- Hypothesis Testing Overview

Hypothesis testing is used to determine whether a statement about the value of a population parameter should or should not be accepted. More specifically, hypothesis testing uses data from a sample to test the two competing statements indicated in the null/alt. hypotheses. A null hypothesis (Ho) is a tentative assumption about a population. The alternative hypothesis (Ha) is the opposite of what is stated in the null hypothesis.

It is not always obvious how the null and alternative hypothesis should be formulated. Context is very important when determining Ho and Ha. Many applications of hypothesis testing involve research; as such, the outcome that stays the same is Ho, and the outcome that is different is Ha. Refer to the narrative below to view an example.

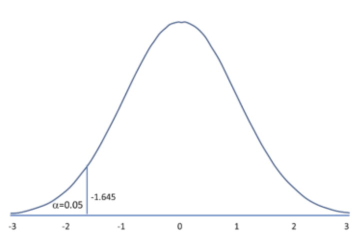
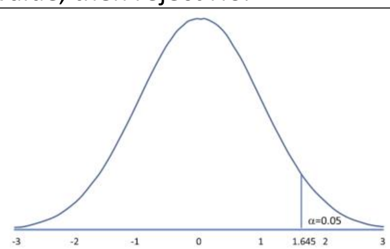
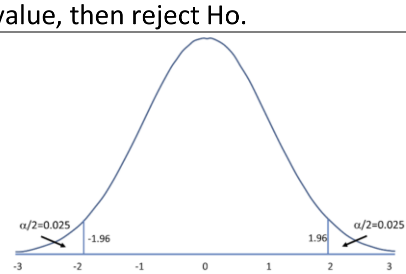
A new teaching method is developed that is believed to be better than the old method. The null and alternative hypotheses are:

- Ho: The new method is no better than the old method.
- Ha: New teaching method is better.

The steps for developing a null hypothesis are as follows:

1. Develop null and alternative hypotheses.
2. Specify the level of significance.
3. Collect sample data and compute value of test statistic.

- Summary of Forms for Null/Alt. Hypotheses
  - μ0: hypothesized value of population mean (variable entered by the problem)
  - μ: instance of μ (Do not replace. Just use symbol)

Lower Tail	Upper Tail	Two Tail
Ho: μ >= μ0 Ha: μ < μ0	Ho: μ <= μ0 Ha: μ > μ0	Ho: μ = μ0 Ha: μ <> μ0
If p-value or test statistic is lower than alpha or critical value, then reject Ho.	If p-value or test statistic is higher than alpha or critical value, then reject Ho.	If p-value or test statistic is lower or higher than alpha or critical value, then reject Ho.
 Rejection Region for Lower-Tailed Z Test (H_1: $\mu < \mu_0$) with α =0.0	 Rejection Region for Upper-Tailed Z Test (H_1: $\mu > \mu_0$) with α=0.0	 Rejection Region for Two-Tailed Z Test (H_1: $\mu \neq \mu_0$) with α =0.05

- Types of Errors in Hypothesis Testing
  - Type I – Rejecting Ho when Ho is true (alpha (a))
  - Type II – Not rejecting Ho when Ho is false (Beta (B))

The probability of making a type I error when the null hypothesis is true is called the level of significance. Applications of hypothesis testing that only control for the type I error are often called significance tests.

The probability of correctly rejecting Ho when it is false is called the power of the test. For any particular value of μ, the power is 1 – B.

- Relationship Among a, B, and n

Only one of the three needs to be known. For a given level of significance (a), increasing n will decrease B. For a given sample size n, decreasing a will increase B, and vice versa. A chart depicting the relationship between the three variables can be found below.

Constant	Change
a	n^, B˅
a	n˅, B^
n	a^, B˅
n	a˅, B^
B	a^, n˅
B	a˅, n^

- Two Approaches to Hypothesis Testing
  - P-value: measures the support, or lack of support, provided by the sample for the null hypothesis. Computed using a test statistic. If p-value is less than alpha value, then you reject the null hypothesis. Alpha value is the probability of committing a type I error.
  - Critical Value: A function of the sample data on which the decision is to be based. Calculate a test statistic. If test statistic is in the rejection zone, then reject the null hypothesis. The test statistics come in two types: z score and t-tests. The equations for each can be found below. As an overview, the test statistic is calculated using x-bar, hypothesized value of the mean, standard deviation, and the square root of the sample size.
    - Z score(Use if population mean is known)
      - $\frac{xbar - \mu 0}{\frac{\sigma}{\sqrt{n}}}$
    - T-test (Use if population mean is unknown)
      - $\frac{xbar - \mu 0}{\frac{s}{\sqrt{n}}}$
      - This test statistic has a t-distribution with n-1 degrees of freedom (D.F.). D.F. is an input into the equation for transferring a test statistic into a p-value, and vice versa.

- Differences between Z scores and t-statistics can be found below

Z Score	T-Statistic
Normal distribution	When the population distribution is normal, then approximations will be exact. When the population is not normally distributed, the statistic is an approximation.
Population standard deviation is known	Population standard deviation is unknown.
N >= 30. Or any sample size if population is not normally distributed but roughly symmetric. Sample sizes as small as 15 can be expected to provide acceptable results.	n >= 30 will provide good results. If the population is approximately normal, n <= 15 can provide good results. If the population is skewed or contains outliers, sample sizes approaching 50 are recommended.

- Hypothesis Testing for Population Variance

Hypothesis testing for population variance is the same concept as hypothesis testing for population/sample means. There are two main differences: first, a chi square distribution is used in lieu of a normal distribution; and second, the formula for calculating the test statistic is different. Refer to the formula below.

- X^2 = $\frac{\left((n-1)s^2\right)}{\sigma\text{^}2}$
- S^2 = Sum $\frac{\left((xi-xbar)^2\right)}{n-1}$