Ex Consider a population that grows with dynamics

$$\frac{dP}{dt} = kP.$$

If the population is 790 in 1750 and 1260 in 1850, what is it in 1950?

$$\frac{dP}{dt} = kP \qquad \text{We need to find } k.$$

separable equation!

$$\frac{dP}{P} = k\, dt$$

integrate

$$\int \frac{dP}{P} = \int k\, dt \implies \ln P = kt + c$$

$$\implies P = P_0 e^{kt}$$

let $P_0 \equiv$ pop in $1750 = 790$

$\quad t \equiv$ years since 1750

$$P(0) = P_0 = 790$$

$$P(100) = 1260 = 790\, e^{k100}$$

find k

$$\frac{1260}{790} = e^{k100}$$

$$\Rightarrow \quad \ln \frac{1260}{790} = k\,100$$

$$\Rightarrow \quad k = \frac{1}{100} \ln \frac{1260}{790}$$

in 1950, $t = 200$

$$P(200) = 790\, e^{k200} = 790\, e^{\frac{200}{100} \ln \frac{1260}{790}}$$

$$= 790\, e^{2 \ln \frac{1260}{790}}$$

$$= 790\, e^{\ln \left(\frac{1260}{790}\right)^2}$$

$$= 790 \left(\frac{1260}{790}\right)^2$$

$$= \frac{1260^2}{790}$$