> = 1500rpm, w<sub>2</sub> = 700rpm, centre distance of the chain drive system > 500mm,

K<sub>s</sub> = 1.2, N<sub>1</sub> = 19, duplex strand.

Step 9. Number of links (even number)

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$