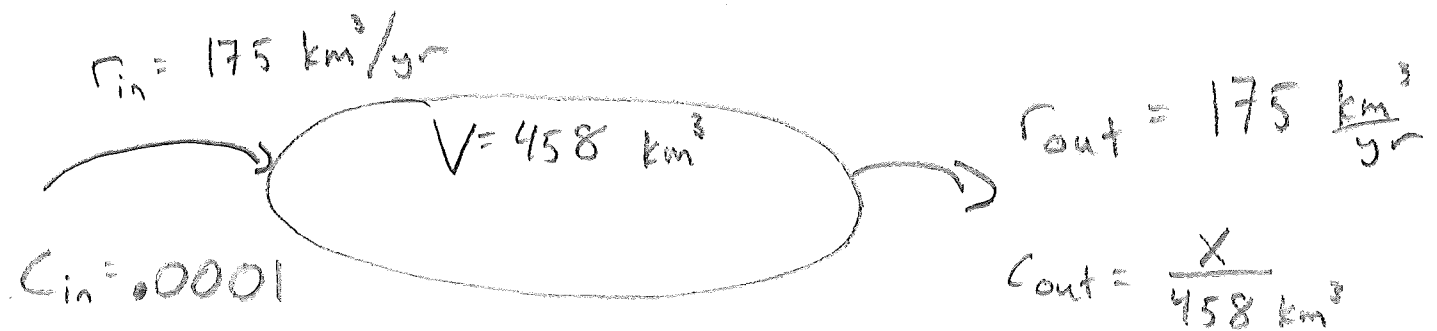


# Ex Lake Erie pollution



let  $x(t)$  = volume of pollution in lake at time  $t$ .

$$x(0) = 0.0005 (458) = 0.2290 \text{ km}^3$$

we want to find when  $x(t) = 0.0002(458)$   
 $= 0.0916 \text{ km}^3$

$$\frac{dx}{dt} = \text{rate in} - \text{rate out}$$

$$\frac{dx}{dt} = 0.0001(175) - \frac{x}{458} (175)$$

$$\frac{dx}{dt} = 0.0175 - 0.3821 x$$

we can rewrite as

$$\frac{dx}{dt} + 0.3821 x = 0.0175$$

1<sup>st</sup> order, linear,  
nonhomogeneous

We can use IFM to solve this.

$$\lambda = e^{\int .3821 dt} = e^{.3821 t}$$

$$e^{.3821 t} \left[ \frac{dx}{dt} + .3821 x \right] = .0175 e^{.3821 t}$$

$$\frac{d}{dt} \left( e^{.3821 t} x \right) = .0175 e^{.3821 t}$$

$$\begin{aligned} \Rightarrow e^{.3821 t} x &= \int .0175 e^{.3821 t} dt \\ &= \frac{.0175}{.3821} e^{.3821 t} + C \end{aligned}$$

$$\Rightarrow x(t) = .0458 + C e^{-.3821 t}$$

use IC  $x(0) = 0.2290$

$$0.2290 = .0458 + C$$

$$\Rightarrow C = 0.1832$$

$$X(t) = .0458 + .1832 e^{-.3821 t}$$

we want to find  $t$  such that

$$X(t) = .0916$$

$$.0916 = .0458 + .1832 e^{-.3821 t}$$

$$\Rightarrow t = \frac{-1}{.3821} \ln \left( \frac{.0916 - .0458}{.1832} \right) \approx 3.63 \text{ yrs}$$