

Example 5 Radioactive Decay ($k < 0$)

$$M = -k \int_0^{\infty} t e^{kt} dt$$

$$= \lim_{b \rightarrow \infty} -k \int_0^b t e^{kt} dt$$

IBP $u=t \quad du=e^{kt} dt$
 $dv=dt \quad v=\frac{1}{k} e^{kt}$

$$= \lim_{b \rightarrow \infty} -k \left[\frac{1}{k} t e^{kt} \Big|_0^b - \frac{1}{k} \int_0^b e^{kt} dt \right]$$

$$= \lim_{b \rightarrow \infty} -k \left[\frac{b}{k} e^{kb} - 0 - \frac{1}{k^2} e^{kt} \Big|_0^b \right]$$

$$= \lim_{b \rightarrow \infty} -k \left[\frac{b}{k} e^{kb} - \frac{1}{k^2} e^{kb} + \frac{1}{k^2} \right]$$

as $b \rightarrow \infty \quad e^{kb} \rightarrow 0$ since $k < 0$

$$= -k \left(\frac{1}{k^2} \right) = -\frac{1}{k} \quad \text{Note } -\frac{1}{k} > 0$$