Example 2

$$\int \frac{e^x}{e^{2x} + 2e^x - 8} \, dx \qquad \text{let } u = e^x$$
$$du = e^x dx$$

$$\int \frac{du}{u^2 + 2u - 8} = \int \frac{du}{(u+4)(u-2)}$$

use partial fractions $\dfrac{1}{(u+4)(u-2)} = \dfrac{A}{u+4} + \dfrac{B}{u-2}$

$$1 = A(u-2) + B(u+4)$$
$$u = 2 \Rightarrow B = \tfrac{1}{6}$$
$$u = -4 \Rightarrow A = -\tfrac{1}{6}$$

$$\int \frac{du}{u^2 + 2u - 8} = \int -\frac{1}{6} \frac{1}{u+4} \, du + \int \frac{1}{6} \frac{1}{u-2} \, du$$

$$= -\frac{1}{6} \ln|u+4| + \frac{1}{6} \ln|u-2| + C$$

$$= -\frac{1}{6} \ln|e^x + 4| + \frac{1}{6} \ln|e^x - 2| + C$$