Formula Sheet: **Formulas for Project 2 Start on page 6**

Definition 1.1: Mean

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Definition 1.2: Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2$$

Definition 1.3: Standard Deviation

$$s = \sqrt{s^2} \text{ and } \sigma = \sqrt{\sigma^2}$$

Subset, Union, Intersection,

$$A \subset B \, , A \cup B \, , A \cap B$$

DeMorgan's Law

$$\overline{(A \cap B)} = \overline{A} \cup \overline{B} \, and \, \overline{(A \cap B)} = \overline{A} \cap \overline{B}$$

Definition 2.3: Sample Space

$$S = \{E_0, E_1, E_2, \ \dots\}$$

Definition 2.6:

$$P(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n) = \sum_{i=1}^{n} P(A_i)$$

$$P(S) = P(E_1 \cup E_2 \cup \dots \cup E_6) = P(E_1) + P(E_2) + \dots + P(E_6) = 1$$

$$P(A) = P(E_1 \cup E_3 \cup E_5) = P(E_1) + P(E_3) + P(E_5) = 1/2$$

Theorem 2.2: Permutation

$$P_r^n = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

Theorem 2.3

$$N = \frac{n!}{n_1!n_2!\ldots n_k!} \equiv \binom{n}{n_1 n_2 \ldots n_k}$$

Theorem 2.4: Combination

$$C_r^n = \frac{P_r^n}{r!} = \frac{n!}{r!\,(n-r)!}$$

Definition 2.9: Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Definition 2.10: Independent Events

$P(A|B) = P(A)$, $P(B|A) = P(B)$, or $P(A \cap B) = P(A)P(B)$

Theorem 2.5: Multiplicative Law of Probability

$$P(A \cap B) = P(A)P(B|A)$$

$$P(A \cap B) = P(B)P(A|B)$$

IF A and B are independent, then $P(A \cap B) = P(A)P(B)$

Theorem 2.6: The Additive Law of Probability

$$P(A \cup B) = P(A)\,P(B) - P(A \cap B)$$

IF A and B are mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

Theorem 2.7

$$P(A) = 1 - P(\overline{A})$$

Theorem 2.8

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Theorem 2.9: Baye's Theorem

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

Definition 3.2

$$P(Y = y)$$

Definition 3.3

$$p(y) = P(Y = y)\ for\ all\ y$$

Theorem 3.1

$$0 \leq p(y) \leq 1\ for\ all\ y$$

$$\sum y \ \ p(y) = 1$$

**For definition 3.4, Theorem 3.2, and Binomial Probability Definition, the equation calculator would not accept a blank on top of the Σ symbol, so I put 0s there instead. **

Definition 3.4

$$E(Y) = \sum_{y}^{0} yp(y)$$

Theorem 3.2

$$E[g(Y)] = \sum_{all\, y}^{0} g(y)p(y)$$

Definition 3.5

$$V(Y) = E[(y - \mu)^2]$$

Theorem 3.4

$$E[cg(Y)] = cE[g(Y)]$$

Theorem 3.5

$$E[g_1(Y) + g_2(Y) + \dots + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)] + \dots + E[g_k(Y)]$$

Theorem 3.6

$$V(Y) = \sigma^2 = E[(Y - \mu)^2] = E(Y^2) - \mu^2$$

Definition 3.7

$$p(y) = C_y^n\, p^y q^{n-y},\ y = 0,\ 1,\ 2,\ \dots,\ n\ and\ 0 \leq p \leq 1$$

Binomial Probability Definition

$$\sum_{y}^{0} p(y) = \sum_{y=0}^{n} (_y^n\, p^y q^{n-y} = (q + p)^n = 1^n = 1$$

Theorem 3.7

$$\mu = E(Y) = np \quad and \quad \sigma^2 = V(Y) = npq$$

Definition 3.8

$$p(y) = q^{y-1}p, \quad y = 1, 2, 3, \ldots, 0 \leq p \leq 1$$

Geometric Probability Definition

$$p(Y) = P(Y = y) = q^{y-1} \cdot p$$

Theorem 3.8

$$\mu = E(Y) = \frac{1}{p} \; and \; \sigma^2 = V(Y) = \frac{1-p}{p^2}$$

Extra Formulas

A success occurs on or before the nth trial:

$$P(X \leq n) = 1 - (1 - p)^n$$

A success occurs before the nth trial:

$$P(X < n) = 1 - (1 - p)^{n-1}$$

A success occurs on or after the nth trial:

$$P(X \geq n) = (1 - p)^{n-1}$$

A success occurs after the nth trial:

$$P(X > n) = (1 - p)^n$$

Definition 3.9

$$p(y) = \binom{y-1}{r-1} p^r q^{y-r}, \quad y = r, r+1, r+2$$

Theorem 3.9

$$\mu = E(y) = \frac{r}{p} \; and \; \sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

Definition 3.10

$$p(y) \;=\; \frac{}{\{\vdash_{y}^{r})\vdash_{n-y}^{N-r}}\{\vdash_{n}^{N})$$

Word didn’t like this one, see definition 3.9 for same issue.

$$p(y) \;=\; \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

Theorem 3.10

$$\mu \;=\; E(Y) \;=\; \frac{nr}{N} \;\; and \;\; \sigma^2 \;=\; V(Y) \;=\; n\vdash_{N}^{r}\vdash_{N}^{N-r}\vdash_{N-1}^{N-n}$$

Similar issue here,

$$\sigma^2 \;=\; V(Y) \;=\; n\binom{r}{N}\binom{N-r}{N}\binom{N-n}{N-1}$$

**** Project 2 Definitions Start Here ****

Definition 3.11

$$p(y) \;=\; \frac{\lambda^y}{y!}e^{-\lambda},\; y \;=\; 0,\,1,\,2,\; \ldots\,,\, \lambda \;>\; 0$$

Theorem 3.11

$$\mu \;=\; E(Y) \;=\; \lambda \, and \, \sigma^2 \;=\; V(Y) \;=\; \lambda$$

Theorem 3.14: Tchebysheff's Theorem

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \; or \; P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Definition 4.1

$$F(y) = P(Y \leq y) \; for \; -\infty < y < \infty$$

Theorem 4.1 Properties of a Distribution Function

$$1. \; F(-\infty) \equiv \lim_{y \to \infty} F(y) = 0$$

$$2. \; F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$$

Definition 4.2

A random variable Y with distribution function $F(Y)$ is said to be *continuous* if $F(Y)$ is continuous, for $-\infty < y < \infty^2$

Definition 4.3

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

Theorem 4.2 Properties of a Density Function

$$1.\ f(y)\ \geq\ 0\ for\ all\ y,\ -\infty\ <\ y\ <\ \infty$$

$$2.\ \int_{-\infty}^{\infty} f(y)dy\ =\ 1$$

Theorem 4.3

$$P(a\ \leq\ Y\ \leq\ b)\ =\ \int_{a}^{b} f(y)dy$$

Definition 4.5

$$E(Y)\ =\ \int_{-\infty}^{\infty} yf(y)\ dy$$

Theorem 4.4

$$E[g(Y)]\ =\ \int_{-\infty}^{\infty} g(y)f(y)dy$$

$$f(y)\ =\ \left\{\frac{1}{\theta_2\ -\ \theta_1},\ \theta_1\ \leq\ y\ \leq\ \theta_2\right\}$$

0, elsewhere

Theorem 4.6

$$\mu = E(Y) = \frac{\theta_1 + \theta_2}{2} \; and \; \sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$$

Definition 4.8

$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)}, \; -\infty < y < \infty$

Definition 5.1

$$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2), \; -\infty < y_1 < \infty, \; -\infty < y_2 < \infty$$

Theorem 5.1

$$1. \; p(y_1, y_2) \geq 0 \; for \; all \; y_1, y_2$$

$$2. \; \sum_{y_1 y_2} p(y_1, y_2) = 1$$

Definition 5.3

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2 \, dt_1$$

Theorem 5.2

$$1.\ f(y_1,\ y_2) \geq 0\ for\ all\ y_1,\ y_2$$

$$2.\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y_1,\ y_2)dy_1\,dy_2 = 1$$

Definition 5.4

$$a.\ p_1(y_1) = \sum_{all\ y_2}^{0} p(y_1,\ y_2)\ and\ p_2(y_2) = \sum_{all\ y_1}^{0} p(y_1,\ y_2)$$

$$b.\ f_1(y_1) = \int_{-\infty}^{\infty} f(y_1,\ y_2)dy_2\ and\ f_2(y_2) = \int_{-\infty}^{\infty} f(y_1,\ y_2)dy_1$$

Definition 5.5

$$p(y_1 \mathrm{I}\, y_2) = P(Y_1 = y_1 \mathrm{I}\, Y_2 = y_2) = \frac{P(Y_1 = y_1,\ Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(y_1,\ y_2)}{p_2(y_2)}$$

Definition 5.6

$$F(y_1 \mathrm{I}\, y_2) = P(Y_1 \leq y_1 \mathrm{I}\, Y_2 = y_2)$$

Definition 5.7

$$f(y_1 \mathrm{I}\, y_2) = \frac{f(y_1,\ y_2)}{f_2(y_2)}$$

Definition 5.8

$$F(y_1, y_2) = F_1(y_1)F_2(y_2)$$

Theorem 5.4

$$p(y_1, y_2) = p_1(y_1)p_2(y_2)$$