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# Terminal velocity of a shuttlecock in vertical fall[a)]

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We have performed a straightforward vertical fall experiment for a case where the effects of air resistance are important and directly measurable. Using a commonly available badminton shuttlecock, a tape measure, and a millisecond timer, the times required for the shuttlecock to fall given distances (up to almost ten meters) were accurately measured. The experiment was performed in an open stairwell. The experimental data was compared to the predictions of several models. The best fit was obtained with the model which assumes a resistive force quadratic in the instantaneous speed of the falling object. This model was fitted to the experimental data enabling us to predict the terminal velocity of the shuttlecock (6.80 m/sec). The results indicate that, starting from rest, the vertically falling shuttlecock achieves 99% of its terminal velocity in 1.84 sec, after falling 9.2 m. The relative ease in collecting the data, as well as the excellent agreement with theory, make this an ideal experiment for use in physics courses at a variety of levels.

## I. INTRODUCTION

In discussing the effects of air resistance on the motion of a vertically falling object, it is generally accepted that the resistive force is proportional to the first or second power of the instantaneous speed.[1] Further, it is generally recognized that the Reynolds number $\mathcal{R}$ plays a crucial role in determining whether the linear or the quadratic force law must be used. For $\mathcal{R} < 1$, linear resistance applies while for $1 < \mathcal{R} < 10^5$, quadratic resistance applies.[2] In this paper we report on a rather straightforward experiment in which the correct resistive-force law for a vertically falling badminton shuttlecock was determined quite conclusively.[3] The shuttlecock was chosen primarily because of its small mass (small gravitational force) and large area (large resistive force). One expects that these factors should result in velocities of fall low enough to permit accurate measurements using equipment readily available in the laboratory. The results of our experiment fully confirmed this expectation. The shuttlecock is ideal for studying the effects of air resistance in the laboratory.

## II. EXPERIMENT

Using a commonly available badminton shuttlecock, a tape measure, and a millisecond timer, we were able to measure the times required for the shuttlecock to fall given distances. The experiment was performed in an open stairwell. Ten trials were made for each data point and the average times (given in Table I) were calculated. Standard deviations ranged from 0.008 sec for the longest time measured to 0.01 sec for the shortest time measured.[4] A schematic of the experimental apparatus used is given in Fig. 1.

For convenience in releasing the shuttlecock, a fine thread was attached to the inside of the shuttlecock enabling it to be supported before release by the pressure exerted on the string when the spring-loaded microswitch was pressed in the closed position. Opening the microswitch released the shuttlecock and simultaneously started the millisecond timer. After falling through a measured distance $y$, a collision between the falling shuttlecock and the cardboard target dislodged the flattened metal rod held by two bar magnets, breaking that circuit and stopping the millisecond timer. Specifically, the millisecond timer begins counting when the trigger voltage is about 1.5 V, and stops when the trigger is grounded. The two bar magnets and the rod form a switch. With the rod in place, the trigger voltage (across $R_2$) is approximately 1.5 V, thereby enabling the timer to run. With the shuttlecock suspended by its thread and the microswitch pressed in the closed position, the trigger will remain grounded (the timer stopped) until the switch is opened (the shuttlecock released, the timer running). When the falling shuttlecock strikes the target and dislodges the rod, the trigger voltage falls to zero, stopping the timer. The two bar magnets were held in place by a ring stand with insulated clamps. Moving the two magnets closer together resulted in a more sensitive apparatus. In an effort to minimize our timing errors, the distance between the two magnets was reduced until the slightest pressure on the target dislodged the rod.

Table I. Experimental data for a vertically falling shuttlecock.

| Distance fallen $y$ (m) | Time elapsed $t$ (sec) |
|---|---|
| 0.61 | 0.347 |
| 1.00 | 0.470 |
| 1.22 | 0.519 |
| 1.52 | 0.582 |
| 1.83 | 0.650 |
| 2.00 | 0.674 |
| 2.13 | 0.717 |
| 2.44 | 0.766 |
| 2.74 | 0.823 |
| 3.00 | 0.870 |
| 4.00 | 1.031 |
| 5.00 | 1.193 |
| 6.00 | 1.354 |
| 7.00 | 1.501 |
| 8.50 | 1.726 |
| 9.50 | 1.873 |

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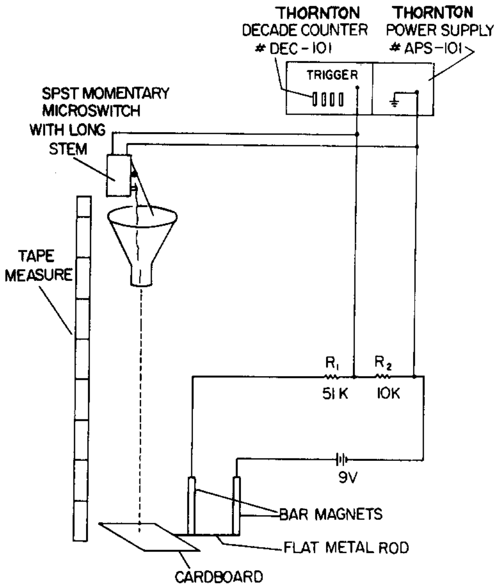


Fig. 1. Experimental apparatus. The bar magnets and the flat metal rod form a switch. With the microswitch closed and the rod in place, the millisecond timer will not run. When the microswitch is opened, releasing the shuttlecock, the timer starts counting and continues counting until the metal rod is dislodged by the collision between the falling shuttlecock and the cardboard target.

## III. THEORY

Assuming a resistive force of the form

$$F_{\text{res}} = -kmv^2, \tag{1}$$

we obtain from Newton's second law

$$mg - kmv^2 = m\frac{dv}{dt}, \tag{2}$$

where we have taken the downward direction to be positive. Clearly, at terminal velocity the acceleration becomes zero and

$$v_T^2 = g/k. \tag{3}$$

If we assume that $v = 0$ at $t = 0$, we may integrate Eq. (2) to obtain[5]

$$t = \frac{v_T}{g}\tanh^{-1}\left(\frac{v}{v_T}\right). \tag{4}$$

Solving Eq. (4) for the velocity as a function of time, we get

$$v = v_T\tanh\left(\frac{gt}{v_T}\right). \tag{5}$$

Equation (5) may be integrated in turn to obtain the position with respect to time as follows,[5]

$$y = \frac{v_T^2}{g}\ln\left[\cosh\left(\frac{gt}{v_T}\right)\right], \tag{6}$$

where we have taken $y = 0$ at $t = 0$.

For the case of a quadratic resistance force, we see from Eqs. (1) and (3) that the terminal velocity is a parameter which must be specified for the particular object and medium in question. In Fig. 2, we plot the experimental data from Table I as well as the theoretical result, Eq. (6), using for the terminal velocity $v_T$ the value which gave the best fit to the data. That value was determined from a least-squares analysis to be $v_T = 6.80$ m/sec. As can be seen from Fig. 2, the agreement between theory and experiment is most striking if one assumes a resistive force quadratic in the speed.[6] Also shown in this figure are the curves for the ideal case (no resistance) and the case of a resistive force linear in the instantaneous speed.[7] For this latter case, one assumes a resistive force of the form

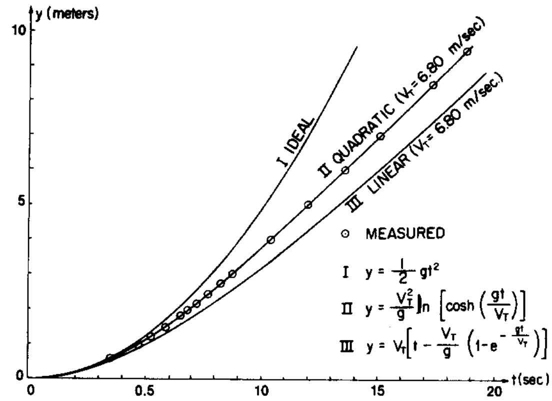


Fig. 2. Comparison of experiment and theory. I: A plot of distance fallen versus time elapsed for the ideal case of no resistance. II: The experimental data points plus a plot of distance fallen versus time elapsed for the case of a resistance force quadratic in the speed of the falling object. A least-squares analysis was done to determine the best fit to the experimental data. The terminal velocity which gave the best fit was found to be $v_T = 6.80$ m/sec. III: A plot of distance fallen versus time elapsed for the case of a resistance force linear in the speed of the falling object, using for the parameter $v_T$ the value 6.80 m/sec.

$$F_{\text{res}} = -(mg/v_T)v, \tag{7}$$

which leads, after straightforward integration, to the results[5]

$$v = v_T(1 - e^{-gt/v_T}) \tag{8}$$

and

$$y = v_T[t - (v_T/g)(1 - e^{-gt/v_T})], \tag{9}$$

where once again we have taken $v = y = 0$ at $t = 0$.

In Fig. 3, we plot the instantaneous velocity of the shut-

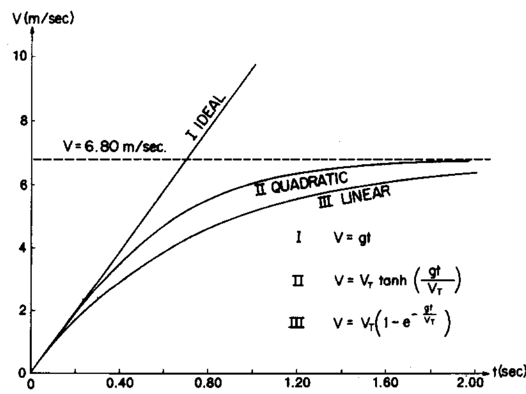


Fig. 3. Velocity versus time curves (theoretical) for vertical fall. I: A plot of instantaneous velocity versus time elapsed for the ideal case of no resistance ($v = gt$). II: A plot of instantaneous velocity versus time elapsed for the case of a resistance force quadratic in the speed of the falling object, Eq. (5), using for the parameter $v_T$ the value 6.80 m/sec. III: A plot of instantaneous velocity versus time for the case of a resistance force linear in the speed of the falling object, Eq. (8), using for the terminal velocity the value $v_T = 6.80$ m/sec.

tlecock versus time as given by Eq. (5). Also shown are the curves for the ideal case as well as the case of a resistance force linear in the speed, Eq. (8). From Eqs. (5) and (6) we find that, according to the quadratic model, the vertically falling shuttlecock achieves 99% of its terminal velocity in 1.84 sec, after falling 9.2 m. As can be seen from Figs. 2 and 3, the effects of quadratic resistance are minimal for $t < 0.2$ sec (i.e., for the first 20 cm of fall), although these effects become substantial for $t \approx 1$ sec, resulting in a rapid approach to terminal velocity for $t > 1$ sec (i.e., $y > 4$ m). At first glance, the relative location of the curves for the linear and quadratic models in Figs. 2 and 3 might appear to be incorrect in that the quadratic model seems to be "closer" to the ideal case than the linear model. This somewhat paradoxical result stems from the fact that for a given terminal velocity $v_T$ and a given instantaneous velocity $v$, the ratio of quadratic resistive force to linear resistive force is equal to the ratio of instantaneous velocity to terminal velocity, i.e.,

$$\frac{F_{\text{quad}}}{F_{\text{lin}}} = \frac{-(mg/v_T^2)v^2}{-(mg/v_T)v} = \frac{v}{v_T}. \qquad (10)$$

For the case of vertical fall from rest, $v < v_T$ and hence $F_{\text{quad}} < F_{\text{lin}}$. Hence, in the regime $v < v_T$ the quadratic resistance force will always be less than the linear resistance force (for a *given* terminal velocity) and, as a result, the quadratic curves will always be closer to the ideal case than the linear curves.

## IV. CONCLUSION

The experiment reported on in this paper shows quite conclusively that the correct resistive force law for the shuttlecock in vertical fall is the one which is quadratic in the instantaneous speed of the object [viz., Eq. (1)]. Our experiment also showed that the shuttlecock, although nonspherical, nevertheless satisfies the claimed connection between Reynolds number and appropriate force law stated in the Introduction.[8] We believe the most striking result obtained to be the excellent agreement between experiment and theory, specifically, the fact that 16 experimental data points could be fitted to the quadratic model with a correlation coefficient approaching unity. This agreement must be attributed, at least in part, to the inherent precision of the timing apparatus which we devised, an apparatus which is ideally suited to the experiment which we performed and which is readily adaptable to many other experiments.

In summary, the vertically falling shuttlecock experiment is a worthwhile one for physics students to perform. The apparatus is generally available, the data easy to collect (and surprisingly reproducible) but best of all, the agreement between theory and experiment is wonderfully reassuring!

[a]This paper was presented at the Second Annual Meeting of the South Eastern Pennsylvania Section of the AAPT, March 17, 1979.

[1]See, for example, Jerry B. Marion, *Classical Dynamics of Particles and Systems*, 2nd ed. (Academic, New York, 1970), p. 53.

[2]G. W. Parker, Am. J. Phys. **45**, 606 (1977), and references cited therein. For an elegant discussion of the role played by the Reynolds number, see R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1964), Vol. II, Sec. 41-3.

[3]Following the notation of Parker in Ref. 2, the Reynolds number for a sphere of radius $r$ moving with velocity $V$ in a fluid of density $\rho$ and viscosity $\eta$ is given by $\mathcal{R} = 2r\rho V/\eta$. Taking $r = 3$ cm and $V = 680$ cm/sec for the shuttlecock and $\eta/\rho = 0.15$ cm$^2$/sec for air, we calculate for our (admittedly nonspherical) shuttlecock a Reynolds number of $\mathcal{R} = 2.7 \times 10^4$. On the basis of this, one would expect that the quadratic rather than the linear force law would apply. This expectation is clearly borne out by the experimental results which we obtained.

[4]These numbers indicate timing errors of about 3% for the shortest times measured to about 0.5% for the longest times measured. As shown in Sec. III, it is the long-time data which determines the terminal velocity (See Ref. 7). Fortunately, it is this long-time data which we are able to measure most precisely. For a general discussion of the kind of error analysis suitable for our experiment (including least-squares principle and correlation), see H. T. Epstein, *Elementary Biophysics: Selected Topics* (Addison-Wesley, Reading, MA, 1963), Chap. 1.

[5]See, for example, R. S. Burington, *Handbook of Mathematical Tables and Functions* (Handbook, Sandusky, 1956).

[6]To appreciate just how striking the agreement is, consider the correlation coefficient defined by

$$\gamma = \{1 - [\sum(y_{\text{exp}} - y_{\text{th}})^2]/[\sum(y_{\text{exp}} - y_{\text{av}})^2\}^{1/2},$$

where $y_{\text{exp}}$ are the experimentally determined distances for given times, $y_{\text{th}}$ the theoretical values for corresponding times, and $y_{\text{av}}$ is the numerical average of all the $y_{\text{exp}}$. Using the 16 experimental data points in Table I, and the theoretical prediction of the quadratic model [Eq. (6) with $v_T = 6.80$ m/sec], one calculates for the correlation coefficient the value $\gamma = 0.99996$! For a discussion of correlation coefficients and their relation to experimental data, see Ref. 4.

[7]It is interesting to note that if one considers just the first ten data points in Table I (i.e., $t < 1$ sec), it is possible to get a good fit using the linear model provided one takes for $v_T$ an artificially high value of 13.4 m/sec. The apparent agreement is fortuitous, however, as seen by the fact that the last three data points in Fig. 2 fall on a straight line with slope (i.e., terminal velocity) equal to 6.80 m/sec. As Parker points out in Ref. 2, and as our shuttlecock experiment confirms, the linear resistance model [Eq. (7)] is of little relevance for objects having sizes and speeds of practical interest.

[8]As a check on whether sphericity (or the lack of it) would affect our results, we subsequently performed a vertical fall experiment using an ordinary ping-pong ball. Interestingly enough, the ping-pong ball also satisfied the quadratic resistive force law. Our nine data points were fitted to the quadratic model and yielded a terminal velocity of $v_T = 6.94$ m/sec with a correlation coefficient $\gamma = 0.995$. We did observe, however, that the motion of the ping-pong ball was somewhat erratic compared to that of the shuttlecock for comparable distances of fall. For example, when dropped from heights near 10 m, the ping-pong ball would often miss the target entirely, even though the release point was not varied from trial to trial. It appears likely that the "knuckleball effect" was responsible for this erratic sideways deflection of the ping-pong ball. For a discussion of this effect as it relates to nonrotating (or slowly rotating) baseballs, see R. G. Watts and E. Sawyer, Am. J. Phys. **43**, 960 (1975). For a description of a laboratory experiment on vertically falling (and nonrotating) ping-pong balls, see R. Terrell, Sports Illustrated **10**(26), 14 (1959).