

Evaluate  $\int \frac{8-3x}{(x+1)(x^2-4x+6)} dx$

First try to factor  $x^2 - 4x + 6$ .

Note that  $x^2 - 4x + 6 = 0$  has roots  $\frac{4 \pm \sqrt{16 - 4(6)}}{2}$

which are imaginary.

(The term under the square root is negative.)

Thus it is irreducible.

Now use Partial Fractions.

$$\frac{8-3x}{(x+1)(x^2-4x+6)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-4x+6}$$

$$8-3x = A(x^2-4x+6) + (Bx+C)(x+1)$$

if  $x = -1$

$$8-3x = 11 = A(1+4+6) + 0$$

$$\Rightarrow A = 1$$

if  $x = 0$

$$8-3x = 8 = 1(6) + C \Rightarrow C = 2$$

if  $x = 1$

$$8-3x = 5 = 1(1-4+6) + (B+2)(2)$$

$$5 = 3 + 2B + 4 \Rightarrow B = -1$$

$$\int \frac{8-3x}{(x+1)(x^2-4x+6)} dx = \int \frac{1}{x+1} + \frac{-x+2}{x^2-4x+6} dx$$

$$u = x^2 - 4x + 6$$

$$du = (2x-4) dx$$

$$(x-2) dx = \frac{1}{2} du$$

$$= \ln|x+1| - \frac{1}{2} \ln|x^2-4x+6| + C$$