

### Example 4 Joule-Lenz law

$$Q = .24 R \int_0^T I_0^2 \sin^2\left(\frac{2\pi t}{T} - \phi\right) dt$$

$$= .24 R I_0^2 \int_0^T \sin^2\left(\frac{2\pi t}{T} - \phi\right) dt$$

$$= .24 R I_0^2 \int_0^T \frac{1 - \cos\left(\frac{4\pi t}{T} - 2\phi\right)}{2} dt$$

$$= .24 R I_0^2 \left[ \frac{1}{2} T - \frac{1}{2} \int_0^T \cos\left(\frac{4\pi t}{T} - 2\phi\right) dt \right]$$

$$= .24 R I_0^2 \left[ \frac{1}{2} T - \frac{1}{2} \frac{\sin\left(\frac{4\pi t}{T} - 2\phi\right)}{\frac{4\pi}{T}} \Big|_0^T \right]$$

$$= .24 R I_0^2 \left[ \frac{1}{2} T - \frac{1}{8\pi} T (\sin(4\pi - 2\phi) - \sin(-2\phi)) \right]$$

$$= \frac{.24 R I_0^2 T}{2}$$