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Probability and Statistics Applied

Hands of Blackjack Data Set Research

The popular casino game, Blackjack or 21, has thousands of different variations possible from hand to hand. How the game works is it is player vs dealer, and the object is for the sum of your cards to be as close to 21 as possible without going over. The dealer hands one card to each person at the table and then one to himself, face up. Then the dealer gives the rest of the table their cards and the dealer's second card is placed face down. The values of the cards are the same as their number, with Jack, Queen, and King being 10, and Ace being 1 or 11. Each player then each has a decision to "hit" or receive another card, or "stand" and stay with whatever total they have. If you hit and your sum goes over 21, you "bust" and automatically lose the round. Once each player makes their decision, the dealer flips his second card. If the dealer's total is 17-21, he automatically stands and each player either wins or loses depending on what they have. If the dealer's sum of both cards is 16 or less, they must hit until they reach at least 17. If they bust, then every player at the table who had not previously busted, wins the round. There are a lot more rules and complexity to this, but this is the very basis of how the game is played. Based on the thousands of hands played over many years, a "strategy" has been formed by people particularly good at math which is known to be the most profitable way of playing blackjack. Even when a player plays this strategy perfectly, though, the house still has a 2% edge over the player, meaning that the casino expects to make roughly 2% profit on each iteration of the game.

Below is the link of the excel file I used for the data set. This is 900,000 hands of blackjack, accounting for 6 players at a table. The data set assumes each player plays perfect strategy and "hits" or "stands" accordingly. For the columns at the end, "blkjck" counts if a player hit exactly 21 on their first two cards, "winloss" counts if the player won or lost the hand, "plybustbeat" counts if the player got beat by the dealer, the dealer busted on the round, or won the hand, "dlbustbeat" counts if the dealer won the hand, got beat, or the player busted. The data set assumes each player bets $10 on each hand, so the "plwinamt" details how much the player won each hand. Since blackjack pays 2:1 for normal wins and 3:2 for blackjack, most wins will amount to a $20 win, or $10 profit. Also meaning the "dlwinamt" shows howe much the dealer wins per player.

blkjckhands.csv

1) From a sample of 7 hands of blackjack, it was found that 4 hands had a sum of above 16, one player won the hand, 3 got beat straight up, and 3 busted

   Find the number of these hands that were:

Got beat straight up and had a sum of over 16

Answer: 3

Got beat straight up and had a sum under 16

Answer: 0

In this question we can assume the dealer had a high sum like 19 or 20, meaning that only one of the player sums over 16 won the hand, and the rest of the hands over 16 stood on their hand and were beat straight up. That leaves the last 3 hands, who had a sum of less than 16, meaning they hit and subsequentially busted, as stated 3 hands busted.

2) Suppose two cards are dealt to a player in any hand of blackjack that result in a sum of 17 or above.

List the sample points for this experiment, disregarding the suit of the card

{(10, 7), (J, 7), (Q, 7), (K, 7), (A, 6), (10, 8), (J, 8), (Q, 8), (K, 8), (A, 7), (10, 9), (J, 9), (Q, 9), (K, 9), (A, 8), (10, 10), (10, J), (10, Q), (10, K), (J, J), (J, Q), (J, K), (Q, Q), (Q, K), (K, K), (10, A), (J, A), (Q, A), (K, A)}

Assign a reasonable probability to each sample point

1/29 or .0344 or 3.4% chance for each combination.

Let A denote the event that exactly one King is observed and B the event that at least one Ace is observed. List the sample points in both A and B

A: {(K, 7), (K, 8), (K, 9), (10, K), (J, K), (Q, K), (K, A)

B: {(A, 6), (A, 7), (A, 8), (10, A), (J, A), (Q, A), (K, A)

Find P(A), P(B), and P(A ∩ B)

P(A): 7/29 or 24.1%

P(B): 7/29 or 24.1%

P(A ∩ B): 1/29 or 3.4%

3) Three hands of blackjack are to be ranked from lowest to highest based on the sum of the two cards they are dealt. That is, the hand with the highest sum will be identified as the best, 2nd highest sum as second best, and the lowest as worst.

Describe one sample point for this experiment

Let us consider H1 is the best hand, H2 second best, and H3 worst hand

Therefore the sample point is (H1, H2, H3)

List the sample space

S = {(H1, H2, H3), (H1, H3, H2), (H2, H1, H3), (H2, H3, H1), (H3, H1, H2), (H3, H2, H1)}

Imagine the hands are now ranked randomly, and one of the hand's sum is much higher than the others. What is the probability they are ranked with the best hand being no worse than second best?

{(H1, H2, H3), (H1, H3, H2), (H2, H1, H3), (H3, H1, H2)}

P(B) = 4/6 or 66.7%

4) An experiment consists of drawing two cards from a deck to form a hand of blackjack, disregarding the suit of the card.

Use the combinatorial theorems to determine the number of sample points in the sample space S.

Since we are disregarding the suit of the card, there are 14 possibilities for card 1 and again 14 possibilities for card 2. This means that the total number of sample points in the sample space is 14x14 or 196.

Find the Probability that the sum of the numbers appearing on the cards is equal to 21.

A = {(A, K), (A, Q), (A, J), (A, 10), (K, A), (Q, A), (J, A), (10, A)

P(A) = 8/196 or 4.1%

5) Assume there is one deck of 52 cards being used for one hand of blackjack that has 6 players and a dealer.

If the first two cards that are dealt, one to player 1 and one to player 2, both have the value of 10, what is the probability that the next 3 cards also have the value of 10?

There are a total of 16 cards with the value of 10 in a 52 card deck. If we remove any 2 of them, then there are 14 left. The probability of choosing 3 cards with the value of 10 out of 50 remaining cards then is = 14C3 / 50C3

= 364 / 19,600 = .0186 or 1.86%

If the first 4 cards all have the value of 10, what is the probability that the next card also has the value of 10

The probability of getting a card with value 10 out of 48 remaining cards is

12/48, or 25%

6) There are two hands of blackjack, one with a sum of 20, giving the probability of H1 to win .70, and another hand with 17, giving the probability of H2 to win .20, and the probability of the dealer to win on any given hand is .58

If the dealer's face card has the value of 10, what is the probability that either H1 or H2 will win the hand.

32%

Find the probability that neither win the hand

10%

7) A player and dealer play a hand of blackjack against each other. If the sum of the player's two cards is closer to 21, they win $5, if the dealer's hand is closer to 21, the player loses $5, if their sum is the same, the player loses $0. Give the probability distribution for the players winnings, Y, on a single hand of blackjack

The probability of the player winning = .45

The probability of the dealer winning = .45

The probability of a tie = .10

y	Player winning	Dealer winning	Tie
P(y)	.45	.45	.10
Player winnings	$5	-$5	$0

8) The probability that a player wins a hand of blackjack is .42. Supposed 20 people play one hand of blackjack against a dealer. What is the probability that

Exactly 14 players win?

P(Y = 14) = $\binom{20}{14}(.42^{14})((1-.42)^{6}$

= .0078 or 0.78%

9) Suppose a player was counting cards at the blackjack table and three hands were dealt with the players chance of winning p = .55

Use the formula for the binomial distribution to calculate the probabilities associated with Y = 0, 1, 2, 3

P(Y = 0) = .091

P(Y = 1) = .334

P(Y = 2) = .408

P(Y = 3) = .499

Find the expected value and standard deviation of Y, using the formulas E(Y) = np and V(Y) = npq

Mean = np = 3 * .55 = 1.65

Variance = np(1 – p) = 3(.55)(.45) = 0.7425, then sqrt = 0.862

10) Upon studying the data set of blackjack hands, one can find that the normal distribution of two cards in a hand is between 4 and 21. Find the probability that the next hand is

Below 11?

P(Y < 11) = .412 or 41.2%

Is greater than 18?

P(Y > 18) = .176 or 17.6%

The results of this research confirm what most people already knew, the house always wins. There really is no winning when it comes to playing games in a casino, and blackjack is no exception. If you look through the dataset provided of hundres of thousands of hands, you can easily tell that the dealer wins more money than any of the players win. It was not easy to relate the entirety of the data set to questions in the text, because there are a lot of factors that go into just one single hand of blackjack, and it is hard to be accounted for when doing these types of probability questions. Overall, though, I thought this research was interesting, especially to me who has played a lot of blackjack, and just reaffirms my knowledge that you can never win in the long term when playing against a casino.