Consider driving down a road.

At time t position given parametrically

$$x(t) = \frac{2}{t^2+1} - 1 \qquad t \geq 0$$

$$y(t) = \frac{2t}{t^2+1}$$

Can you calculate how far you have driven down road in time, T.

$$L(T) = \int_0^T \sqrt{\left(\frac{-4t}{(t^2+1)^2}\right)^2 + \left(\frac{2-2t^2}{(t^2+1)^2}\right)^2}\, dt$$

$$= \int_0^T \sqrt{\frac{16t^2 + 4 - 8t^2 + 4t^4}{\left((t^2+1)^2\right)^2}}\, dt$$

$$= \int_0^T \sqrt{\frac{4t^4 + 8t^2 + 4}{\left(t^2+1)^2\right)^2}}\, dt$$

$$= \int_0^T \sqrt{\frac{4(t^2+1)^2}{\left((t^2+1)^2\right)^2}}\, dt$$

$$= \int_0^T 2 \frac{1}{t^2+1}\, dt$$

$$= 2 \tan^{-1} t \Big|_0^T$$

$$= 2\tan^{-1}(T) - \cancelto{0}{2\tan^{-1}(0)}$$

$$L(T) = 2\tan^{-1}(T)$$

$$T = \tan\left(\frac{L}{2}\right)$$

position as a fcn of distance, L, down road instead of time, t,

$$x = \frac{2}{\tan^2\left(\frac{L}{2}\right)+1} - 1$$

$$y = \frac{2\tan\left(\frac{L}{2}\right)}{\tan^2\left(\frac{L}{2}\right)+1}$$

~~$X(t)$~~

$$\frac{2}{\tan^2\left(\frac{L}{2}\right)+1} - 1 = \frac{2}{\sec^2\left(\frac{L}{2}\right)} - 1$$

$$= 2\cos^2\left(\frac{L}{2}\right) - \cos^2\frac{L}{2} - \sin^2\frac{L}{2}$$

$$= \cos^2\left(\frac{L}{2}\right) - \sin^2\left(\frac{L}{2}\right)$$

$$= \cos(L)$$

$y(t)$

$$\frac{2\tan\left(\frac{t}{2}\right)}{\tan^2\left(\frac{t}{2}\right)+1} = \frac{2\,\frac{\sin t/2}{\cos t/2}}{\sec^2\left(\frac{t}{2}\right)}$$

$$= \frac{2\cos^2\left(\frac{t}{2}\right)\sin\left(\frac{t}{2}\right)}{\cos\left(t/2\right)}$$

$$= 2\cos\left(\frac{t}{2}\right)\sin\left(\frac{t}{2}\right)$$

$$= \sin\left(\frac{t}{2}+\frac{t}{2}\right)$$

$$= \sin(t)$$