Topic 1 - Series Convergence

A) $\sum_{n=1}^{\infty} \frac{1}{n} - \sin\left(\frac{2n^2}{3n^5+n^2}\right)$

Use a limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

Note $\lim_{n\to\infty} \frac{\frac{1}{n} - \sin\left(\frac{2n^2}{3n^5+n^2}\right)}{1/n} = \lim_{n\to\infty} \frac{1/n - \sin(0)}{1/n}$

$= 1 > 0$ and finite

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} - \sin\left(\frac{2n^2}{3n^5+n^2}\right)$ diverges b/c $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

B) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

Note $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

p-series with $p = 3/2 > 1$

$\Rightarrow$ series converges

C) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$ see factorial think ratio test

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{\frac{e^{n+1}}{(n+1)!}}{\frac{e^n}{n!}} = \lim_{n\to\infty} \frac{e^{n+1}}{(n+1)!} \frac{n!}{e^n}$$

$$= \lim_{n\to\infty} \frac{e}{n+1} = 0 < 1$$

$\Rightarrow$ series converges by ratio test

D) $\sum_{n=1}^{\infty} (7+2^n)^{-4}$

Use limit comparison test to $\sum_{n=1}^{\infty} \left(\frac{1}{2^4}\right)^n$
which is a convergent geometric series.

$$\lim_{n\to\infty} \frac{(7+2^n)^{-4}}{\left(\frac{1}{2^4}\right)^n} = \lim_{n\to\infty} \frac{(2^4)^n}{(7+2^n)^4}$$

as $n \to \infty$ looks like $7+2^n \to 2^n$

$$= \lim_{n\to\infty} \frac{(2^4)^n}{(2^n)^4} = 1 > 0 \text{ and finite}$$

$\Rightarrow$ series converges